Karlsruher Institut für Technologie

Non-factorisable corrections to *t*-channel single-top production at the LHC

Chiara Signorile-Signorile

LFC22

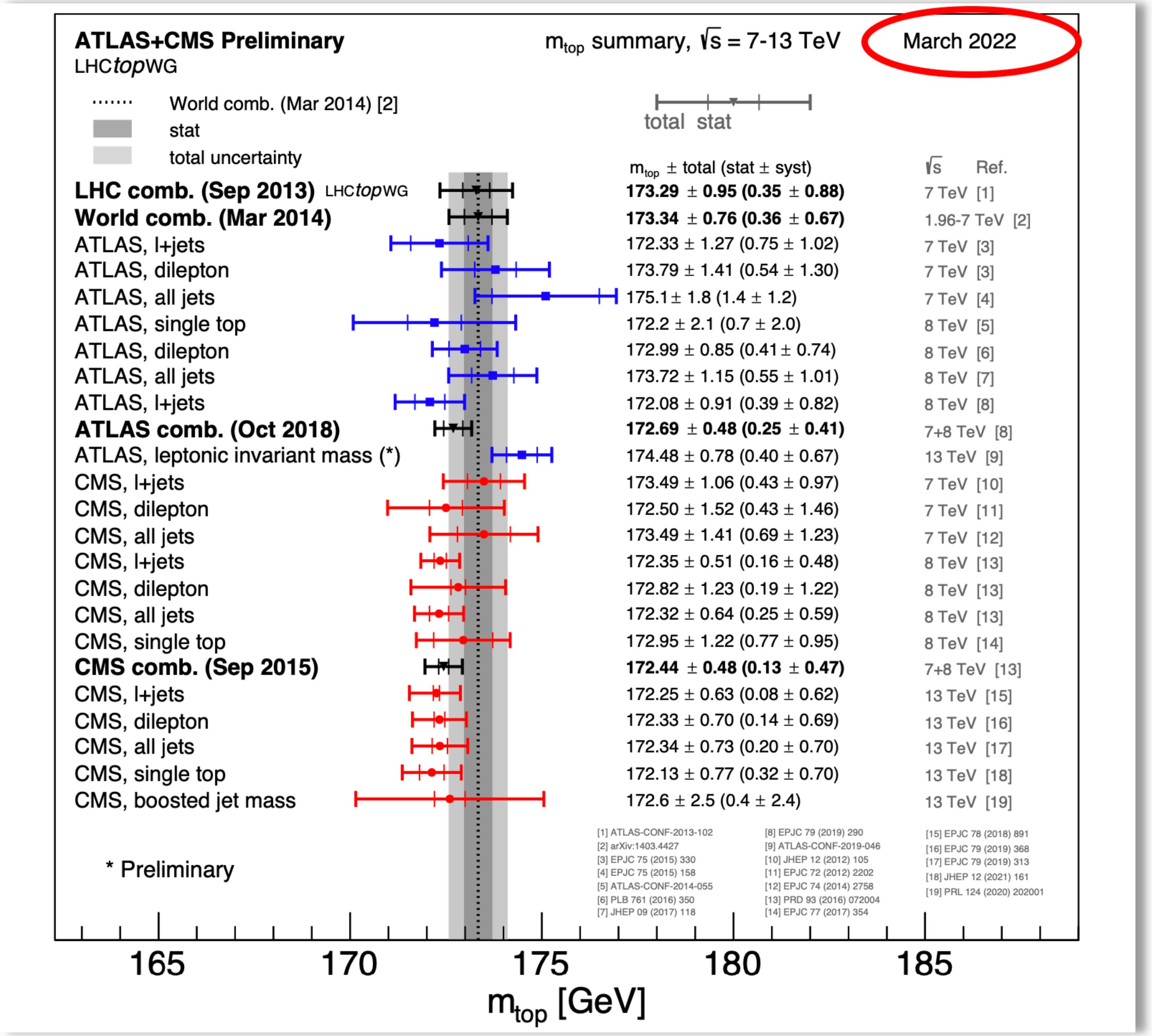
Trento, 31/08/2022

In collaboration with: Christian Brønnum-Hansen, Kirill Melnikov, Jérémie Quarroz and Chen-Yu Wang

Based on: arXiv 2204.05770

Motivation: why top quark?

- **Heaviest observed particle**
  - $m_t = (173.34 \pm 0.76)$ GeV [World Combination 14, ATLAS, CDF, CMS, D0]
- **Substantial Yukawa coupling**
  - $Y_t = \sqrt{2}\,\dfrac{m_t}{v} \sim 1$
- **Special relation with SM Higgs Boson**
- **Short lifetime** → decay before bound states can be formed

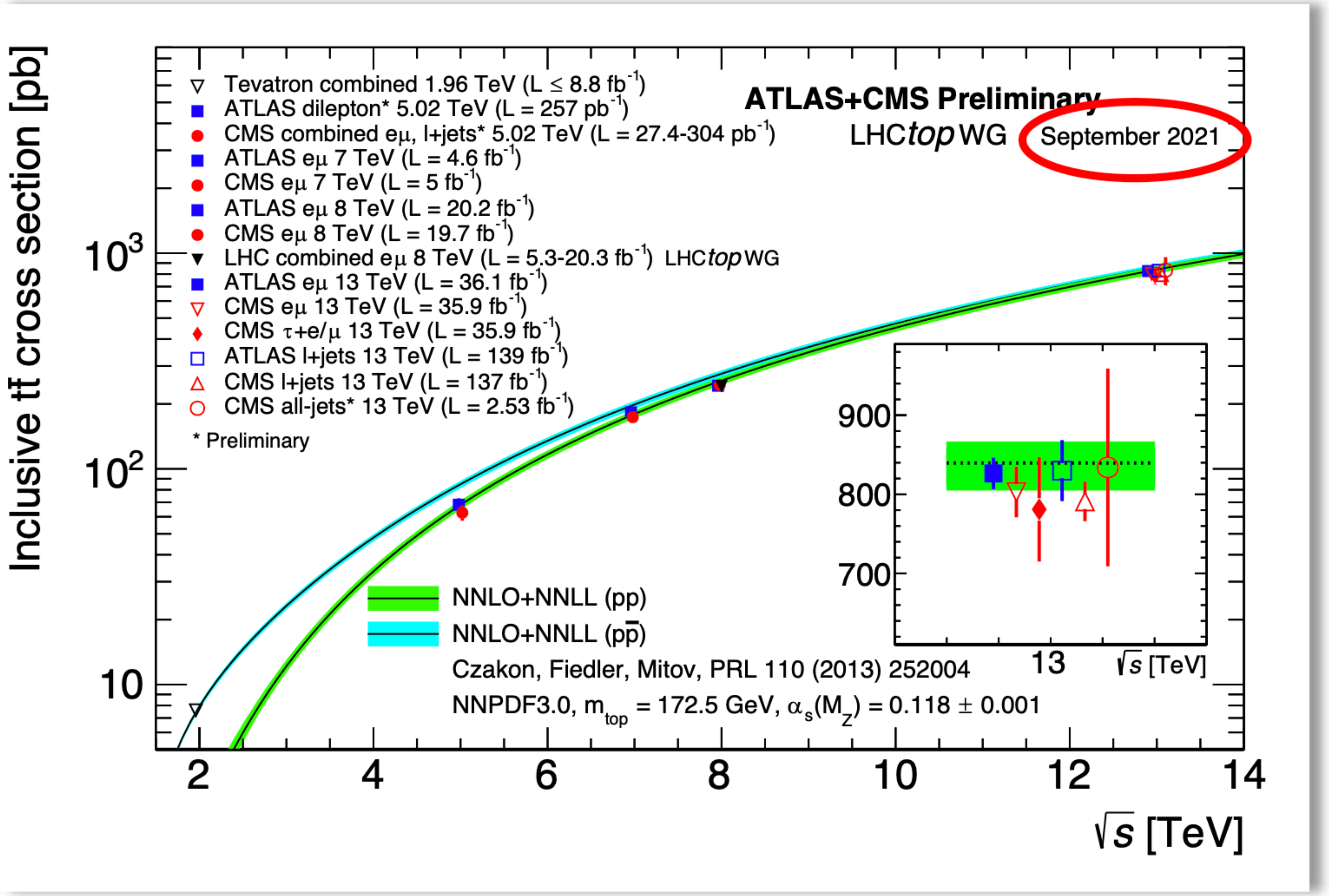


Precision top-quark Physics

- **Extracting SM parameters**
- **Constraining PDFs**
- **Examining (anomalous) couplings**
- **Better understanding of EW symmetry breaking**
- **Hints for heavy New Physics**

See Victor Miralles's talk!

Motivation: single-top production

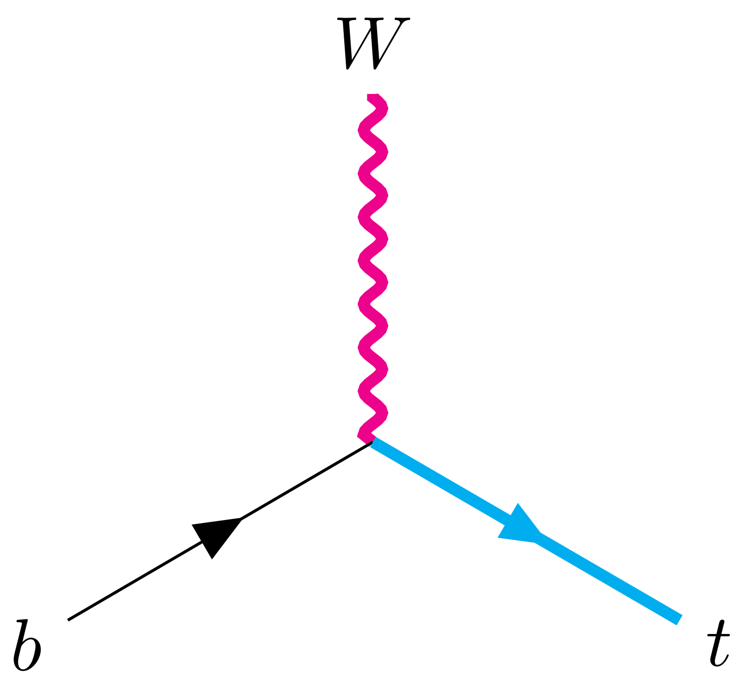


- At LHC **top quarks are mainly produced in pairs** via strong interactions
  - **Large production rate**
  - **Advanced theoretical predictions**:

    NLO QCD *[Nason, Dawson, Ellis '88]* and NLO EW corrections *[Kuhn, Scharf, Uwer '06]*

    total and fully differential NNLO QCD corrections in the NW approximation *[Czakon, Fiedler, Mitov '13, Behring, Czakon, Mitov et al. '19, Czakon, Mitov, Poncelet '21]*

- **Single-top production** also relevant
  - Production rate of the same order of magnitude as $t\bar{t}$: $\sigma_t \sim 1/4\,\sigma_{t\bar{t}}$
  - Electroweak mediated

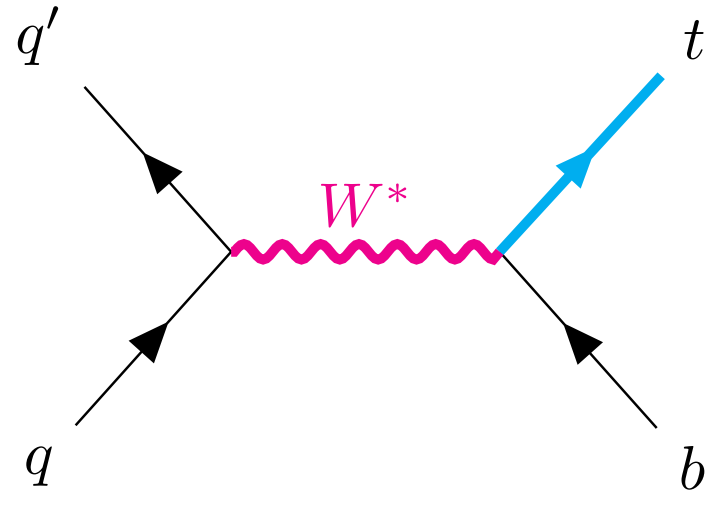


Determination of the **CKM matrix** V_{bt}

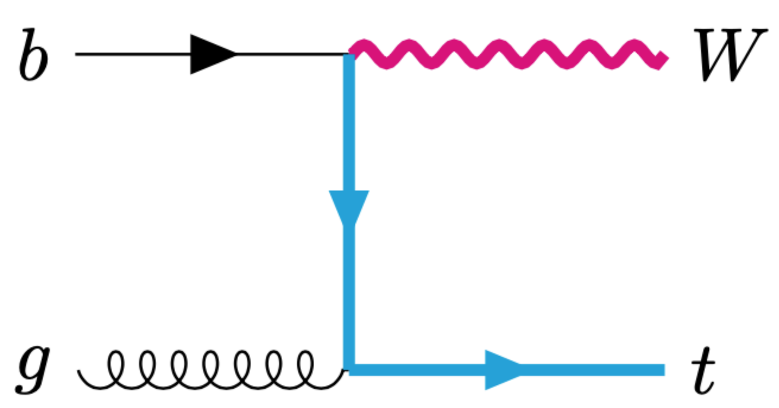
Indirect determination of **width** Γ_t and **mass** m_t

Bottom-quark PDF

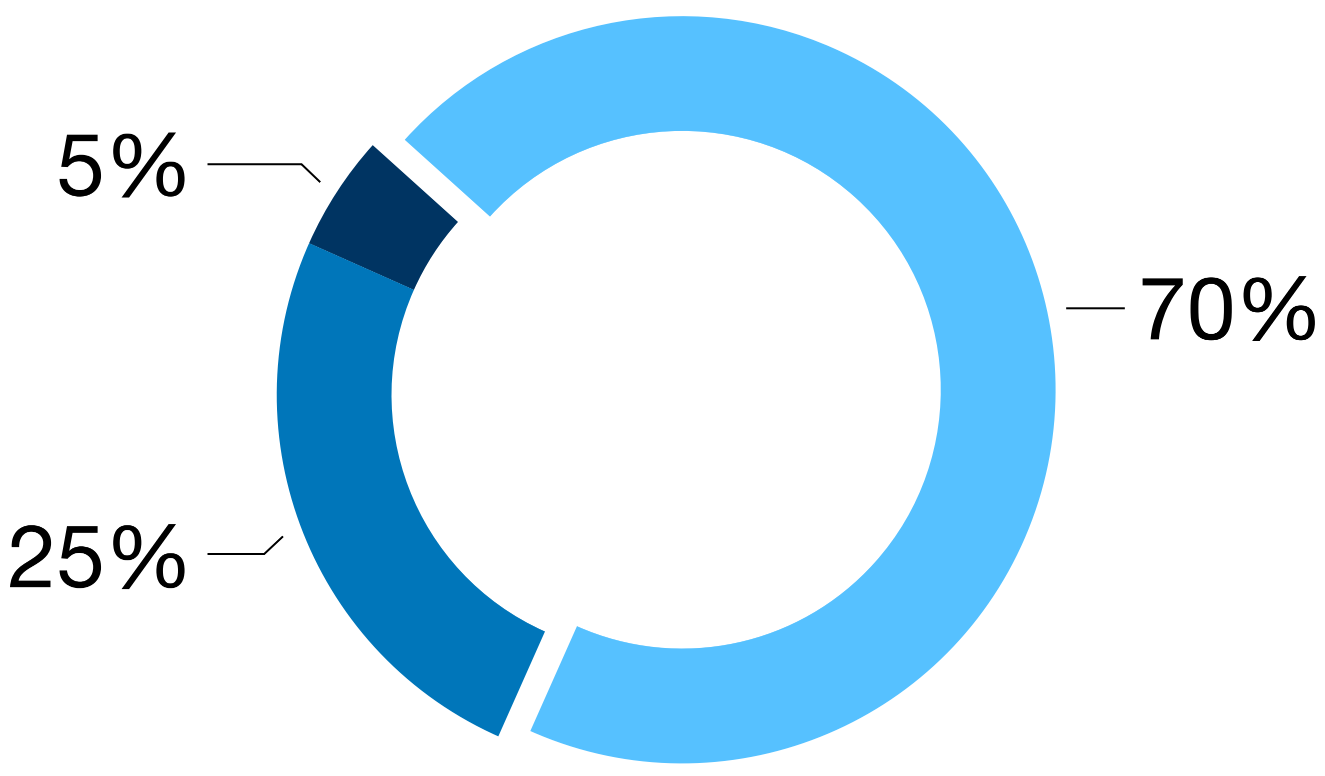
Single-top production: different modes



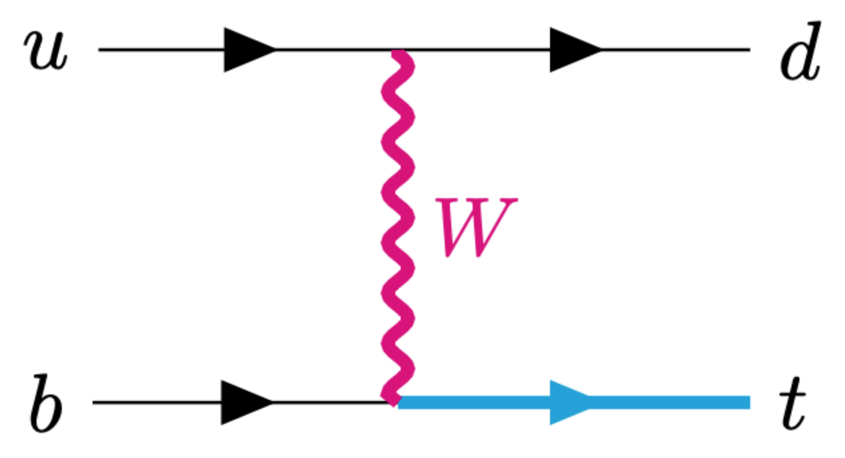
s-channel: $q\bar{q} \to W^* \to t\bar{b}$



associated production: $gb \to Wt$



t-channel: $qb \to q't$



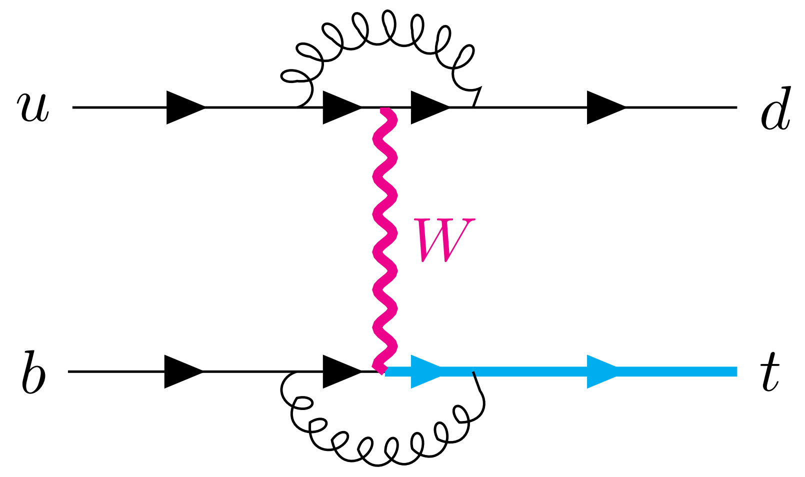
- **Two main topologies** contribute to the t-channel, single-top production:
  - **Factorisable contributions**

