New Physics in Flavour Observables?

B anomalies in semi-leptonic penguins

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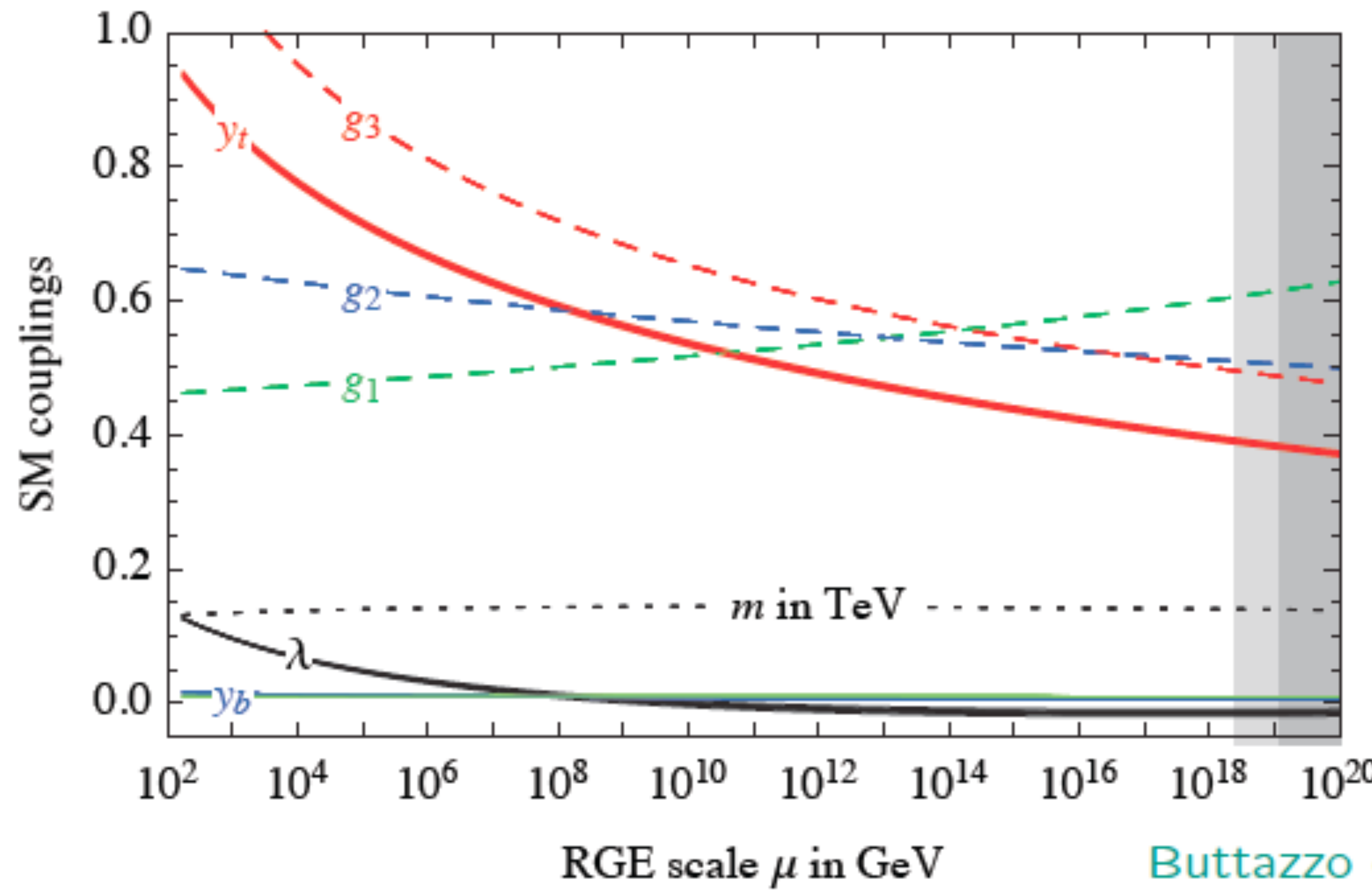
Prologue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)

- Neutrino masses (Dirac or Majorana masses ?)

