

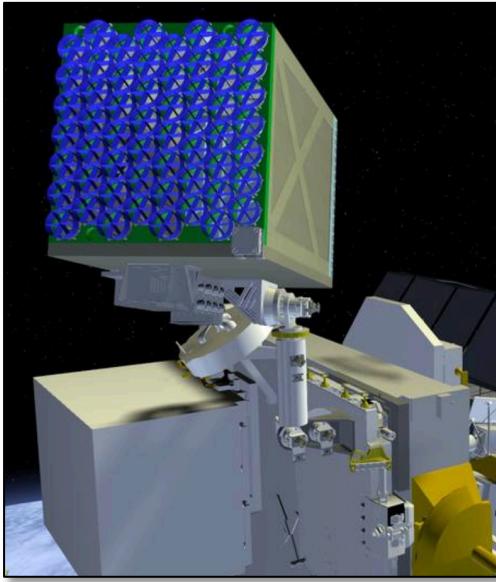
Constraints on the nuclear equation of state from neutron star observations

Jeremy Holt
Texas A&M, College Station

Supported by:



Next-generation observational campaigns of neutron stars

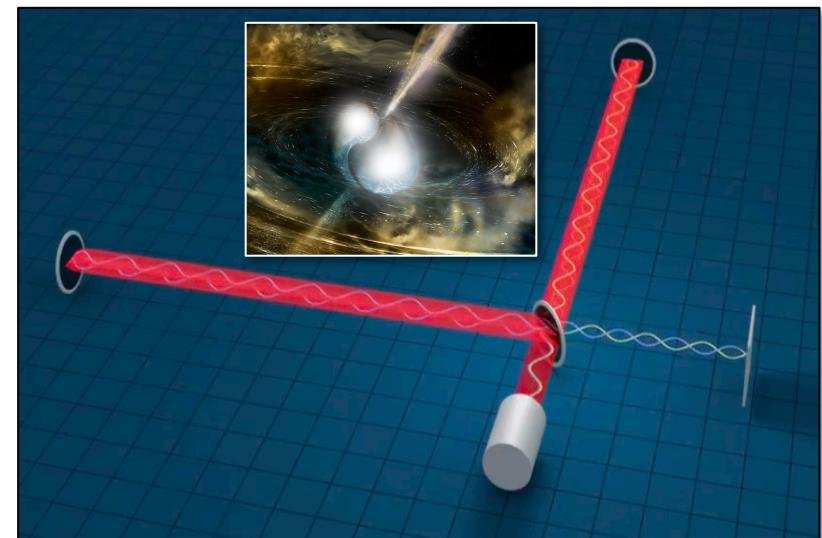


Neutron Star Interior Composition Explorer (NICER)

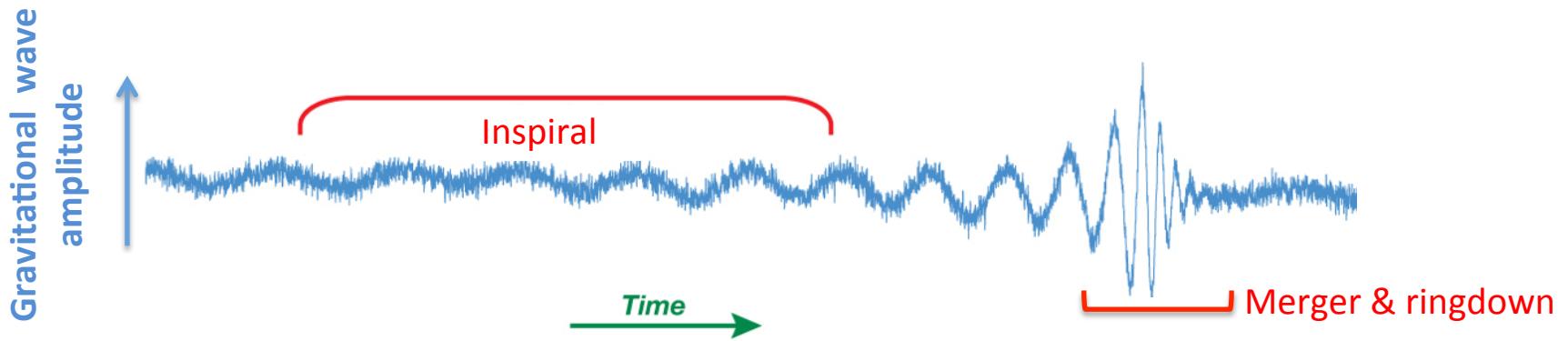
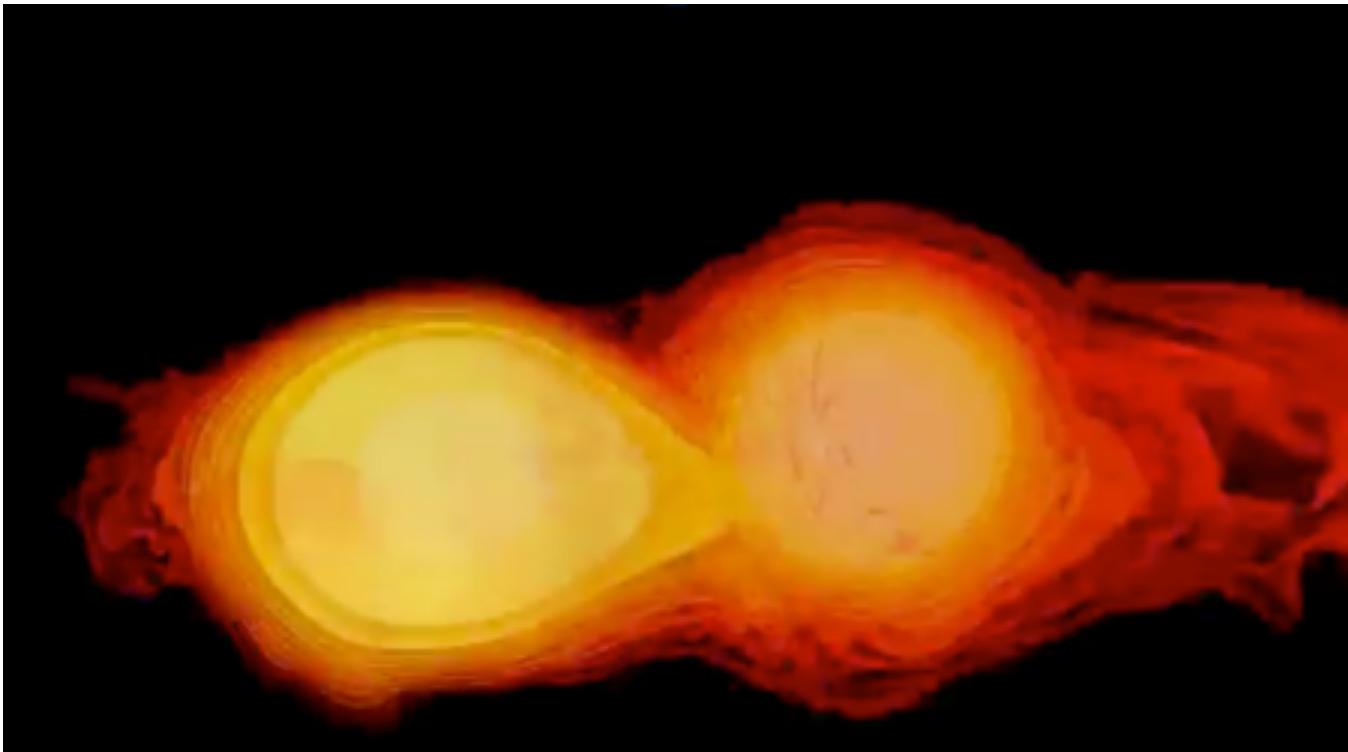
- Combined timing and spectral resolution in the soft X-ray band
- First dedicated targets: $\left\{ \begin{array}{l} \text{PSR_J0437-4715} \\ \text{PSR_J0030+0451} \end{array} \right.$
- Neutron star radii: $\pm 5\%$
- Neutron star masses: $\pm 10\%$

LIGO/VIRGO

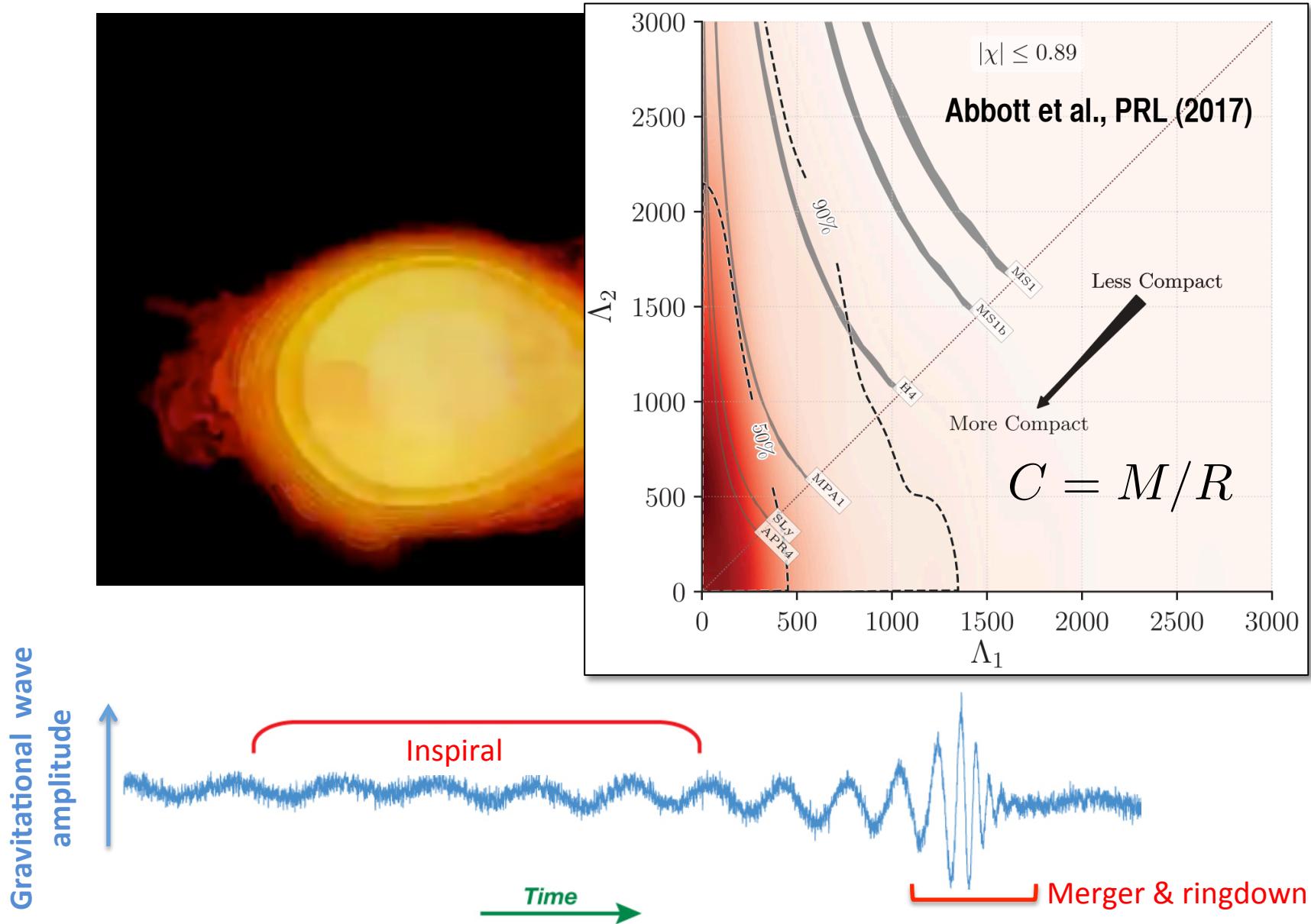
- Late-inspiral gravitational waveform related to neutron star tidal deformability
- Poster-merger peak frequency sensitive to neutron star radius



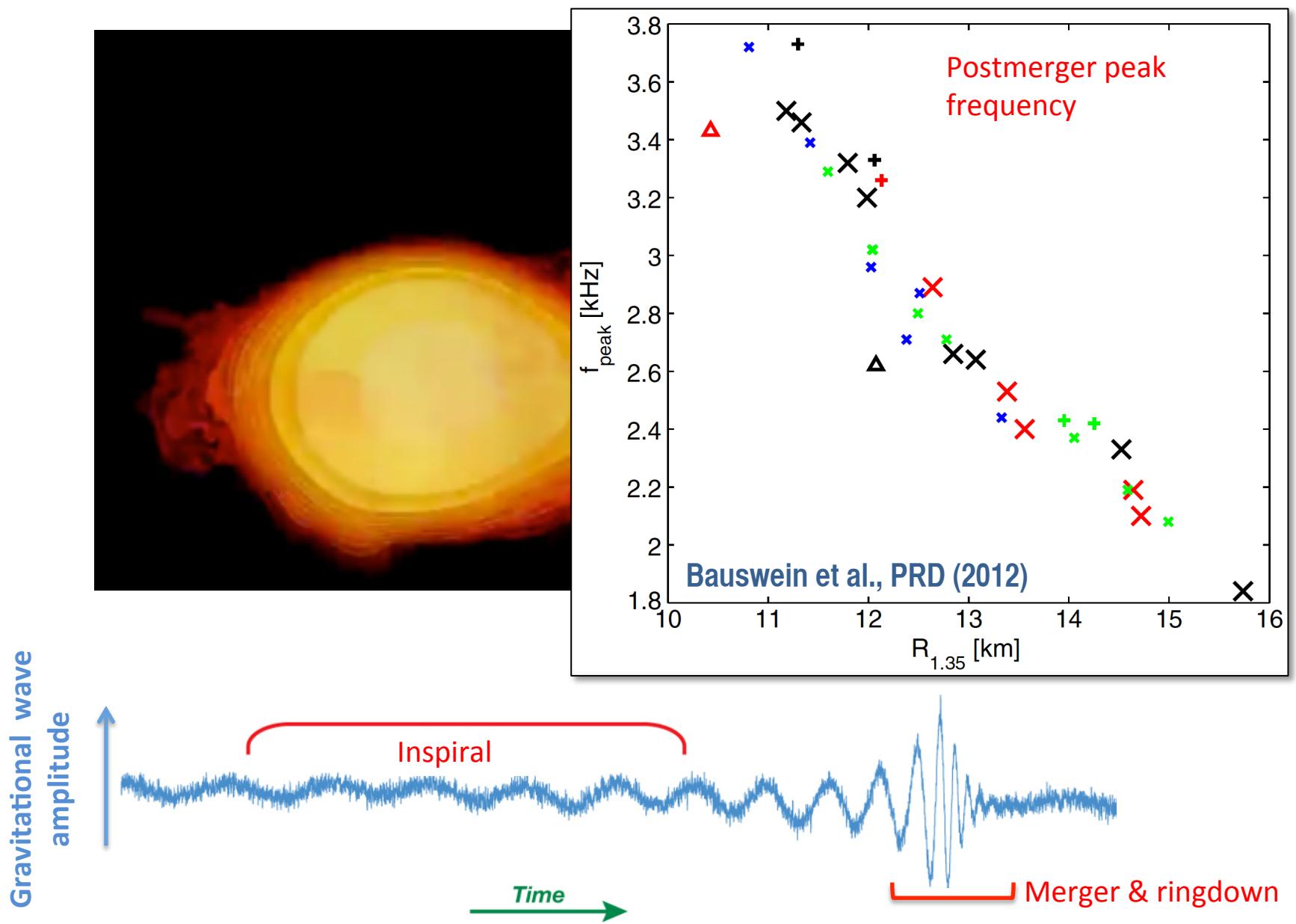
Neutron star radii from neutron star mergers



