## Models of partial compositeness in four dimensions

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- Pioneering papers:
- Composite Higgs: D. B. Kaplan and H. Georgi, Phys. Lett. B 136 (1984) 183.
- Partial compositeness: D. B. Kaplan, Nucl. Phys. B 365 (1991) 259.
- Our work: (trying to combine the two) 2203.07270, 2202.00037, 2106.12615, 1710.11142, 1610.06591, 1604.06467, 1404.7137, 1312.5330 with various combinations of: A. Banerjee, A. Belyaev, D. B. Franzosi, G. Cacciapaglia, H. Cai, X. Cid Vidal, A. Deandrea, G. Ferretti, T. Flacke, B. Fuks, D. Karateev, M. Kunkel, L. Panizzi, A. Parolini, W. Porod, H. Serodio, C. Vázquez Sierra


## The story so far...

The Higgs boson is ten years old and looking very Standard Model-like.

Still, some of us think there are new phenomena not far above the electro-weak scale.

The reason for this is the fact that the Higgs mass is not "natural" and I will unapologetically embrace this argument.
In this spirit, I will discuss some ideas on compositeness concentrating on 4D models in which the Higgs is realized as a (pseudo) Nambu-Goldstone boson and (at least) the top is partially composite.

I usually joke saying that this idea is so old that it appears new, but there are new ingredients, because we need to take into account the constraints coming from Higgs physics and precision tests.

So, what's the idea?
The idea is to start with the Higgsless (thus massless) Standard Model

$$
\mathcal{L}_{\mathrm{SM} 0}=-\frac{1}{4} \sum_{F=G W B} F_{\mu \nu}^{2}+i \sum_{\psi=Q u d L e} \bar{\psi} \not \supset \psi
$$

with gauge group $G_{S M}=S U(3) \times S U(2) \times U(1)$ and couple it to a theory $\mathcal{L}_{\text {comp. }}$ with hypercolor gauge group $G_{\mathrm{HC}}$ and global symmetry structure $G_{\mathrm{F}} \rightarrow H_{\mathrm{F}}$ such that $h \in G_{\mathrm{F}} / H_{\mathrm{F}}$ and

$$
\mathcal{L}_{\text {comp. }}+\mathcal{L}_{\text {SM } 0}+\mathcal{L}_{\text {int. }} \quad \longrightarrow \quad \mathcal{L}_{\text {SM }}+\cdots
$$

( $\mathcal{L}_{\mathrm{SM}}+\cdots$ is the full SM plus possibly light extra matter from bound states of $\mathcal{L}_{\text {comp. }}$.)

Since we are taking the ultra-conservative approach "Give me Naturalness or give me Death (a.k.a. Landscape)", in the construction of $\mathcal{L}_{\text {comp. }}$ we are only going to use fermions (collectively denoted by $\lambda$ ) and hypercolor gauge fields (field strength $G$ ). Light scalars need to be realized as pNGBs.

Dropping all gauge, spinor, chirality and Lorentz indices:

|  | $G_{\mathrm{SM}}(F)$ | $G_{\mathrm{HC}}(G)$ |
| :--- | :---: | :---: |
| $f=(l, q)(\mathrm{SM})$ | $R_{\mathrm{SM}}$ |  |
| $\lambda=(\psi, \chi, \cdots)(\mathrm{BSM})$ | $R_{1}$ | $R_{2}$ |

The renormalizable part of the full Lagrangian is completely fixed by gauge invariance. (There can be mass terms for $\lambda \mathrm{s}$, if allowed.) Let me just flash for completeness all the $4<\operatorname{dim} \leq 6$ terms.

- vectors only, dim=6: $F^{3}, F^{2} G, F G^{2}, G^{3}$
- fermions only, dim=6: $f^{4}, f^{3} \lambda, f^{2} \lambda^{2}, f \lambda^{3}, \lambda^{4}$
- mixed, dim=5: fFFf, $\lambda F f, f G f, \lambda F \lambda, \lambda G f, \lambda G \lambda$
- mixed, dim=6: fFDf, $\lambda F Đ f, f G Đ f, \lambda F D \lambda, \lambda G D f, \lambda G D \lambda$

1. Potentially troublesome $\operatorname{dim}=6 \mathrm{SM}$ terms $f^{4}$, $(f F D f)$
2. Fully BSM term $\lambda^{4}$ could have destabilizing effects on the pNGB masses.
3. Terms $f^{2} \lambda^{2}, f \lambda^{3}$ which are what the doctor ordered to implement the composite Higgs+partial compositeness mechanism.