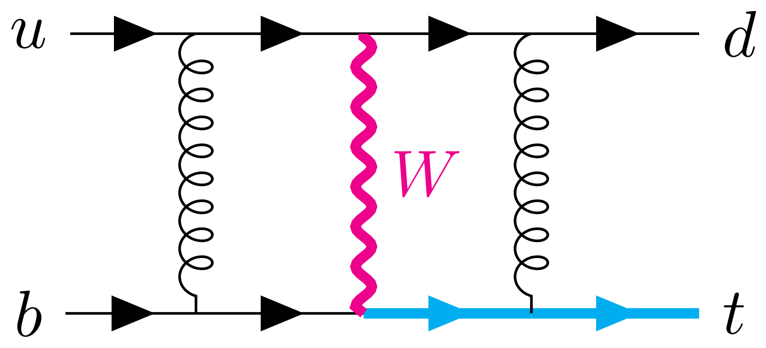
    Extensively investigated:

    NLO QCD *[Bordes, van Eijk '95][Campbell, Ellis, Tramontano '04][Cao, Yuan '05][Cao, Schwienhorst, Benitez, Brock, Yuan '05][Harris, Laenen, Phaf, Sullivan, Weinzierl '02] [Schwienhorst, Yuan, Mueller, Cao '11]*

    NNLO QCD *[Brucherseifer, Caola, Melnikov '14] [Berger, Gao, Yuan, Zhu '16] [Campbell, Neumann, Sullivan '21]*

  - **Non-factorisable contributions** → **cross talk between different quark lines**



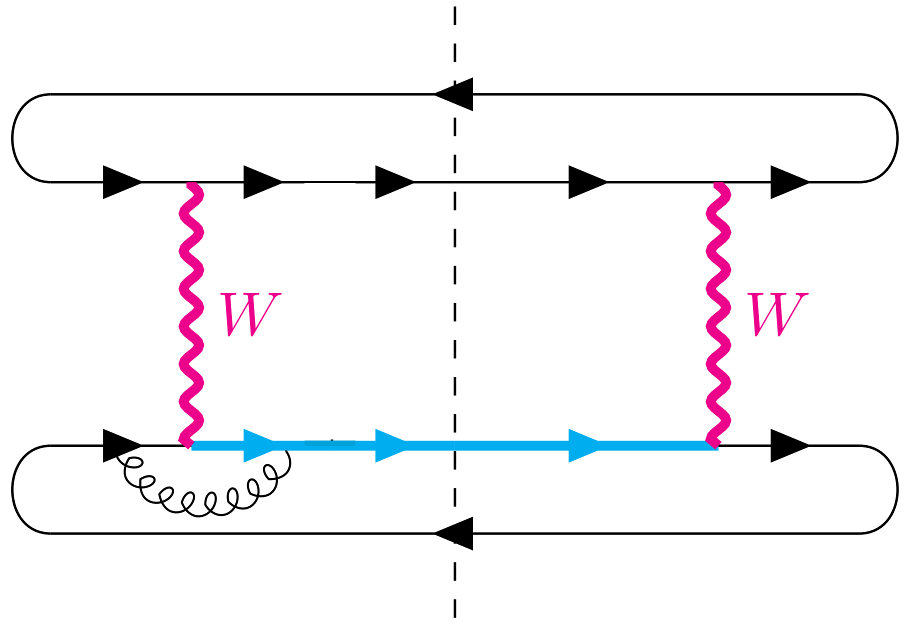


Factorisable *vs* non-factorisable corrections

[thanks to Jérémie and Christian for the nice picture in the slides]

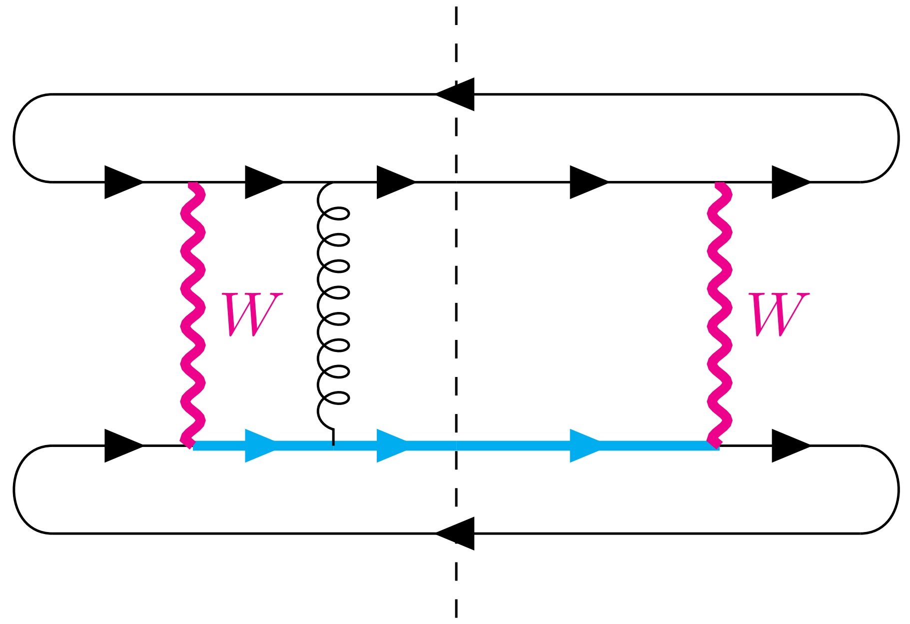
NLO

Factorisable contributions



$$\propto \mathrm{Tr}\left(t^a t^a\right) = \frac{N_c^2 - 1}{2}$$

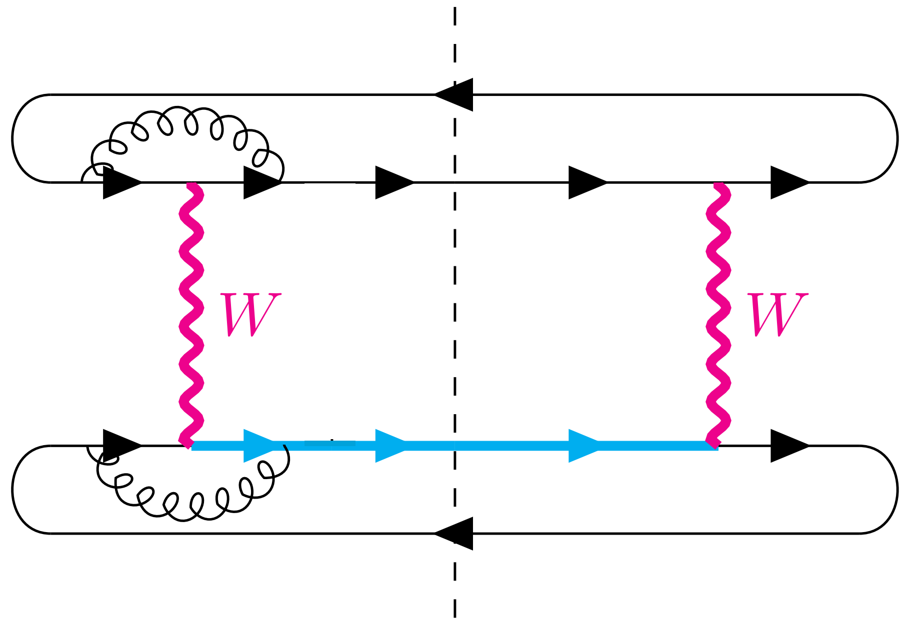
Non-factorisable contributions



$$\propto \mathrm{Tr}\left(t^a\right) = 0$$

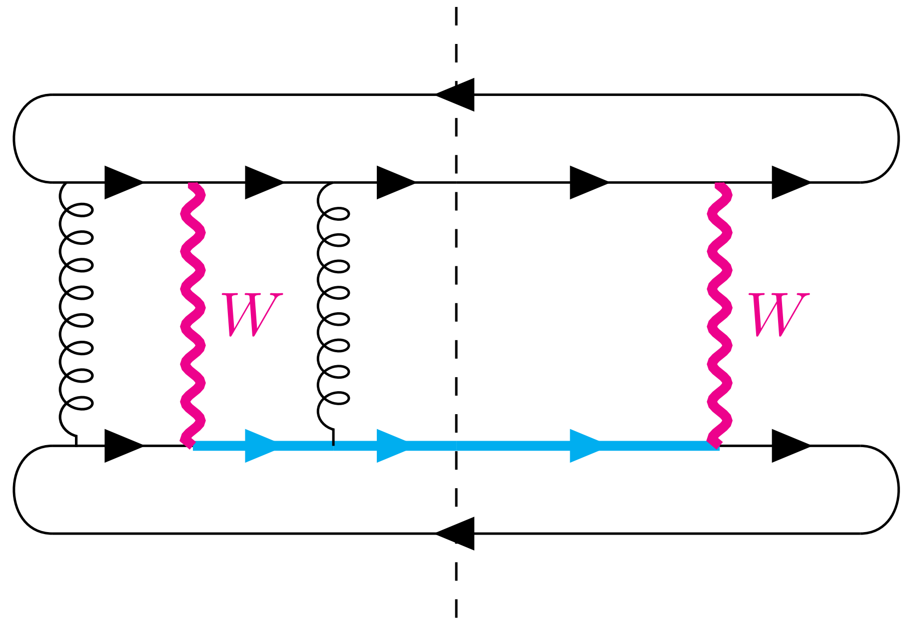
NNLO

Factorisable contributions



$$\propto \mathrm{Tr}\left(t^a t^a\right) \mathrm{Tr}\left(t^b t^b\right) = \frac{(N_c^2 - 1)^2}{4}$$

Non-factorisable contributions



$$\propto \mathrm{Tr}\left(t^a t^b\right) \mathrm{Tr}\left(t^a t^b\right) = \frac{N_c^2 - 1}{4}$$

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

Non-factorisable corrections: why?

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

However:

- Non-factorisable corrections could be enhanced by a factor $\pi^2 \sim 10$ due to the Glauber phase

  $\rightarrow$ proven for Higgs production in weak boson fusion in the eikonal approximation *[Liu, Melnikov et al. '19]*

- The actual size of NNLO non-factorisable corrections cannot be inferred from NLO contributions, since they vanish

Even though **non-factorisable contributions are suppressed by colour it is not guaranteed that they are actually negligible.**

In addition:

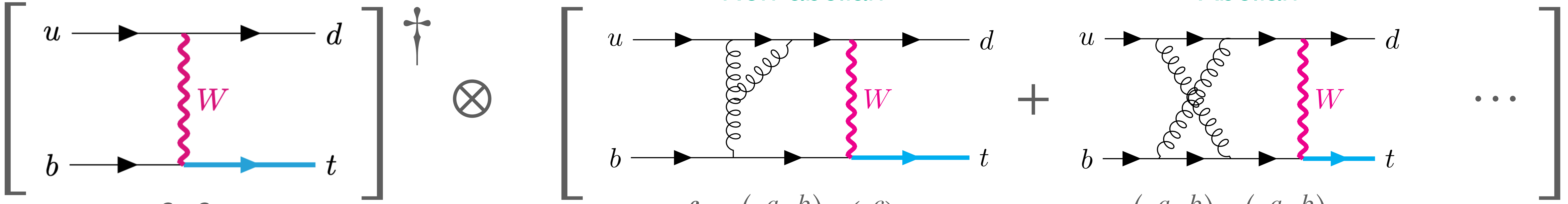
- Inclusive factorisable corrections are very small *[Brucherseifer et al. '14] [Berger et al. '16] [Campbell, Neumann et al. '21]*

$$\delta\sigma_{\text{fact.}}^{\text{NLO}} \sim (2-3\%)\,\sigma^{\text{LO}} \qquad \delta\sigma_{\text{fact.}}^{\text{NNLO}} \sim (1-3\%)\,\sigma^{\text{NLO}}$$

Non-factorisable corrections: main properties

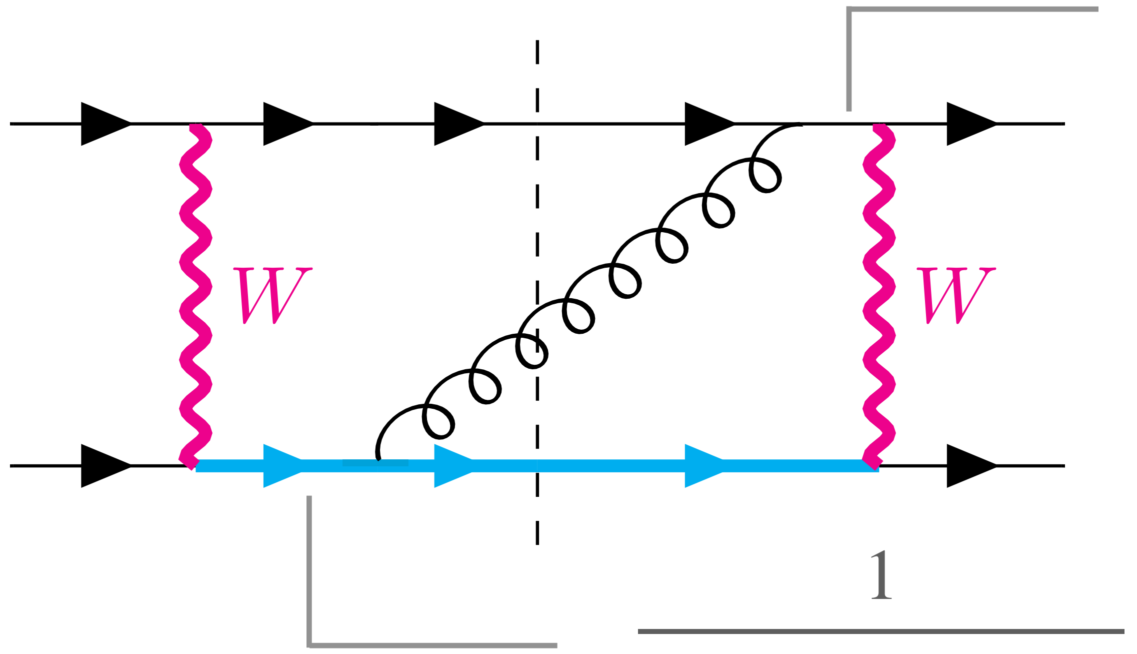
Non-factorisable contributions have to **connect upper an lower quark line**, and are effectively **Abelian**

Non-abelian   Abelian



$\delta_{ij}\,\delta_{kl}$ $f_{abc}\left(t^a t^b\right)_{ij}\left(t^c\right)_{kl} \rightarrow 0$ $\left(t^a t^b\right)_{ij}\left(t^a t^b\right)_{kl} \rightarrow \frac{N_c^2-1}{4}$

The **infrared structure is simplified**: **no collinear singularities**

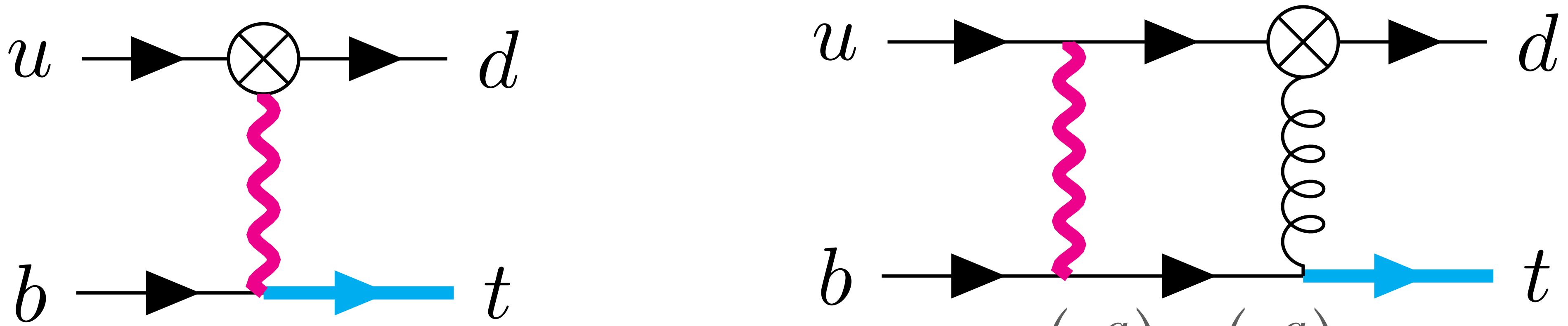


$$\frac{1}{E_g E_q (1-\cos\theta_{gq})}$$

$$\frac{1}{E_g E_t (1-\cos\theta_{gt})}$$

All IR singularities are of soft origin.

