Initial-state QED radiation at NLL accuracy for future e^+e^- colliders

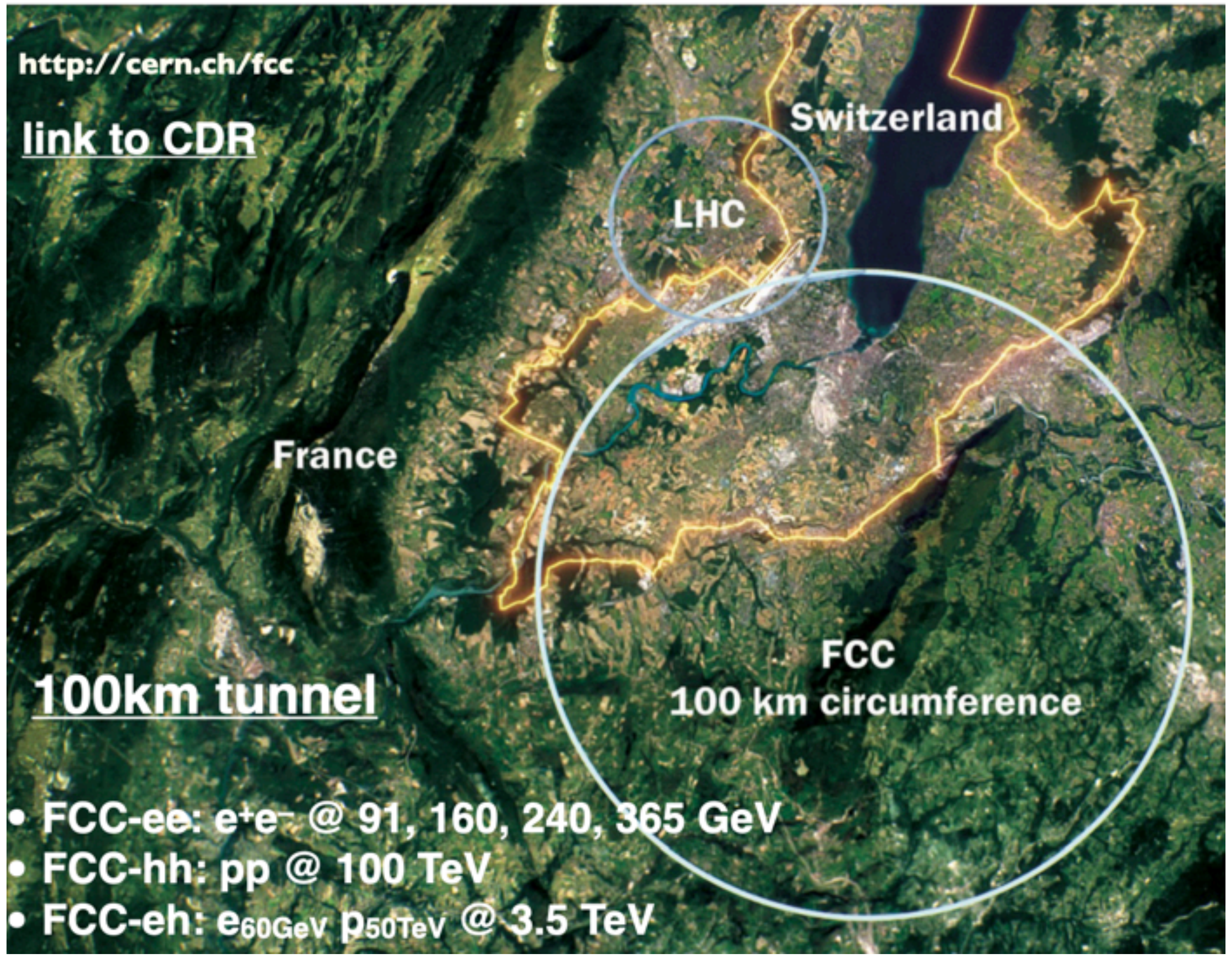
Giovanni Stagnitto
(University of Zurich)

in collaboration with:
V. Bertone, M. Cacciari, S. Frixione, M. Zaro, X. Zhao

LFC22, ECT* Trento, 29.08.22-02.09.22

Introduction

Future Circular Collider



M. Mangano, “Why FCC?”
Theory Colloquium, 15 June 2022, CERN
https://indico.cern.ch/event/1155782/

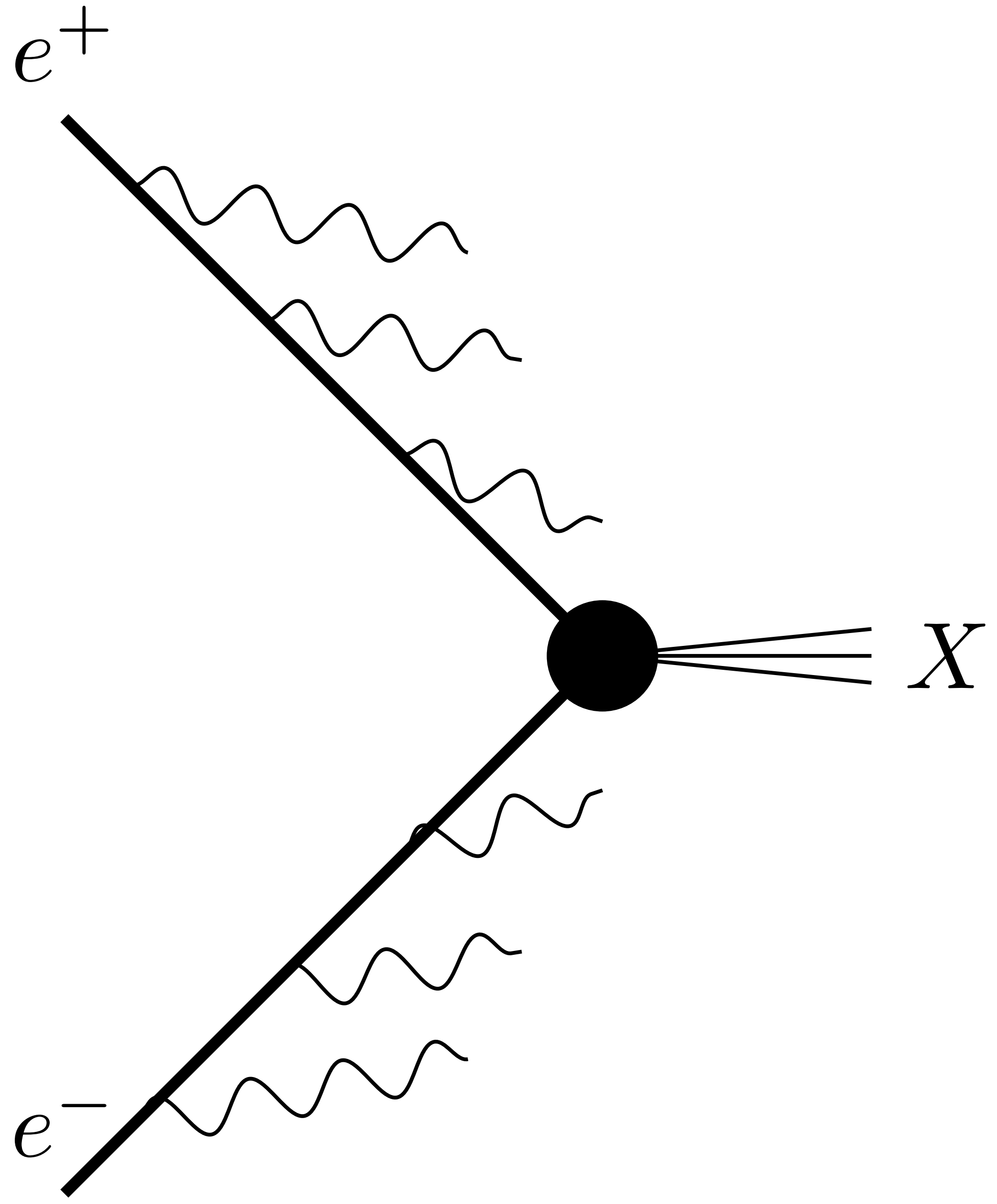
Electro weak precision observables

	Experiment uncertainty			Theory uncertainty
	Current	CEPC	FCC-ee	Current
M_W[MeV]	15	0.5	0.4	4
Γ_Z[MeV]	2.3	0.025	0.025	0.4
$R_b[10^{-5}]$	66	4.3	6	10
$\sin^2\theta^l_{\rm eff}[10^{-5}]$	16	< 1	0.5	4.5

Summary slides of week 1
“Precision calculations for future e+e– colliders: targets and tools”
7-17 June 2022, CERN
https://indico.cern.ch/event/1140580/

Initial state radiation (ISR)

Problem: presence in the cross section $d\sigma_{e^+e^-}$ of **potentially large logarithms**, due to **collinear photon emissions** in the initial state



$$\simeq \alpha^b \sum_{n=0}^{\infty} \alpha^n \left(c_0^{(n)} + c_1^{(n)} L + \ldots + c_n^{(n)} L^n \right) \quad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

b: power of the α in the Born process, m_e: electron mass

Q^2: typical hard scale of the process e.g. c.o.m. energy squared s

Basically all precision observables at e^+e^- colliders affected by ISR!

