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# Precision measurement of the $W$ boson mass: status and prospects

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Lorenzo Bianchini  
*University of Pisa*



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European Research Council  
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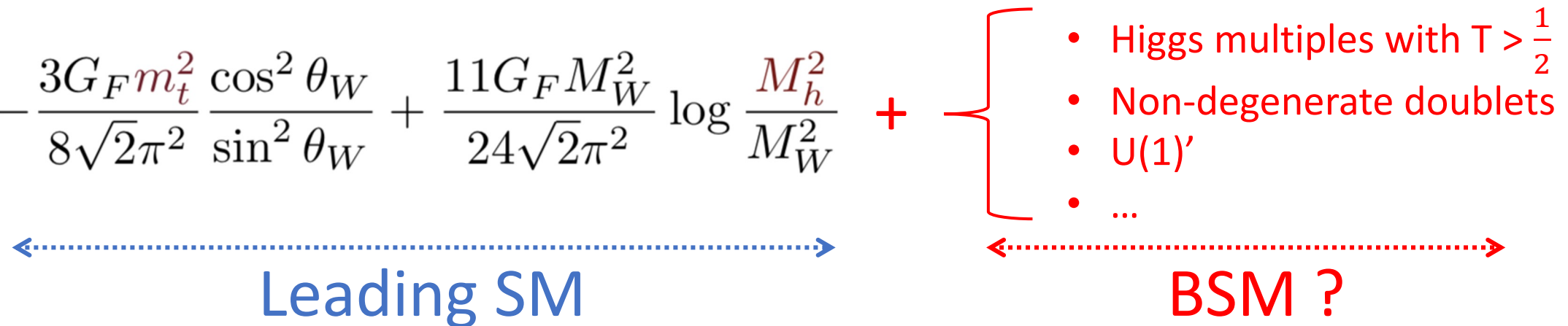
# What we can learn from $M_W$

- In the SM at tree-level,  $M_W$  is a function of 3 parameters  $\rightarrow G_F, M_Z, \alpha_{EM}$

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha_{EM}(M_Z)}{\sqrt{2} G_F (1 - \Delta r)},$$

- **Quantum corrections** provide a shift  $\Delta r \approx 3.6\% \rightarrow +500 \text{ MeV}$
- This relation entails *custodial symmetry* and the breaking of it:

$$\Delta r = -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \frac{\cos^2 \theta_W}{\sin^2 \theta_W} + \frac{11G_F M_W^2}{24\sqrt{2}\pi^2} \log \frac{M_h^2}{M_W^2} + \left\{ \begin{array}{l} \bullet \text{ Higgs multiples with } T > \frac{1}{2} \\ \bullet \text{ Non-degenerate doublets} \\ \bullet \text{ U(1)'} \\ \bullet \dots \end{array} \right.$$



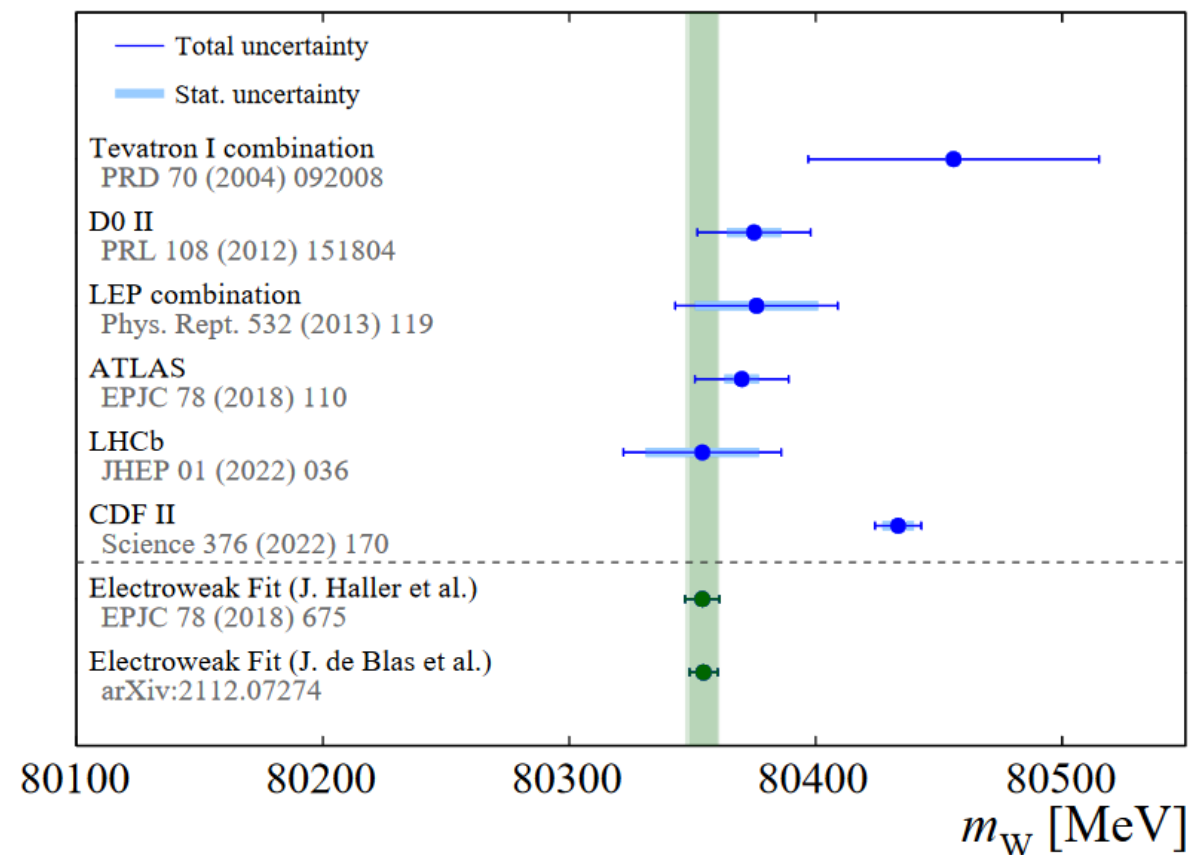
# State-of-the-art

- After Higgs boson discovery (2012)  
→  $\Delta M_W = \mathbf{6-7 MeV}$

- M.H.O. → 4 MeV
- $m_{\text{top}}$  → 4 MeV
- $M_Z$  → 2.6 MeV
- $\alpha_s$  → 2.6 MeV
- $\Delta\alpha_{\text{had}}^{(5)}$  → 2.4 MeV

- Latest CDF-II result (2022) in neat tension ( $7.0\sigma$ ) with SM
- **BSM interpretations are possible** and do not contradict other EWPT
  - my focus will be on the experiments

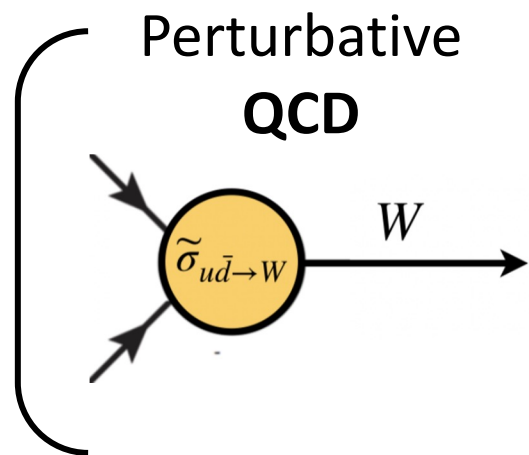
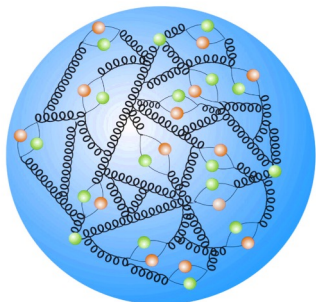
LHCb-FIGURE-2022-00



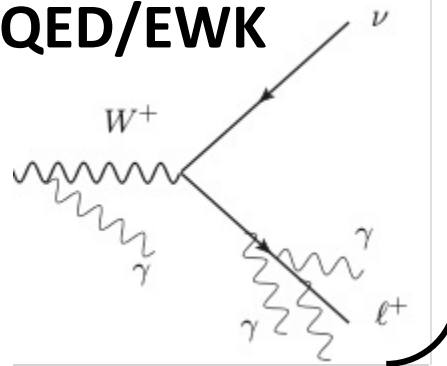
*Dozens of pre-prints in hep-ph  
See e.g. arXiv:2204.04191*

