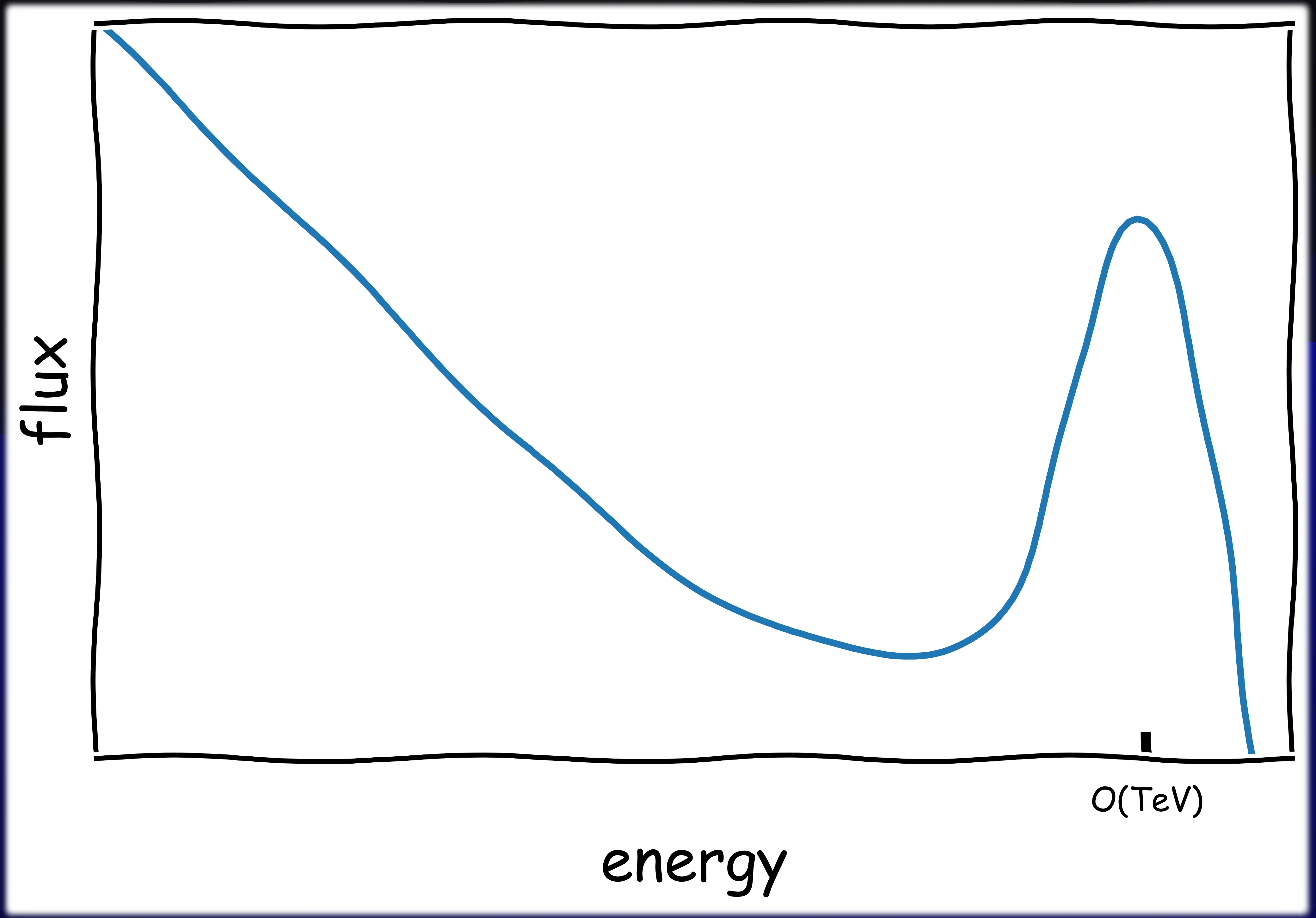
Resummation of large electroweak terms for indirect Dark Matter detection

LFC22: Strong interactions from QCD to new strong dynamics at LHC and Future Colliders. Trento

Martin Vollmann — Uni Tuebingen

The Problem

Gamma rays signals from dark matter in the center of the Milky Way



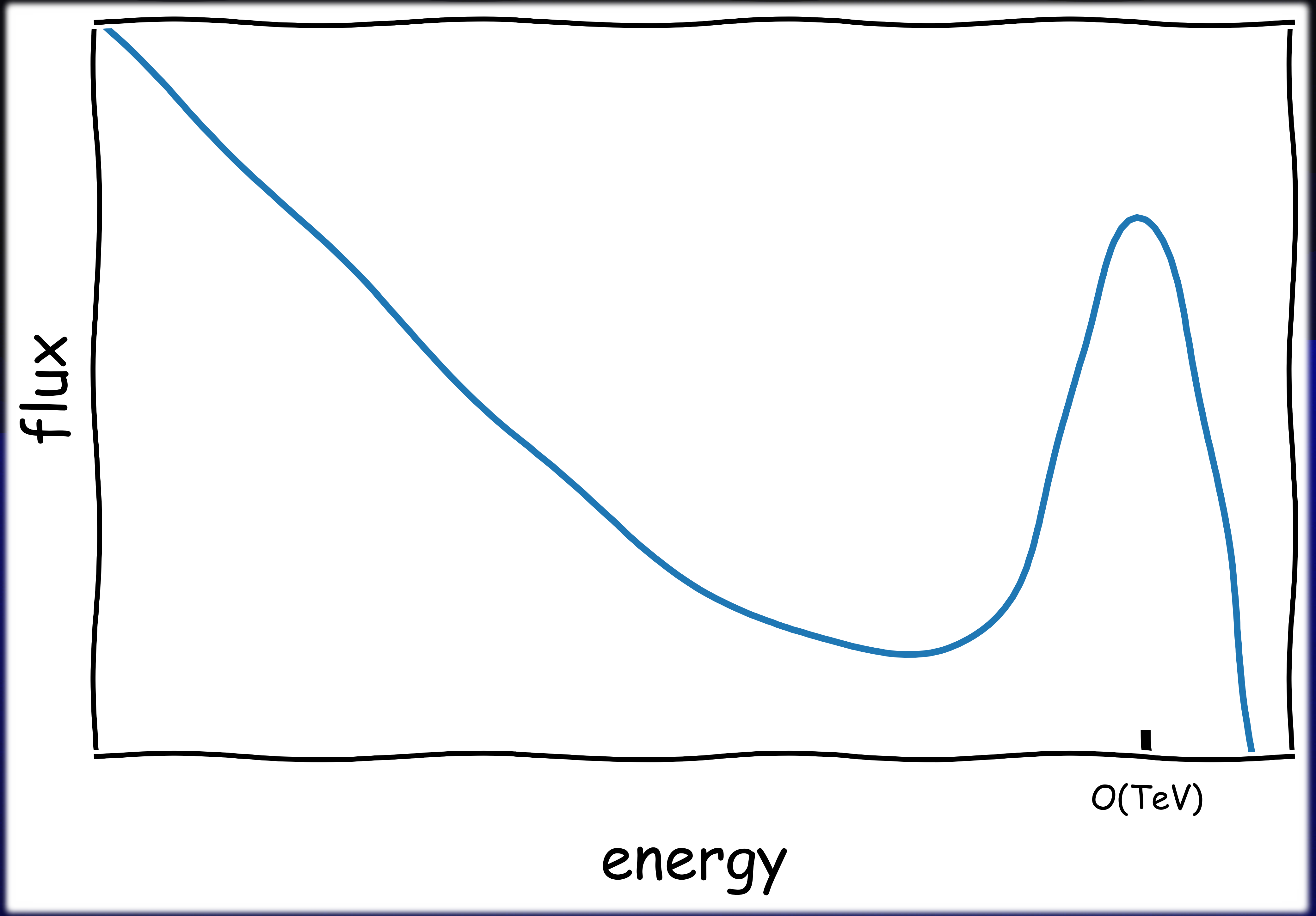
The problem

- “*Bumpy*” endpoint (spectral line)

  Smoking gun

- Non-trivial theoretical prediction

  (Resummable) higher-order effects



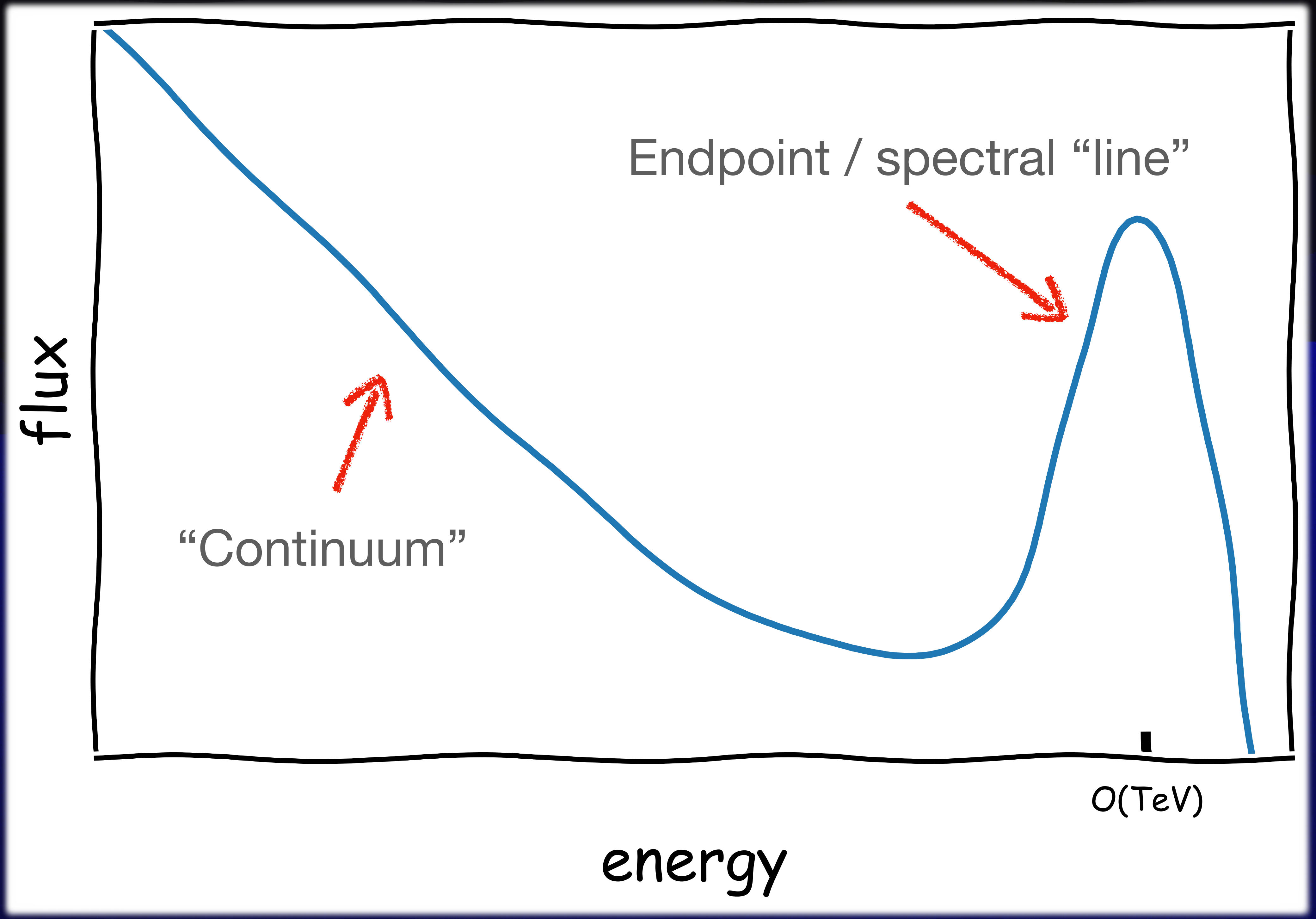
The solution

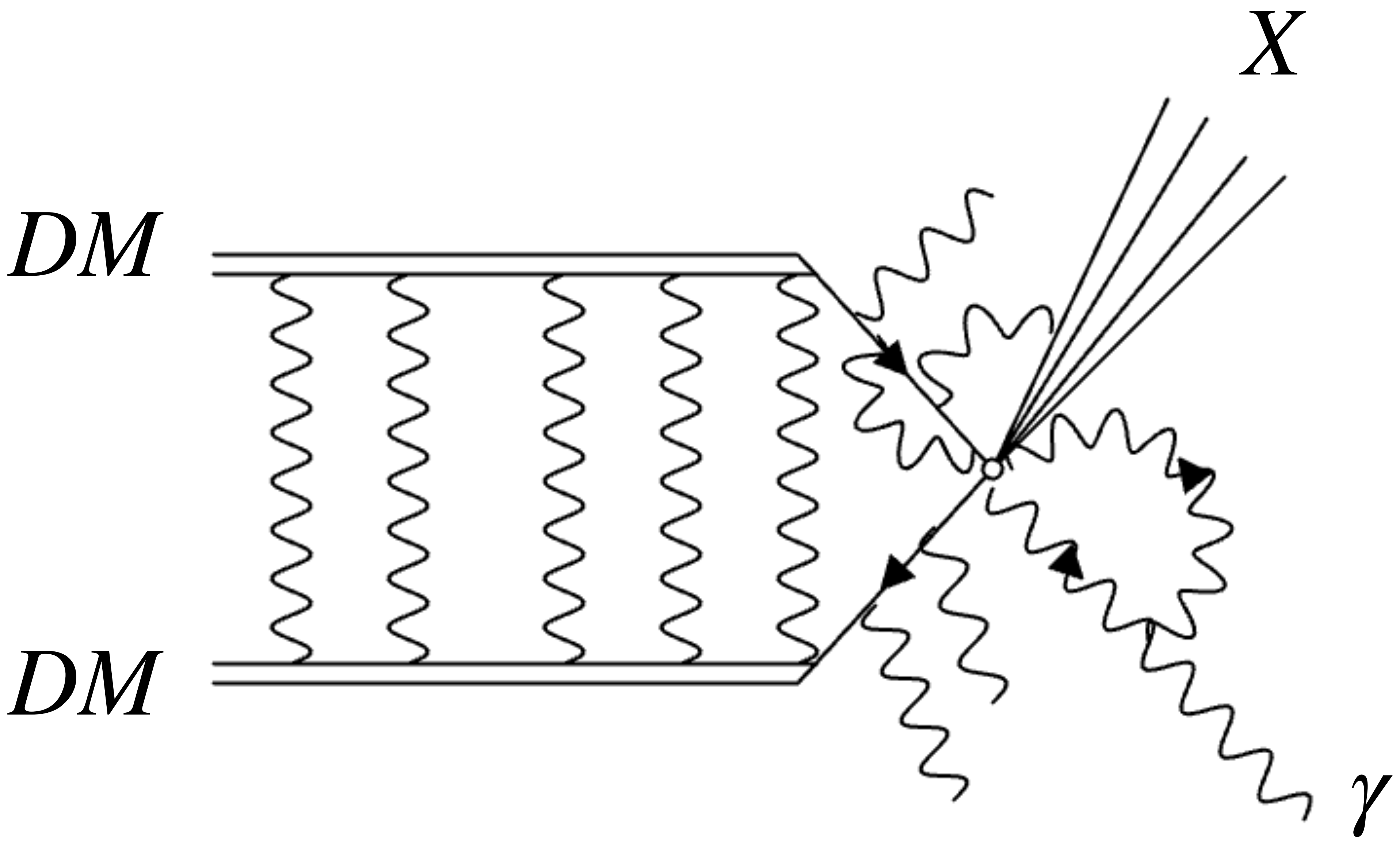
- Continuum

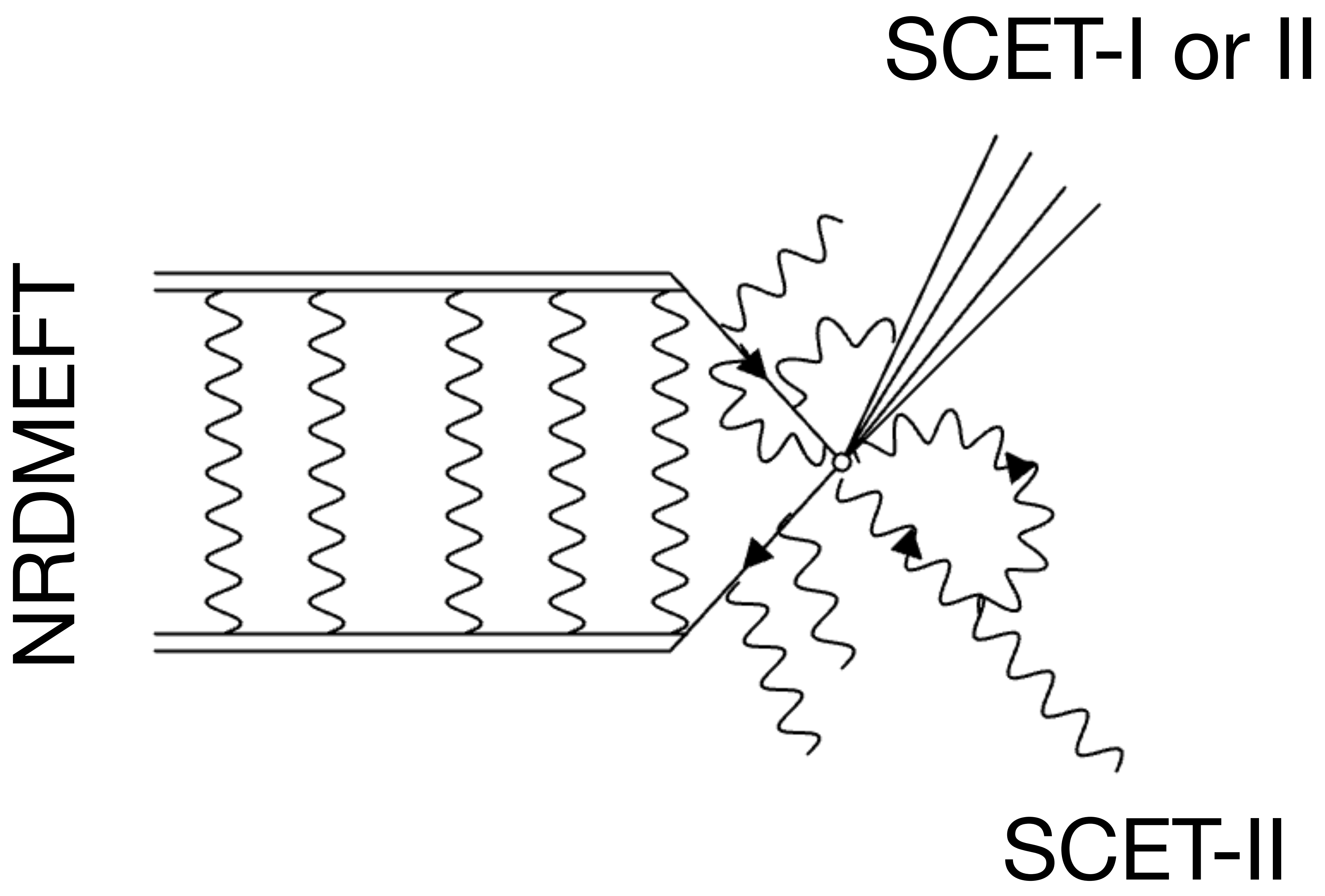
  Fixed-order + Parton showers + NREFT

- Endpoint

  Non-relativistic (NREFT) and soft-collinear (SCET) effective theories




DM
DM
X
γ



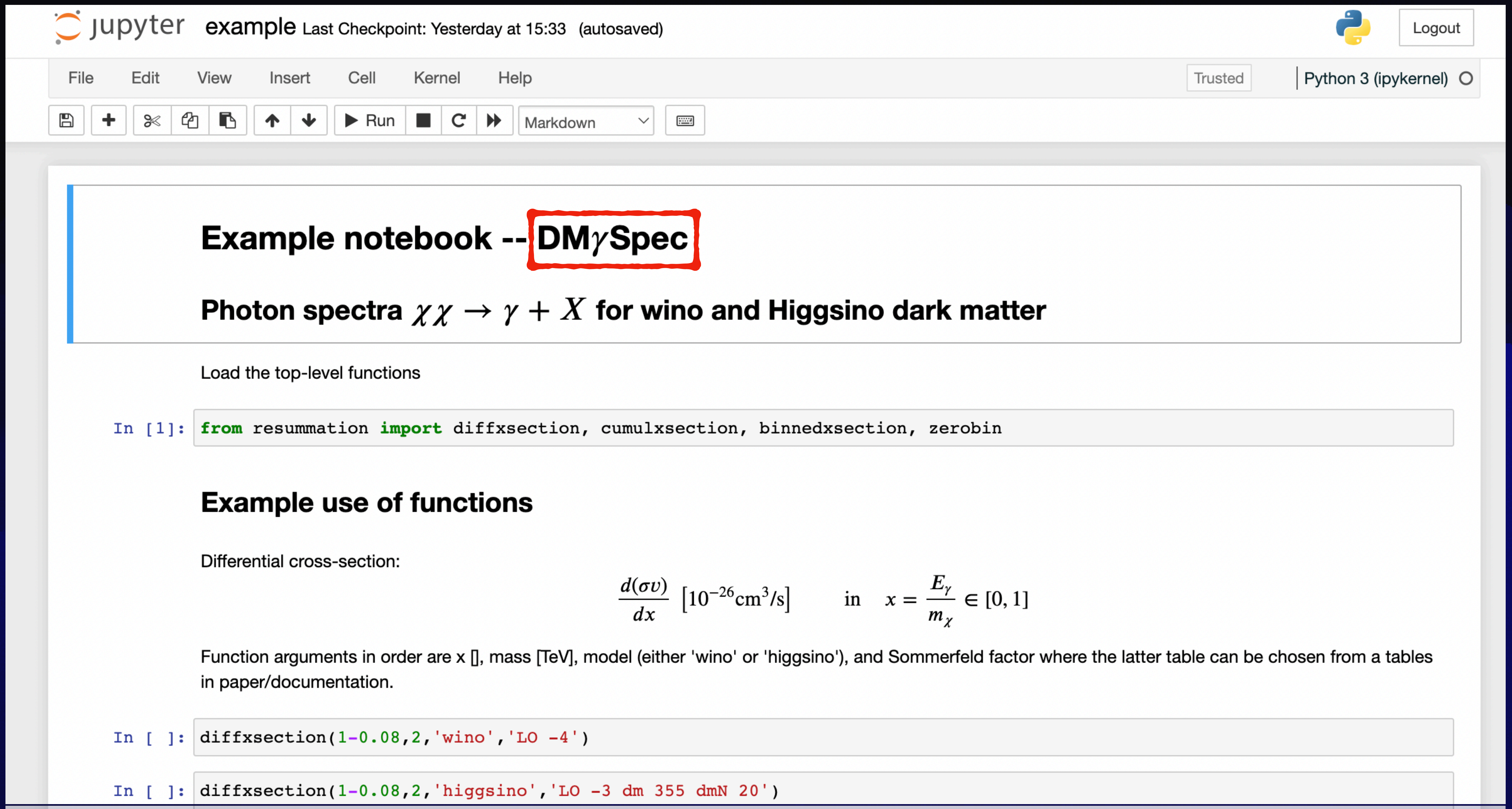
Semi-inclusive processs

Short advertisement

DMɣSpec

- Public python package to compute gamma ray spectra from pure wino and higgsino annihilation
  - **O(1%)** theoretical uncertainty in the endpoint region
- Available on HEPForge:

  https://dmyspec.hepforge.org/



Example notebook -- DMγSpec

Photon spectra $\chi\chi \to \gamma + X$ for wino and Higgsino dark matter

Load the top-level functions

```
In [1]: from resummation import diffxsection, cumulxsection, binnedxsection, zerobin
```

Example use of functions

Differential cross-section:

$$\frac{d(\sigma v)}{dx}\ [10^{-26}\mathrm{cm}^3/\mathrm{s}] \quad \text{in} \quad x = \frac{E_\gamma}{m_\chi} \in [0,1]$$

Function arguments in order are x [], mass [TeV], model (either 'wino' or 'higgsino'), and Sommerfeld factor where the latter table can be chosen from a tables in paper/documentation.

```
In [ ]: diffxsection(1-0.08,2,'wino','LO -4')
```

```
In [ ]: diffxsection(1-0.08,2,'higgsino','LO -3 dm 355 dmN 20')
```

Based on

Mainly:

- *Matching resummed endpoint and continuum γ-ray spectra from dark-matter annihilation*.
  Beneke, Vollmann, Urban — 2022
  arXiv:2203.01692

- *Resummed photon spectrum from dark matter annihilation for intermediate and narrow energy resolution*.
  Beneke, Broggio, Hasner, Vollmann, Urban — 2019 (~100 pages)
  arXiv:1903.08702

But also:

- *Precise yield of high-energy photons from Higgsino dark matter annihilation*.
  Beneke, Hasner, Vollmann, Urban — 2019
  arXiv:1912.02034

Outline

Why?

