Phenomenology of small-x resummation

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LFC22

Strong interactions from QCD to new strong dynamics at LHC and Future Colliders

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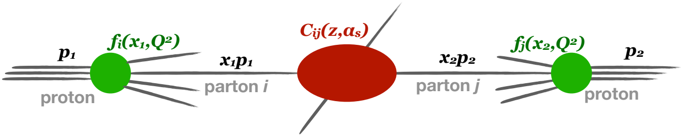
QCD collinear factorization:

$$y = Y - \frac{1}{2}\log\frac{x_1}{x_2}$$

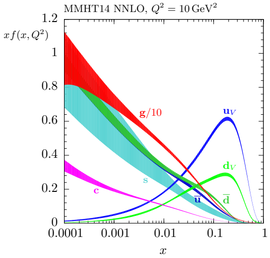
$$\frac{d\sigma}{dQ^2 dY dp_t \ldots} = \sum_{i,j=g,q} \int_\tau^1 dx_1 \int_\tau^1 dx_2\, f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, C_{ij}\left(\frac{\tau}{x_1 x_2}, y, p_t, \ldots, \alpha_s\right)$$

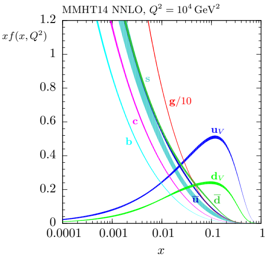


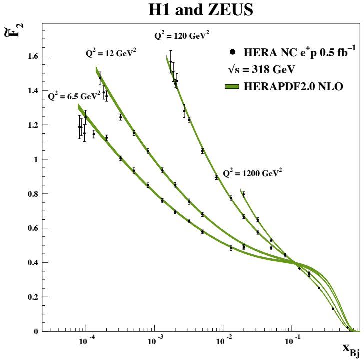
$x_1, x_2, \frac{\tau}{x_1 x_2}$ can get as small as $\tau = \frac{Q^2}{s}$ (note: typical values $x_1, x_2 \sim \sqrt{\tau}$)

τ	Higgs	Z, W	low mass DY	$c\bar{c}$
LHC (13 TeV)	10^{-4}	5×10^{-5}	$\sim 10^{-6}$	$\sim 10^{-7}$
FCC-hh (100 TeV)	1.5×10^{-6}	8×10^{-7}	$\sim 10^{-8}$	$\sim 10^{-9}$

FCC-hh probes roughly two orders of magnitude smaller x







Gluon and sea-quark PDFs grow at small x ⇒ cross sections grow

At sufficiently small x, the density of partons becomes too high for linear evolution to be still valid ⇒ **saturation**

Moreover, at small x the presence of $\log\frac{1}{x}$ contributions in perturbative coefficients make fixed-order results unreliable ⇒ **small-x resummation**

Small-x logarithms in the context of collinear factorization

$$\frac{d\sigma}{dQ^2\,dY\,dp_t\ldots} = \sum_{i,j=g,q} \int_\tau^1 dx_1 \int_\tau^1 dx_2\, f_i(x_1,Q^2)\, f_j(x_2,Q^2)\, C_{ij}\left(\frac{\tau}{x_1 x_2}, y, p_t, \ldots, \alpha_s\right)$$

DGLAP evolution: $$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \int_x^1 \frac{dz}{z}\, P_{ij}(z,\alpha_s(\mu^2))\, f_j\left(\frac{x}{z},\mu^2\right)$$

Heavy-quark matching: $$f_i^{[n_f+1]}(x,\mu_m^2) = \int_x^1 \frac{dz}{z}\, A_{ij}(z,\alpha_s(\mu_m^2))\, f_j^{[n_f]}\left(\frac{x}{z},\mu_m^2\right)$$

Any object with a perturbative expansion can exhibit a logarithmic enhancement:

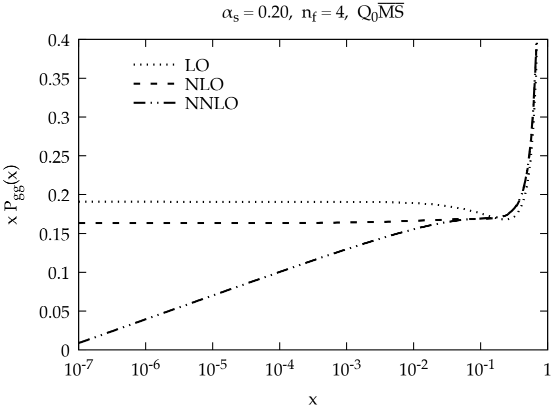
- observable: coefficient functions $C(x,y,p_t,\ldots,\alpha_s)$
- evolution: splitting functions $P(x,\alpha_s)$ and matching conditions $A(x,\alpha_s)$

Small-x logarithms: single logs $\alpha_s^n \frac{1}{x} \log^k \frac{1}{x} \quad (0 \le k \le n-1)$

When $\alpha_s \log \frac{1}{x} \sim 1$ perturbativity is spoiled $\to$ **all-order resummation needed**

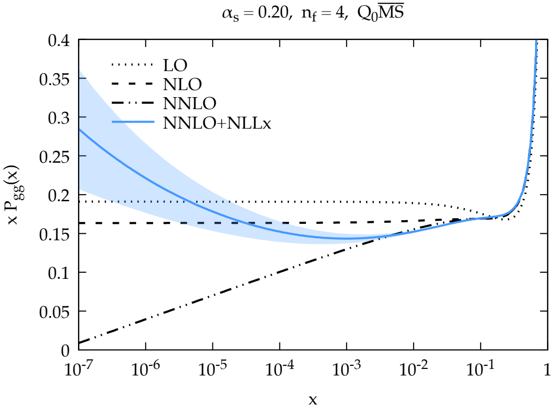
In $\overline{\text{MS}}$ and related schemes, both coefficient $C(x,\alpha_s)$ and splitting $P(x,\alpha_s)$ functions, and also matching conditions $A(x,\alpha_s)$, are logarithmically enhanced at small x (in the singlet sector)