# $M_W$ at colliders

Proton **PDFs**



Perturbative **QED/EWK**

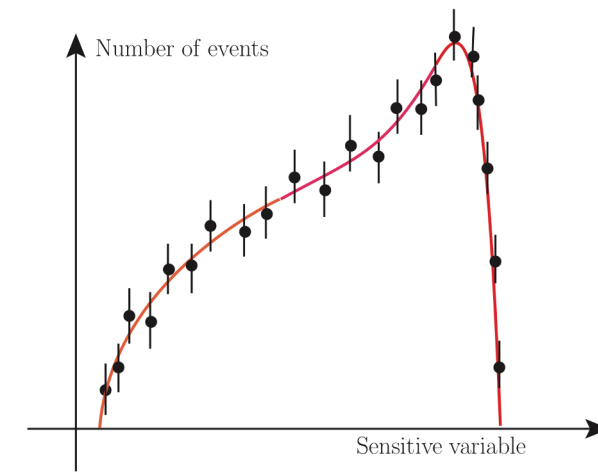


$$\frac{d^6 \sigma}{d^3 \mathbf{p}_l d^3 \mathbf{p}_\nu}$$



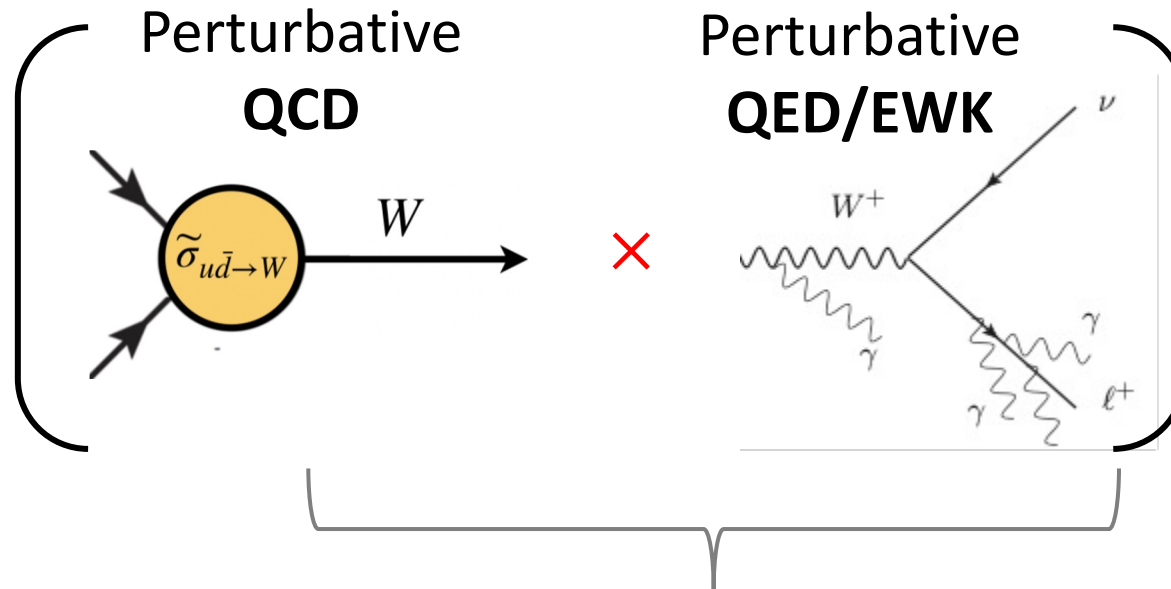
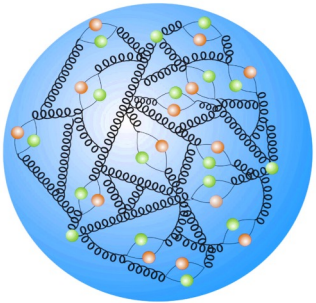
**DETECTOR**

*p.d.f.* of sensitive variable



# $M_W$ at colliders

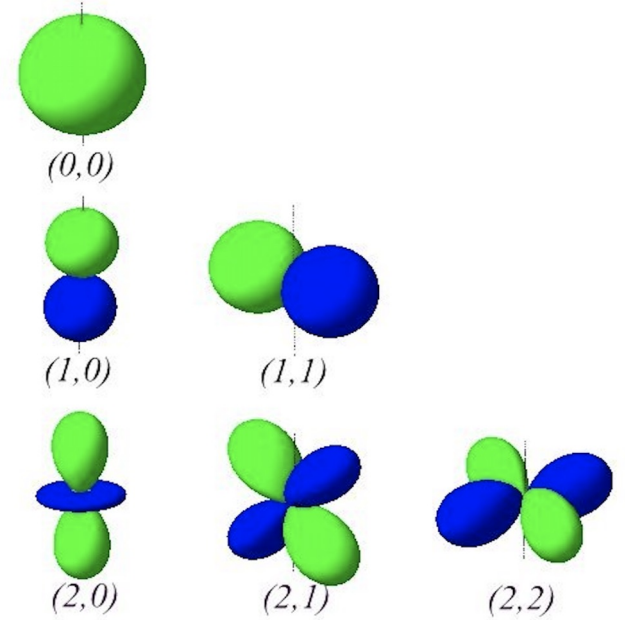
Proton **PDFs**



This is unlike other mass measurement which only depend on the **propagator** and **FSR radiation**

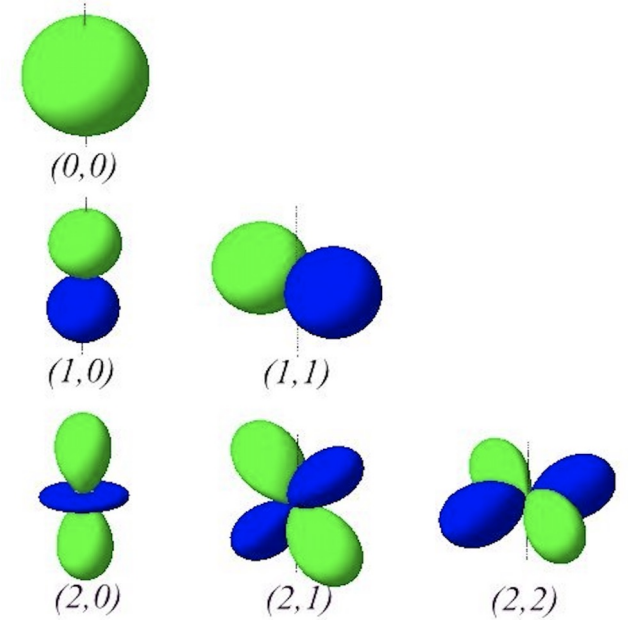
# Generalities of $W$ and $Z$ production

$$\frac{d\sigma}{dp_T^W dy dM d\cos\vartheta d\varphi} = \frac{3}{16\pi} \frac{d\sigma^{\text{unpol.}}}{dp_T^W dy dM} \left\{ (1 + \cos^2\vartheta) + A_0 \frac{1}{2} (1 - 3\cos^2\vartheta) + A_1 \sin 2\vartheta \cos\varphi \right. \\ \left. + A_2 \frac{1}{2} \sin^2\vartheta \cos 2\varphi + A_3 \sin\vartheta \cos\varphi + A_4 \cos\vartheta \right. \\ \left. + A_5 \sin^2\vartheta \sin 2\varphi + A_6 \sin 2\vartheta \sin\varphi + A_7 \sin\vartheta \sin\varphi \right\}$$



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- $d\sigma^{\text{unpol}}$  and  $A_i$  can be determined in pQCD (up to NP effects)
  - PDF-dependent
  - known at NNLO<sub>QCD</sub> + NLO<sub>EWK</sub>
  - Resummation-improved  $d\sigma^{\text{unpol}}$  and  $A_4$  available at N<sup>3</sup>LL+NNLO. N<sup>4</sup>LL just arrived

arXiv:2207.07056

# Main differences between colliders

- **TeVatron** is a  $p\bar{p}$  collider at  $\sqrt{s} = 1.96$  TeV, with  $\langle\mu\rangle = 2-3$ 
  - Perfect +/- symmetry
  - Preponderance of valence  $u/d$  quark – *well known from DIS*
  - Better recoil resolution
  - Simpler detector (i.e. less material)
- **LHC** is a  $pp$  collider at  $\sqrt{s} = 7-13$  TeV, with  $\langle\mu\rangle = 20-50$ 
  - asymmetric between +/-
  - Large contribution from gluon and  $c/s$  sea – *less well known*
  - Poor recoil resolution
  - Larger material budget & harsher data taking conditions

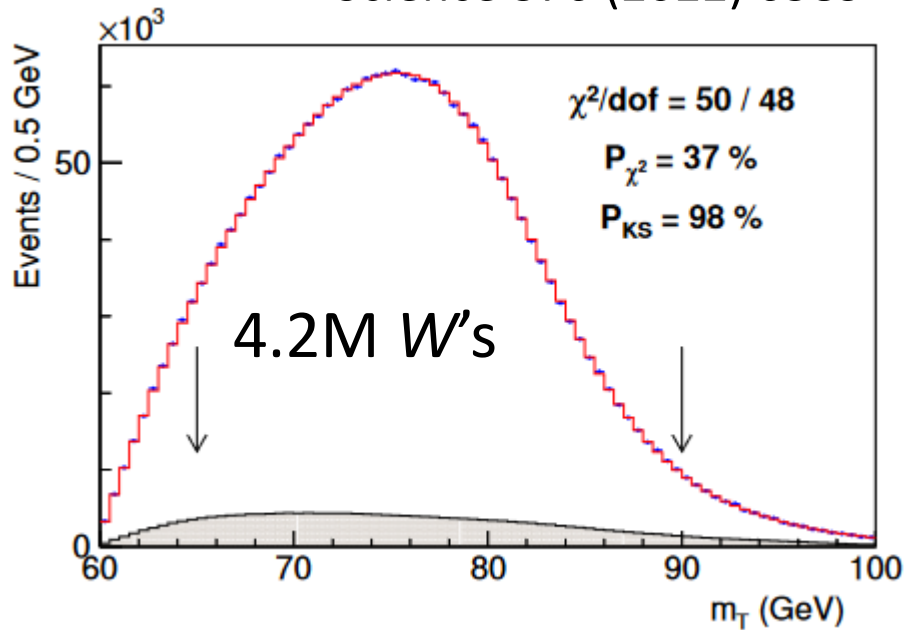


# Common experimental aspects

- All measurements rely on theoretical modeling of  $W$  production
  - Ultimately limited by **model-uncertainties: PDFs,  $p_T^W$  and  $A_i$ .**
- The **absolute energy scale** has to be determined from quarkonia and/or  $Z$ 
  - But:  $\langle p_T^l \rangle_{quarkonia,Z} \neq \langle p_T^l \rangle_W$  so some extrapolation is needed
- $M_W$  is extracted from a chi2 fit of data to **MC-based templates**
  - Need for large-scale MC simulations – most challenging at the LHC

# CDF-II

Science 376 (2022) 6589

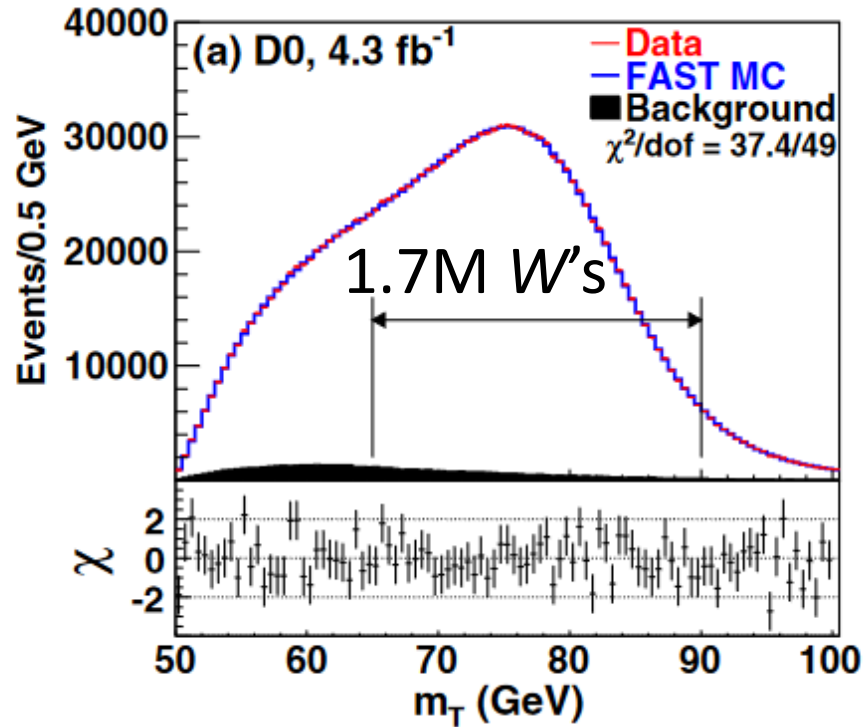


$$\Delta M_W = 9 \text{ MeV}$$

- Physics modeling: CTEQ6M+ResBosP(\* $p_T^Z$ )+Photos
- Detector modeling: custom MC simulation
- Calibration: data matched to  $J/\Psi$ ,  $\Upsilon(1s)$ ,  $Z$ .
- BLUE comb. of 6 channels:  $(p_T^l, m_T, p_T^{\nu}) \times (e, \mu)$
- Cross-checks:  $M_Z$ , data-taking, +/-, detector region

