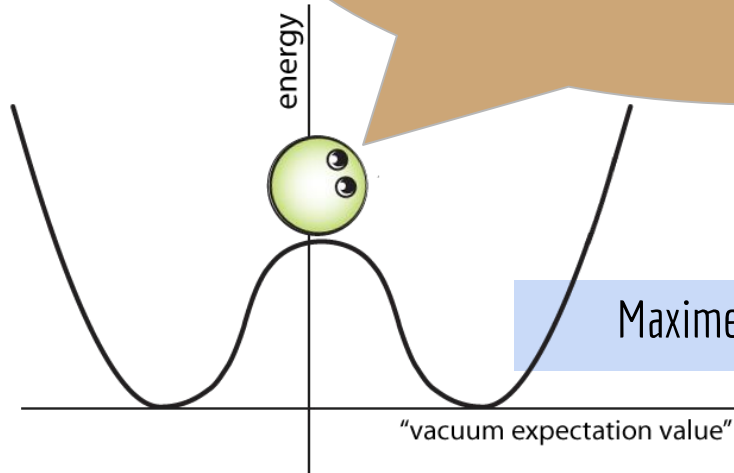


# Higgs-boson potential at colliders



# Preamble



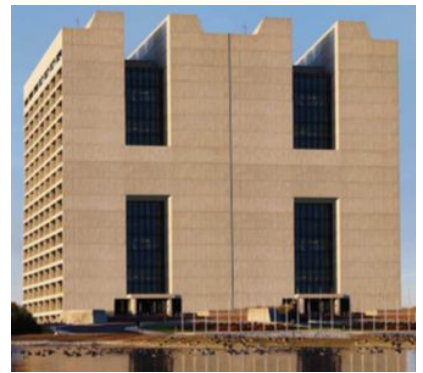
## 2020 UPDATE OF THE EUROPEAN STRATEGY FOR PARTICLE PHYSICS

**by the European Strategy Group**

The exploration of significantly higher energies than the LHC will make it possible to study the production of Higgs boson pairs and thus to explore the particle's interaction with itself, which is key to understanding the fabric of the universe.

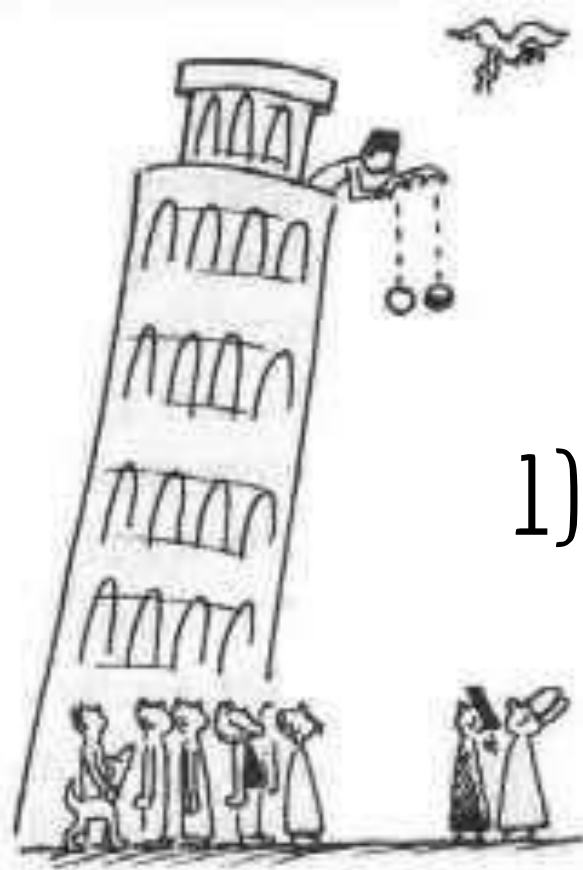
# Roadmap

- 1) Theoretical bases for the BEH potential
- 2) LHC results
- 3) Projections for Future colliders



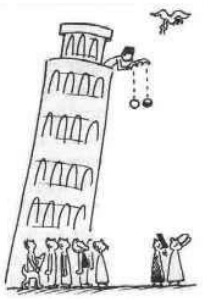
This presentation is based on:

- 1) The HH white paper: “**Higgs boson potential at colliders: Status and perspectives**”: <https://arxiv.org/abs/1910.00012> published in REVIP in Nov. 2020.
- 2) Higgs pair workshop 2022 in Dubrovnik:  
<https://indico.cern.ch/event/1001391/timetable/>
- 3) LHC publications with full Run II dataset including “Jubileum” paper in Nature.



# 1) Theoretical bases of the BEH potential

# 1.1) The BEH potential in SM



BEH was introduced in the SM as a minimalistic function (quartic polynomial) with a non-trivial minimum (inspired for example by Landau-Ginzburg potential of Superconductors):

$$V(h) = -m_H^2 h^2 + \lambda h^4$$

before EWSB with “h” an EW doublet

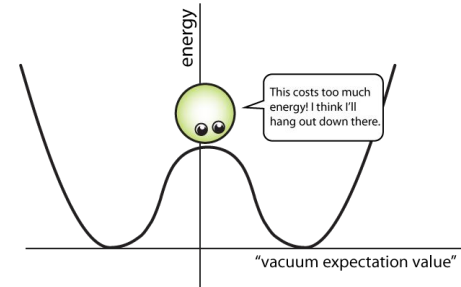
$$V(H) = \frac{1}{2}m_H^2 H^2 + \lambda \nu H^3 + \frac{1}{4}\lambda H^4 + V_0$$

after EWSB with H physical boson

$$“h = \nu + H”$$

The potential is defined by 3 parameters, 2 being independent:

$$m_H, \lambda, \text{ et } \nu = \frac{m_H}{\sqrt{2}\lambda}$$



# 1.2) Potential parameters within SM paradigm

$$\lambda = \frac{m_H^2}{2v^2}$$

Expérimentalement:

- Vev is measured via muon life time that provides Fermi constant with high precision

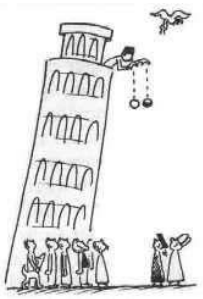
$$v = (\sqrt{2}G_F)^{-1/2} \simeq 246.22 \text{ GeV} \quad \frac{G_F}{(\hbar c)^3} = 1,166\,378\,7(6) \times 10^{-5} \text{ GeV}^{-2}$$

- Higgs boson mass is known with 0.1% precision from the LHC:

$$m_H = 125.38 \pm 0.14$$

- Self-coupling within SM paradigm is known with precision of 0.2%.

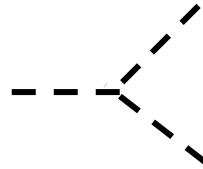
# 1.3) Higgs Self-coupling: a direct handle



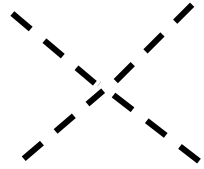
$$V(H) = \frac{1}{2}m_H^2 H^2 + \lambda \nu H^3 + \frac{1}{4}\lambda H^4 + V_0$$



Mass term



Triple  
self-coupling



Quartic  
self-coupling

# 1.4) How potential can be different from SM?

- It is important to remember that BEH potential is a postulate. We need to measure it directly!
- The Higgs boson discovery and its SM-like behaviour in EW sector shows that BEH potential shall be a very good approximation of the real potential.
- The real potential can be written in EFT approach or explicit models as :

$$V = V_{\text{BEH}} + \text{Correction}$$



# 1.5) Example of SM singlet extension

## The Model

Extend the SM by a real scalar particle,  $S$

$$V(H) \quad \Rightarrow \quad V(H, S)$$

Study the trilinear interactions

$$V_H(H) = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$$

$$V_{HS}(H, S) = \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2$$

$$V_S(S) = b_1 S + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4.$$

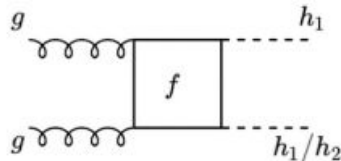
H. Alhazmi

[https://indico.cern.ch/event/1001391/contributions/4842692/attachments/2455023/4207706/Haider\\_Higgs\\_Pairs\\_2022.pdf](https://indico.cern.ch/event/1001391/contributions/4842692/attachments/2455023/4207706/Haider_Higgs_Pairs_2022.pdf)

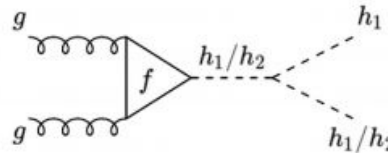
$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}$$

$$V_{\text{self}} = \frac{\lambda_{111}}{3!} h_1^3 + \frac{\lambda_{211}}{2!} h_2 h_1^2 + \frac{\lambda_{221}}{2!} h_2^2 h_1 + \frac{\lambda_{222}}{3!} h_2^3 + \frac{\lambda_{1111}}{4!} h_1^4 + \frac{\lambda_{2111}}{3!} h_2 h_1^3 + \frac{\lambda_{2211}}{4} h_2^2 h_1^2 + \frac{\lambda_{2221}}{3!} h_2^3 h_1 + \frac{\lambda_{2222}}{4!} h_2^4.$$

Chen, Dawson, Lewis  
arXiv:1410.5488  
and many others

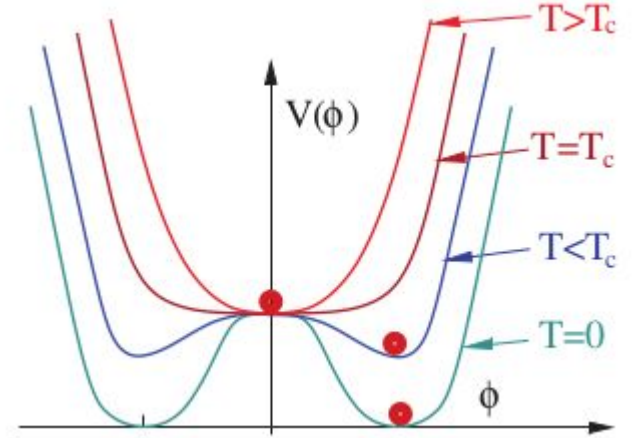
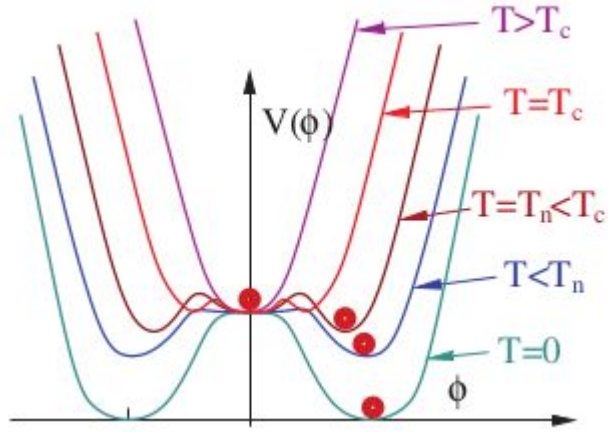