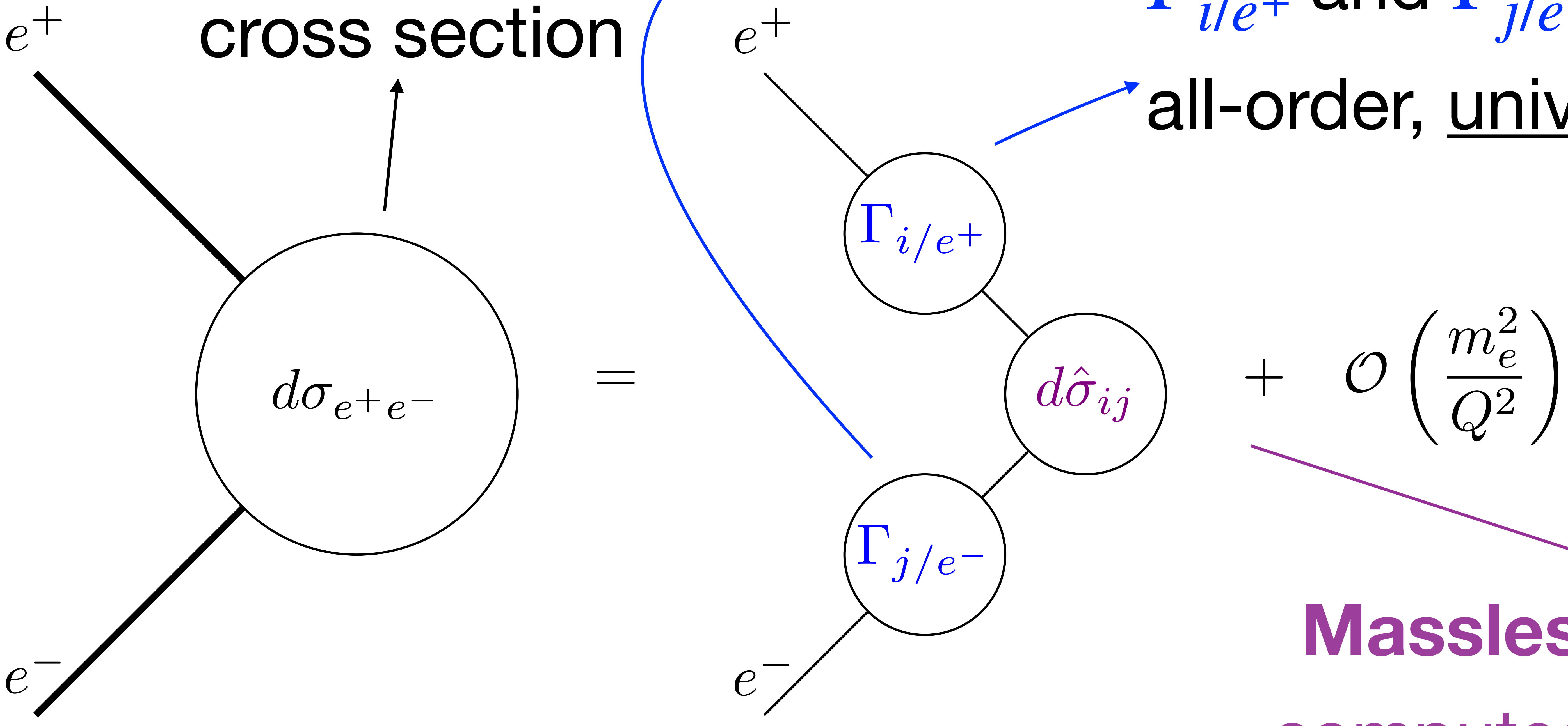
Collinear factorisation

Massive particle-level cross section

QED Parton Distribution Functions (PDFs)

Γ_{i/e^+} and Γ_{j/e^-}, *partons* $i, j = e^+, e^-, \gamma, \ldots$

all-order, universal, resumming $\log(Q^2/m_e^2)$



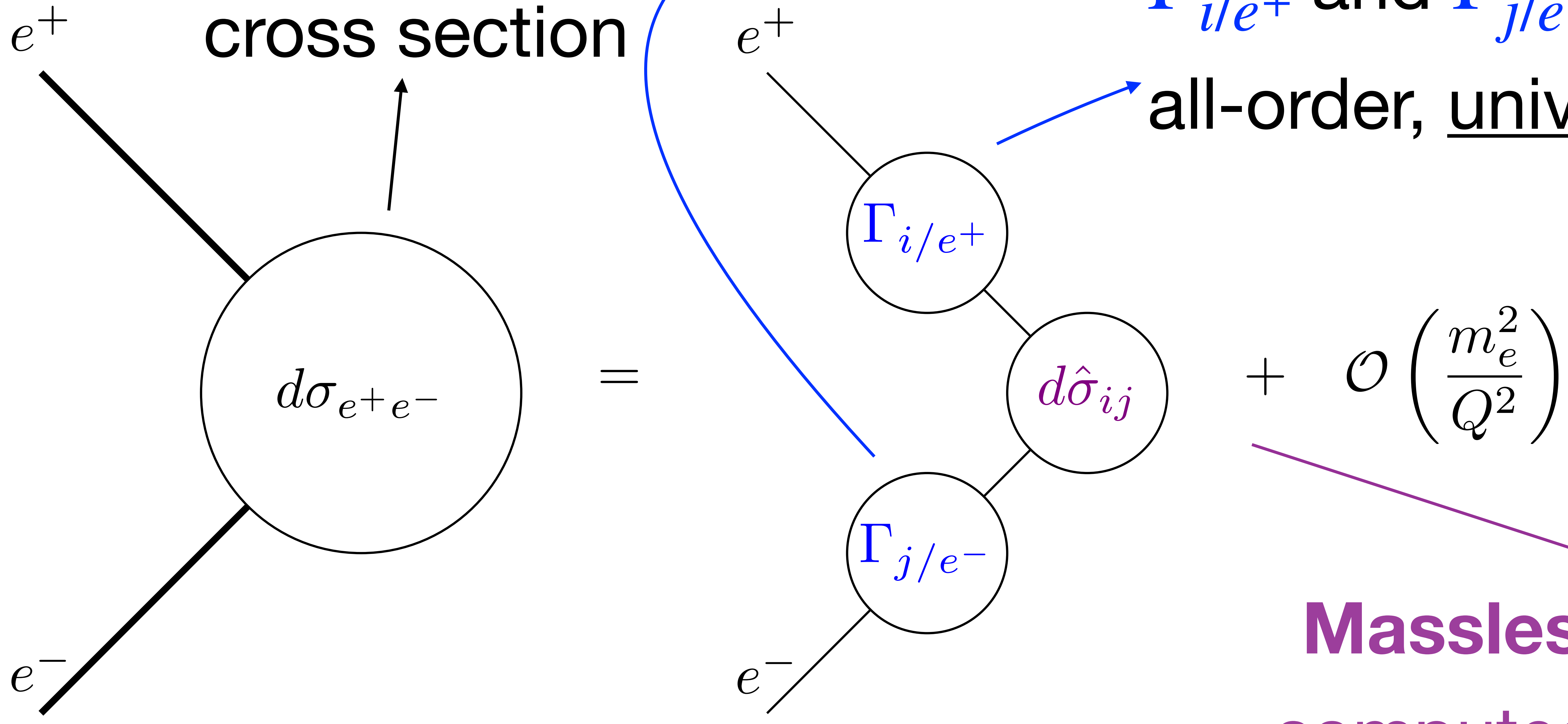
Massless parton-level cross section $d\hat{\sigma}_{ij}$

computed order by order, process-specific

$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/e^+}(z_+, \mu^2, m_e^2) \, \Gamma_{j/e^-}(z_-, \mu^2, m_e^2) \, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

Collinear factorisation

Massive particle-level cross section



QED Parton Distribution Functions (PDFs)

Γ_{i/e^+} and Γ_{j/e^-}, *partons* $i, j = e^+, e^-, \gamma, \ldots$

all-order, universal, resumming $\log(Q^2/m_e^2)$

leading log (LL): $(\alpha\, L)^k$

next-to-leading log (NLL): $\alpha(\alpha\, L)^{k-1}$

...