We'll come back to 1 . and 2 .

As for $f^{2} \lambda^{2}, f \lambda^{3}$, going to the IR:
$\lambda^{3} \rightarrow a \Lambda_{\mathrm{IR}}^{3} \Psi$ and $\lambda^{2} \rightarrow a^{\prime} \Lambda_{\mathrm{IR}}^{2} H$, where $a \Lambda_{\mathrm{IR}}^{3}$ and $a^{\prime} \Lambda_{\mathrm{IR}}^{2}$ are the overlap probability densities $|\psi(0)|^{2}$ of the hyperfermions inside the composite fermions/bosons. $\Psi$ and $H$ are the interpolating fields.

To fix the ideas:

- $\Lambda_{\mathrm{IR}} \approx 4 \pi f_{h} \approx 10 \mathrm{TeV}$ the scale at which the theory confines. ( $f_{h}$ is the Higgs pNGB "decay constant" that cannot be much smaller than 1 TeV in order to satisfy EWPT.)
- The four-fermi terms $f^{2} \lambda^{2}, f \lambda^{3}$ are generated by integrating out some mediator field at a scale $\Lambda_{\mathrm{UV}} \gg \Lambda_{\mathrm{IR}}$ and the theory is "assumed" to be nearly conformal in that range.

We can put more structure on these models by asking how the $f^{4}$ terms are generated.

- B or L violating terms $f^{4}=q q q l$ require a scale $>10^{12} \mathrm{TeV}$ so they must be protected by an (accidental!) symmetry of the full theory (but this is easily done).
- FCNC or CP violating terms $f^{4}=q q q q$ require a scale $>10^{3} \mathrm{TeV}$ so there is some room to play.

Say the mediator mass is $M=\Lambda_{\mathrm{UV}}$ and denote the UV couplings generically by $c_{\mathrm{UV}}$, not indicating the flavor indices


So the parameters that control the IR physics are of the type
$\kappa=a c_{\mathrm{UV}}^{2}\left(\frac{\Lambda_{\mathrm{IR}}}{\Lambda_{\mathrm{UV}}}\right)^{2-\gamma_{\Psi}}$ (pre-Yukawa terms)


$$
\approx \frac{\kappa_{i} \kappa_{j} \kappa_{l} \kappa_{m}}{\Lambda_{\mathrm{IR}}^{2}}<1 /\left(10^{3} \mathrm{TeV}\right)^{2}
$$

Note that the similar term generated in the UV by integrating out the mediators is quite small


$$
: \frac{c_{\mathrm{UV}}^{4}}{16 \pi^{2} \Lambda_{\mathrm{UV}}^{2}}
$$

The real issue is to get $\kappa_{t}=a_{t} c_{\mathrm{UV} t}^{2}\left(\frac{\Lambda_{\mathrm{IR}}}{\Lambda_{\mathrm{UV}}}\right)^{2-\gamma_{\Psi}}$ big enough.

The mediator could be a vector (Hard! requires partial unification between $G_{\mathrm{SM}}$ and $G_{\mathrm{HC}}$ ) or a scalar.

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Now it's time to be more concrete. First I quickly recall an old set of models giving the required d.o.f. in the IR. They require at least two types of BSM fermions $\lambda=\psi, \chi$, one for the Higgs (EW) coset and one more for the (colored) top-partners. Then I present some (unfinished) ideas on how to couple it to the SM in the spirit above.

As far as the EW sector is concerned, the possible minimal custodial cosets of this type are generated by $\left\langle\tilde{\psi}^{i} \psi_{j}\right\rangle$ or $\left\langle\psi^{i} \psi^{j}\right\rangle$ for

| $4(\psi, \tilde{\psi}) \in$ Complex irrep | $S U(4) \times S U(4)^{\prime} / S U(4)_{D}$ |
| :---: | :---: |
| $4 \psi \in$ Pseudoreal irrep | $S U(4) / S p(4)$ |
| $5 \psi \in$ Real irrep | $S U(5) / S O(5)$ |