- Baryon asymmetry of the Universe (new sources of CP violation needed)

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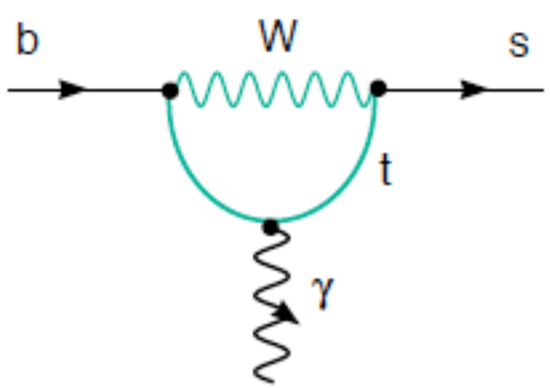
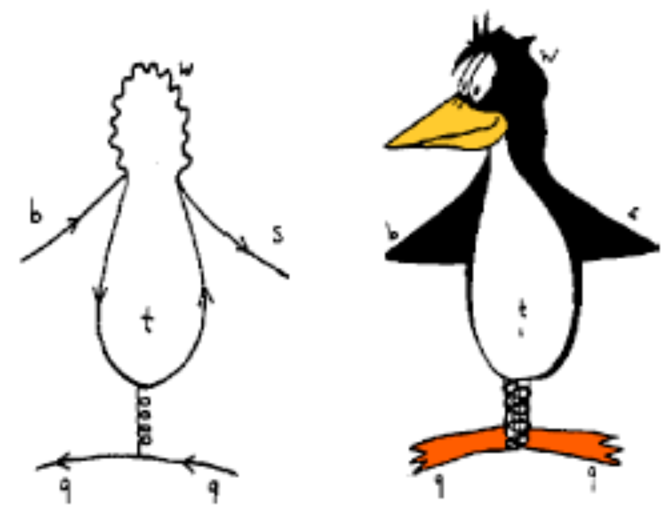
Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

Indirect exploration of higher scales via flavour

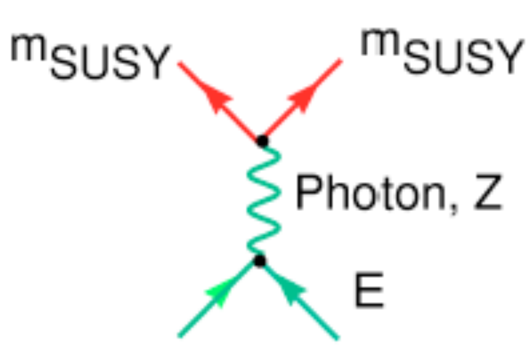
- Flavour changing neutral currrent processes like $b \to s\gamma$ or $b \to s\ell^+\ell^-$ directly probe the SM at the one-loop level.



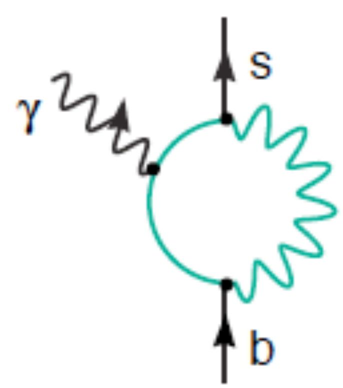


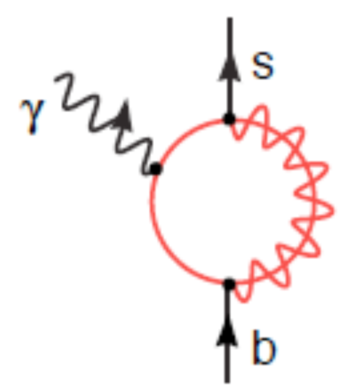
- Indirect search strategy for new degrees of freedom beyond the SM

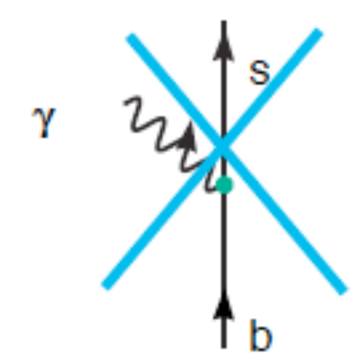
Direct:



Indirect:



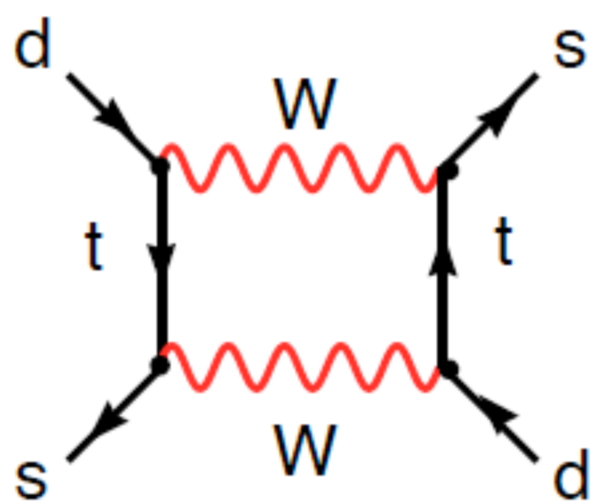


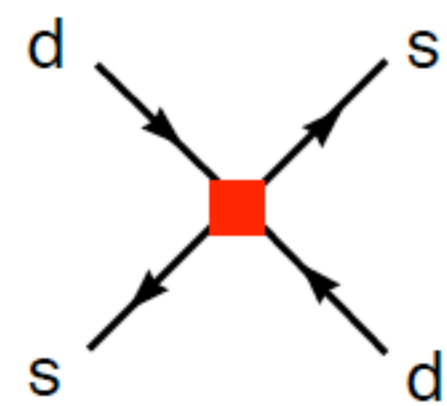


Flavour problem of New Physics or how FCNCs hide?

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \sum_i \frac{c_i^{New}}{\Lambda} \mathcal{O}_i^{(5)} + \ldots$$

- SM as effective theory valid up to cut-off-scale Λ
- $K^0 - \bar{K}^0$-mixing $\mathcal{O}^6 = (\bar{s}\,d)^2$: $c^{SM}/M_W^2 \times (\bar{s}\,d)^2 + c^{New}/\Lambda^2 \times (\bar{s}\,d)^2 \Rightarrow \Lambda > 10^4$ TeV





- Natural stabilisation of Higgs boson mass $\Rightarrow \Lambda \sim 1$TeV

Ambiguity of new physics scale from flavour data

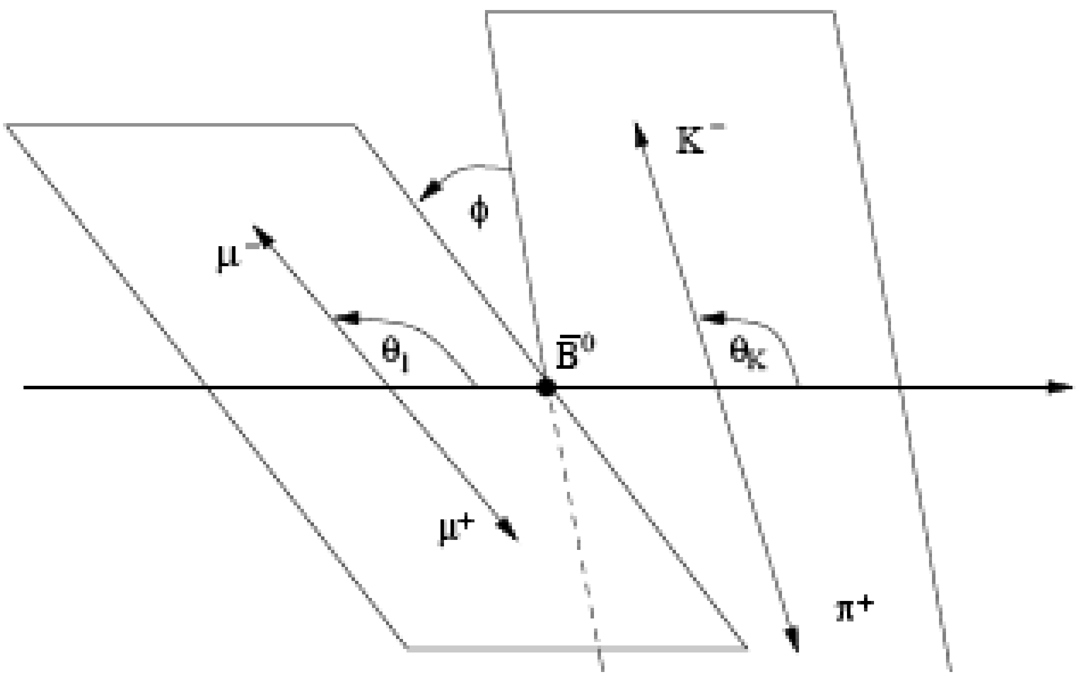
$$(C^i_{\mathrm{SM}}/M_W + C^i_{\mathrm{NP}}/\Lambda_{\mathrm{NP}}) \times \mathcal{O}_i$$