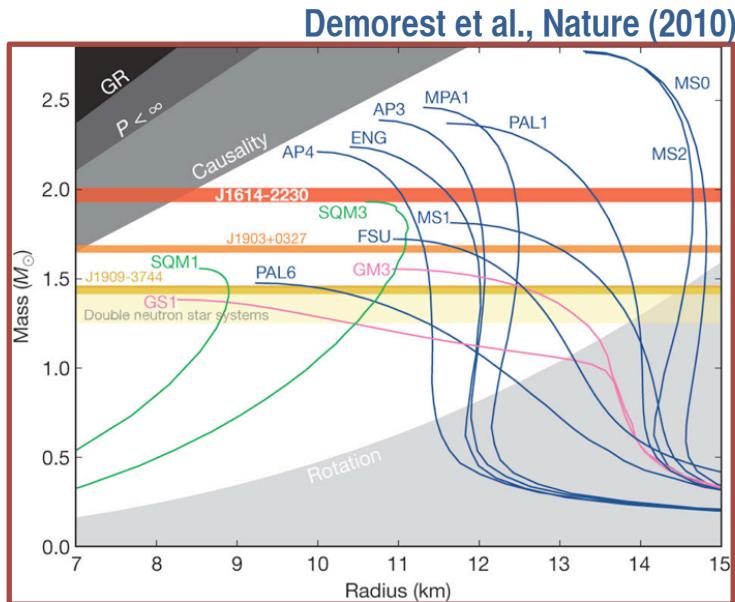
Neutron star radii from late inspiral



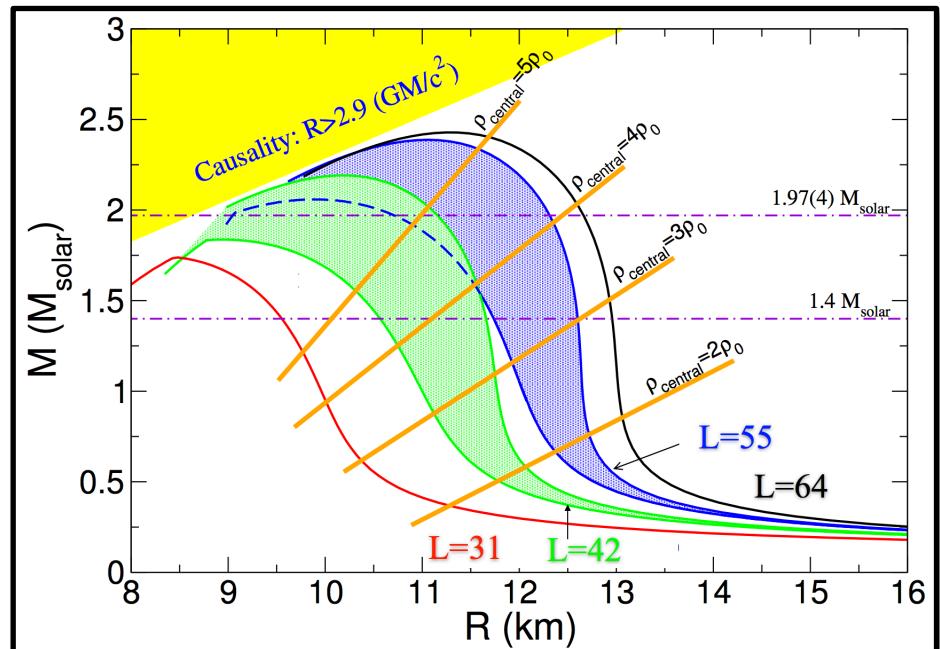
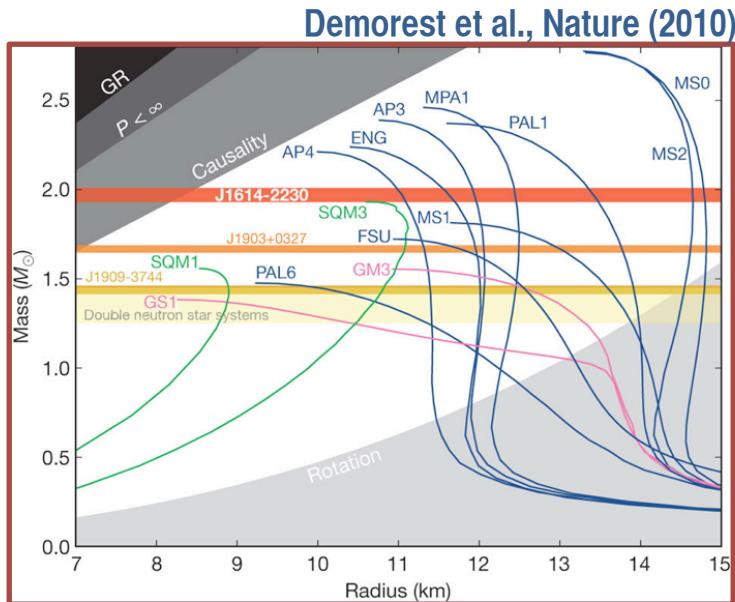
Neutron star radii from post-merger signal



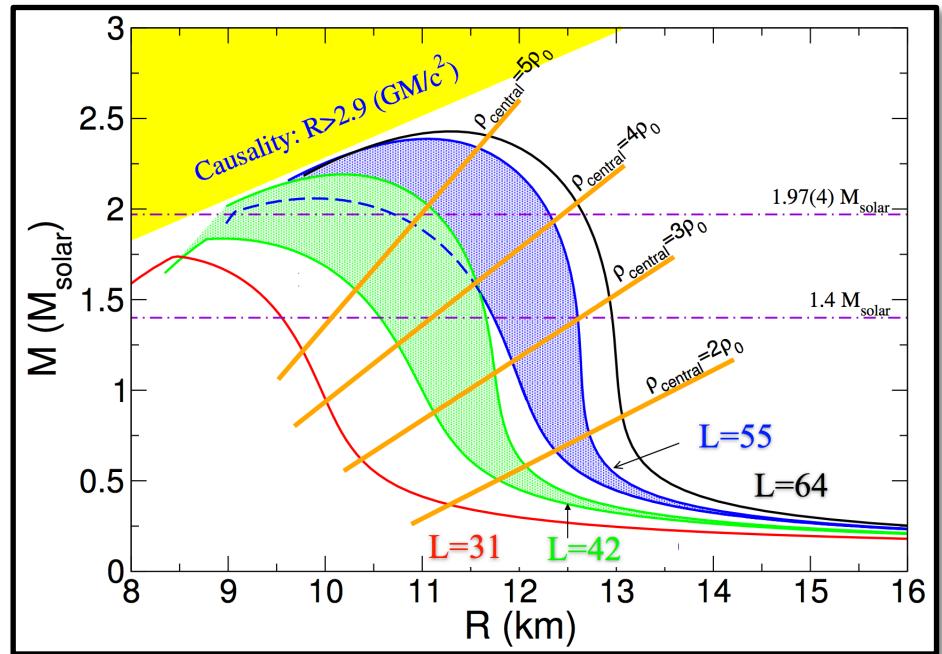
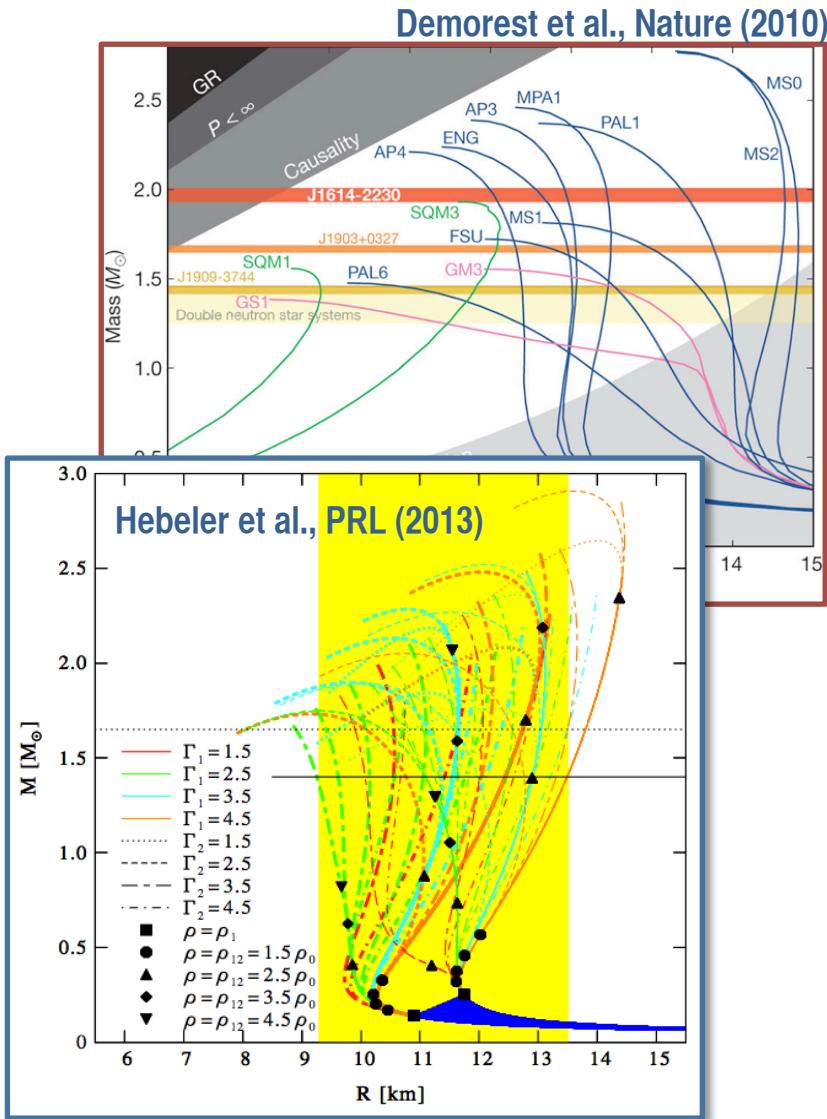
Equation of state constraints from simultaneous neutron star mass and radius measurements



Equation of state constraints from simultaneous neutron star mass and radius measurements

