ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Condensed Dark Matter with a Yukawa interaction

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R. Garani, M. Redi and A. Tesi (JHEP 12 (2021) 139, 2105.03429)

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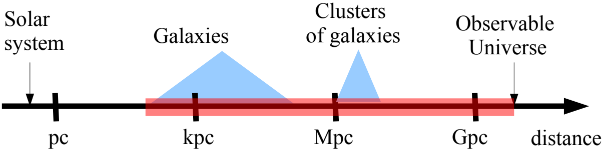
Matter

Dark Matter and visible matter in the Universe



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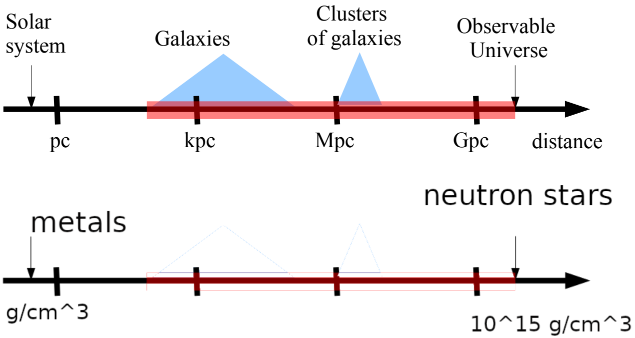

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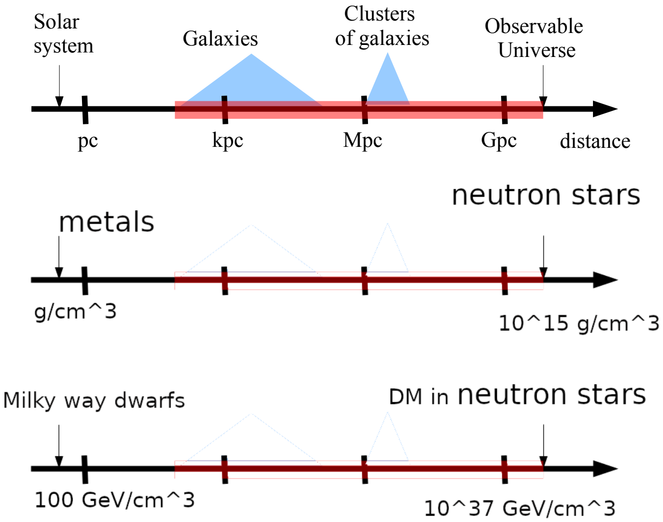


What is new here?

Outline

- Fermion asymmetric DM with yukawa interaction for dark sector. Going beyond non-interacting scenario Domcke & Urbano '14, Randall et al. '16, Gresham &Zurek '18
- Consistent description of in-medium effect
- Delimiting possible phases of the Yukawa theory
- Generalized 'gap equations' and equation of state for arbitrary mediator masses. Note this regime not encountered in the lab.

Phases in the Yukawa theory

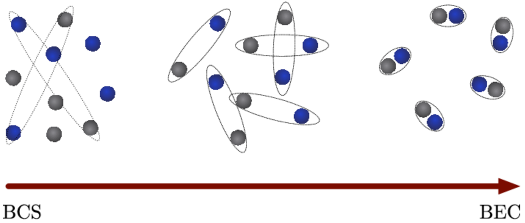
The model

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + \mu\bar{\psi}\gamma^0\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - g\,\bar{\psi}\psi\phi\ .$$

- 4 free parameters: m, m_ϕ, g and the density μ
- Dark particles singlets under SM. The fermion ψ charged under $U(1)_{\text{dark}}$ global
- Fermi energy $E_F = \mu \equiv \sqrt{m^2 + k_F^2}$, number density $n = N/V \equiv \langle\bar{\psi}\gamma_0\psi\rangle$

Phases in the Yukawa theory

Scattering in the Yukawa theory



- The scattering length effectively captures the short distance properties of a potential

$$\lim_{k\to 0} k \cot \delta_0(k) = -\frac{1}{a} \; .$$

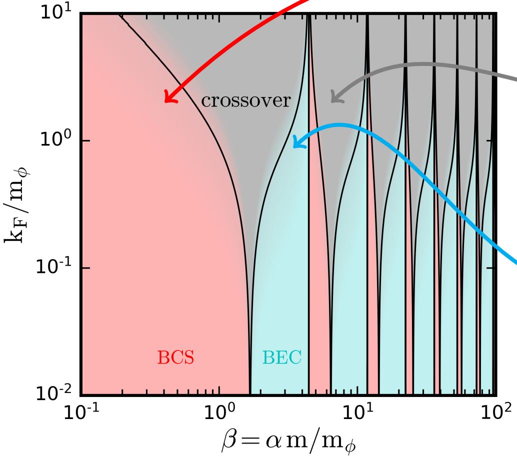
Computable for dilute gases in the non-relativistic limit.