$P_{gg}(x, \alpha_s)$ splitting function at fixed order



Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability** for $x \lesssim 10^{-3}$
(for $Q \sim 5$GeV)

$\boldsymbol{P_{gg}(x, \alpha_s)}$ splitting function at fixed order



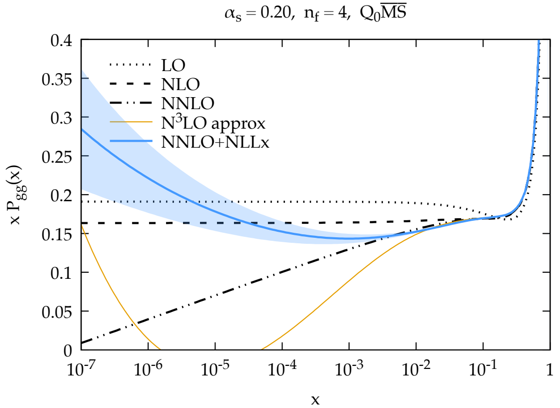
Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability** for $x \lesssim 10^{-3}$ (for $Q \sim 5$GeV)

Resummation obtained with the `HELL` public code

[MB,Marzani,Peraro 1607.02153] [MB,Marzani,Muselli 1708.07510] [MB,Marzani 1805.06460]

Small-x logarithms in gluon-gluon splitting function

$P_{gg}(x, \alpha_s)$ splitting function at fixed order

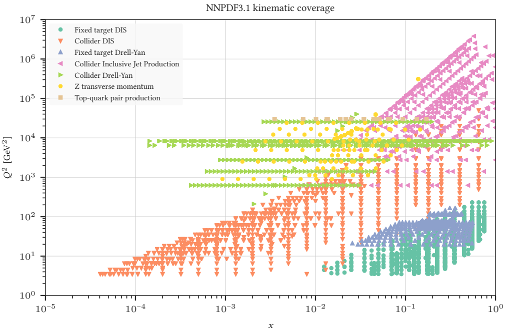


Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability** for $x \lesssim 10^{-3}$ (for $Q \sim 5$GeV)

N^{3}LO splitting functions are much more unstable at small x → need resummation!

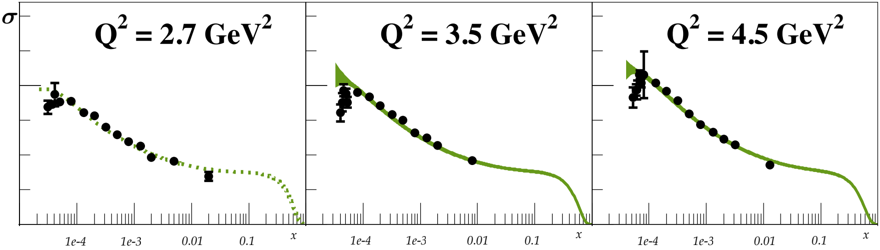
Do we experience the need for small-x resummation?

Hint: look at PDF fits...

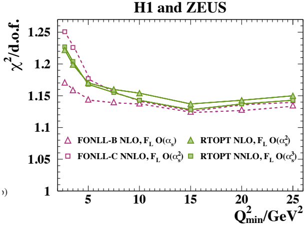


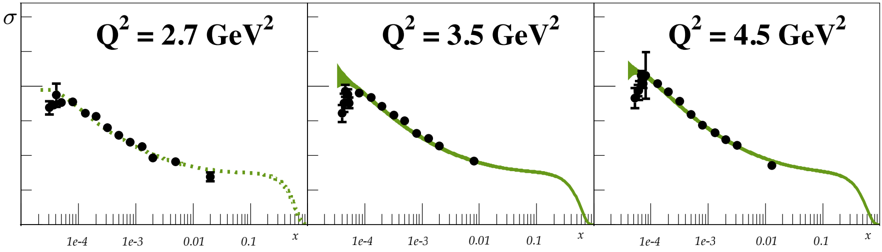
Deep-inelastic scattering (DIS) data from HERA extend down to $x \sim 3 \times 10^{-5}$ in the "perturbative region" $Q^2 > 2$GeV^2

Tension between HERA data at low Q^2 and low x with fixed-order theory



Also leads to a deterioration of the χ^2 of PDF fits when including low-Q^2 data





These data are at low x but also at low Q^2

Possible explanations:

- Higher twist contributions

  [Abt,Cooper-Sarkar,Foster,Myronenko,Wichmann,Wing 1604.02299]

- Small-x resummation

  [Ball,Bertone,MB,Marzani,Rojo,Rottoli 1710.05935] [xFitter+MB 1802.00064]

- Maybe saturation already?

The role of the longitudinal structure function

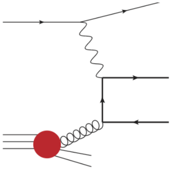
The HERA data are reduced cross sections, given by

$$\sigma_{r,\mathrm{NC}} = F_2(x, Q^2) - \frac{y^2}{1 + (1-y)^2} F_L(x, Q^2) \qquad\qquad y = \frac{Q^2}{x\, s}$$

in terms of the structure functions F_2, F_L

The turnover can be explained by a larger F_L, contributing mostly at small x

The other option, a turnover in F_2, seems unlikely (requires peculiar PDF shape)



Note that $F_L = \mathcal{O}(\alpha_s)$, and it is gluon dominated

It plays a key role in DIS at small x

$\Rightarrow$ having good measurements of F_L is very important!

