Soft Logarithms in Processes with Heavy Quarks

Andrea Ghira
in collaboration with D.Gaggero, S.Marzani, G.Ridolfi
Università degli Studi di Genova,
INFN, Genova

ECT* Trento, 30 Agosto 2022

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Introduction

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Fragmentation Function Approach

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Massive Scheme Approach

Full mass dependence taken into account.

Introduction

In the fragmentation function approach

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \sum_i \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, \frac{\mu^2}{q^2}\right) D_i(z, \mu^2, m^2) + \mathcal{O}\left(\frac{m^2}{q^2}\right)$$

$$\xrightarrow[N-\text{space}]{} \widetilde{\Gamma}(N,\xi) = \sum_i \widetilde{C}_i\left(N, \alpha_s, \frac{\mu^2}{q^2}\right) \widetilde{D}_i\left(N, \alpha_s, m^2\right) + \mathcal{O}\left(\frac{m^2}{q^2}\right).$$

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The fragmentation functions D_i (**process independent**) fulfil the DGLAP evolution equations in N-space:

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\,\mu^2}\widetilde{D}_i(N,\mu^2,m^2) = \sum_j \gamma_{ij}\left(N,\alpha_s(\mu^2)\right)\widetilde{D}_j(N,\mu^2,m^2),$$

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- Powers of $\log \frac{m^2}{q^2}$ resummed to all orders by solving the evolution equation

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Massive Scheme Approach

- Kinematics treated correctly but $\log m^2/q^2$ are not resummed
- Difficult calculations at higher order

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The initial condition grows to order α_s as $\alpha_s \log^2 N \leftrightarrow \alpha_s \left(\frac{\log(1-x)}{1-x}\right)_+$, as shown in [1].

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The coefficient functions (process-dependent) $C_i \sim \alpha_s^k \log^{2k} N$ may cancel this double logarithmic behaviour, but this is not always the case.

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$\widetilde{\Gamma}(N, \xi)$ may have double logs$\rightarrow$ soft (soft gluon radiation) and massless limit ($m^2 \ll q^2$) do not commute

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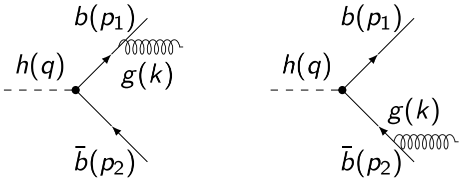
Interplay between soft and massless limit in $H \to b\bar{b}$

We focus on a specific process in the massive scheme at NLO:

$$h(q) \to b(p_1) + \bar{b}(p_2) + g(k), \qquad \xi \equiv \frac{m^2}{q^2}$$



then we compute the small mass (necessary for the FF approach) and the $x \equiv \frac{2p_1 \cdot q}{q^2} \to 1$ limit (soft emission).

Interplay between soft and massless limit in $H \to b\bar{b}$

$$\lim_{\xi \to 0} \lim_{x \to 1} \frac{1}{\Gamma_0} \frac{d\Gamma}{dx} = -\frac{2\alpha_s C_F}{\pi} \left[\frac{1 + \log \xi}{1 - x} + ... \right],$$

$$\lim_{x \to 1} \lim_{\xi \to 0} \frac{1}{\Gamma_0} \frac{d\Gamma}{dx} = -\frac{\alpha_s C_F}{\pi} \left[\frac{\log \xi}{1 - x} + \frac{\log(1 - x)}{1 - x} + \frac{7}{4} \frac{1}{1 - x} + ... \right],$$

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- In the first case we have a mass logarithm multiplied by a soft one ($\frac{1}{1-x} \leftrightarrow \log N$)
- In the second one we have an additional term which corresponds to a $\log^2 N$ in Mellin space
- The overall coefficient is halved

Interplay between soft and massless limit in $H \to b\bar{b}$

A measurment of x fixes the invariant mass $(p_2 + k)^2 = m^2_{g\bar{b}}$ thus screening one of the collinear (mass) logs.

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Furthermore:

$$\int_{-1}^{1} \frac{1}{1 - \beta_1 \cos\theta} \,\mathrm{dcos}\,\theta \simeq \log \frac{x^2}{\xi(1-x)}, \qquad \beta_1 = \frac{x\sqrt{1 - 4\xi/x^2}}{x - 2\xi},$$

where β_1 is the quark velocity in the $\vec{p_2} + \vec{k} = 0$ frame

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We expect this behaviour to arise if look at a differential distribution which is directly related to the virtuality of one of the propagators, here $m^2_{g\bar{b}}$