# D0

PRD 89 (2014) 012005

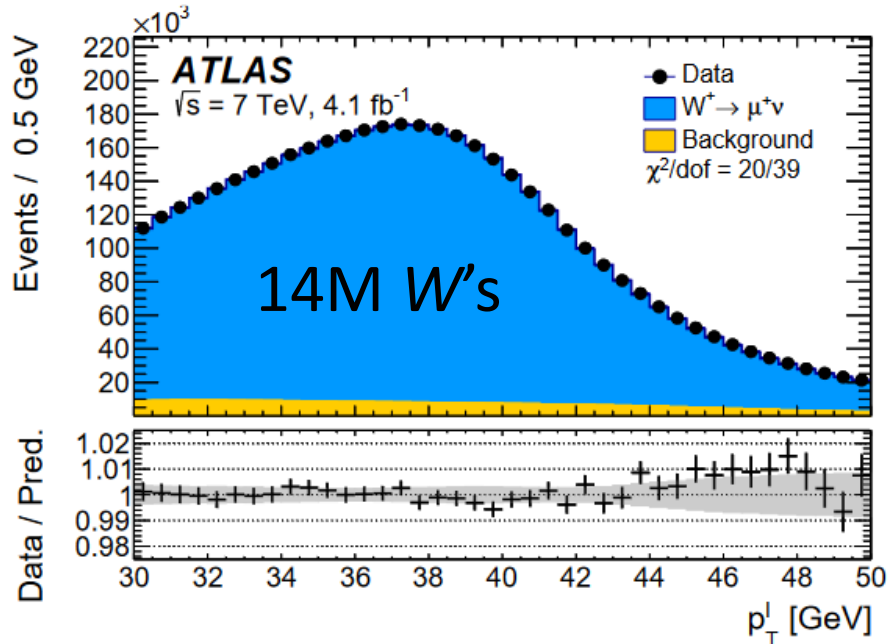


$$\Delta M_W = 23 \text{ MeV}$$

- Physics modeling: CTEQ6.6+ResBosCP(\* $p_T^Z$ )+Photos
- Detector modeling: custom MC simulation
- Calibration: data matched to Z
- BLUE comb. of 3 channels:  $(p_T^l, m_T, p_T^{\nu}) \times e$
- Cross-checks: data-taking, detector region

# ATLAS

EPJC 78 (2018) 110

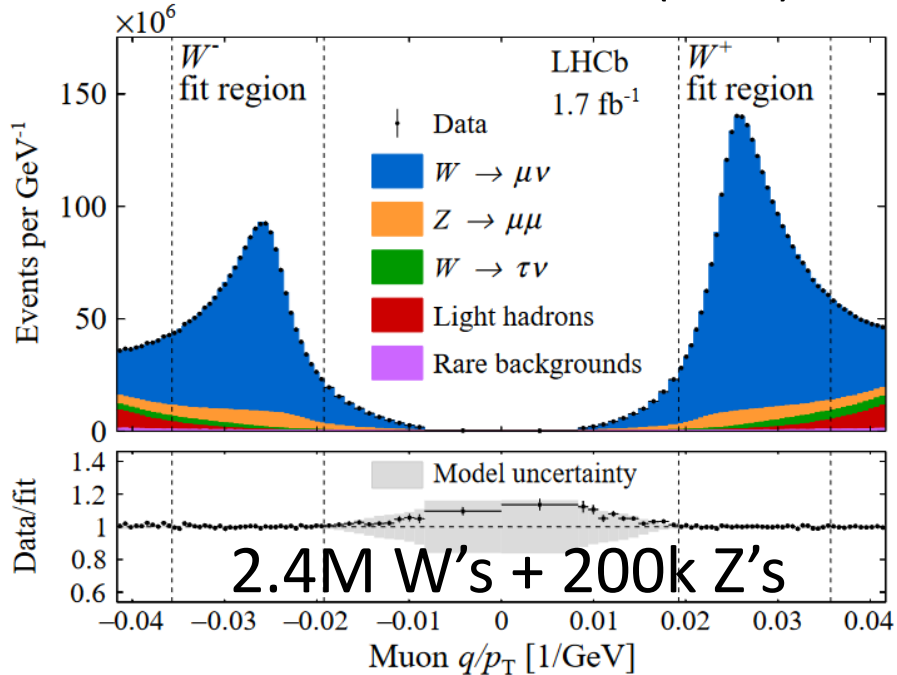


$$\Delta M_W = 19 \text{ MeV}$$

- Physics modeling:  
CT10+Powheg(\*DYNNLO)+Pythia(\* $p_T^Z$ )+Photos
- Detector modeling: full MC simulation
- Calibration: simulation matched to  $Z$  data
- BLUE comb. of 28 channels:  $(p_T^l, m_T, p_T^{\nu}) \times (e, \mu) \times \eta^l$  bin
- Cross-checks: detector region, +/-

# LHCb

JHEP 01 (2022) 036



$$\Delta M_W = 32 \text{ MeV}$$

- Physics modeling:  
NNPDF31+Powheg(\*DYTurbo)+Pythia(\* $\phi_{ll}^*$ )+Photos
- Detector modeling: full MC simulation
- Calibration: simulation matched to  $J/\Psi$ ,  $\Upsilon(1s)$ ,  $Z$
- Measurement: simultaneous fit to  $q/p_T^l$  and  $\phi_{ll}^*$
- Cross-checks: polarity, detector region,  $W$ -like  $M_Z$

# Towards a combined result

- Need to refine/corroborate PDG average: **80379 ± 12 MeV**
  - Use of different models complicate things
  
- Joint LHC-TeVatron effort to define **updates to published results** in sight of a proper combination
  - focus on QCD modeling and PDFs
  
- How to combine largely inconsistent measurements will be another story...

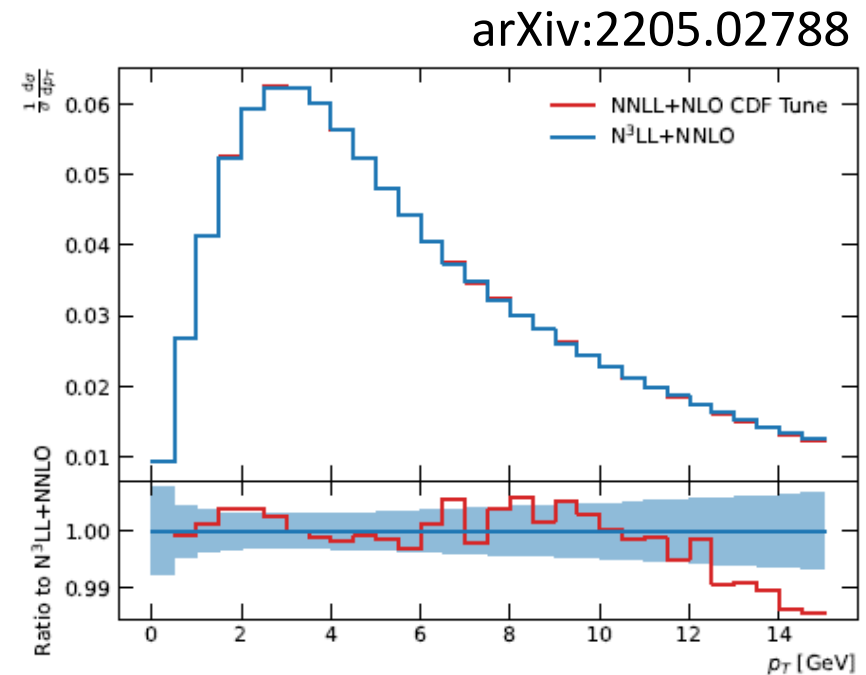
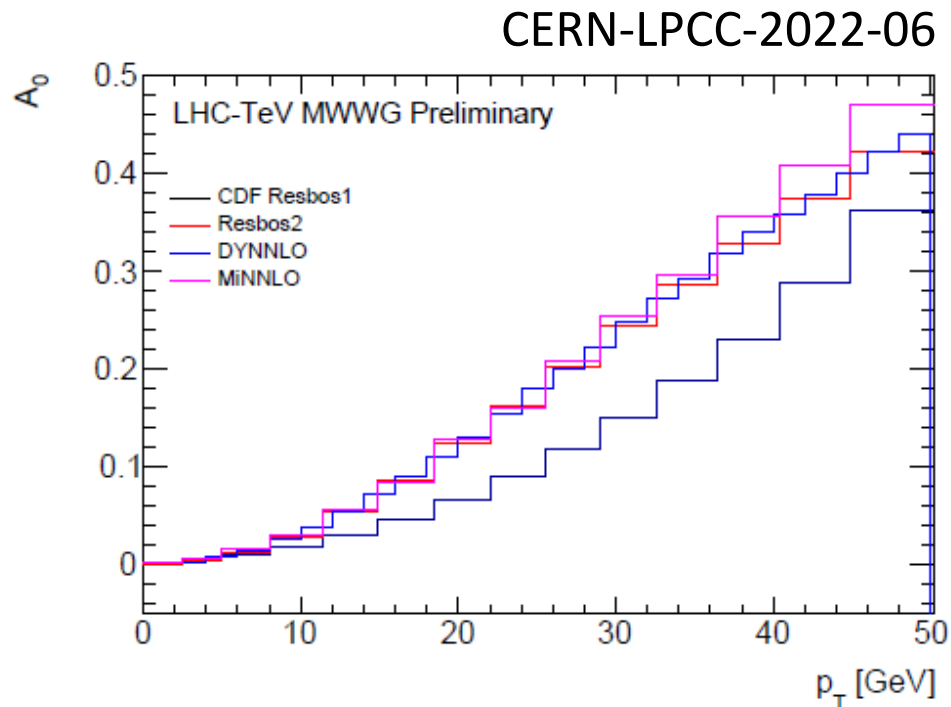
$$m_W^{\text{updated}} = m_W^{\text{ref}} - \delta m_W^{\text{QCD}} - \delta m_W^{\text{PDF}}$$