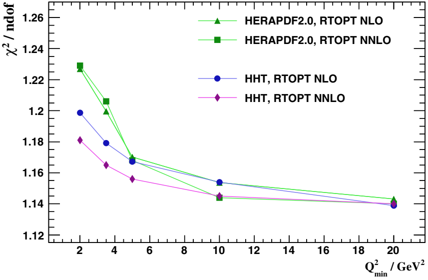
Future ep colliders (LHeC, FCC-eh) could provide precise F_L measurements!!

Higher twist explanation of HERA low-x data

$$F_L \to F_L \times \left(1 + \frac{A_L}{Q^2}\right)$$

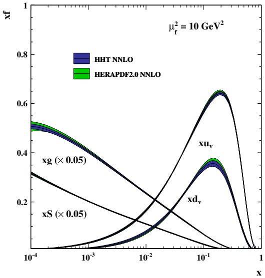
with A_L fitted from data

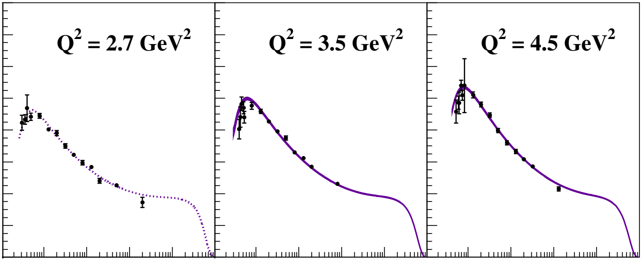
Improved description, but χ^2 still grows



PDFs unaffected

↓ ↓ ↓ ↓ ↓





[Abt,Cooper-Sarkar,Foster,Myronenko,Wichmann,Wing 1604.02299]

The first two PDF fits with small-x resummation

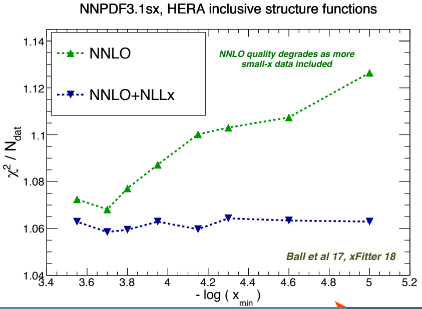
HELL $\rightarrow$ makes possible a PDF fit with small-x resummation

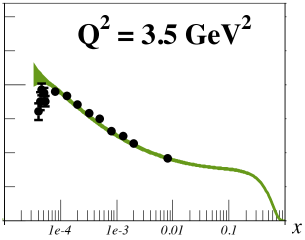
NNPDF3.1sx [1710.05935]	xFitter [1802.00064, see also 1902.11125]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC ($\sim$ **4000** datapoints)	only HERA data ($\sim$ **1200** datapoints)
NLO, NLO+NLLx, NNLO, NNLO+NLLx	NNLO, NNLO+NLLx

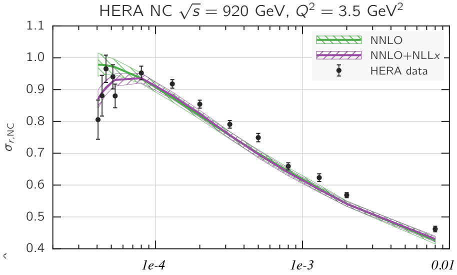
The quality of the fit improves substantially including small-x resummation

χ^2/N_{dat}	NNLO	NNLO+NLLx
xFitter	**1.23**	**1.17**
NNPDF3.1sx	**1.130**	**1.100**
		smaller!

Stable upon inclusion of low-x data $\rightarrow$



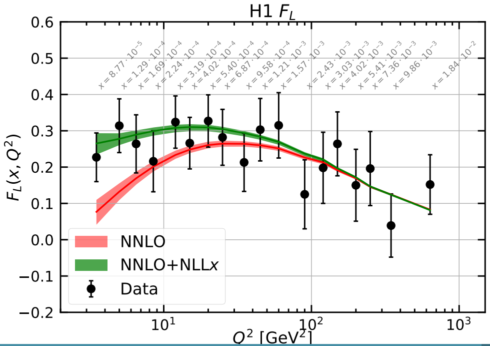




Improved description of the data, turnover well reproduced

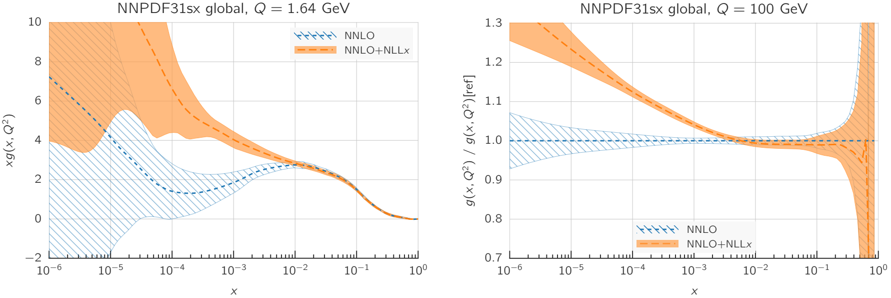
The better description mostly comes from a larger resummed F_L

Note: no extra parameters in the fit, just improved theory



Impact of small-x resummation on PDFs: the gluon

Small-x resummation mostly affects the gluon PDF (and the total quark singlet)



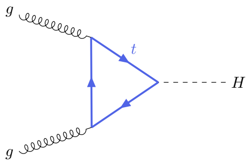
Dramatic effect of resummation on the gluon PDF at $x \lesssim 10^{-3}$

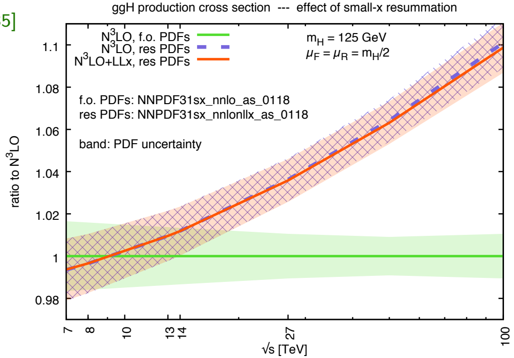
Persists at higher energy scales $\Rightarrow$ impact for LHC and FCC-hh phenomenology

Note that the gluon PDF obtained with small-x resummation grows faster
$\rightarrow$ saturation at some point!