Phenomenology

Resummations

DMySpec

Conclusions

Why?

Dark Matter exists

Electroweak interactions exist

Why Dark Matter?

and why winos/higgsinos?

- Successful Standard Model of Cosmology (Λ**CDM**)
  - Observational evidence all the way down to galactic scales
- **Freeze-out mechanism** (FOM): **electroweak sector** ⇔ **dark matter**
  - **Supersymmetry** worthy of 50yrs of research even if we don't find it…
  - Naturalness aside, pure winos and higgsinos ➙ still very **good** dark-matter
    - **eluded detection**: have to be **heavy** for the FOM to work out.
    - **minimal BSM field content**!

Why indirect detection?

and why winos/higgsinos?

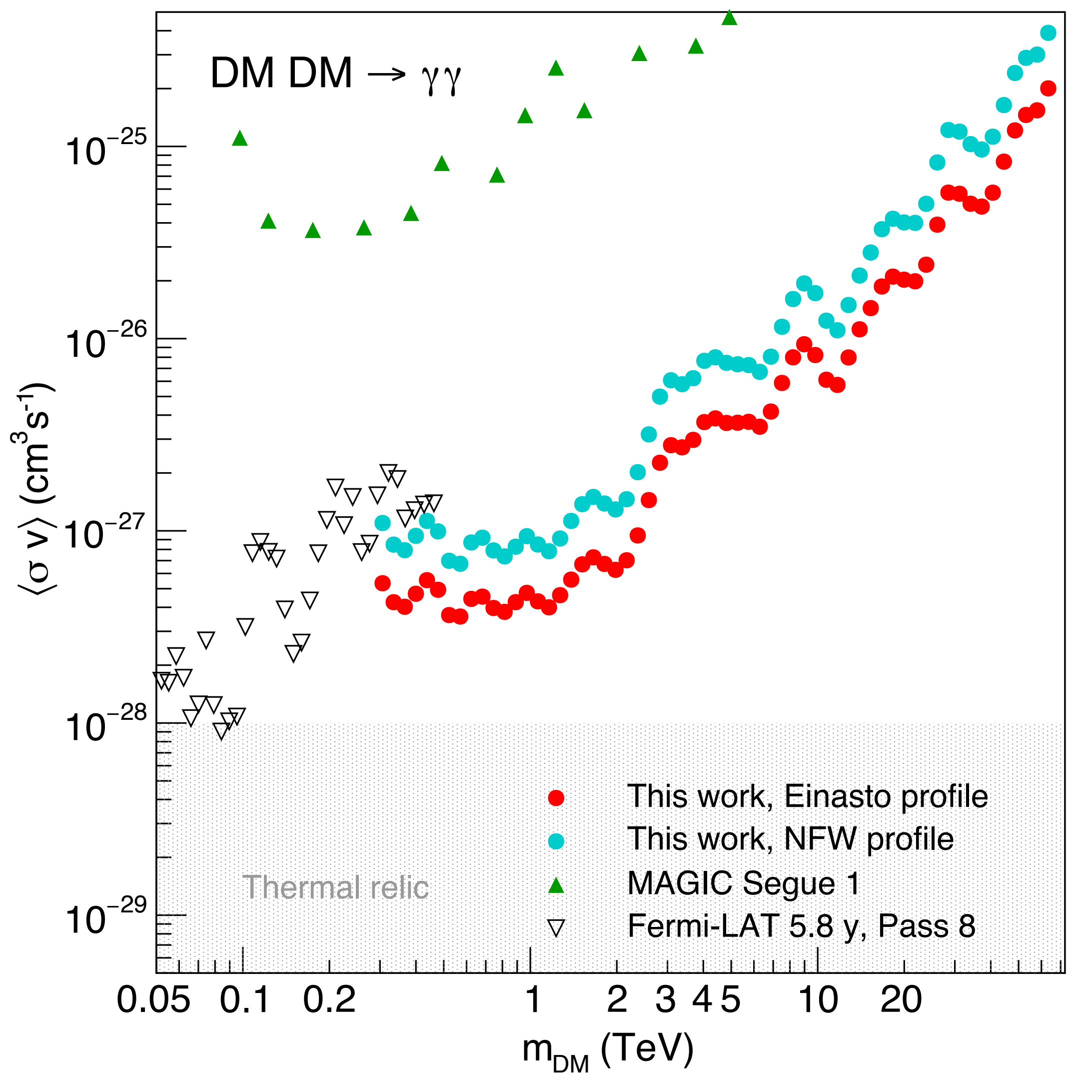
Winos and higgsinos (as dark matter candidates)

  - too heavy for the **LHC**
  - don't couple at Born level with the SM fermions ⇒ ~~direct detection~~

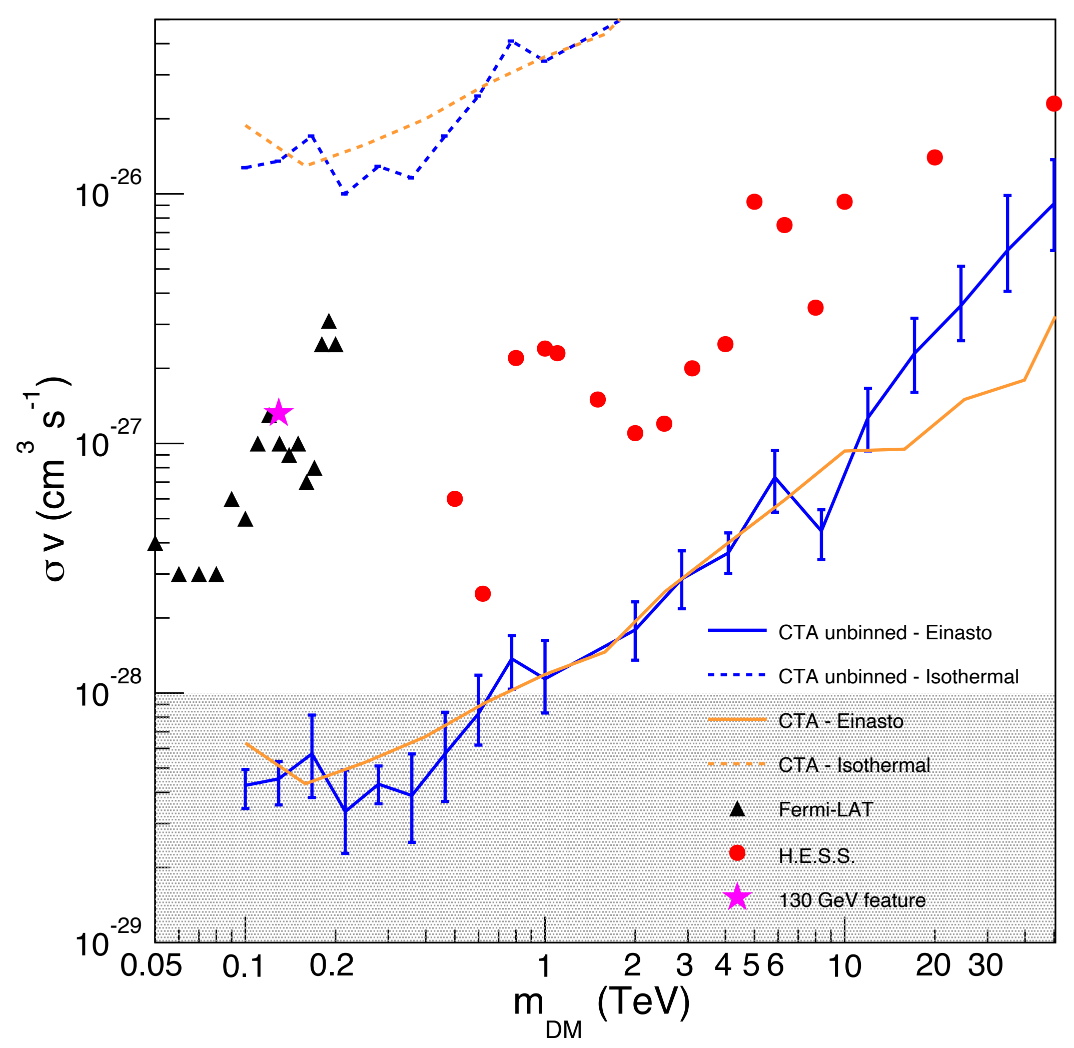
- Imaging Atmospheric Cherenkov Telescopes (such as HESS, MAGIC, VERITAS, HAWC, etc. or the next generation CTA, LHAASO) can search for TeV-scale spectral lines
- Sommerfeld effect: enhancements by factors of 10^3 to 10^6 !!!

Why indirect detection?

HESS
(arXiv:1805.05741)

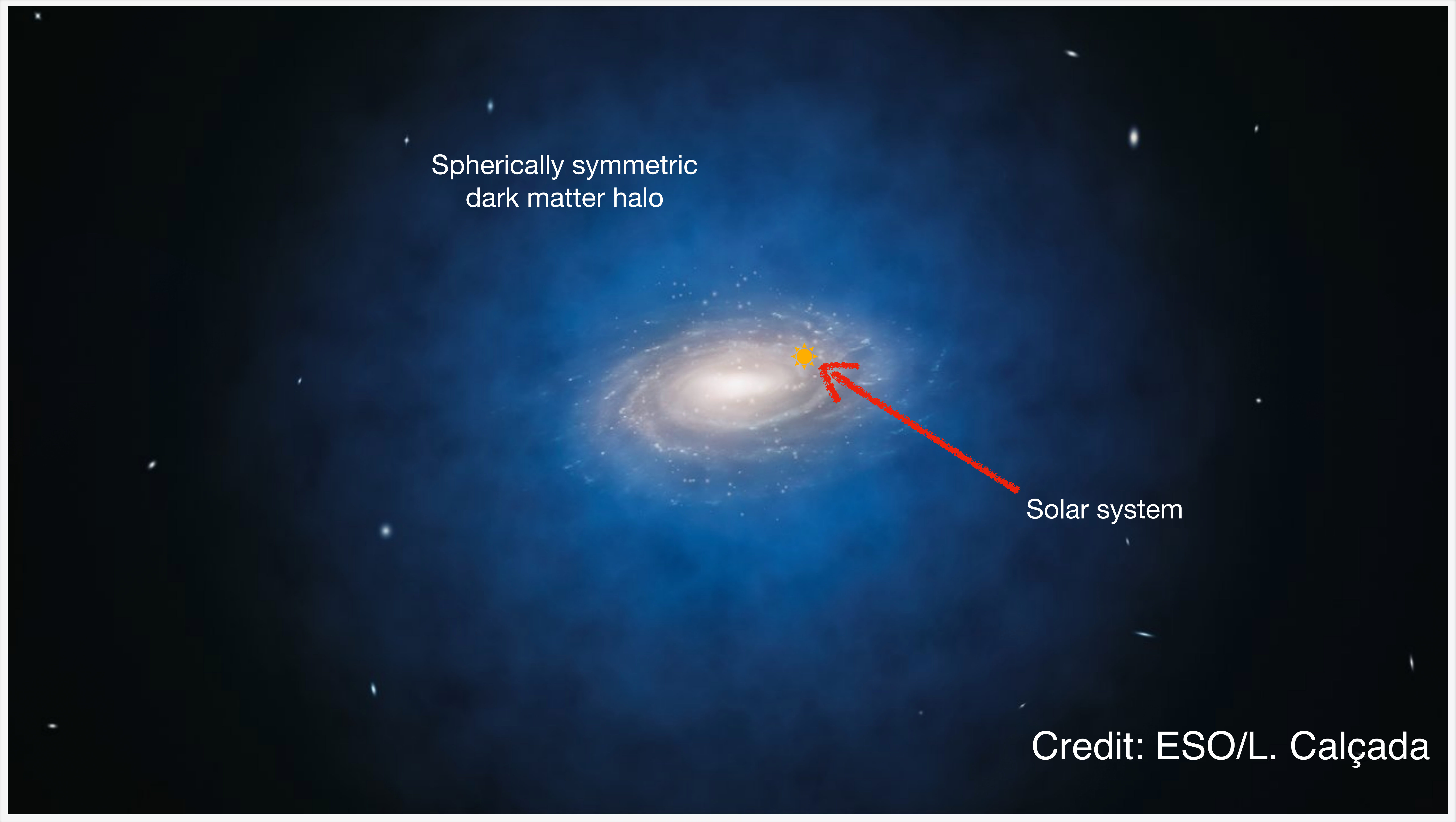


Projected (500hrs) CTA
(arXiv:1709.07997)



Phenomenology

Modern (ΛCDM) picture of the Milky Way

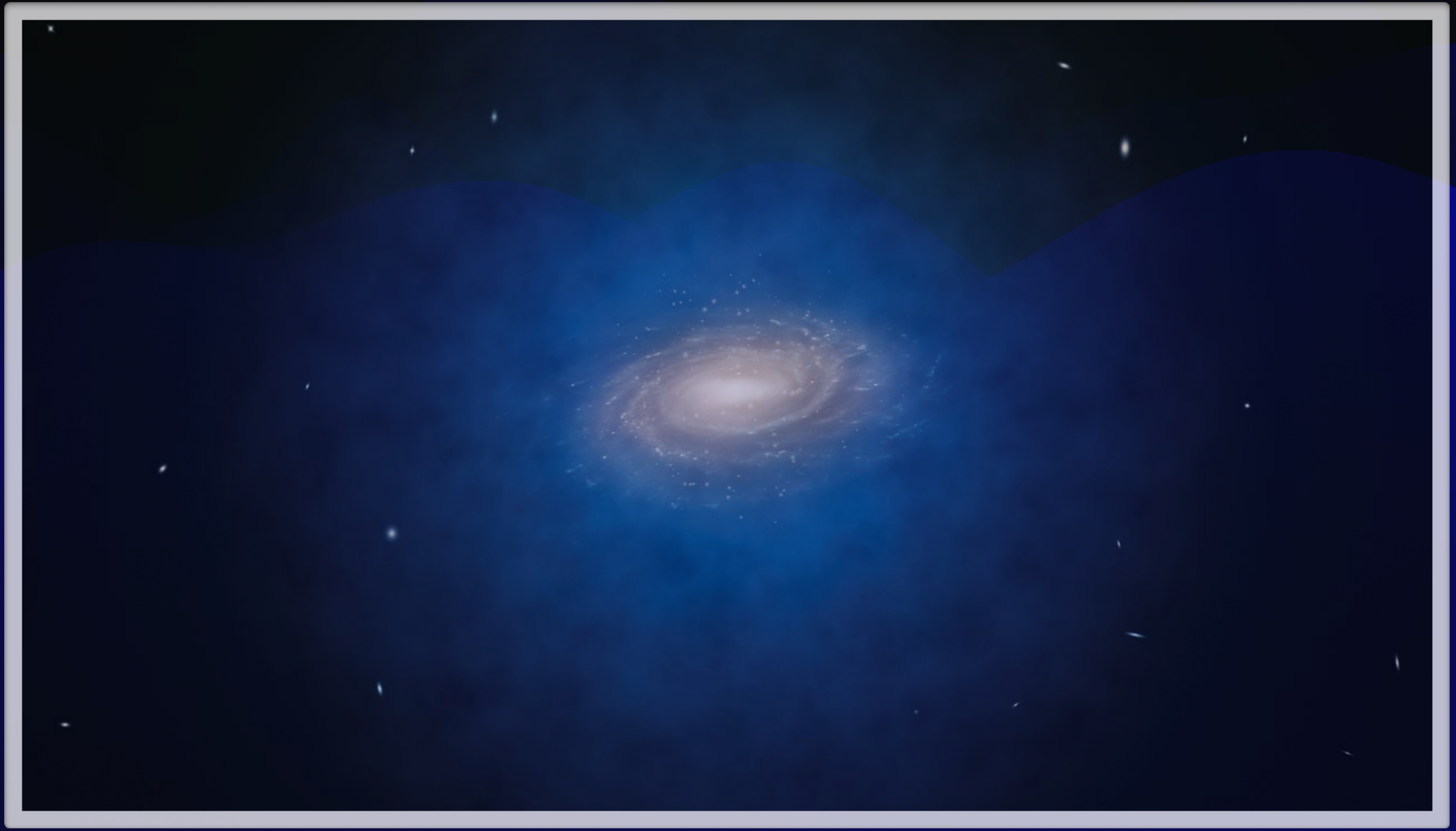


Credit: ESO/L. Calçada

Milky Way DM halo

- Most of the dark matter is in the innermost regions of the Galaxy
  - Very uncertain, though. E.g.

	Cusp	Core
$\rho(r) \underset{r \to 0}{\propto}$	$\frac{1}{r}$	$\text{const}\,.$



Dark Matter annihilation

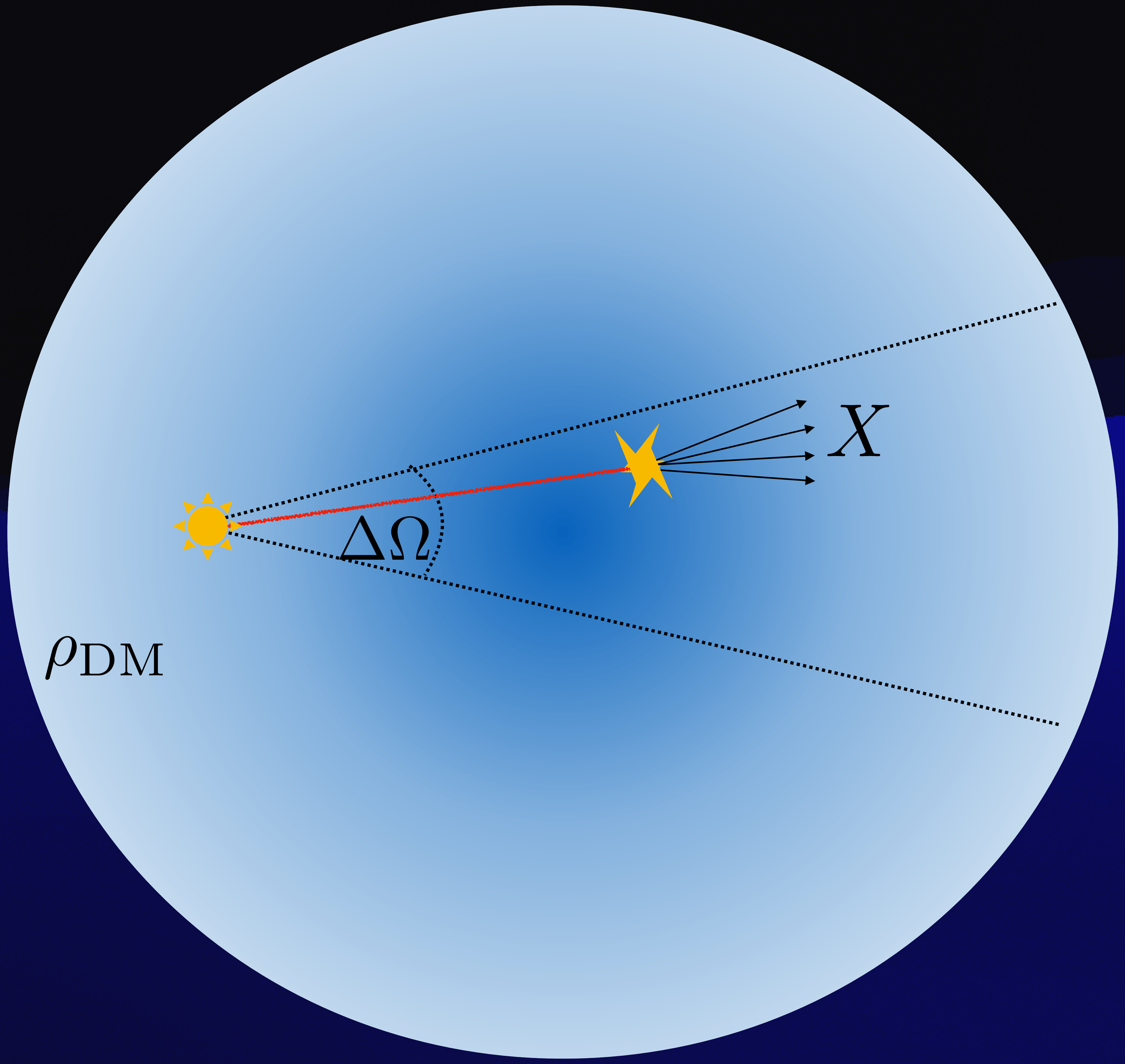
Prompt gamma rays

- "Count" the number of rays subtended in $\Delta\Omega$

$$\Phi_\gamma = \int_{\Delta\Omega} \mathrm{d}\Omega\, I_\gamma\,, \quad I_\gamma = \int_{\mathrm{l.o.s.}} \mathrm{d}s\, \frac{1}{4\pi} S_\gamma$$

- Rate sensitive to the (unknown) number density of DM particles
  - DM mass density ρ (if uncertain) is the available quantity

$$S_\gamma = \frac{1}{2} n_\chi^2 \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma} = \frac{1}{2\, m_\chi^2} \rho_{\mathrm{DM}}^2 \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma}$$