+

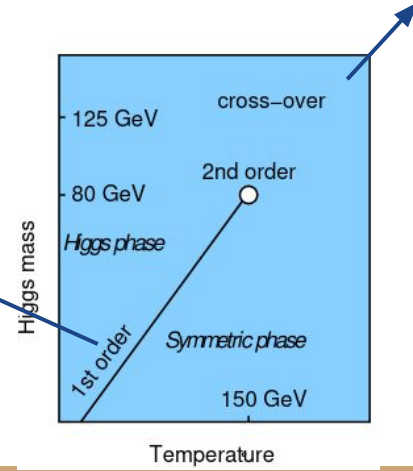


3

# 1.6) Connection to cosmology



- 1<sup>st</sup> order phase transition can generate baryon asymmetry.
- Requires lighter Higgs boson :(

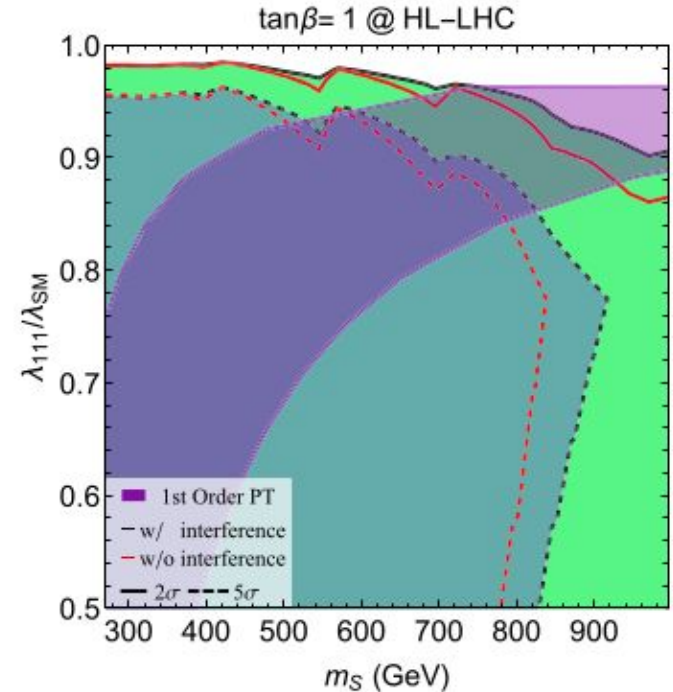


<https://arxiv.org/pdf/2008.09136.pdf>

# 1.6) Connection to cosmology

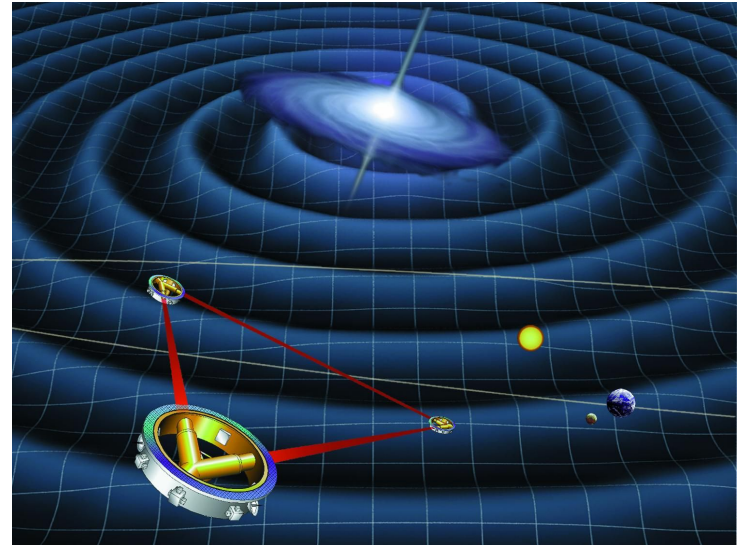
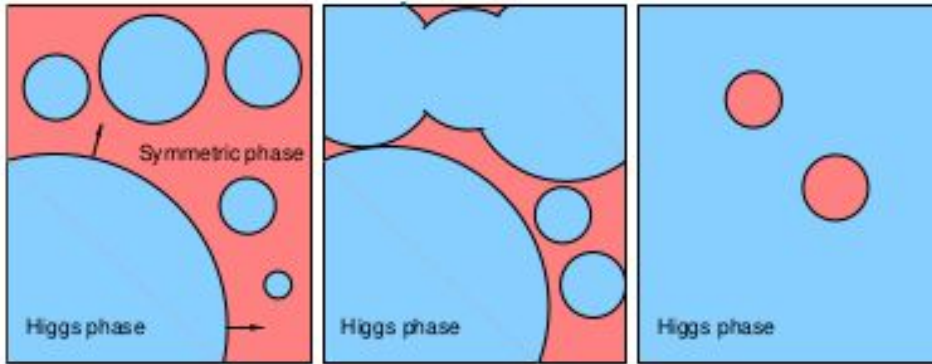
<https://doi.org/10.1103/PhysRevD.97.095032>

- Singlet extension can preserve the 1<sup>st</sup> order phase transition.
- It leads also to a different effective  $k_\lambda$  parameter of BEH potential.

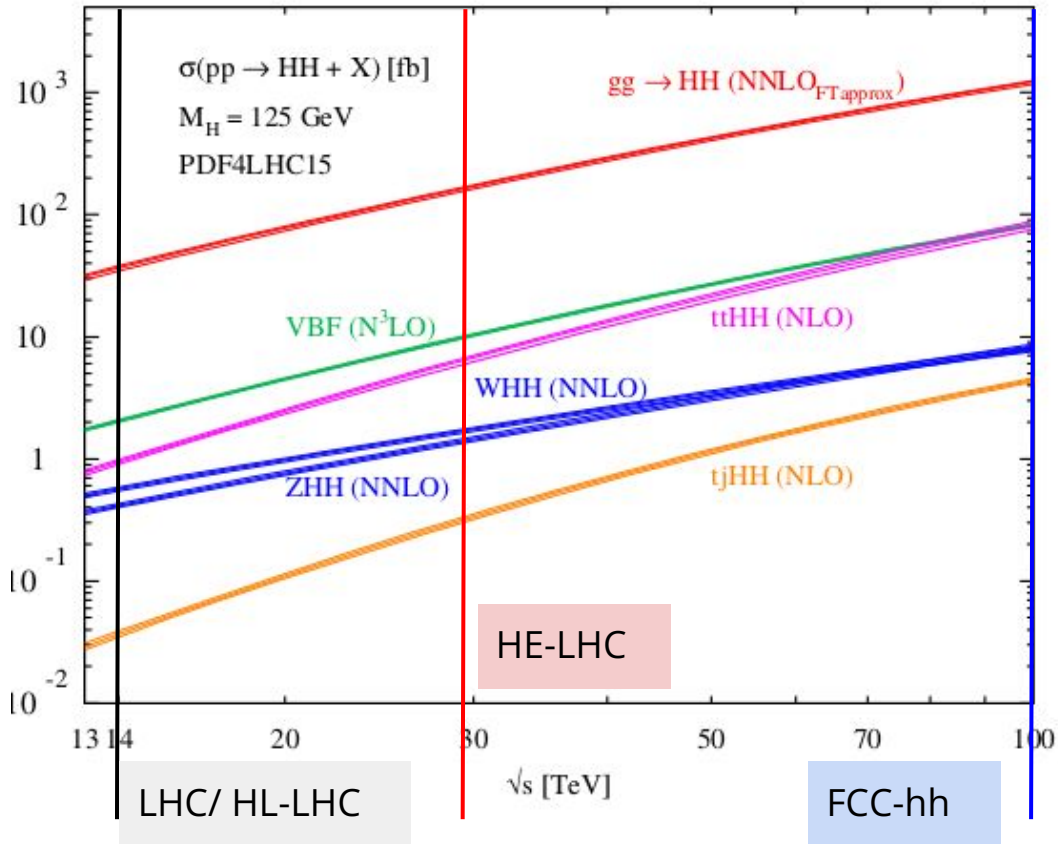


# 1.6) Connection to cosmology

- 1<sup>st</sup> order phase transition can be also observed in large scale primary gravitational waves.

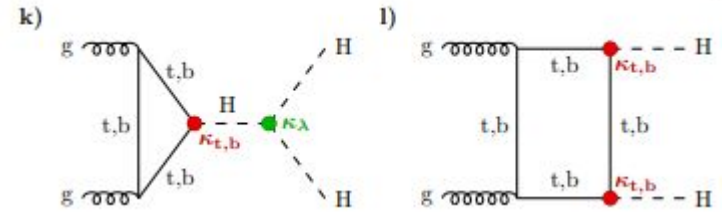


# 1.7) Production cross sections

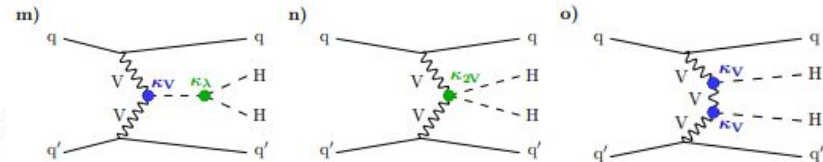


- Cross section rises as power law with  $\sqrt{s}$

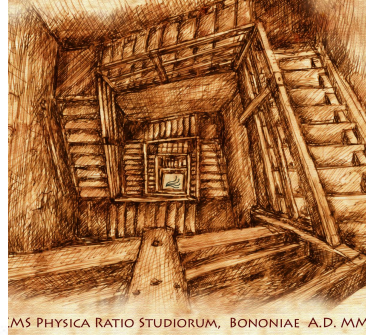
- Dominant production mode:  
 $gg \rightarrow HH$



- - Subdominant O(5%): VBF HH

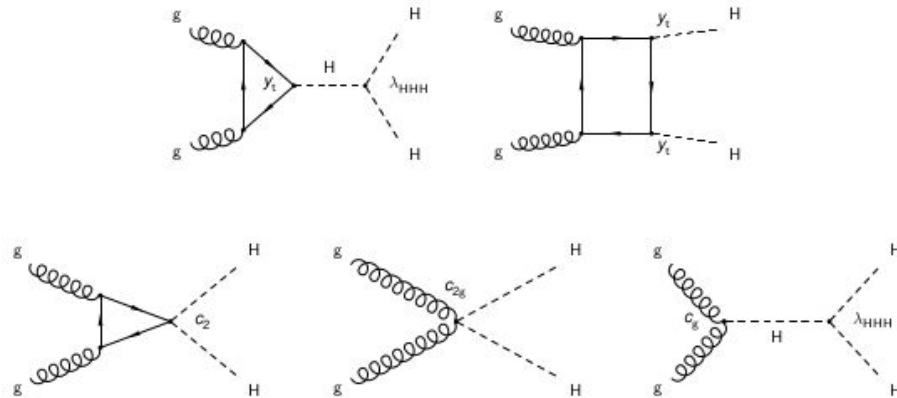


# 1.8) HH in HEFT



Potential part related to HH production\* in gluon-gluon fusion:

$$\mathcal{L}_{HH} = \kappa_\lambda \lambda_{HHH}^{\text{SM}} v H^3 - \frac{m_t}{v} \left( \kappa_t H + \frac{c_2}{v} H^2 \right) (\bar{t}_L t_R + \text{h.c.}) + \frac{1}{4} \frac{\alpha_s}{3\pi v} \left( c_g H - \frac{c_{2g}}{2v} H^2 \right) G^{\mu\nu} G_{\mu\nu}$$