$gg \to H$ inclusive cross section

[MB,Marzani 1802.07758] [MB 1805.08785]

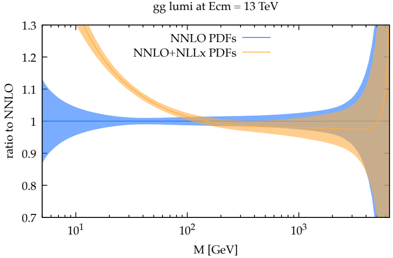




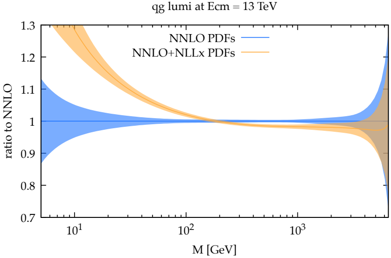
ggH cross section at FCC-hh $\sim$ **10%** larger than expected!

At LHC $+1\%$ effect; larger effect expected at differential level

Other recent works on Higgs production [Hentschinski,Kutak,vanHameren 2011.03193] [Celiberto,Ivanov,Mohammed,Papa 2008.00501] gg:

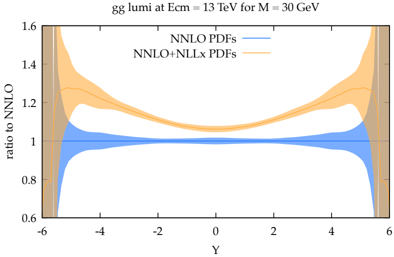


qg:

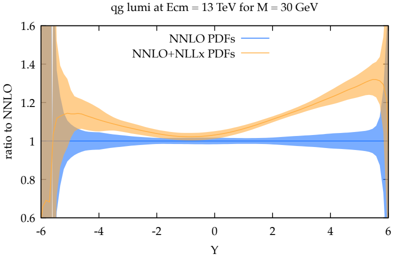


Difference more pronounced in differential distributions at large rapidity

gg:

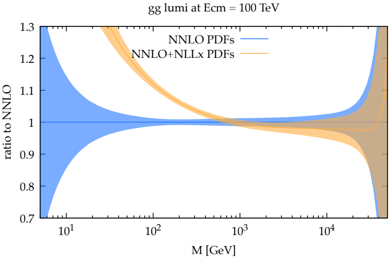


qg:

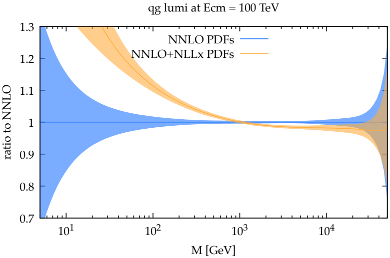


Parton luminosities at FCC-hh

gg:

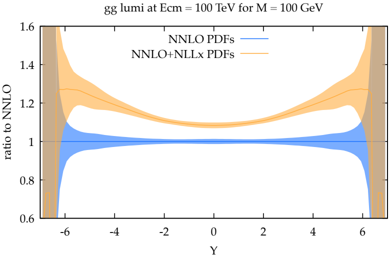


qg:

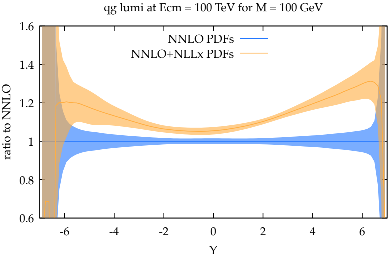


Large effects also at the EW scale, especially at large rapidities

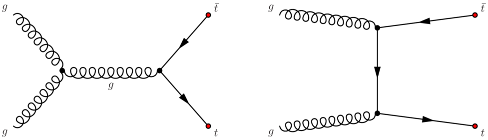
gg:



qg:



Fully differential heavy-quark pair production [MB,Silvetti (in preparation)]



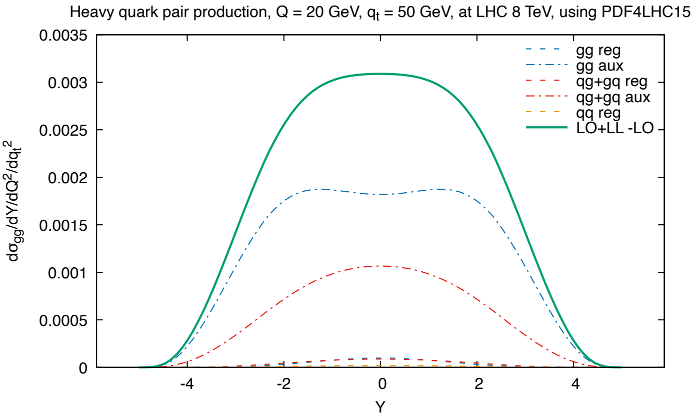
We have resummed the cross section for different kinematics:

- heavy-quark pair $\frac{d\sigma}{dQ^2\, dY\, dq_t}$ $\rightarrow$ e.g. J/ψ, Υ production
- single heavy quark $\frac{d\sigma}{dY\, dp_t}$ $\rightarrow$ e.g. D, B meson production

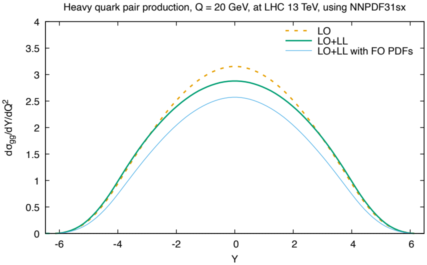
Small-x resummation crucial for charm and bottom production

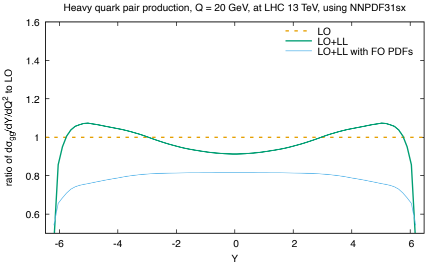
- sensitive to very small x $\rightarrow$ constrain the PDFs [Gauld, Rojo 1610.09373]
- key process at a forward physics facility (FPF) [Feng et al 2203.05090]

Hadron-level purely-resummed results for $\frac{d\sigma}{dQ^2\,dY\,dq_t}$ (pair kinematics)

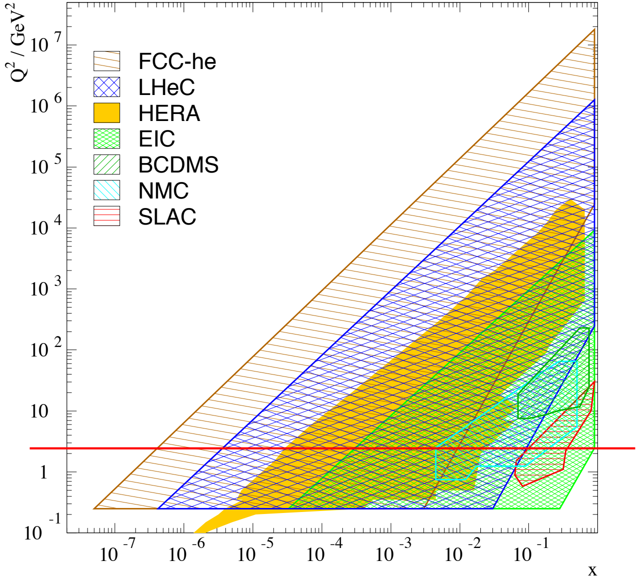


Hadron-level resummed results for $\frac{d\sigma}{dQ^2\, dY}$ (pair kinematics)

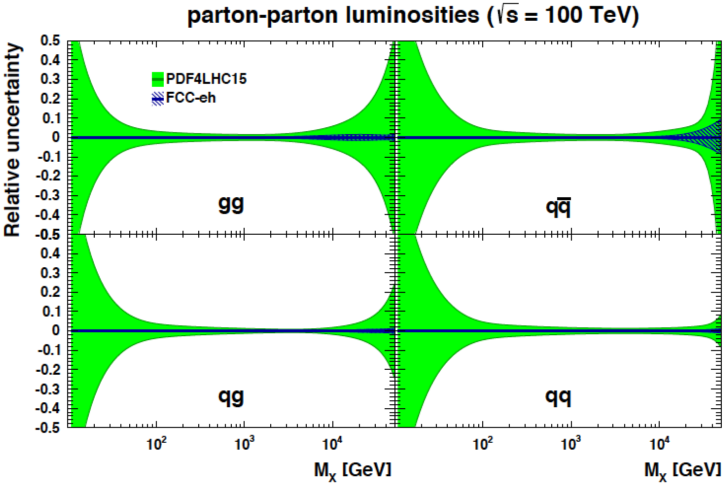




What can we gain from future ep colliders?



The role of FCC-eh (and LHeC): impact on parton luminosities for FCC-hh



Dramatic reduction of PDF uncertainties with FCC-eh, especially at low and high x

Note: all PDFs from a single experiment!

Conclusions

Interest in small x:

- **opportunity**: exploring and understanding new regimes of QCD
- **tool**: fundamental ingredient for FCC-hh (and low-Q^2 LHC) phenomenology

QCD at small x:

- small-x resummation (BFKL regime)
- non-linear behaviour (saturation regime)
- crucial to understand how QCD works at small x to provide reliable predictions for present and future pp colliders

Where we are and where we go:

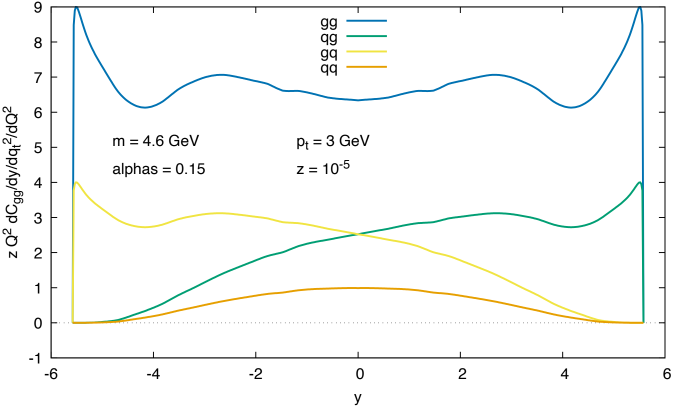
- resummation of evolution, inclusive cross sections (DIS, Higgs, ...) ✓
- resummation of differential distributions ($Q\bar{Q}$ ✓, Drell-Yan ongoing, ...)
- extension beyond LLx (attempts ongoing)

A word on future colliders:

- ep colliders (LHeC, FCC-eh) are crucial for exploring QCD at small x and to provide precise PDFs for pp colliders $\rightarrow$ let's consider them seriously!