E.g. $S U(4) / S O(4)$ is not acceptable since the pNGB are only in the symmetric irrep $(\mathbf{3}, \mathbf{3})$ of $S O(4)=S U(2)_{L} \times S U(2)_{R}$ and thus we do not get the Higgs irrep (2, 2).
pNGB content under $S U(2)_{L} \times S U(2)_{R}$ :

- Ad of $S U(4)_{D} \rightarrow(\mathbf{3}, \mathbf{1})+(\mathbf{1}, \mathbf{3})+2 \times(\mathbf{2}, \mathbf{2})+(\mathbf{1}, \mathbf{1})$
- $\mathbf{A}_{2}$ of $\operatorname{Sp}(4) \rightarrow(\mathbf{2}, \mathbf{2})+(\mathbf{1}, \mathbf{1})$
$-\mathbf{S}_{2}$ of $S O(5) \rightarrow(\mathbf{3}, \mathbf{3})+(\mathbf{2}, \mathbf{2})+(\mathbf{1}, \mathbf{1})$

We also want a top-partner $\Psi$. This requires additional hyper-fermions $\chi$ carrying color, schematically $\Psi \approx \psi \chi \psi$ or $\chi \psi \chi$.
Since we have introduced a new set of hyper-fermions, we also need to embed the color group $S U(3)_{c}$ into the unbroken global symmetry of $\mathcal{L}_{\text {comp. }}$.
The choices of minimal field content allowing an anomaly-free embedding of unbroken $S U(3)_{c}$ are

| $3(\chi, \tilde{\chi}) \in$ Complex irrep | $S U(3) \times S U(3)^{\prime} \rightarrow S U(3)_{D} \equiv S U(3)_{c}$ |
| :---: | :---: |
| $6 \chi \in$ Pseudoreal irrep | $S U(6) \rightarrow S p(6) \supset S U(3)_{c}$ |
| $6 \chi \in$ Real irrep | $S U(6) \rightarrow S O(6) \supset S U(3)_{c}$ |

We narrowed it down to a list of twelve models likely to be outside the conformal window but with still enough matter to realize the mechanism of partial compositeness: [1604.06467,1610.06591]

| $G_{\mathrm{HC}}$ | $\psi$ | $\chi$ | $G_{\mathrm{F}} / H_{\mathrm{F}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{SO}(7)$ | $5 \times \mathbf{F}$ | $6 \times$ Spin | $\frac{S U(5)}{S O(5)} \frac{S U(6)}{S O(6)} U(1)$ |
| $\mathrm{SO}(9)$ | $5 \times \mathbf{F}$ | $6 \times$ Spin |  |
| $S O(7)$ | $5 \times$ Spin | $6 \times \mathbf{F}$ |  |
| $S O(9)$ | $5 \times$ Spin | $6 \times \mathbf{F}$ |  |
| $S p(4)$ | $5 \times \mathbf{A}_{2}$ | $6 \times \mathbf{F}$ | $\frac{S U(5)}{S O(5)} \frac{S U(6)}{S p(6)} U(1)$ |
| $S U(4)$ | $5 \times \mathbf{A}_{2}$ | $3 \times(\mathbf{F}, \overline{\mathbf{F}})$ | $\frac{S U(5)}{S O(5)} \frac{S U(3) \times S U(3)^{\prime}}{S U(3)_{D}} U(1)$ |
| $S O(10)$ | $5 \times \mathbf{F}$ | $3 \times($ Spin, $\overline{\text { Spin }})$ |  |
| $S p(4)$ | $4 \times \mathbf{F}$ | $6 \times \mathbf{A}_{2}$ | $\frac{S U(4)}{S p(4)} \frac{S U(6)}{S O(6)} U(1)$ |
| $S O(11)$ | $4 \times$ Spin | $6 \times \mathbf{F}$ |  |
| $S O(10)$ | $4 \times($ Spin, $\overline{\text { Spin }})$ | $6 \times \mathbf{F}$ | $\frac{S U(4) \times S U(4)^{\prime}}{S U(4)_{D}} \frac{S U(6)}{S O(6)} U(1)$ |
| $S U(4)$ | $4 \times(\mathbf{F}, \overline{\mathbf{F}})$ | $6 \times \mathbf{A}_{2}$ |  |
| $S U(5)$ | $4 \times(\mathbf{F}, \overline{\mathbf{F}})$ | $3 \times\left(\mathbf{A}_{2}, \overline{\mathbf{A}}_{2}\right)$ | $\frac{S U(4) \times S U(4)^{\prime}}{S U(4)_{D}} \frac{S U(3) \times S U(3)^{\prime}}{S U(3)_{D}} U(1)$ |