Dark Matter annihilation

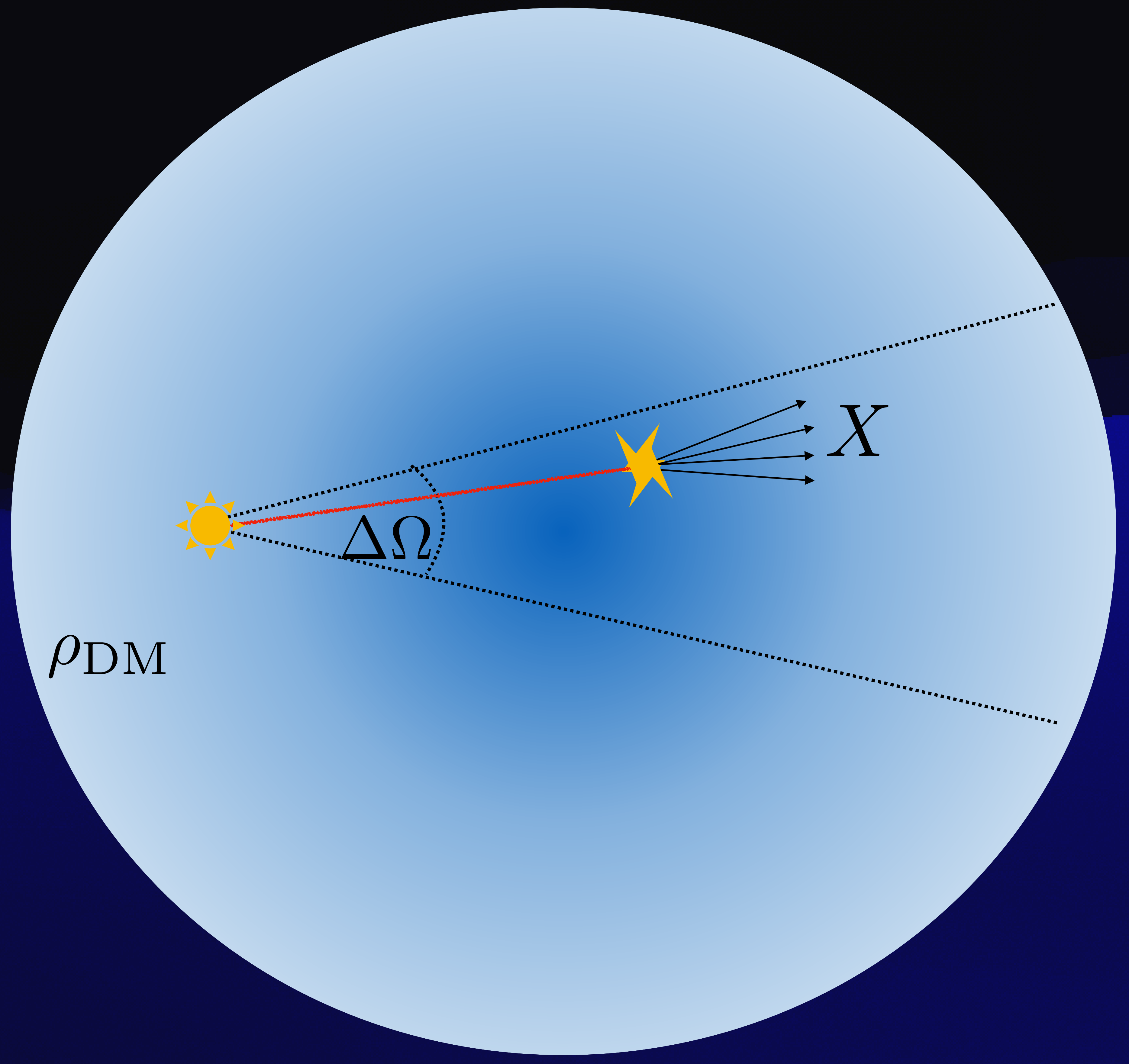
Astrophysics factored out

- Putting things together:

$$\Phi_\gamma = \frac{1}{8\pi m_\chi^2} \times J \times \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma},$$

where the “J” factor is defined as

$$J = \int \mathrm{d}\Omega \int_{l.o.s.} \mathrm{d}s\, \rho_{\mathrm{DM}}^2$$



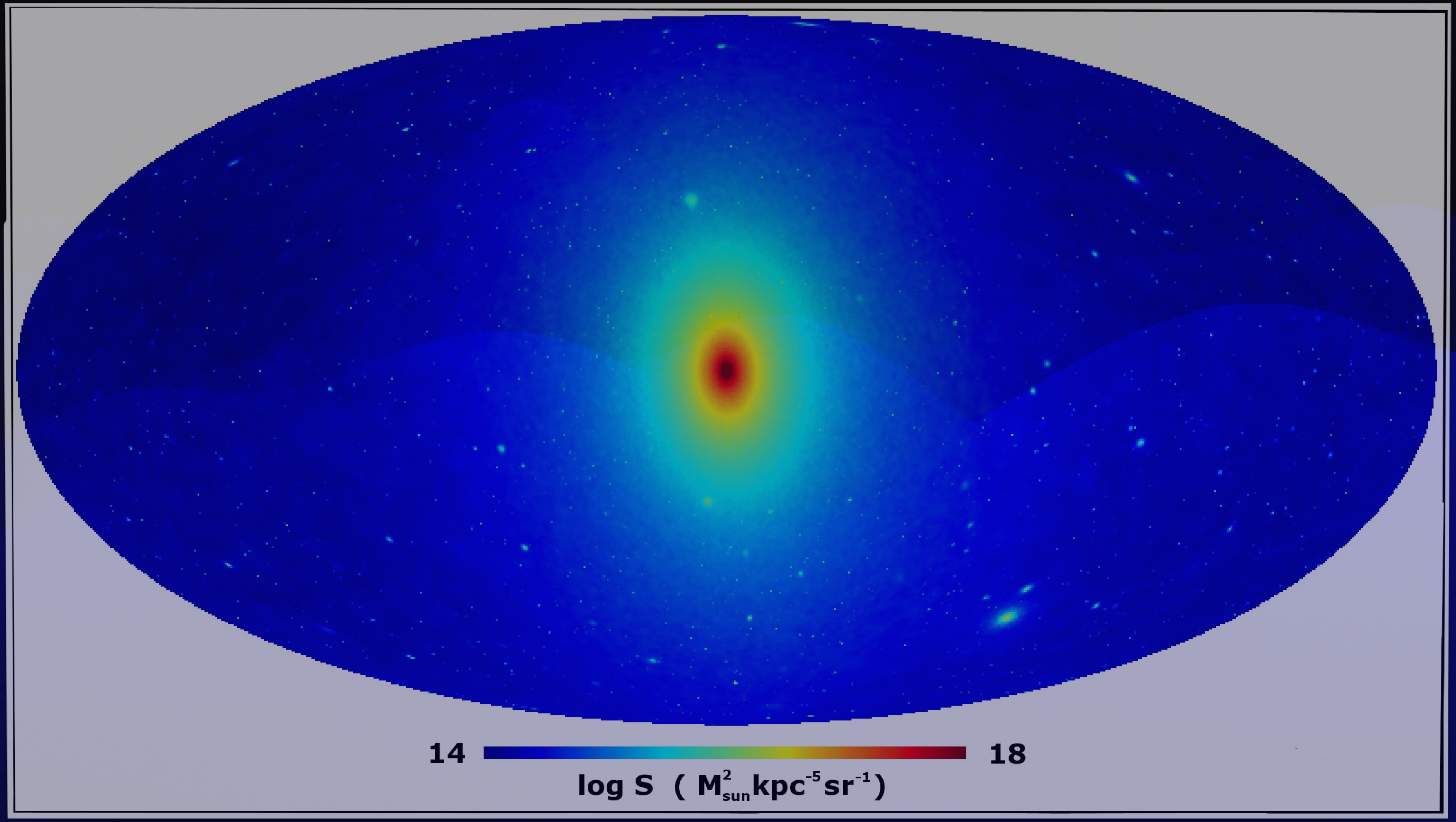
Dark Matter annihilation

Astrophysics factored out

- Dark-matter annihilation map of a Milky-Way-like galaxy from the Aquarius (Aq-A-1) simulation:

$$S = \int_{\mathrm{l.o.s.}} \mathrm{d}s\, \rho_{\mathrm{DM}}^2$$

- Same map for all γ-ray energies!
- ρ_{DM}: highly unconstrained especially in innermost regions
  - Simplified analytical benchmarks (NFW, Einasto, etc.)

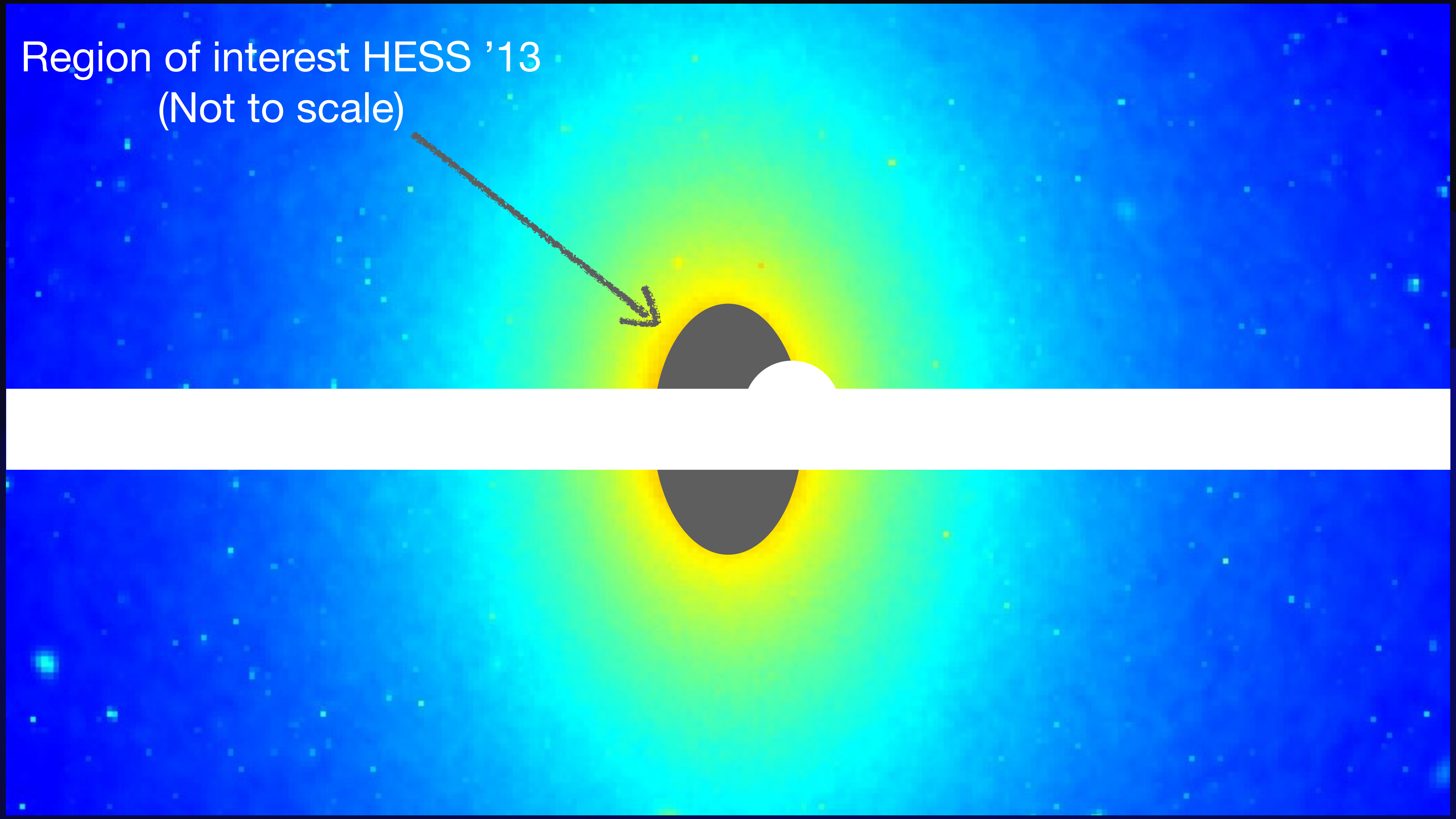


https://wwwmpa.mpa-garching.mpg.de/aquarius/

Dark Matter annihilation

Astrophysics factored out

- "J" factor in this region of interest (ROI) ($J = \int_{\mathrm{ROI}} \mathrm{d}\Omega\, S$) varies from 1.1 all the way up to 8.0 times $10^{21}\mathrm{GeV}^2/\mathrm{cm}^5$ for NFW and Einasto halo parametrizations respectively
    - γ-ray fluxes uncertain by a factor of ca. 10 !!



https://wwwmpa.mpa-garching.mpg.de/aquarius/

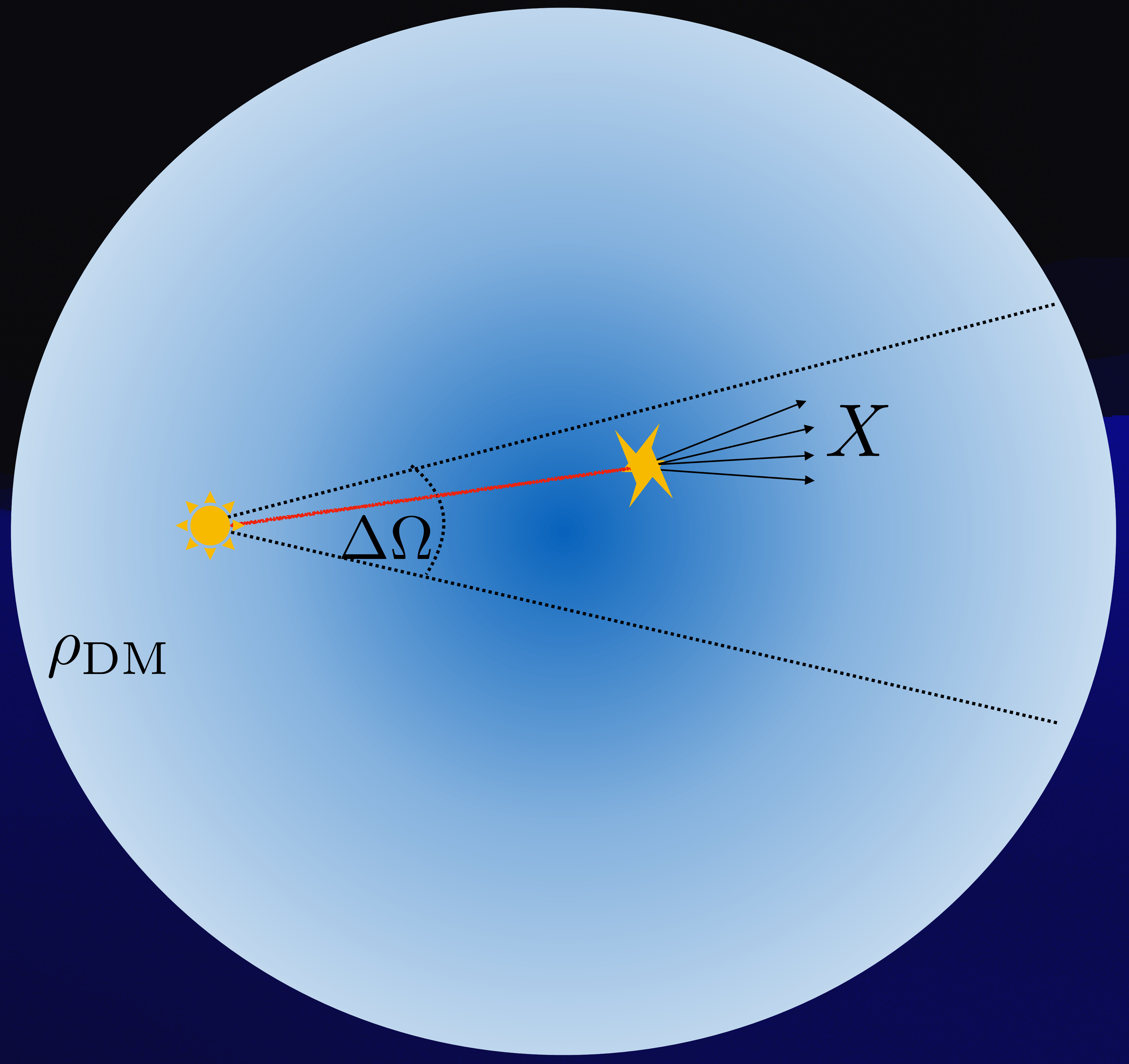
Dark Matter annihilation

Prompt gamma rays

- γ-ray flux via dark-matter annihilation

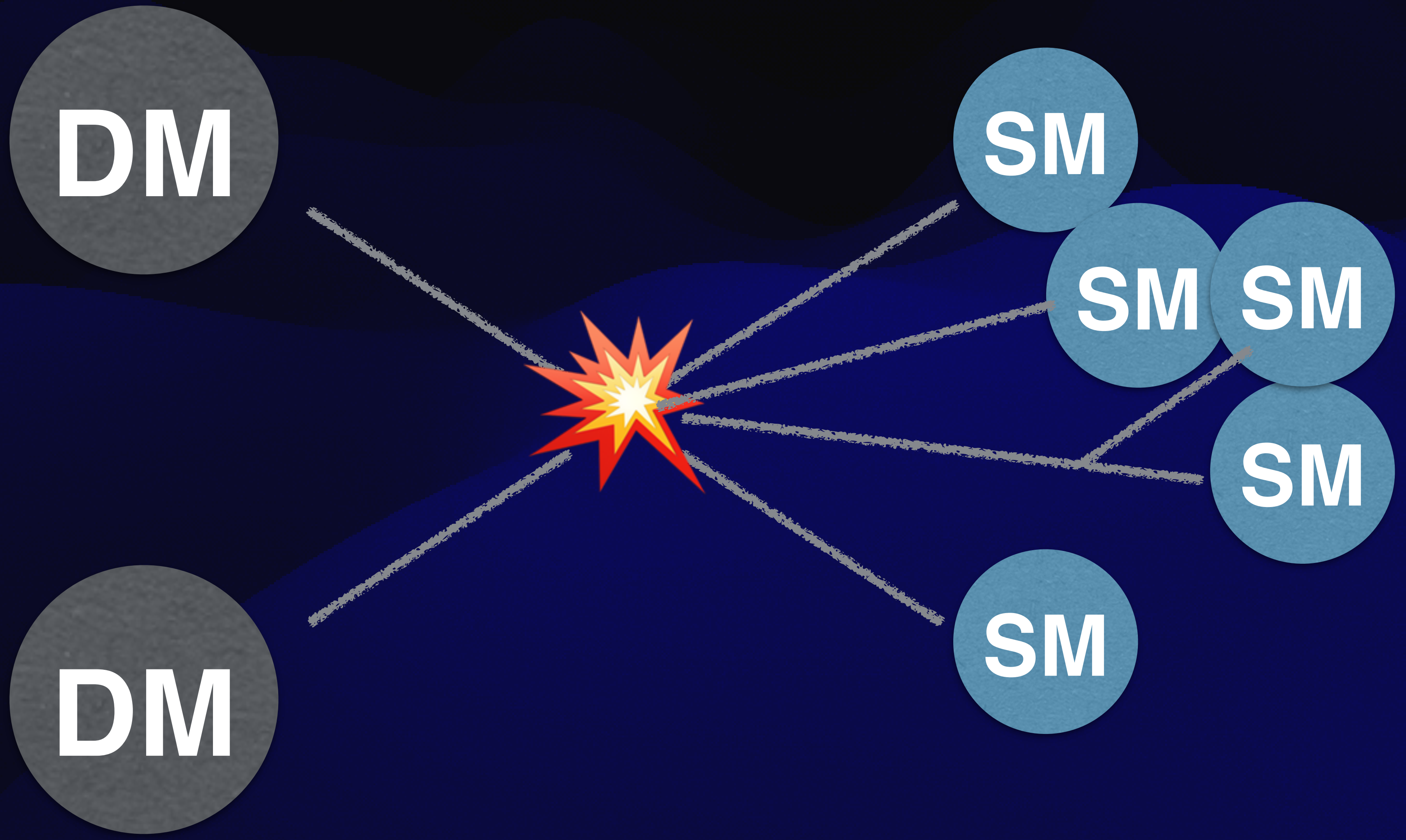
$$\Phi_\gamma = \frac{1}{8\pi m_\chi^2} \times J \times \boxed{\frac{\mathrm{d}\langle \sigma v \rangle}{\mathrm{d}E_\gamma}}$$

➡ Focus on a particle physics problem!