\* Neglecting some operators there : chromomagnetic, coupling to b quark. More details in J. Lang talk

[https://indico.cern.ch/event/1001391/contributions/4842688/attachments/2454956/420758/2/JannisLang\\_HiggsPairs2022.pdf](https://indico.cern.ch/event/1001391/contributions/4842688/attachments/2454956/420758/2/JannisLang_HiggsPairs2022.pdf)

# 1.9) Analytical parametrization

- One can build an analytical parametrization of HH production as function of these parameters.

$$R_{\text{HH}} \equiv \frac{\sigma_{\text{gg} \rightarrow \text{HH}}}{\sigma_{\text{gg} \rightarrow \text{HH}}^{\text{SM}}} \stackrel{\text{LO}}{=} A_1 \kappa_t^4 + A_2 c_2^2 + (A_3 \kappa_t^2 + A_4 c_g^2) \kappa_\lambda^2 + A_5 c_{2g}^2$$

$$+ (A_6 c_2 + A_7 \kappa_t \kappa_\lambda) \kappa_t^2 + (A_8 \kappa_t \kappa_\lambda + A_9 c_g \kappa_\lambda) c_2$$

$$+ A_{10} c_2 c_{2g} + (A_{11} c_g \kappa_\lambda + A_{12} c_{2g}) \kappa_t^2$$

$$+ (A_{13} \kappa_\lambda c_g + A_{14} c_{2g}) \kappa_t \kappa_\lambda + A_{15} c_g c_{2g} \kappa_\lambda.$$

$$\sigma_{\text{gg} \rightarrow \text{HH}} = \sigma_{\text{HH,NNLO+NNLL}}^{\text{SM}} \cdot R_{\text{HH}},$$

For NLO parameterization have a look at L. Skyboz talk:

[https://indico.cern.ch/event/1001391/contributions/4827320/attachments/2453035/4203719/HiggsPairs-2022\\_Scyboz.pdf](https://indico.cern.ch/event/1001391/contributions/4827320/attachments/2453035/4203719/HiggsPairs-2022_Scyboz.pdf)

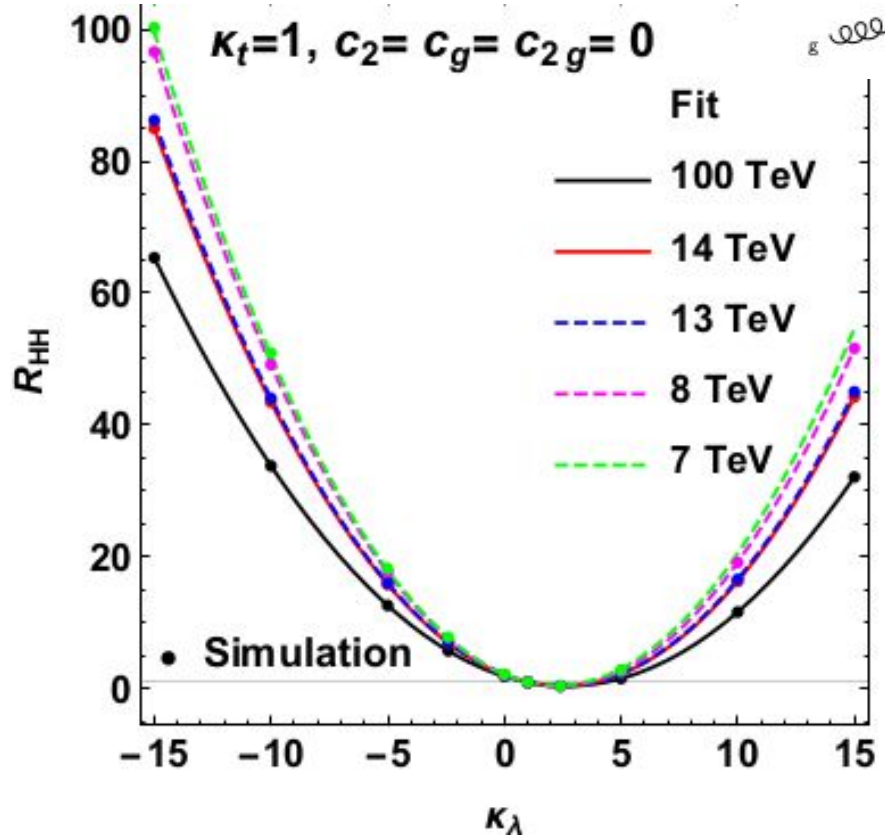
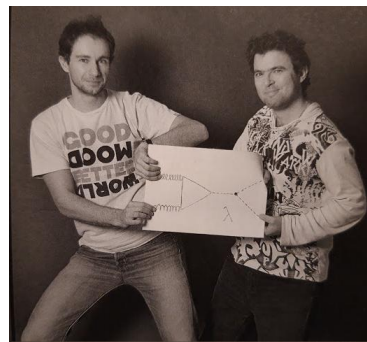
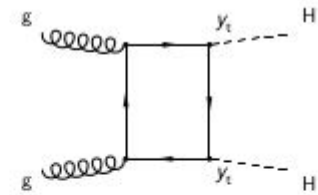
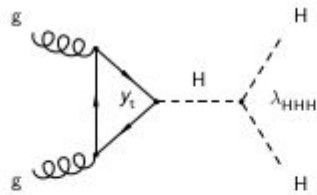


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LHCHSWG-2016-001 and CERN YR4

$\sqrt{s}$	7 TeV	8 TeV	13 TeV	14 TeV	100 TeV
$A_1$	2.21	2.18	2.09	2.08	1.90
$A_2$	9.82	9.88	10.15	10.20	11.57
$A_3$	0.33	0.32	0.28	0.28	0.21
$A_4$	0.12	0.12	0.10	0.10	0.07
$A_5$	1.14	1.17	1.33	1.37	3.28
$A_6$	-8.77	-8.70	-8.51	-8.49	-8.23
$A_7$	-1.54	-1.50	-1.37	-1.36	-1.11
$A_8$	3.09	3.02	2.83	2.80	2.43
$A_9$	1.65	1.60	1.46	1.44	3.65
$A_{10}$	-5.15	-5.09	-4.92	-4.90	-1.65
$A_{11}$	-0.79	-0.76	-0.68	-0.66	-0.50
$A_{12}$	2.13	2.06	1.86	1.84	1.30
$A_{13}$	0.39	0.37	0.32	0.32	0.23
$A_{14}$	-0.95	-0.92	-0.84	-0.83	-0.66
$A_{15}$	-0.62	-0.60	-0.57	-0.56	-0.53

# 1.10) HH “SM”-like



- Strong dependence on  $\kappa_\lambda$  with an interference minimum at  $\kappa_\lambda^{\min} = 2.2$ .
- Strong interference and symmetry around  $\kappa^{\min}$ .

$$R_{HH} = A_1(2.09) \kappa_t^4 + A_3(1.33) \kappa_t^2 \kappa_\lambda^2 + A_7(-1.37) \kappa_t^3 \kappa_\lambda$$

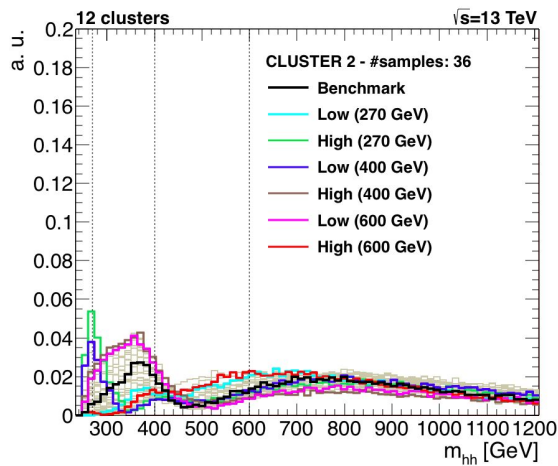


# 1.11) Shape benchmarks: how to distinguish between different EFT parameters

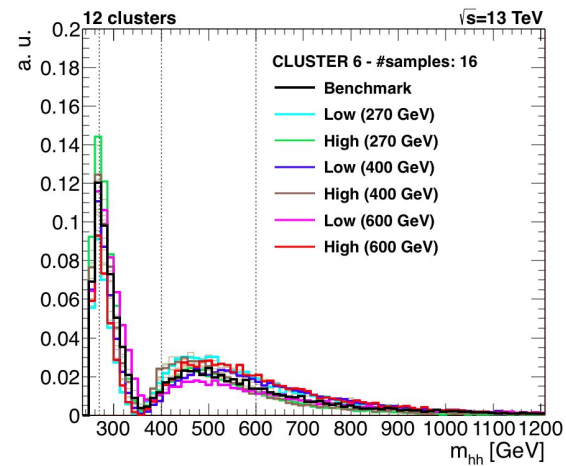


- Different operators have different  $m_{HH}$  spectra -> Many operators complex interference patterns.
- At LO 12 typical benchmarks (arXiv:1507.02245, LHCHSWG-2016-001, CERN YR4).  
More recently 7 BM an NLO including experimental constraints (arXiv:2204.13045).
- Any BSM point shape can be mapped to one of the benchmarks.

Benchmark	$\kappa_\lambda$	$\kappa_t$	$c_2$	$c_g$	$c_{2g}$
1	7.5	1.0	-1.0	0.0	0.0
2	1.0	1.0	0.5	-0.8	0.6
3	1.0	1.0	-1.5	0.0	-0.8
4	-3.5	1.5	-3.0	0.0	0.0
5	1.0	1.0	0.0	0.8	-1.0
6	2.4	1.0	0.0	0.2	-0.2
7	5.0	1.0	0.0	0.2	-0.2
8	15.0	1.0	0.0	-1.0	1.0
9	1.0	1.0	1.0	-0.6	0.6
10	10.0	1.5	-1.0	0.0	0.0
11	2.4	1.0	0.0	1.0	-1.0
12	15.0	1.0	1.0	0.0	0.0
SM	1.0	1.0	0.0	0.0	0.0

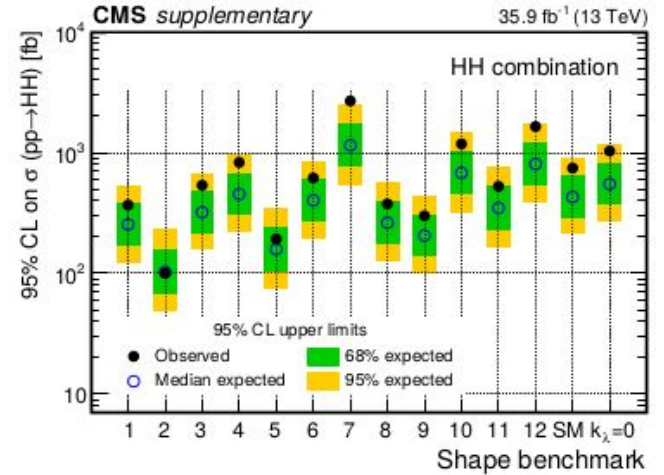
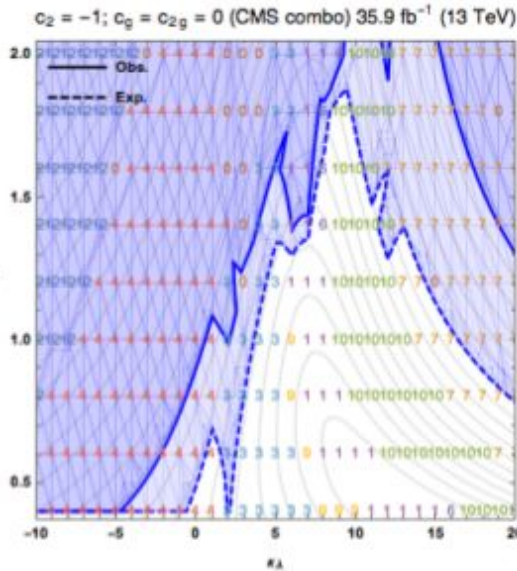


**BSM with long UV tail**



**Maximal interference**

# 1.12) How to use benchmarks



EFT parameter	Method	allowed interval at 95% CL	
		observed	expected
$\kappa_\lambda$	Benchmarks	-11 -20	-6 - 12
	CMS	-12 - 19	-7.1 - 14
$\kappa_t$	Benchmarks	-2.05 -2.3	
$c_2$	Benchmarks	-1.35 -1.45	

- Experiments provides limits on benchmarks. They can be mapped into EFT spaces or into explicit theories.