Non-factorisable contributions are **UV finite**



$\delta_{ij}\,\delta_{kl}$ $\left(t^a\right)_{ij}\left(t^a\right)_{kl}$

Renormalisation simply consists of $\alpha_s^{\text{bare}}\,\mu_0^{2\epsilon} = \alpha_s\,\mu^{2\epsilon}\,S_\epsilon$

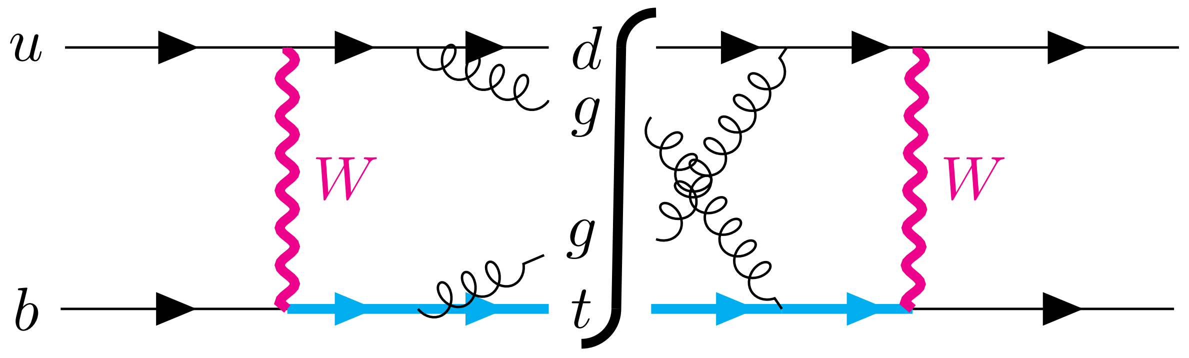
Non-factorisable corrections: ingredients of the calculation

Three terms contribute to the non-factorisable cross section

$$d\sigma^{\rm nf}_{pp\to X+t} = \sum_{i,j} \int dx_1\, dx_2\, f_i(x_1,\mu_F)\, f_j(x_2,\mu_F)\, d\hat{\sigma}^{\rm nf}_{ij\to X+t}(x_1,x_2)$$

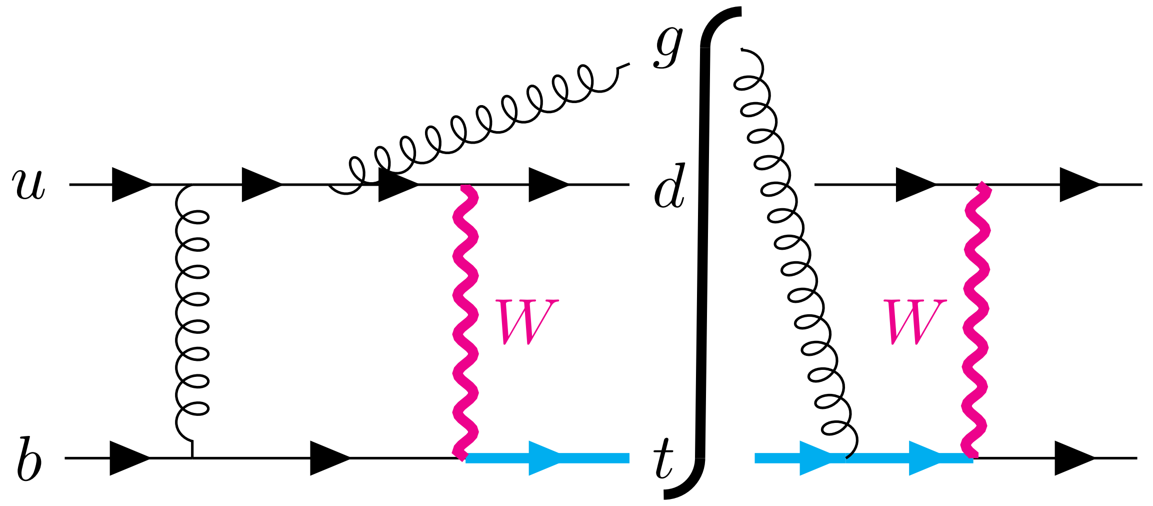
$$d\hat{\sigma}^{\rm nf}_{\rm NNLO} = d\hat{\sigma}^{\rm nf}_{\rm RR} + d\hat{\sigma}^{\rm nf}_{\rm RV} + d\hat{\sigma}^{\rm nf}_{\rm VV}$$

$$d\hat{\sigma}^{\rm nf}_{\rm RR} \; : \; \mathscr{A}^{(0)}_6 \otimes \mathscr{A}^{(0)}_6$$

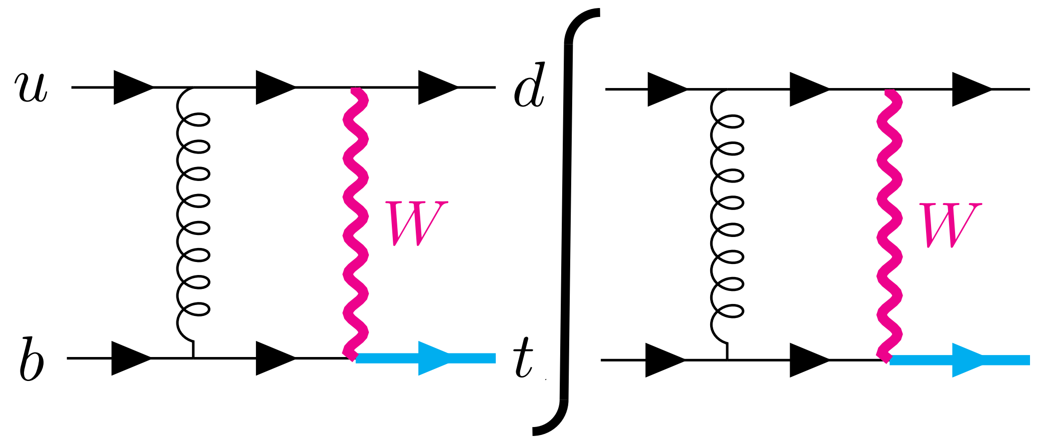


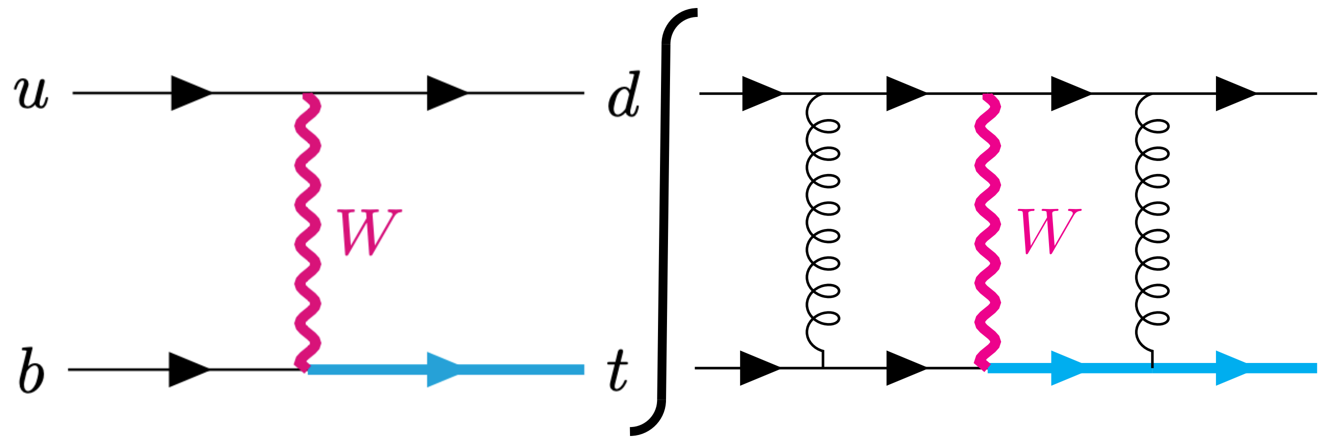
Each ingredient requires **specific treatment** and encodes **difficulties** to overcome

$$d\hat{\sigma}^{\rm nf}_{\rm RV} \; : \; \mathscr{A}^{(1)}_5 \otimes \mathscr{A}^{(0)}_5$$



$$d\hat{\sigma}^{\rm nf}_{\rm VV} : \mathscr{A}^{(1)}_4 \otimes \mathscr{A}^{(1)}_4 \,,\, \mathscr{A}^{(0)}_4 \otimes \mathscr{A}^{(2)}_4$$



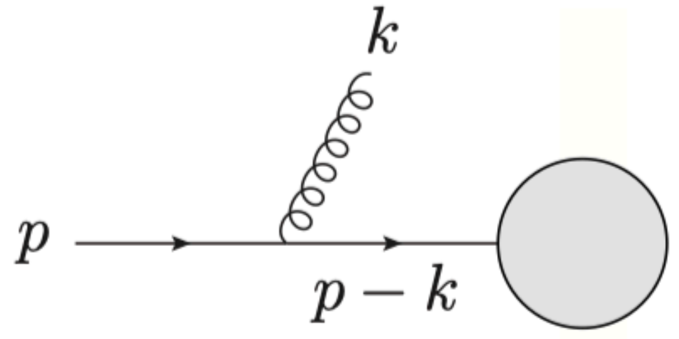


IR singularities

Higher-order corrections are affected by **infrared singularities arising from unresolved radiation**.

- Virtual corrections:
  - Explicit IR singularities from loop integrations $\rightarrow$ poles in $1/\epsilon$

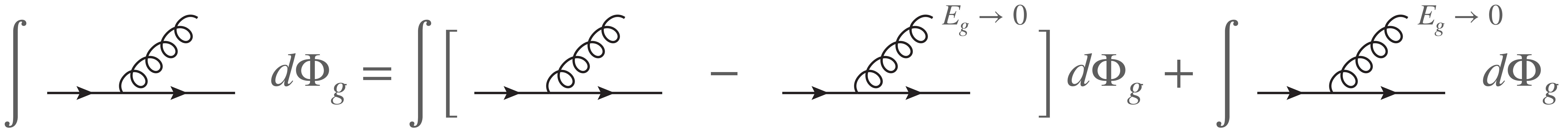
- Real corrections:
  - Singularities after integration over full phase space of radiated parton



$$\sim \frac{1}{(p-k)^2} = \frac{1}{2E_p E_k(1-\cos\theta)} \xrightarrow[\substack{E_k \to 0 \\ \textbf{or} \\ \theta \to 0}]{} \infty\,. \qquad \int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}2E_k}\,|M(\{p\},k)|^2 \underset{\substack{E_k \to 0 \\ \theta \to 0}}{\sim} \int \frac{\mathrm{d}E_k}{E_k^{1+2\epsilon}}\,\frac{\mathrm{d}\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{4\epsilon^2}\,.$$

  - **Integrating implies losing kinematic information** (needed for distributions, kinematic cuts, …)
  - For **non-factorisable** corrections **only soft** limits are relevant $\rightarrow$ only $1/\epsilon$ poles

$\longrightarrow$ **Subtraction scheme**: extract singularities without integrating over full phase space of radiated partons



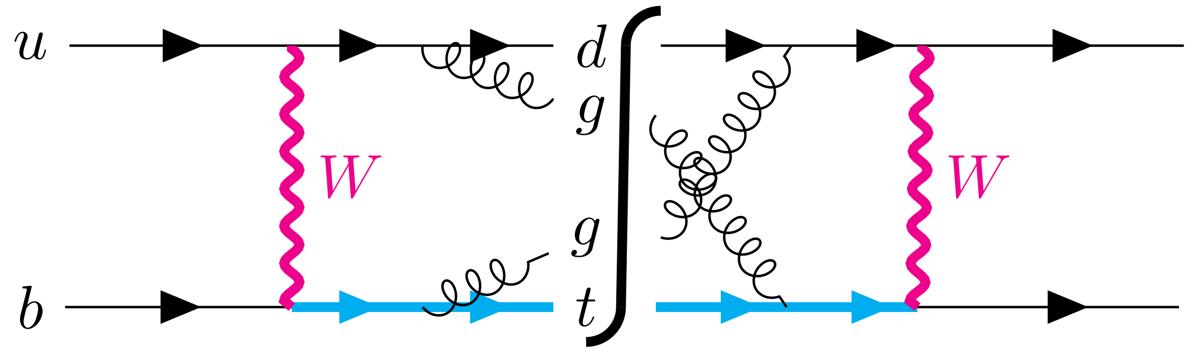
Finite in d=4, integrable numerically

exposes the same $1/\epsilon$ poles as the virtual correction

Double-real emission

Main issue of the **double-real contribution: extract and regularise IR singularities** preserving the fully-differential nature of the calculation

→ **Nested soft-collinear subtraction scheme** *[Caola, Melnikov, Röntsch 1702.01352]*



$$F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right) = \mathcal{N}\int {\rm dLips}_{34}\,(2\pi)^d\,\delta^{(d)}\left(p_1+p_2-\sum_{i=3}^{6}p_i\right)\times\left|\mathcal{A}_0\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right|^2_{\rm nf}$$

Integration over potentially unresolved phase space

$$[{\rm d}p] = \frac{{\rm d}^{d-1}p}{(2\pi)^{d-1}2E_p}\,\theta\left(E_{\rm max}-E_p\right)$$

$$2s\cdot\sigma_{\rm RR}^{\rm nf} = \frac{1}{2!}\int[{\rm d}p_5][{\rm d}p_6]\,F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\equiv\left\langle F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$

Separate the **soft-divergent part** from the **soft-finite contribution**

$$\left\langle F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle = \left\langle S_5\,S_6\,F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$ → **Double-soft** counterterm

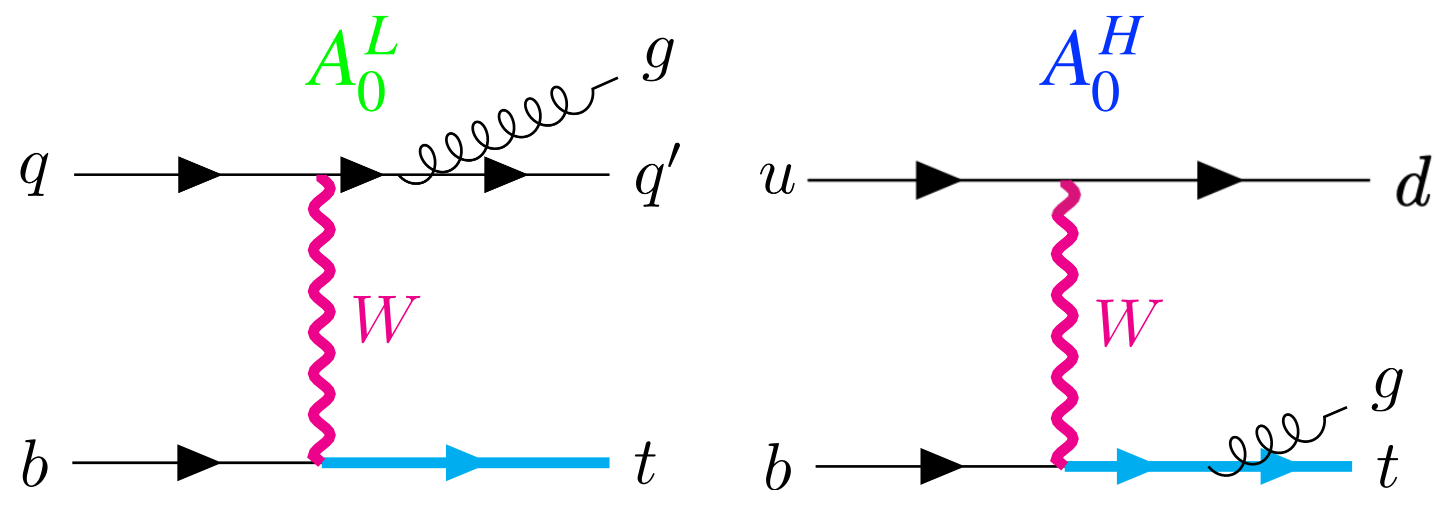
$$+2\left\langle S_6\,(I-S_5)\,F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$ → **Single-soft** counterterm

$$+\left\langle (I-S_5)\,(I-S_6)\,F_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g,6_g\right)\right\rangle$$ → **Resolved** contribution