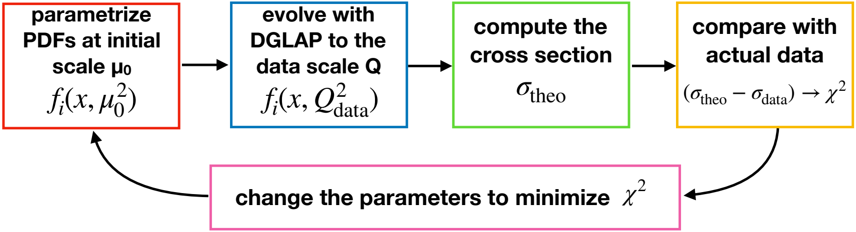
Backup slides

Parton-level purely-resummed results for $\frac{d\sigma}{dY\,dp_t}$ (single-quark kinematics)



Preliminary!!

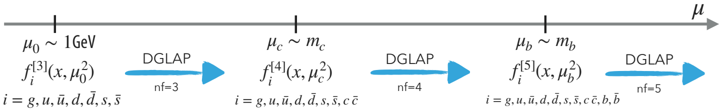
Strategy: fit $f_i(x, \mu_0^2)$ by comparison with (many) data



Quality of extracted PDFs depends on the accuracy of the experimental data **and of the theoretical input**

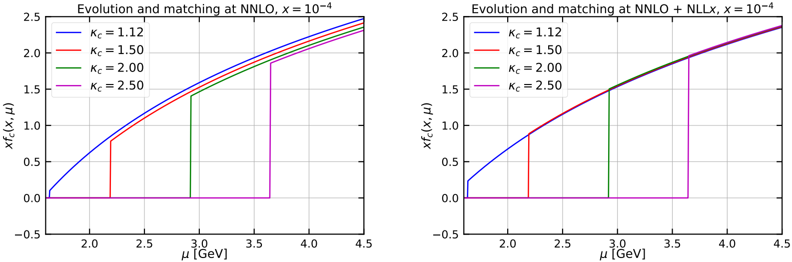
Variable flavour number scheme: charm matching conditions

The number n_f of "active" flavours changes during the evolution (factorization scheme choice to resum large collinear mass logarithms from heavy quark pair production)



Matching relation between PDFs in schemes with different n_f

$$f_i^{[n_f+1]}(\mu^2) = \sum_{j=\text{light}} A_{ij}(m^2/\mu^2) \otimes f_j^{[n_f]}(\mu^2) \qquad A_{ij} = \text{perturbative matching coefficients}$$



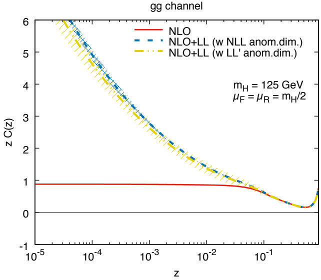
The perturbatively generated charm PDF is much less dependent on the (unphysical) matching scale when small-x resummation is included!

Why is the effect of resummation mostly driven by the PDFs?

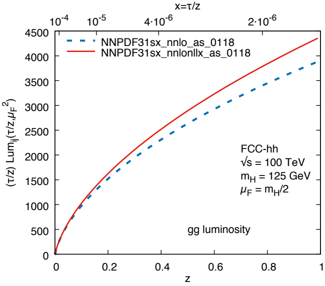
$$\frac{d\sigma}{dQ^2 dY...} = \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, f_i\left(\sqrt{\frac{\tau}{z}} e^{\hat{y}}, Q^2\right) f_j\left(\sqrt{\frac{\tau}{z}} e^{-\hat{y}}, Q^2\right) C_{ij}(z, Y - \hat{y}, ..., \alpha_s)$$

The small z integration region, where logs in C are large, is weighted by the PDFs at large momentum fractions $x = \sqrt{\frac{\tau}{z}} e^{\pm \hat{y}}$
Since PDFs die fast at large x, especially the gluon, the small-z region is suppressed!

Rather, the large z region is enhanced by the gluon-gluon luminosity
In that region, the difference between fixed-order and resummed PDFs is large