- Anologous to contact interactions in low temperature physics, phases delimitted by dimenionless $k_F a$.

Phases in the Yukawa theory

Phases in the Yukawa theory

RG, M.H.G Tytgat and J. Vandecasteele '22



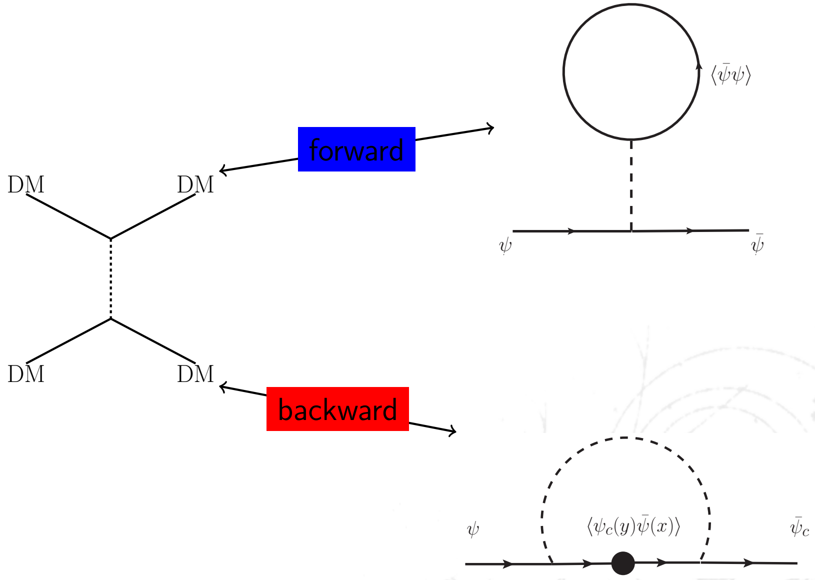
BCS phase: $(k_F\, a)^{-1} <$ -1

Crossover: -1 $\leq (k_F\, a)^{-1} \leq$ 1

BEC phase: $(k_F\, a)^{-1} >$ 1

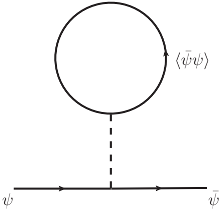
The BCS phase

Cooper pairing and in-medium effects



Full forward scattering

Scalar density condensate

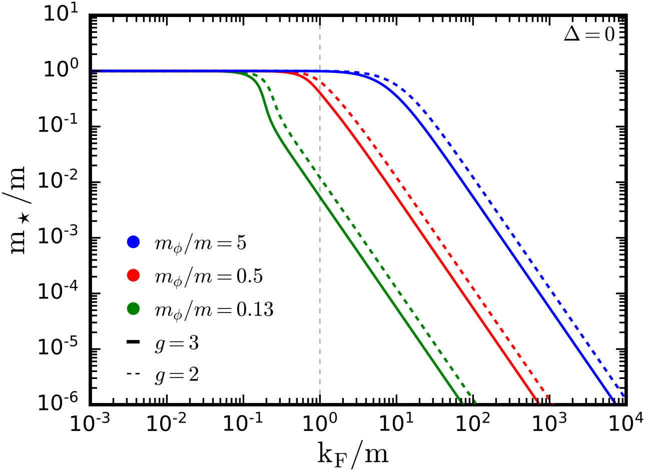


- Tadpole $\neq 0$ when $\mu \neq 0$
- The scalar operator $\bar{\psi}\psi$ has a non-zero mean, $n_s = \langle\bar{\psi}\psi\rangle > 0$ Waleck '74, Gresham et al. '18. $\implies n_s$ sources the scalar field due to its Yukawa interactions with the fermions
    - $\frac{\delta\mathcal{L}}{\delta\phi} = 0 \rightarrow m_\phi^2\langle\phi\rangle + g\langle\bar{\psi}\psi\rangle = 0$
    - $m_* = m + g\langle\phi\rangle \rightarrow m_* = m - \frac{g^2}{m_\phi^2} n_s(m_*)$