Dealing with soft limits

Consider **single emission**: simpler bookkeeping, clear procedure

1. Decompose the amplitude into **colour-stripped, sub-amplitudes**



$$\left\langle c \,|\, \mathscr{A}_0\left(1_q, 2_b, 3_{q'}, 4_t; 5_g\right)\right\rangle = g_{s,b}\left[t^{c_5}_{c_3c_1}\delta_{c_4c_2}\, A_0^L\left(5_g\right) + t^{c_5}_{c_4c_2}\delta_{c_3c_1}\, A_0^H\left(5_g\right)\right]$$

2. Under **soft limit**, sub-amplitudes factorise into **universal eikonal factors** and **lower-multiplicity amplitudes**

$$S_5 \;\; A_0^{L/H}\left(5_g\right) = \varepsilon_\mu^{(\lambda)}(5)\; J^\mu(3,1;5)\; A_0\left(1_q, 2_b, 3_{q'}, 4_t\right)$$

$$J^\mu(i,j;k) = \frac{p_i^\mu}{p_i \cdot p_k} - \frac{p_j^\mu}{p_j \cdot p_k}$$

3. **Contract sub-amplitudes** to connect different quark lines

$$S_5\; 2\mathrm{Re}\left[A_0^L\left(5_g\right) A_0^{H\,*}\left(5_g\right)\right] = \sum_\lambda \varepsilon_\mu^{(\lambda)*}(5)\, \varepsilon_\nu^{(\lambda)}(5)\; J^\mu(3,1;5)\; J^\nu(4,2;5)\; |A_0(1_q, 2_b, 3_{q'}, 4_t)\,|^2$$

$$= -\boxed{\mathrm{Eik}_{\mathrm{nf}}\left(1_q, 2_b, 3_{q'}, 4_t; 5_g\right)}\; |A_0\left(1_q, 2_b, 3_{q'}, 4_t\right)|^2$$

$$\mathrm{Eik}_{\mathrm{nf}}(1_q, 2_b, 3_{q'}, 4_t; k_g) = J^\mu(3,1;k)\, J_\mu(4,2;k) = \sum_{\substack{i\,\in\,[1,3] \\ j\,\in\,[2,4]}} \frac{\lambda_{ij}\; p_i \cdot p_j}{(p_i \cdot p_k)(p_j \cdot p_k)}$$

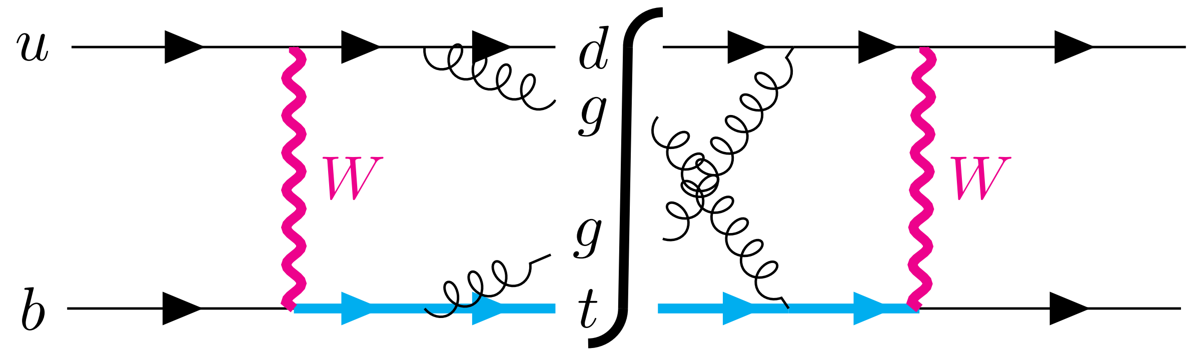
Dealing with soft limits

4. Integrate the eikonal factor over the radiation phase space

$$g_{s,b}^2 \int [\mathrm{d}p_k]\, \mathrm{Eik}_{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;k_g\right) \equiv \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon} K_{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;\epsilon\right) = \frac{\alpha_s}{2\pi}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon}\left[\frac{1}{\epsilon}\,\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right)+\mathcal{O}(\epsilon^0)\right]$$

Double-real correction treated in the same fashion:

- Independent emissions
- Factorised double-soft limit



$$S_6\left|\mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g,6_g)\right|^2_{\mathrm{nf}} = -\,g_{s,b}^4\,\frac{N^2-1}{2}\,\mathrm{Eik}_{\mathrm{nf}}(6_g)\left[A_0^L(5_g)\,A_0^{H*}(5_g)+\mathrm{c.c.}\right]$$

$$S_5\,S_6\left|\mathscr{A}_0(1_q,2_b,3_{q'},4_t;5_g,6_g)\right|^2_{\mathrm{nf}} = g_{s,b}^4\,(N^2-1)\,\mathrm{Eik}_{\mathrm{nf}}(5_g)\,\mathrm{Eik}_{\mathrm{nf}}(6_g)\left|A_0(1_q,2_b,3_{q'},4_t)\right|^2$$

Double-real at **cross-section level** results in a remarkably simple object

Colour-stripped sub-amplitudes

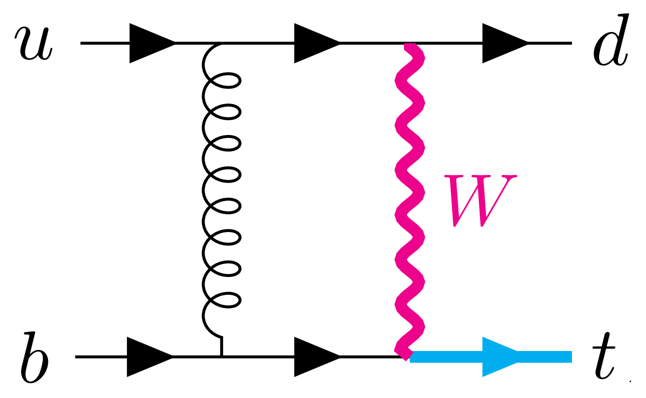
$$2s\cdot\sigma_{RR} = \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2N^2}\left(\frac{2E_{\max}}{\mu}\right)^{-4\epsilon}\left\langle K_{\mathrm{nf}}^2(\epsilon)\,F_{\mathrm{LM}}(1_q,2_b,3_{q'},4_t)\right\rangle$$

$$-\left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left(\frac{2E_{\max}}{\mu}\right)^{-2\epsilon}\left\langle K_{\mathrm{nf}}(\epsilon)\,(I-S_5)\;\widetilde{F}^{\mathrm{nf}}_{\mathrm{LM}}(1_q,2_b,3_q,4_t;5_g)\right\rangle + \left\langle (I-S_5)(I-S_6)\;F^{\mathrm{nf}}_{\mathrm{LM}}(1_q,2_b,3_{q'},4_t;5_g,6_g)\right\rangle$$

Double-virtual contribution

Extract IR singularities from virtual radiation and compute finite contributions.

1. One-loop correction to the 4-point amplitude



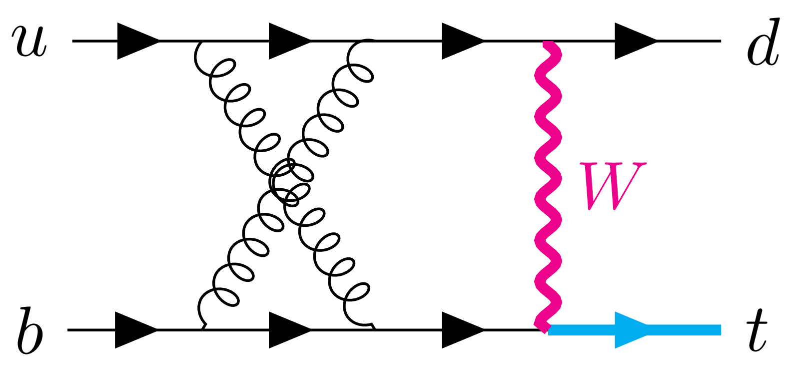
$$\left\langle c \,|\, \mathscr{A}_1(1_q,2_b,3_{q'},4_t)\right\rangle = \frac{\alpha_s}{2\pi}\left(\ldots + t^a_{c_3c_1}\, t^a_{c_4c_2}\, B_1\left(1_q,2_b,3_{q'},4_t\right)\right)$$

B_1 is UV-finite, but IR-divergent: the abelian nature of the correction leads to the simple pole structure

$$B_1\left(1_q,2_b,3_{q'},4_t\right) = I_1(\epsilon)\, A_0\left(1_q,2_b,3_{q'},4_t\right) + B_{1,\text{fin}}\left(1_q,2_b,3_{q'},4_t\right)$$

$$I_1(\epsilon) \equiv I_1(1_q,2_b,3_{q'},4_t;\epsilon) = \frac{1}{\epsilon}\left[\log\left(\frac{p_1\cdot p_4\; p_2\cdot p_3}{p_1\cdot p_2\; p_3\cdot p_4}\right) + 2\pi i\right]$$

2. Two-loop correction to the 4-point amplitude

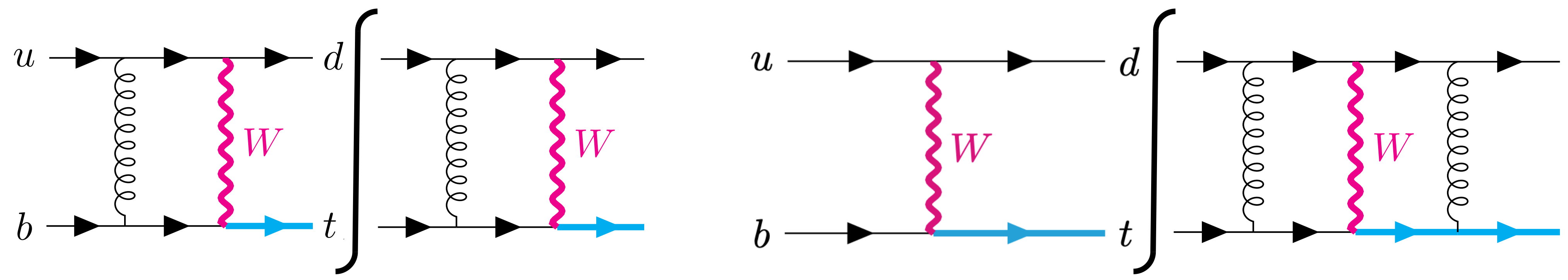


$$\left\langle c \,|\, \mathscr{A}_2(1_q,2_b,3_{q'},4_t)\right\rangle = \left(\frac{\alpha_s}{2\pi}\right)^2\left(\ldots + \frac{1}{2}\{t^a,t^b\}_{c_3c_1}\,\frac{1}{2}\{t^a,t^b\}_{c_4c_2}\, B_2\left(1_q,2_b,3_{q'},4_t\right)\right)$$

$$B_2\left(1_q,2_b,3_{q'},4_t\right) = -\frac{I_1^2(\epsilon)}{2}\, A_0\left(1_q,2_b,3_{q'},4_t\right) + I_1(\epsilon)\, B_1\left(1_q,2_b,3_{q'},4_t\right) + B_{2,\text{fin}}\left(1_q,2_b,3_{q'},4_t\right)$$

Double-virtual contribution

Complete double-virtual cross section cast in a very compact expression



$$2s \cdot \sigma_{\mathrm{VV}} = \langle F^{\mathrm{nf}}_{\mathrm{LVV}}(1_q,2_b,3_{q'},4_t) \rangle = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{4} \left[\frac{2}{N^2} \left\langle \left(\mathrm{Re}\,[I_1(\epsilon)]\right)^2 F_{\mathrm{LM}}(1_q,2_b,3_{q'},4_t) \right\rangle \right.$$

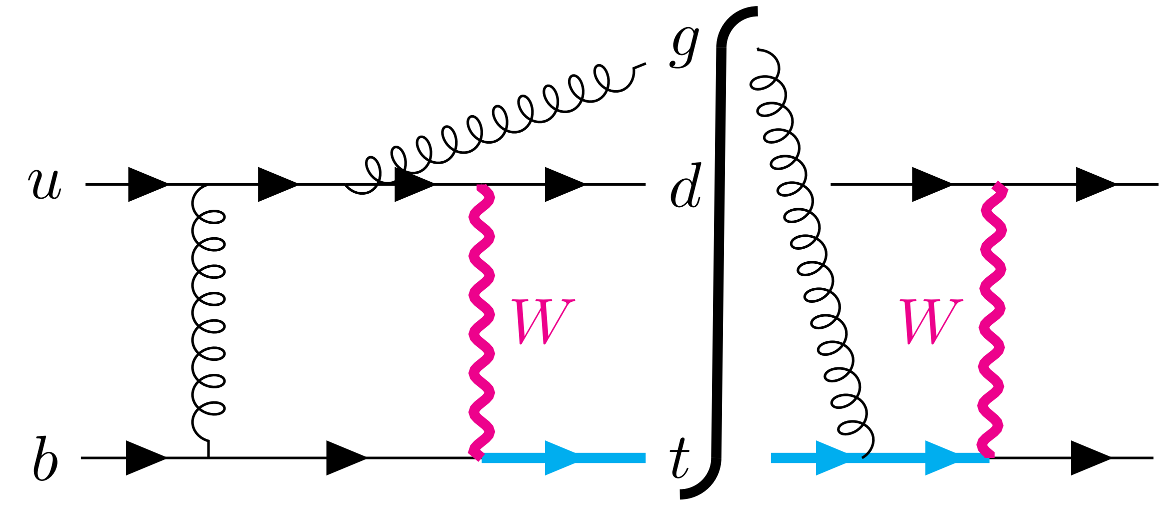
$$\left. + 2 \left\langle \mathrm{Re}\,[I_1(\epsilon)]\, \widetilde{F}^{\mathrm{nf}}_{\mathrm{LV,fin}}(1_q,2_b,3_{q'},4_t) \right\rangle + \left\langle \widetilde{F}^{\mathrm{nf}}_{\mathrm{VV,fin}}(1_q,2_b,3_{q'},4_t) \right\rangle \right]$$

Finite contributions built on one- and two-loop color stripped amplitudes

$$\widetilde{F}^{\mathrm{nf}}_{\mathrm{LV,fin}}(1_q,2_b,3_{q'},4_t) = \mathcal{N} \int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4)\; 2\mathrm{Re}\left[A_0^*(1_q,2_b,3_{q'},4_t)\, B_{1,\mathrm{fin}}(1_q,2_b,3_{q'},4_t)\right]$$

$$\widetilde{F}^{\mathrm{nf}}_{\mathrm{VV,fin}}(1_q,2_b,3_{q'},4_t) = \mathcal{N} \int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_1+p_2-p_3-p_4) \left\{ \left| B_{1,\mathrm{fin}}(1_q,2_b,3_{q'},4_t) \right|^2 + 2\mathrm{Re}\left[A_0^*(1_q,2_b,3_{q'},4_t) B_{2,\mathrm{fin}}(1_q,2_b,3_{q'},4_t)\right] \right\}$$

Real-virtual contribution



Extract **IR singularities both** from **real *and* virtual radiation**

$$F_{\mathrm{LV}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g\right) = \mathcal{N}\int \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}\Big(p_1+p_2-\sum_{i=3}^{5}p_i\Big)2\mathrm{Re}\Big[\mathcal{A}_0^*\left(1_q,2_b,3_{q'},4_t;5_g\right)\mathcal{A}_1\left(1_q,2_b,3_{q'},4_t;5_g\right)\Big]_{\mathrm{nf}}$$

1. **First** extract **phase-space singularities**

$$2s\cdot\sigma_{\mathrm{RV}} = \int [dp_5]\,F_{\mathrm{LV}}^{\mathrm{nf}}\left(1_q,2_b,3_{q'},4_t;5_g\right) = \left\langle S_5\,F_{\mathrm{LV}}^{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;5_g)\right\rangle + \left\langle (I-S_5)\,F_{\mathrm{LV}}^{\mathrm{nf}}(1_q,2_b,3_{q'},4_t;5_g)\right\rangle$$

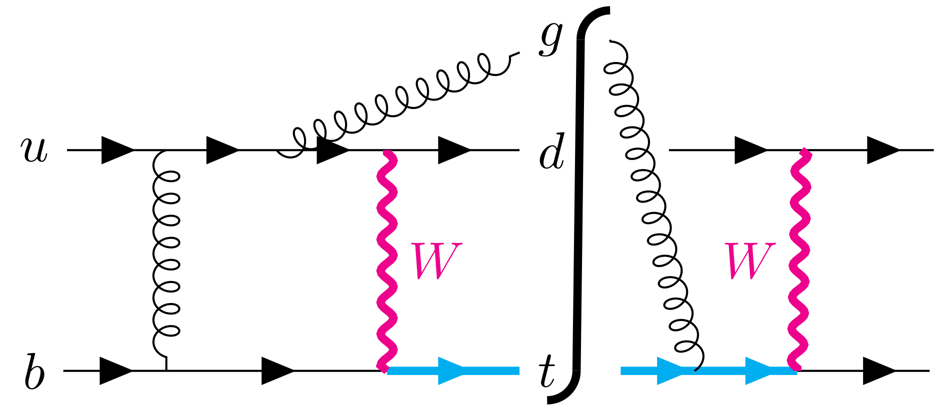
Soft-divergent Soft-regulated

2. Notice that **both contributions** contain **explicit poles in** ϵ

IR **divergencies of one-loop** amplitudes do **not depend on the real radiation** → same $I_1(\epsilon)$ as VV