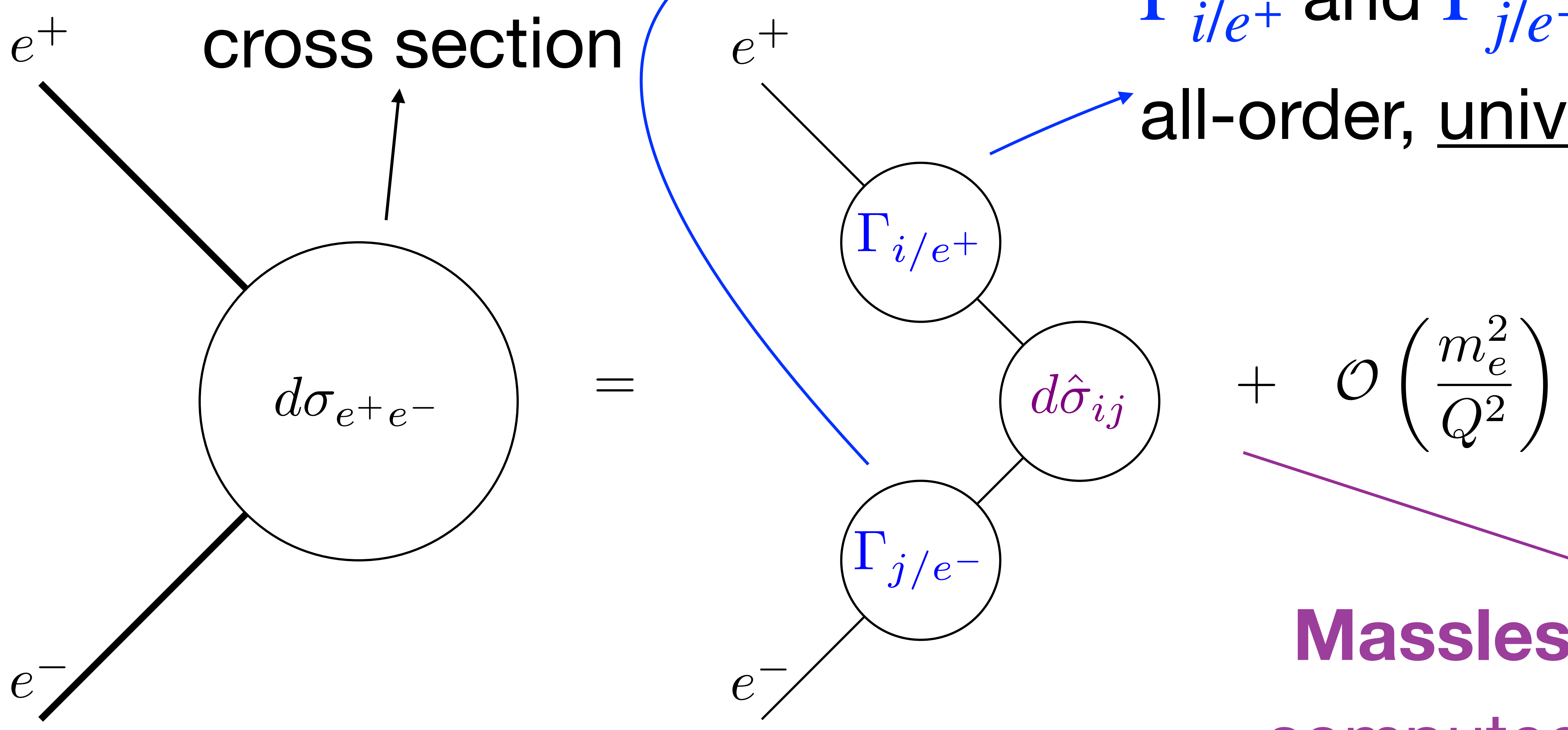
Massless parton-level cross section $d\hat{\sigma}_{ij}$
computed order by order, process-specific

$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_-\, \Gamma_{i/e^+}(z_+, \mu^2, m_e^2)\, \Gamma_{j/e^-}(z_-, \mu^2, m_e^2)\, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

Collinear factorisation

Charge-conjugation implies
$\Gamma_{\alpha/e^-} = \Gamma_{\bar{\alpha}/e^+} \equiv \Gamma_\alpha$

Massive
particle-level
cross section



QED Parton Distribution Functions (PDFs)

Γ_{i/e^+} and Γ_{j/e^-}, *partons* $i, j = e^+, e^-, \gamma, \ldots$

all-order, universal, resumming $\log(Q^2/m_e^2)$

leading log (LL): $(\alpha L)^k$

next-to-leading log (NLL): $\alpha(\alpha L)^{k-1}$

...

Massless parton-level cross section $d\hat{\sigma}_{ij}$
computed order by order, process-specific

$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/e^+}(z_+, \mu^2, m_e^2) \, \Gamma_{j/e^-}(z_-, \mu^2, m_e^2) \, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

Aim: soft resummation for:

slide stolen from S. Frixione

$$\left\{ e^+(p_1) + e^-(p_2) \longrightarrow X(p_X) + \sum_{i=0}^{n} \gamma(k_n) \right\}_{n=0}^{\infty}$$

Achieved with:

Yennie, Frautschi, Suura Ann.Phys.13(61)379

$$d\sigma(L,\ell) \quad = \quad e^{Y(p_1,p_2,p_X)} \sum_{n=0}^{\infty} \beta_n \left(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^{n} \right) d\mu_{X+n\gamma}$$

Jadach, Ward, Was hep-ph/0006359

- Y essentially universal (process dependence only through kinematics); resums ℓ
- The soft-finite β_n are process-specific, and are constructed by means of local subtractions involving matrix elements and eikonals (i.e. *not* BN)

$$\beta_n = \alpha^b \sum_{i=0}^{n} \alpha^i \sum_{j=0}^{i} c_{n,i,j} L^j$$

- For a given n, matrix elements have different multiplicities, hence the need for the kinematic mapping $\mathcal{R}$

Another approach: YFS

$$l = \log \frac{Q^2}{\langle E_\gamma \rangle^2}$$

Soft log

$$L = \log \frac{Q^2}{m^2}$$

Collinear log

ISR methods compared in:
Frixione, Laenen et al. 2203.12557

Evolution operator formalism

Collinear Logarithms resumed by mean of DGLAP equation:

$$\frac{\partial \Gamma(z,\mu^2)}{\partial \log \mu^2} = \frac{\alpha(\mu)}{2\pi}[\mathbb{P} \otimes \Gamma](z,\mu^2)$$

In Mellin space, $f_N = \int_0^1 \mathrm{d}z\, z^{N-1} f(z)$, it becomes <u>multiplicative</u>

$$\Gamma_N(\mu^2) = \mathbb{E}_N(\mu^2,\mu_0^2)\, \Gamma_N(\mu_0^2)$$

Evolution operator

Initial condition
(fully perturbative in QED!)

We end up with an **equation for the evolution operator**

$$\frac{\partial \mathbb{E}_N(\mu^2,\mu_0^2)}{\log \mu^2} = \frac{\alpha(\mu)}{2\pi}\left[\mathbb{P}_N^{[0]} + \frac{\alpha(\mu)}{2\pi}\mathbb{P}_N^{[1]}\right]\mathbb{E}_N(\mu^2,\mu_0^2) + \mathcal{O}(\alpha^2)$$

Here α fixed, but LL with α running also available

QED PDFs $\Gamma_\alpha(z,\mu^2)$ at LL

Well-known LL result for Γ_{e^-}, evolving $\Gamma(z,\mu_0^2) = \delta(1-z)$ at scale $\mu_0^2 \simeq m_e^2$:

$$\Gamma_{e^-}^{\mathrm{LL}}(z,\mu^2) = \frac{\exp\left[(3/4-\gamma_E)\eta\right]}{\Gamma(1+\eta)}\eta(1-z)^{-1+\eta} - \frac{1}{2}\eta(1+z) + \mathcal{O}(\alpha^2)\,, \qquad \eta = \frac{\alpha}{\pi}\log\frac{\mu^2}{m_e^2} \equiv \frac{\alpha}{\pi}L$$

All-order large-z bulk

Gribov, Lipatov 1972

Obtained by exploiting $z \to 1 \iff N \to \infty$ in N-space and then invert back to z-space

Fixed-order all-z terms (known up to high order)

Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini

Obtained by recursively solving the DGLAP equation or by fixed order calculations

In view of future colliders, LL accuracy is insufficient and systematics not well defined at LL (e.g. which α?)

NLL-accurate QED PDFs

(Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265)

- NLO initial conditions at scale $\mu_0^2 = m_e^2$ **evolved at NLL up to** μ^2 **with all fermion families** (lepton and quarks), in a variable flavour number scheme.

- PDFs in **three different renormalisation schemes**: $\overline{\mathrm{MS}}$ (where α runs), $\alpha(m_Z)$ and G_μ(where α is fixed); **two different factorisation schemes**: $\overline{\mathrm{MS}}$ and Δ (DIS-like, with NLO initial condition maximally simplified).

- Solution built out of a numerical evolution, with a **switch to analytical expressions for** $z \to 1$, where the electron PDF Γ_{e^-} features a power-like integrable singularity.

- **Photon-initiated partonic contributions** (through the photon PDF Γ_γ) naturally included in the collinear framework at NLL.