$\implies$ the fermion mass is reduced in the medium! (similar to NJL model of chiral symmetry breaking)

Full forward scattering

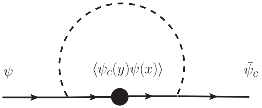
Results for scalar density condensate RG, M.H.G Tytgat and J. Vandecasteele '22



Full backward scattering

Cooper pairing and superfluidity: BCS argument

- Free enrgy for N particles $\Omega_N = E - \mu N$
- Add a particle $\implies \Omega_{N+1} = E_{+1} - \mu\,(N+1)$
- If attractive interactions $\Omega_{N+1} < \Omega_N$
- Formation of many bosonic Cooper pairs which condensate $\sim \langle \psi\psi \rangle$ (Leon Cooper '57). Pairing in ^1S channel.
    - Object that gets a vev $\sim \left\langle \psi_C(y)\bar{\psi}(x) \right\rangle$, a 4×4 quantity



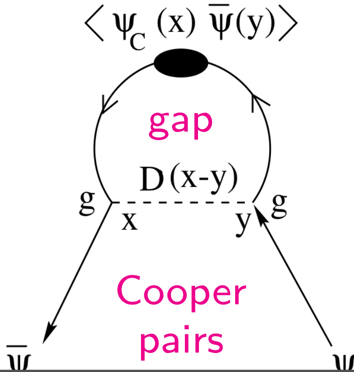
Full backward scattering

Qualitative physics

Yukawa theory when $m_\phi \gg m$: $4-$fermion interaction Schmitt '14

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \gamma^0\mu - m)\psi + G_\phi \bar{\psi}\bar{\psi}\psi\psi$$

$$\approx \begin{pmatrix}\bar{\psi} & \bar{\psi}_C\end{pmatrix} \begin{pmatrix} k\!\!\!/ + \mu\gamma^0 - m & \left\langle \psi\bar{\psi}_C \right\rangle \times G_\phi \\ \left\langle \psi_C\bar{\psi} \right\rangle \times G_\phi & k\!\!\!/ - \mu\gamma^0 - m \end{pmatrix} \begin{pmatrix}\psi \\ \psi_C\end{pmatrix}$$



The consistent set of gap equation

Gap structure and dsipersion RG, M.H.G Tytgat and J. Vandecasteele '22

Δ has fermionic indices, respects Fermi statistics

$$\Delta_{\alpha\beta} \equiv \left\langle \psi_{C,\alpha}(x)\, \bar{\psi}_{\beta}(y) \right\rangle$$

Ansatz for the Yukawa theory Pisarski and Rischke '99

$$\Delta = \Delta_1 \gamma_5 + \Delta_2 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{k}} \gamma_0 \gamma_5 + \Delta_3 \gamma_0 \gamma_5$$

$$\mathcal{L} = \begin{pmatrix} \bar{\psi} & \bar{\psi}_C \end{pmatrix} \underbrace{\begin{pmatrix} \not{k} + \mu\gamma^0 - m & \Delta(k) \\ \Delta(k) & \not{k} - \mu\gamma^0 - m \end{pmatrix}}_{\text{inverse propagator}} \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$$

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In BCS, $\Delta \ll \mu$

$$\epsilon_\pm^2 \approx (\omega \pm \mu)^2 + \left(\Delta_1 \pm \left(\frac{k}{\omega}\Delta_2 + \frac{m}{\omega}\Delta_3\right)\right)^2$$

Like standard BCS theory but non-trivial momentum dependence

The consistent set of gap equation

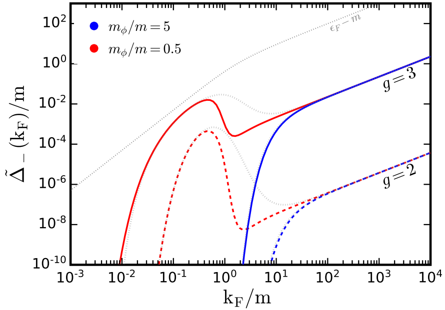
$$\Sigma(0) = \frac{-g^2}{m_\phi^2}\sum_\eta \int \frac{d^3k}{(2\pi)^3}\left\{\frac{m_*}{\omega_k}\left(\frac{\omega_k+\eta\mu}{\epsilon_\eta(k)}-1\right)-\eta\frac{k}{\omega_k}\frac{\tilde{\kappa}(k)}{\omega_k}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}\right\},$$