Real radiation in the **soft limit** exposes the **usual eikonal factor** → same $\mathrm{Eik}_{\mathrm{nf}}\,(k_g)$ as RR

Real-virtual contribution



Complete double-virtual cross section cast in a very compact expression

$$2s \cdot \sigma_{\rm RV} = \int [dp_5]\, F_{\rm LV}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right) = \left\langle S_5\, F_{\rm LV}^{\rm nf}(1_q,2_b,3_{q'},4_t;5_g)\right\rangle + \left\langle (I-S_5)\, F_{\rm LV}^{\rm nf}(1_q,2_b,3_{q'},4_t;5_g)\right\rangle$$

$$\left\langle S_5\, F_{\rm LV}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle = -\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{N^2}\left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon}\left\langle K_{\rm nf}(\epsilon)\,{\rm Re}[I_1(\epsilon)]\, F_{\rm LM}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{N^2-1}{2}\left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon}\left\langle K_{\rm nf}(\epsilon)\, \widetilde{F}^{\rm nf}_{\rm LV,fin}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle$$

$$\left\langle (I-S_5)\, F_{\rm LV}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle = \left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left\langle {\rm Re}\,[I_1(\epsilon)]\,(I-S_5)\,\widetilde{F}^{\rm nf}_{\rm LM}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{4}\left\langle (I-S_5)\,\widetilde{F}^{\rm nf}_{\rm LV,fin}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle$$

Finite contributions built on one-loop, 4-point and 5-point color stripped amplitudes

$$\widetilde{F}^{\rm nf}_{\rm LV,fin}\left(1_q,2_b,3_{q'},4_t\right) = \mathcal{N}\int {\rm dLips}_{34}\,(2\pi)^d\,\delta^{(d)}\left(p_1+p_2-p_3-p_4\right)\; 2{\rm Re}\left[A_0^*\left(1_q,2_b,3_{q'},4_t\right) B_{1,{\rm fin}}\left(1_q,2_b,3_{q'},4_t\right)\right]$$

$$\widetilde{F}^{\rm nf}_{\rm LV,fin}(1_q,2_b,3_{q'},4_t;5_g) = \mathcal{N}\int {\rm dLips}_{34}\,(2\pi)^d\,\delta^{(d)}\left(p_1+p_2-\sum_{i=3}^{5}p_i\right)g_{s,b}^2\left(A_0^{L*}(5_g)\; B_{1,{\rm fin}}^{sH}(5_g) + A_0^{H*}(5_g)\; B_{1,{\rm fin}}^{sL}(5_g) + {\rm c.c.}\right)$$

Pole cancellation

Manifestly finite result: combination of RR, VV and RV → split into contributions of **different multiplicities**

$$\sigma_{\rm nf} = \sigma_{\rm nf}^{(2g)} + \sigma_{\rm nf}^{(1g)} + \sigma_{\rm nf}^{(0g)}$$

$\sigma_{\rm nf}^{(2g)}$: **fully resolved**, implemented numerically

$\sigma_{\rm nf}^{(1g),\,(0g)}$: finite due to **cancellation of real and virtual singularities**

$$\mathscr{W}(1_q,2_b,3_{q'},4_t) = \left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon} K_{\rm nf}(\epsilon) - {\rm Re}[I_1(\epsilon)] = \mathcal{O}\left(\epsilon^0\right)$$

$$2s \cdot \sigma_{\rm nf}^{(1g)} = -\left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{2}\left\langle \mathscr{W}\left(1_q,2_b,3_{q'},4_t\right)(I-S_5)\ \widetilde{F}_{\rm LM}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right)\frac{N^2-1}{4}\left\langle (I-S_5)\ \widetilde{F}_{\rm LV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t;5_g\right)\right\rangle$$

$$2s \cdot \sigma_{\rm nf}^{(0g)} = \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2N^2}\left\langle \mathscr{W}^2\left(1_q,2_b,3_{q'},4_t\right) F_{\rm LM}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle - \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{2}\left\langle \mathscr{W}(1_q,2_b,3_{q'},4_t)\ \widetilde{F}_{\rm LV,fin}^{\rm nf}\left(1_q,2_b,3_{q'},4_t\right)\right\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2\frac{N^2-1}{4}\left\langle \widetilde{F}_{\rm VV,fin}^{\rm nf}(1_q,2_b,3_{q'},4_t)\right\rangle$$

In the entire procedure the only amplitude which must be expanded to $\mathcal{O}(\epsilon)$ is $B_1(1_q,2_b,3_{q'},4_t)$ as it is needed to extract the two-loop finite remainder $B_{2,\rm fin}(1_q,2_b,3_{q'},4_t)$.

Amplitude evaluation

Diagrams generated with QGRAPH and processed with FORM.

W boson forces **light quark** to be **left-handed** and we decompose the **massive momentum into 2 massless momenta**

$$p_4 = p_4^\flat + \frac{m_t^2}{2n \cdot p_4} n$$

$$\bar{u}_L(p_4) = \langle 4^\flat | + \frac{m_t}{[n4^\flat]}[n| \quad , \quad \bar{u}_R(p_4) = [4^\flat | + \frac{m_t}{\langle n4^\flat \rangle}\langle n|$$

The auxiliary momentum n can be appropriately chosen to simplify the result

RV: one-loop five-point amplitude

- 24 diagrams: 8 pentagons and 16 boxes
- 7 kinematic scales

VV: two-loop four-point amplitude *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

- 18 diagrams: all topologies maximal
- 4 kinematic scales: s, t, m_t^2, m_W^2
- 428 master integrals evaluated numerically using the auxiliary mass flow method to $20 \text{digits in} \sim 30 \text{min}$ on a single core
- 10 sets of 10^4 points extracted from a grid prepared on the Born squared amplitude

Amplitude evaluation

Diagrams generated with QGRAPH and processed with FORM.

W boson forces **light quark** to be **left-handed** and we decompose the **massive momentum into 2 massless momenta**

$$p_4 = p_4^\flat + \frac{m_t^2}{2n \cdot p_4} n$$

$$\bar{u}_L(p_4) = \langle 4^\flat | + \frac{m_t}{[n4^\flat]}[n| \quad , \quad \bar{u}_R(p_4) = [4^\flat | + \frac{m_t}{\langle n4^\flat \rangle}\langle n|$$

The auxiliary momentum n can be appropriately chosen to simplify the result

RV: one-loop five-point amplitude

- 24 diagrams: 8 pentagons and 16 boxes
- 7 kinematic scales

VV: two-loop four-point amplitude *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

- 18 diagrams: all topologies maximal

	ϵ^{-2}	ϵ^{-1}
$\langle \mathcal{A}^{(0)} \| \mathcal{A}^{(2)}_{\mathrm{nf}} \rangle$	$-229.0940408654660 - 8.978163333241640i$	$-301.1802988944764 - 264.177359652950 5i$
IR poles	$-229.0940408654665 - 8.978163333241973i$	$-301.1802988944791 - 264.177359652953 5i$

Results

Differential cross section:

pp collision: $\sqrt{s} = 13\text{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379\text{GeV}$, $m_t = 173.0\text{GeV}$, $\alpha_s(m_t) = 0.108$, $\mu_F = m_t$.

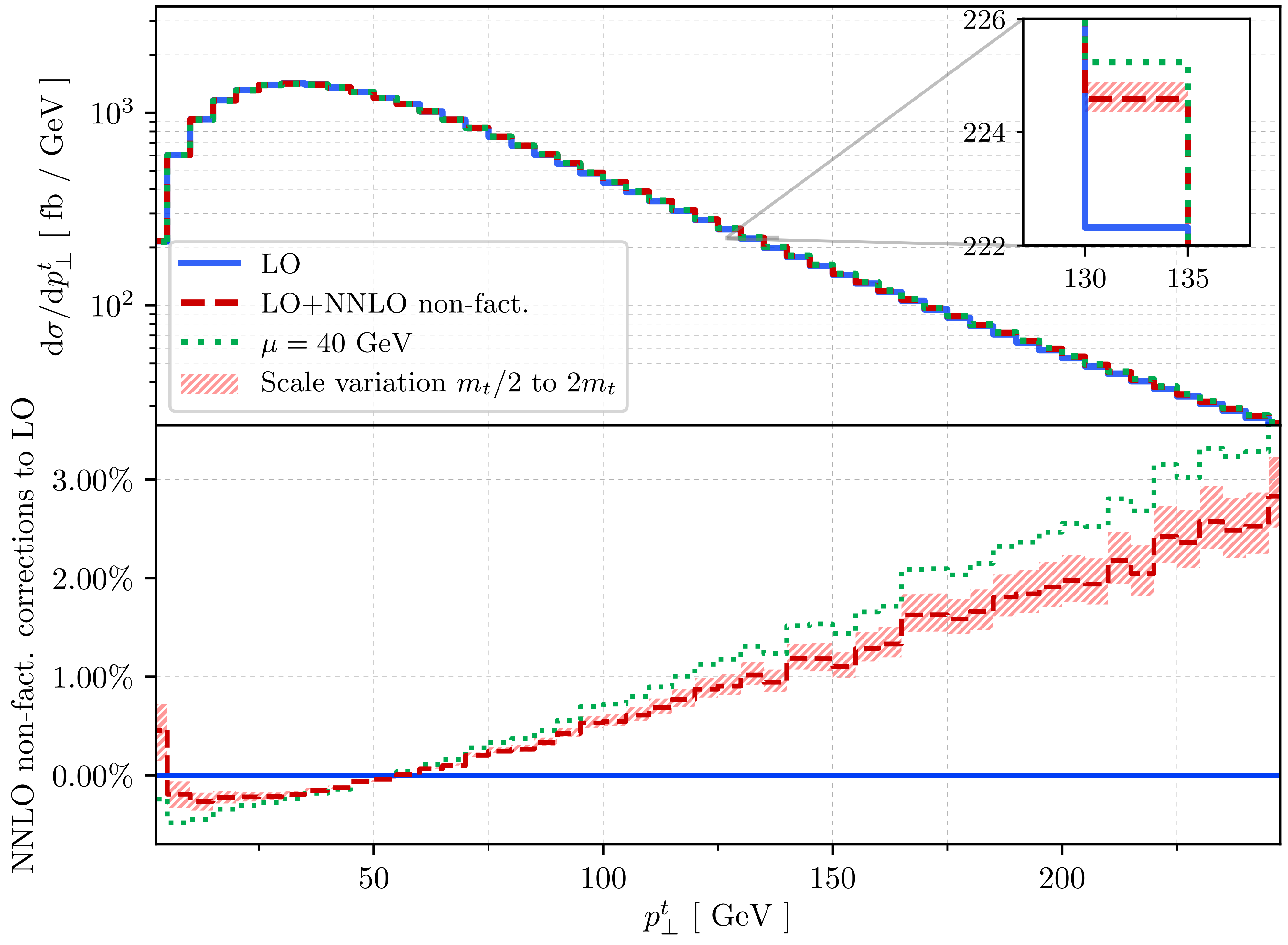
$$\frac{\sigma_{pp\to X+t}}{1\ \text{pb}} = 117.96 + 0.26\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

1. Non-factorisable corrections are $0.22^{-0.04}_{+0.05}\ \%$ LO for $\mu_R = m_t$.
2. **Theoretical uncertainties** are estimated through **scale variation**: $2m_t, m_t/2$.
3. **Unclear optimal scale choice**: non-factorisable corrections appear for the first time at NNLO → no indication from lower orders.
4. For $\mu_R = 40\text{GeV}$ (typical **transfer momentum scale** of top quark) non-factorisable corrections are $0.35\ \%$ LO.
5. In comparison, **NNLO factorisable** corrections to **NLO cross section** are around $0.7\ \%$.

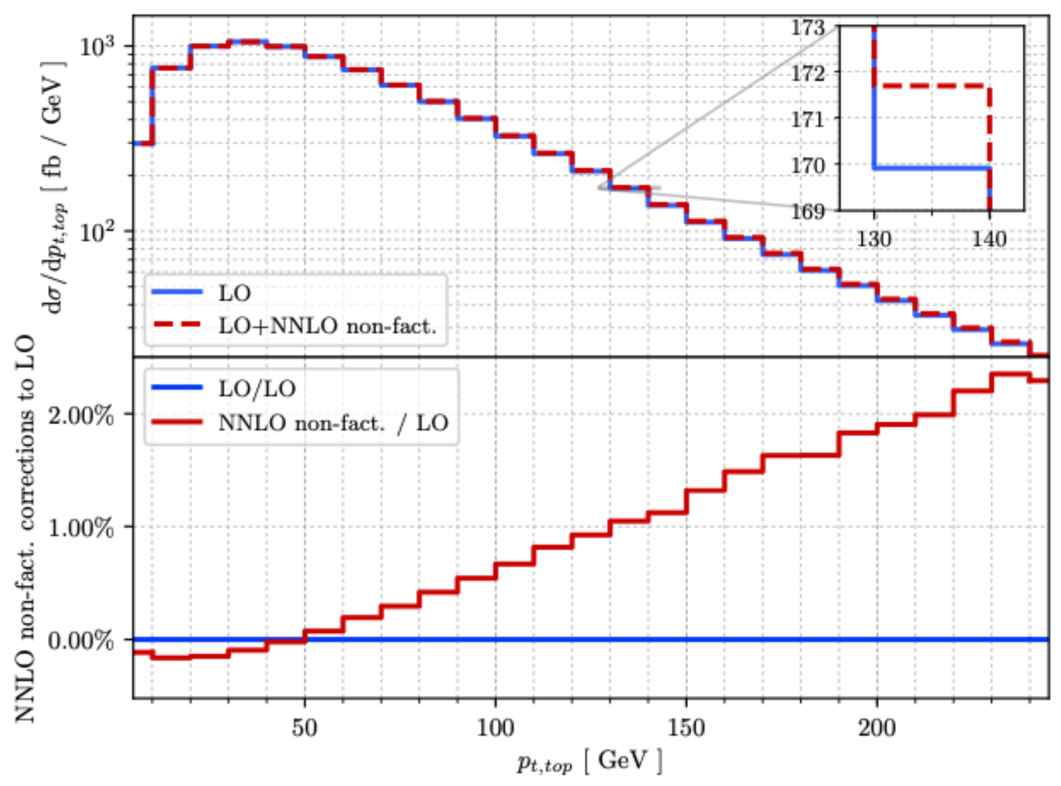
Results

Differential cross section:

pp collision: $\sqrt{s} = 13$TeV, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379$GeV, $m_t = 173.0$GeV, $\alpha_s(m_t) = 0.108$.



1. Non-factorisable corrections are $p_\perp^t$-dependent.
2. Non-factorisable corrections are small and negative at low values of $p_\perp^t$. They vanish at $p_\perp^t \sim 50$GeV [in agreement with results for virtual corrections]



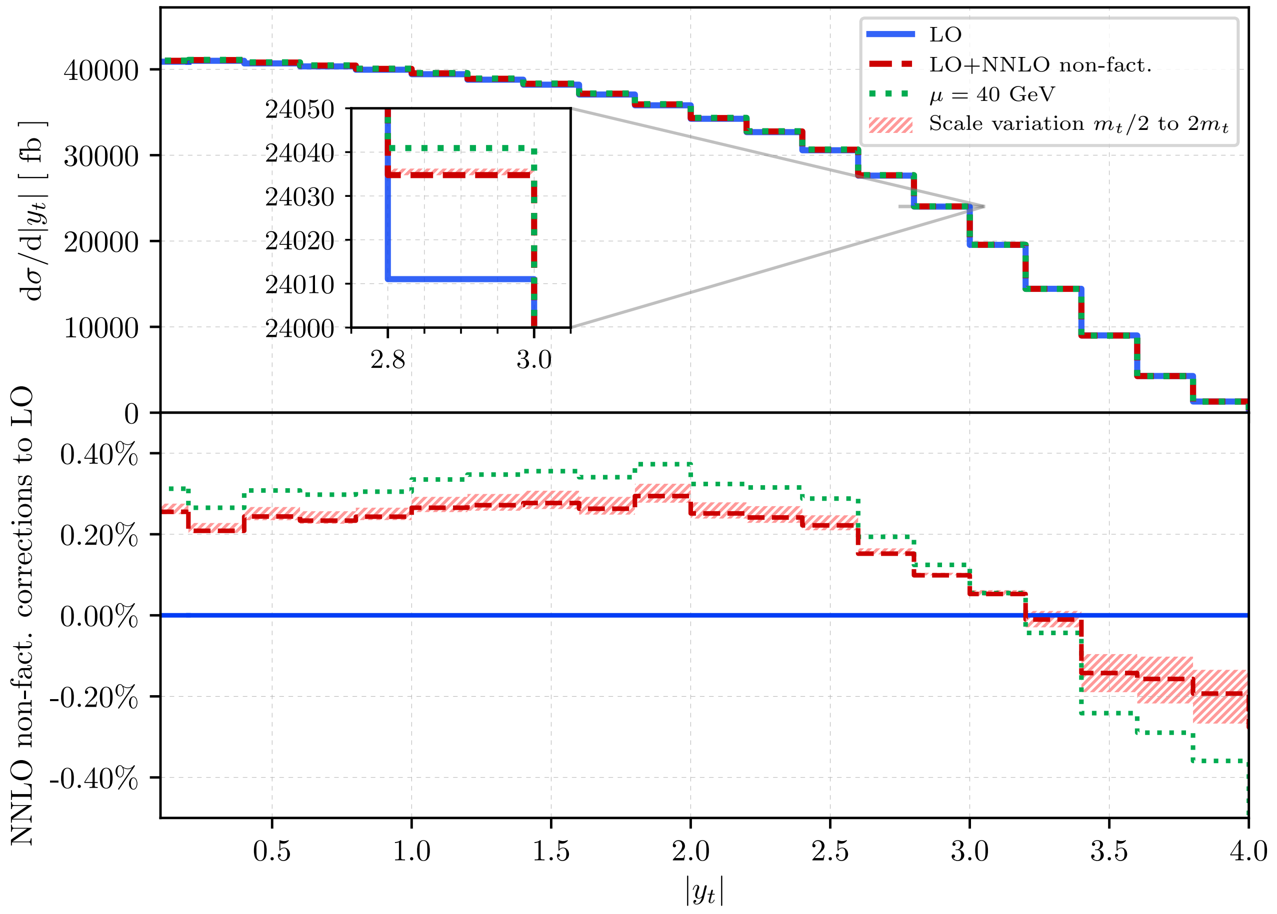
[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

3. **Factorisable corrections** vanish around $p_\perp^t \sim 30$GeV.
4. Factorisable and non-factorisable corrections are comparable in the region around the maximum of the $p_\perp^t$ distribution.