The b > s Anomalies

Differential decay rate of $B \to K^*\ell\ell$

Assuming the $\bar{K}^*$ to be on the mass shell, the decay $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ described by the lepton-pair invariant mass, s, and the three angles θ_l, θ_{K^*}, ϕ.

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



$$J(q^2, \theta_l, \theta_K, \phi) =$$

$$= J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+ J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l$$
$$+ J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi$$

Large number of independent angular obervables

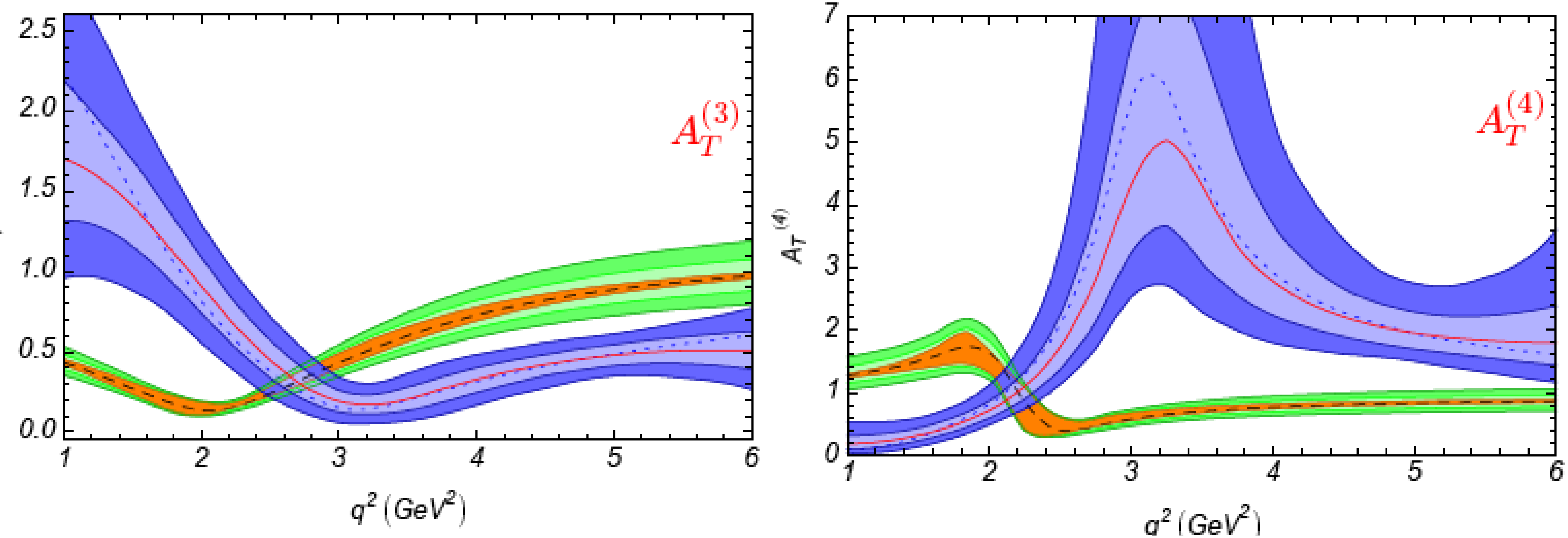
Careful design of theoretical clean angular observables