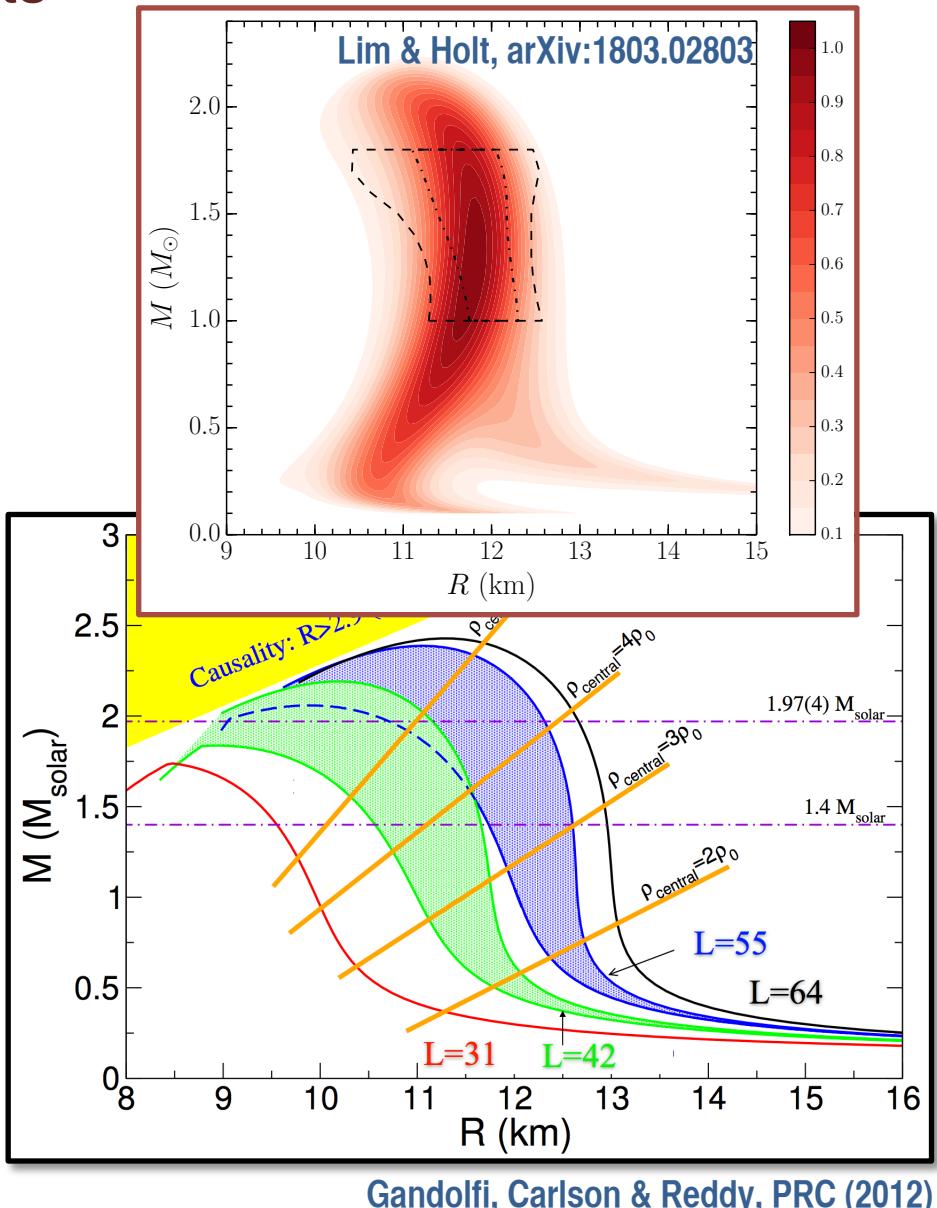
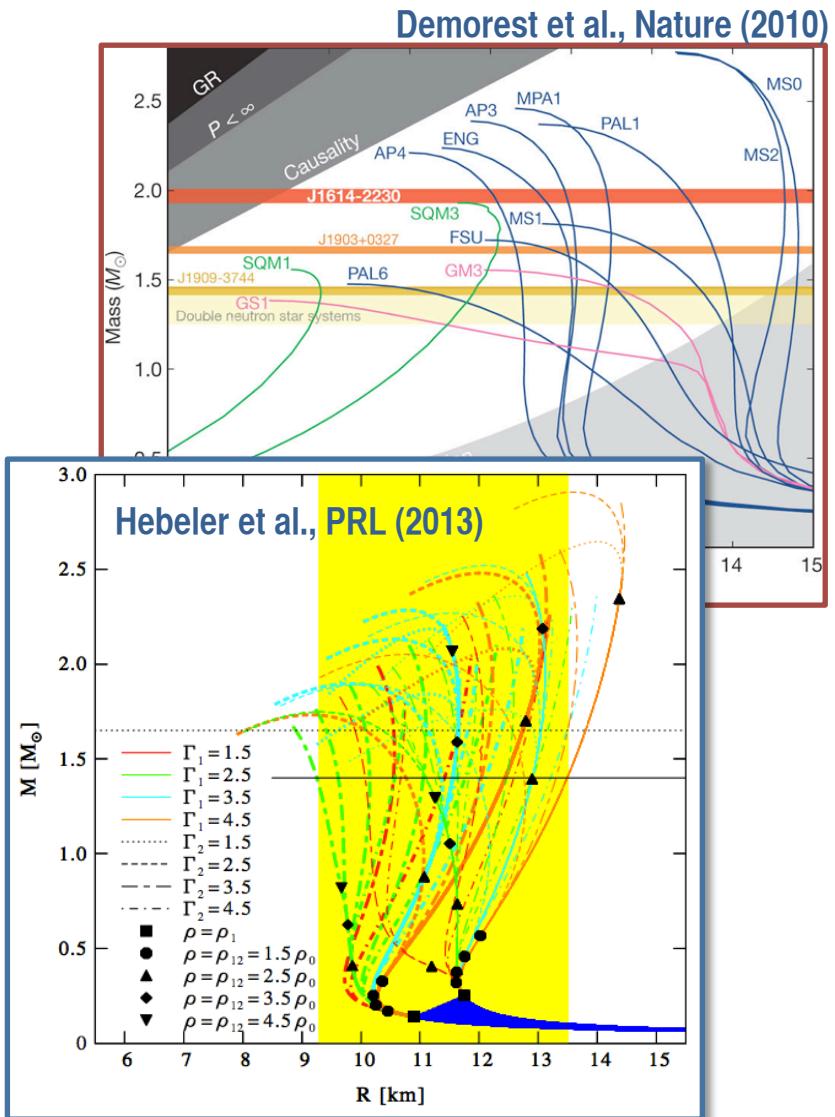


Equation of state constraints from simultaneous neutron star mass and radius measurements



Gandolfi, Carlson & Reddy, PRC (2012)

Equation of state constraints from simultaneous neutron star mass and radius measurements



Question: How to infer properties of the neutron star equation of state from a precise mass vs. radius measurement?

- Construct a model with parameters \vec{a}
- Bayes' Theorem:

$$P(\vec{a}|data) \sim \underbrace{P(data|\vec{a})}_{\text{Likelihood of data given a probability distribution for } \vec{a}} \underbrace{P(\vec{a})}_{\text{Beliefs about parameters } \vec{a} \text{ before measurements ("Prior")}}$$

The diagram illustrates the components of Bayes' Theorem. It features three boxes arranged horizontally. The first box, with a black border, contains the text "Posterior". The second box, with a blue border, contains the text "Likelihood of data given a probability distribution for \vec{a} ". The third box, with a red border, contains the text "Beliefs about parameters \vec{a} before measurements ("Prior")". Three arrows point upwards from each box to their respective terms in the equation: a black arrow from "Posterior" to the first term, a blue arrow from "Likelihood" to the second term, and a red arrow from "Prior" to the third term.

- Strategy:
 - Find useful parametrizations for the equation of state
 - Obtain priors from chiral EFT predictions
 - Use laboratory measurements of finite nuclei to obtain likelihood functions and posteriors

Parametrizing the zero-temperature equation of state

$$\frac{E}{A}(\rho, \delta_{np}) = A_0(\rho) + S_2(\rho)\delta_{np}^2 + \underbrace{\sum_{n=2}^{\infty} (S_{2n} + L_{2n} \ln |\delta_{np}|) \delta_{np}^{2n}}_{\text{Small}}$$

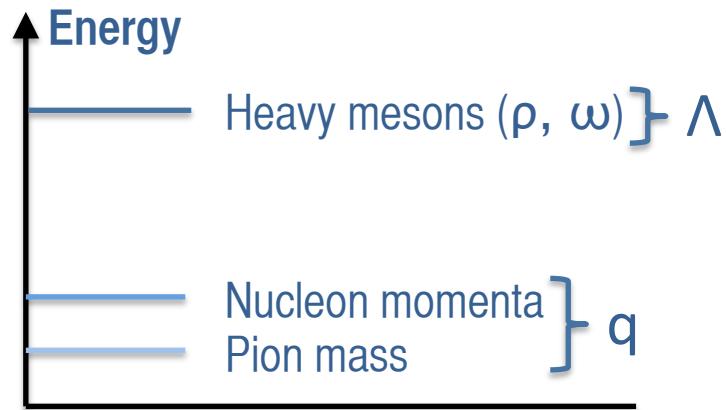
- Expand about a small reference Fermi momentum k_r

$$\frac{E}{A}(\rho, \delta = 0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(a_0 + a_1 \beta + \frac{1}{2}a_2 \beta^2 + \frac{1}{6}a_3 \beta^3 \right)$$
$$\beta = \frac{k_F - k_r}{k_r}$$

$$\frac{E}{A}(\rho, \delta = 1) = 2^{2/3} \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(b_0 + b_1 \beta + \frac{1}{2}b_2 \beta^2 + \frac{1}{6}b_3 \beta^3 \right)$$

Modern theory of nuclear forces

NATURAL SEPARATION OF SCALES



Pions weakly-coupled at low momenta

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2f_\pi^2} (\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi} \gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \right) N$$

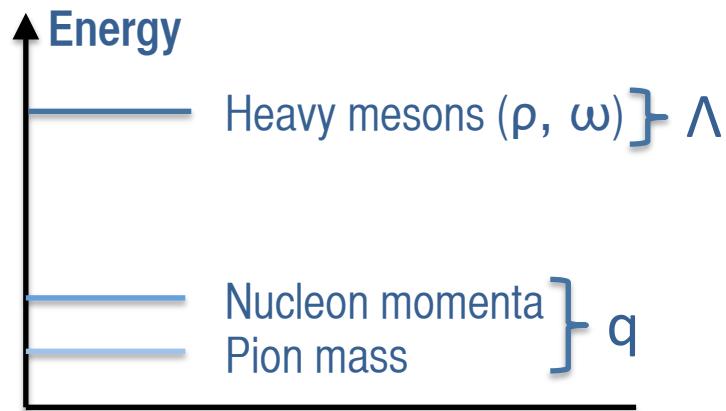
CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions

	2N force	3N force	4N force
$(q/\Lambda)^0$	$N \times \pi$		Systematic expansion
$(q/\Lambda)^2$			
$(q/\Lambda)^3$			
$(q/\Lambda)^4$			

Modern theory of nuclear forces

NATURAL SEPARATION OF SCALES



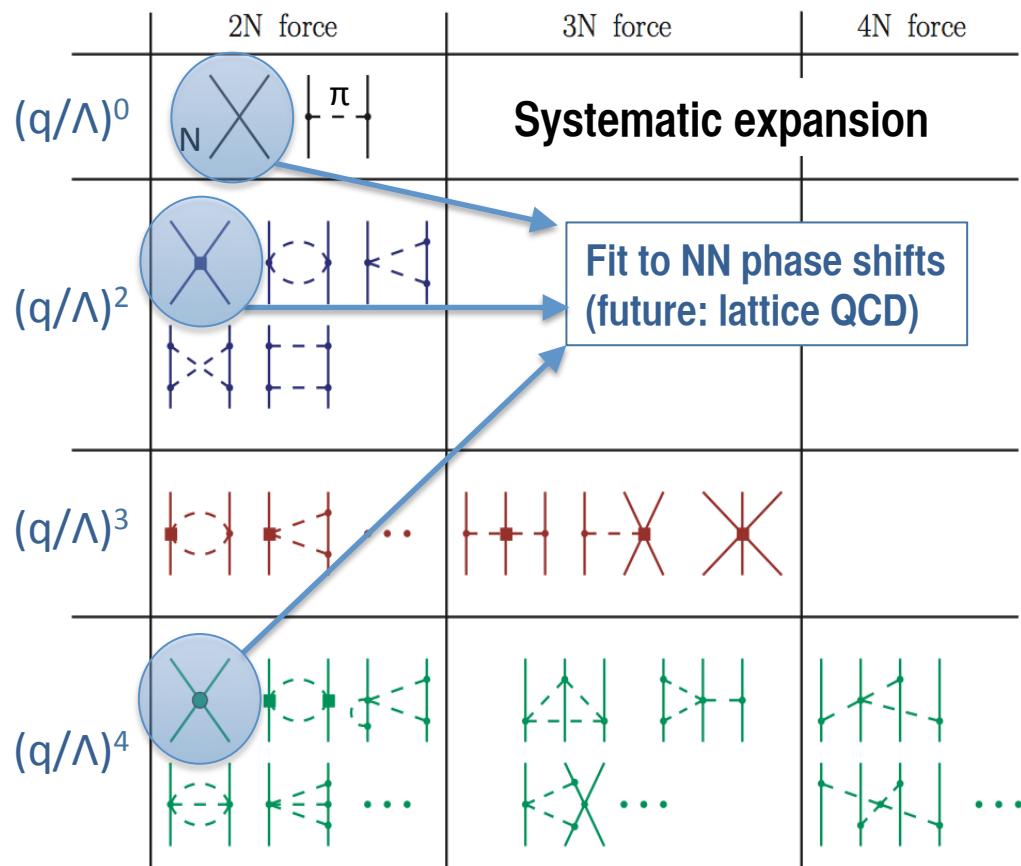
Pions weakly-coupled at low momenta

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2f_\pi^2} (\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi} \gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \right) N$$

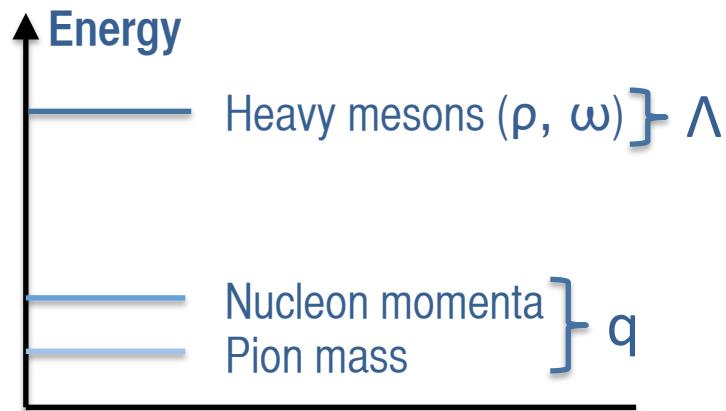
CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



Modern theory of nuclear forces

NATURAL SEPARATION OF SCALES



Pions weakly-coupled at low momenta

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2f_\pi^2} (\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi} \gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \right) N$$

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions

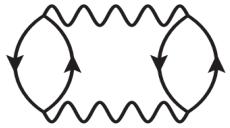
	2N force	3N force	4N force
$(q/\Lambda)^0$	$N \times \frac{\pi}{\Lambda}$		Systematic expansion
$(q/\Lambda)^2$			
$(q/\Lambda)^3$			
$(q/\Lambda)^4$			

Fit to 3H binding energy and lifetime

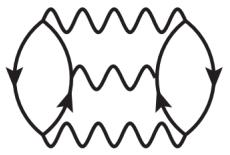
Priors from chiral EFT EOS calculations



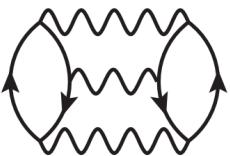
$$\rho E^{(1)} = \frac{1}{2} \sum_{12} n_1 n_2 \langle 12 | (\bar{V}_{NN} + \bar{V}_{NN}^{\text{med}}/3) | 12 \rangle,$$