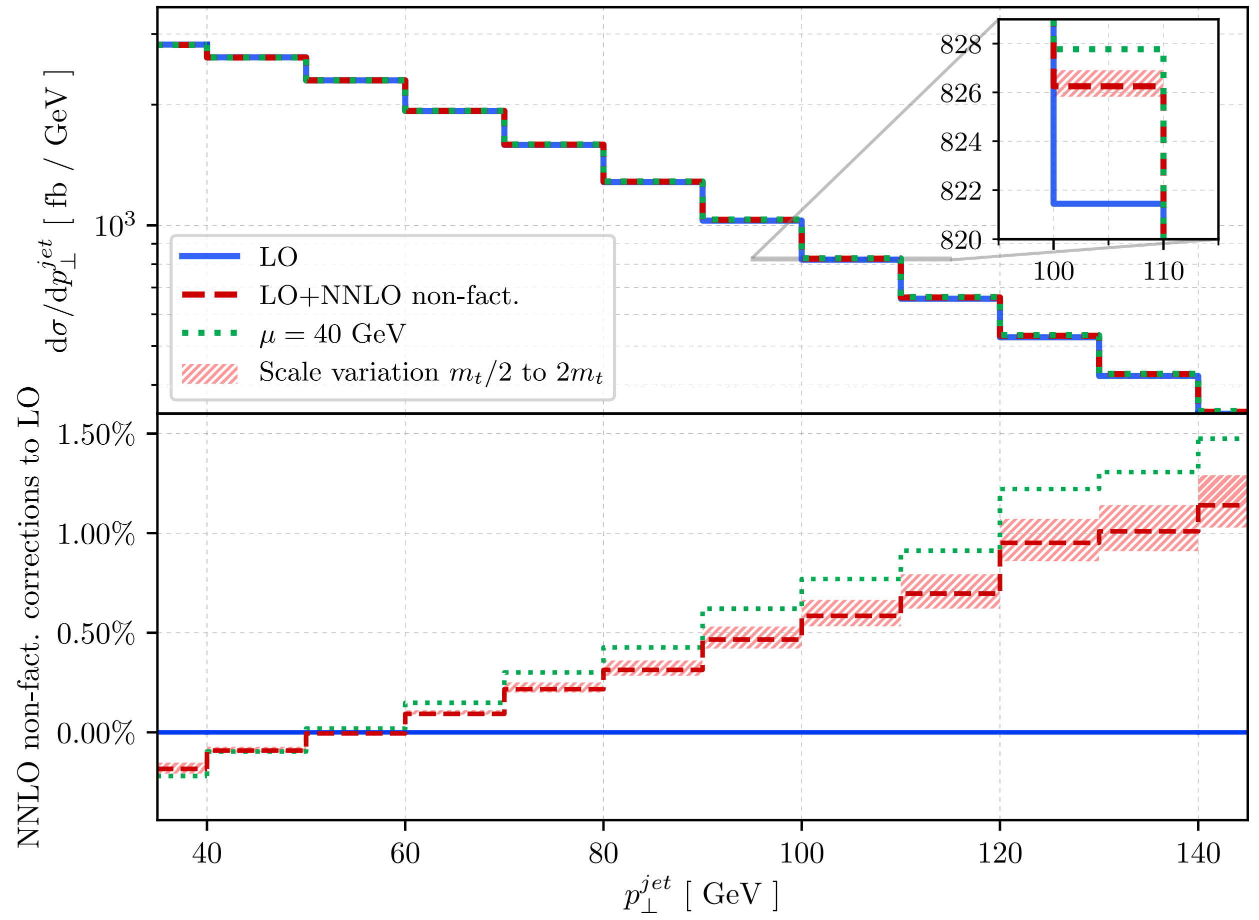
Results

Differential cross section:

pp collision: $\sqrt{s} = 13$TeV, PDFs: $\texttt{CT14\_lo}$@LO, $\texttt{CT14\_nnlo}$@NNLO, $m_W = 80.379$GeV, $m_t = 173.0$GeV, $\alpha_s(m_t) = 0.108$.



1. Relative non-factorisable **correction** to **top-quark rapidity fairly flat** for $|y_t| < 2.5$, $\mathcal{O}(0.25\%)$.
2. **Sign change** around $|y_t| \sim 3$
3. **Factorisable** corrections change sign around $|y_t| \sim 1.2$
4. For some top-quark rapidity values, **factorisable and non-factorisable** correction become **quite comparable**.



5. k_t**-algorithm** to define jets $p_\perp^{jet} > 30$GeV, R=0.4.
6. **Non-factorisable** corrections reach $1.2\,\%$ at $p_\perp^{jet} \sim 140$GeV.

Conclusions

1. We **complete** the calculation of **NNLO corrections** to the ***t*-channel single-top production**: the **non-factorisable corrections**.

2. The **auxiliary mass flow method** was used for integral evaluation in VV. It is sufficiently robust to produce results relevant for phenomenology.

3. **Non-factorisable** corrections are **smaller** than, but **quite comparable** to, the **factorisable** ones.

4. If **percent precision** in single-top studies can be reached, the **non-factorisable effects will have to be taken into account**.

Thank you for your attention!

Backup

Double-virtual contribution

The pole structure of the two-loop amplitude is well studied, and can be easily cross-checked against literature
[Catani '98] [Aybat, Dixon, Sterman '06][Becher, Neubert '09][Czakon, Mitov, Sterman '09][Mitov, Sterman, Sung '09, '10][Ferroglia, Neubert, Pecjak, Yang '09]

$$|\mathscr{A}\rangle = \mathbf{Z}\,|\mathscr{F}\rangle\,, \qquad \mu\frac{d}{d\mu}\mathbf{Z} = -\,\mathbf{\Gamma}\,\mathbf{Z}$$

$$\begin{aligned}\mathbf{\Gamma}\big(\{p_i\},m_i,\mu\big) = &\sum_{(i,j)}\frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\gamma_{\rm cusp}(\alpha_s)\,\log\Big(\frac{\mu^2}{-s_{ij}}\Big) + \sum_{(I,j)}\mathbf{T}_I\cdot\mathbf{T}_j\,\gamma_{\rm cusp}(\alpha_s)\,\log\Big(\frac{m_I\,\mu}{-s_{Ij}}\Big)\\ &+\sum_{(I,J)}\frac{\mathbf{T}_I\cdot\mathbf{T}_J}{2}\gamma_{\rm cusp}(v_{IJ},\alpha_s) + \sum_i\gamma^i(\alpha_s) + \sum_I\gamma^I(\alpha_s)\\ &+\sum_{(I,J,K)} if^{abc}\,T_I^a\,T_J^b\,T_K^c\,F_1\big(v_{IJ},v_{JK},v_{KI}\big) + \sum_{(I,J)}\sum_k if^{abc}\,T_I^a\,T_J^b\,T_k^c\,f_2\Big(v_{IJ},\log\Big(\frac{-\sigma_{Jk}v_J\cdot p_k}{-\sigma_{Ik}v_I\cdot p_k}\Big)\Big) + \mathcal{O}\big(\alpha_s^3\big)\,.\end{aligned}$$

Collinear

n>1 massive

Non-abelian

Several simplifications occur when only non-factorisable corrections are considered *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

$$\boxed{\mathbf{\Gamma}_{\rm nf} = \Big(\frac{\alpha_s}{4\pi}\Big)\mathbf{\Gamma}_{0,\rm nf} = \Big(\frac{\alpha_s}{4\pi}\Big)4\left[\mathbf{T}_1\cdot\mathbf{T}_2\,\log\Big(\frac{\mu^2}{-s-i\epsilon}\Big) + \mathbf{T}_2\cdot\mathbf{T}_3\,\log\Big(\frac{\mu^2}{-u-i\epsilon}\Big) + \mathbf{T}_1\cdot\mathbf{T}_4\,\log\Big(\frac{\mu\,m_t}{m_t^2-u-i\epsilon}\Big) + \mathbf{T}_3\cdot\mathbf{T}_4\,\log\Big(\frac{\mu\,m_t}{m_t^2-s-i\epsilon}\Big)\right]}$$

$$\langle\mathscr{A}^{(0)}|\mathscr{A}^{(2)}_{\rm nf}\rangle = -\frac{1}{8\epsilon^2}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}^2_{0,\rm nf}|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}_{0,\rm nf}|\mathscr{A}^{(1)}_{\rm nf}\rangle + \langle\mathscr{A}^{(0)}|\mathscr{F}^{(2)}_{\rm nf}\rangle$$

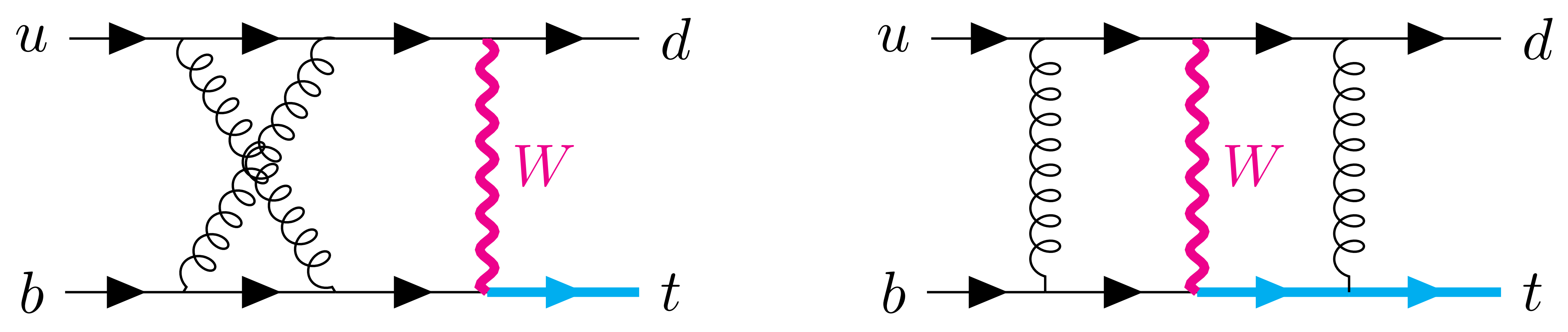
$$\langle\mathscr{A}^{(1)}_{\rm nf}|\mathscr{A}^{(1)}_{\rm nf}\rangle = -\frac{1}{4\epsilon^2}\langle\mathscr{A}^{(0)}|\,|\mathbf{\Gamma}_{0,\rm nf}|^2\,|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(1)}_{\rm nf}|\mathbf{\Gamma}_{0,\rm nf}|\mathscr{A}^{(0)}\rangle + \frac{1}{2\epsilon}\langle\mathscr{A}^{(0)}|\mathbf{\Gamma}^\dagger_{0,\rm nf}|\mathscr{A}^{(1)}\rangle + \langle\mathscr{F}^{(1)}_{\rm nf}|\mathscr{F}^{(1)}_{\rm nf}\rangle$$

Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1,2,3\ , \qquad p_4^2 = m_t^2, s, t, m_W^2$

Dimensions: $d = 4 - 2\epsilon$



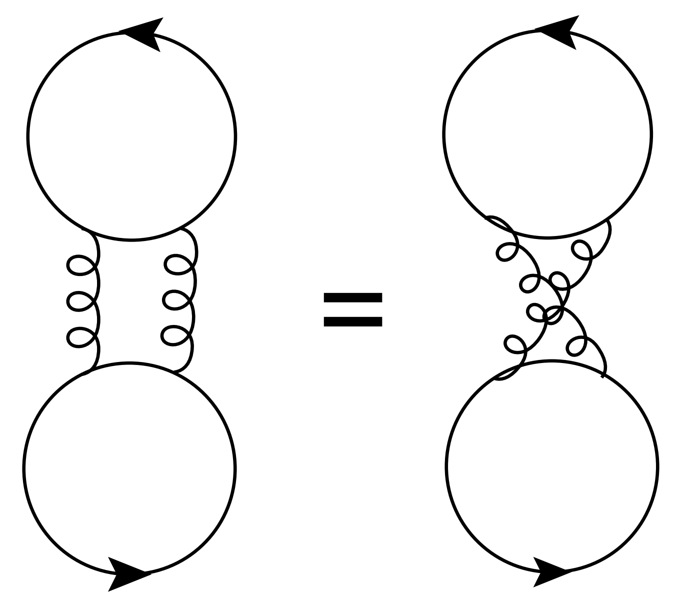
Planar and non-planar amplitudes appear at 2-loop order. However, only a particular combination of them do actually contribute

$$\mathscr{A}^{(2)}_{\rm nf} = \frac{1}{4}\{T^a, T^b\}_{c_3c_1}\,\{T^a, T^b\}_{c_3c_1}\left(\mathrm{A}^{(2),\,\rm pl}_{\rm nf} + \mathrm{A}^{(2),\,\rm npl}_{\rm nf}\right) + \dots$$

$$\mathrm{A}^{(2)}_{\rm nf} = \mathrm{A}^{(2),\,\rm pl}_{\rm nf} + \mathrm{A}^{(2),\,\rm npl}_{\rm nf}$$

Upon interference with tree-level amplitude the colour distinction between planar and non-planar diagrams disappears

(Abelian nature of non-factorisable corrections)



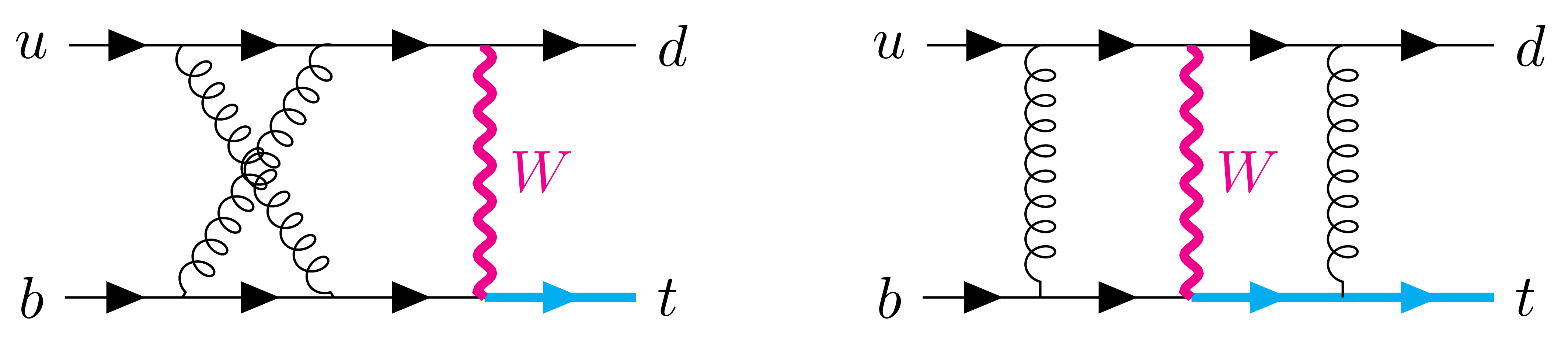
$$\sum_{\rm color} \mathscr{A}^{(0)\,*}\,\mathscr{A}^{(2)}_{\rm nf} = \frac{1}{4}\left(N_c^2 - 1\right)\mathrm{A}^{(0)\,*}\,\mathrm{A}^{(2)\,*}_{\rm nf}$$