NLL-accurate QED PDFs

(Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265)

- PDFs in **three different renormalisation schemes**: $\overline{\mathrm{MS}}$ (where α runs), $\alpha(m_Z)$ and G_μ(where α is fixed); **two different factorisation schemes**: $\overline{\mathrm{MS}}$ and Δ (DIS-like, with NLO initial condition maximally simplified).

$$\Gamma_{e^-}^{[0],\overline{\mathrm{MS}}}(z,\mu_0^2) = \Gamma_{e^-}^{[0],\Delta}(z,\mu_0^2) = \delta(1-z)$$

$$\Gamma_{e^-}^{[1],\overline{\mathrm{MS}}}(z,\mu_0^2) = \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ , \quad \Gamma_{e^-}^{[1],\Delta}(z,\mu_0^2) = \log\frac{\mu_0^2}{m^2}\left[\frac{1+z^2}{1-z}\right]_+$$

Evolution operator and short-distance cross section modified, such that $\hat{\sigma}_N(\mu^2)\,E_N(\mu^2,\mu_0^2)\,\Gamma_N(\mu_0^2)$ independent on the fact. scheme (up to NLO)

Large-z analytical expressions for Γ_{e^-}

$$\Gamma_{e^-}^{\mathrm{NLL}}(z,\mu^2) = \frac{e^{-\gamma_E \xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)} \xi_1 (1-z)^{-1+\xi_1} h(z,\mu^2)$$

$$\xi_1 = 2t + \mathcal{O}(\alpha^2)$$

$$\hat{\xi}_1 = \frac{3}{2} t + \mathcal{O}(\alpha^2)$$

$$t = \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)}$$

$$h^{\overline{\mathrm{MS}}}(z,\mu^2) = 1 + \frac{\alpha(\mu_0)}{\pi} \left[\left(\log \frac{\mu_0^2}{m^2} - 1 \right) \left(A(\xi_1) + \frac{3}{4} \right) - 2B(\xi_1) + \frac{7}{4} + \left(\log \frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1) \right) \log(1-z) - \log^2(1-z) \right]$$

$$h^{\Delta}(z,\mu^2) = \frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi} \log \frac{\mu_0^2}{m^2} \left(A(\xi_1) + \log(1-z) + \frac{3}{4} \right)$$

$$A(\xi_1) = \frac{1}{\xi_1} + \mathcal{O}(\xi_1)$$

$$B(\xi_1) = -\frac{\pi^2}{6} + 2\zeta_3 \xi_1 + \mathcal{O}(\xi_1^2)$$

Logarithmic terms artefacts of the $\overline{\mathrm{MS}}$ fac. scheme, **absent in the Δ scheme.**

Here shown in the $\overline{\mathrm{MS}}$ ren. scheme and with a single-fermion family; evolution with multiple fermion families with their mass thresholds and different ren. schemes (e.g. $\alpha(m_Z)$, G_μ) amount to a redefinition of ξ_1 and $\hat{\xi}_1$.

NLL-accurate QED PDFs

(Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265)

- PDFs in **three different renormalisation schemes**: $\overline{\rm MS}$ (where α runs), $\alpha(m_Z)$ and G_μ(where α is fixed); **two different factorisation schemes**: $\overline{\rm MS}$ and Δ (DIS-like, with NLO initial condition maximally simplified).

$$\alpha_R = \alpha_{\overline{\rm MS}}(m_Z) - \Delta_{\overline{\rm MS}\to R}\,\alpha^2_{\overline{\rm MS}}(m_Z) + \mathcal{O}(\alpha^3)$$

Modified evolution to reabsorb the running of alpha, leading to $\mathcal{O}(\alpha^3)$ w.r.t. $\overline{\rm MS}$ results → naively neglecting the running of α leads to $\mathcal{O}(\alpha^2)$ differences w.r.t. $\overline{\rm MS}$

$$\mathbb{P}_R^{[0,k]} = \mathbb{P}_{\overline{\rm MS}}^{[0,k]}$$

$$\mathbb{P}_R^{[1,k]} = \mathbb{P}_{\overline{\rm MS}}^{[1,k]} + \left(2\pi b_0^{(k)} \log\frac{\mu^2}{m_{k+1}^2} + D^{(k)}\right)\mathbb{P}_{\overline{\rm MS}}^{[0,k]}$$

$$D^{(k)} = 2\pi \sum_{i=k+1}^{M} b_0^{(i)} \log\frac{m_i^2}{m_{i+1}^2} + 2\pi\Delta_{\overline{\rm MS}\to R}$$

NLL-accurate QED PDFs

(Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265)

- NLO initial conditions at scale $\mu_0^2 = m_e^2$ **evolved at NLL up to** μ^2 **with all fermion families** (lepton and quarks), in a variable flavour number scheme
- Solution built out of a numerical evolution, with a **switch to analytical expressions for** $z \to 1$, where the electron PDF Γ_{e^-} features a power-like integrable singularity.

Public code eMELA: https://github.com/gstagnit/eMELA

Numerical evolution in Mellin space with a discretised path-ordered product.

Runtime evaluation too slow → grids in LHAPDF format

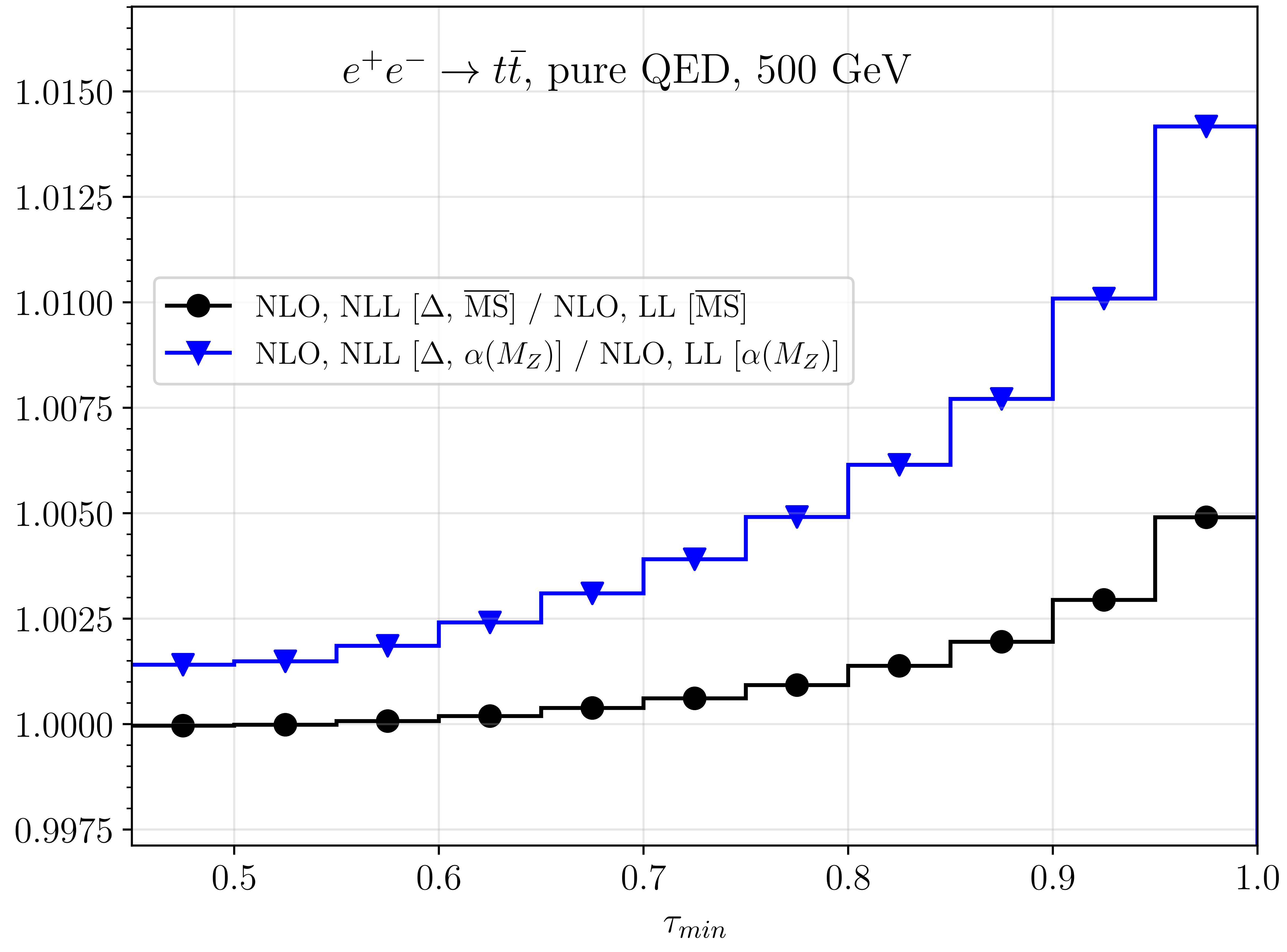
Even with grids, eMELA always switches to the analytical solution for $z \to 1$