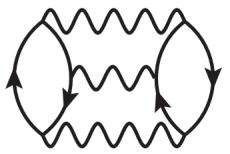
$$\rho E^{(2)} = -\frac{1}{4} \sum_{1234} |\langle 12 | \bar{V}_{\text{eff}} | 34 \rangle|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2},$$



$$\begin{aligned} \rho E_{\text{pp}}^{(3)} &= \frac{1}{8} \sum_{123456} \langle 12 | \bar{V}_{\text{eff}} | 34 \rangle \langle 34 | \bar{V}_{\text{eff}} | 56 \rangle \langle 56 | \bar{V}_{\text{eff}} | 12 \rangle \\ &\quad \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 \bar{n}_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)}, \end{aligned}$$

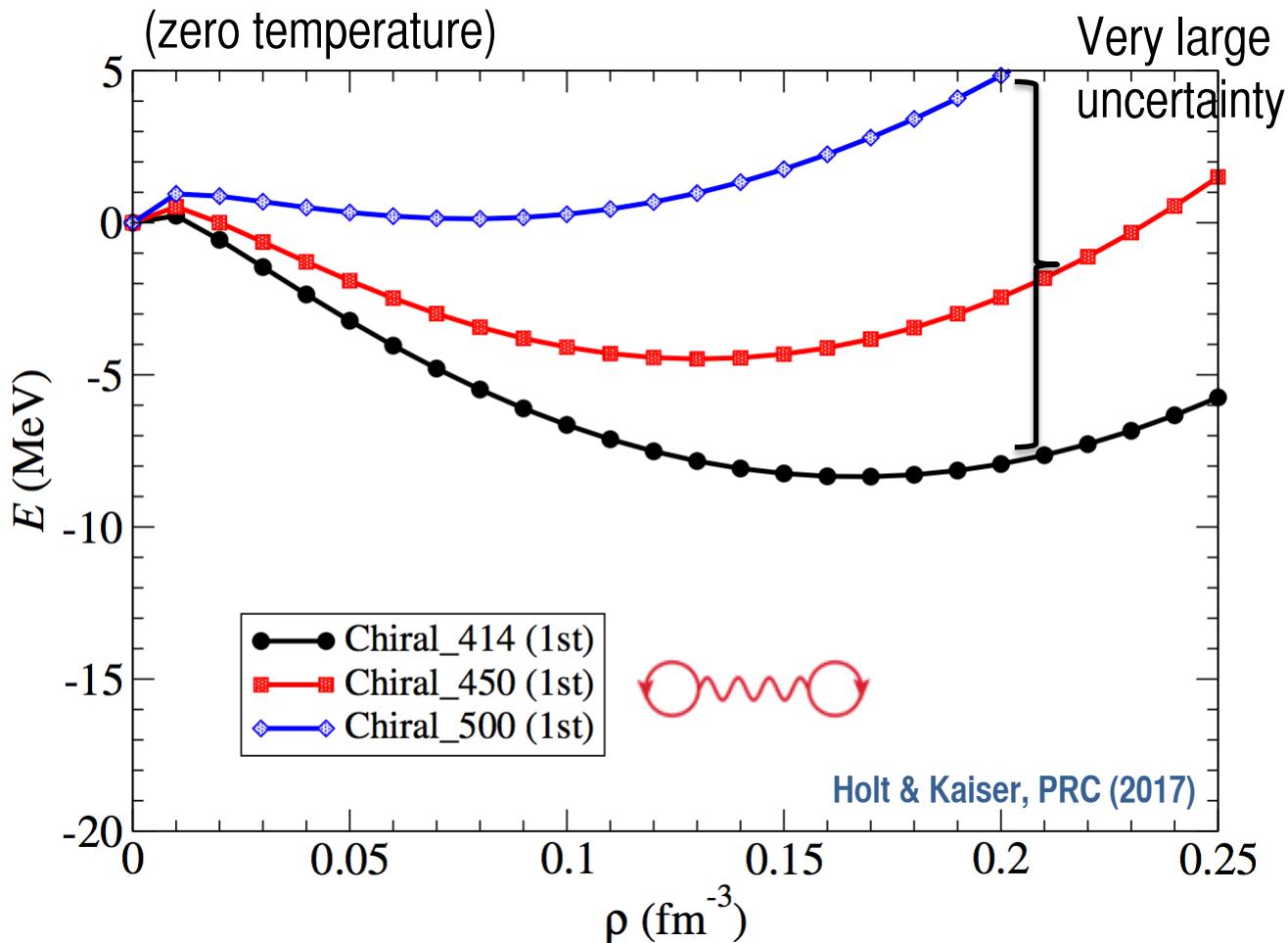


$$\begin{aligned} \rho E_{\text{hh}}^{(3)} &= \frac{1}{8} \sum_{123456} \langle 12 | \bar{V}_{\text{eff}} | 34 \rangle \langle 34 | \bar{V}_{\text{eff}} | 56 \rangle \langle 56 | \bar{V}_{\text{eff}} | 12 \rangle \\ &\quad \times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)}, \end{aligned}$$

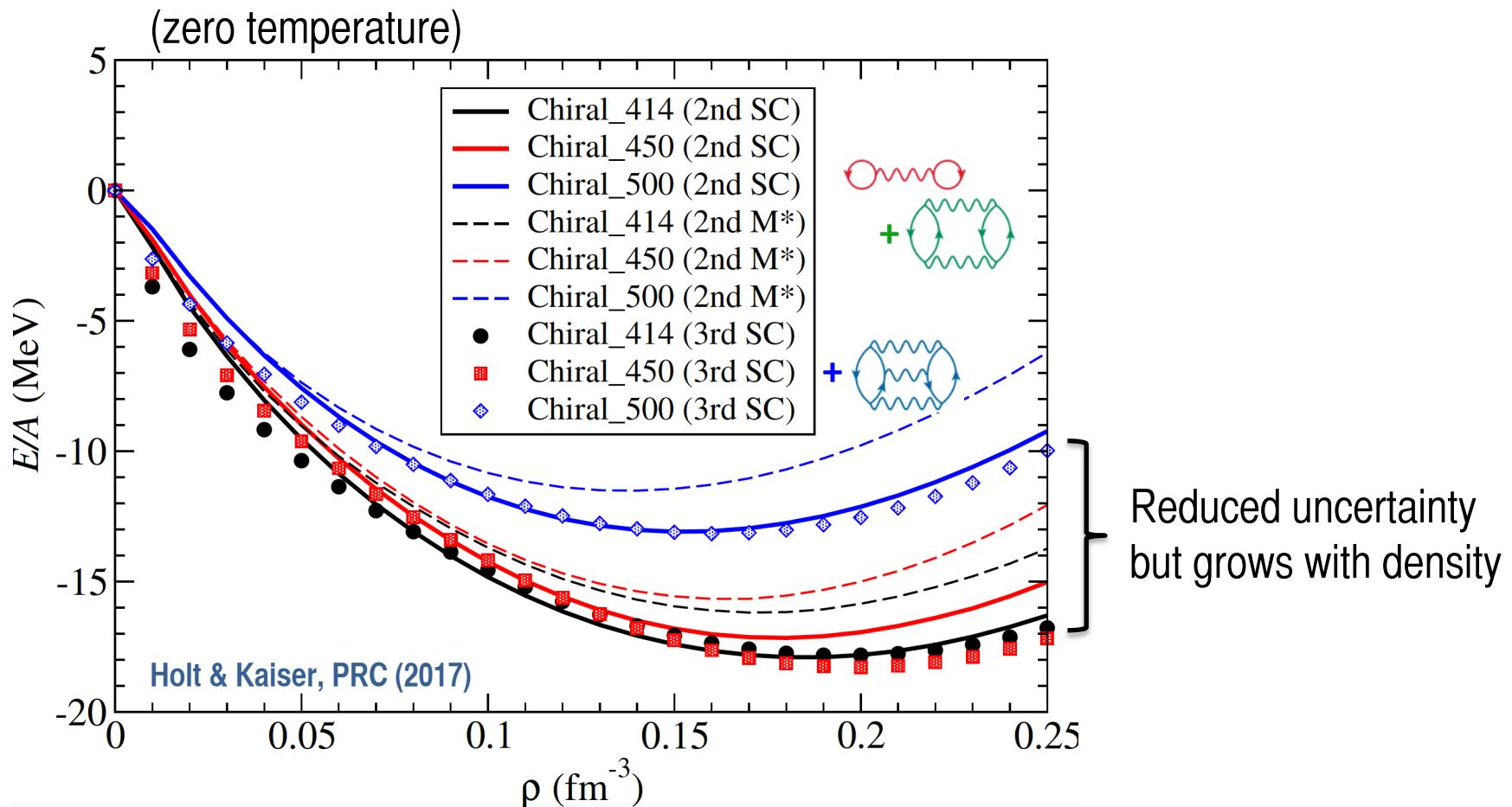


$$\begin{aligned} \rho E_{\text{ph}}^{(3)} &= - \sum_{123456} \langle 12 | \bar{V}_{\text{eff}} | 34 \rangle \langle 54 | \bar{V}_{\text{eff}} | 16 \rangle \langle 36 | \bar{V}_{\text{eff}} | 52 \rangle \\ &\quad \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 n_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)}, \end{aligned}$$

Symmetric nuclear matter at Hartree-Fock level

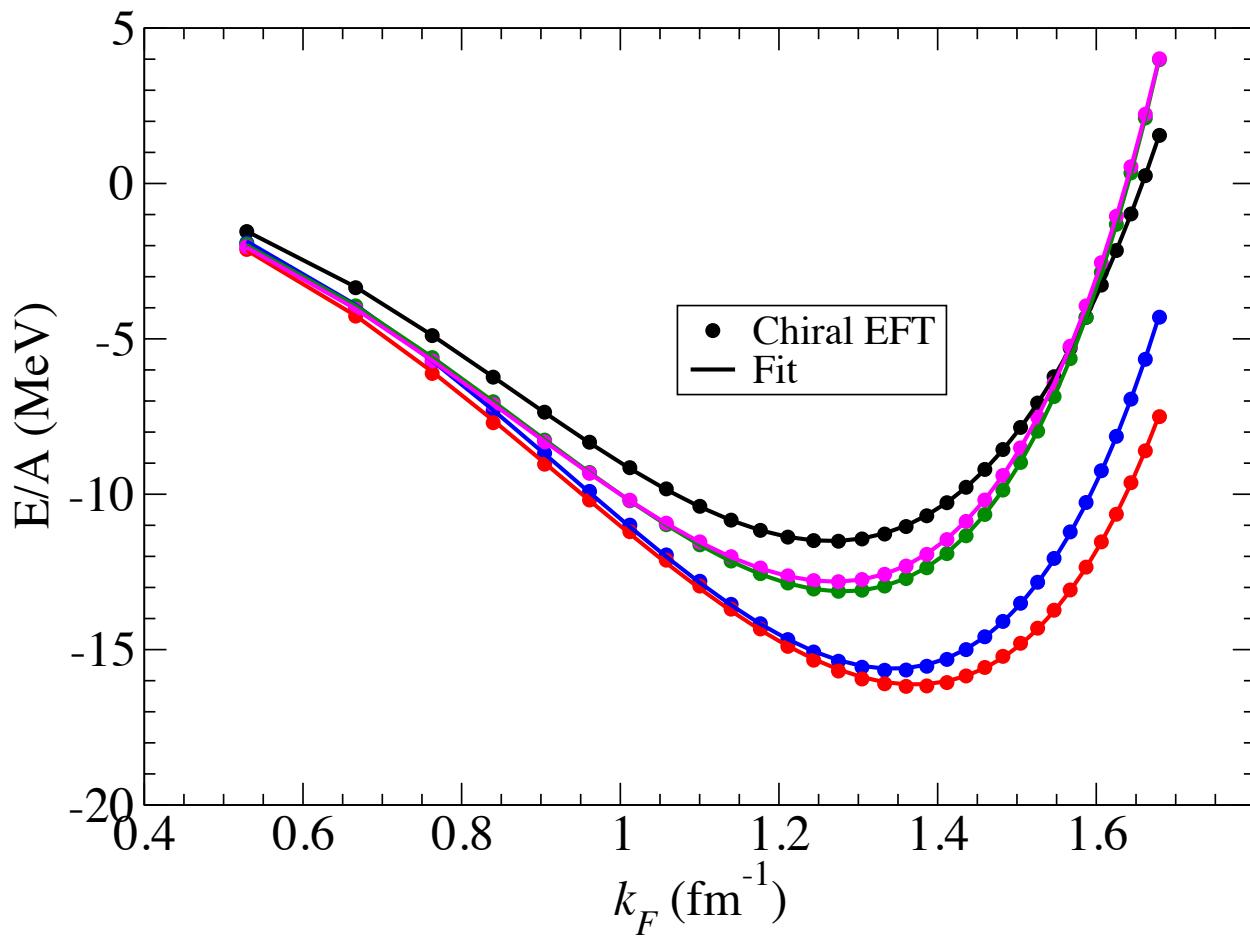


Symmetric nuclear matter equation of state



Quality of parametrization

$$\frac{E}{A}(\rho, \delta = 0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(a_0 + a_1 \beta + \frac{1}{2}a_2 \beta^2 + \frac{1}{6}a_3 \beta^3 \right)$$