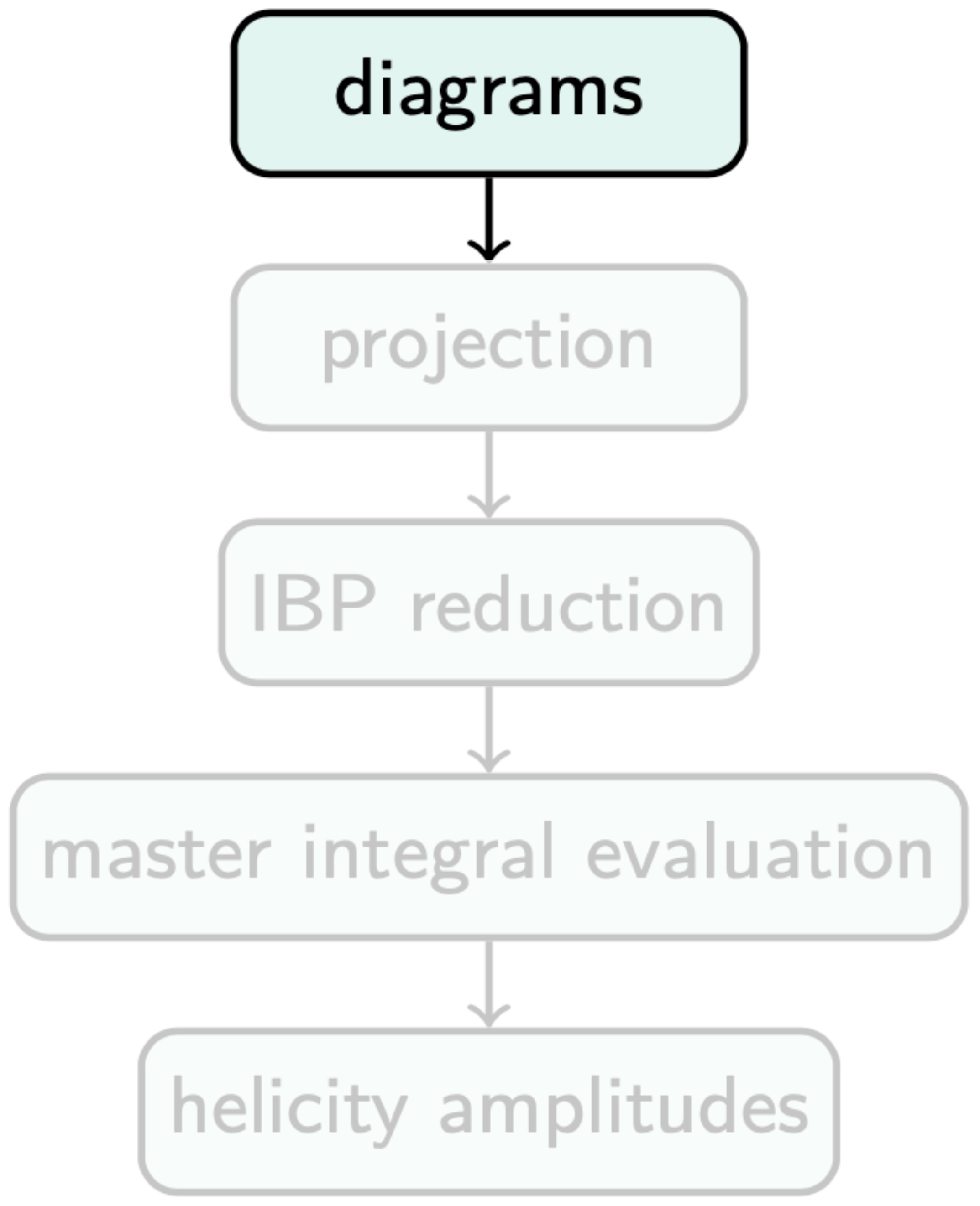
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1,2,3\ ,\qquad p_4^2 = m_t^2, s, t, m_W^2$

Dimensions: $d = 4 - 2\epsilon$





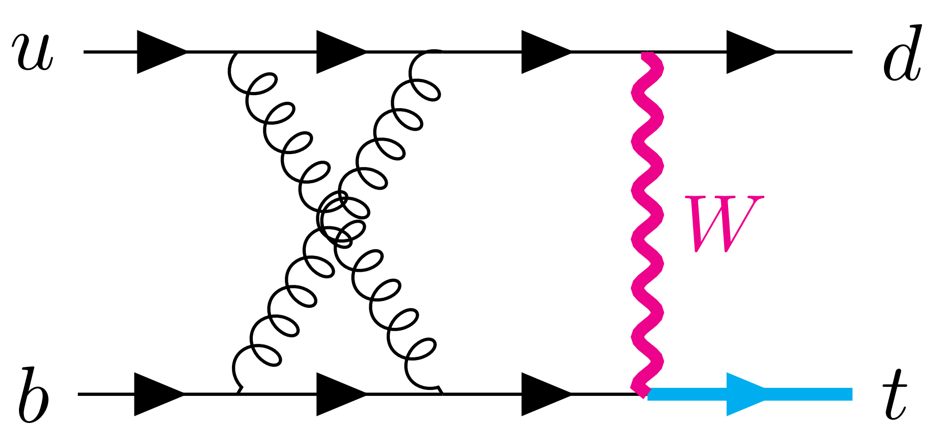
- **18 diagrams**: generated with QGRAPH *[Nogueira '93]* and processed with FORM *[Vermaseren '00] [Kuipers et al. '15] [Ruijl et al. '17]*. All **topologies maximal**.

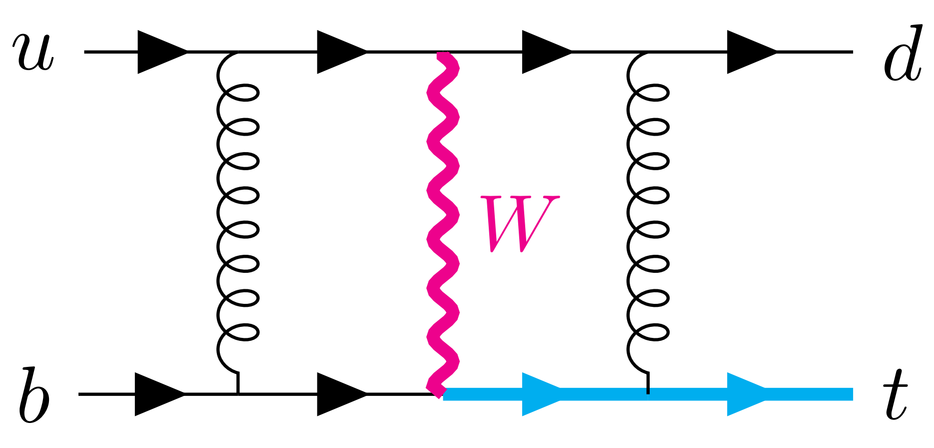
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

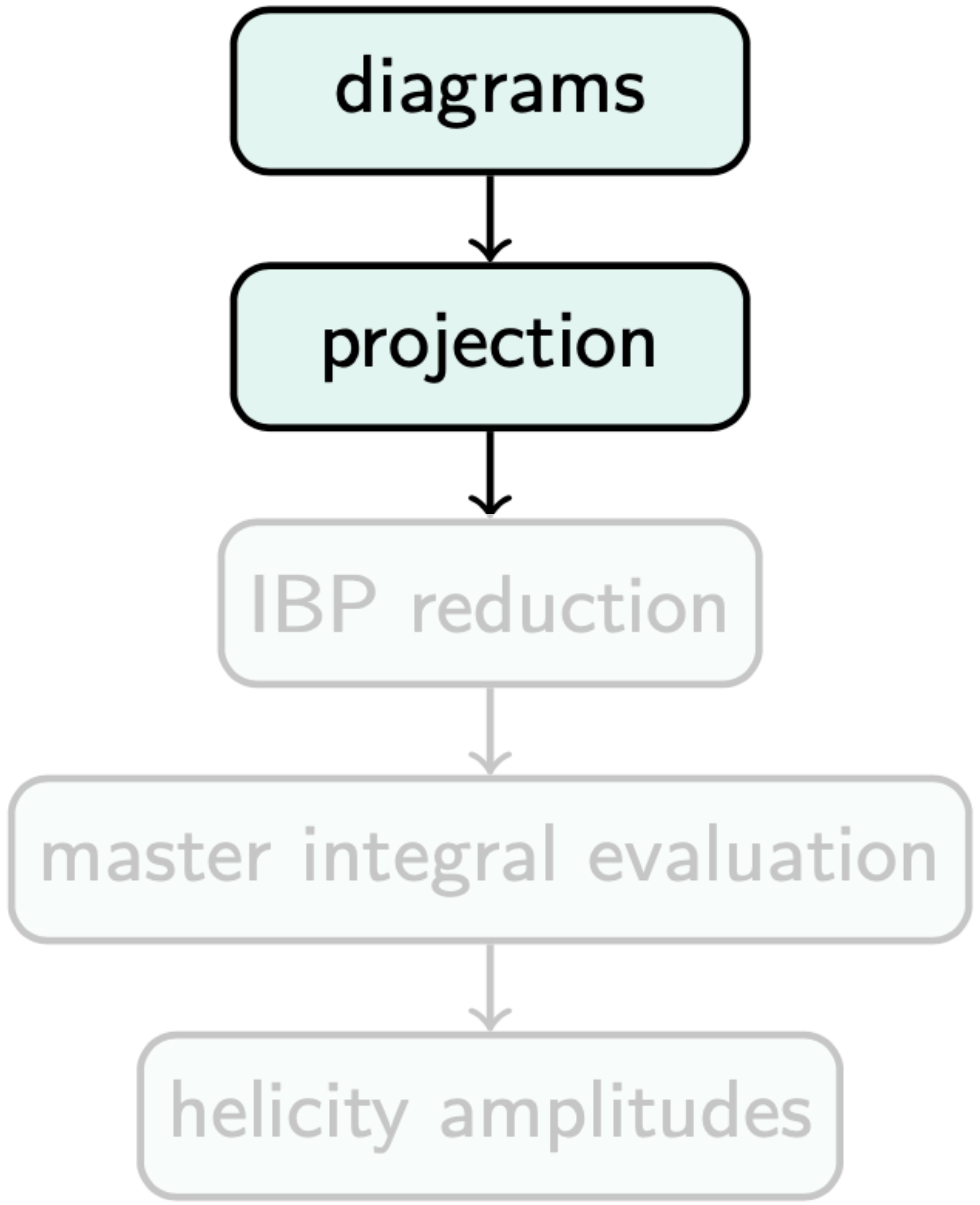
Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1,2,3\ , \qquad p_4^2 = m_t^2, s, t, m_W^2$

Dimensions: $d = 4 - 2\epsilon$







- One- and two-loop **amplitudes** are written in terms of **invariant form factors** and **independent Lorentz structure**

- γ_5 enters through **charged weak currents** (left-handed projectors)

- Use **anti-commuting prescription** for γ_5 and move **left-handed projectors** to act **on** external **massless fermions.**

- **11 structures** $\mathcal{S}_i(\lambda)$ and corresponding **form factor (FF)**

$$Q_i = \sum_{\vec{\lambda}} \mathcal{S}_i^\dagger(\vec{\lambda})\, \mathrm{A}_{\mathrm{nf}}^{(2)}(\vec{\lambda})\ , \qquad i = 1,\ldots,11$$

- **FF do not depend on helicities of external particles** $\to$ **vector current part**

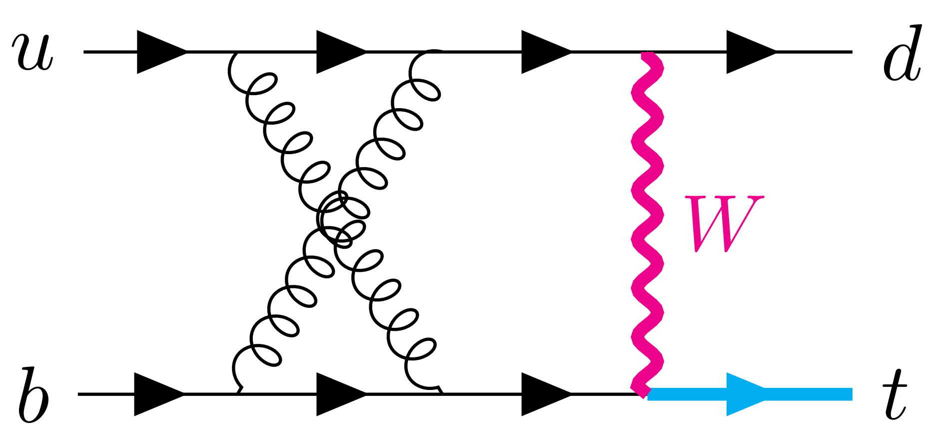
- **Polarisation sum** returns independent **traces** $\to$ scalar products of loop and external momenta **(no external spinor)**

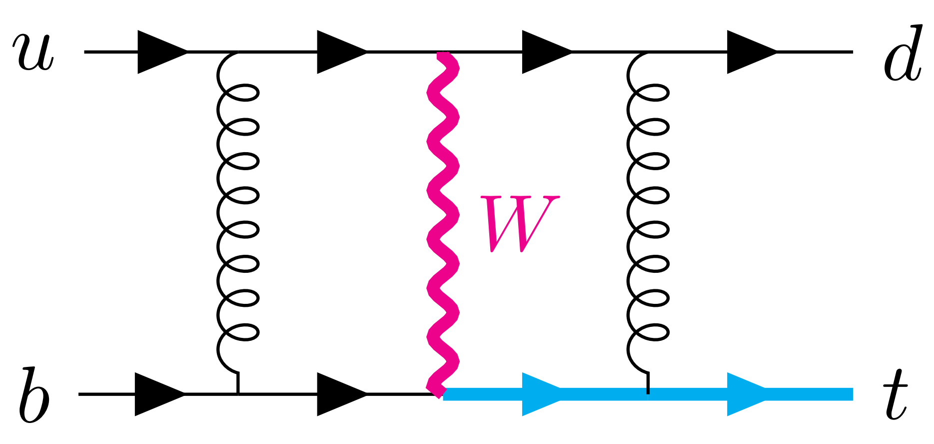
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

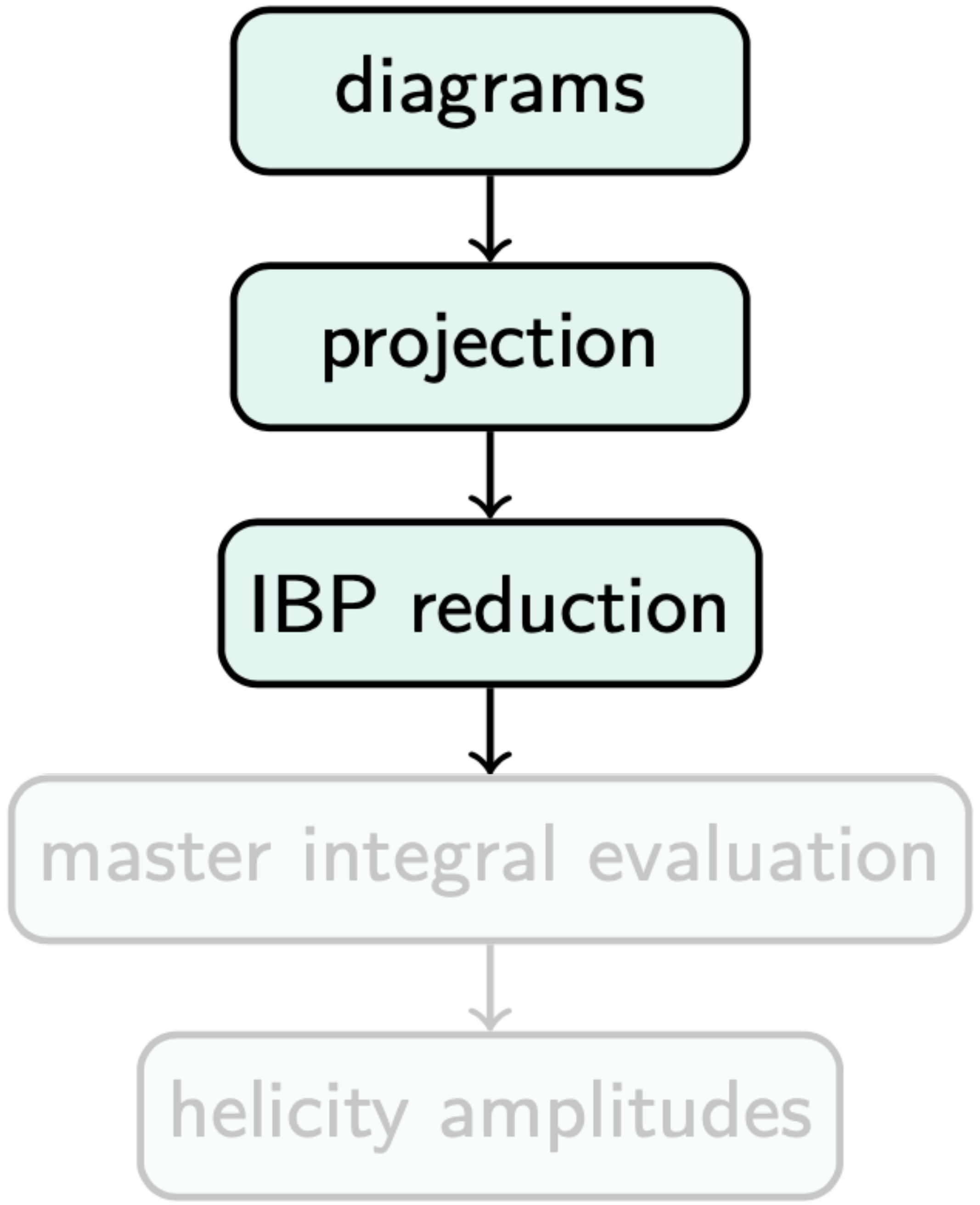
Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1{,}2{,}3\ ,\qquad p_4^2 = m_t^2, s, t, m_W^2$

Dimensions: $d = 4 - 2\epsilon$







- **Reduction** performed **analytically** with KIRA2.0 *[Klappert, Lange et al. '20]* and FireFly *[Klappert, Lange '20] [Klappert, Klein et al. '21]*

$$\left\langle \mathscr{A}^{(0)} \,\middle|\, \mathscr{A}^{(2)}_{\mathrm{nf}} \right\rangle = \frac{1}{4}(N_c^2 - 1) \sum_{i=1}^{428} c_i(d, s, t, m_t, m_w)\, I_i$$

- Most complicated took 4 days on 20 cores.