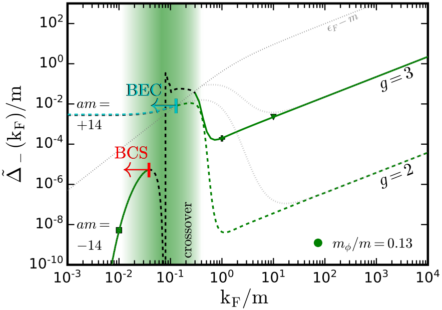
$$\tilde{\Delta}_\pm(p) = \frac{g^2}{32\pi^2}\sum_\eta \int_0^\infty dk\frac{k}{p}\left\{\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\mp\eta\right.$$
$$\frac{kp}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)$$
$$\left.\pm\eta\frac{m_*^2}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)},$$

$$\tilde{\kappa}(p) = \frac{g^2}{32\pi^2}\sum_\eta \int_0^\infty dk\frac{k}{p}\left\{-\eta\frac{m_*k}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)\right.$$
$$\left.-\eta\frac{m_*p}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}.$$

The consistent set of gap equation

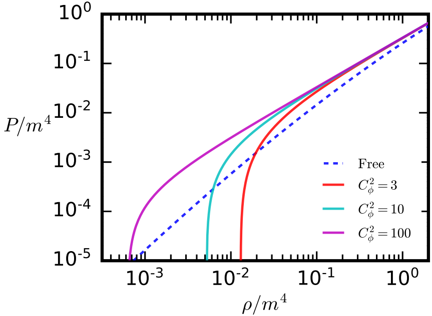
Solution to gap equations

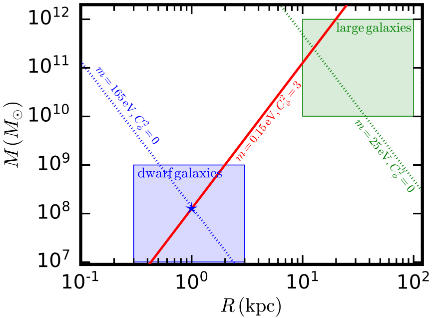




Equation of state

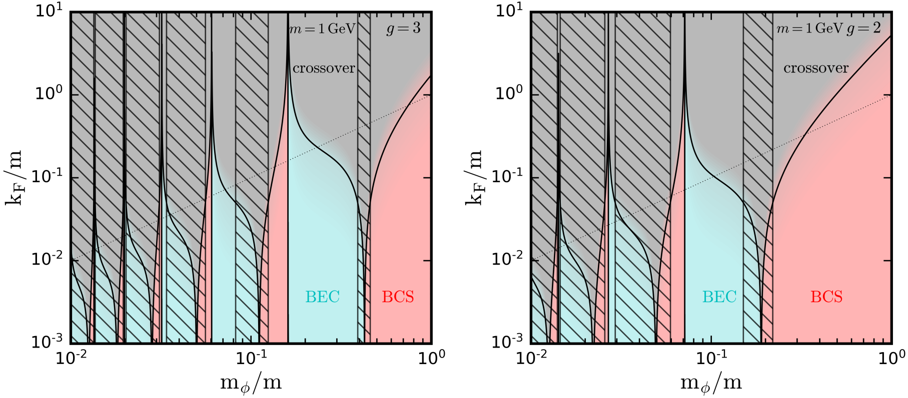
Application to Halos





Bullet cluster constraints

$\sigma/m \approx$ barn/GeV



A cosmological dark-QCD model

The model

- Can dark matter (DM) be a baryon/pion of new confining dark sectors? $\Longrightarrow$ composite DM Bai, Hill '10 + Boddy et.al. '14 +Gresham, Lou, Zurek '17 + Bai, Long, Lu '18 + many more

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- Cosmologically accidentally stable, like protons $\Longrightarrow$ dark baryon number Antipin et.al. '15 + Niel et.al. '16 + Mitradate et.al '17 + Contino et.al. '18 + Redi et.al '18

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- Here we focus on $SU(3)$

$$\int d^4x\,\sqrt{-g}\left[\mathcal{L}_{\rm SM}-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}+\bar{\psi}_i\left(\not{D}-m_i\right)\psi_i+\sum\frac{\mathcal{O}_{\rm SM}\mathcal{O}_{\rm dark}}{M_{\rm pl}^{\#}}\right].$$

with possibility of both dark-pion and -baryon DM!