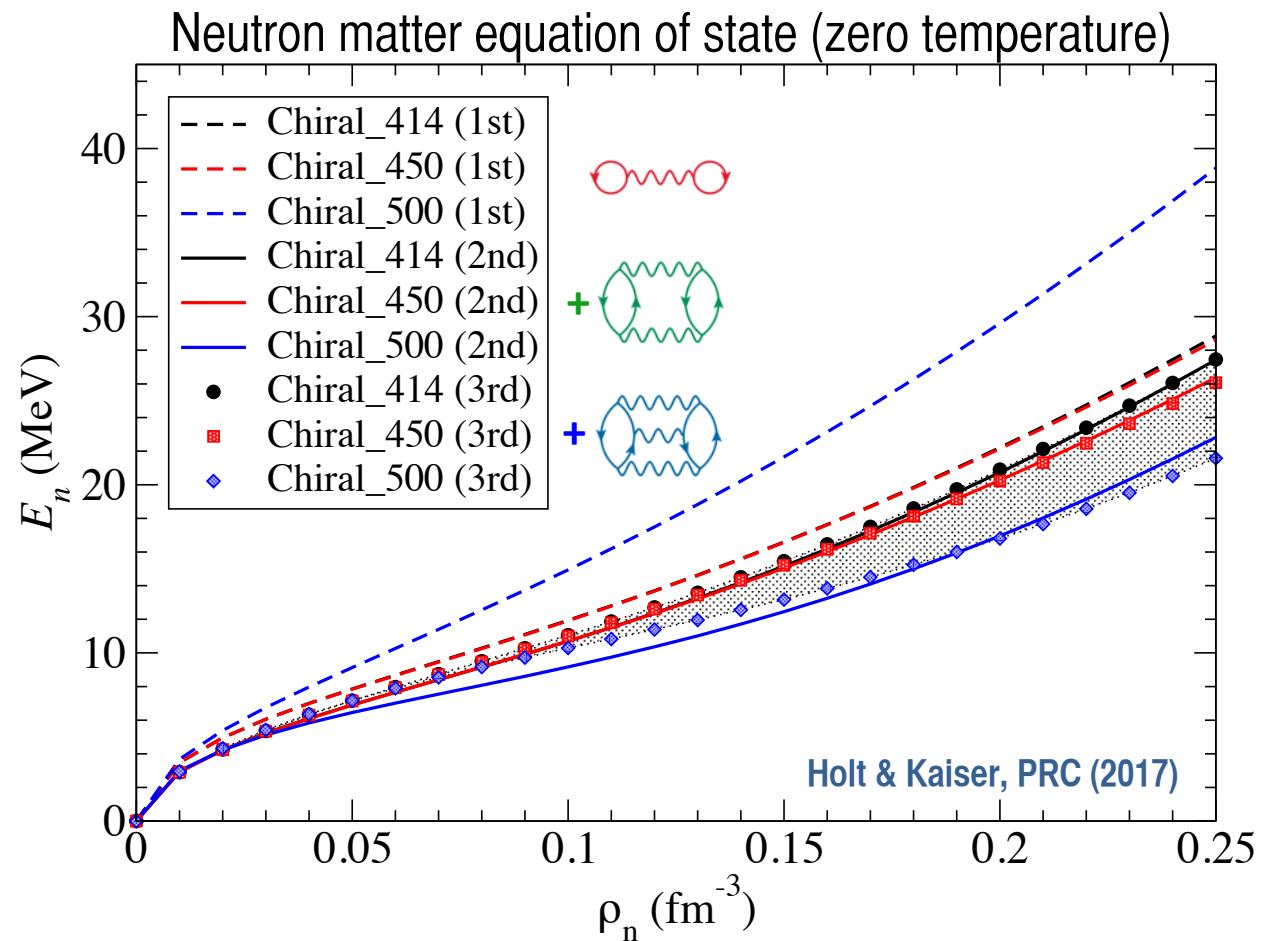
$$a_0 = -3.41 \pm 0.20 \text{ fm}^2$$

$$a_1 = 6.44 \pm 0.25 \text{ fm}^2$$

$$a_2 = -1.02 \pm 0.96 \text{ fm}^2$$

$$a_3 = 21.92 \pm 8.98 \text{ fm}^2$$

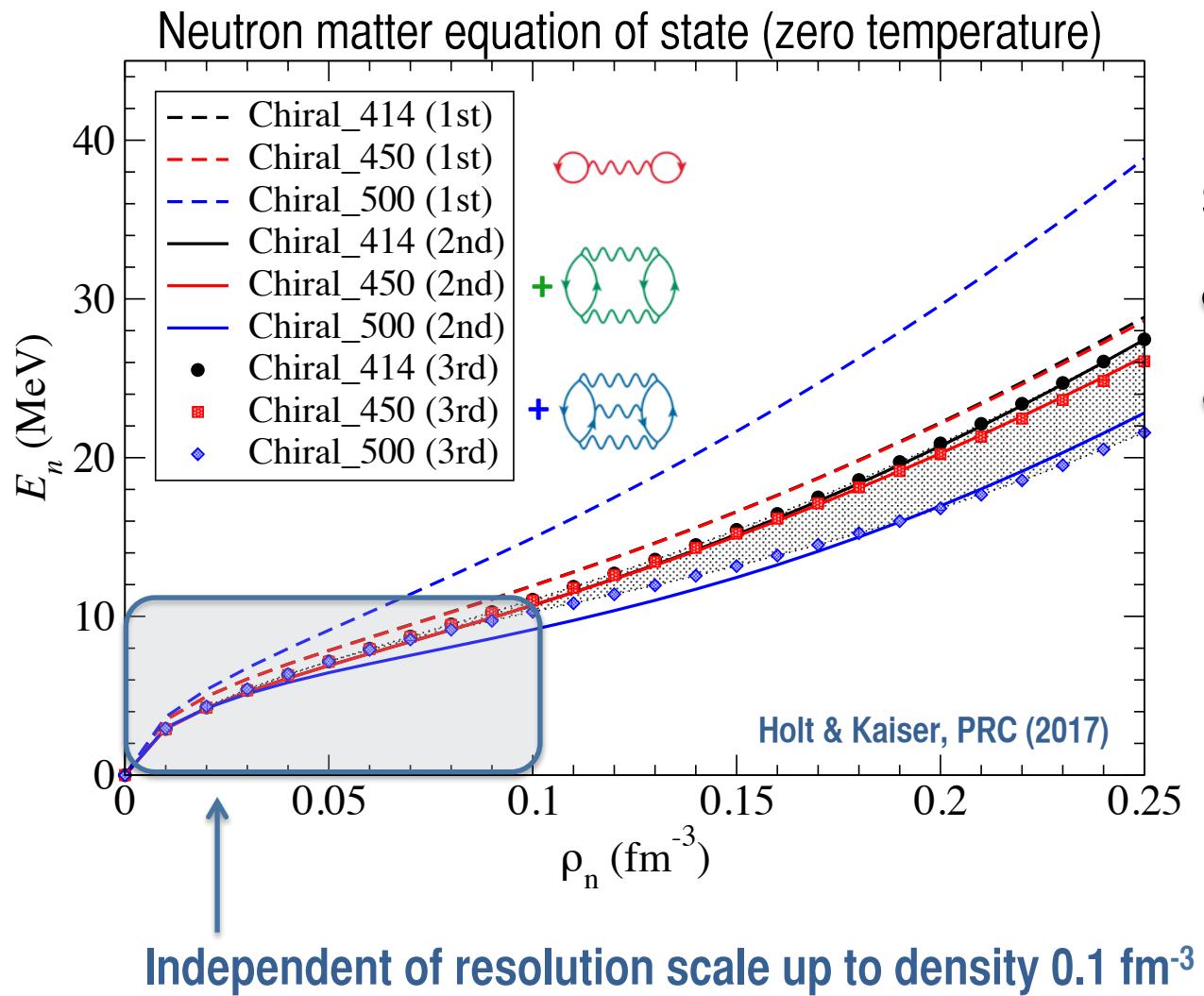
Pure neutron matter uncertainty estimates



Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory

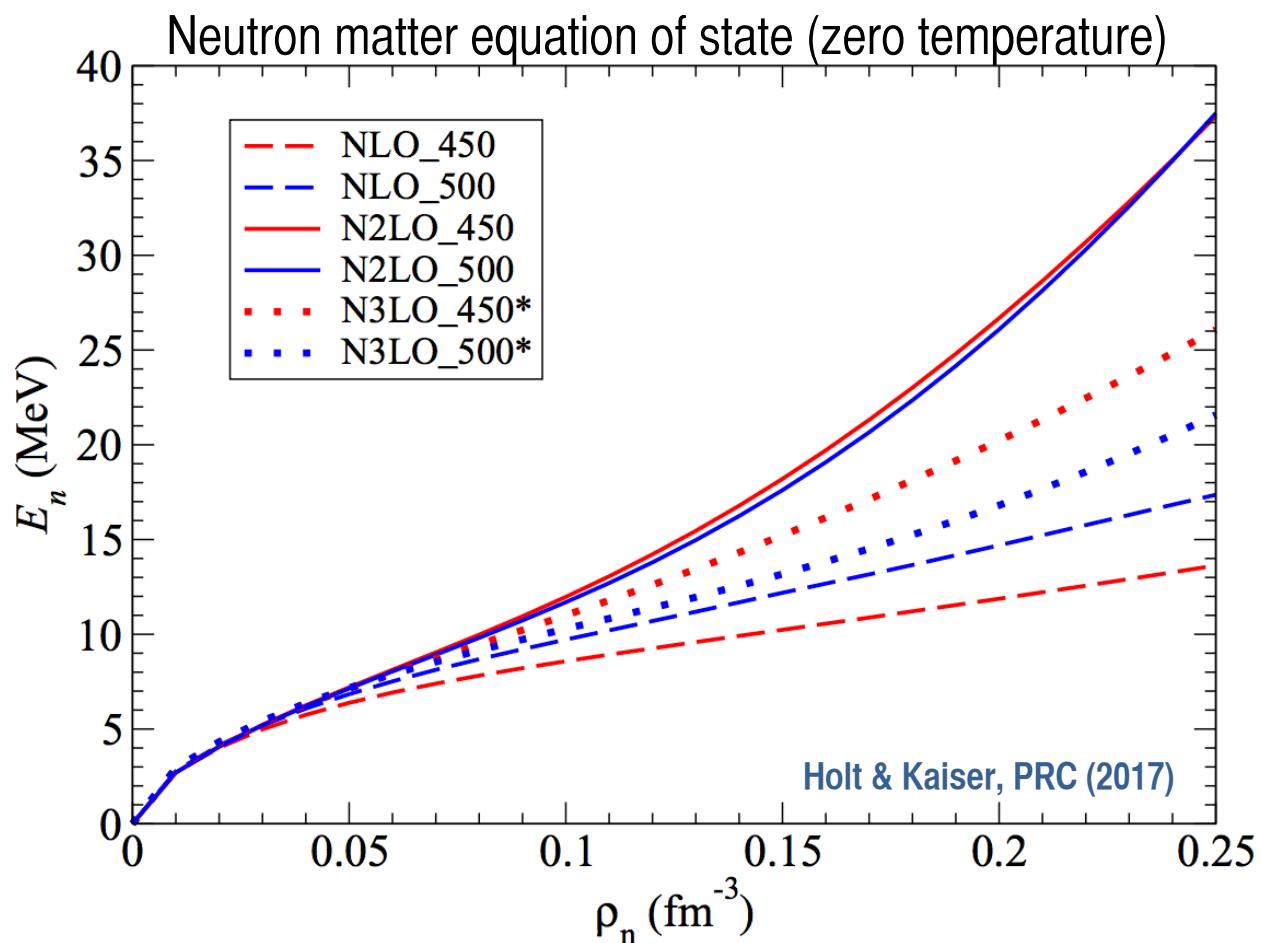
Pure neutron matter uncertainty estimates



Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory

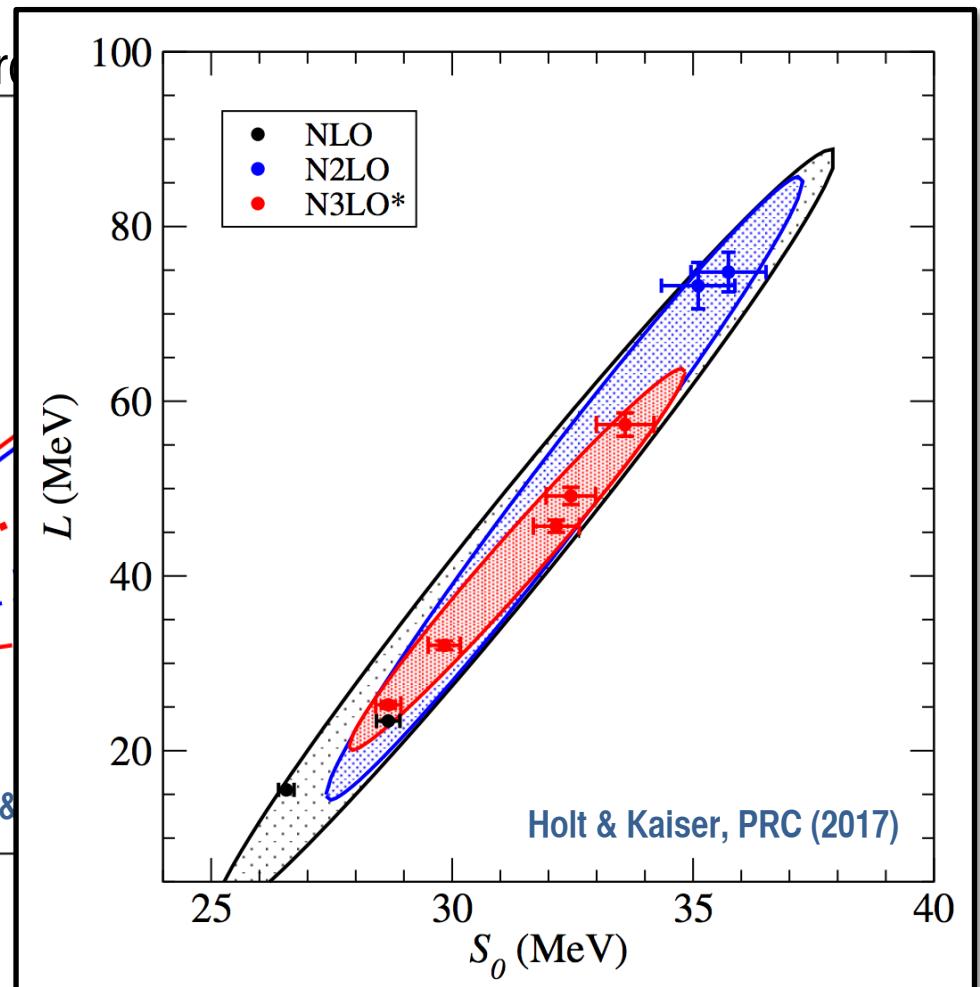
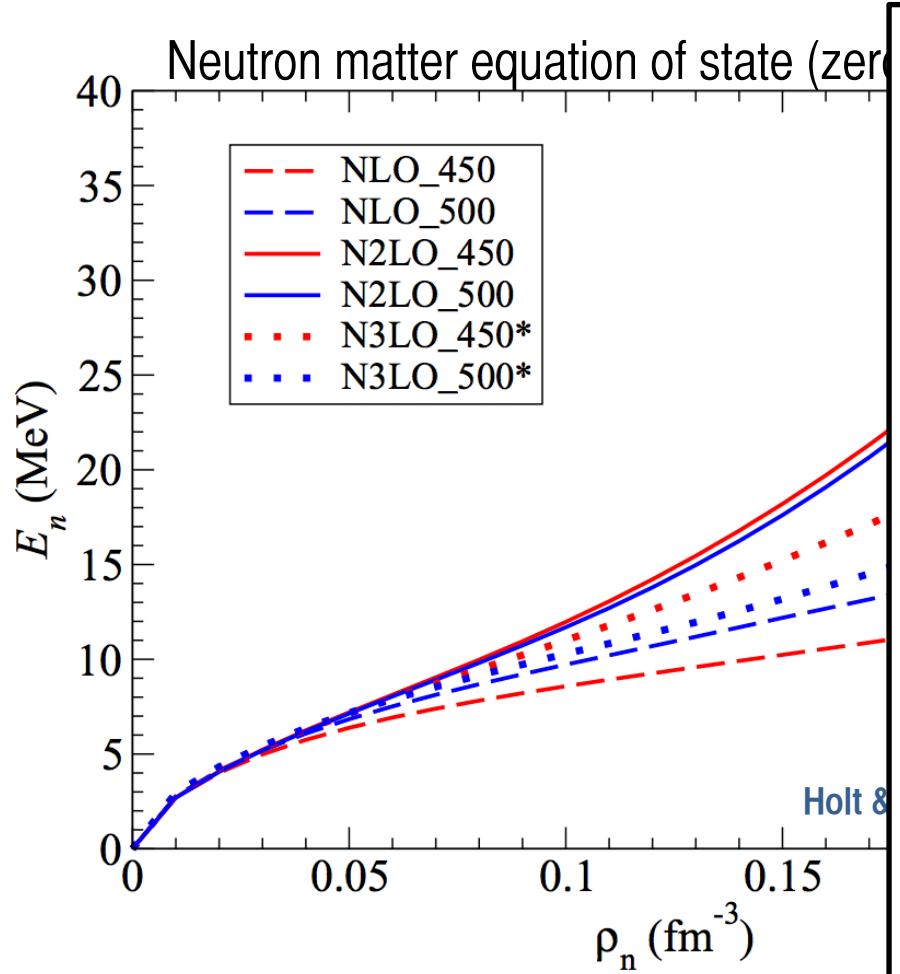
Pure neutron matter convergence in the chiral expansion