Correction	$\delta m_W^{\text{QCD}}$ [MeV]					
	$p_T^W$ -constrained			No constraint		
	$p_T^\ell$	$m_T$	$p_T^\gamma$	$p_T^\ell$	$m_T$	$p_T^\gamma$
Invariant mass	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1
Rapidity	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1
$A_0$	7.6	10.0	15.8	16.0	12.6	19.5
$A_1$	-2.4	-1.9	-1.8	-1.2	-1.6	-1.4
$A_2$	-3.0	-2.6	2.9	-4.2	-3.0	2.3
$A_3$	2.9	1.6	-0.5	3.5	1.8	-0.2
$A_4$	2.4	-0.1	-0.5	0.1	-0.7	-1.0
$A_0 - A_4$	7.6	7.0	16.0	14.1	9.1	18.9
Total	7.6	7.0	16.0	14.1	9.1	18.9
RESBos2	7.3±1.1	8.4±1.0	16.6±1.2	13.9±1.1	10.3±1.0	19.8±1.2
Non-closure	-0.3±1.1	1.4±1.0	0.6±1.2	-0.2±1.1	1.2±1.0	0.9±1.2

*Example of ResBos1 → ResBos2 shifts for the D0 measurement*

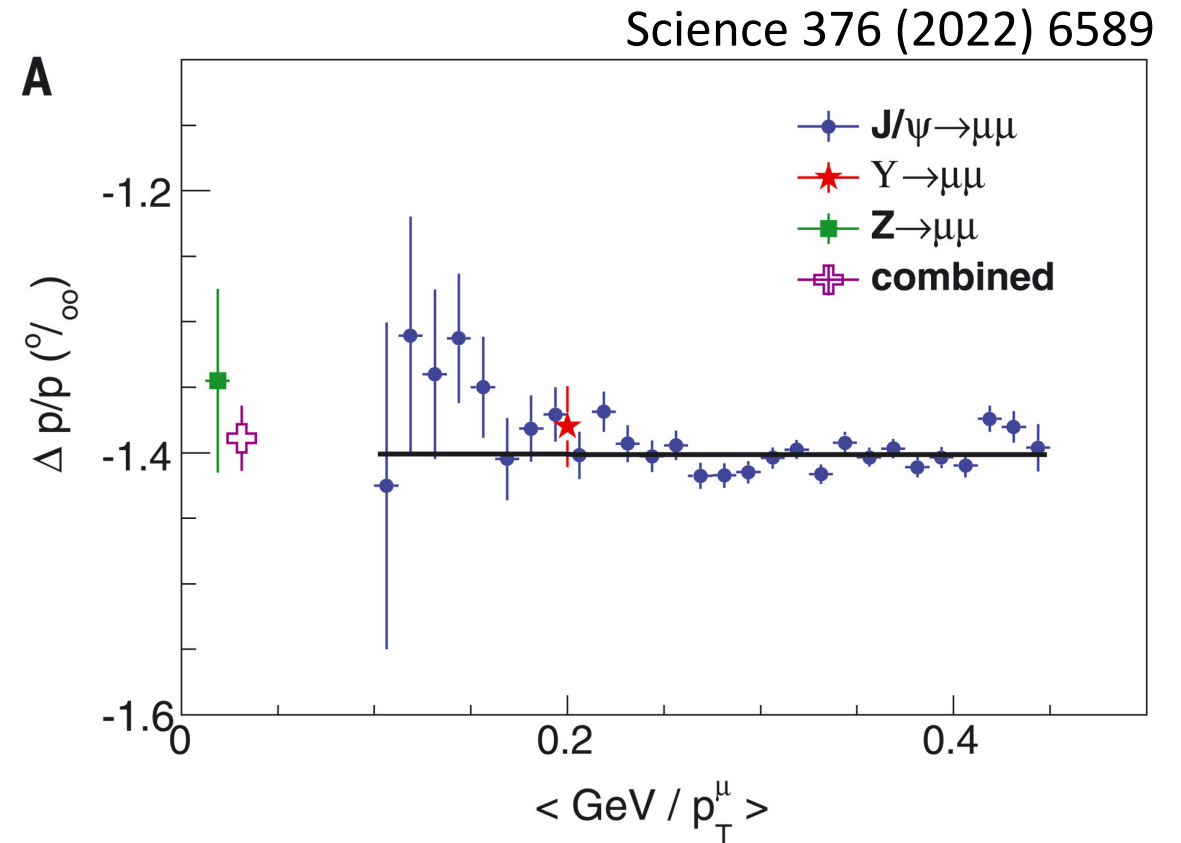
# Some known criticism to CDF-II

- Physics modeling based on **outdated ResBos** version
  - known bugged values of  $A_{0\dots3}$   $\rightarrow$  up to 14 MeV shifts in the D0 setup
  - Reduced logarithmic accuracy  $\rightarrow$  O(10) MeV shift when moving to N<sup>3</sup>LL



# Some known criticism to CDF-II

- Correction to **momentum scale** is  $\frac{\Delta p}{p} = -1393 \pm 26$  ppm, i.e. 3 MeV on  $M_W$ 
  - 0.1% correction to absolute scale is large and so far unexplained
  - is **linear extrapolation** from  $\langle p_T^l \rangle_{quarkonia,Z}$  to  $\langle p_T^l \rangle_W$  fully justified?





# CDF-II vs ATLAS

Source	Final CDF Run 2 (MeV)	ATLAS (MeV)
Lepton uncertainties	3.5	9.2
Recoil energy scale & resolution	2.2	2.9
Backgrounds	3.3	4.5
Model theoretical uncertainties	3.5	9.9
PDFs	3.9	9.2
Statistical	6.4	6.8
Total	9.4	18.5

- Same statistical uncertainties, but much larger **experimental** and **theoretical** uncertainties at LHC
  - Only partly due to environmental effects. Individual choices play a role...

# What if?

- LHCb had applied...

- NLO  $\mu_R$  variations on  $A_3$  vs. freely floating  $A_3$  norm.  $\rightarrow \Delta M_W = 30$  vs.  $10$  MeV
- same  $\alpha_s$  for W & Z vs two independent values of  $\alpha_s$   $\rightarrow \delta M_W = +39$  MeV

- ATLAS had applied...

- $\mu_F$  variations deuncorrelated vs correlated  $\rightarrow \Delta M_W = 30$  vs.  $5$  MeV
- resummed corrections to  $p_T^W$  vs tuned pythia  $\rightarrow$  **likely a disaster**

- CDF-II has used...

- ResBos2 vs ResBos1  $\rightarrow$  two independent sources of **O(10) MeV shifts** identified

# Missing systematics

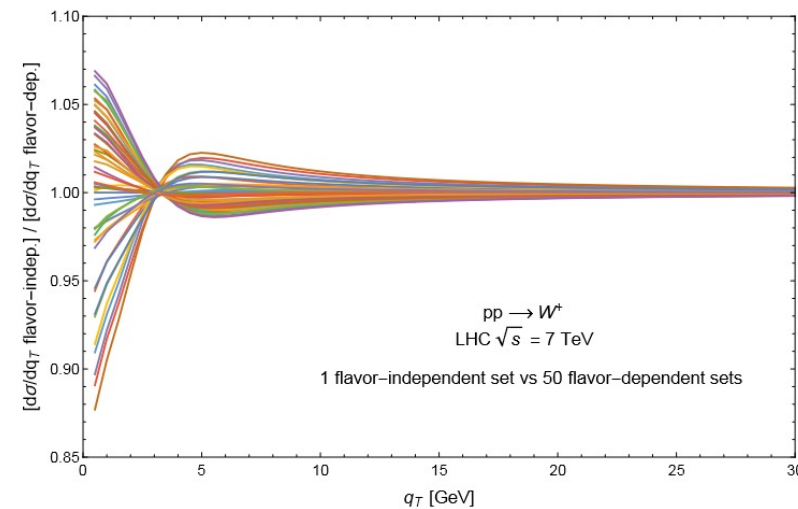
- Mixed QCD ⊗ EWK corrections do DY have been computed
  - Not yet included by experiments
  - Some (crude) estimates of their effect in the literature:

corrections cause bigger shifts in  $m_W$ . For example, we estimate that the cuts employed by the ATLAS collaboration in their recent extraction of the  $W$  mass [5] may lead to a shift of about  $\mathcal{O}(17)$  MeV due to unaccounted mixed QCD-electroweak effects in the production process.

PRD 103, 113002 (2021)

- Impact of non-perturbative corrections to  $p_T^{W/Z}$  yet to be understood
  - Assuming flavour non-universality of NP models can bring to additional  $\mathcal{O}(10)$  MeV shifts