# 1.13) HEFT reinterpretation with explicit models

- SM-like:  $\kappa_\lambda$  et  $\kappa_t$
- 2HDM, Vector Like Quarks... :  $\kappa_\lambda, \kappa_t$  et  $c_2$
- Colored fermions in loops:  $c_g, c_{2g}$ .

Linear EFT realisation (Higgs doublet of EW):  $c_g = c_{2g}$

Fund. Parameters	$\kappa_\lambda$	$\kappa_t$	$c_2$
$\alpha, m_2, \lambda_\alpha$	real scalar singlet with explicit $Z_2$ breaking [73] [74]		
	$1 - \frac{3}{2}t_\alpha^2$	$1 - \frac{t_\alpha^2}{2}$	$-\frac{t_\alpha^2}{2}$
	$+t_\alpha^2 (\lambda_\alpha - t_\alpha \frac{m_2}{v}) / \lambda_{SM}$		
$\alpha$	real scalar singlet with spontaneous $Z_2$ breaking [73]		
	$1 - \frac{3}{2}t_\alpha^2$	$1 - \frac{t_\alpha^2}{2}$	$-\frac{t_\alpha^2}{2}$
$\beta, Z_6, m_H$	2HDM (addtl. scalars heavy + $Z_2$ ) [75]		
	$1 - \frac{3Z_6^2}{2\lambda_{SM}} \frac{v^2}{m_H^2}$	$1 - \frac{Z_6}{t_\beta} \frac{v^2}{m_H^2}$	$-\frac{3Z_6}{2t_\beta} \frac{v^2}{m_H^2}$

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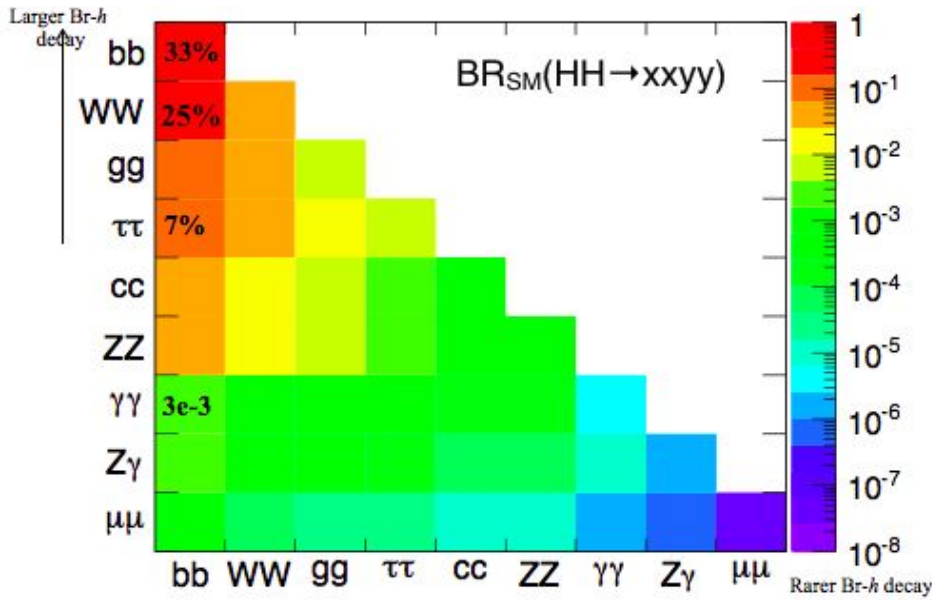
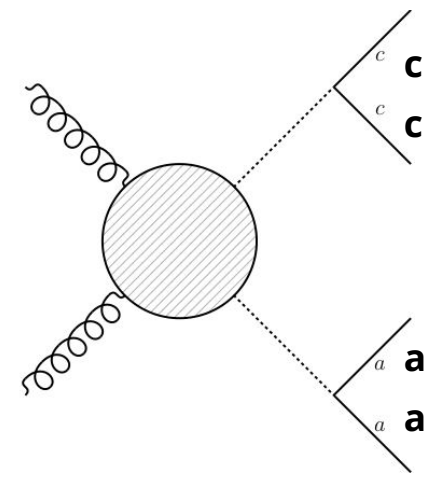
An abstract painting of a cityscape, viewed from a high angle looking down. The buildings are rendered in a variety of colors including red, blue, green, yellow, and grey, with thick, expressive brushstrokes. The composition is dense and geometric, with many rectangular and triangular shapes. A semi-transparent white box is overlaid in the center, containing the title text in yellow. In the bottom right corner, there is a small white rectangular label with the artist's name and the year.

# LHC results and HL-LHC projections

Danielle Monico, 2021

DANIELLE  
MONICO  
Mai 2021 -

# 2.1) Higgs - squared



- Very challenging search:  
 $\sigma_{HH} < \sigma_H / 1000$

- Golden channels

- **cc = bb**  $\rightarrow$  BR(H $\rightarrow$ bb) = 57%
- **aa =  $\gamma\gamma, \tau\tau, WW/ZZ$**   $\rightarrow$  trigger and QCD background reduction.
- **aa = bb**  $\rightarrow$  highest rate, but large background.

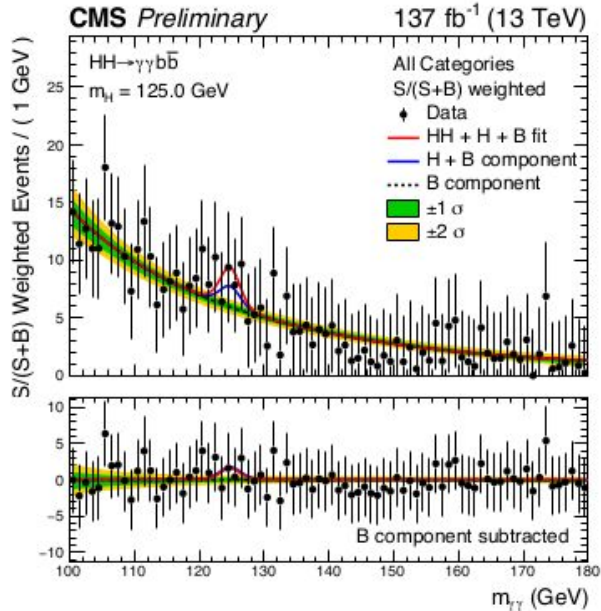
- Second level channels:

- **$\gamma\gamma WW$ , multileptons**: lower sensitivity, but many of them combined can bring some info.

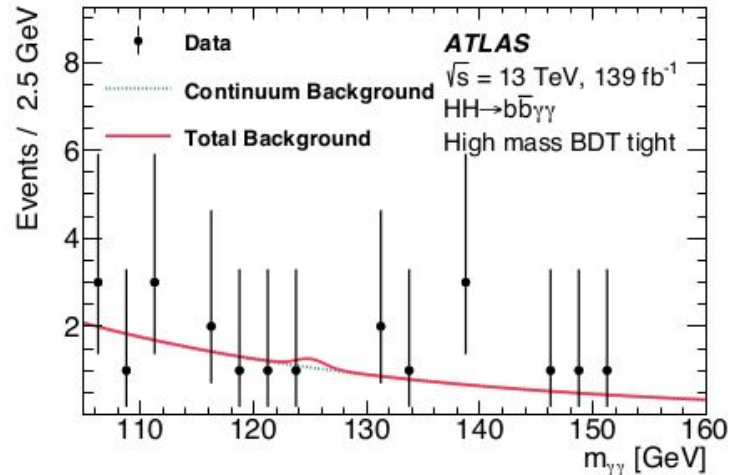
Nice experimental summary of HH production from L. Cadamuro

[https://indico.cern.ch/event/1001391/contributions/4842925/attachments/2455652/4208916/HH\\_experimental\\_summary.pdf](https://indico.cern.ch/event/1001391/contributions/4842925/attachments/2455652/4208916/HH_experimental_summary.pdf)