Sources of uncertainty

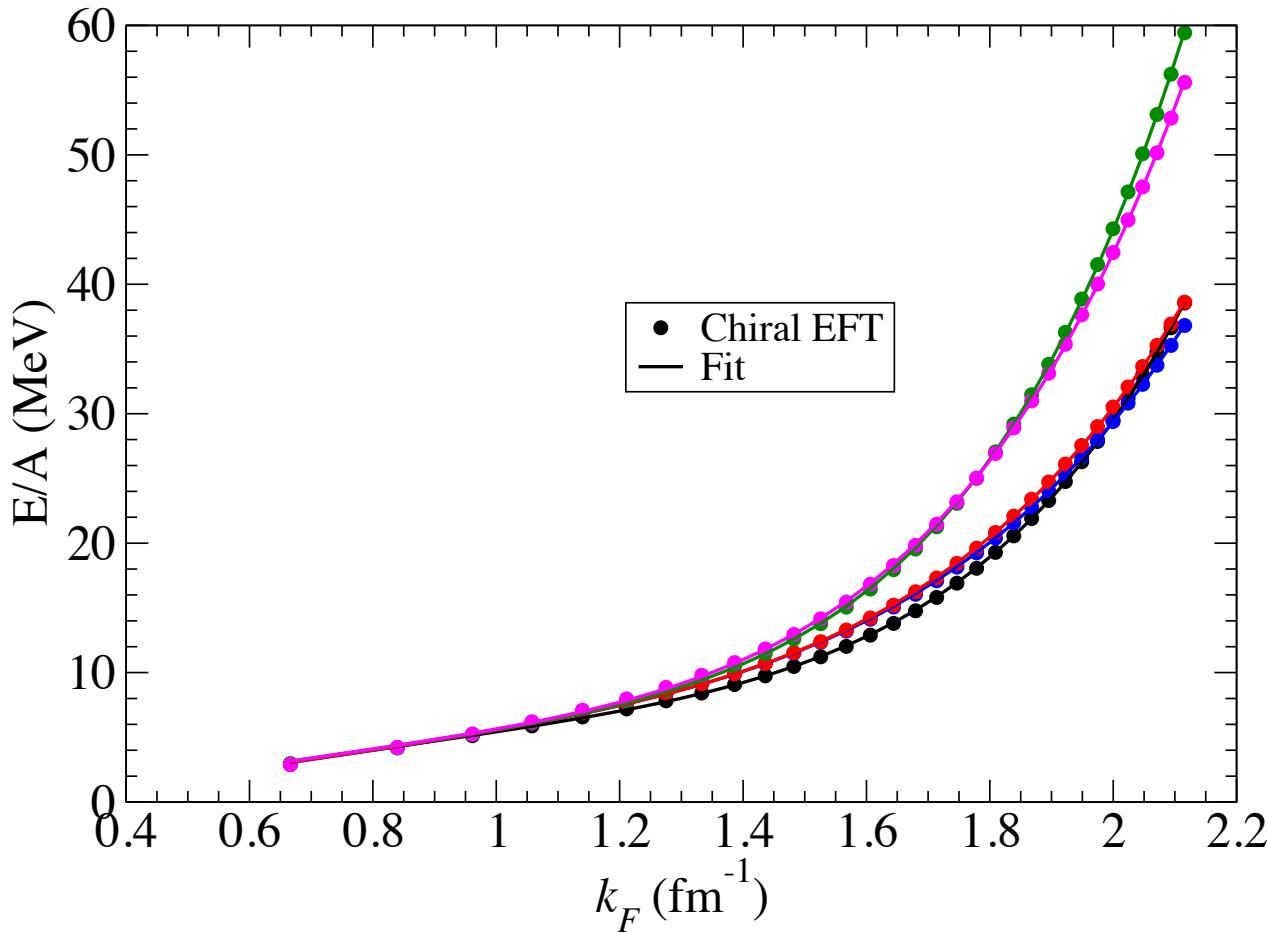
- Scale dependence
- Convergence in many-body perturbation theory
- Convergence in chiral expansion

Pure neutron matter convergence in the chiral expansion



Quality of parametrization

$$\frac{E}{A}(\rho, \delta = 1) = 2^{2/3} \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(b_0 + b_1 \beta + \frac{1}{2} b_2 \beta^2 + \frac{1}{6} b_3 \beta^3 \right)$$



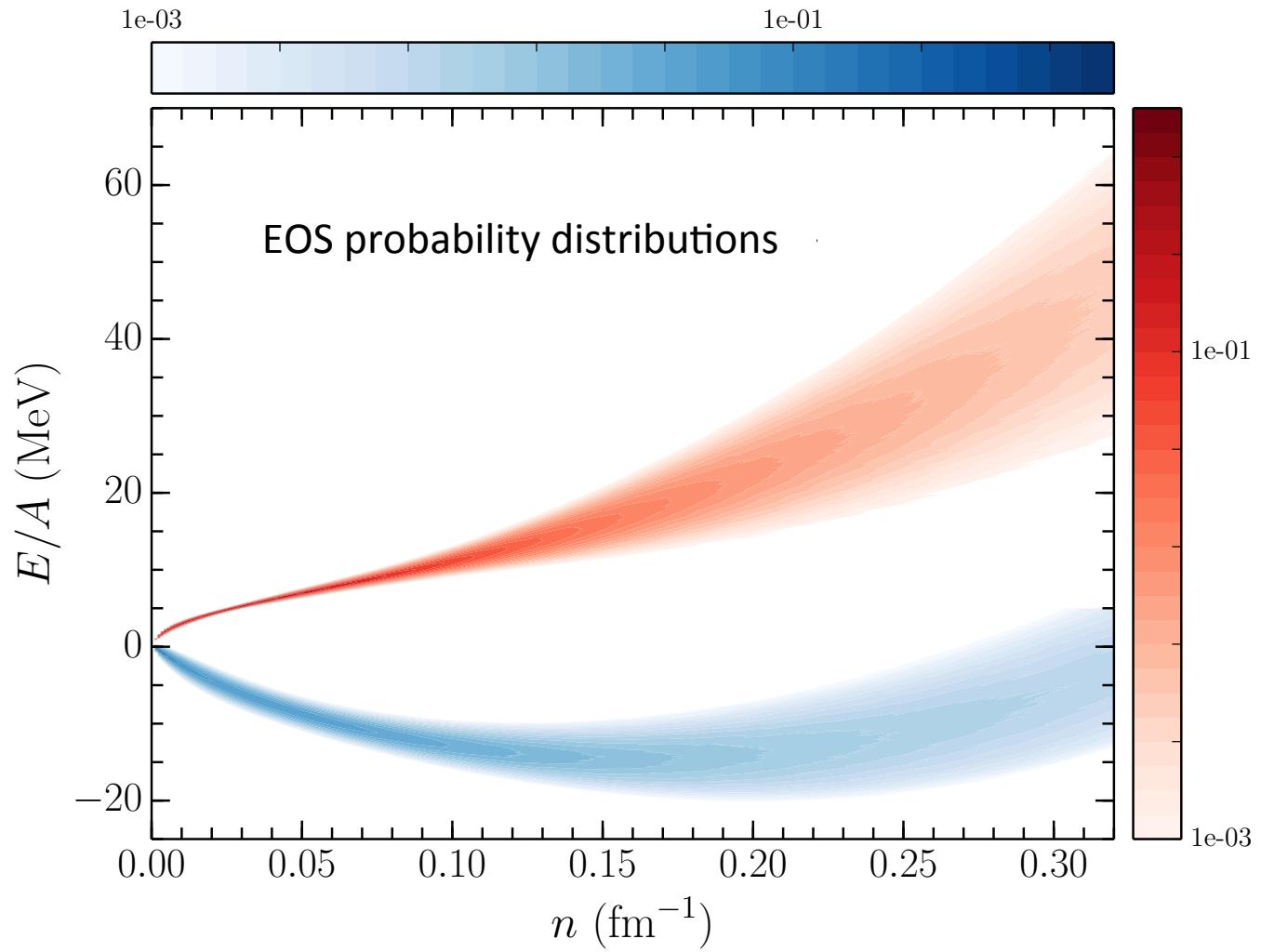
$$b_0 = -1.68 \pm 0.22 \text{ fm}^2$$

$$b_1 = 4.14 \pm 0.90 \text{ fm}^2$$

$$b_2 = 3.81 \pm 2.56 \text{ fm}^2$$

$$b_3 = 5.11 \pm 2.84 \text{ fm}^2$$

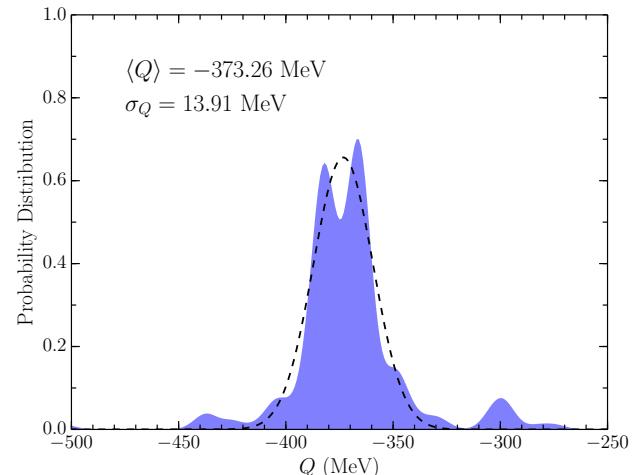
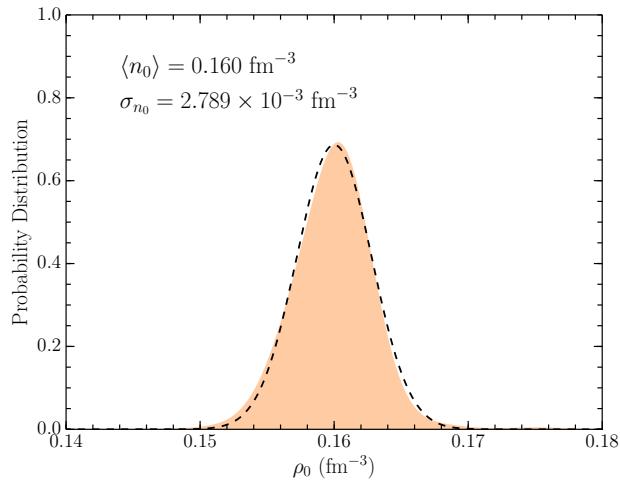
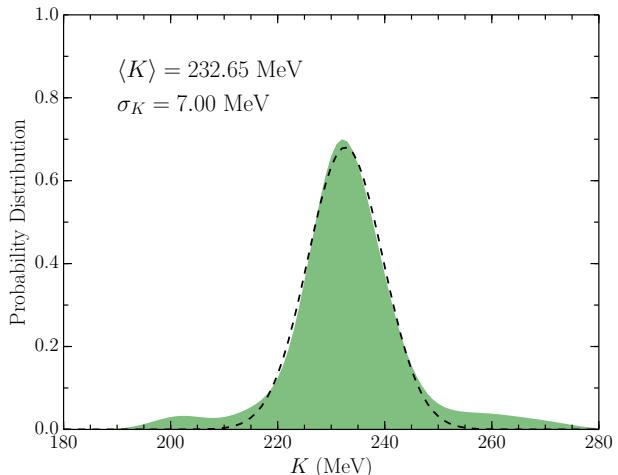
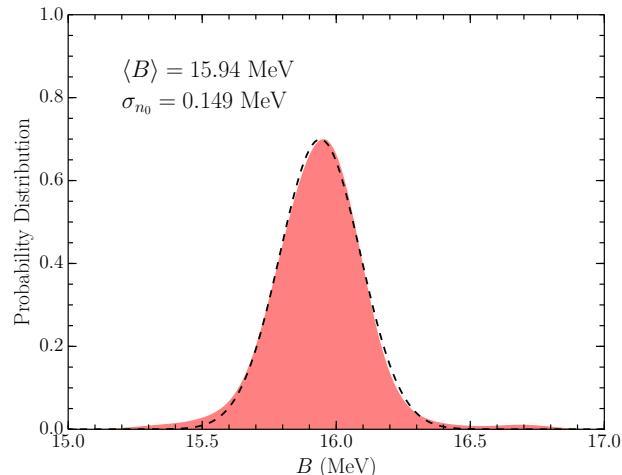
Equations of state from chiral EFT priors