In the search for a possible scalar mediator $\Phi$ we take $\Phi$ to transform under a full irrep of $\left(G_{\mathrm{HC}}, G_{\psi}, G_{\chi}\right)$.
$E . g$. for M8 [1311.6562], $G H C=\operatorname{Sp}(4)$ with $4 \psi \in \mathbf{F}$, and $6 \chi \in \mathbf{A d}_{2}$ : $\left(G_{\mathrm{HC}}, G_{\psi}, G_{\chi}\right)=(S p(4), S U(4), S U(6))$.
With SM fields $q^{f}, u^{f}, d^{f}$ spurions for $q_{L}^{f}, u_{R}^{(c) f}, d_{R}^{(c) f}$ (LH Weyl notation), take $\Phi \in(\mathbf{4}, \mathbf{4}, \mathbf{6})$. (recall that $\left\langle\Phi \Phi^{\dagger}\right\rangle \neq 0$ ).

$$
\mathcal{L}_{\mathrm{ppY}}=c_{(q)}^{f_{i}} q^{f} \Phi^{i} \psi+c_{(u)}^{f i} u^{f} \Phi^{i \dagger} \psi+c_{(d)}^{i} d^{f} \Phi^{i \dagger} \psi+\xi^{i} \Phi^{i \dagger} \psi \chi+\text { h.c. }
$$

NB: Asymptotic freedom is lost above the $\Phi$ mass.

- We get all bilinear terms $c_{(q)}^{f_{i}} c_{(u)}^{f^{\prime} i} q^{f} u^{f^{\prime}} \psi \psi$, with $q^{f} \in(\overline{\mathbf{6}}, \overline{\mathbf{6}}) \ni(\mathbf{2}, \mathbf{2})$ of $\left(G_{\psi}, G_{\chi}\right), u^{f}, d^{f} \in(\mathbf{1 5}, \mathbf{6}) \ni(\mathbf{1}, \mathbf{3})$, custodial. (The Yukawa matrix is $y^{f f^{\prime}} \propto c_{(q)}^{f i} f_{(u)}^{f^{\prime} i}$.
- Since we need all the spurions to have $U(1)_{X}$ charge $=2 / 3$ and because of the $Z b b$ "custodial" symmetry, the acceptable $G_{\psi}$ irreps are those containing the $S U(2)_{L} \times S U(2)_{R}$ irreps:
- $(\mathbf{2}, \mathbf{2})$ for $q_{L}$
- $(\mathbf{1}, \mathbf{1})$ or $(\mathbf{1}, \mathbf{3})$ for $t_{R}^{(c)}$
- $(\mathbf{1}, \mathbf{3})$ for $b_{R}^{(c)}$

This means that all models with partners of the type $\chi \psi \chi$ (M1, M2, M5, M6, M7) require a more complicated mediator sector to give a mass to the bottom quark.

- The " $\lambda^{4}$ " terms are ok, since

$$
\xi^{i} \Phi^{i \dagger} \psi \chi+\text { h.c. } \rightarrow \frac{\xi^{i} \xi^{i *}}{\Lambda_{\mathrm{UV}}^{2}} \psi \chi \psi^{\dagger} \chi^{\dagger}=-\frac{\xi^{i} \xi^{i *}}{2 \Lambda_{\mathrm{UV}}^{2}} \psi^{\dagger} \sigma^{\mu} \psi \chi \bar{\sigma}_{\mu} \chi^{\dagger}
$$

product of two $G_{\mathrm{HC}}$ invariant currents. (No vev and no anomalous dimensions.)

- The generic structure of the top (and bottom) mass matrix ( $\bar{T}_{L} \mathcal{M} T_{R}+$ h.c.) is

$$
\mathcal{M}=\left(\begin{array}{cc}
y^{f f^{\prime}} v & \omega_{L}^{f} \\
\omega_{R}^{f^{\prime}} & M \mathbf{1}
\end{array}\right)
$$

with
$y^{. f f^{\prime}} \propto c_{(q)}^{f_{i}} c^{f^{\prime} i}(u),($ rank $=$ range of $i)$
$\omega_{L}^{f} \propto c_{(q)}^{f} \xi^{i}$, and $\omega_{R}^{f^{\prime}} \propto c_{(u)}^{f^{\prime} i *} \xi^{i}$, (rank one)
Clearly a large number of unknown parameters! (There could even be more than one M1.)