Dark Matter annihilation

Prompt gamma rays

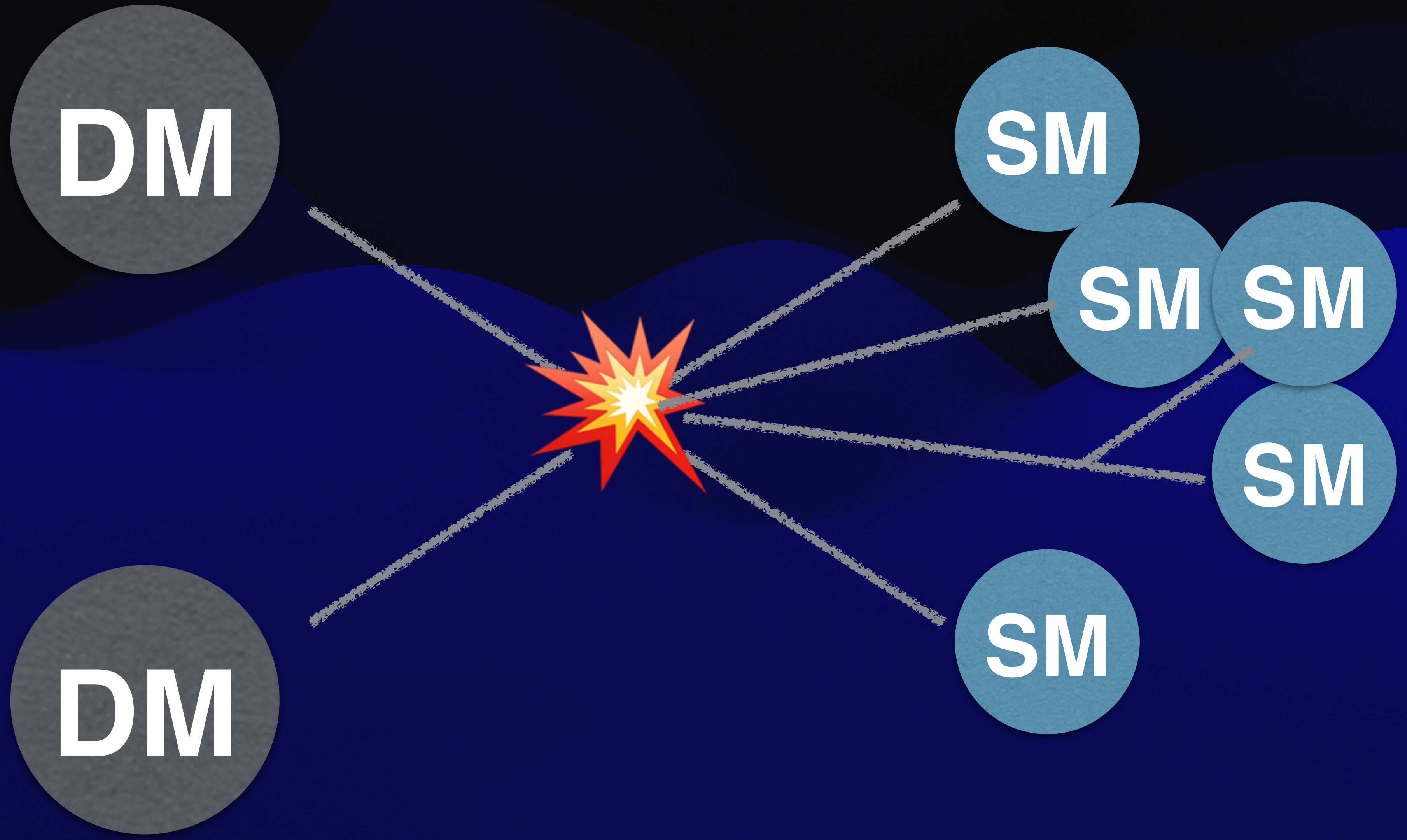


Dark Matter annihilation

Prompt gamma rays

- Simple kinematics (The dark matter is cold → non relativistic)
  - Lab frame ≃ CoM frame

$$\sqrt{s} = 2\,m_\chi + \mathcal{O}(m_\chi v^2)$$



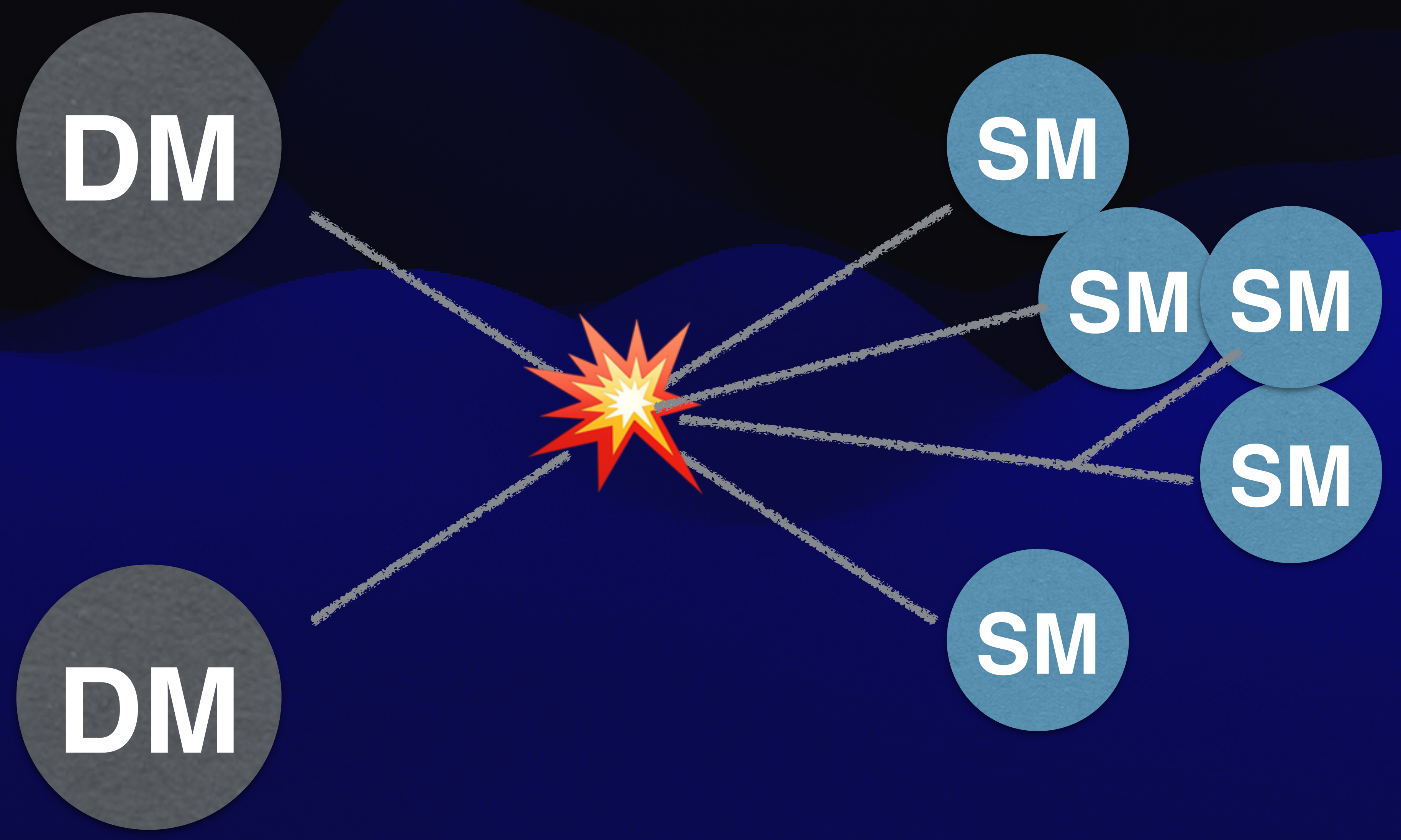
Dark Matter annihilation

Endpoint spectrum

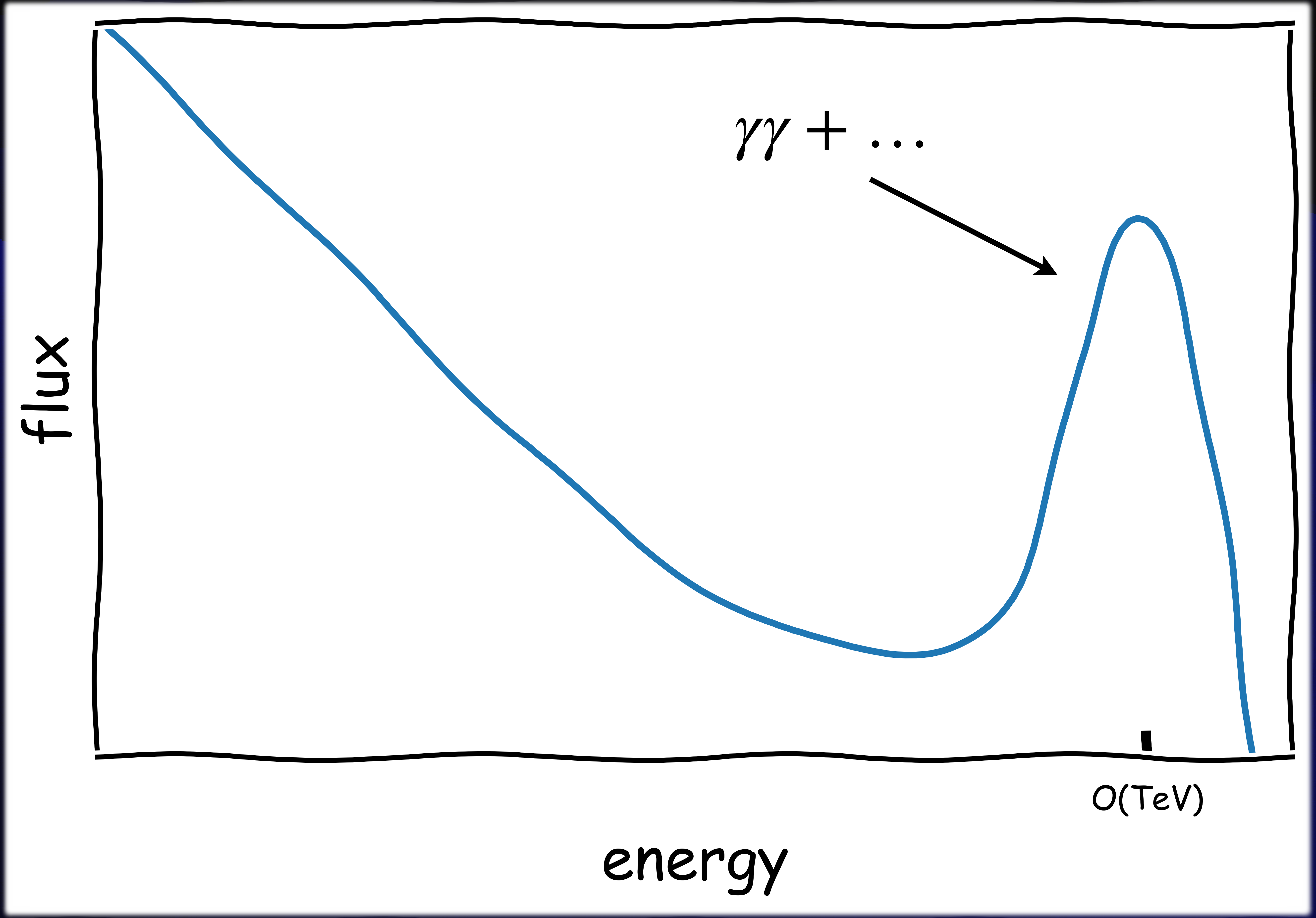
- Consider the fully-exclusive process $\chi_0\chi_0 \to \gamma\gamma$

$$E_\gamma = m_\chi$$

  - Back-to-back monochromatic TeV-scale photons

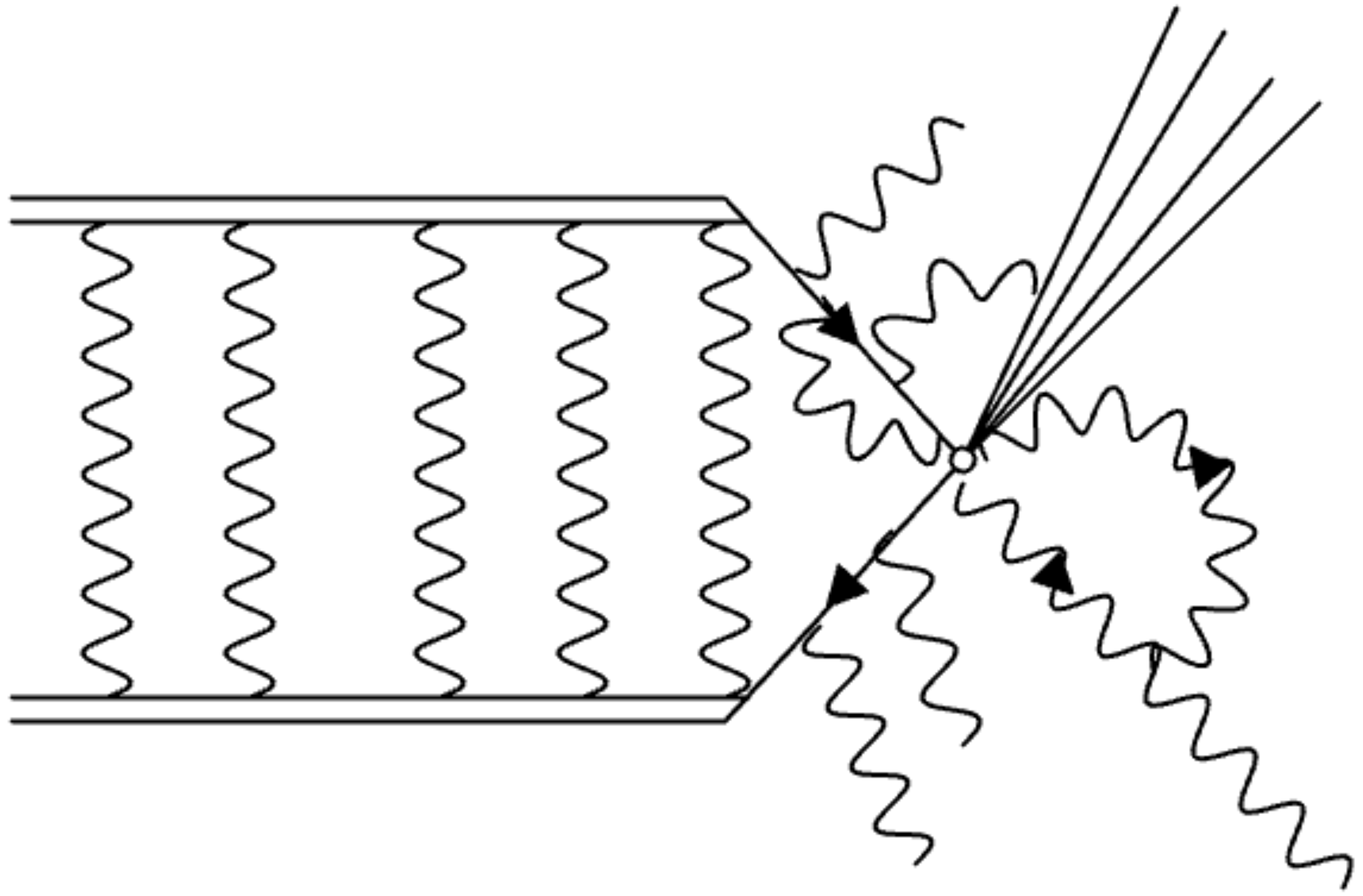


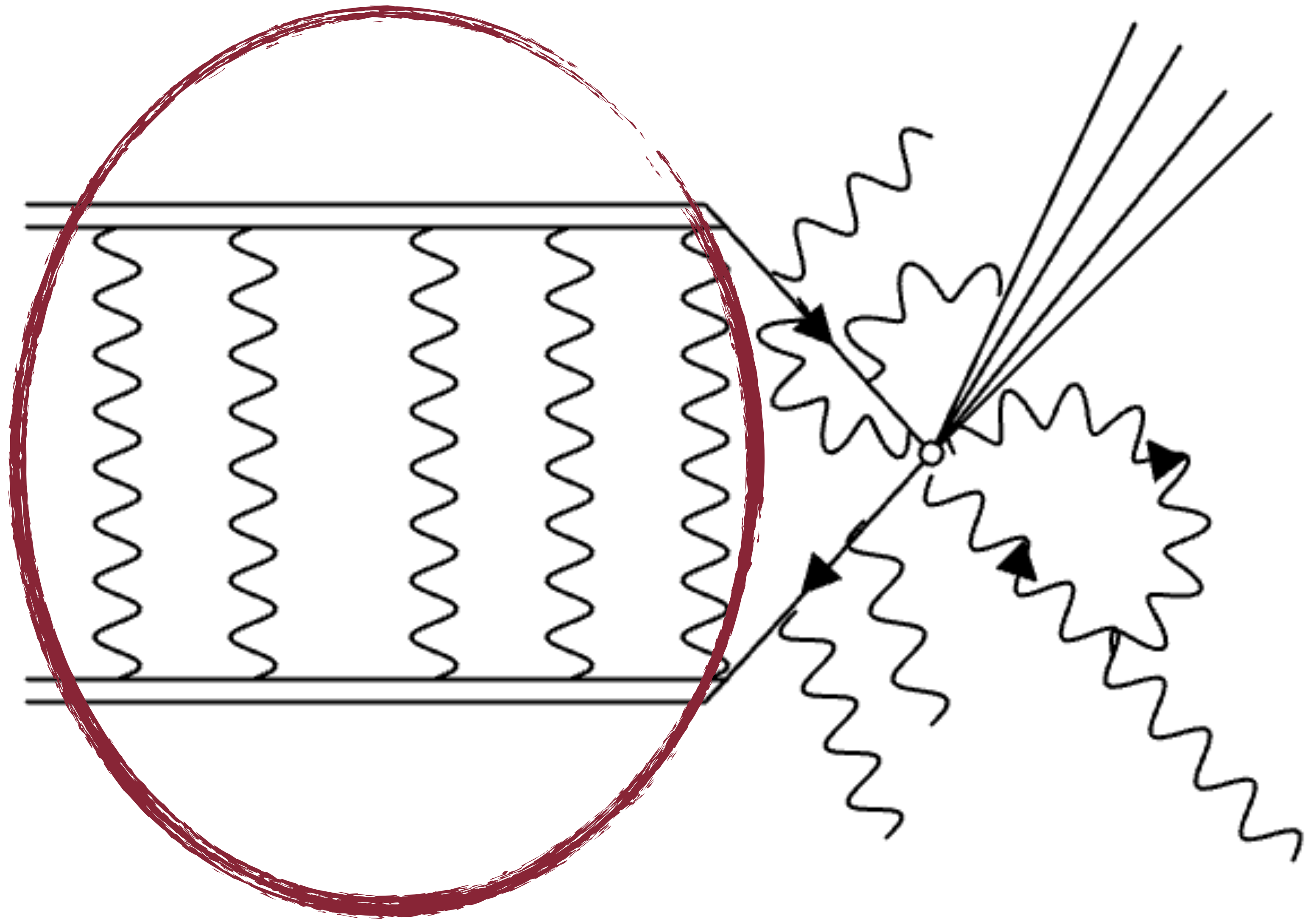
Quasi-monochromatic spectral line



Gamma-ray telescopes

Resummations





Sommerfeld enhancement

Sommerfeld effect

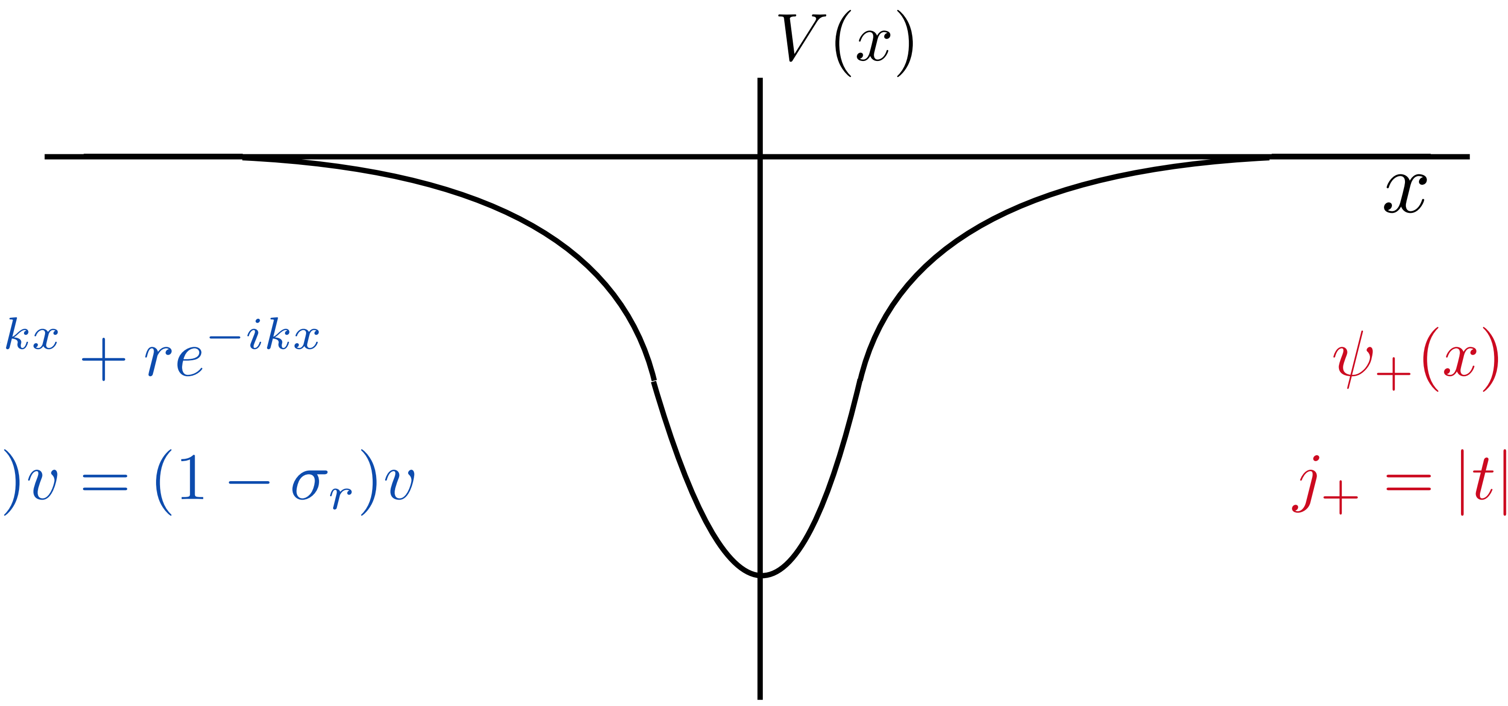
The wave function of a two-wimp system

$$\left(-\frac{1}{m_\chi}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V(x)\right)\psi(x)=E\psi(x)$$

$$j(x)=\frac{i}{m_\chi}\left[\psi(x)\psi'^*(x)-\psi^*(x)\psi'(x)\right]$$ = const.