## 2.2) Example of HH search: $HH \rightarrow 2b2\gamma$

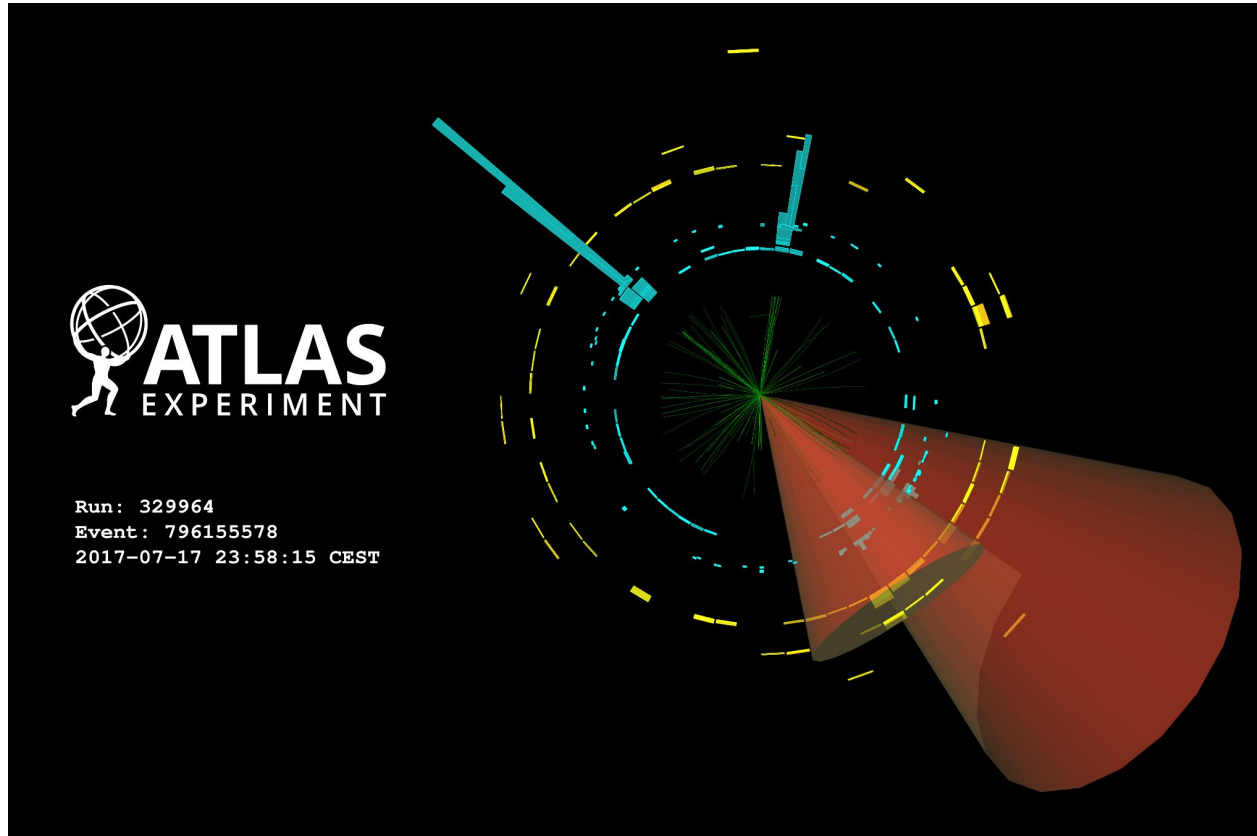


◀ All categories  $S/(S+B)$  weighted  
 ▼ Most sensitive category



- Similar flows of ATLAS and CMS analyses
- Purity and  $m_{HH}$  categories for maximal sensitivity
- Powerful signature from the  $H \rightarrow \gamma\gamma$  decay used to search for a signal
- Sensitivity limited by stat. uncertainty

## 2.2) Example of HH search: $HH \rightarrow 2b2\gamma$

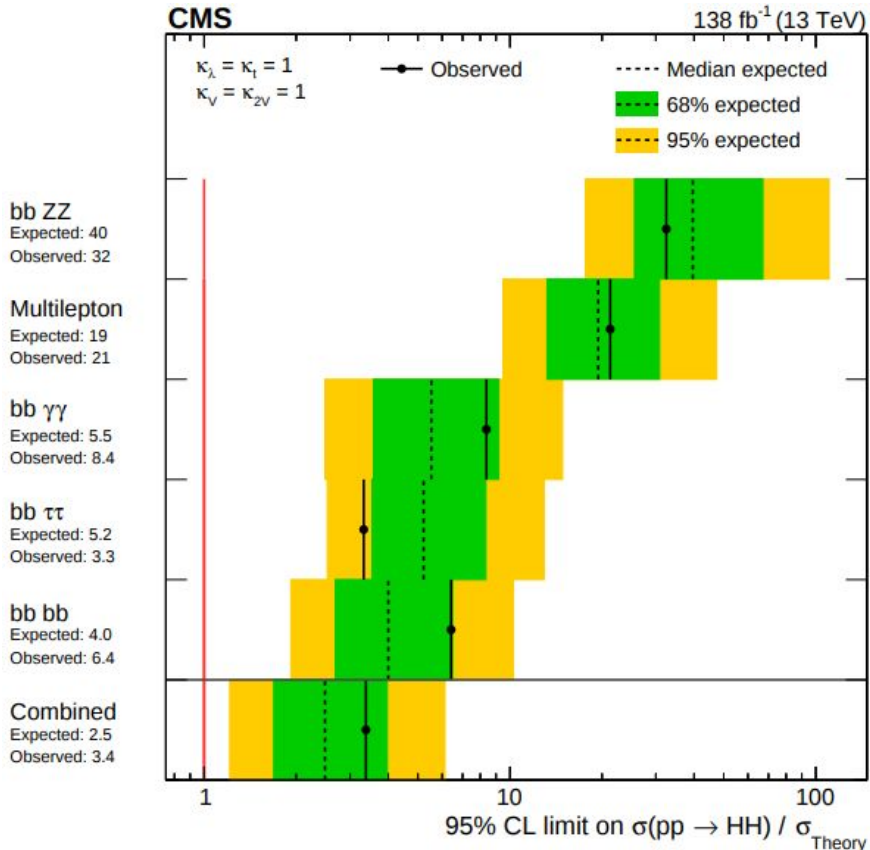


## 2.3) Information provided by the LHC analyses

- 1) Limit on total  $gg \rightarrow HH$  cross section assuming SM-like combination of  $k_L$  and  $k_T$
- 2) Exclusion range for  $k_L$  ( $gg \rightarrow HH$  and VBF  $HH$ )
- 3) Exclusion range for  $k_L \times k_T$
- 4) Exclusion range for BSM operators:  $c_2$
- 5) Exclusion /  $gg \rightarrow HH$  BSM benchmark
- 6) Exclusion range:  $c_{2v} - c_v$  (VBF  $HH$ )
- 7) Limits on resonant production:  $gg \rightarrow X \rightarrow HH$  or  $gg \rightarrow X \rightarrow YH$



## 2.4) Higgs Jubilee paper in Nature: SM HH



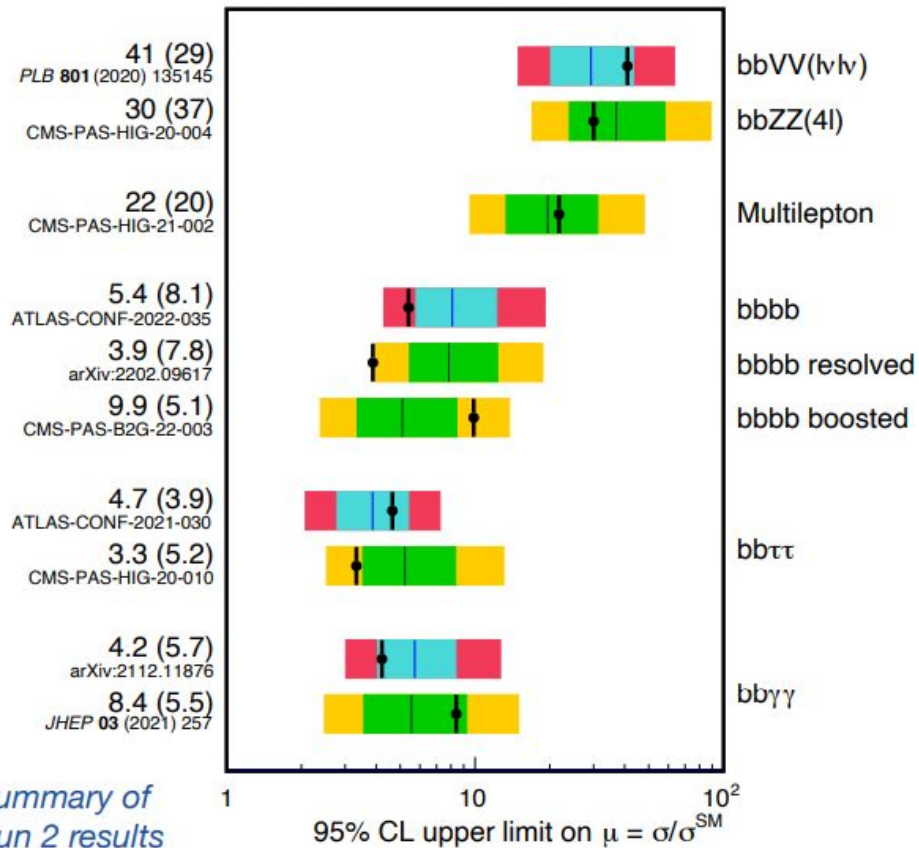
- ATLAS and CMS published in Nature 2 papers describing the portrait of the Higgs boson.
- CMS included part of the HH results with full Run II (138 fb<sup>-1</sup>) and their combination.

→ We are getting closer and closer to SM HH.

[Nature 607 \(2022\) 60-68](#)

# 2.5) ATLAS vs CMS comparison

ATLAS CMS



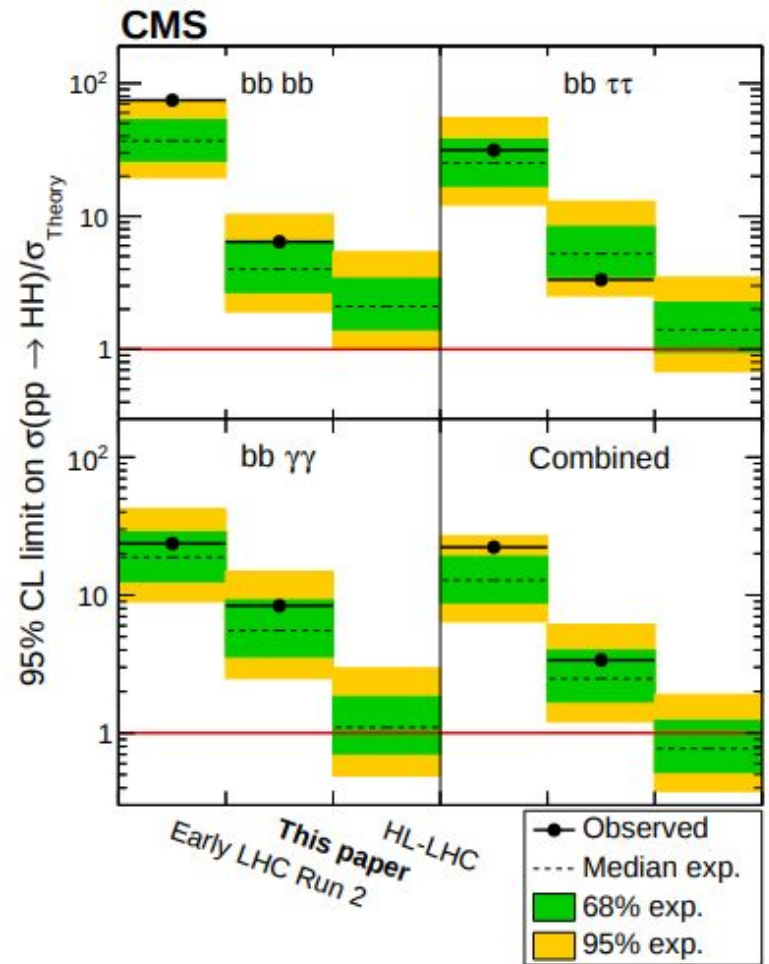
- CMS and ATLAS results for different channels are comparables.

→ Including more channels and combining between collaborations the limit  $< 2$  SM is within the reach of Run II.

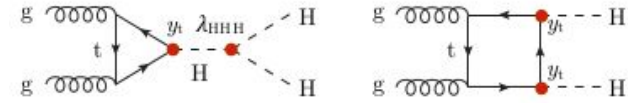
## 2.6) Improvements over time

	Early anal.	Run II	Run III + Run II	HL-LHC
Lumi	36	138	400	3000
sqrt (S) scaling wrt to early analysis	1	2	3.3	10

Run II analysis is improving much faster than simple luminosity scaling.  
Run III + Run II may tackle SM HH once combining ATLAS + CMS.