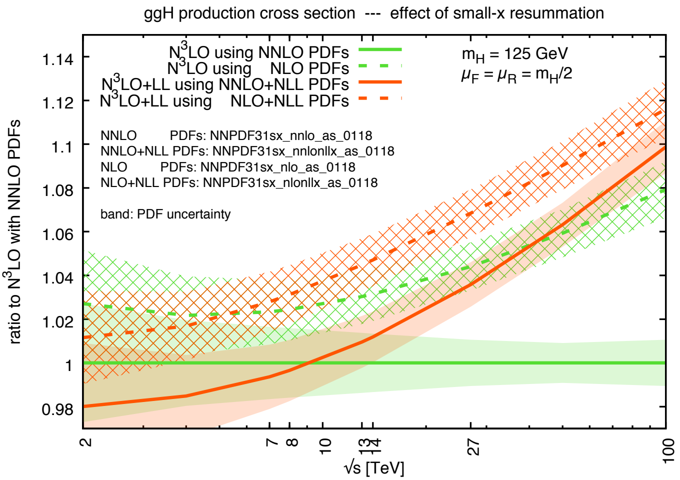


gg partonic cross section



gg luminosity

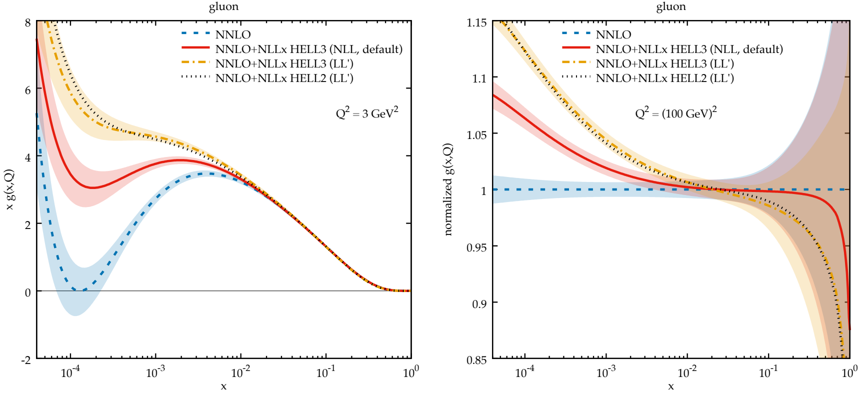
Higgs production: LHC and future colliders



The large effect of the resummation is due to the NNLO being perturbatively unstable at small x, leading to a smaller NNLO gluon at small x

First fit with `HELL 3.0` [MB,Giuli 1902.11125]

Red and yellow curves differ by subleading logs



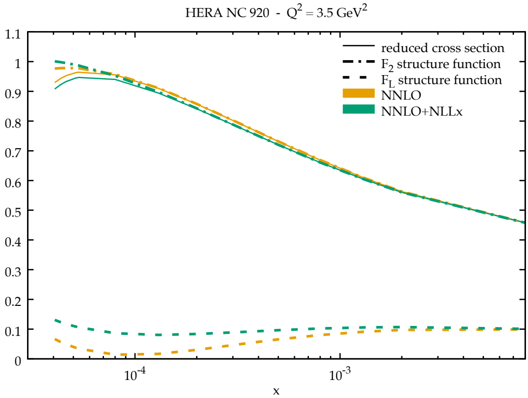
Achieved with a new parametrization, more flexible at small x

$$xf(x,\mu_0^2) = A\,x^B(1-x)^C\left[1 + Dx + Ex^2 + F\log x + G\log^2 x + H\log^3 x\right]$$

Improved description of the low x data even at fixed order

Impact of subleading logs (with xFitter)

The good agreement obtained at fixed order with the low x HERA data is achieved in a different way with respect to the resummed case [MB,Giuli 1902.11125]



At resummed level, both F_L and F_2 grow

At fixed order, F_L grows below $x \sim 10^{-4}$ and F_2 decreases, due to the sudden growth of the gluon PDF

Future F_L measurements

Measured DIS cross section

$$\sigma_{r,\mathrm{NC}} =$$

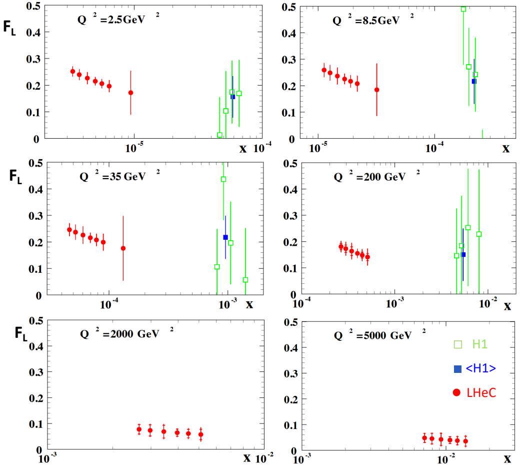
$$= F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$

$$y = \frac{Q^2}{x\,s} = 1 - \frac{E_e'}{E_e}$$

Extraction of F_L requires changing s at same x, Q^2, at sufficiently large y, namely small E_e'

LHeC projections assuming three electron beam energies: $E_e = 60, 30, 20$ GeV

Much higher precision than HERA!



Similar results expected for FCC-eh as well, but one order of magnitude smaller x

In standard linear approach (DGLAP) we consider parton splittings only

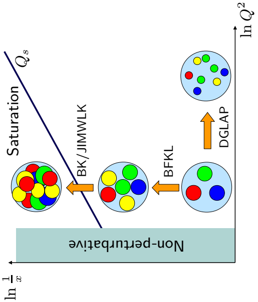
At high density, non negligible probability that partons *recombine* $\to$ non-linear behaviour (restoration of unitarity)

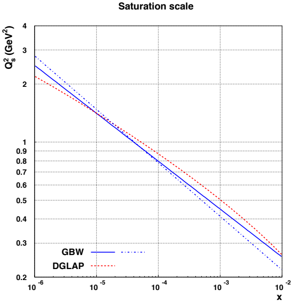
Saturation models [Bartels,Golec-Biernat,Kowalski hep-ph/0203258] [Iancu,Itakura,Munier hep-ph/0310338] [Golec-Biernat,Sapeta 1711.11360]