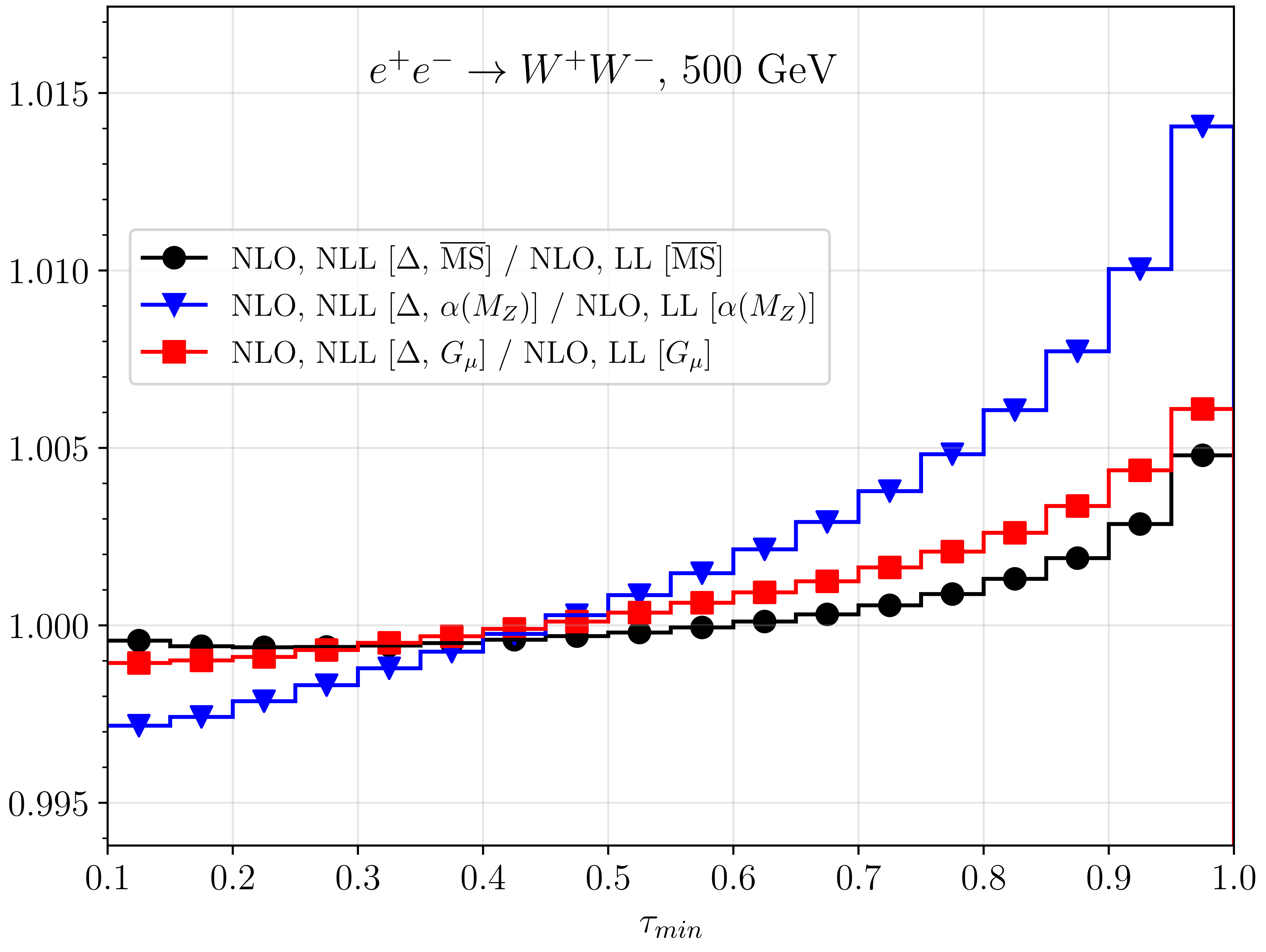
Studies on physical cross sections

- Computed in the MG5_aMC framework, at NLO (EW) + NLL in e^+e^- collisions. MG5_aMC with NLO EW in e^+e^- will be released soon.
- Processes:
  - $e^+e^- \to q\bar{q}(\gamma)$ [pure QED, with real and virtual radiation limited to initial state]
  - $e^+e^- \to W^+W^-(X)$ [full EW]
  - $e^+e^- \to t\bar{t}(X)$ [full EW] and $e^+e^- \to t\bar{t}(X)$ [pure QED]
- $\mu = \sqrt{s} = 500$ GeV (qualitatively similar results in the range 50-500 GeV)
- We focus on the cumulative cross section:

$$\sigma(\tau_{min}) = \int d\sigma\, \Theta(\tau_{min} \leq M^2_{p\bar{p}}/s)\,, \quad p = q, t, W^+$$

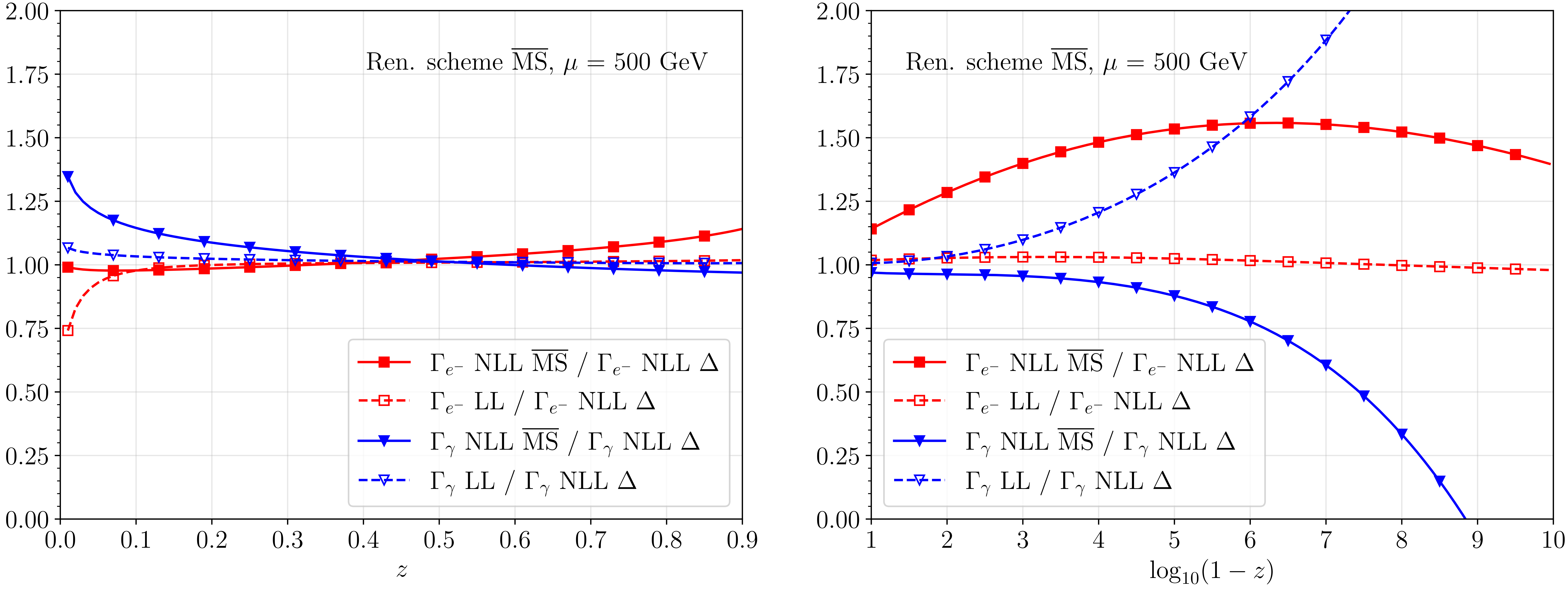
Impact of NLL





Non trivial pattern, impossible to account in some universal manner.
NLL-accurate PDFs are phenomenologically important for precision studies.