$\psi_-(x)=e^{ikx}+re^{-ikx}$

$j_-=(1-|r|^2)v=(1-\sigma_r)v$



$\psi_+(x)=te^{ikx}$

$j_+=|t|^2v=\sigma_t v$

Unitarity: $j_-=j_+ \quad\rightarrow\quad \sigma_r+\sigma_t=1$ (No absorption)

Sommerfeld effect

The wave function of a two-wimp system

$$\left(-\frac{1}{m_\chi}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V(x)+\frac{i}{2}\sigma_a^{(0)}v\delta(x)\right)\psi(x)=E\psi(x)$$

Unitarity-violating term $\rightarrow$ $j_+ = j_- + |\psi(0)|^2\sigma_a v$

$$\sigma_r+\sigma_t+\boldsymbol{\sigma_a}=1$$

$$\sigma_a=|\boldsymbol{\psi(0)}|^{\mathbf{2}}\sigma_a^{(0)}$$

Resummed cross section	=	Sommerfeld factor (“long” range NR physics)	X	QFT cross section (short range physics)

Sommerfeld enhancement

Concrete example: pure wino

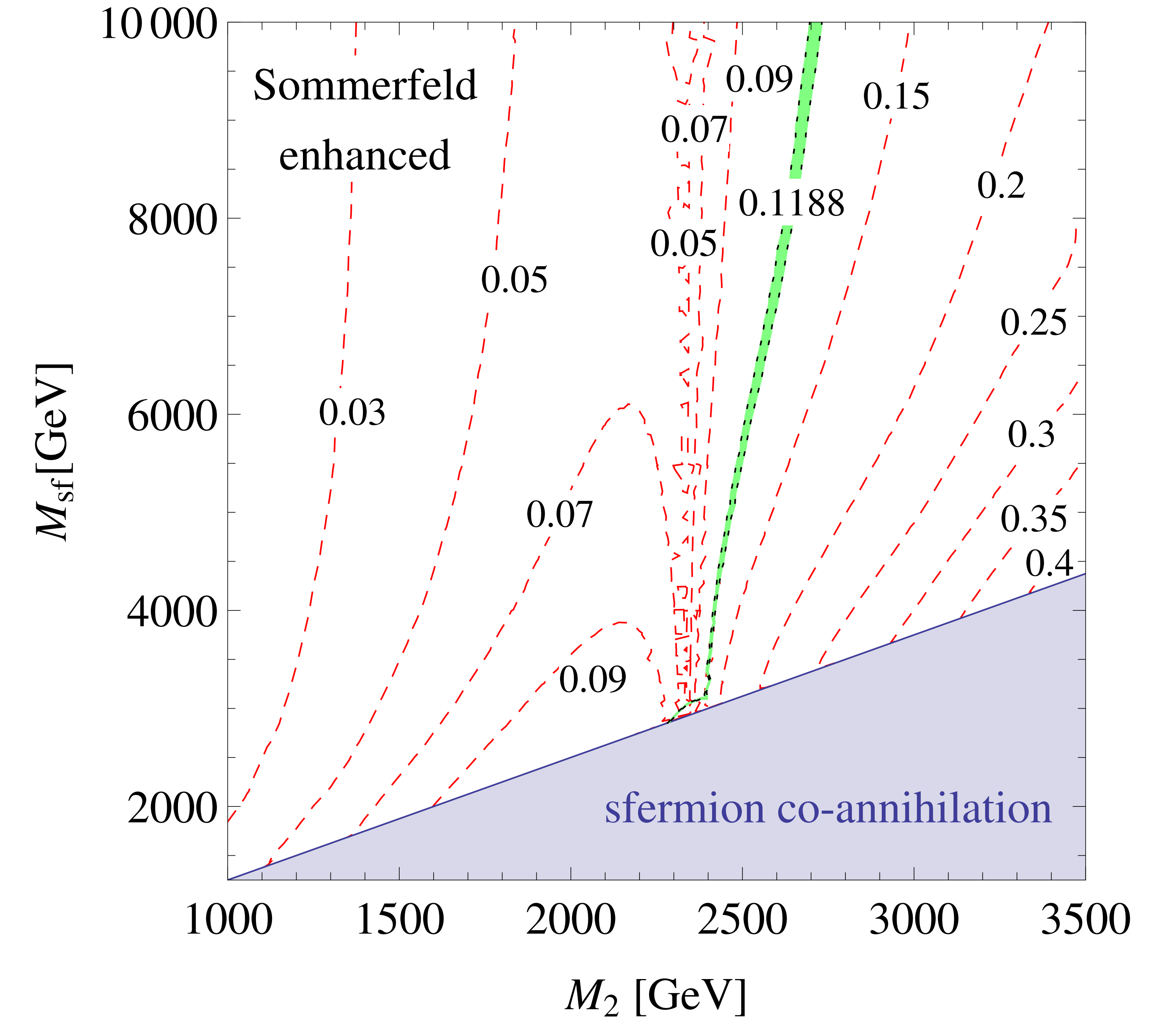
SM + Majorana SU(2) triplet

$$\delta\mathcal{L}_{\rm Wino} = \frac{1}{2}\bar{\chi}(i\gamma^\mu {\rm D}_\mu - m_\chi)\chi$$

$$\xrightarrow{\cancel{\text{EW}}}$$

Q=0 *Majorana* DM
Q=1 *Dirac* chargino

- $m_{\chi^+} - m_{\chi^0} \simeq 164\text{MeV}$
- DM stable through a $\mathbb{Z}_2$ symmetry
- Suitable WIMP for $m_{\chi^0} \simeq 3\text{TeV}$
- Super-partner of the SU(2) gauge bosons in SUSY



Sommerfeld enhancement

Concrete example: pure wino

$$\frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma} = 2\sum_{I,J} S_{IJ} \frac{\mathrm{d}(\sigma v)_{IJ}}{\mathrm{d}E_\gamma}$$

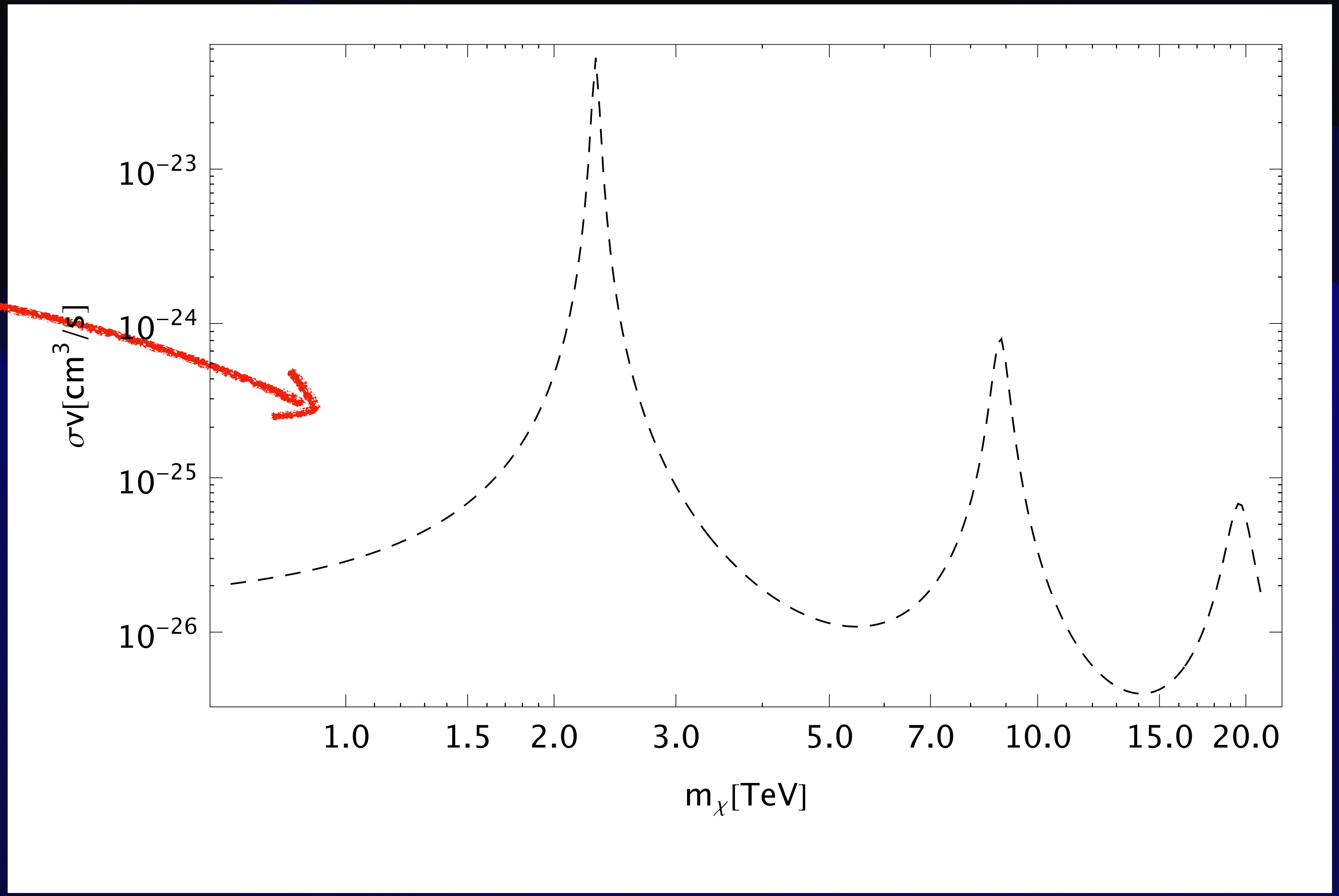
Sommerfeld **matrix**
I, *J* = (χ0χ0) or (χ+χ-)

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{\mathrm{e}^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{\mathrm{e}^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{\mathrm{e}^{-m_Z r}}{r} \end{pmatrix}$$

Sommerfeld enhancement

“Explosive” Dark Matter annihilation

$$\boxed{\left.\frac{\mathrm{d}(\sigma v)}{\mathrm{d}E_\gamma}\right|_{\text{Somm}} = 2\sum_{I,J} S_{IJ}\frac{\mathrm{d}(\sigma v)^{\text{tree}}_{IJ}}{\mathrm{d}E_\gamma}}$$

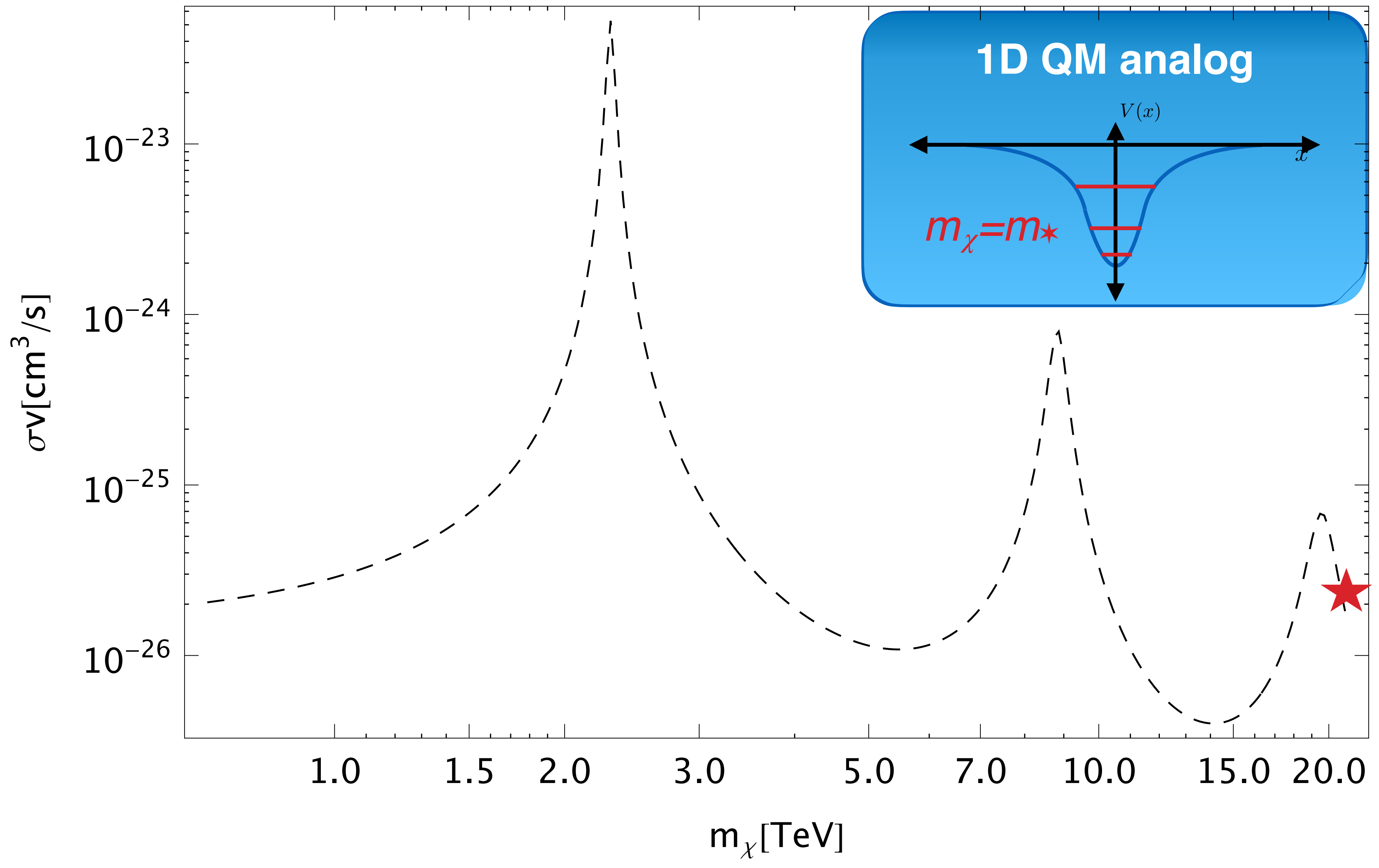


$$\frac{\mathrm{d}(\sigma v)^{\text{tree}}_{(00)(00)}}{\mathrm{d}E_\gamma} = \frac{\mathrm{d}(\sigma v)^{\text{tree}}_{(+-)(00)}}{\mathrm{d}E_\gamma} = 0$$

$$\frac{\mathrm{d}(\sigma v)^{\text{tree}}_{(+-)(+-)}}{\mathrm{d}E_\gamma} = \frac{2\pi\alpha_2^2 s_W^4}{m_\chi^2}\delta(E_\gamma - m_\chi) + \frac{2\pi\alpha_2^2 s_W^2 c_W^2}{m_\chi^2}\delta(E_\gamma - E_0^{\gamma Z})$$