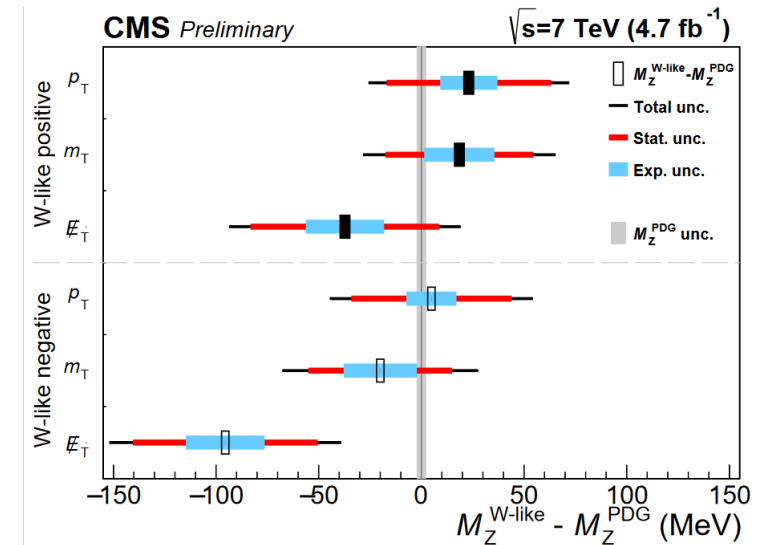
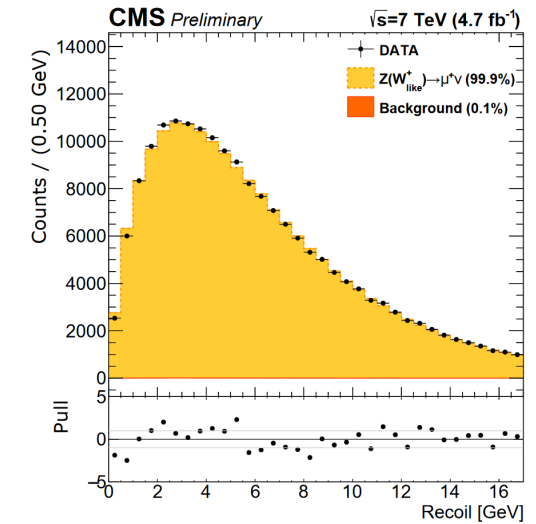
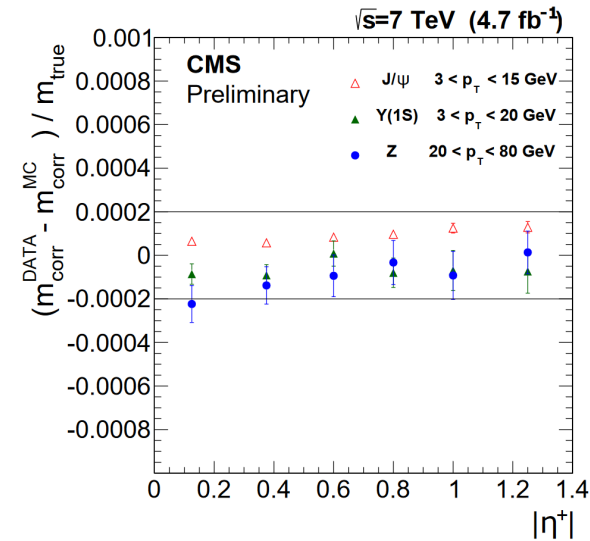
AHE. 2019 (2019) 2526897



	$\Delta m_{W^+}$		$\Delta m_{W^-}$	
Set	$m_T$	$p_T^\ell$	$m_T$	$p_T^\ell$
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3
6	1	0	-1	4
7	2	-1	-1	0
8	0	2	1	7
9	0	4	-1	0

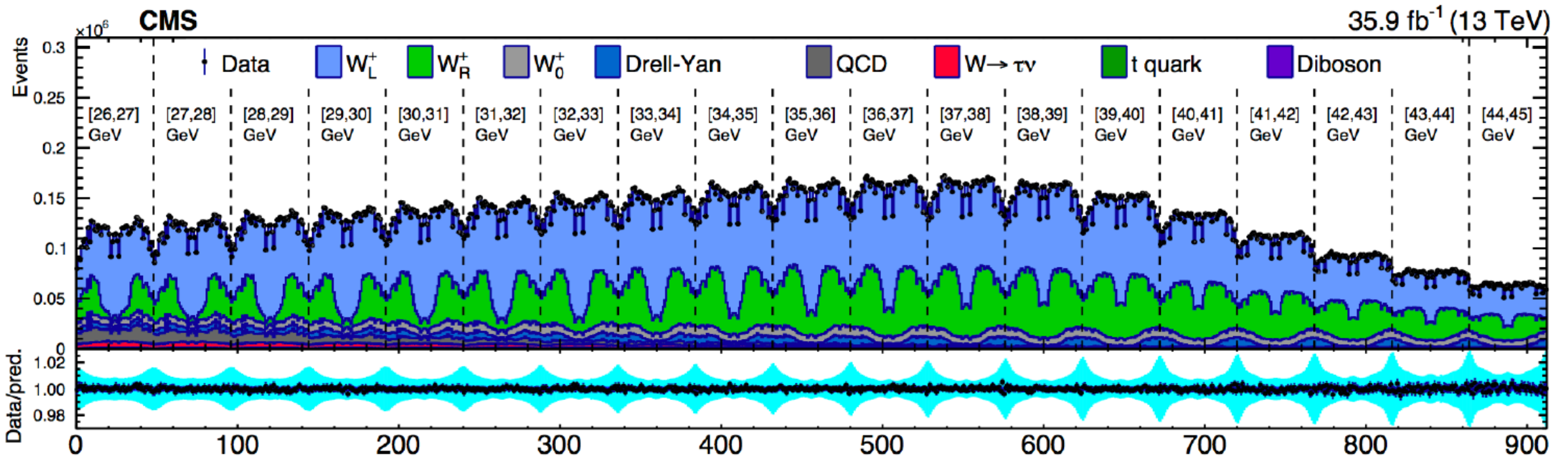
# And CMS?

- CMS still missing but has all the potentialities to do the measurement.
  - Proof-of-principle measurement of  $W$ -like  $M_Z$  at 7 TeV with **competitive muon and recoil calibration**
  
- Since then, the guideline has been to **reduce model-uncertainty by using**
  - **more (-differential) data → in situ constraint**
  - **state-of-the art calculations**



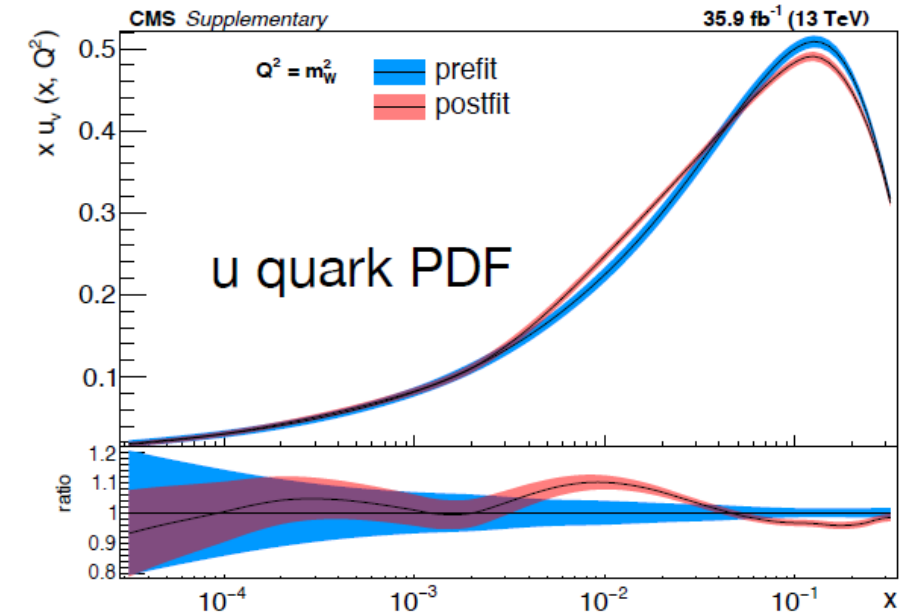
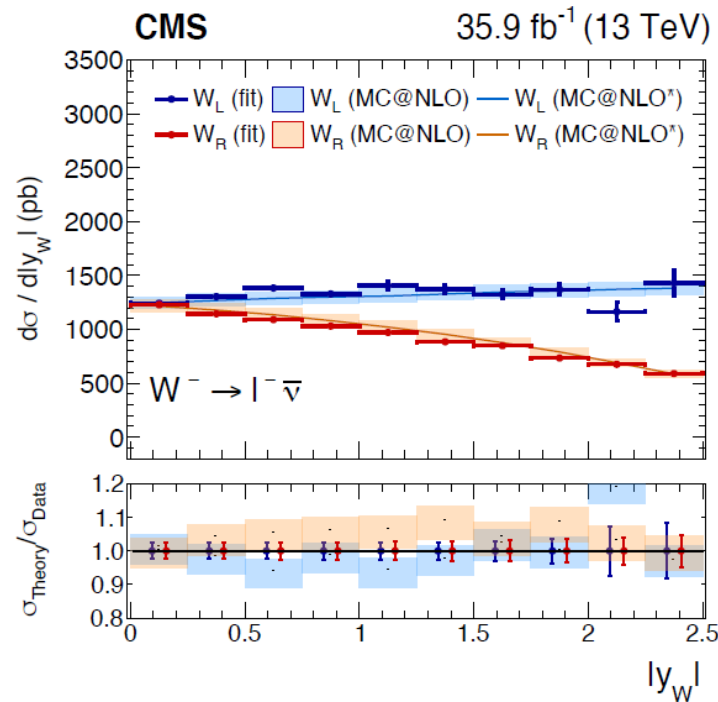
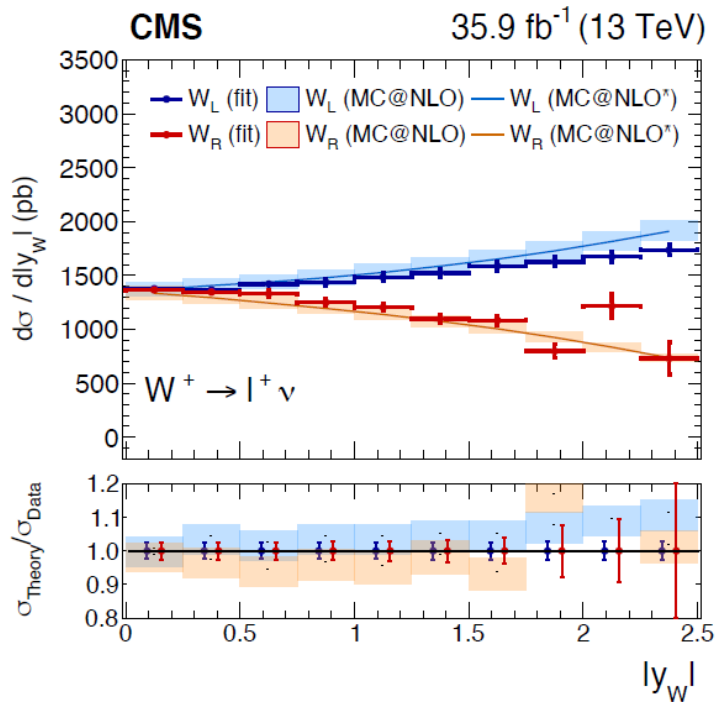
# Intermediate milestone

- O(100 M)  $W$  events within detector fiducial acceptance in just 2016 data-taking  
 → Differential measurement of  $W$  rapidity, helicity, and charge asymmetry