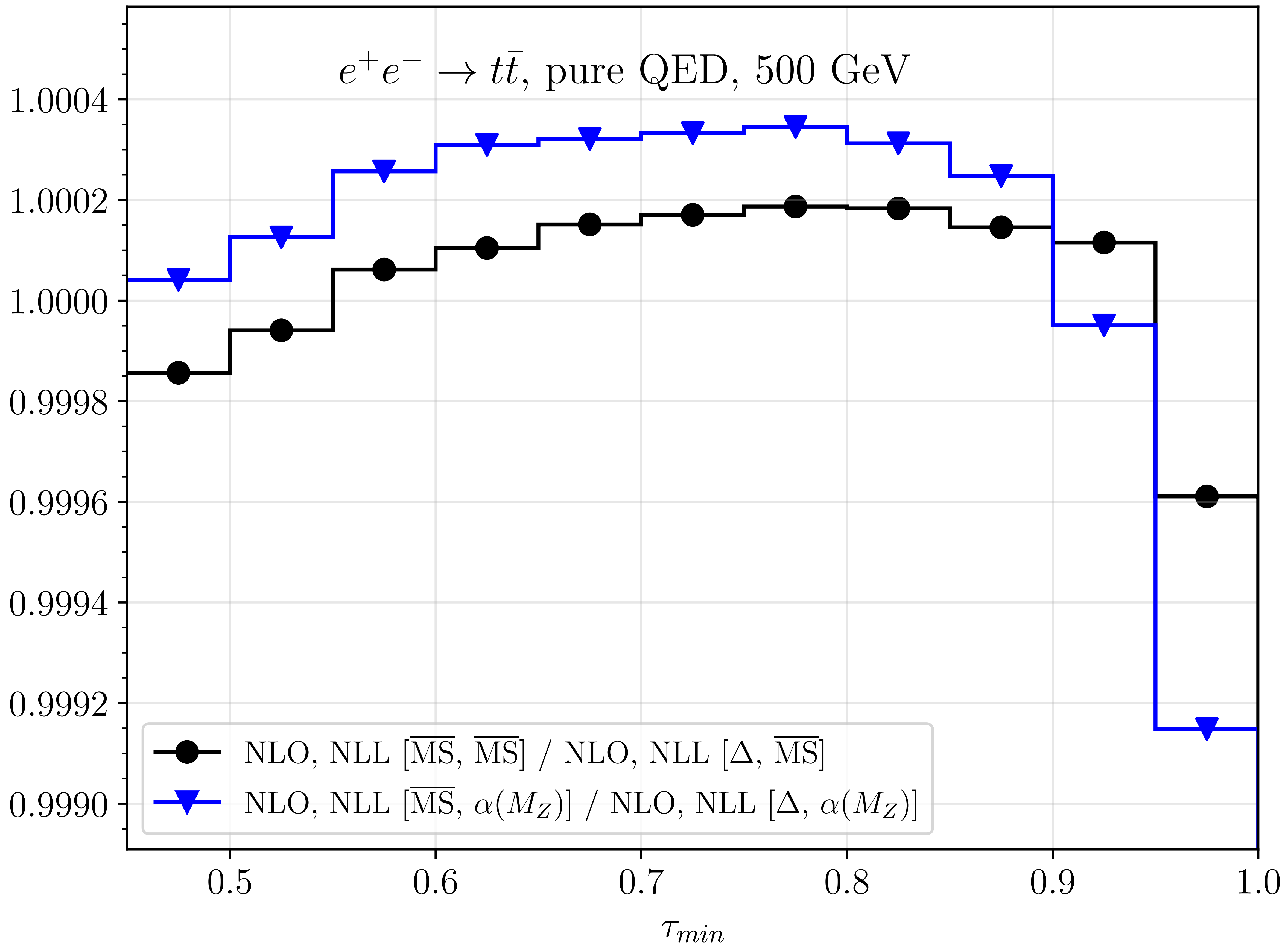
Dependence on factorisation scheme

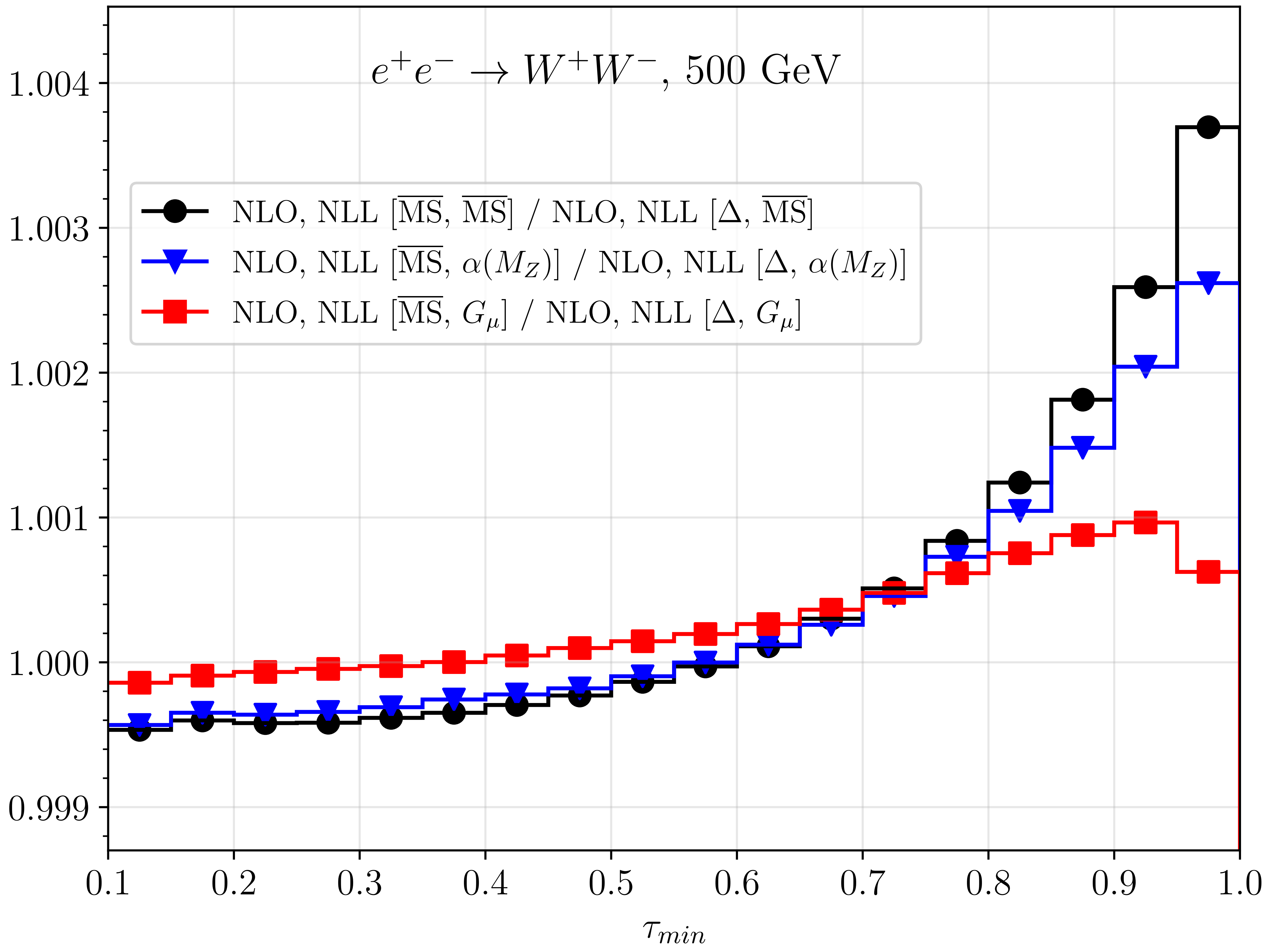


At the PDF level, $\mathcal{O}(1)$ difference between $\overline{\text{MS}}$ and Δ scheme.

Electron at NLL in the Δ scheme closer to the LL value.