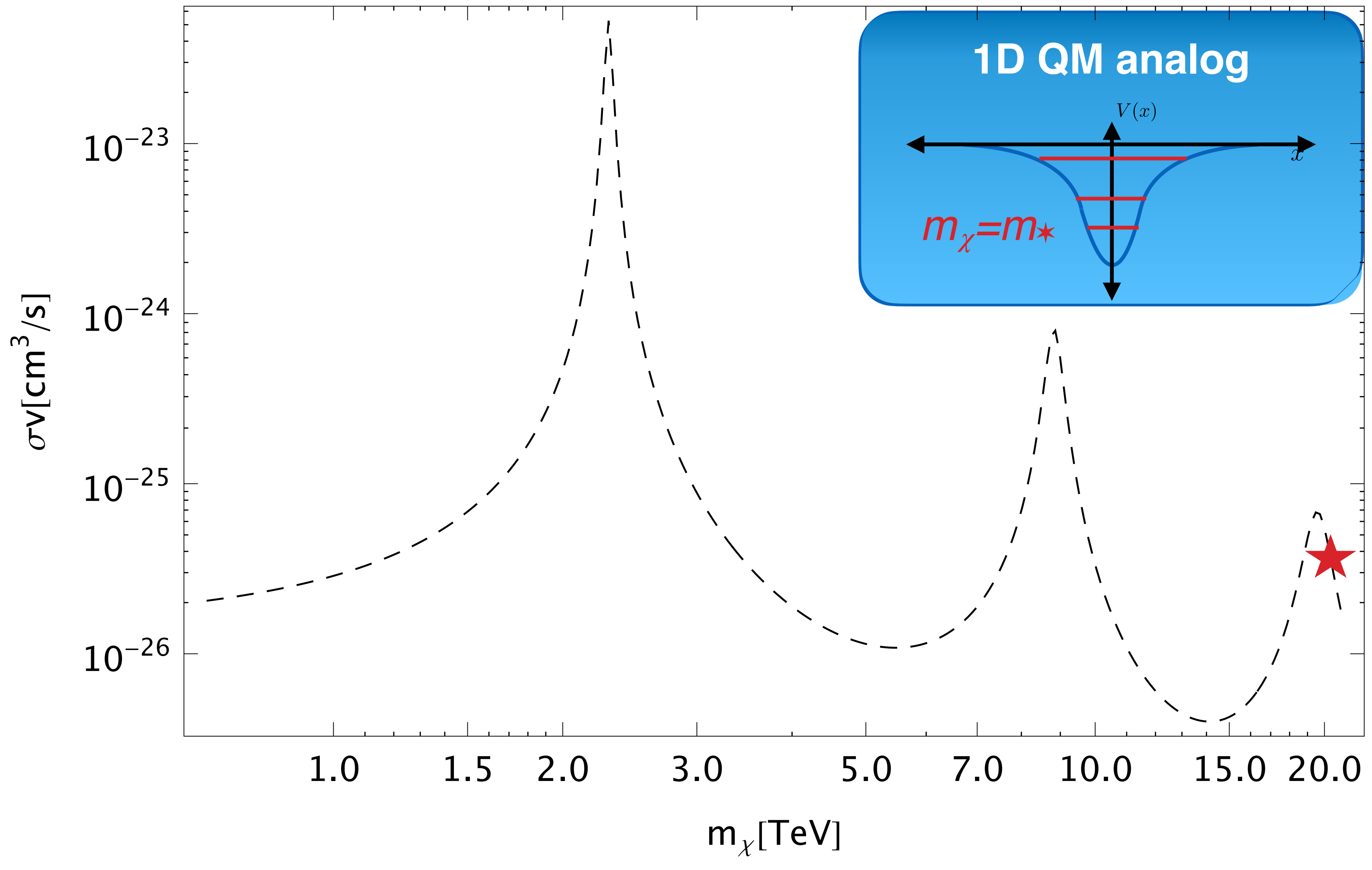
Sommerfeld enhancement

Bound states? … Not quite



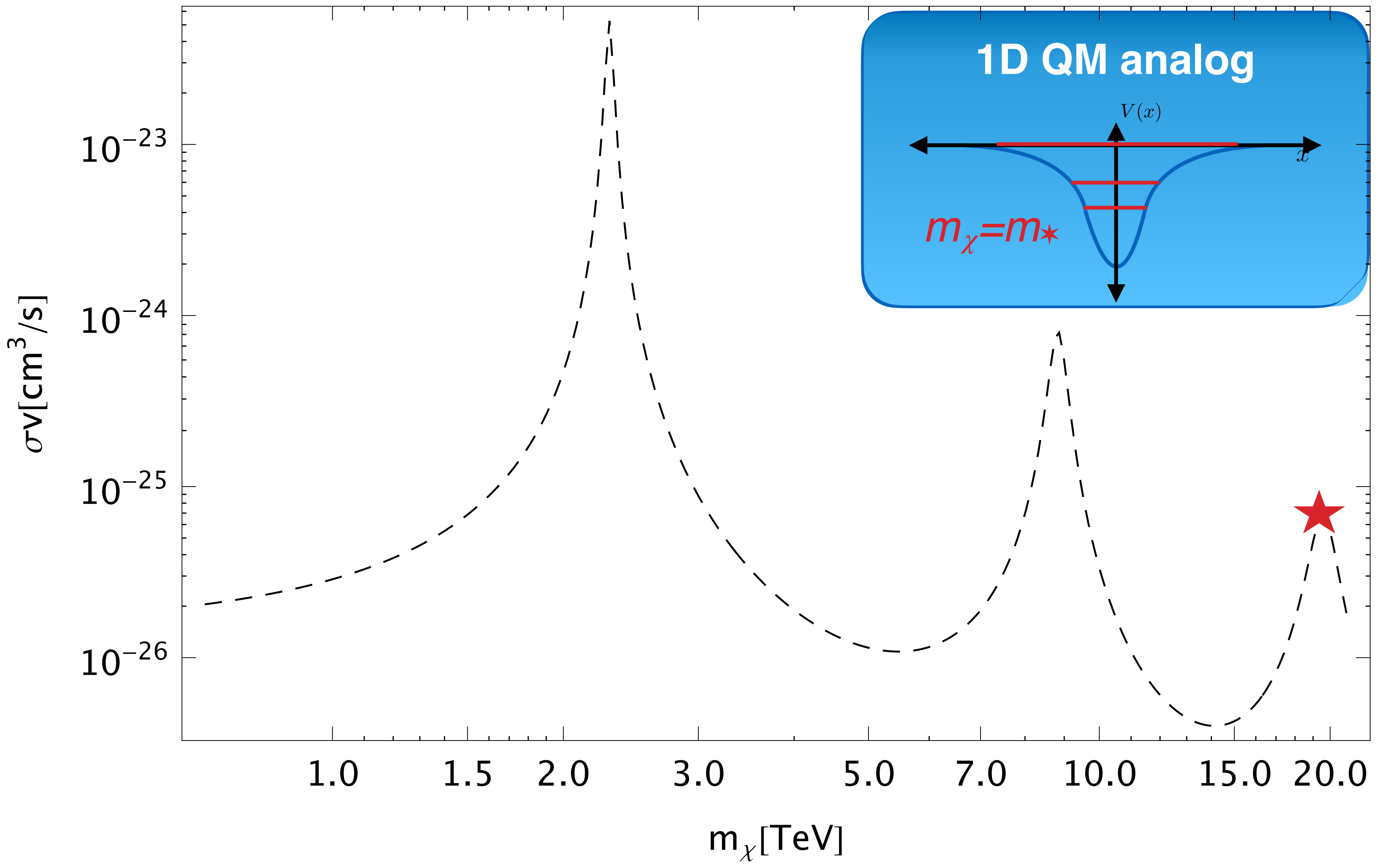
Sommerfeld enhancement

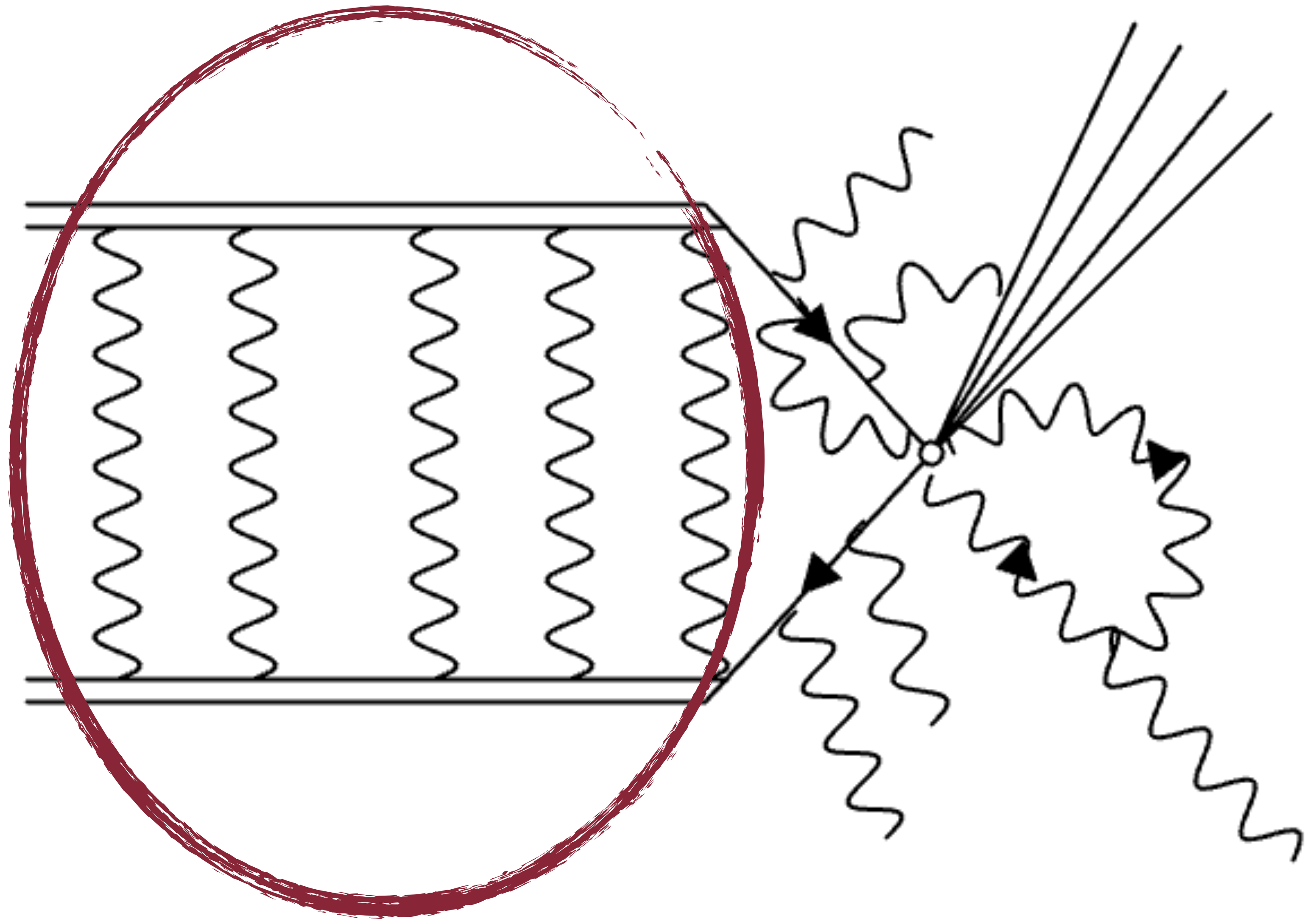
Bound states? … Not quite



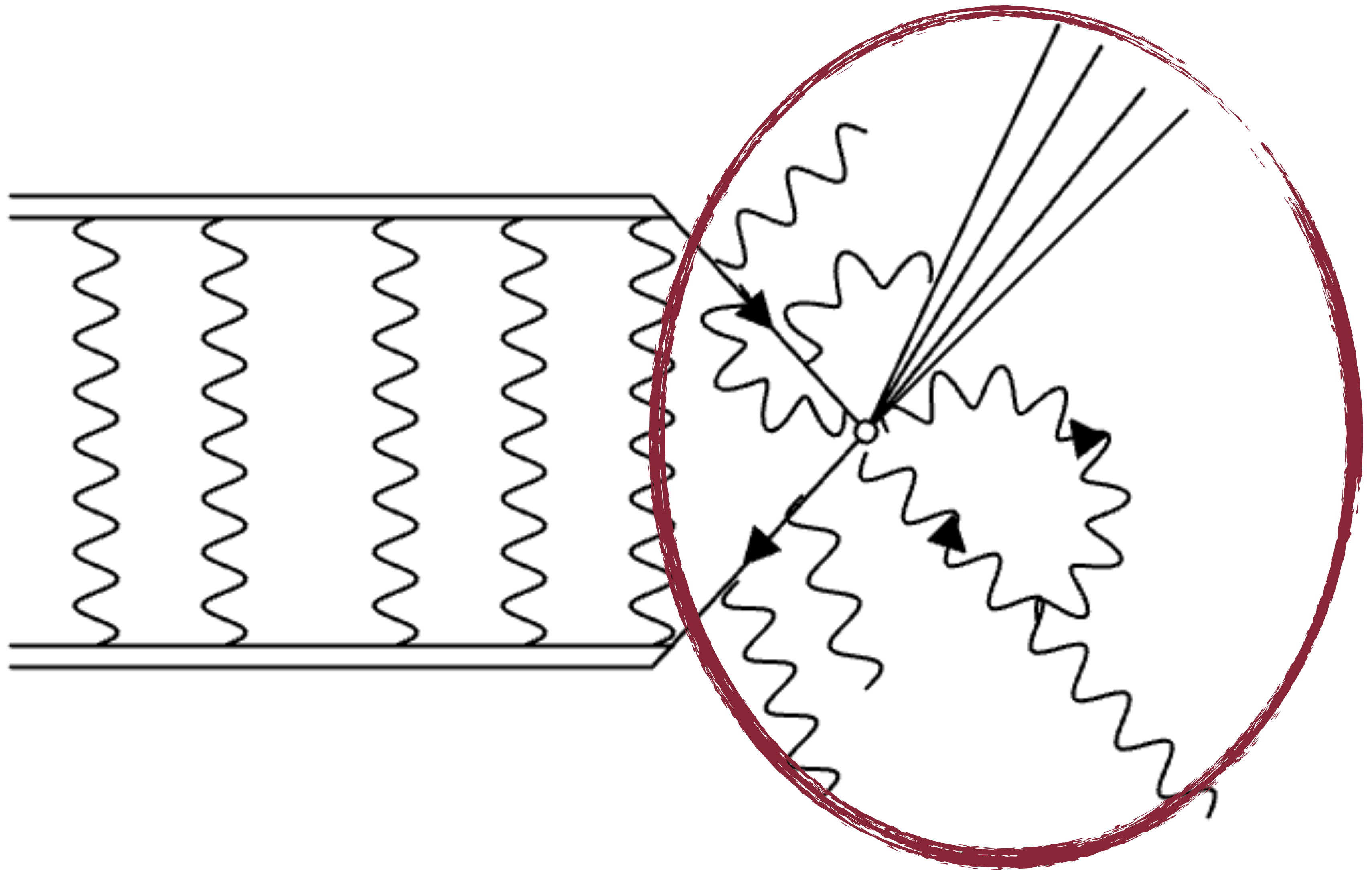
Sommerfeld enhancement

Bound states? … Not quite



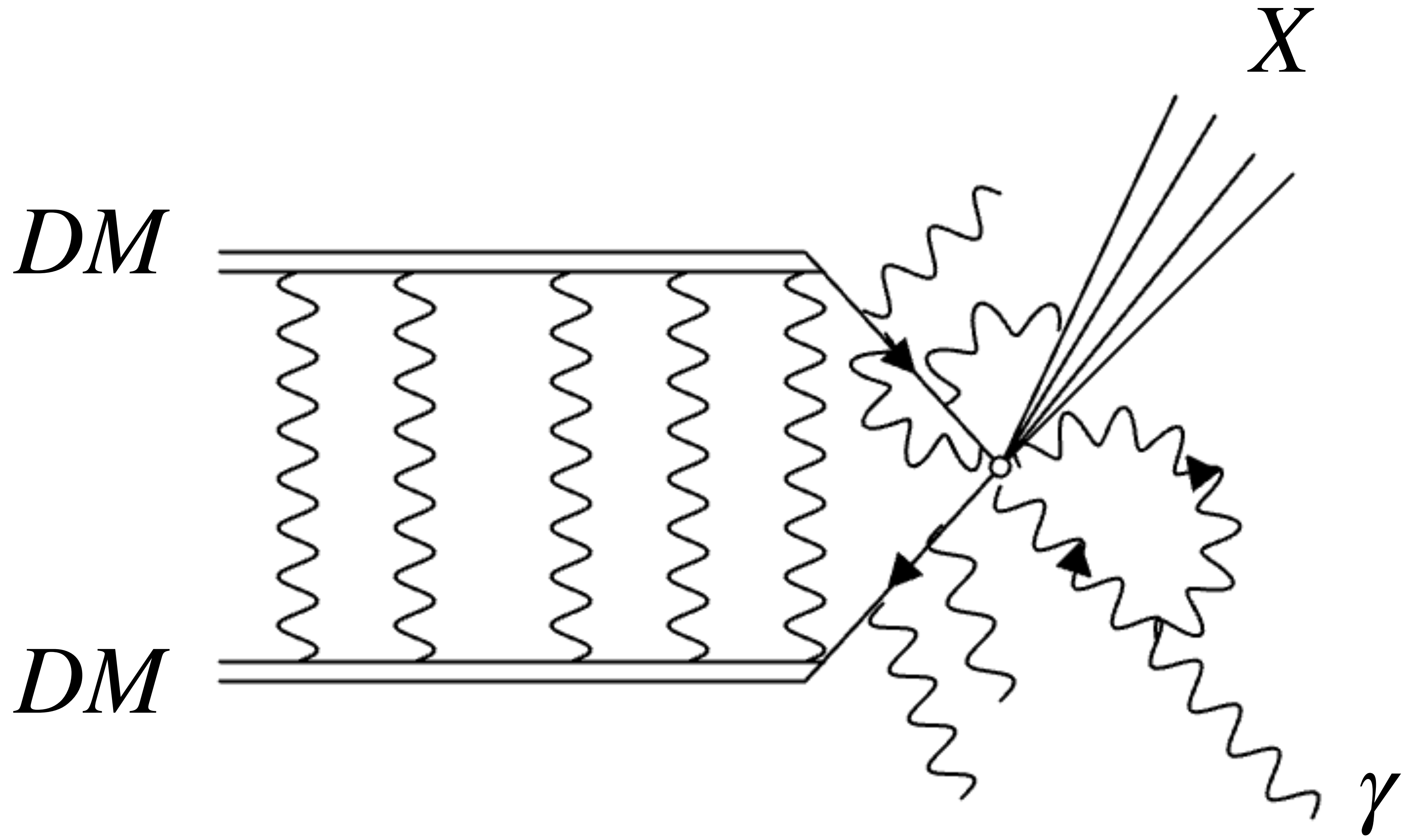


Sommerfeld enhancement



Sudakov double logs

$$m_X^2 \ll s = 4\, m_{\mathrm{DM}}^2$$



Sudakov double logs

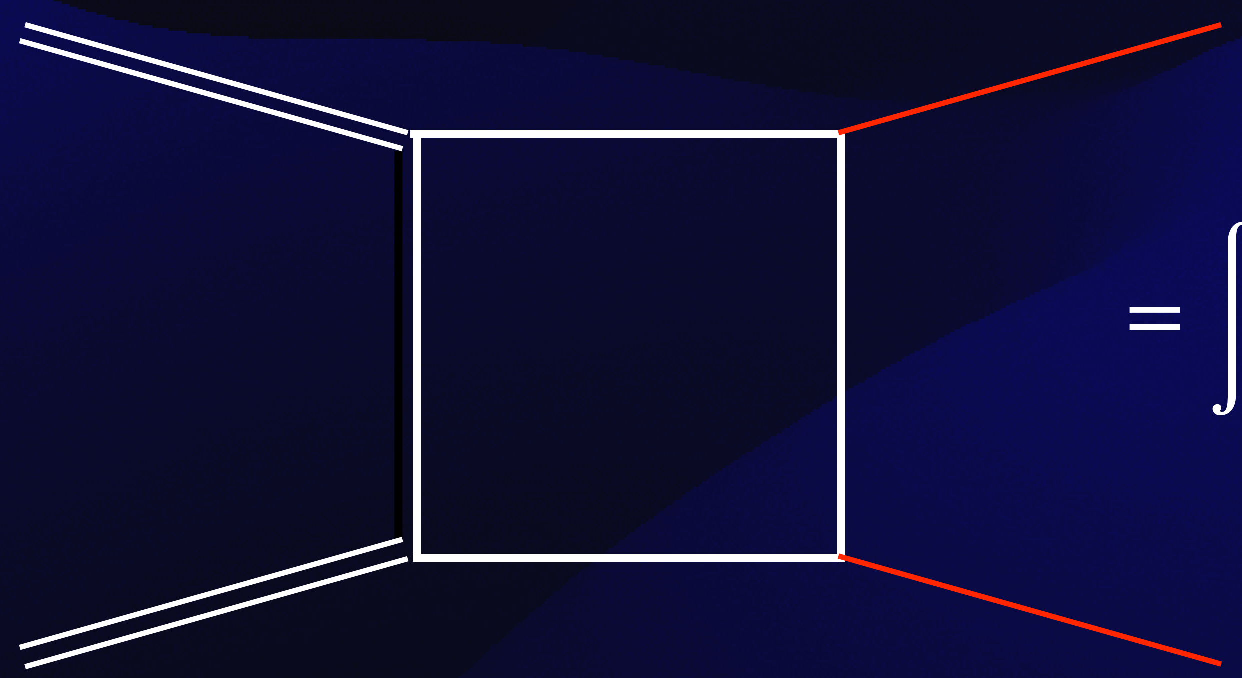
Soft Collinear Effective Field Theory (SCET)

Method of regions

$p_1 = m_\chi(1, \vec{0})$

$k_3 = m_\chi(1, \hat{n}) \equiv m_\chi n$

$I_\text{example} =$



$$= \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q+k_3-p_1)^2 - m_\chi^2} \frac{1}{(q+k_3)^2 - m_W^2} \frac{1}{q^2 - m_W^2} \frac{1}{(q-k_4)^2 - m_W^2}$$

$p_2 = m_\chi(1, \vec{0})$

$k_4 = m_\chi(1, -\hat{n}) \equiv m_\chi \bar{n}$

SCET for indirect DM detection

Method of regions

$$I_{\text{ex.}} = \square = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q+k_3-p_1)^2 - m_\chi^2} \frac{1}{(q+k_3)^2 - m_W^2} \frac{1}{q^2 - m_W^2} \frac{1}{(q-k_4)^2 - m_W^2}$$

Light-cone coordinates $q = q_c n + q_{\bar{c}} \bar{n} + q_\perp \longrightarrow (q_c, q_{\bar{c}}, q_\perp)$

Expand propagators in according to 4 different momentum scalings

$$q_h \sim m_\chi (1,1,1) \qquad q_s \sim m_W (1,1,1)$$

$$q_c \sim \left(\frac{m_W^2}{m_\chi}, m_\chi, m_W \right) \qquad q_{\bar{c}} \sim \left(m_\chi, \frac{m_W^2}{m_\chi}, m_W \right)$$

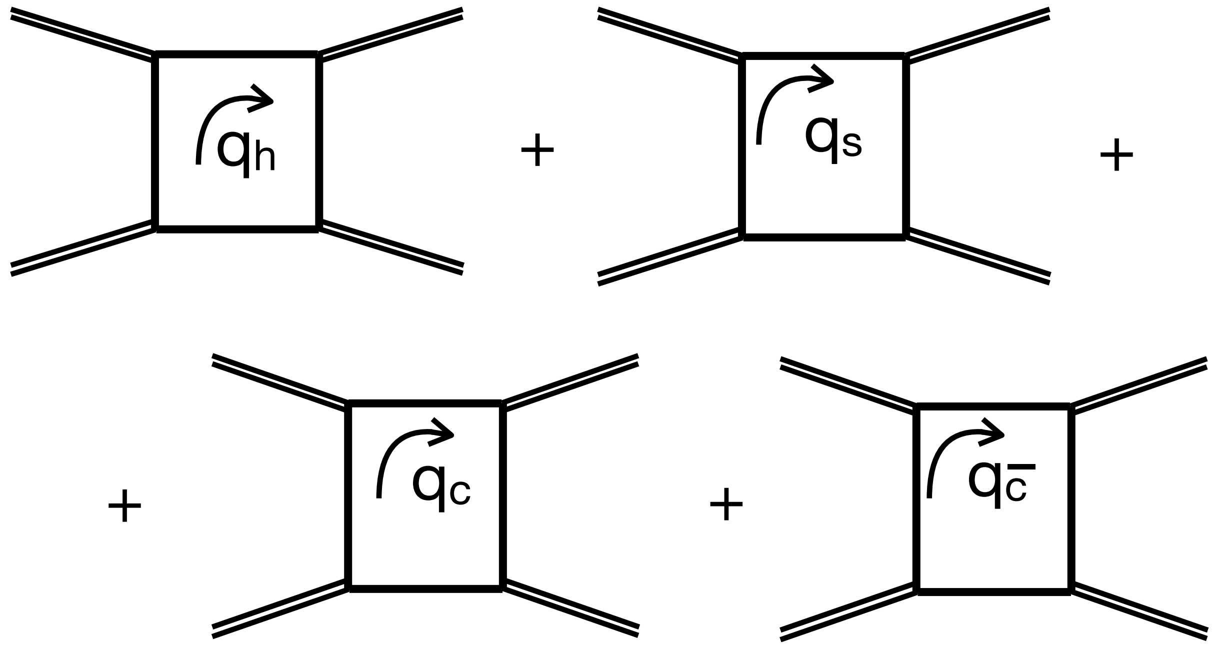
For example: $I_h = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q+k_3-p_1)^2 - m_\chi^2} \frac{1}{(q+k_3)^2} \frac{1}{q^2} \frac{1}{(q-k_4)^2}$

SCET for indirect DM detection

Method of regions

Let the magic happen:

$I_{\text{ex.}} =$

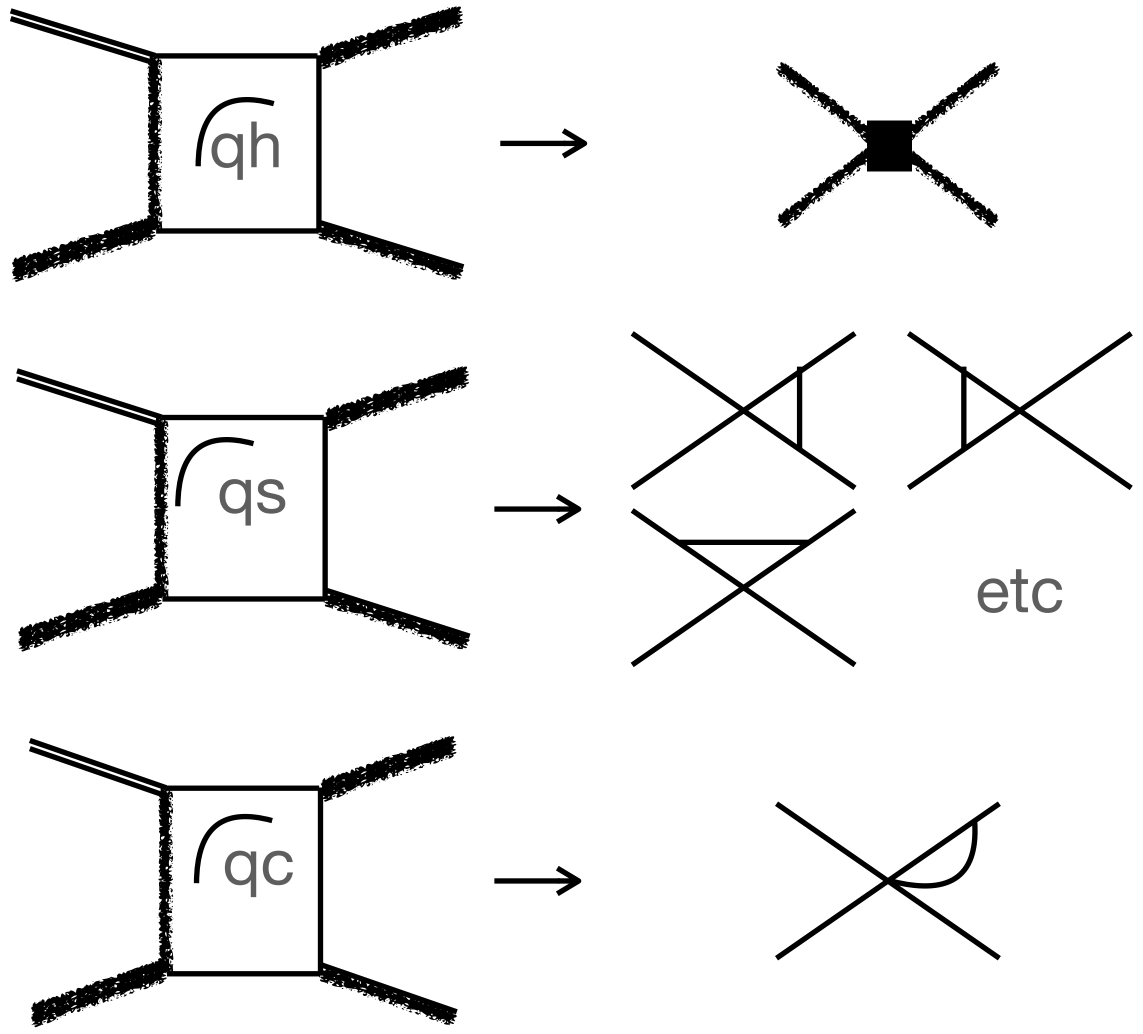


+ power corrections

SCET for indirect DM detection

Method of regions

1. Organize/formalize this procedure: SCET
2. Factorize



Factorization (after including all diagrams)

Wilson coefficients	only depend on the hard scale m_χ
Soft functions	only depend on the soft scale m_W
Jet functions	only depends on the typical jet scale ⚠

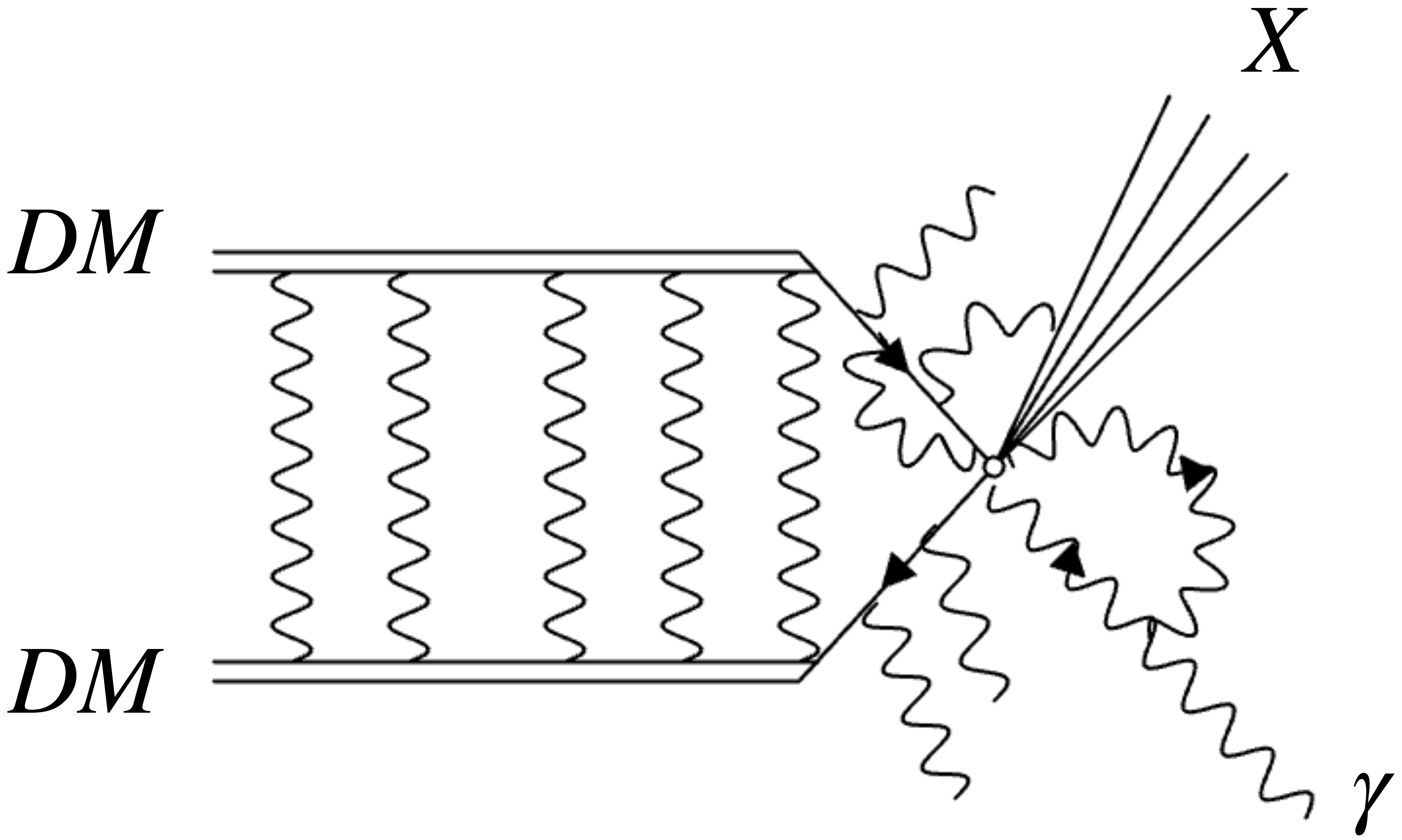
Fully resummed result

NREFT × SCET-II for indirect dark-matter detection

$$\frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v] = |\psi(0)|^2 \times |C|^2(\mu) \times Z_\gamma(\mu,\nu) \times J(\mu,\nu) \otimes W(\mu,\nu)$$

DMySpec

$$m_X^2 \ll s = 4\, m_{\mathrm{DM}}^2$$



Sudakov double logs

Endpoint

Regimes

- Narrow ‘nrw’: $4m_\chi^2 \gg m_X^2 \sim m_W^2$ (or $1 \gg 1-x \sim m_W^2/m_\chi^2$)
  - Beneke, Broggio, Hasner, MV — 1805.07367 — NLL’ for wino
  - Beneke, Hasner, MV, Urban — 1912.02034 — NLL’ for higgsino
- Intermediate ‘int’: $4m_\chi^2 \gg m_X^2 \sim 2\,m_\chi m_W$ (or $1-x \sim m_W/m_\chi$)
  - Beneke, Broggio, Hasner, MV, Urban — 1903.08702 — NLL’ for wino
  - Beneke, Hasner, MV, Urban — 1912.02034 — NLL’ for higgsino
- Wide: $4m_\chi^2 \gg m_X^2 \gg m_\chi m_W$ (or $1 \gg 1-x \gg m_W/m_\chi$)
  - Baumgart, Cohen, Moulin, Moult, Rinchiuso, Rodd, Slatyer, Stewart, Vaidya — 1808.08956 — NLL for wino
- Continuum: E_γ and $m_\chi - E_\gamma$ of $\mathcal{O}(m_\chi)$ (or $1-x$ of $\mathcal{O}(1)$)

Factorization formulas (Sudakov-log resumm.)