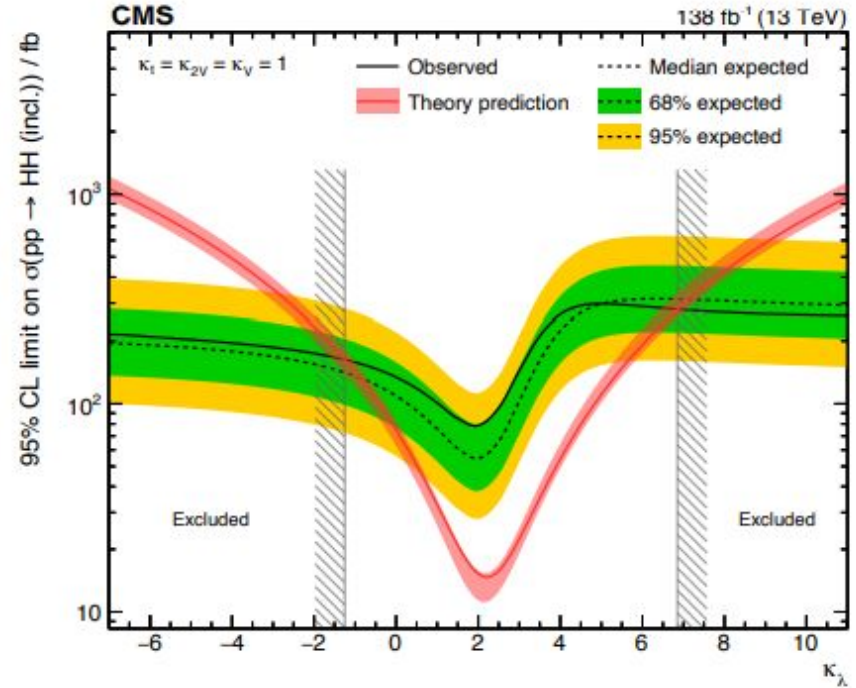
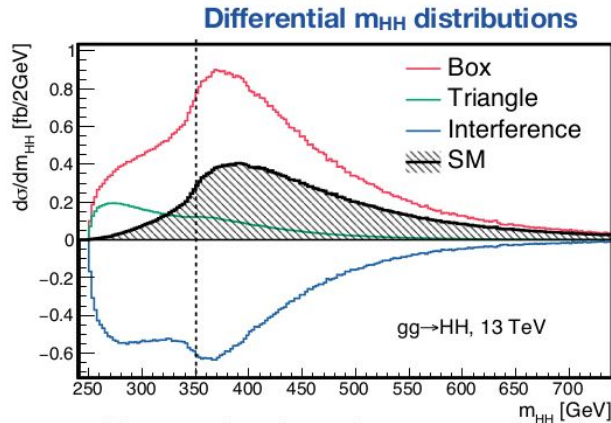
# 2.7) Self-coupling extraction Run II: $k_\lambda$



SM  $k_\lambda$  is complicated to measure:

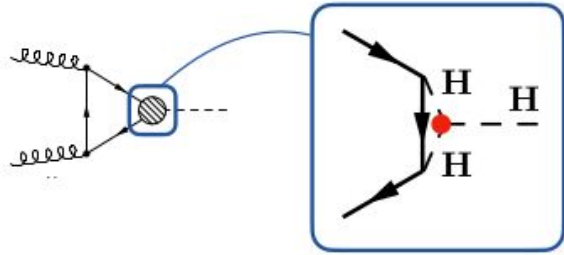
- HH production is dominated by top box diagram: 90% of the total
- Softer spectrum for  $k_\lambda \gg |1|$ 
  - lower reconstruction efficiency,
  - larger cross section.
- Very strong negative interference for  $0 < k_\lambda < 4$ 
  - harder spectrum
  - smaller cross section

$$R_{HH} = A_1(2.09) \kappa_t^4 + A_3(1.33) \kappa_t^2 \kappa_\lambda^2 + A_7(-1.37) \kappa_t^3 \kappa_\lambda$$



# 2.8) "Global fit"

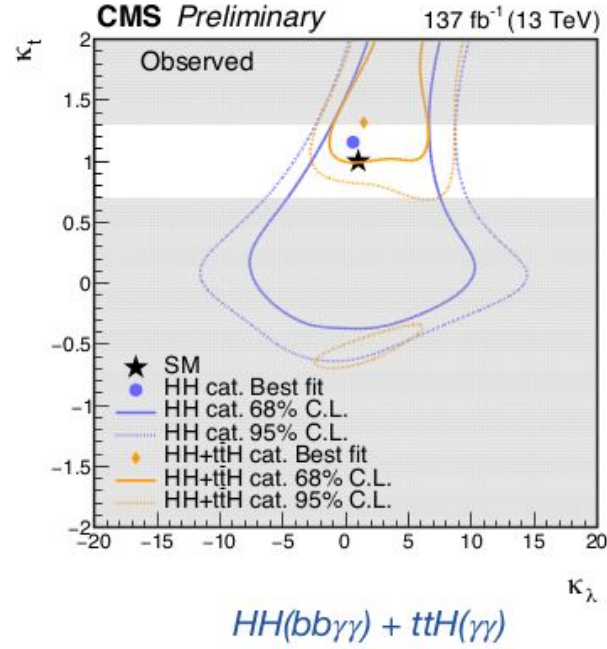
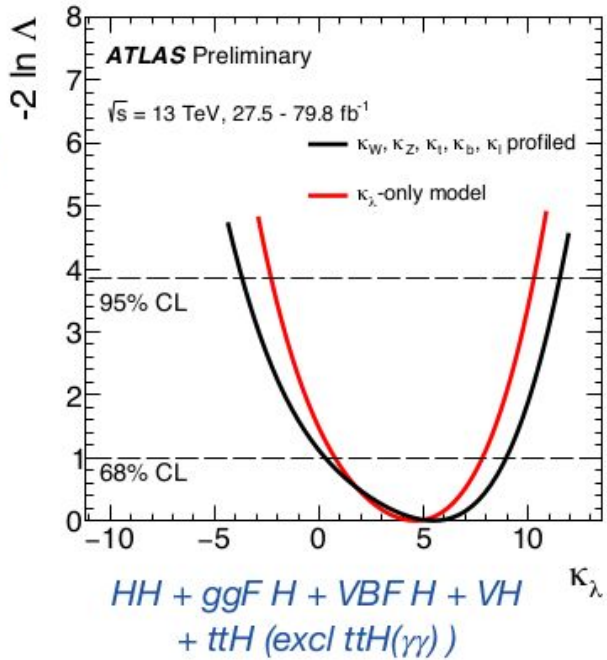
$$R_{HH} = A_1(2.09) \kappa_t^4 + A_3(1.33) \kappa_t^2 \kappa_\lambda^2 + A_7(-1.37) \kappa_t^3 \kappa_\lambda$$



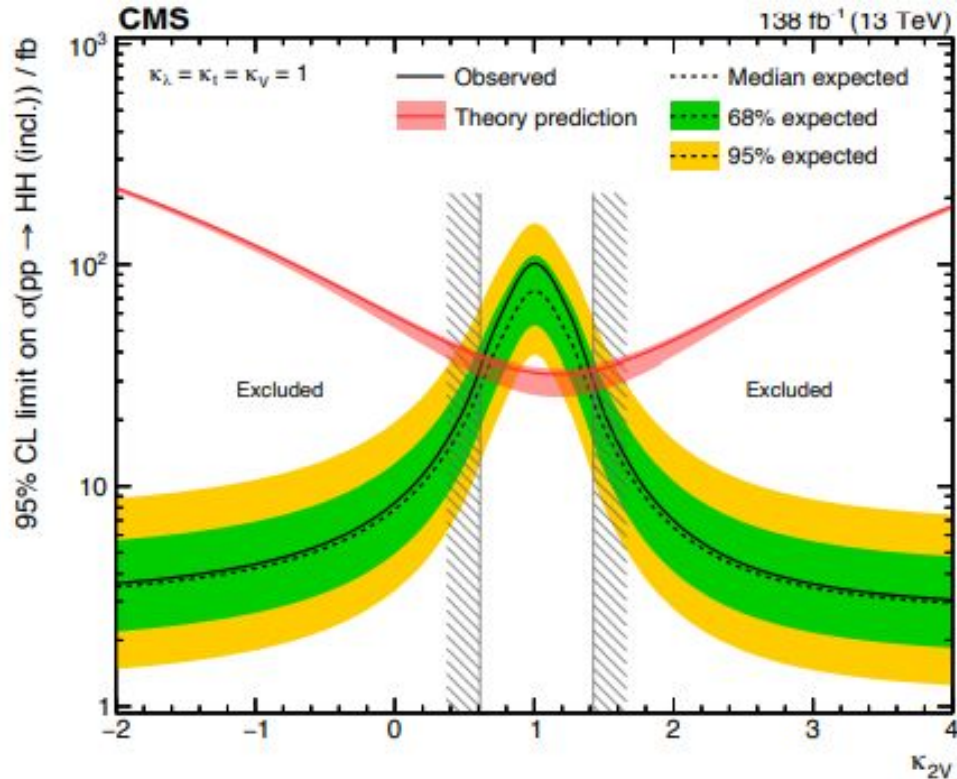
## Global view of $\lambda_{HHH}$ effects in Higgs physics

Sensitivity to  $\lambda_{HHH}$  from loop-level effects

- Experimental challenges from channel overlap
- Strong assumptions in the interpretation ( $\kappa$  framework + NLO effects)
- A global model-independent EFT fit as the next step



# 2.9) First “measured” HH coupling: $k_{2V}$



- Sensitivity to  $k_{2V}$  is obtained through the unitarisation constraint.

$$\mathcal{A}(V_L V_L \rightarrow HH) \simeq \frac{\hat{s}}{v^2} (C_{2V} - C_V^2)$$

- We observe this operator! Significance  $> 5\sigma$  is already reached.