Interplay between soft and massless limit in $H \to b\bar{b}$

Let us consider the differential distribution in $\bar{x} = \frac{(p_1+p_2)^2}{q^2} \to 1$ as $k \to 0$. Performing an explicit calculation:

$$\lim_{\xi\to0}\lim_{\bar{x}\to1}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\bar{x}} = \lim_{\bar{x}\to1}\lim_{\xi\to0}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\bar{x}} =$$

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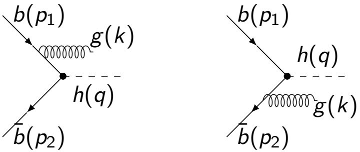
$$\boxed{-\frac{2\alpha_s C_F}{\pi}\frac{1+\log\xi}{1-\bar{x}}} + ..,$$

In this case we have only a single logarithmic enhancement $\implies$ The limits commute!

Higgs Boson Production

We interested in checking the Higgs production $b(p_1) + \bar{b}(p_2) \to h(q)$, differential in $\tau = \frac{(p_1+p_2)^2}{q^2}$, which is not related to the virtuality of the propagators:



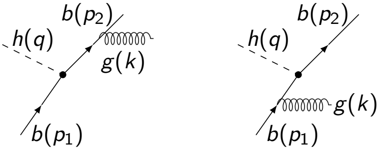
In this case we find that the limits commute:

$$\lim_{\tau\to 1}\lim_{\xi\to 0}\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s C_F}{\pi}\frac{1+\log\xi}{1-\tau}+\dots$$

$$\lim_{\xi\to 0}\lim_{\tau\to 1}\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s C_F}{\pi}\frac{1+\log\xi}{1-\tau}+\dots$$

Higgs Boson DIS

Finally study the real emission correction to the scattering $b(p_1) + h(q) \to b(p_2)$, differential in $x_{\mathrm{B}} = \frac{-q^2}{2p_1 \cdot q}$ which is related to the virtuality $(p2 + k)^2$:



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$$\lim_{\xi\to 0}\lim_{x_{\mathrm{B}}\to 1}\frac{1}{\bar{\sigma}_0}\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\mathrm{B}}} = -\frac{2\alpha_{\mathrm{s}} C_{\mathrm{F}}}{\pi}\frac{1+\log\xi}{1-x_{\mathrm{B}}} + \ldots.$$

Same behaviour as in the decay.

Soft Resummation in the Massive Scheme

In the large N limit [2]:

$$\widetilde{\Gamma}(N,\xi) = \underbrace{C(\xi,\alpha_s)}_{\text{Hard Function}} \underbrace{e^{-2\int_0^1 dx \frac{x^{N-1}-1}{1-x}\gamma_{\text{soft}}\left(\beta,\alpha_s\left((1-x)^2q^2\right)\right)}}_{\text{Soft Function}},$$

γ_{soft} is the massive anomalous soft dimension and we have at most single logs of N. We want to evaluate this expression at NLL.

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We need:

- the two loops expression γ_{soft}:

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- the two loops expression $\gamma_{\rm soft}$:

$$\gamma^{(0)}_{\rm soft} = C_F\left(\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right).$$

  while the second order was presented in [3].

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$$C(\xi, \alpha_s) = 1 + \frac{\alpha_s}{\pi} C^{(1)}(\xi) + \mathcal{O}\left(\alpha_s^2\right)$$

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$$C(\xi, \alpha_s) = 1 + \frac{\alpha_s}{\pi} C^{(1)}(\xi) + \mathcal{O}\left(\alpha_s^2\right)$$

By definition:

$$\frac{\alpha_s}{\pi} C^{(1)}(\xi) = \lim_{N \to \infty} \left[\widetilde{\Gamma}(N, \xi) - \left(1 + \frac{2\alpha_s}{\pi} \gamma^{(0)}_{\mathrm{soft}}(\beta) \log \frac{1}{\bar{N}} \right) \right],$$

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It receives contributions from the virtual corrections and from the end point of the one real emission diagrams.

Soft Resummation in the Massive Scheme

Writing the real emission differential decay rate as:

$$\frac{\mathrm{d}\Gamma^{(R)}}{\mathrm{d}x} = \frac{\alpha_s C_F}{\pi} \Gamma_0^{(d)} \frac{f_\varepsilon\left(x, \xi, \frac{q^2}{\mu^2}\right)}{(1-x)^{1+2\epsilon}}.$$