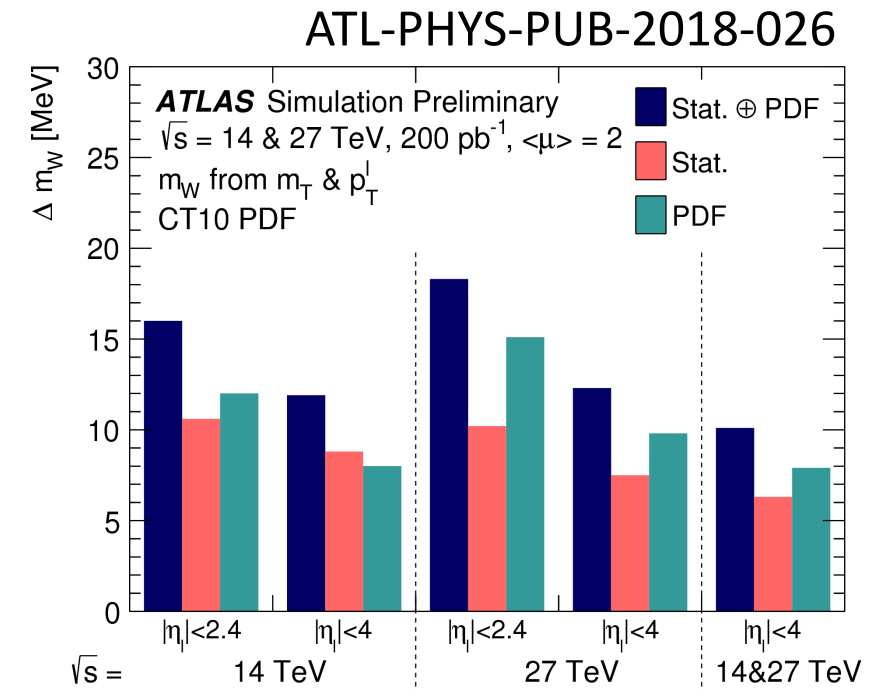
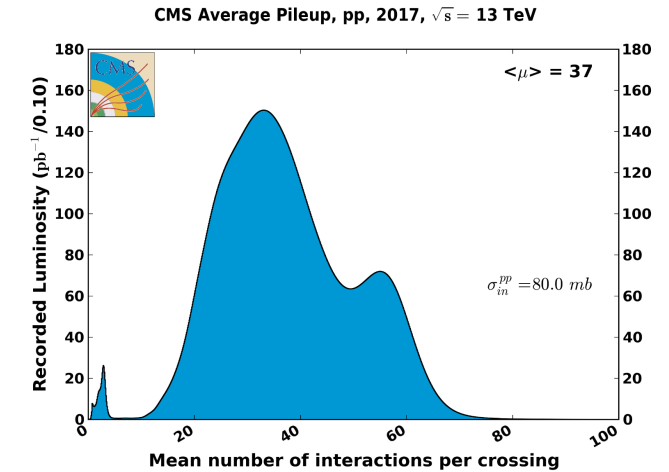
# W-helicity and PDFs

- Constraining power of data verified by effective **reduction of PDF uncertainties**
  - Obtained via profiling of eigenvectors



# Opportunities from low-PU runs

- Dedicated low-PU runs delivered in 2017 (~200/pb).
- About 5M  $W$  events needed to reach 6 MeV stat-only uncertainty (as for CDF-II)
  - That is, **> 1/fb of low-PU data**, i.e.  $\sim 15/\text{fb}$  of lost high-PU data
- Further improvements expected from planned detector upgrades



# Model-independent measurement

- In alternative to a low-PU measurement, consider unconventional ways of measuring  $M_W$  from full LHC data
  - Something has to be done to evade the model-uncertainty
- Do so by fitting the  $(p_T^l, \eta^l)$  spectrum to a **theory-agnostic model** written in the basis of helicity components

$$\frac{\Delta^2 \sigma}{\Delta p_T^l \Delta \eta^l} = \sum_{\Delta q_T, \Delta |y|} \frac{\Delta^2 \sigma_{-1}}{\Delta q_T \Delta |y|} \left( T_{-1}(p_T, \eta | M_W) + \sum_{i=0 \dots 4} A_{i, \Delta q_T, \Delta |y|} \times T_i(p_T, \eta | M_W) \right)$$

- $M_W$  is a parameter of the templates; it is to be determined together with  $d\sigma^{unpol}$  and  $A_i$



# Projecting an agnostic measurement to full Run2+3

- This measurement will be pursued in the next years as part of an ERC-funded project
- The challenge is to control **experimental uncertainties** over 6 years of data taking
  - $\Delta M_W$  in [8,11] MeV range appears feasible

Projection to 300/fb (*private work*)

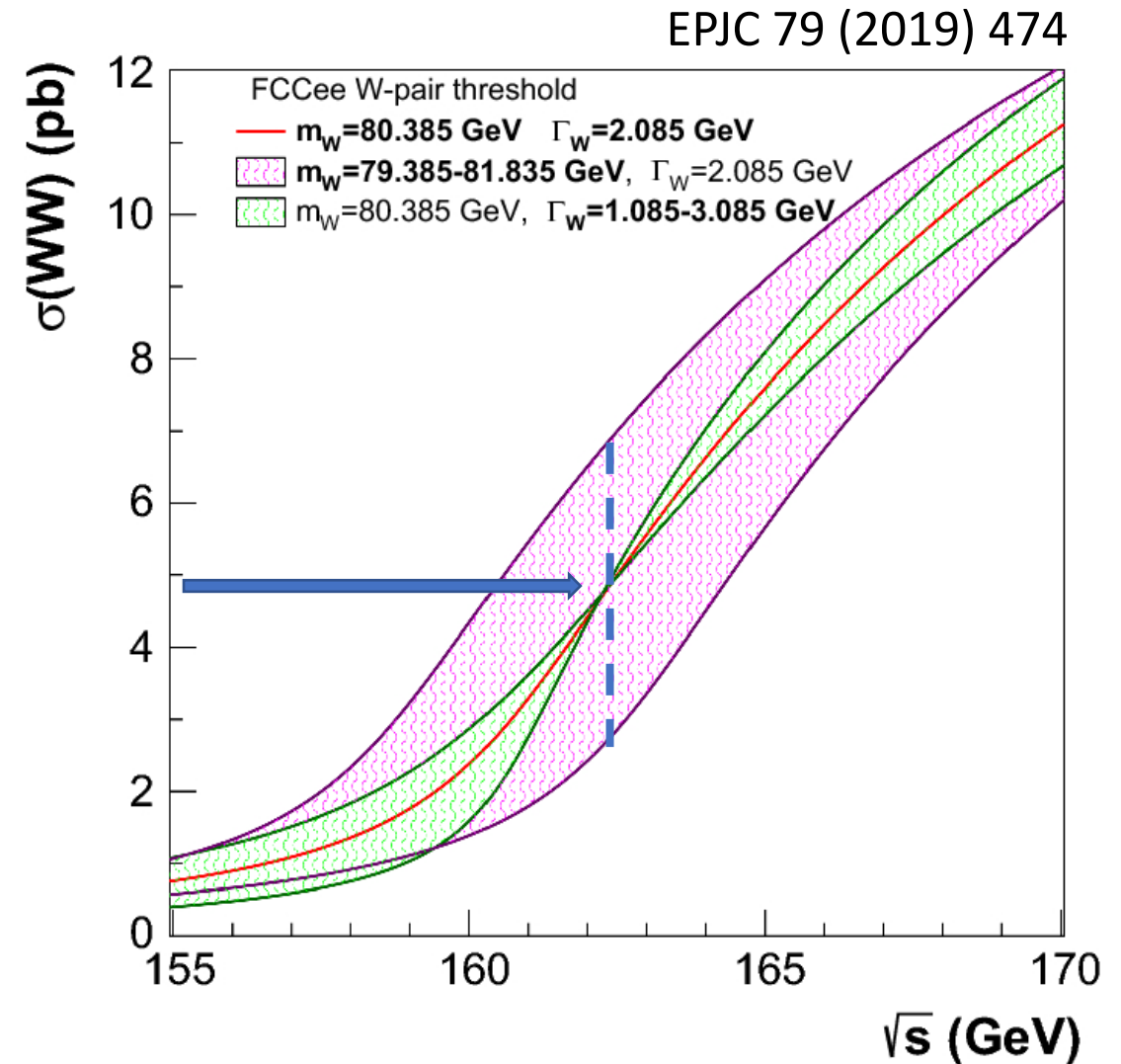
		Stat.	Exp.	Bkg.	QCD	EW	PDF	Tot.
Reference (ATLAS @7 TeV)		7	6	5	8	6	9	19
<b>ASYMOW</b>	Conservative	4	8	5	3	3	3	11
	Intermediate	4	4 / 8	5 / 3	3	3	3	9 / 10
	Aggressive	4	4	3	3	3	3	8



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# Ultimate future for $M_W$

- **Ultimate precision** from next-generation of lepton colliders (>2040)
  - FCC-ee + 2y at threshold → 0.5 MeV
- Beyond any conceivable reach of hadron colliders 😊



# Conclusions

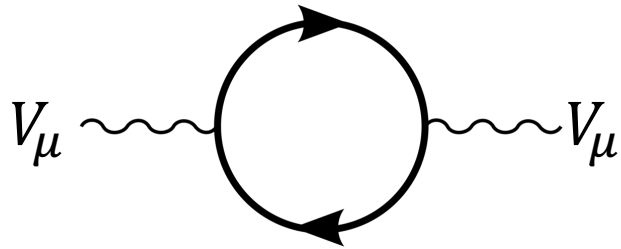
- CDF-II changed a **tempting excess** into a **stunning anomaly**
  - BSM interpretations of such anomaly are consistent with other precision data
  - CDF-II is inconsistent with current LHC (D0) at  $3.5\sigma$  ( $2.5\sigma$ ) and barely consistent with its superseded measurement ( $\sim 2\sigma$ ).
- CMS is expected to deliver a first measurement soon.
- Modeling of  $W$  production remains ***the bottleneck*** of this analysis
  - An important effort of the TH community is ongoing (e.g. *LHC EW precision WG*)
  - **More LHC data in the future can do the difference**