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- Global chiral symmetry SU(N_F)$\times$ SU(N_F) $\rightarrow$ SU(N_F)

$$\mathcal{L}_\pi = \frac{f^2}{4}\mathrm{Tr}(\partial_\mu U)^2 + b\mathrm{Tr}[MU + h.c.] + \mathrm{WZW}, \qquad U = \exp[i\pi/f]$$

and $M_{ij} = m_i\delta_{ij}$. Resulting in $N_F^2 - 1$ goldstone bosons in the adjoint

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- Stability: not absolute. Violated by

$$\frac{1}{\Lambda_5}\bar{\Psi}^i\gamma^5\Psi^j|H|^2 + \frac{1}{\Lambda_6^2}\bar{\Psi}^i\gamma^\mu\gamma^5\Psi^j\bar{f}\sigma^\mu f\,.$$

$\langle 0|\bar{\Psi}\gamma^5\Psi|\pi\rangle = c\,4\pi f^2 \implies$ mixing with higgs $\frac{4\pi f^2}{\Lambda_5}|H^2|\pi$

Is emergent phenomenon realized in dark-QCD models?

Yes

- Much richer structure than the Yukawa theory

$$g\bar{\psi}\psi\phi \to g\bar{\psi}_\alpha\gamma^\mu T_a^{\alpha\beta}\psi_\beta A_\mu^a$$

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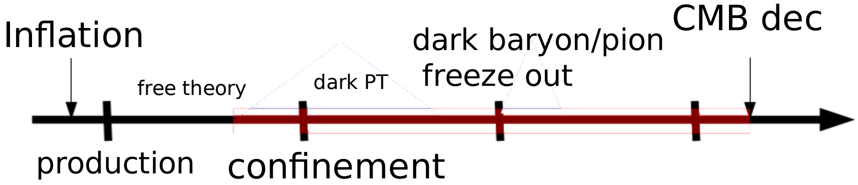
- Wavefunction has to be overall antisymmetric
- Need to be also antisymmetric in flavour ✓

Dark-QCD model

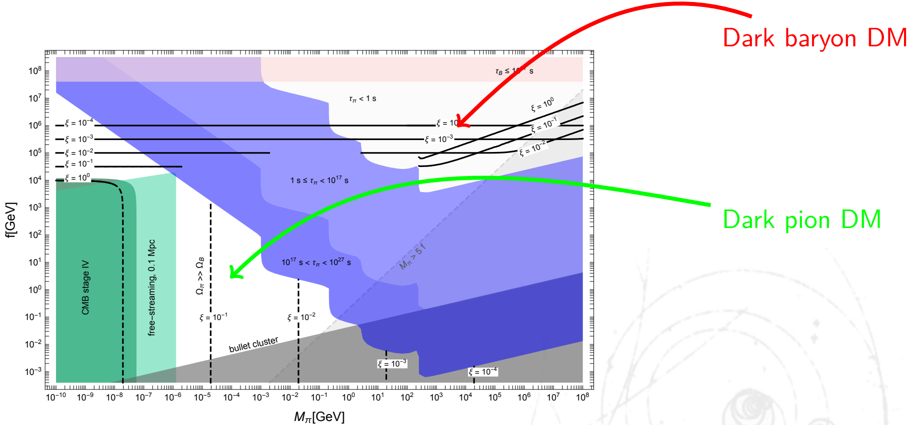
Cosmology



Dark-QCD model

Results

RG, M. Redi, A. Tesi '21



Conclusions and Outlook

- Emergent phenomena can be realized in dark sectors due to DM−DM interactions.
- Many interesting phenomena arise with very little ingredients.
- Using scattering length we have delimitted phases of Yukawa theory.
- Very general framework to describe superfluidity, motivated by DM phenomenology. For arbitrary mediator masses all the way from non-relativistic limit to relativistic limit.
- We are at the crossroad of many areas in physics.
- Construct EoS that corectly interpolates between condensate dominated high density regions and low density Maxwellian regimes $\rightarrow$ realistic description of DM halos at dwarf galaxy scales.