Likelihood functions for symmetric nuclear matter

- Parametrization: $\frac{E}{A}(\rho, \delta = 0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(a_0 + a_1 \beta + \frac{1}{2}a_2 \beta^2 + \frac{1}{6}a_3 \beta^3 \right)$
- Average values of \vec{a} and full covariance matrix from analysis of 200 Skyrme mean field models fitted to nuclear properties

[M. Dutra et al., PRC (2012)]



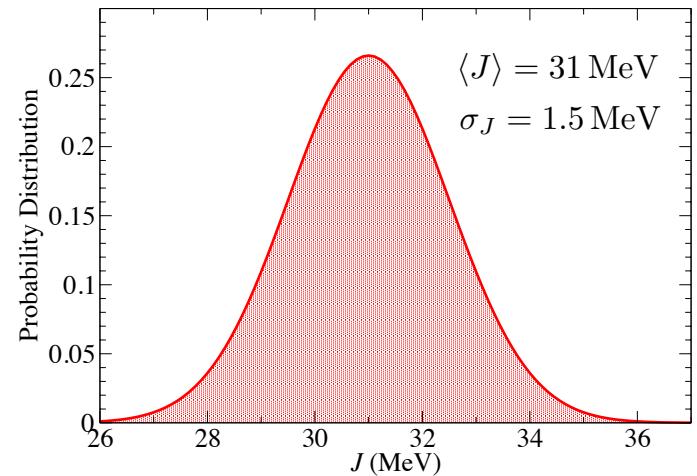
Likelihood functions for pure neutron matter

- Parametrization: $\frac{E}{A}(\rho, \delta = 1) = 2^{2/3} \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(b_0 + b_1 \beta + \frac{1}{2} b_2 \beta^2 + \frac{1}{6} b_3 \beta^3 \right)$

$$S_2(\rho) = J + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{1}{2} K_{\text{sym}} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \dots$$

$$S_2(\rho) = \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2} \underbrace{\left(c_0 + c_1 \beta + \frac{1}{2} c_2 \beta^2 + \frac{1}{6} c_3 \beta^3 \right)}_{}$$

Derive correlations among J, L, K_{sym}



$$\left. \begin{aligned} L &= (3 + \gamma)J - (1 + \gamma)S_0 - \gamma \frac{\rho_0}{6} (c_0 - \eta_1 c_1 + \eta_1 c_2) \\ K_{\text{sym}} &= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma \frac{\rho_0}{6} (c_0 - \eta_2 c_1 + \eta_2 c_2) \end{aligned} \right\}$$

J. W. Holt and Y. Lim,
arXiv:1805.01000

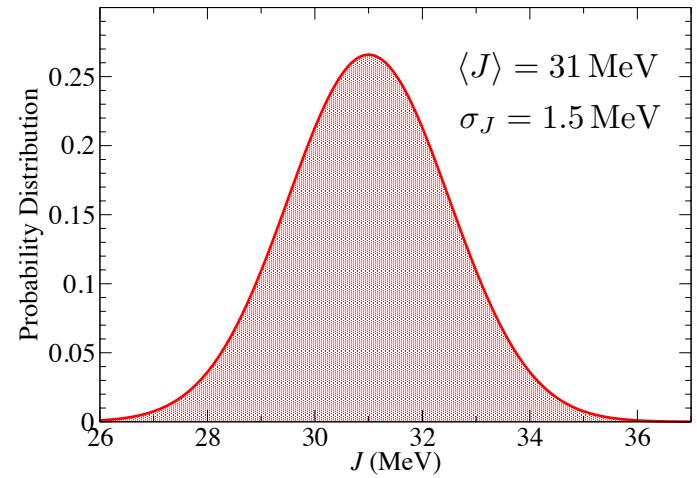
Likelihood functions for pure neutron matter

- Parametrization: $\frac{E}{A}(\rho, \delta = 1) = 2^{2/3} \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(b_0 + b_1 \beta + \frac{1}{2} b_2 \beta^2 + \frac{1}{6} b_3 \beta^3 \right)$

$$S_2(\rho) = J + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{1}{2} K_{\text{sym}} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \dots$$

$$S_2(\rho) = \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2} \underbrace{\left(c_0 + c_1 \beta + \frac{1}{2} c_2 \beta^2 + \frac{1}{6} c_3 \beta^3 \right)}_{}$$

Derive correlations among J, L, K_{sym}



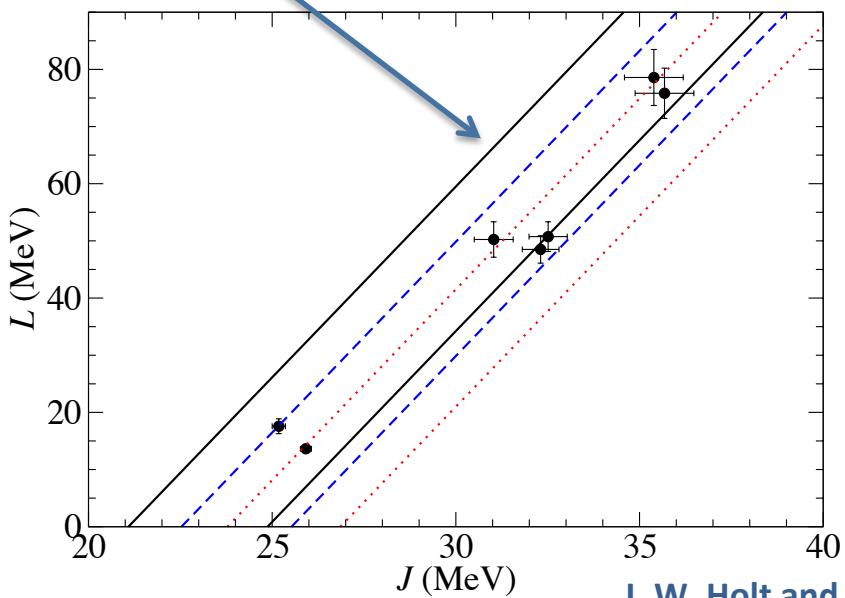
$$\left. \begin{aligned} L &= (3 + \gamma)J - (1 + \gamma)S_0 - \gamma \frac{\rho_0}{6} (c_0 - \eta_1 c_1 + \eta_1 c_2) \\ K_{\text{sym}} &= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma \frac{\rho_0}{6} (c_0 - \eta_2 c_1 + \eta_2 c_2) \end{aligned} \right\}$$