*Thanks for your attention!*

# Backup

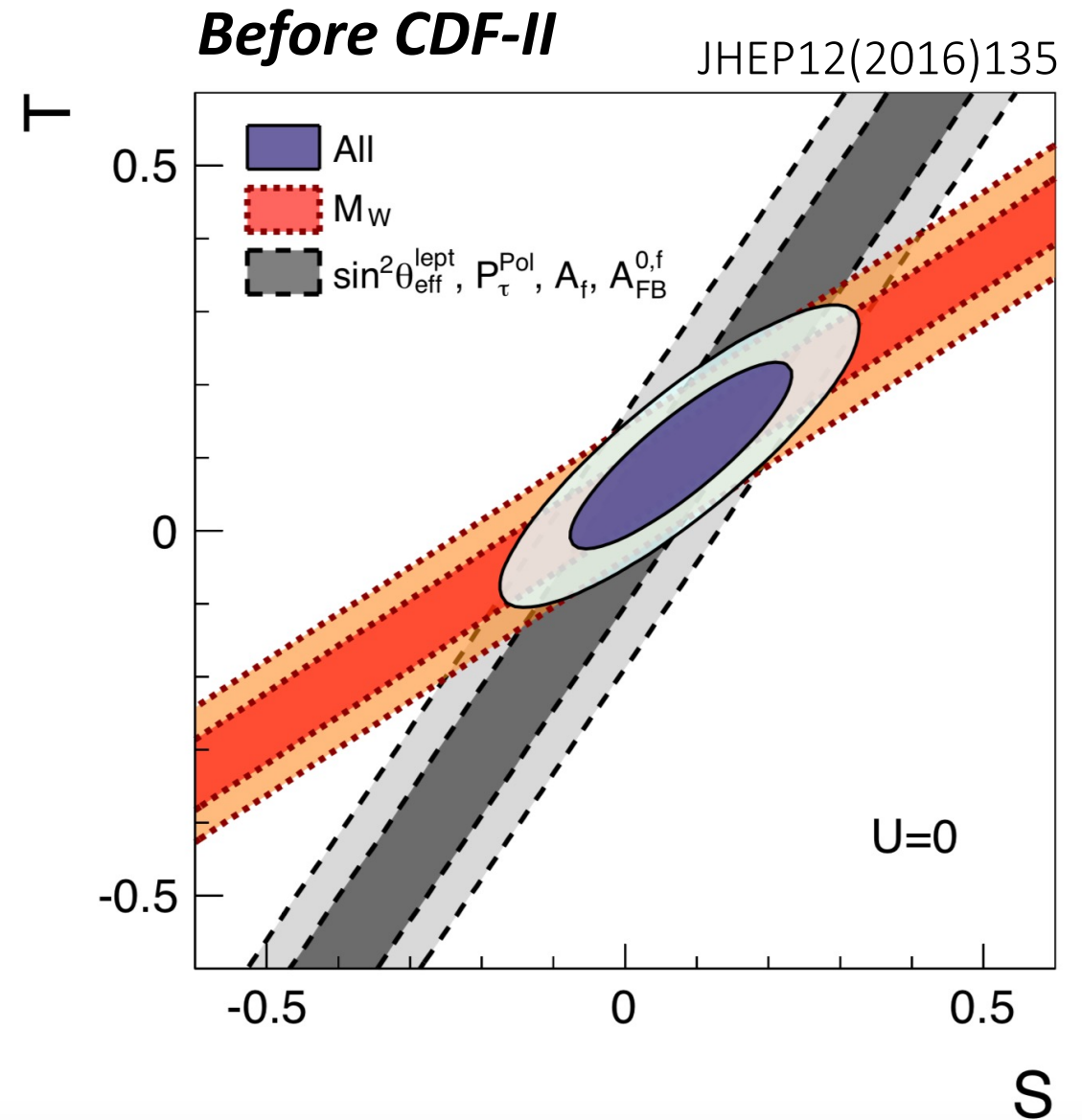
# $M_W$ as a probe of NP

- Pivotal role in determination of **oblique parameters  $S, T, U$** 
  - bounds on universal new physics

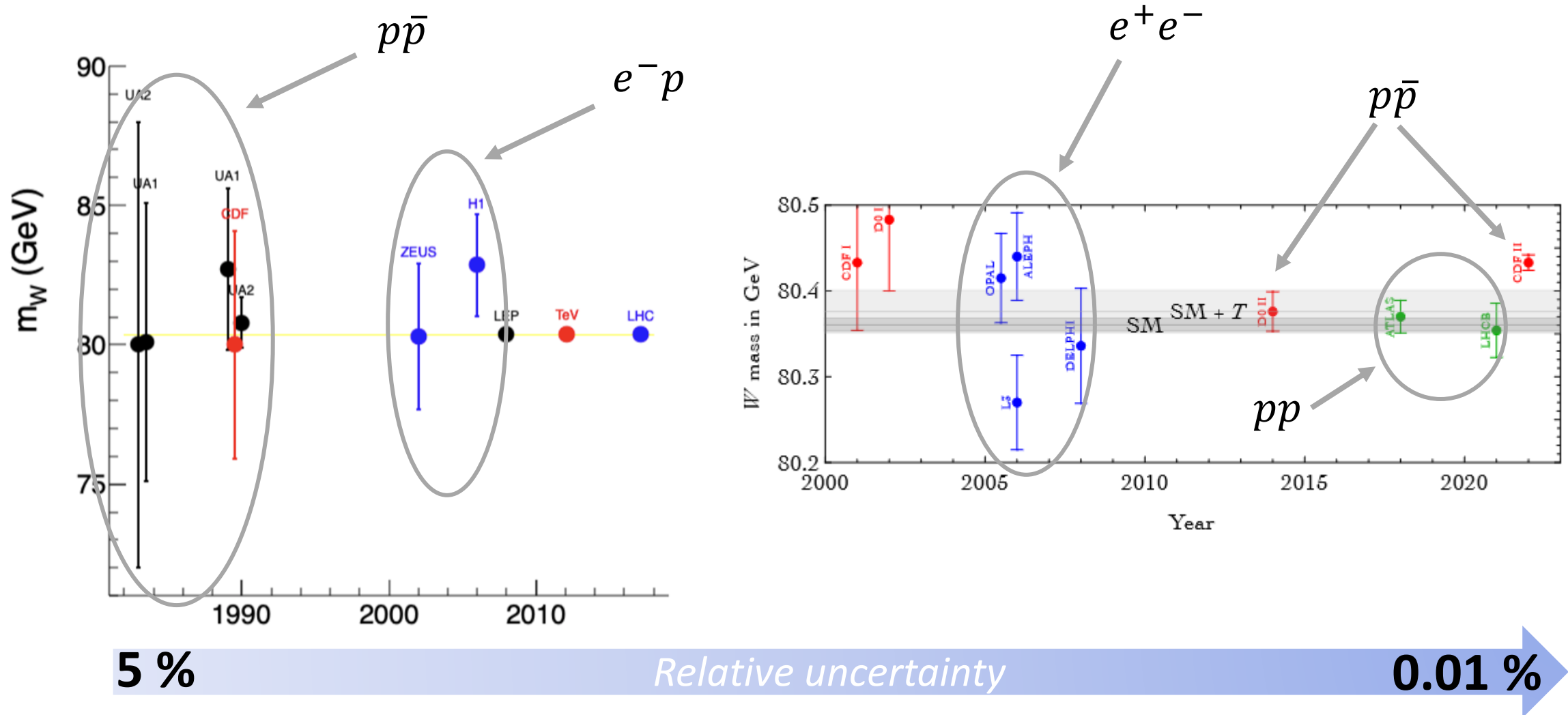


$$M_Z^2 = M_{Z0}^2 \frac{1 - \hat{\alpha}(M_Z)T}{1 - G_F M_{Z0}^2 S / 2\sqrt{2}\pi},$$

$$M_W^2 = M_{W0}^2 \frac{1}{1 - G_F M_{W0}^2 (S + U) / 2\sqrt{2}\pi},$$

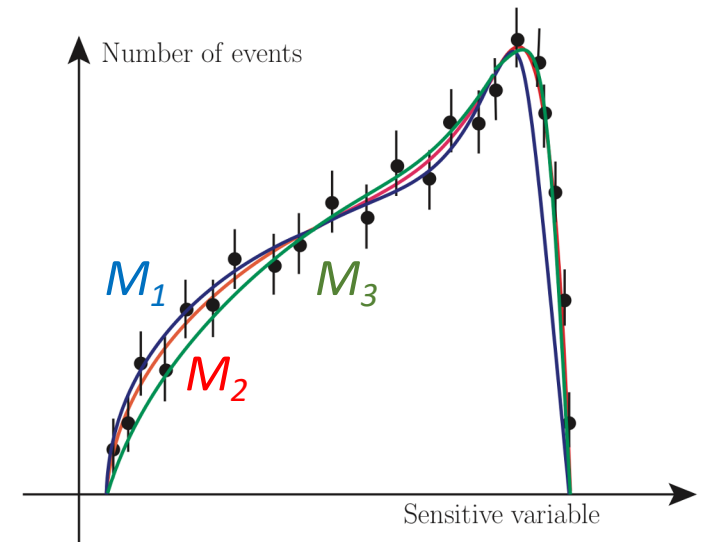
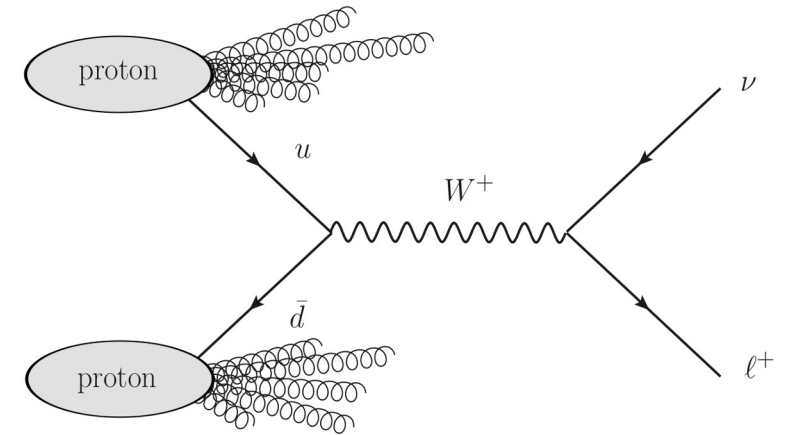