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The coefficient $C^{(1)}$ can be determined using the identity between distributions:

$$\frac{f_\varepsilon(x, \xi, \frac{q^2}{\mu^2})}{(1-x)^{1+2\varepsilon}} = \frac{f_\varepsilon\left(x, \xi, \frac{q^2}{\mu^2}\right) + f_\varepsilon\left(1, \xi, \frac{q^2}{\mu^2}\right) - f_\varepsilon\left(1, \xi, \frac{q^2}{\mu^2}\right)}{(1-x)^{1+2\varepsilon}} =$$

$$\delta(1-x)\left[-\frac{f_0(1,\xi)}{2\varepsilon} + f_0(1,\xi)\log(1-2\sqrt{\xi}) - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\varepsilon} f_\varepsilon\left(1, \xi, \frac{q^2}{\mu^2}\right)\Big|_{\varepsilon=0}\right] + \frac{f_0(x,\xi)}{(1-x)_+} + \mathcal{O}(\varepsilon)\,.$$

Soft Resummation in the Massive Scheme

$$
\begin{aligned}
C^{(1)}(\xi) = \frac{C_F}{2}\Bigg\{ & -2\frac{\gamma^{(0)}_{\text{soft}}(\beta)}{C_F}\Bigg[-2\log\left(1-\sqrt{1-\beta^2}\right)+\log\frac{m^2}{q^2} \\
& +\log\left(\frac{1-\beta^2}{4}\right)+1\Bigg]-2+2L(\beta)\left(\frac{1-\beta^2}{\beta}\right) \\
& +\frac{1+\beta^2}{\beta}\Bigg[\frac{1}{2}L(\beta)\log\left(\frac{1-\beta^2}{4}\right)+2L(\beta)(1-\log\beta) \\
& +2\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)+L(\beta)^2+L(\beta)\log\frac{1-\beta}{2}+\frac{2}{3}\pi^2 \\
& -\frac{1}{2}\left(\mathrm{Li}_2\left(\frac{4\beta}{(1+\beta)^2}\right)-\mathrm{Li}_2\left(\frac{-4\beta}{(1-\beta)^2}\right)\right)\Bigg]\Bigg\},
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In the small ξ limit we find:

$$\alpha_s C^{(1)}(\xi) = \alpha_s C_F \left(\frac{1}{2} \log^2 \xi + \log \xi + \mathcal{O}(\xi^0) \right)$$

Double log of the mass in disagreement with DGLAP.

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Double log of the mass in disagreement with DGLAP.
What is the problem?

Soft Resummation in the Massive Scheme

$$f_\varepsilon(x,\xi,\frac{q^2}{\mu^2}) = f_\varepsilon\left(x,\xi,\frac{q^2}{\mu^2}\right) + \boxed{f_\varepsilon\left(1,\xi,\frac{q^2}{\mu^2}\right) - f_\varepsilon\left(1,\xi,\frac{q^2}{\mu^2}\right)}$$

This relation can be expanded only if ξ is finite

$$f_0(x,\xi) \xrightarrow[\xi\to 0]{} \log\left(1-x\right)$$

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This relation can be expanded only if ξ is finite

$$f_0(x, \xi) \xrightarrow[\xi \to 0]{} \log(1 - x)$$

Non commutativity of the limits

The distribution identity does not hold when $\xi \to 0$ because in this limit $f_0(1, \xi)$ is not defined.

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Double mass logs in the soft limit of the massive scheme $\Longleftrightarrow$ Double soft logs in the fragmentation function approach

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A well defined expression in the massless limit be be obtained rewriting the differential decay rate as:

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx} = \delta(1-x) + \frac{\alpha_s}{\pi}\left[C_F\left(\frac{f_0(x,\xi)}{1-x}\right)_+ + A(\xi)\,\delta(1-x)\right],$$

The delta coefficient has an expected behaviour for $\xi \to 0$

$$A(\xi) = C_F\frac{3}{2}\log\xi + \mathcal{O}(\xi^0),$$

in agreement with DGLAP evolution.

Conclusions and Outlook

We have considered observables with different kinematics focusing on the differential distributions in the massive scheme.

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- Soft and massless do not always commute, in particular in the massless limit the structure of the distributions can radically change: $\Longrightarrow$ presence of double logs of N
- The origin of this particular behaviour can be traced back to the interplay between the observable we are computing and the fermionic propagators in the scattering amplitudes.

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Finally, we have focused on the massive scheme resummation of the process $H \to b\bar{b}$ in the large N limit.

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- We have found that within this approach double logarithms of the mass may appear.
- We have traced back the origin of the disagreement with the DGLAP picture in the non commutativity between the large N and small mass limit

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- We have shown that the logarithmic structure of the two approaches is different, it would be interesting to study numerical differences at collider energies.
- In the context of the heavy quark calculations one combines the two schemes in order to obtain better predictions(e.g. FONLL). However in the case of the soft gluon resummation the merging is far from trivial. An all-order matching procedure that would allow to combine soft resummation in the massive and massless scheme is left to a future work.

Thanks for your attention!