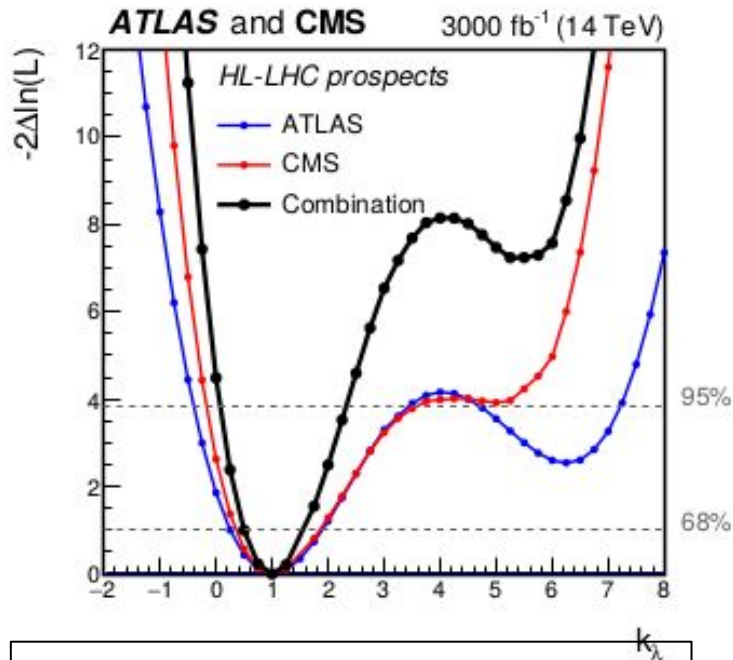
- The specificity there is that we claim the observation by “negative result”, that is less valuable than a



Projections for future  
colliders

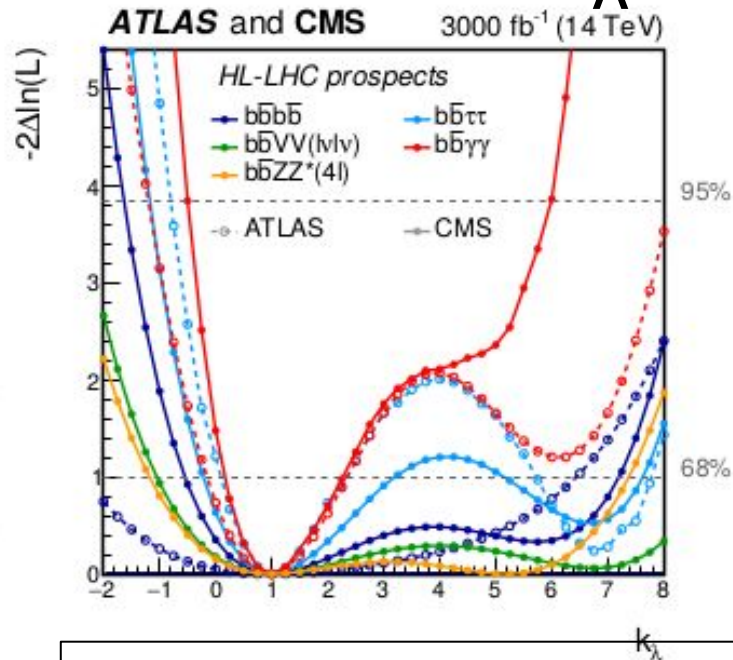
A bit of dream

# 3.1) Self-coupling extraction HL-LHC: $k_\lambda$



$k_\lambda = 1 \pm 50\%$  is expected.

30% precision may be reached with analysis improvements.



Best channels for self-coupling  $HH \rightarrow b\bar{b}\gamma\gamma$  et  $HH \rightarrow b\bar{b}\tau\tau$  sensibles at low  $m_{HH}$ : low background and good low  $p_T$  trigger.

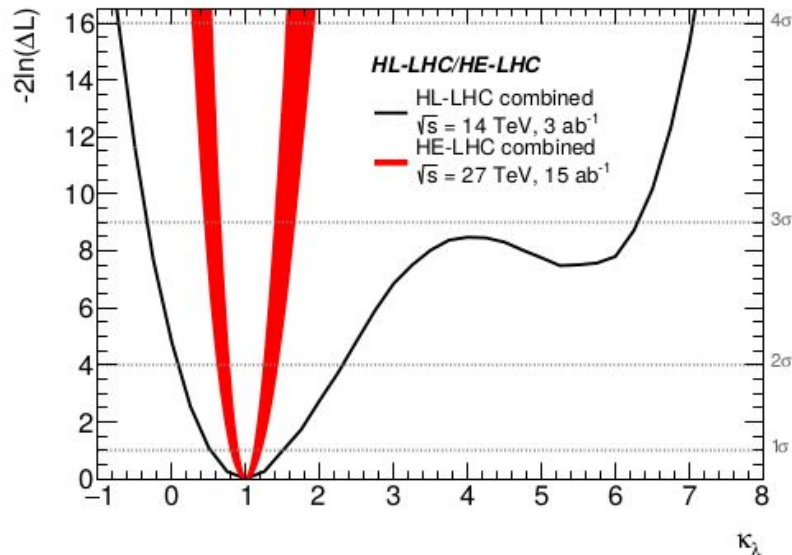


## 3.2) Constraints from future pp colliders

**Table 23**

Expected precision on the direct Higgs self-coupling measurement at future 27 and 100 TeV  $p - p$  colliders.

	HE-LHC (27 TeV, $\mathcal{L} = 15 \text{ ab}^{-1}$ )	FCC-hh (100 TeV, $\mathcal{L} = 30 \text{ ab}^{-1}$ )
$\delta\kappa_\lambda$	10–20%	5–7%

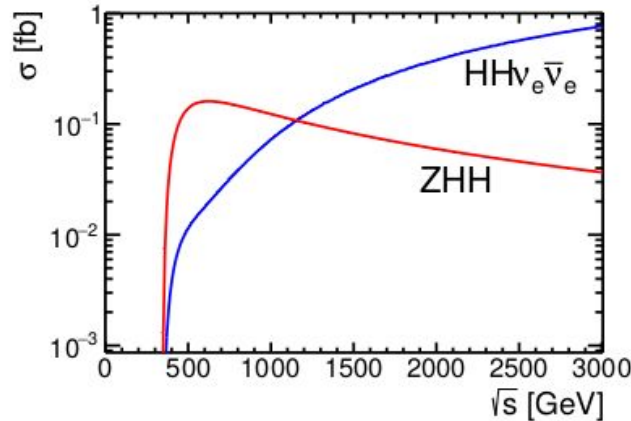


- Higgs self-coupling can be measured with high precision in future pp colliders.

“Higgs Physics at the HL-LHC and HE-LHC”, CERN Yellow Report, <https://arxiv.org/abs/1902.00134>

# 3.3) Constraints from future ee colliders

- The direct HH production in ee colliders require quite high  $\sqrt{s}$



collider	1-parameter	full SMEFT
CEPC 240	18%	-
FCC-ee 240	21%	-
FCC-ee 240/365	21%	44%
FCC-ee (4IP)	15%	27%
ILC 250	36%	-
ILC 250/500	32%	58%
ILC 250/500/1000	29%	52%
CLIC 380	117%	-
CLIC 380/1500	72%	-
CLIC 380/1500/3000	49%	-

- $k\lambda$  can be extracted from EW corrections to single Higgs production in ee colliders.
- Best constraint expected from combination with HL-LHC at least.

# Conclusion

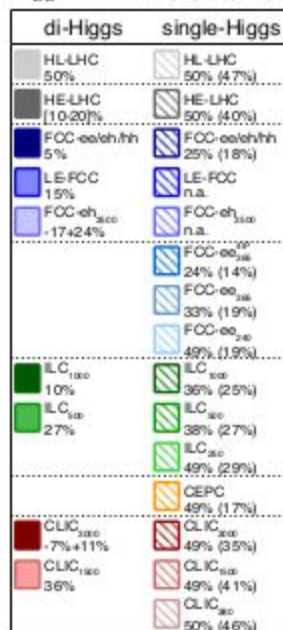
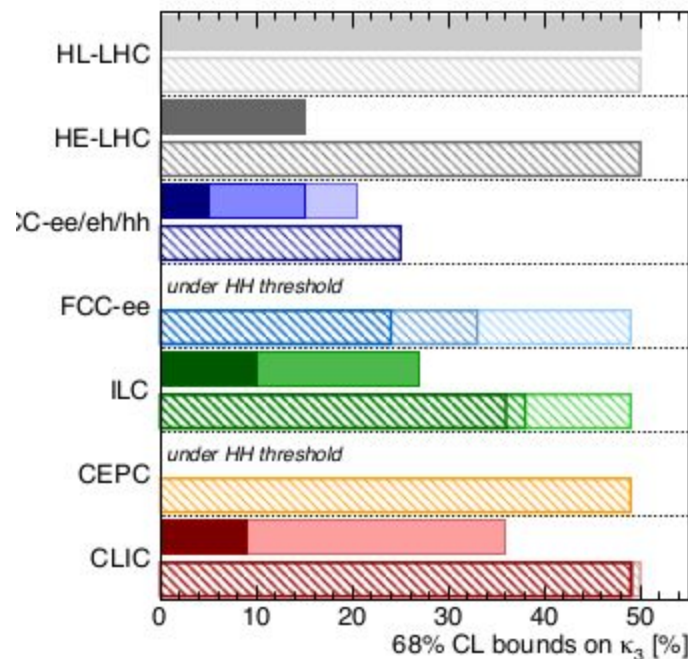
- The LHC program is slowly getting into the BEH potential constraints
- It is a very large scale project O(300) people including experimentalists and theoreticians comparable to the Higgs boson discovery effort.
- BEH potential constraint is the guaranteed and wanted output of any future collider project.





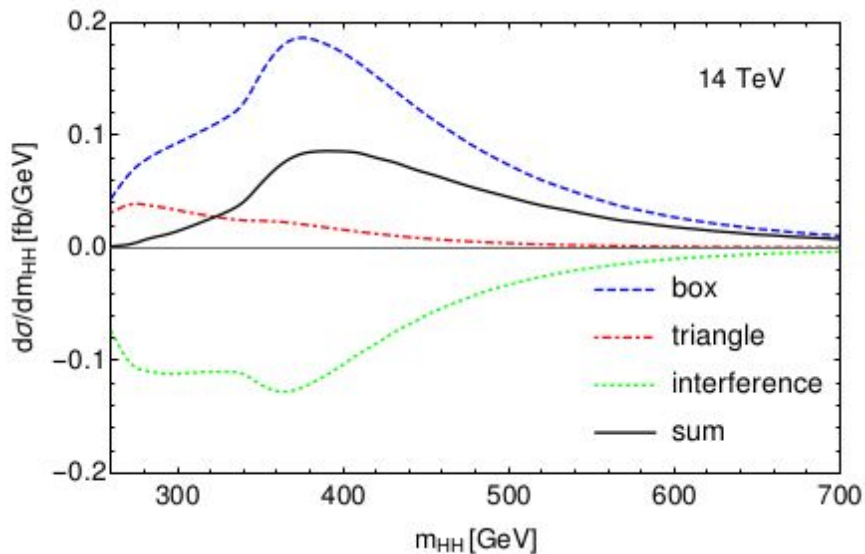
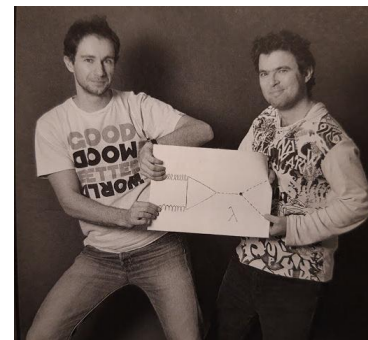
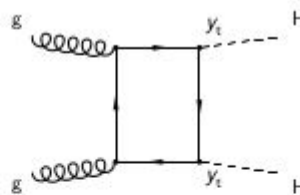
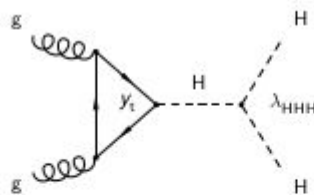
Danielle Monico, 2021

collider	1-parameter	full SMEFT
CEPC 240	18%	-
FCC-ee 240	21%	-
FCC-ee 240/365	21%	44%
FCC-ee (4IP)	15%	27%
ILC 250	36%	-
ILC 250/500	32%	58%
ILC 250/500/1000	29%	52%
CLIC 380	117%	-
CLIC 380/1500	72%	-
CLIC 380/1500/3000	49%	-