# $M_W$ in the history of colliders



# $M_W$ at hadron colliders

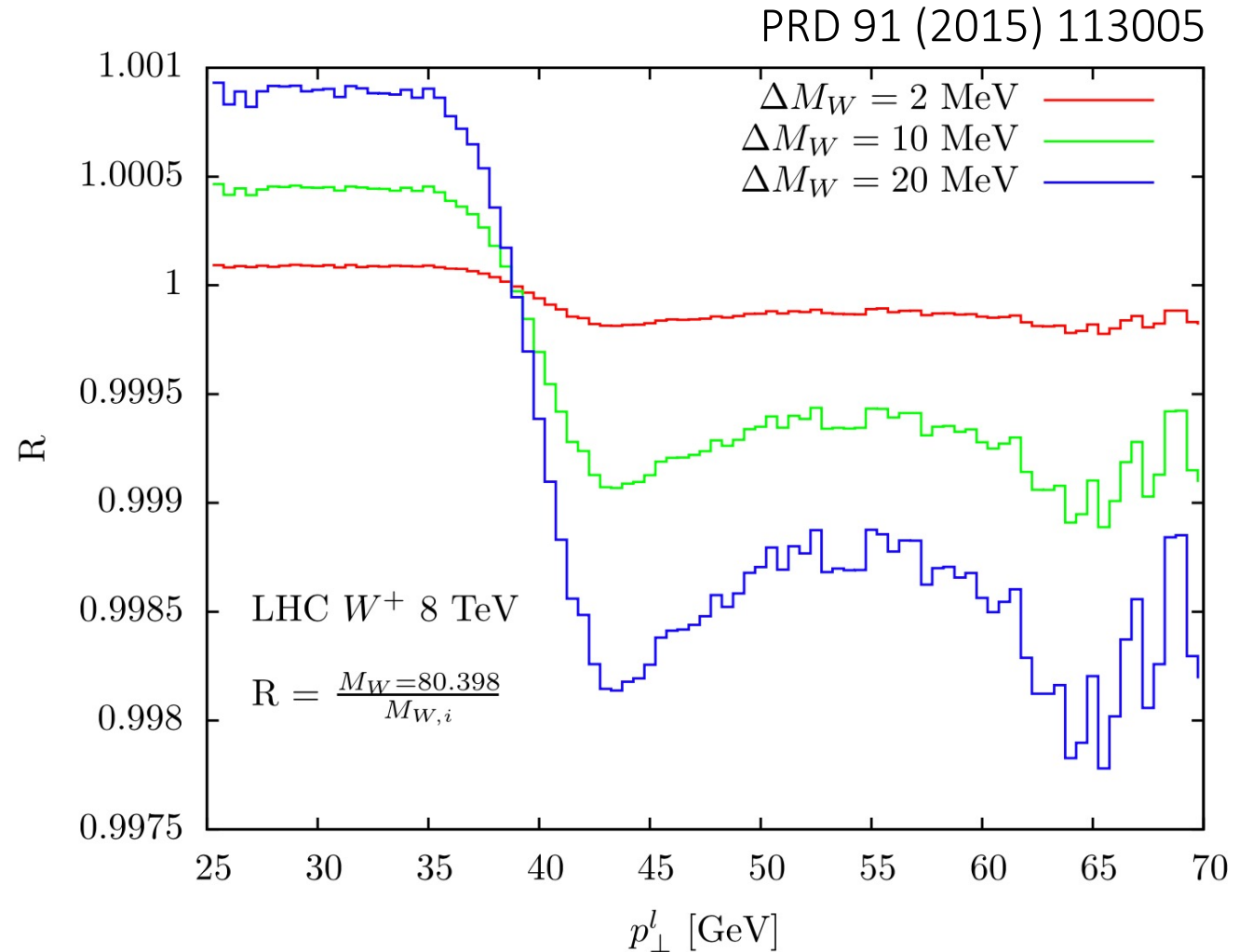
- Direct production:  $pp \rightarrow W^\pm \rightarrow l^\pm \nu$ 
  - Continuous spectrum of  $W$  momenta
  - Neutrino  $p_4$  unreconstructed
- Use of kinematic variables sensitive to  $M_W$  but NOT Lorentz-invariant
- Comparison of experimental distributions to **model-dependent** templates
  - Fit for the “best”  $M_W$



# Experimental accuracy: $p_T^l$

*Impact of a 10 MeV shift of  $M_W$  on the  $p_T^l$  spectrum  $\rightarrow$  **0.1%***

- This is unlike other mass measurement which can rely on neat mass peaks
  - The full  $W$  production x decay chain must be modeled at the **1% level**

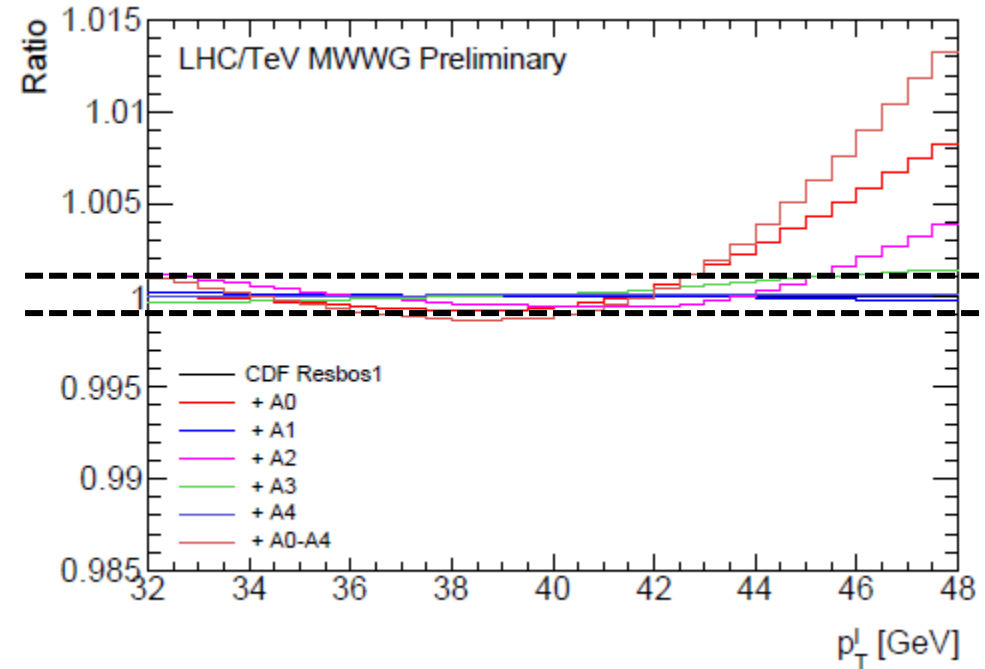
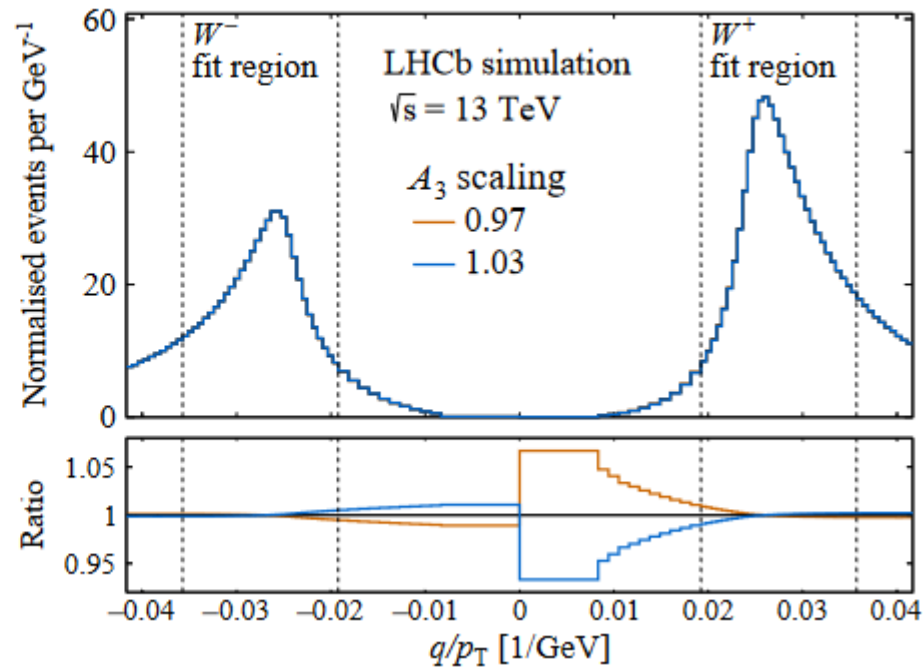




# Impact of angular coefficients

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Source	Section	$m_T$	$p_T^e$	$E_T$
Experimental				
Electron energy scale	VIIC4	16	17	16
Electron energy resolution	VIIC5	2	2	3
Electron shower model	VC	4	6	7
Electron energy loss	VD	4	4	4
Recoil model	VIID3	5	6	14
Electron efficiencies	VIIB10	1	3	5
Backgrounds	VIII	2	2	2
$\sum$ (Experimental)		18	20	24
$W$ production and decay model				
PDF	VIC	11	11	14
QED	VIB	7	7	9
Boson $p_T$	VIA	2	5	2
$\sum$ (Model)		13	14	17
Systematic uncertainty (experimental and model)		22	24	29
$W$ boson statistics	IX	13	14	15
Total uncertainty		26	28	33

# CDF-II

Source of systematic uncertainty	$m_T$ fit			$p_T^\ell$ fit			$p_T^\nu$ fit		
	Electrons	Muons	Common	Electrons	Muons	Common	Electrons	Muons	Common
Lepton energy scale	5.8	2.1	1.8	5.8	2.1	1.8	5.8	2.1	1.8
Lepton energy resolution	0.9	0.3	-0.3	0.9	0.3	-0.3	0.9	0.3	-0.3
Recoil energy scale	1.8	1.8	1.8	3.5	3.5	3.5	0.7	0.7	0.7
Recoil energy resolution	1.8	1.8	1.8	3.6	3.6	3.6	5.2	5.2	5.2
Lepton $u_{  }$ efficiency	0.5	0.5	0	1.3	1.0	0	2.6	2.1	0
Lepton removal	1.0	1.7	0	0	0	0	2.0	3.4	0
Backgrounds	2.6	3.9	0	6.6	6.4	0	6.4	6.8	0
$p_T^Z$ model	0.7	0.7	0.7	2.3	2.3	2.3	0.9	0.9	0.9
$p_T^W/p_T^Z$ model	0.8	0.8	0.8	2.3	2.3	2.3	0.9	0.9	0.9
Parton distributions	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
QED radiation	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Statistical	10.3	9.2	0	10.7	9.6	0	14.5	13.1	0
Total	13.5	11.8	5.8	16.0	14.1	7.9	18.8	17.1	7.4

# LHCb

Source	Size [ MeV]
Parton distribution functions	9
Theory (excl. PDFs) total	17
Transverse momentum model	11
Angular coefficients	10
QED FSR model	7
Additional electroweak corrections	5
Experimental total	10
Momentum scale and resolution modelling	7
Muon ID, trigger and tracking efficiency	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total	32

# ATLAS

Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.	$\chi^2/\text{dof}$ of Comb.
$m_T-p_T^\ell, W^\pm, e-\mu$	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5	29/27