Still, there is some predictivity.

- We can count the number of degenerate partners: $\operatorname{dim}(M 1)-2 \equiv \operatorname{dim}(\mathcal{M})-5$.

$$
\left(\begin{array}{cc}
y v & \omega_{L} \\
\omega_{R} & M \mathbf{1}
\end{array}\right) \rightarrow\left(\begin{array}{ccccc}
\left.\begin{array}{llll}
y^{\prime} v & 0 & 0 & 0
\end{array}\right) & \ldots & \ldots \\
0 & & & x & 0
\end{array}\right)
$$

So we should study a $5 \times 5$ mixing matrix $\left(\right.$| $y^{\prime}$ |  | 0 | 0 |
| :---: | :---: | :---: | :---: |
|  | 0 | 0 |  |
| 0 | 0 | $x^{\prime}$ |  |
| 0 | 0 |  |  |
| 0 | 0 | $x^{\prime \prime}$ | 0 |$)$ (

These are the matrices to use in the study of FCNC, CP, EWPT in these models. (Related work in [2207.14235])

Even the degenerate states are interesting since they are the lightest and get off-diagonal one loop terms in their self energy.


This leads to an interesting quantum interference problem between channels for a pair production process (Early work in [Cacciapaglia, Deandrea, De Curtis 0906.3417].)


$$
\sigma(p p \rightarrow \mathcal{T} \overline{\mathcal{T}} \rightarrow A \bar{B}) \stackrel{\mathrm{NWA}}{=} N_{\mathcal{T}} \sigma(p p \rightarrow \mathcal{T} \overline{\mathcal{T}}) \mathcal{B} \mathcal{R}_{2}(\mathcal{T} \overline{\mathcal{T}} \rightarrow A \bar{B})
$$

Notice that it is not possible to write $\mathcal{B R}_{2}$ as a product of two branching ratios, as one would do if there was no degeneracy. It is however possible to define an effective branching ratio of $\mathcal{T} \rightarrow A$ as

$$
\mathcal{B R}(\mathcal{T} \rightarrow A) \equiv \sum_{\bar{B}} \mathcal{B R}_{2}(\mathcal{T} \overline{\mathcal{T}} \rightarrow A \bar{B})
$$

$$
\text { such that, } \sum_{A} \mathcal{B} \mathcal{R}(\mathcal{T} \rightarrow A)=1
$$

but again $\mathcal{B R}_{2}(\mathcal{T} \overline{\mathcal{T}} \rightarrow A \bar{B}) \neq \mathcal{B R}(\mathcal{T} \rightarrow A) \mathcal{B R}(\overline{\mathcal{T}} \rightarrow \bar{B})$ implying non-trivial correlations between the two final states.

Pheno example: Diphoton signal for a specific model [2202.00037]

M6: $G H C=S U(4)$ with $5 \psi \in \mathbf{A}_{2}$, and $3 \chi \in(\mathbf{F}, \overline{\mathbf{F}})$ : $\left(G_{\mathrm{HC}}, G_{\psi}, G_{\chi}\right)=(S U(4), S U(5), S U(3) \times S U(3))$.


Summing over all channels containing a pNGB decaying to diphoton: $\sigma(p p \rightarrow t / \bar{t} \gamma \gamma+X) \approx$ few fb (depending of course on the benchmark point)
Ongoing search in ATLAS!?

Even more promising is direct production of exotic pNGBs, since fermion partners could very well be inaccessible at LHC. Here the deciding factor is the luminosity since they have a low mass but interact weakly. (No vev.) [Snowmass 2203.07270 ]


## CONCLUSIONS

- Realizing partial compositeness via ordinary 4D gauge theories provides a self contained concrete class of models to address the hierarchy problem.
- There are lots of open questions that go to the heart of strongly coupled theories, such as the range of the conformal window, anomalous dimensions and LEC.
- In spite of the large number of unknown parameters, there is some predictivity, since masses and interaction matrices have structure.
- The search at LHC is not over yet! (I would personally put some money on pNGBs.)


## Thank you for your attention!