Saturation line

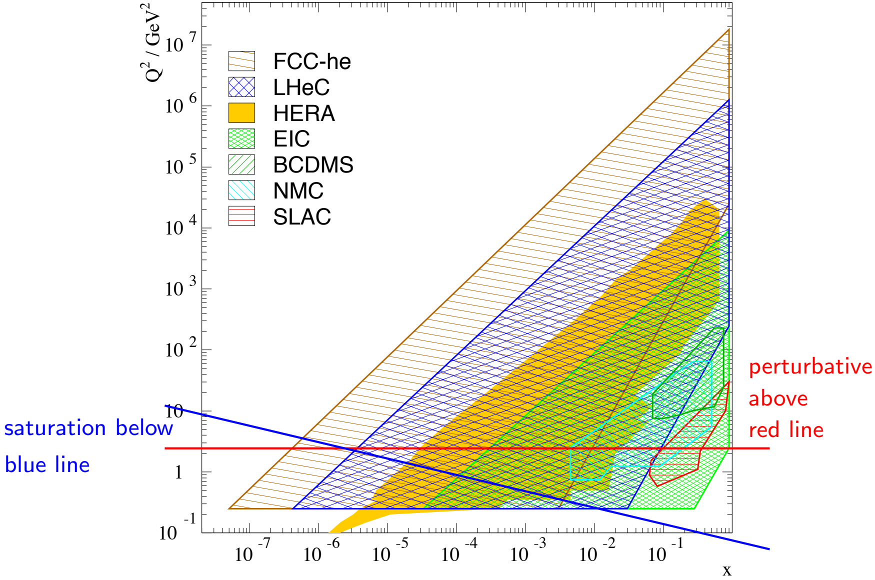
$$Q_s^2 \sim \left(\frac{1}{x}\right)^{\lambda}$$

$$\lambda \sim 0.25$$

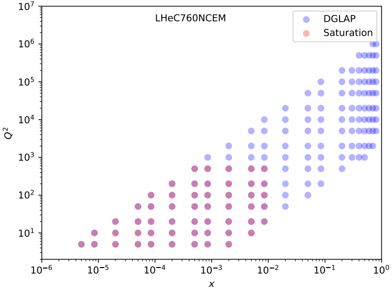




(fits to HERA data, including also data at $Q^2 \lesssim 1\text{GeV}^2$)

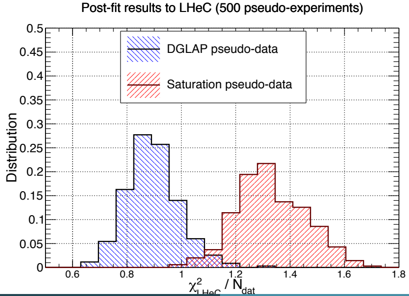

FCC-he
LHeC
HERA
EIC
BCDMS
NMC
SLAC
Q^2 / GeV^2
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
saturation below
blue line
perturbative
above
red line

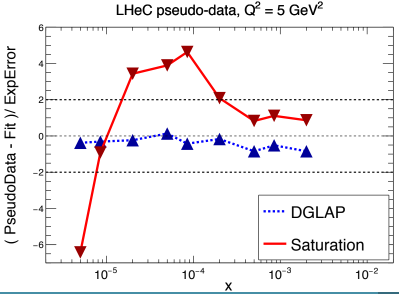
Pseudo-data for LHeC with saturation→



DGLAP fits cannot absorb all the effect of saturation → it is possible to identify saturation effects by distortions in pulls

Possible thanks to the presence of data sensitive to saturation at different Q^2: the fit cannot absorb a non-DGLAP Q^2 dependence





Resummation or saturation? The role of eA experiments

A tension between fixed-order DGLAP fits and data at small-x can also be due to the lack of small-x resummation in the theory

Once small-x resummation is included, it will be much more difficult to distinguish between linear and non-linear dynamics, given that saturation is so much at the border of the perturbative region for the accessible values of x at FCC-eh

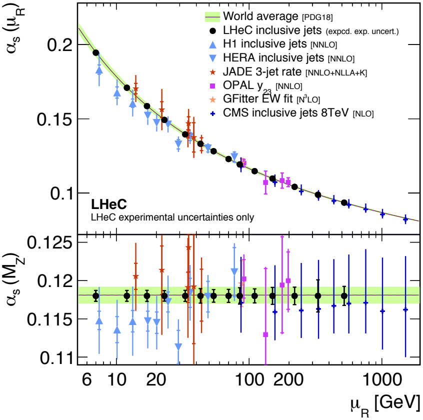
DIS experiment on nuclei will help!

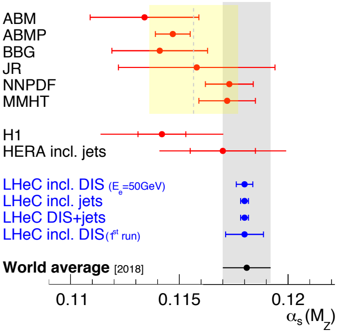
Indeed

- saturation is a density non-linear effect, and we excpect it to be very sensitive on the increased density in nuclei
- resummation is a linear effect $\longrightarrow$ much less sensitive to nuclear effects

$\Rightarrow$ $\boldsymbol{eA}$ program at FCC-eh will be fundamental to disentangle small-x resummation linear dynamics to saturation non-linear dynamics

Note that EIC, due to the limited coverage in x (small energy), will not help for this

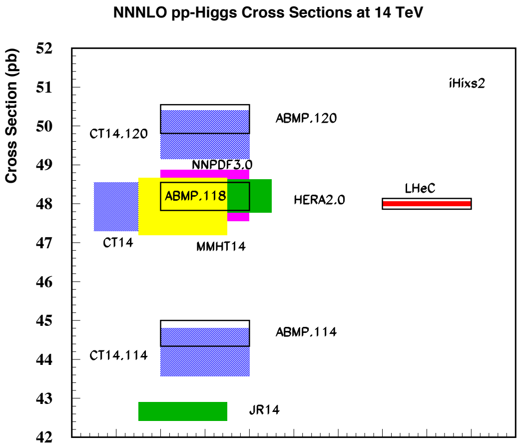




Future ep colliders offer a unique opportunity to determine α_s with high precision (simultaneous determination of α_s and PDFs)

Note: also a low luminosity run will already improve significantly the precision

Direct determination at low Q^2 $\rightarrow$ important also for small x



Red box: PDF uncertainty

Black box: PDF+α_s uncertainty, using α_s extracted from LHeC data