- **428 master integrals** I_i **.**

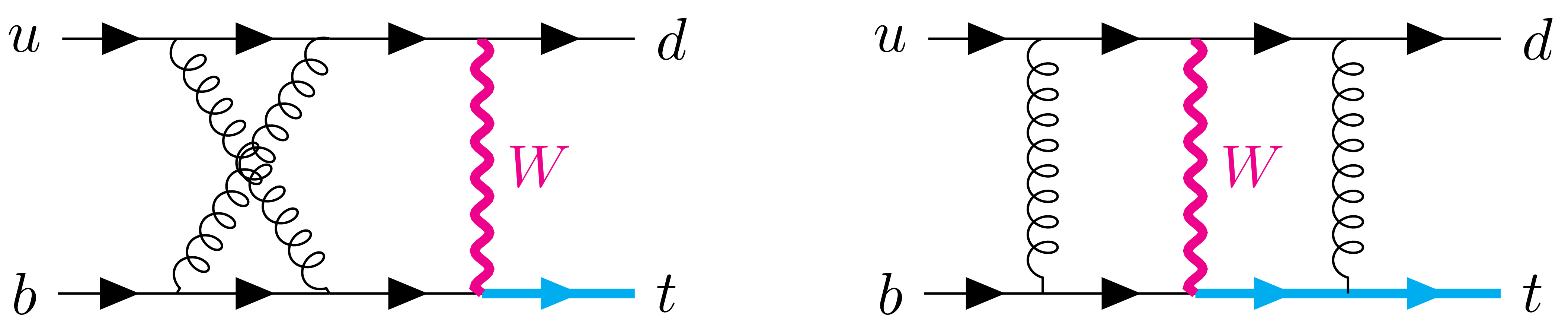
- **Exact dependence on the top-quark mass and the W mass** (very first reduction to master integral performed for the fixed numerical relation $m_t^2 = 14/3\, m_W^2$ *[Assadsolimani et al. '14]*)

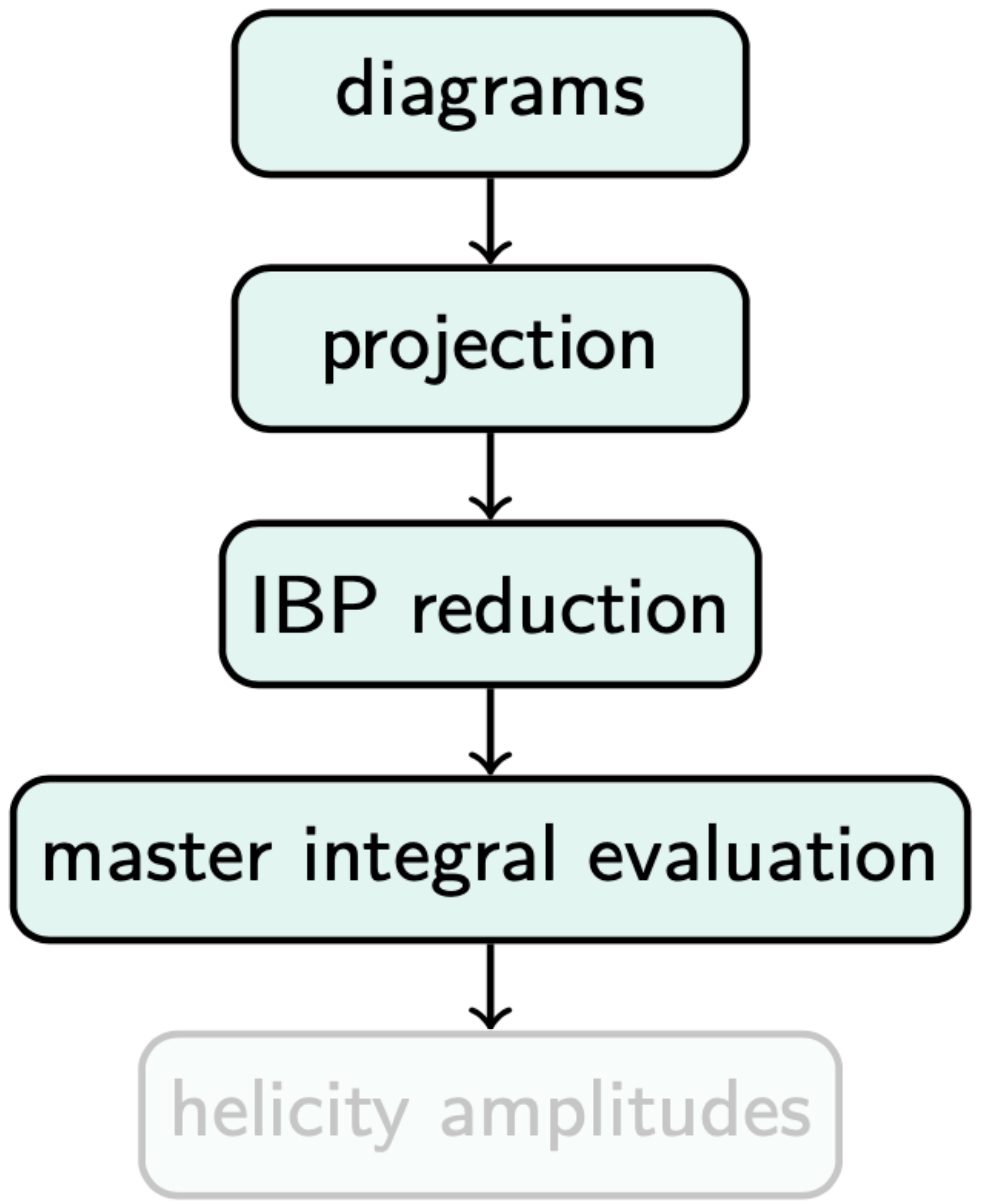
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1{,}2{,}3\ , \qquad p_4^2 = m_t^2, s, t, m_W$

Dimensions: $d = 4 - 2\epsilon$





- Compute **master integrals** using the **auxiliary mass flow method** *[Liu, Ma, Wang '18] [Liu, Ma, Tao, Zhang '21]*

$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d k_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

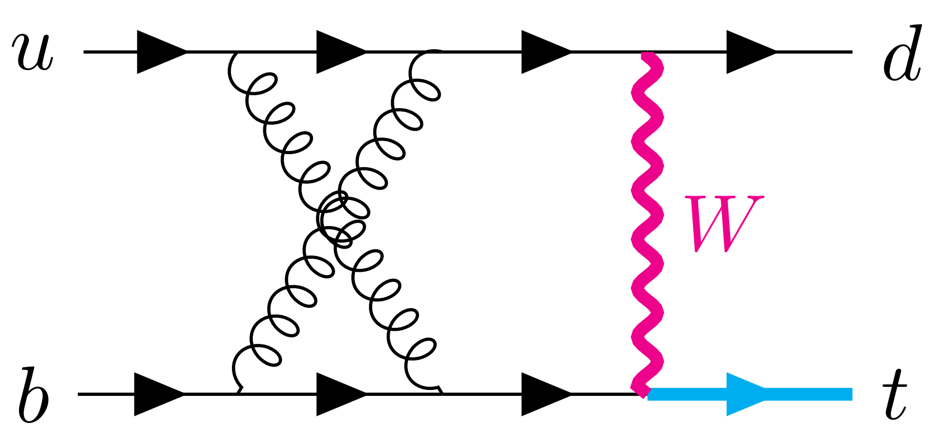
- Add **imaginary part to the W boson mass** $m_W^2 \to m_W^2 - i\eta$
- **Solve** system of **DE** at each **phase space point:** $\partial_x I = MI$, $m_W^2 - i\eta = m_W^2(1+x)$
- **Boundary condition** $x \to -i\infty$**, physical point** $x = 0$**.**
- Some of the **boundary integrals** are **hard to compute:** add **imaginary part to the top mass** $m_t^2 \to m_t^2 - i\eta$
- **DE** in m_t. **Boundary** $\eta \to \infty$. **Physical point** $\eta \to 0$.

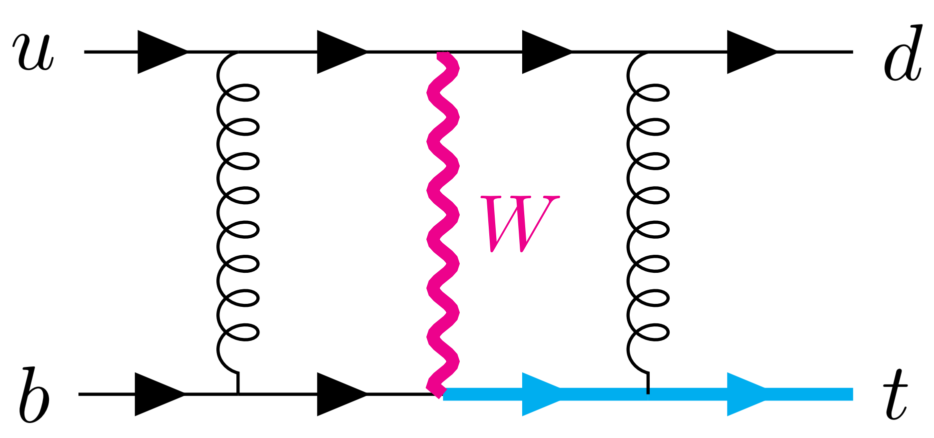
Double-virtual contribution: amplitude evaluation *[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]*

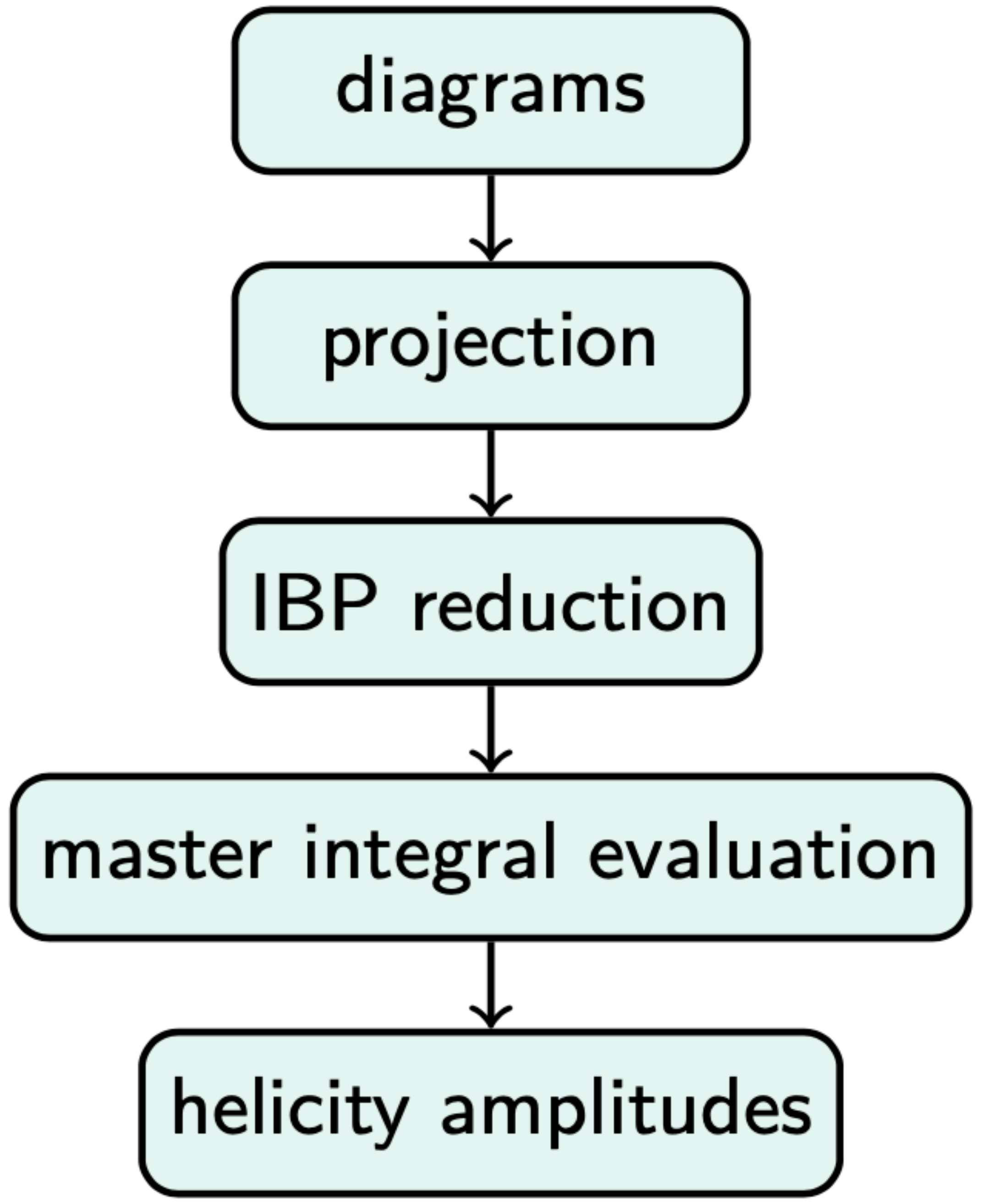
Process: $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0\ , i = 1,2,3\ , \qquad p_4^2 = m_t^2, s, t, m_W$

Dimensions: $d = 4 - 2\epsilon$







- **t'Hooft-Veltman scheme**: external momenta in $d = 4$ and internal momenta in $d = 4 - 2\epsilon$
- To be non-vanishing a matrix events in $d = 4 - 2\epsilon$ between two $d = 4$ spinors requires an even number of matrices with support in -2ϵ space
- ϵ dependence can be explicitly and unambiguously extracted

	7 TeV pp		14 TeV pp		1.96 TeV $\bar{p}p$
	top	anti-top	top	anti-top	$t+\bar{t}$
$\sigma_{\mathrm{LO}}^{\mu=m_t}$	$37.1^{+7.1\%}_{-9.5\%}$	$19.1^{+7.3\%}_{-9.7\%}$	$134.6^{+10.0\%}_{-12.1\%}$	$78.9^{+10.4\%}_{-12.6\%}$	$2.09^{+0.8\%}_{-3.1\%}$
$\sigma_{\mathrm{LO}}^{\mathrm{DDIS}}$	$39.5^{+6.4\%}_{-8.6\%}$	$19.9^{+7.0\%}_{-9.3\%}$	$140.9^{+9.4\%}_{-11.4\%}$	$80.7^{+10.2\%}_{-12.3\%}$	$2.31^{-0.3\%}_{-1.8\%}$
$\sigma_{\mathrm{NLO}}^{\mu=m_t}$	$41.4^{+3.0\%}_{-2.0\%}$	$21.5^{+3.1\%}_{-2.0\%}$	$154.3^{+3.1\%}_{-2.3\%}$	$91.4^{+3.1\%}_{-2.2\%}$	$1.96^{+3.1\%}_{-2.3\%}$
$\sigma_{\mathrm{NLO}}^{\mathrm{DDIS}}$	$41.8^{+3.3\%}_{-2.0\%}$	$21.5^{+3.4\%}_{-1.6\%}$	$154.4^{+3.7\%}_{-1.4\%}$	$91.2^{+3.1\%}_{-1.8\%}$	$2.00^{+3.6\%}_{-3.4\%}$
	PDF $^{+1.7\%}_{-1.4\%}$	PDF $^{+2.2\%}_{-1.5\%}$	PDF $^{+1.7\%}_{-1.1\%}$	PDF $^{+1.9\%}_{-0.9\%}$	PDF $^{+4.3\%}_{-5.3\%}$
$\sigma_{\mathrm{NNLO}}^{\mu=m_t}$	$41.9^{+1.2\%}_{-0.7\%}$	$21.9^{+1.2\%}_{-0.7\%}$	$153.3(2)^{+1.0\%}_{-0.6\%}$	$91.5(2)^{+1.1\%}_{-0.9\%}$	$2.08^{+2.0\%}_{-1.3\%}$
$\sigma_{\mathrm{NNLO}}^{\mathrm{DDIS}}$	$41.9^{+1.3\%}_{-0.8\%}$	$21.8^{+1.3\%}_{-0.7\%}$	$153.4(2)^{+1.1\%}_{-0.7\%}$	$91.2(2)^{+1.1\%}_{-0.9\%}$	$2.07^{+1.7\%}_{-1.1\%}$
	PDF $^{+1.3\%}_{-1.1\%}$	PDF $^{+1.4\%}_{-1.3\%}$	PDF $^{+1.2\%}_{-1.0\%}$	PDF $^{+1.0\%}_{-1.0\%}$	PDF $^{+3.7\%}_{-5.0\%}$

Fully inclusive cross section for pp collision @7TeV and @14TeV (LHC), as well as $p\bar{p}$ collision @1.96TeV (Tevatron) with scales $\mu_R = \mu_F = m_t$ and DDIS scales using $\mathtt{CT14}$ PDFs. *[Campbell, Neumann, Sullivan '21]*