$\gamma = 3.7 \quad \eta_1 = -0.08 \quad \eta_2 = -0.16$

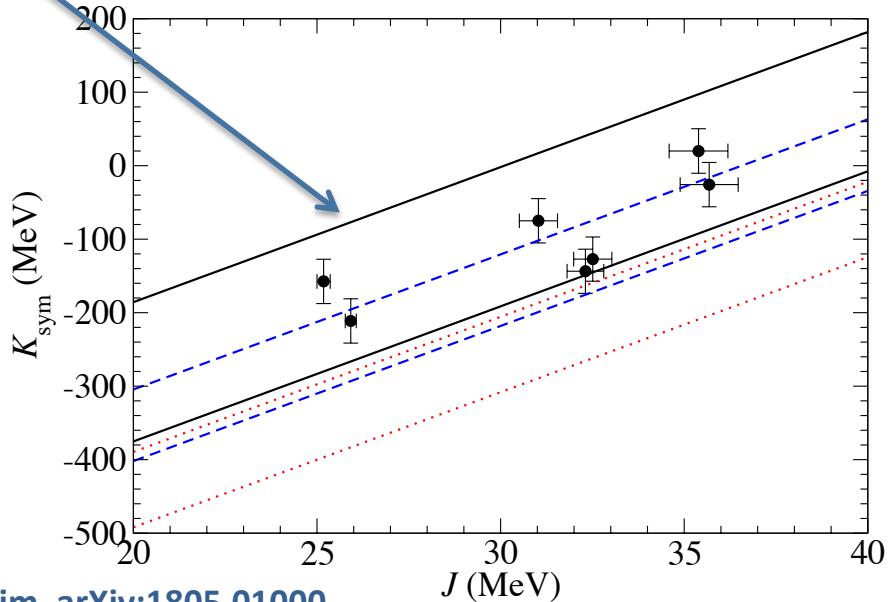
J. W. Holt and Y. Lim,
arXiv:1805.01000

Priors for pure neutron matter

Nearly universal correlations among J, L, K_{sym}

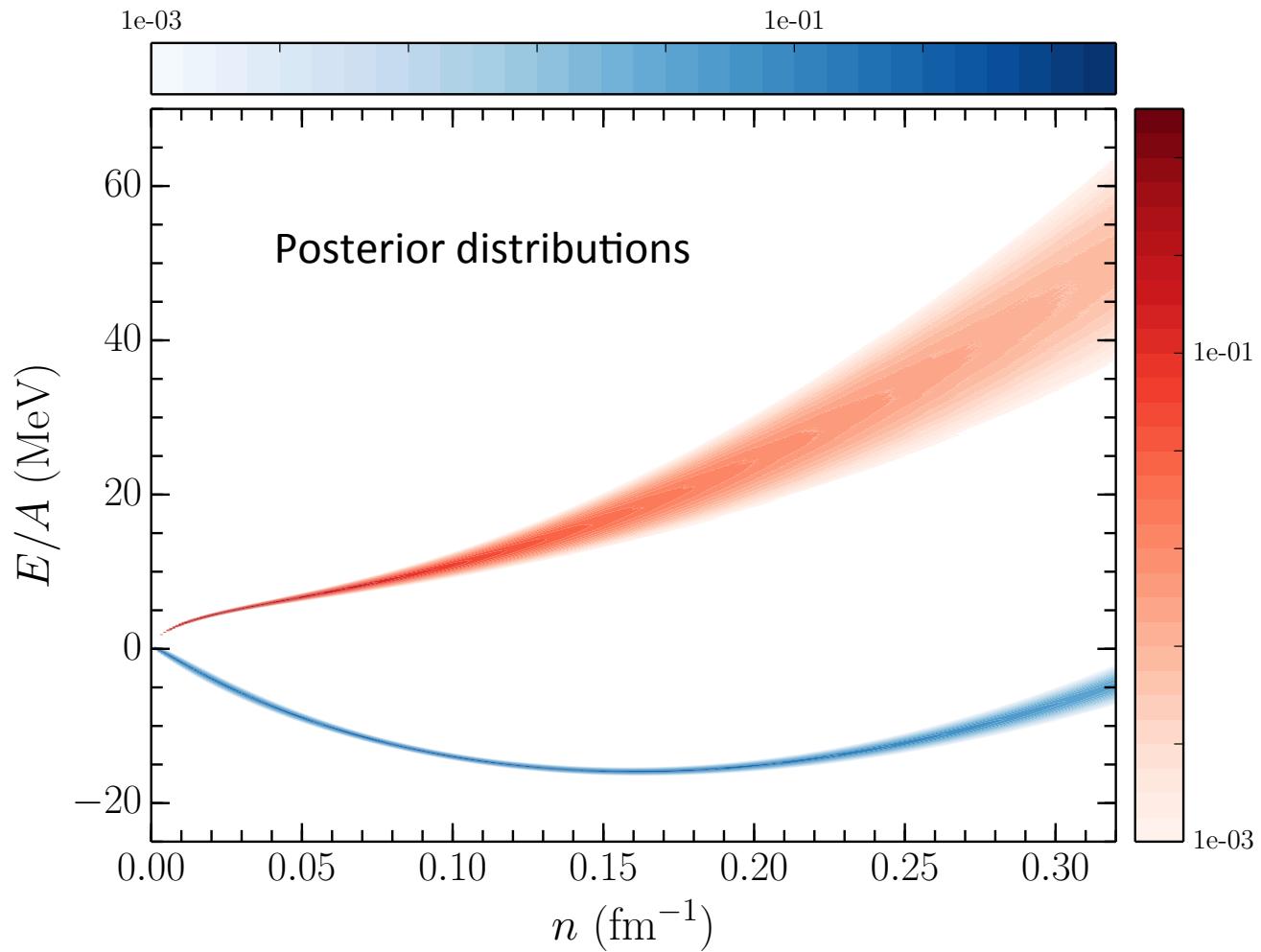


J. W. Holt and Y. Lim, arXiv:1805.01000

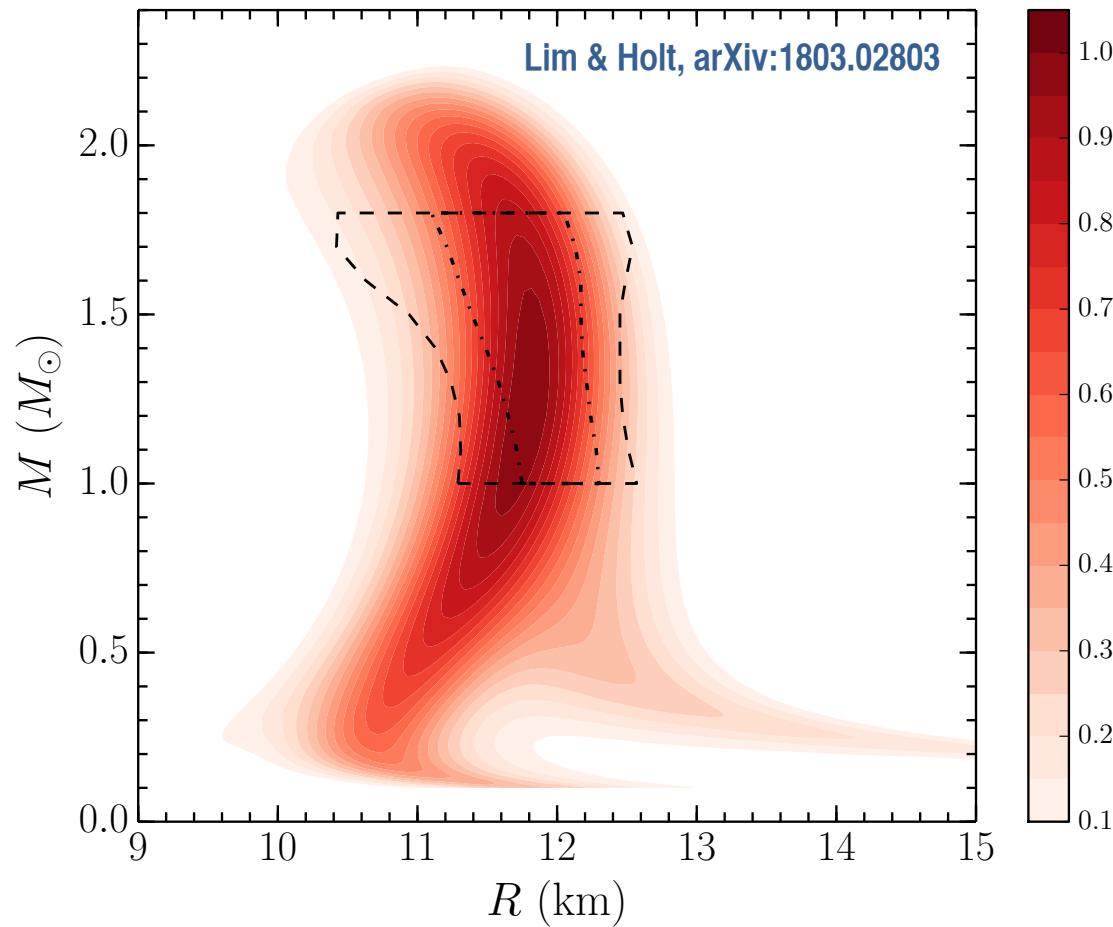


- Reference Fermi momentum: $k_r = 0.9 \text{ fm}^{-1}$
- Derive corresponding correlations among $\{b_0, b_1, b_2, b_3\}$

Equations of state from posterior probability distributions

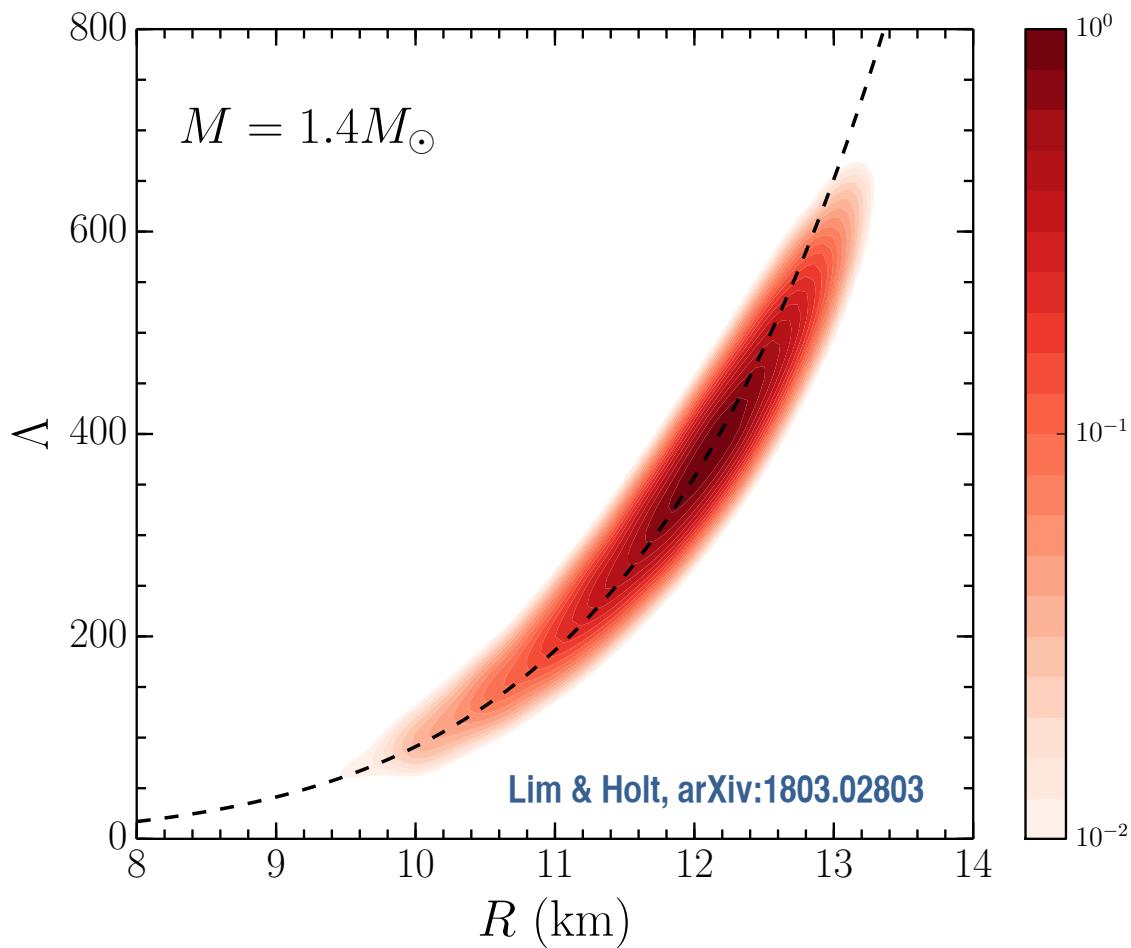


NEUTRON STAR MASS VS. RADIUS



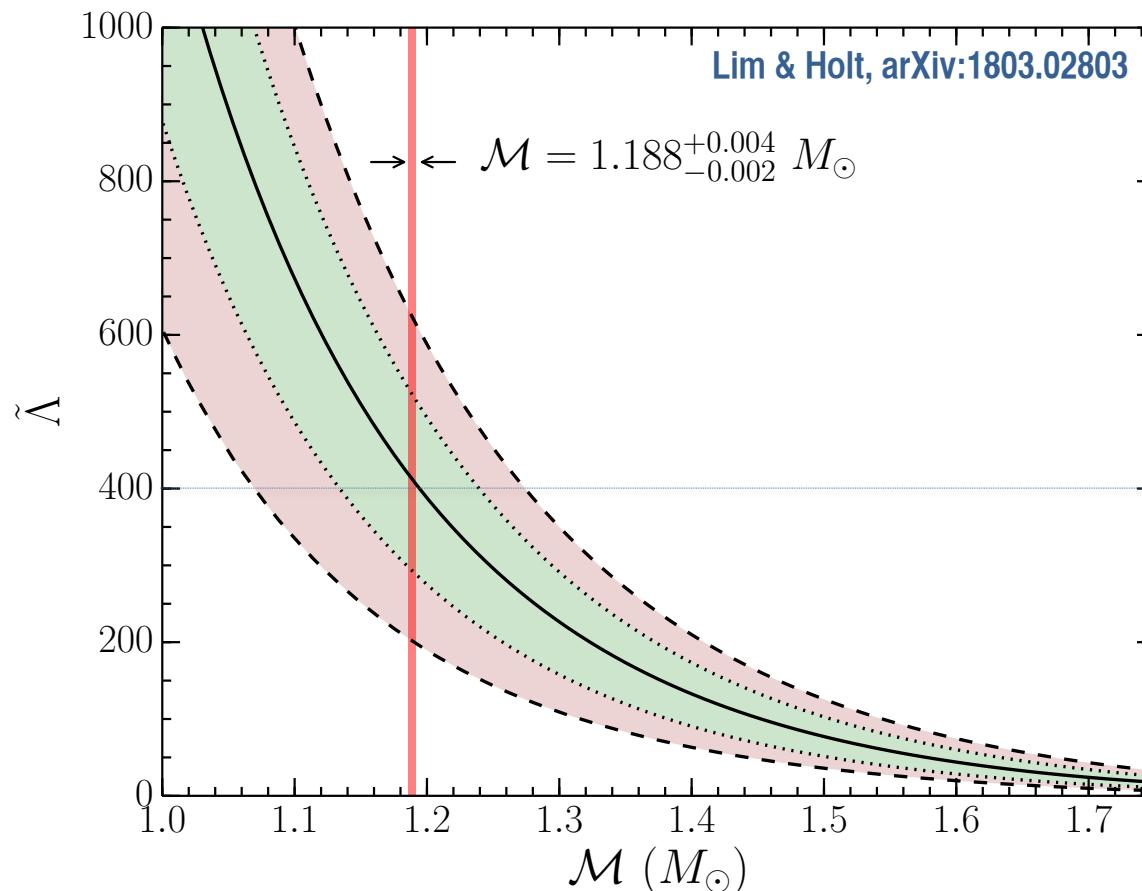
- Small probability for maximum masses greater than $2.2M_{\odot}$

Correlations between radius and tidal deformability



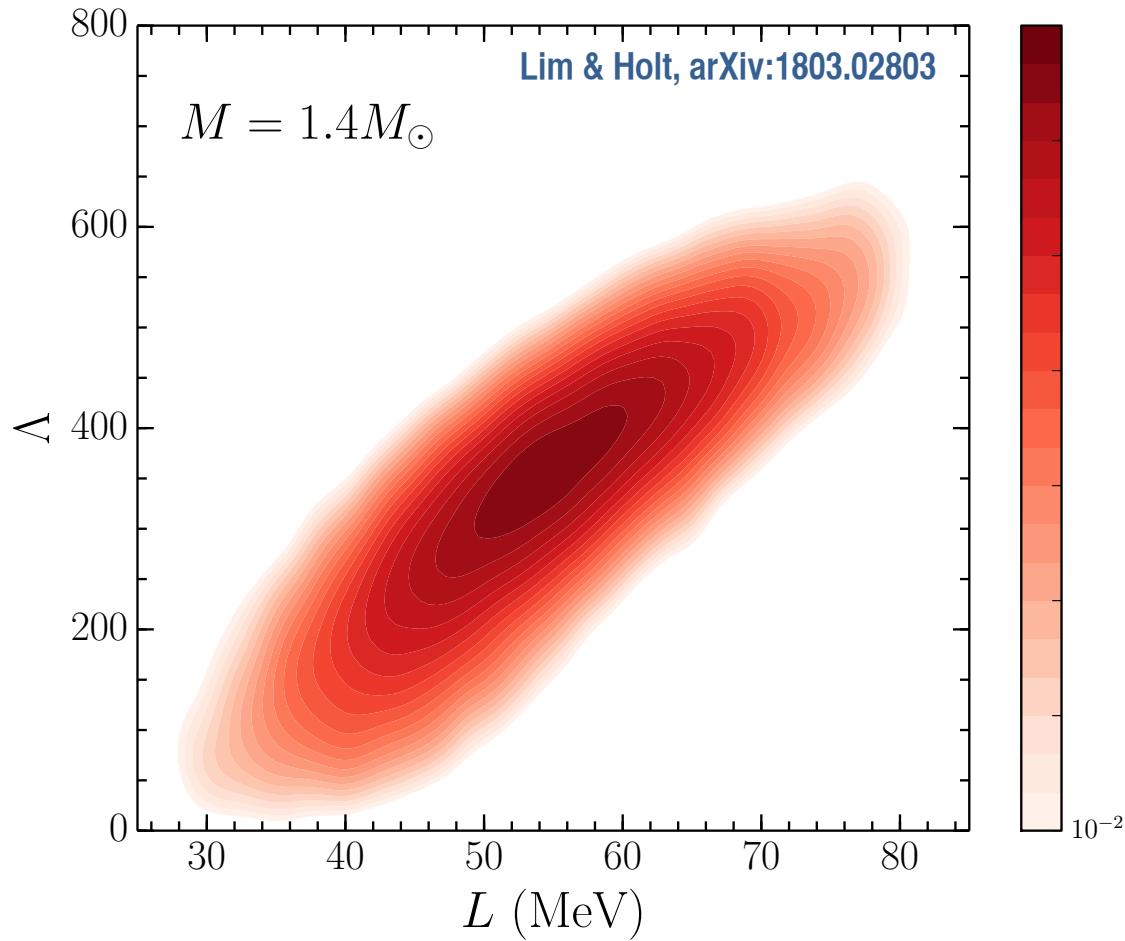
- Constraint on tidal deformabilities will strongly constrain neutron star radii

NEUTRON STAR TIDAL DEFORMABILITIES



- Small tidal deformabilities associated with soft equations of state that will produce prompt black hole formation

Correlations between L and tidal deformability



- Precise measurement of tidal deformability may better constrain the symmetry energy slope parameter L

Summary and outlook

- New era of major observational campaigns to study the properties of neutron stars
- Complementary theoretical models with accurate nuclear physics inputs needed to guide and interpret observations
- Combine properties of finite nuclei with “model independent” predictions from chiral EFT to obtain posterior distribution function for model parameters
- Ultra high-density matter a challenging frontier for *any* theoretical, experimental, or observation investigation

Discussion questions:

- How to appropriately weight the results from lower orders in the chiral expansion or lower orders in many-body perturbation theory?
- Alternative parametrizations of the nuclear equation of state?
- How to ultimately connect back to nuclear forces?
- Other possible priors?