Regime ‘int’

$$\Gamma^{\mathrm{higgsino}}_{IJ}(E_\gamma) = \frac{1}{(\sqrt{2})^{n_{id}}}\frac{1}{4}\frac{2}{\pi m_\chi}\sum_{i,j} C_i(\mu)C_j^*(\mu) \times Z_\gamma^{WY}(\mu,\nu) \times \int \mathrm{d}\omega \left(J^{\mathrm{SU(2)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, W^{\mathrm{SU(2)},ij}_{IJ,WY}(\omega,\mu,\nu) + \right.$$

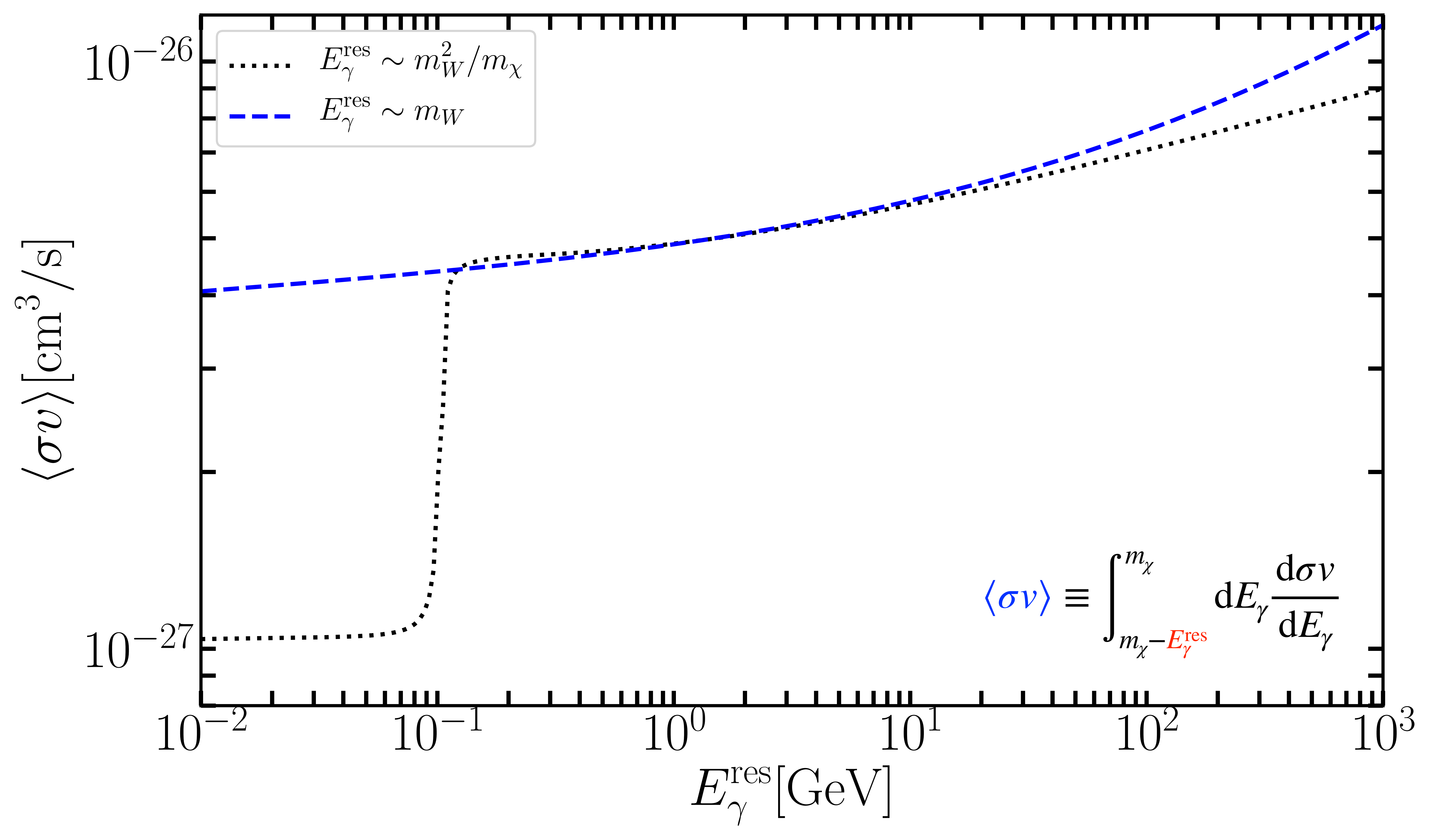
$$\left. + J^{\mathrm{U(1)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, W^{\mathrm{U(1)},ij}_{IJ,WY}(\omega,\mu,\nu) \right)$$

……

$$\Gamma^{\mathrm{wino}}_{IJ}(E_\gamma) = \frac{1}{(\sqrt{2})^{n_{id}}}\frac{1}{4}\frac{2}{\pi m_\chi}\sum_{i,j} C_i(\mu)C_j^*(\mu) \times Z_\gamma^{33}(\mu,\nu) \times \int \mathrm{d}\omega J^{\mathrm{SU(2)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, \tilde{W}^{ij}_{IJ}(\omega,\mu,\nu)$$

Numerical results

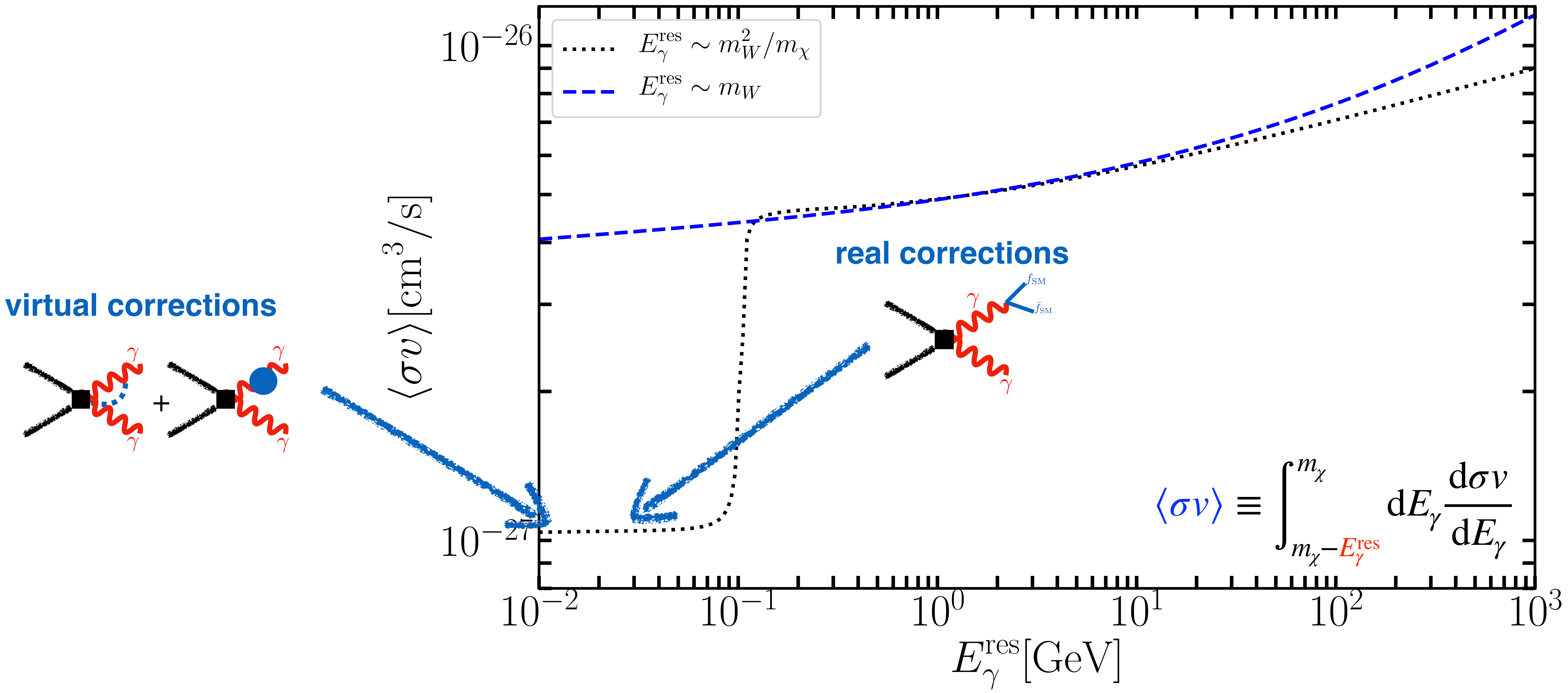
Cumulative cross sections



$$\langle\sigma v\rangle \equiv \int_{m_\chi - E_\gamma^{\rm res}}^{m_\chi} {\rm d}E_\gamma \frac{{\rm d}\sigma v}{{\rm d}E_\gamma}$$

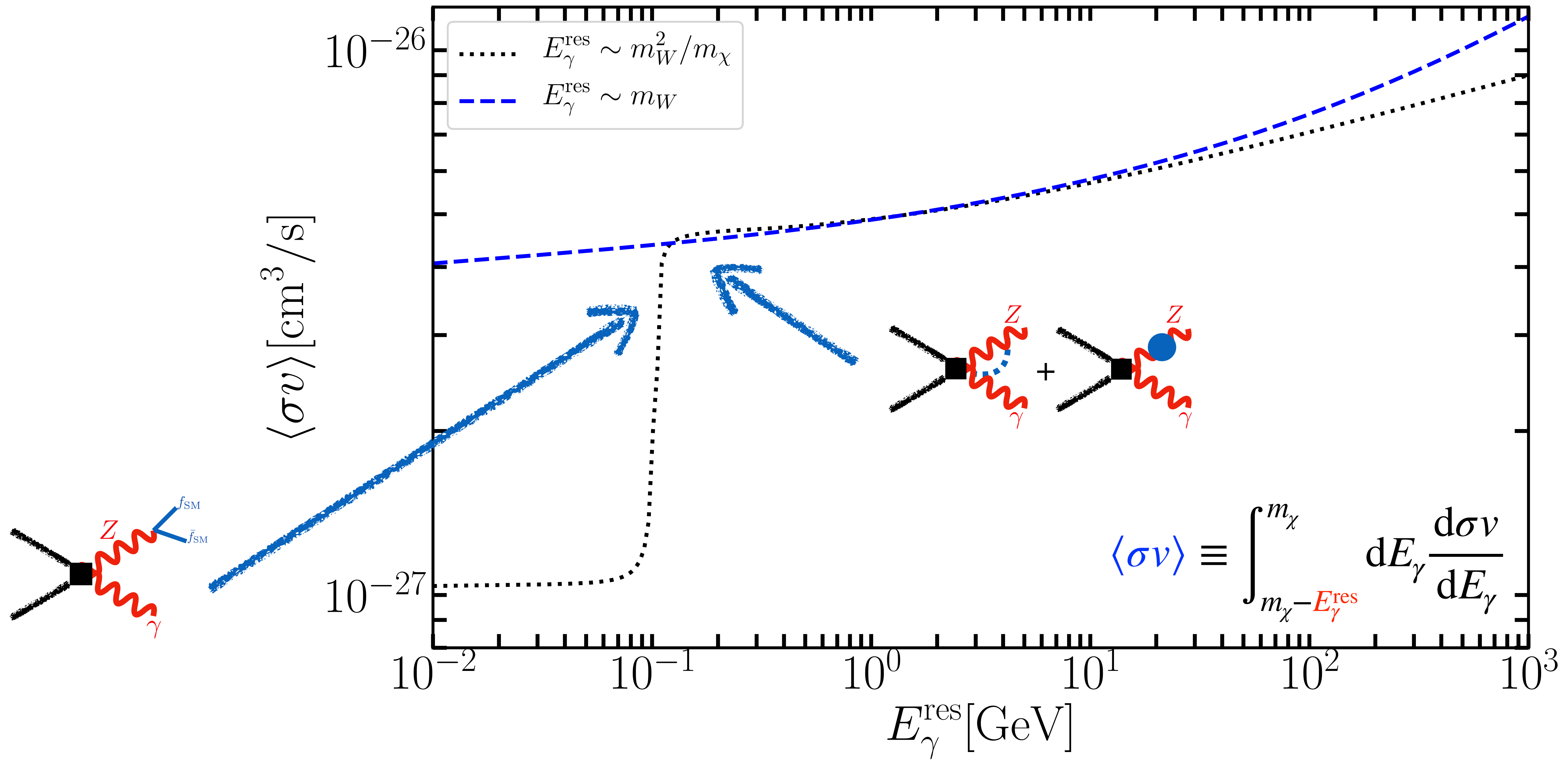
Numerical results

Cumulative cross sections



Numerical results

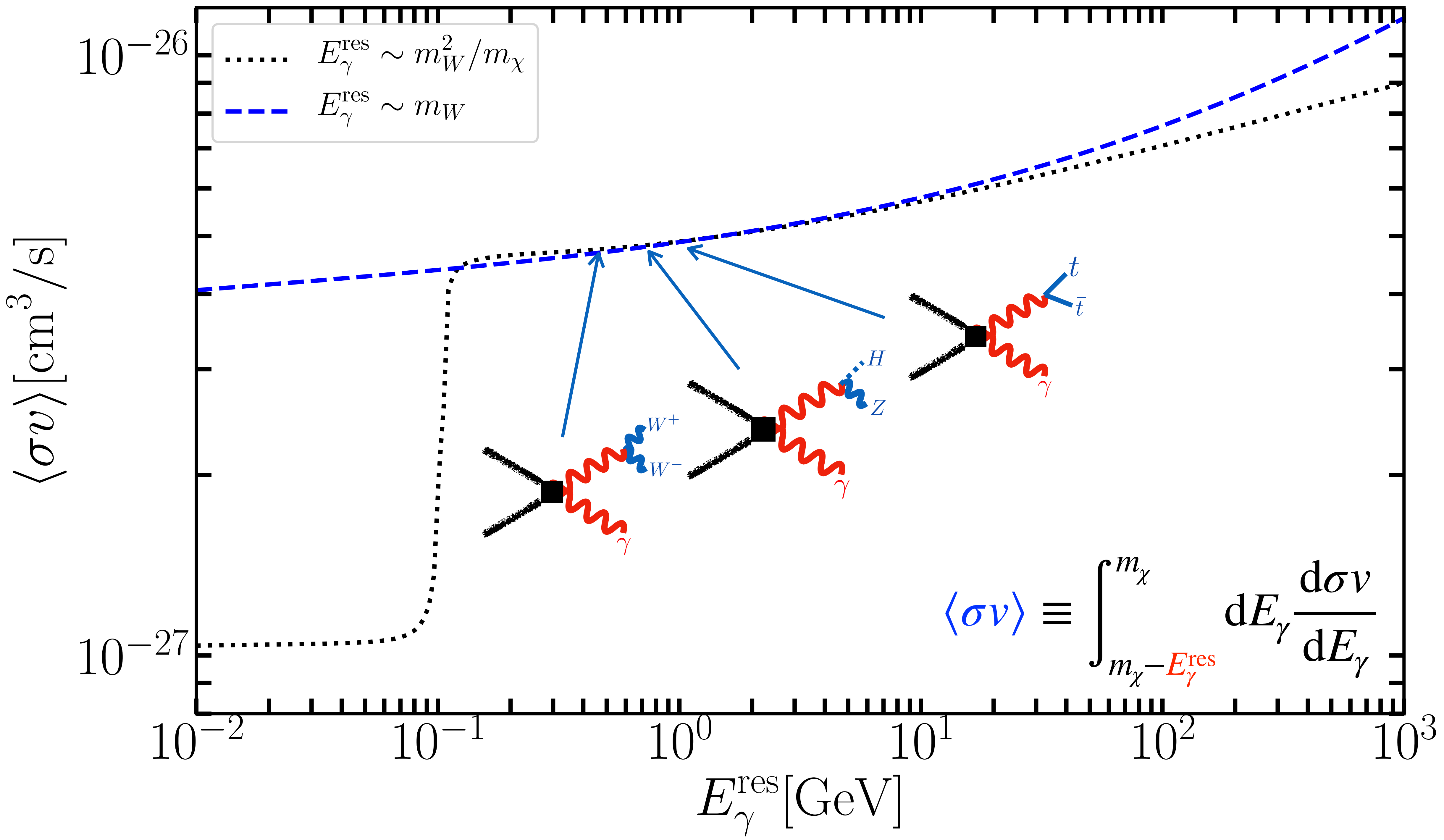
Cumulative cross sections



$$\langle\sigma v\rangle \equiv \int_{m_\chi - E_\gamma^{\rm res}}^{m_\chi} \mathrm{d}E_\gamma \frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma}$$

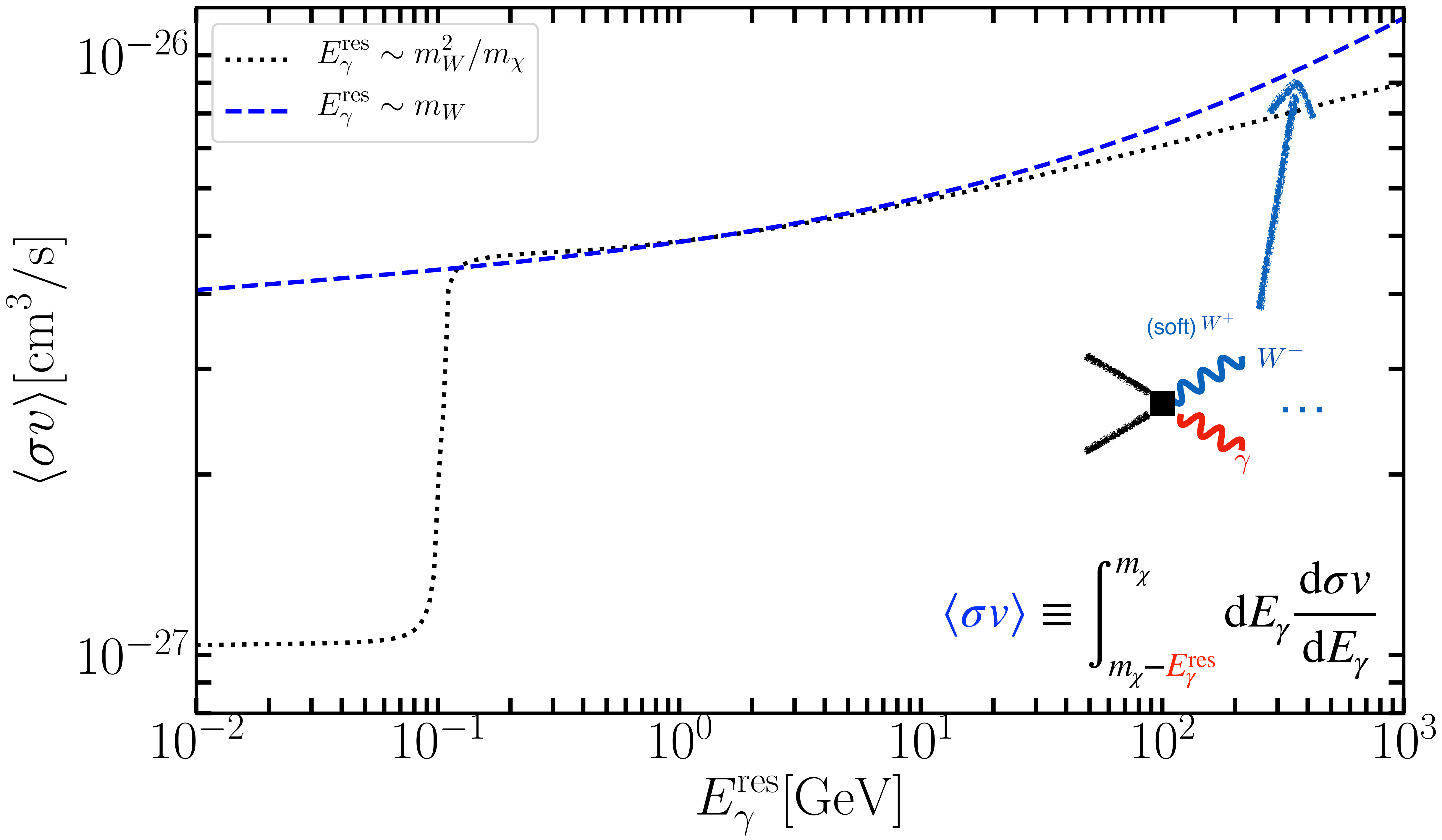
Numerical results

Cumulative cross sections



Numerical results

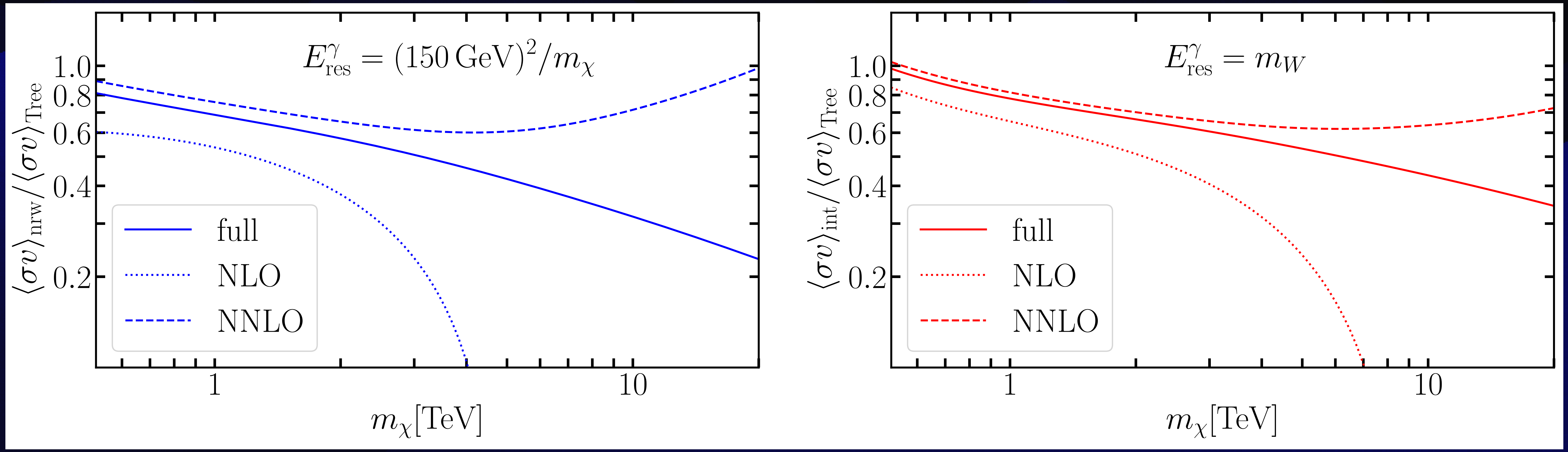
Cumulative cross sections



$$\langle\sigma v\rangle \equiv \int_{m_\chi - E_\gamma^{\mathrm{res}}}^{m_\chi} \mathrm{d}E_\gamma \frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma}$$