Dependence on factorisation scheme

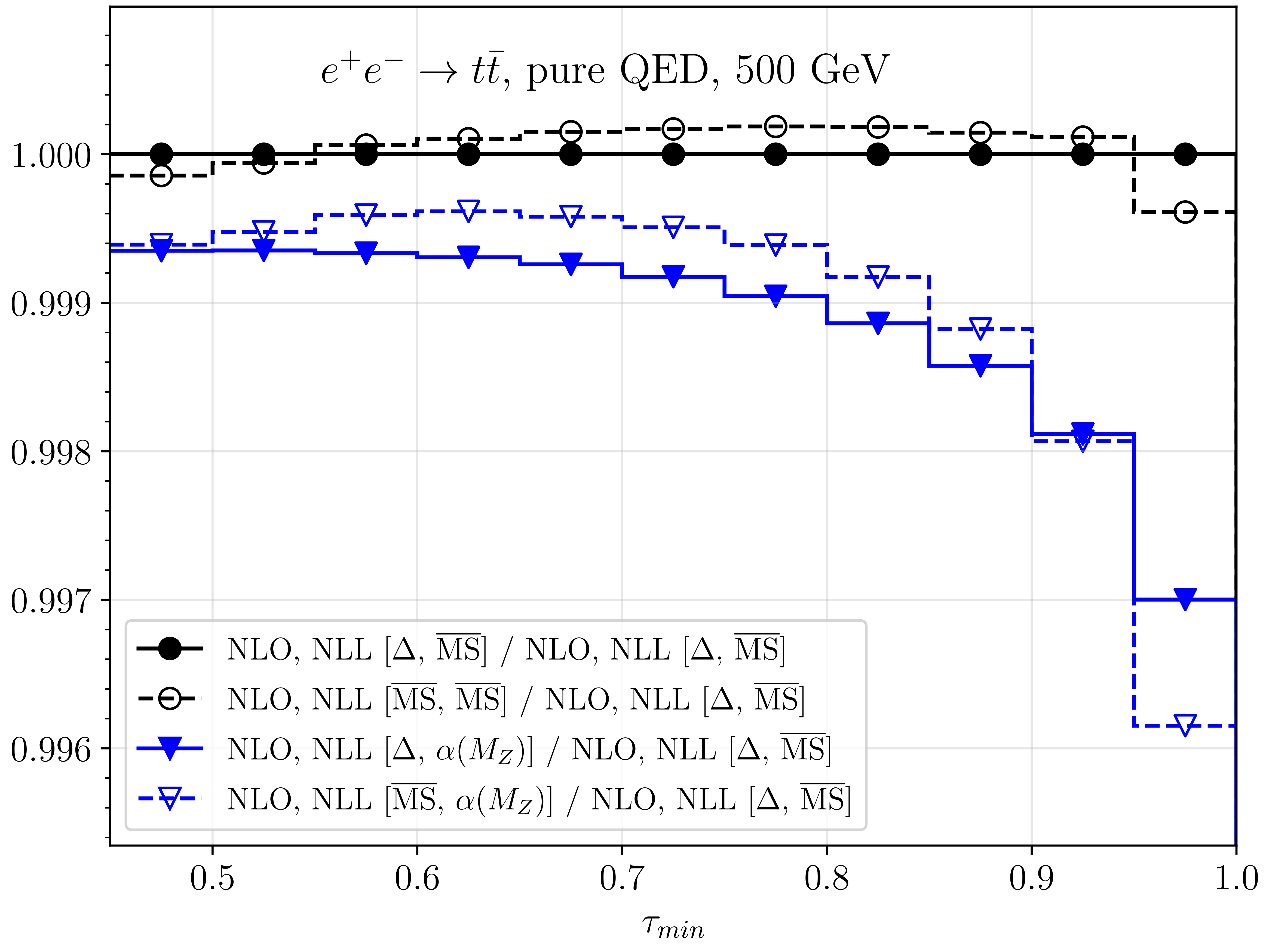


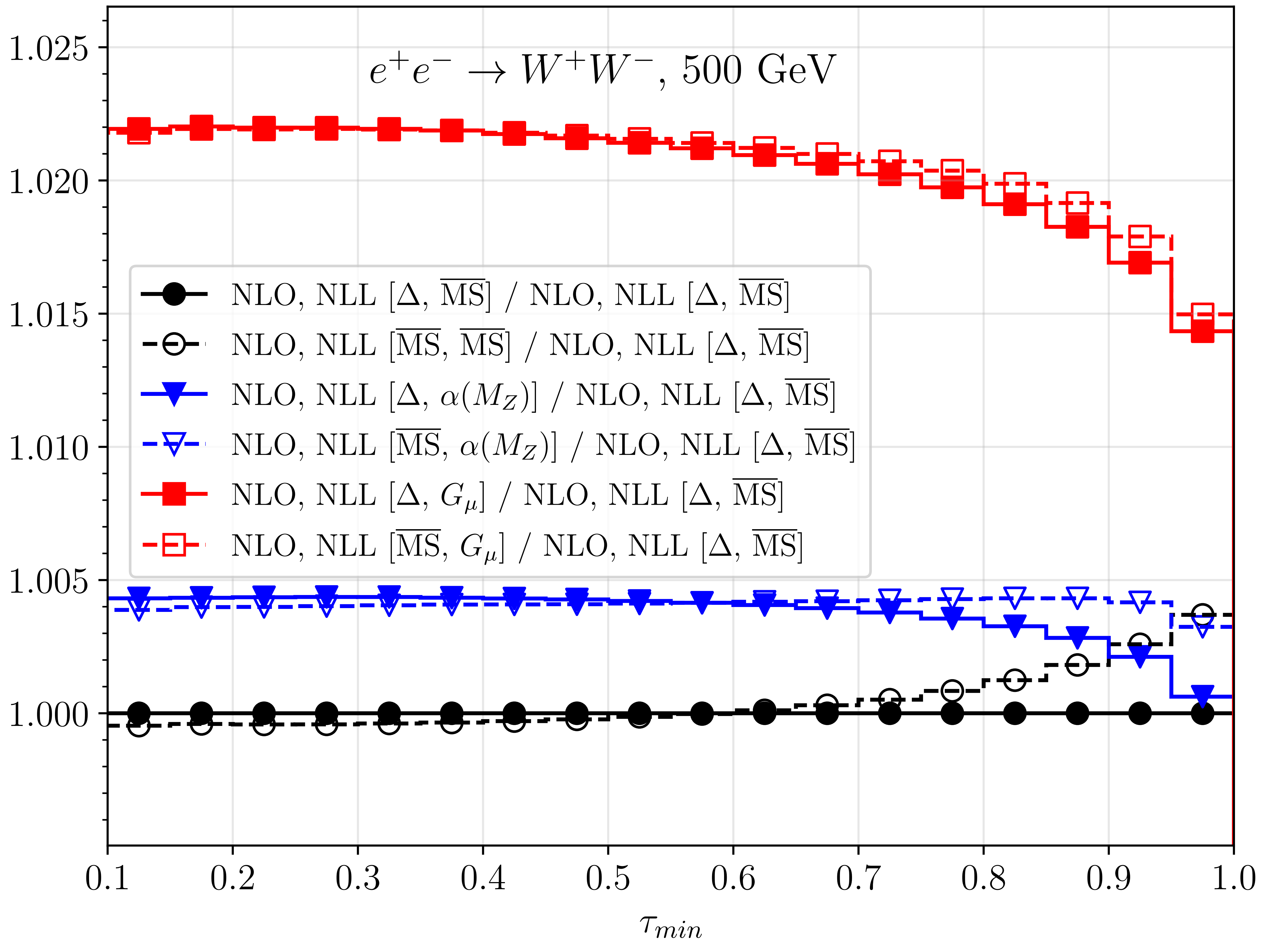


At the cross section level, $\mathcal{O}(10^{-4} - 10^{-3})$ difference between fact. schemes.

Large cancellations in the $\overline{\text{MS}}$ fact. scheme.