All future colliders combined with HL-LHC

# How to distinguish different



- A 13 TeV:  $\sigma_{SM}(HH) \sim 31$  fb
  - 1000 fois moins que  $\sigma_{SM}(H)$ !

$$\sigma_{hh} = \sigma_T + \sigma_B + 2 \cos(\alpha_I) \sqrt{\sigma_T \sigma_B}$$

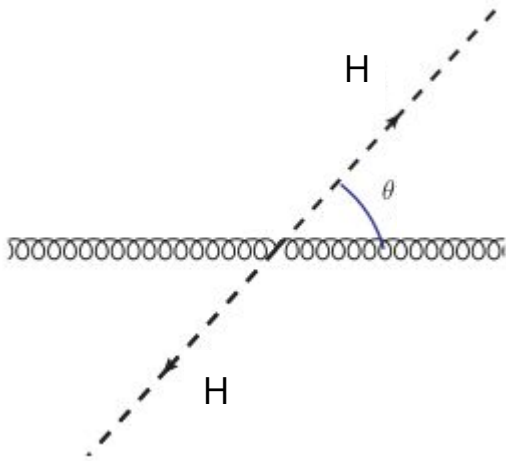
avec  $\cos \alpha_1 \sim -0.9$

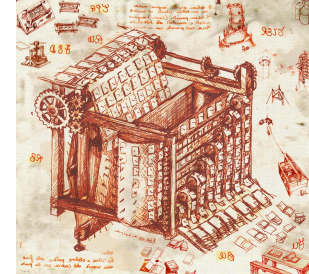
- Très forte dépendance de  $m_{HH} = \sqrt{\hat{s}}$

# Considérations cinématiques simples



- Etat final gg  $\rightarrow$  HH décrit par 8 variables:
  - 2 Higgs sur couche de masse.
  - 2 lois de conservation  $p_{Tx}$ ,  $p_{Ty}$ .
  - 1 Invariance en  $\Phi$
  - $P_z HH \rightarrow$  caractéristique des PDFs
  - 2 Inv. de Lorentz:
    - $\cos \theta_{HH}^*$   $\rightarrow$  La production HH est dominée par la S-wave  $\rightarrow$  uniforme en  $\cos \theta_{HH}^*$
    - $m_{HH} \rightarrow \hat{s}$



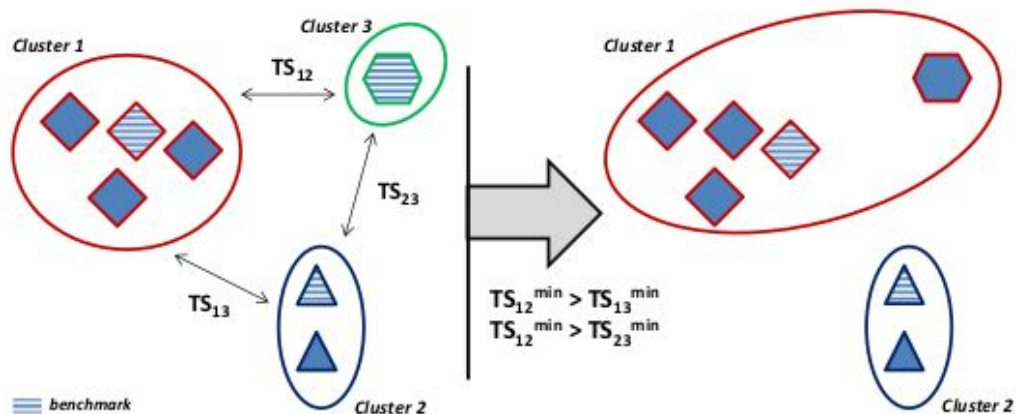


# Analyse en cluster

- Généré 1507 points qui maillent l'espace en LO
- Construit une Statistique des test qui sert de "distance" sur l'espace des spectres  $\sigma = f(m_{HH}, \cos \theta^*)$

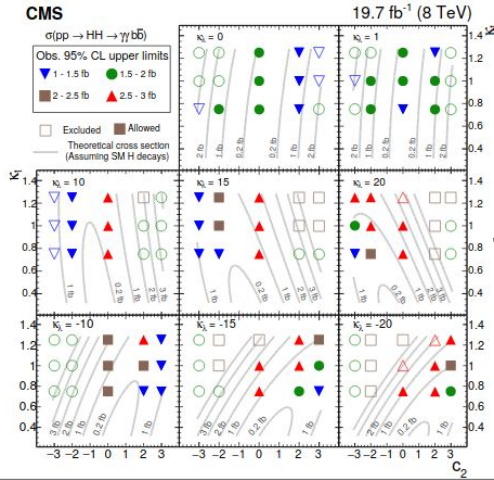
$$TS = 2 \log \left( \frac{L}{L_S} \right) = -2 \sum_{i=1}^{N_{bins}} \left[ \log(n_{i,1}!) + \log(n_{i,2}!) - 2 \log \left( \frac{n_{i,1} + n_{i,2}}{2}! \right) \right]$$

- Tourné un algorithme de partition de l'espace en un nombre prédéfini de clusters

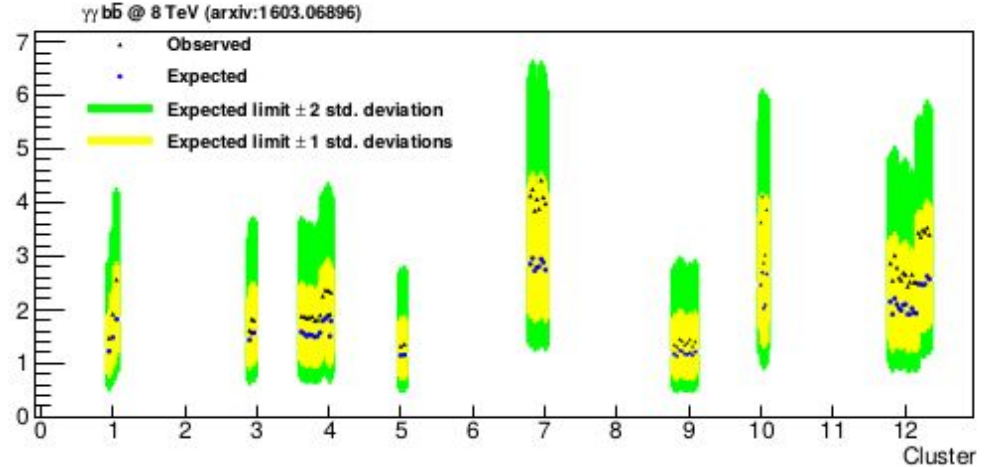




# Preuve du principe



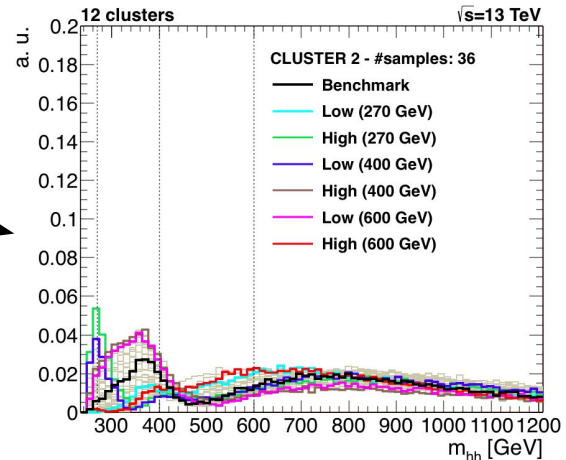
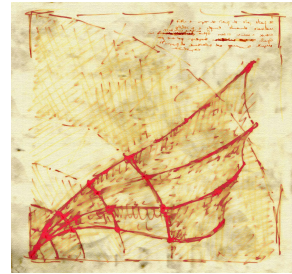
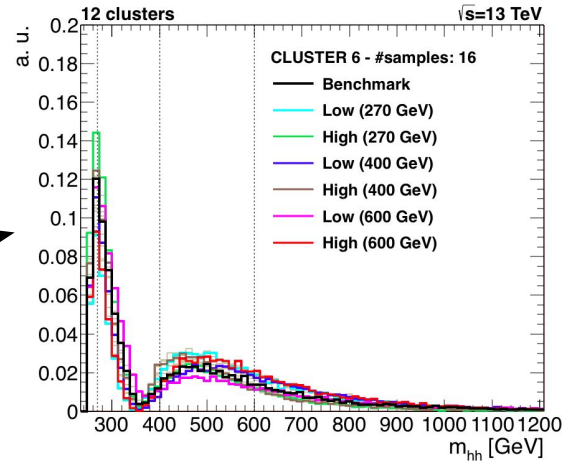
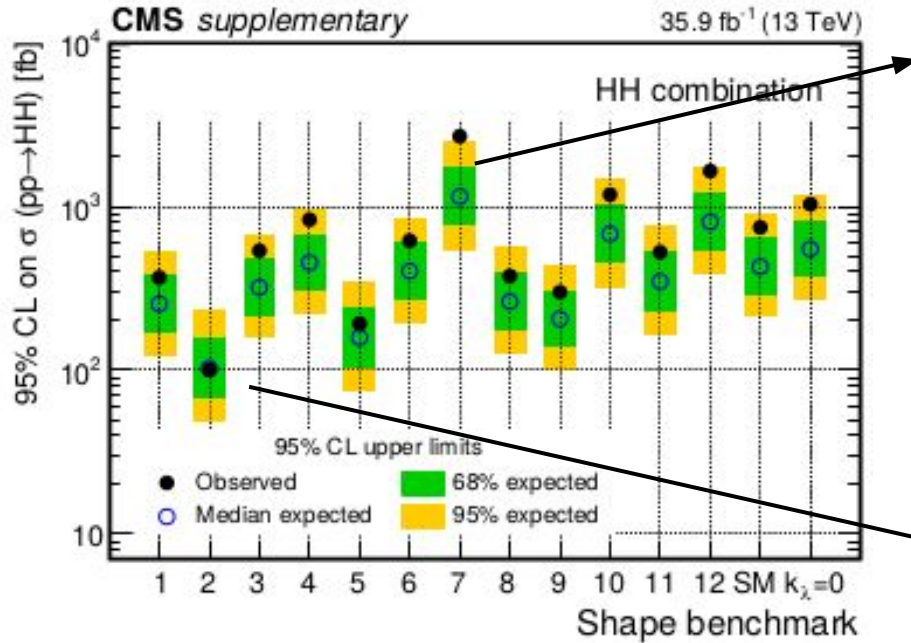
$\sigma(pp \rightarrow HH) \times B(HH \rightarrow \gamma\gamma b\bar{b})$  (fb)



PRD 94, 052012 (2016)

- Preuve expérimentale avec 120 limites obtenus avec de vrai échantillons générés.
- Puis répartis en 12 clusters  
Variance intra clusters ( $\sim 20\%$ )  $\ll$  Variance extra-cluster (Fisher-Snedecor)
- La limite entre clusters peut varier d'un facteur 3!!!

# Utilisation 1/2



- Repérage d'excès possibles