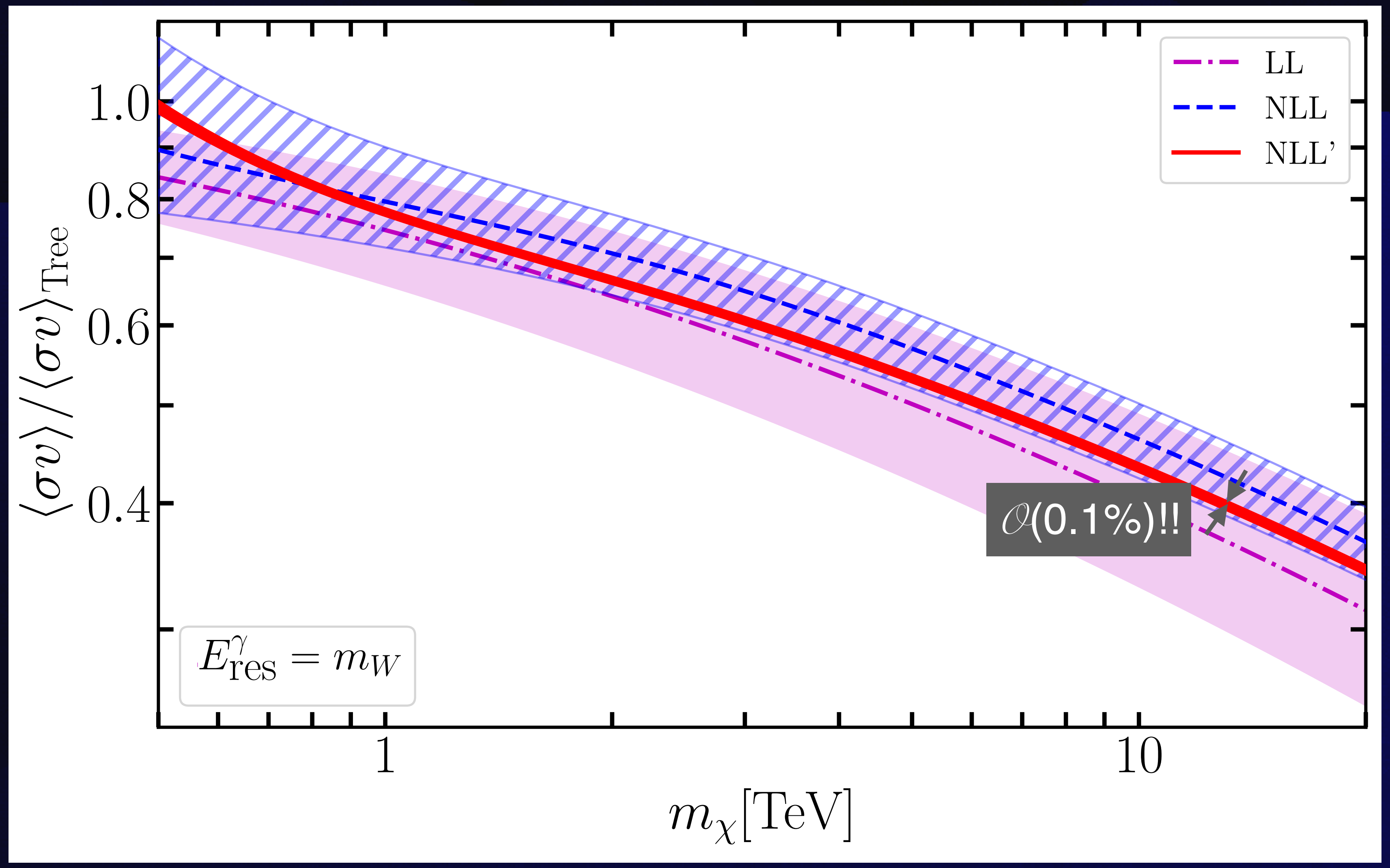
Fixed-order cross sections

Breakdown of the perturbative expansion (after Sommerfeld resummation)



Sudakov suppression

Scale variations



$\mathcal{O}$(0.1%)!!

Matching with the continuum (parton showers)

- Pure wino and higgsino annihilate into gauge bosons:

  Prescription to include Sommerfeld effect into the showering:

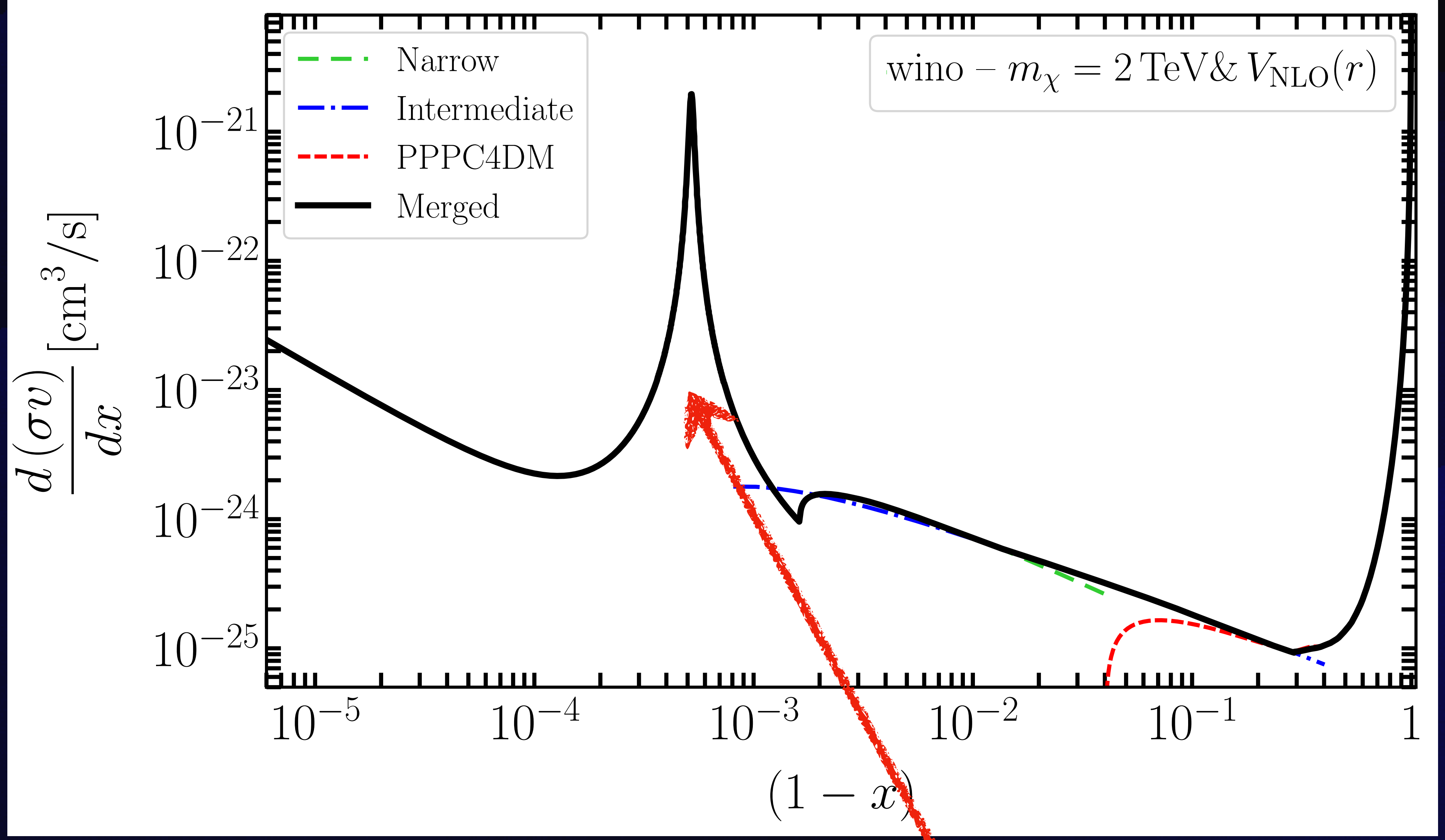
$$\frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma} = 2\sum_{I,J} S_{IJ}\Gamma_{IJ}^{\mathrm{cont.}}(E_\gamma)$$

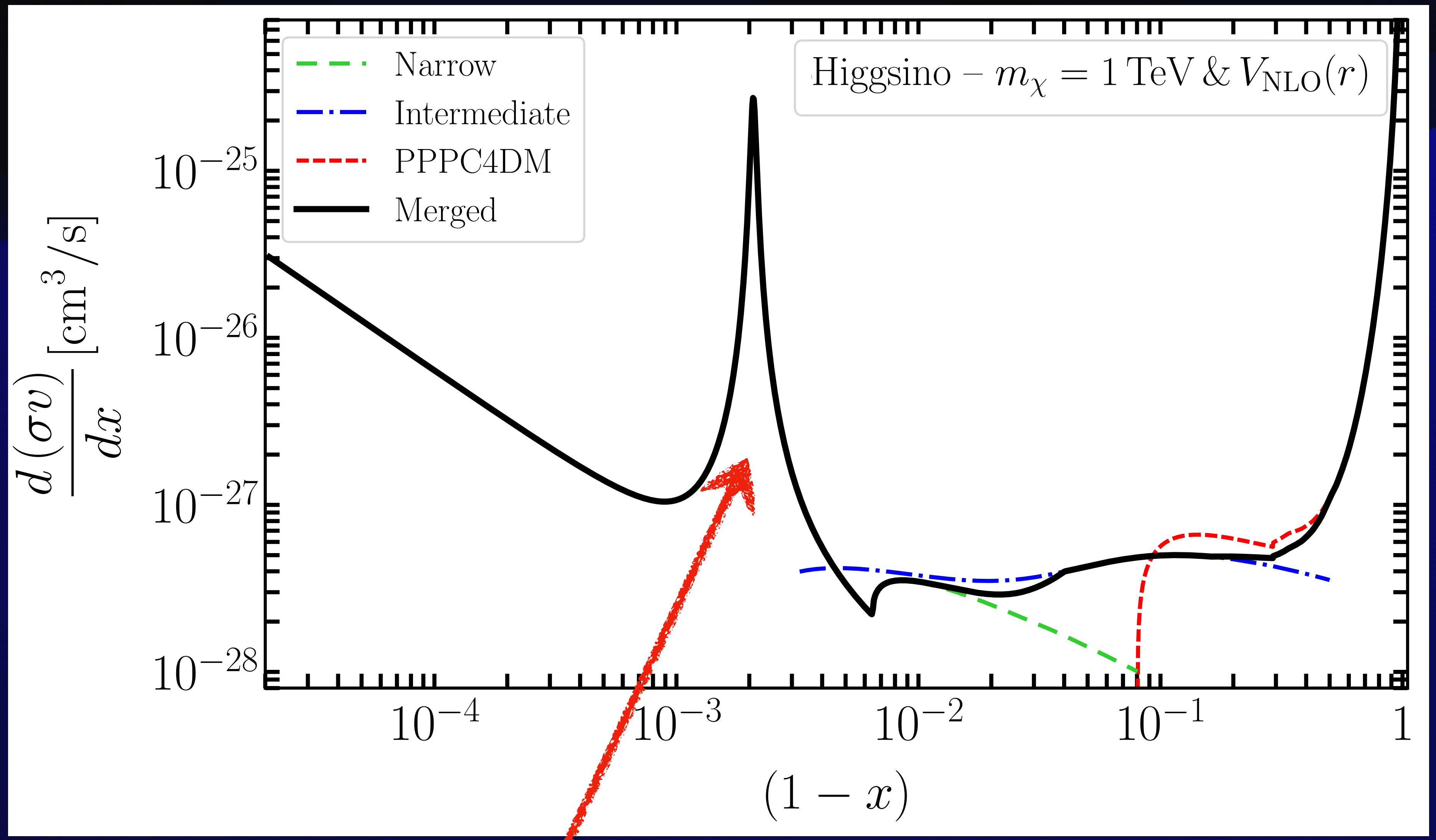
$$\Gamma_{IJ}^{\mathrm{cont.}}(E_\gamma) = [\sigma v]_{IJ}^{W^+W^-}\frac{\mathrm{d}N^{\mathrm{PPPC}}_{W_T^+W_T^-}}{\mathrm{d}E_\gamma} + [\sigma v]_{IJ}^{ZZ}\frac{\mathrm{d}N^{\mathrm{PPPC}}_{Z_TZ_T}}{\mathrm{d}E_\gamma} + [\sigma v]_{IJ}^{\gamma Z}\frac{\mathrm{d}N^{\mathrm{PPPC}}_{\gamma Z}}{\mathrm{d}E_\gamma} + [\sigma v]_{IJ}^{\gamma\gamma}\frac{\mathrm{d}N^{\mathrm{PPPC}}_{\gamma\gamma}}{\mathrm{d}E_\gamma},$$

- PPPC: Poor particle physicist cookbook for indirect dark matter detection.

DMγSpec

Full gamma-ray spectra for indirect wino/higgsino detection

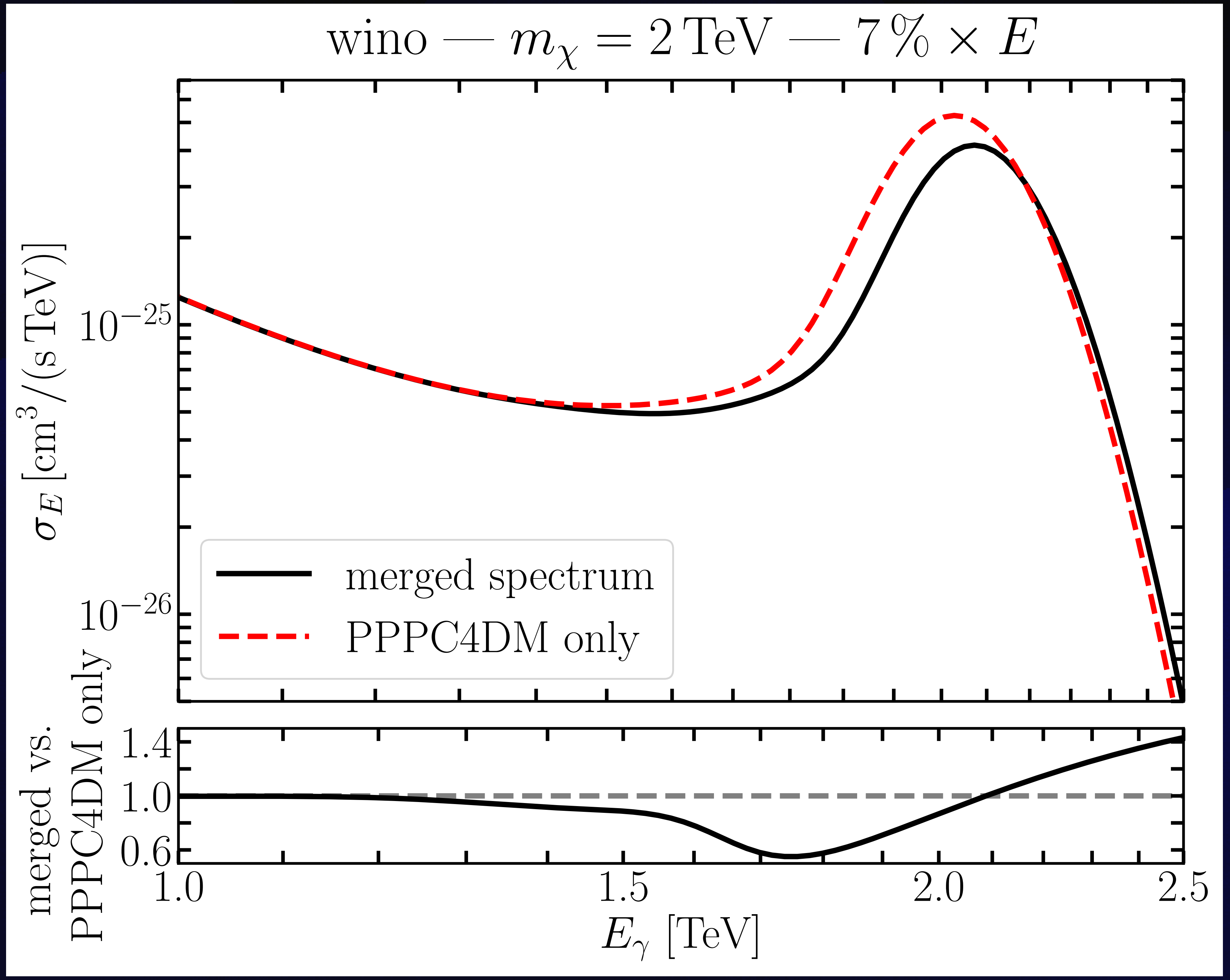


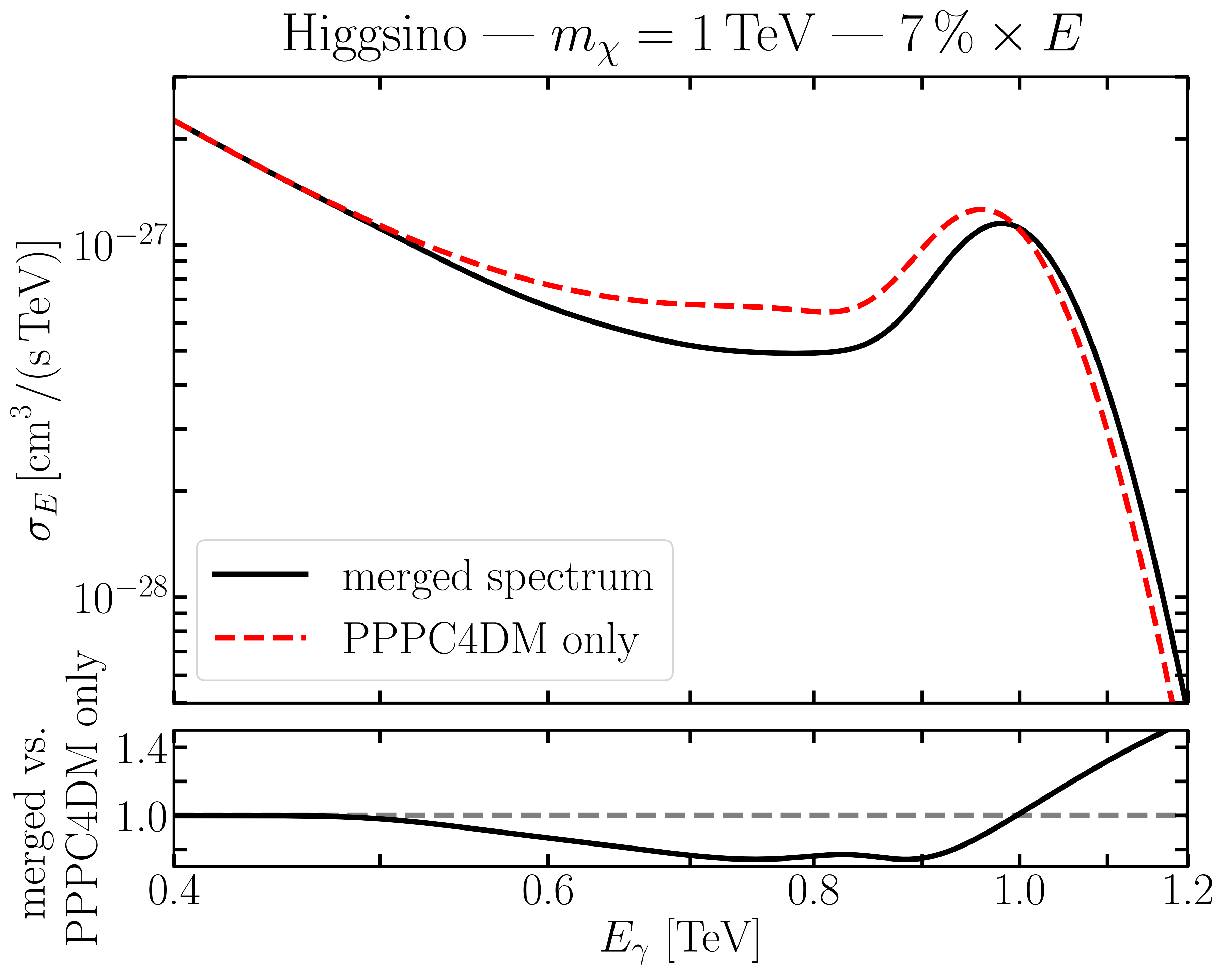


Dyson-resummed Z pole

DMγSpec

Gamma-ray spectra convoluted with an instrument response function





Conclusions

Conclusions

- Unexplored heavy WIMP parameter-space chunk to be probed by indirect detection observations in the near future
- Electroweak effects are extremely important
  - Besides Sommerfeld enhancements, Sudakov-log resummation at the endpoint plays a very important role
- Provided a complete description of prompt gamma-ray spectra from wimp annihilation for the benchmark wino and higgsino models

DMSpec

  - Demonstrated a perfect matching and consistency between different regimes/ calculations apparent in these spectra