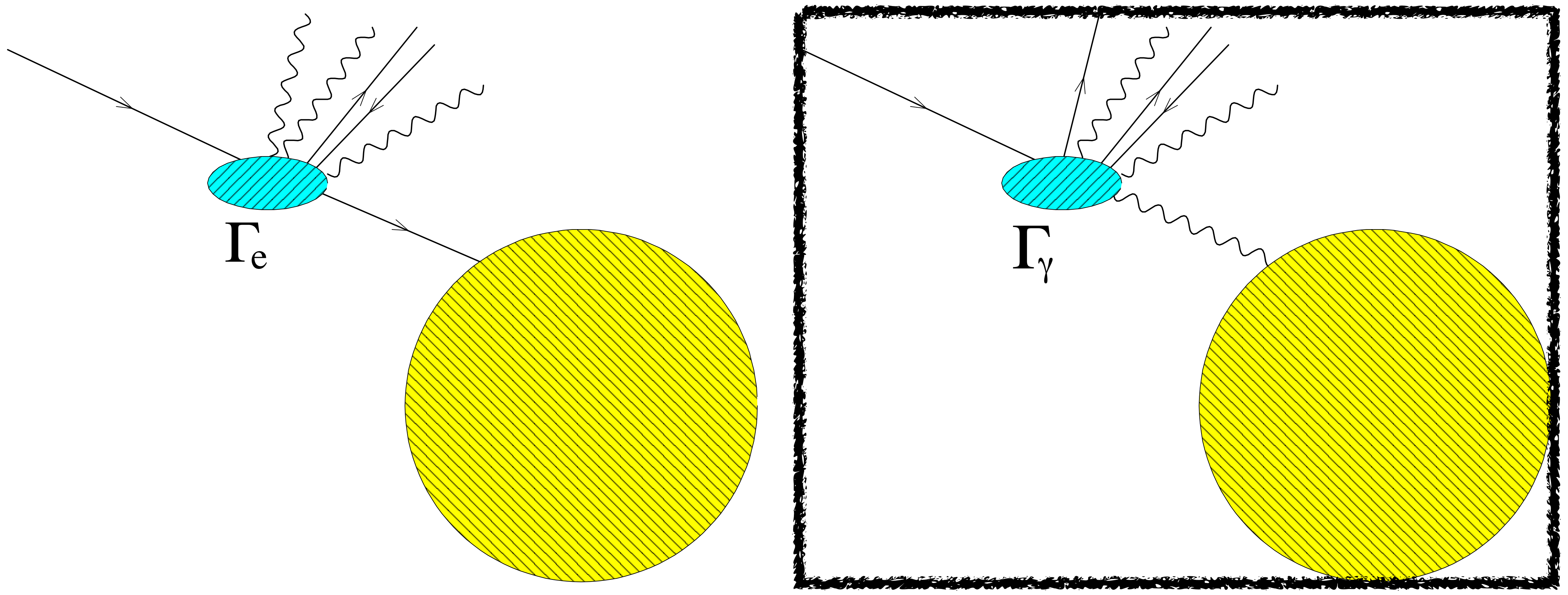
Dependence on renormalisation scheme





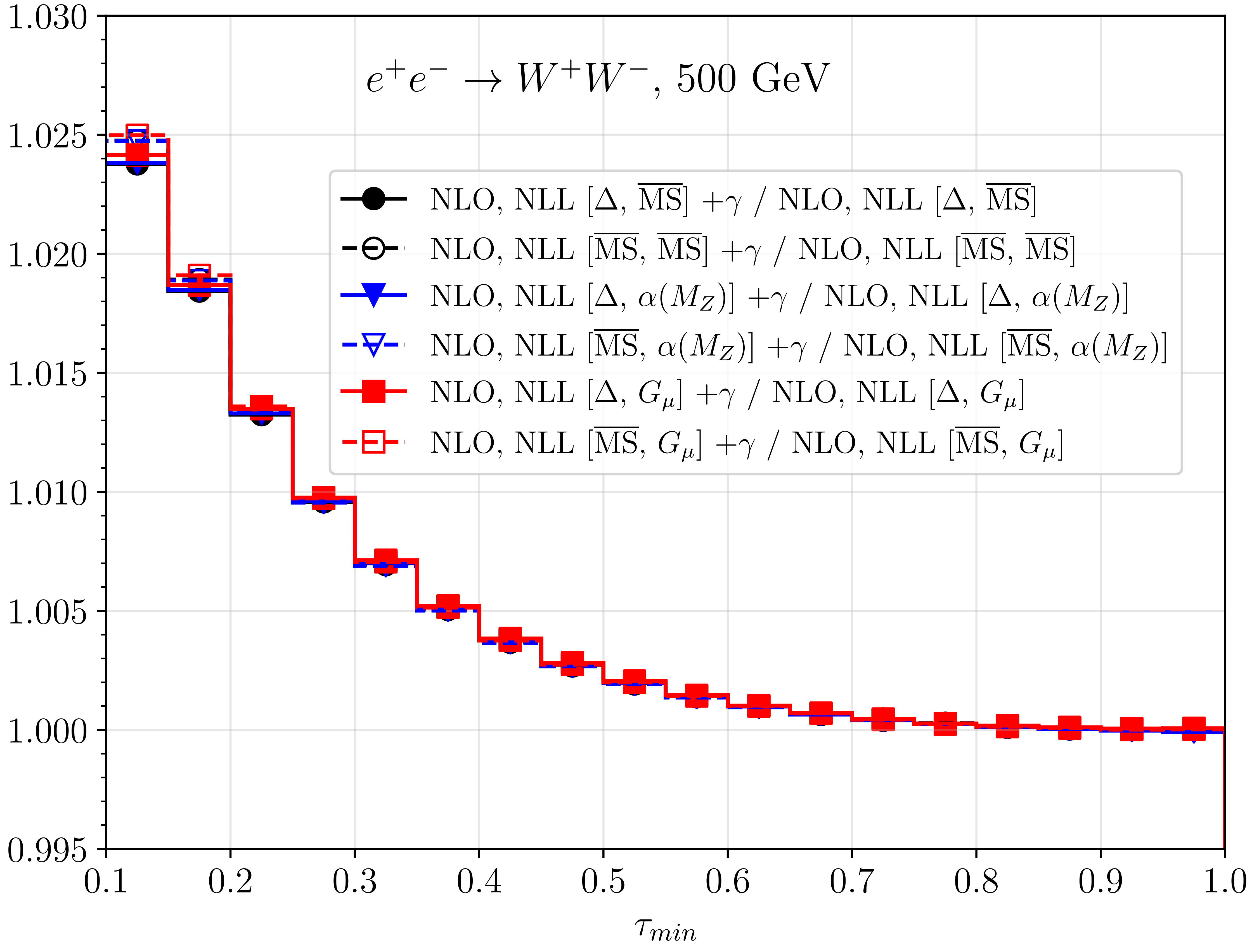
Ren. scheme dependence significantly **larger** than the fact. scheme one.
Mostly a normalisation effect.

Impact of photon-induced contributions



- At LO, i.e. $\mathcal{O}(\alpha^2)$, both W^+W^- and $t\bar{t}$ feature a $\gamma\gamma$ channel.
- Photon PDF Γ_γ only suppressed by a power of α w.r.t. Γ_{e^-}, and peaked at small-z values.

Both effects can lead to **physical effects** e.g. W^+W^- at small τ_{min}.



Beamstrahlung effects

[Frixione, Mattelaer, Zaro, Zhao 2108.10261]

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

$$\mathcal{B}_{kl}(y_+, y_-) \approx \sum_{n=1}^{N} b_{n,kl}^{(e^+)}(y_+) \, b_{n,kl}^{(e^-)}(y_-)$$

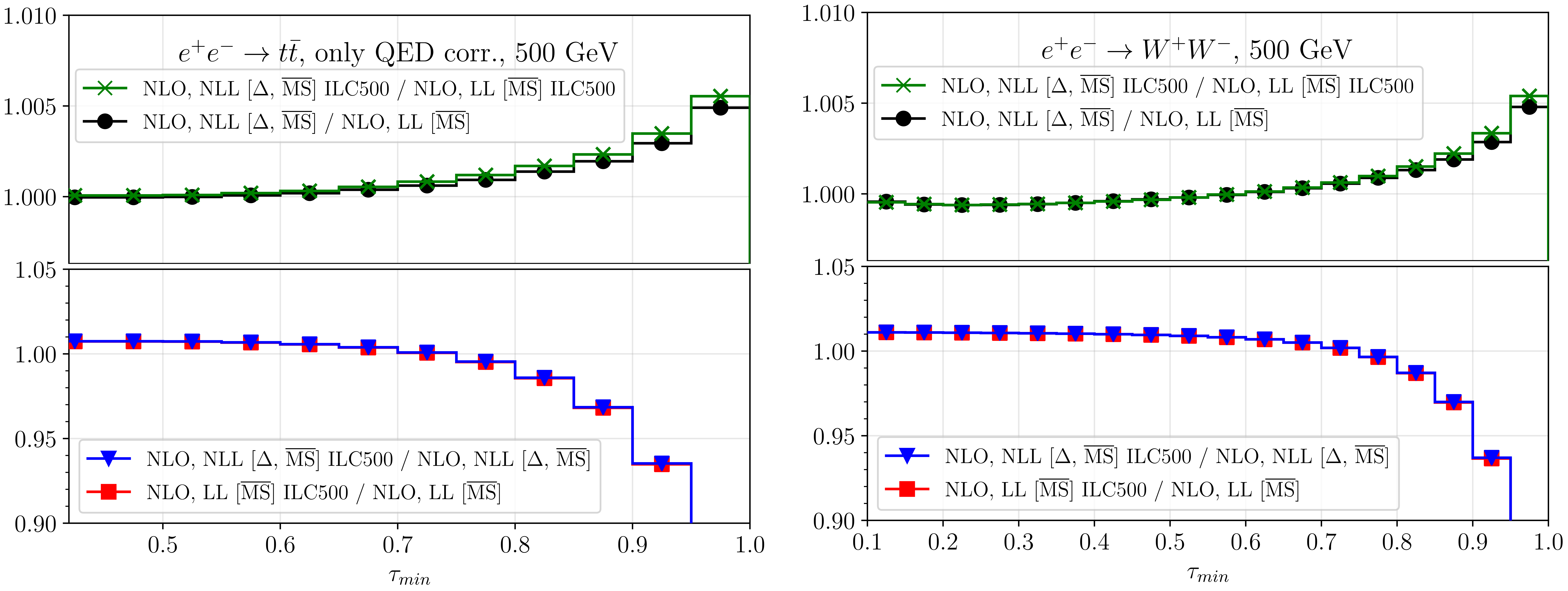
Parameters in b determined through fit to GuineaPig simulations

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{n=1}^{N} \sum_{ijkl} \int dx_+ dx_- \, \phi_{i/k,n,kl}^{(e^+)}(x_+, \mu^2, m^2) \, \phi_{j/l,n,kl}^{(e^-)}(x_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(x_+ P_{e^+}, x_- P_{e^-}, \mu^2, m^2) \,,$$

We can store in the grids also beamstrahlung!

$$\phi_{i/k,n,kl}^{(e^\pm)}(x, \mu^2, m^2) = \int dy \, dz \, \delta(x - yz) \, b_{n,kl}^{(e^\pm)}(y) \, \Gamma_{i/k}(z, \mu^2, m^2)$$

Example of beamstrahlung (ILC500)



Beamstrahlung effects have a clearly visible impact,
however affecting in the same way predictions at NLO+LL and at NLO+NLL

Conclusions

- **First NLO+NLL predictions** at high-energy e^+e^- colliders, improving on accuracy but also important for an assessment of sources of theoretical uncertainties.
- Several options for fact. (Δ, $\overline{\mathrm{MS}}$) and ren. ($\overline{\mathrm{MS}}$, $\alpha(m_Z)$, G_μ) schemes.
  Fact. scheme: systematic uncertainty; ren. scheme: informed choice.
- **Impact of NLL PDFs local** both in shape and size, hence impossible to account in a universal manner.
- **Photon-induced contributions not negligible**, properly included at NLL
- **Public code for NLL PDFs**, eMELA: https://github.com/gstagnit/eMELA.
  PDFs with beamstrahlung effects are also provided.
  MG5_aMC with NLO EW in e^+e^- will be released soon.