Egede, Hurth, Matias, Ramon, Reece, arXiv:0807.2589, arXiv:1005.0571

- Dependence of soft form factors, $\xi_\perp$ and $\xi_\|$, to be minimized !
  form factors should cancel out exactly at LO, best for all s
- unknown Λ/m_b power corrections

$A_{\perp,\|,0} = A^0_{\perp,\|,0}\,(1 + c_{\perp,\|,0})$ vary c_i in a range of ±10% and also of ±5%

Guesstimate



The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

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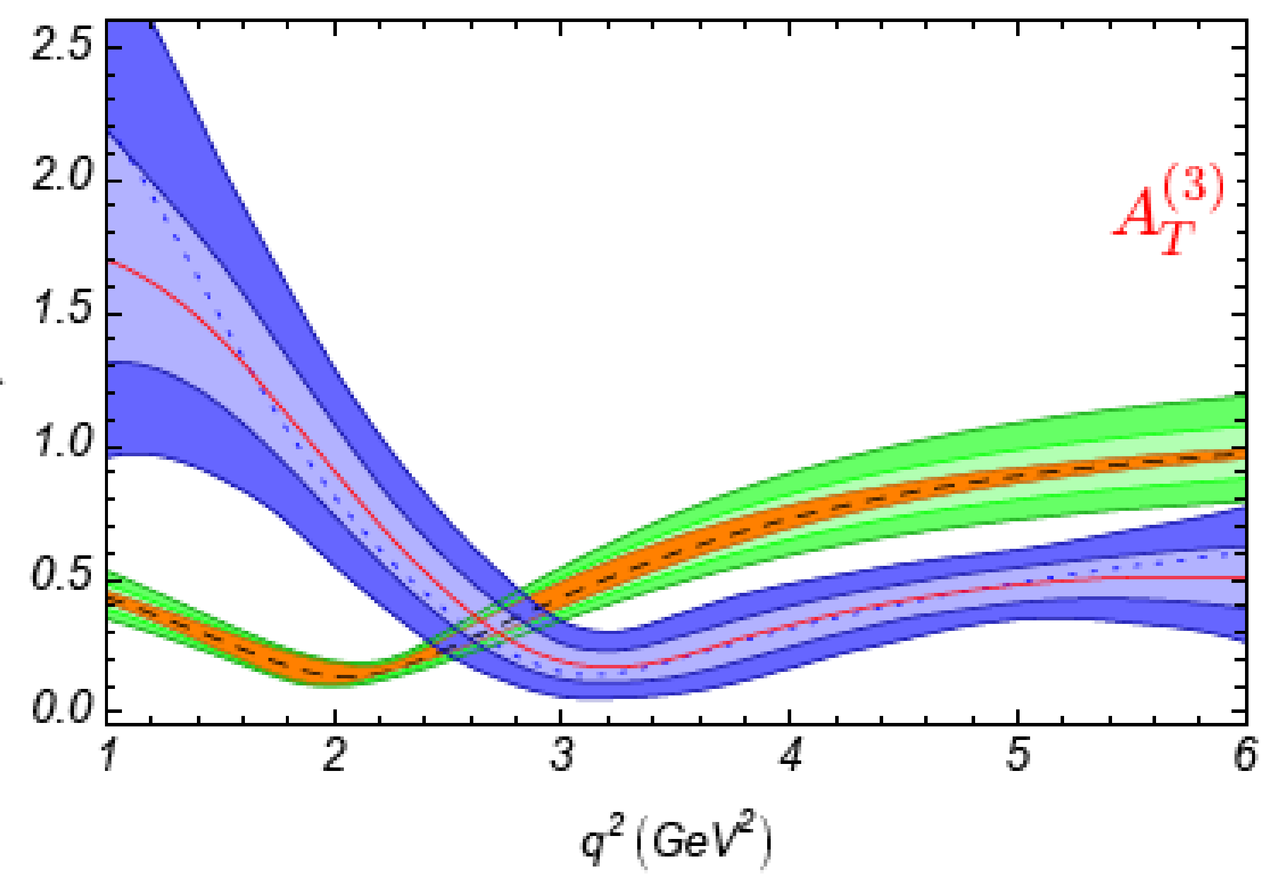
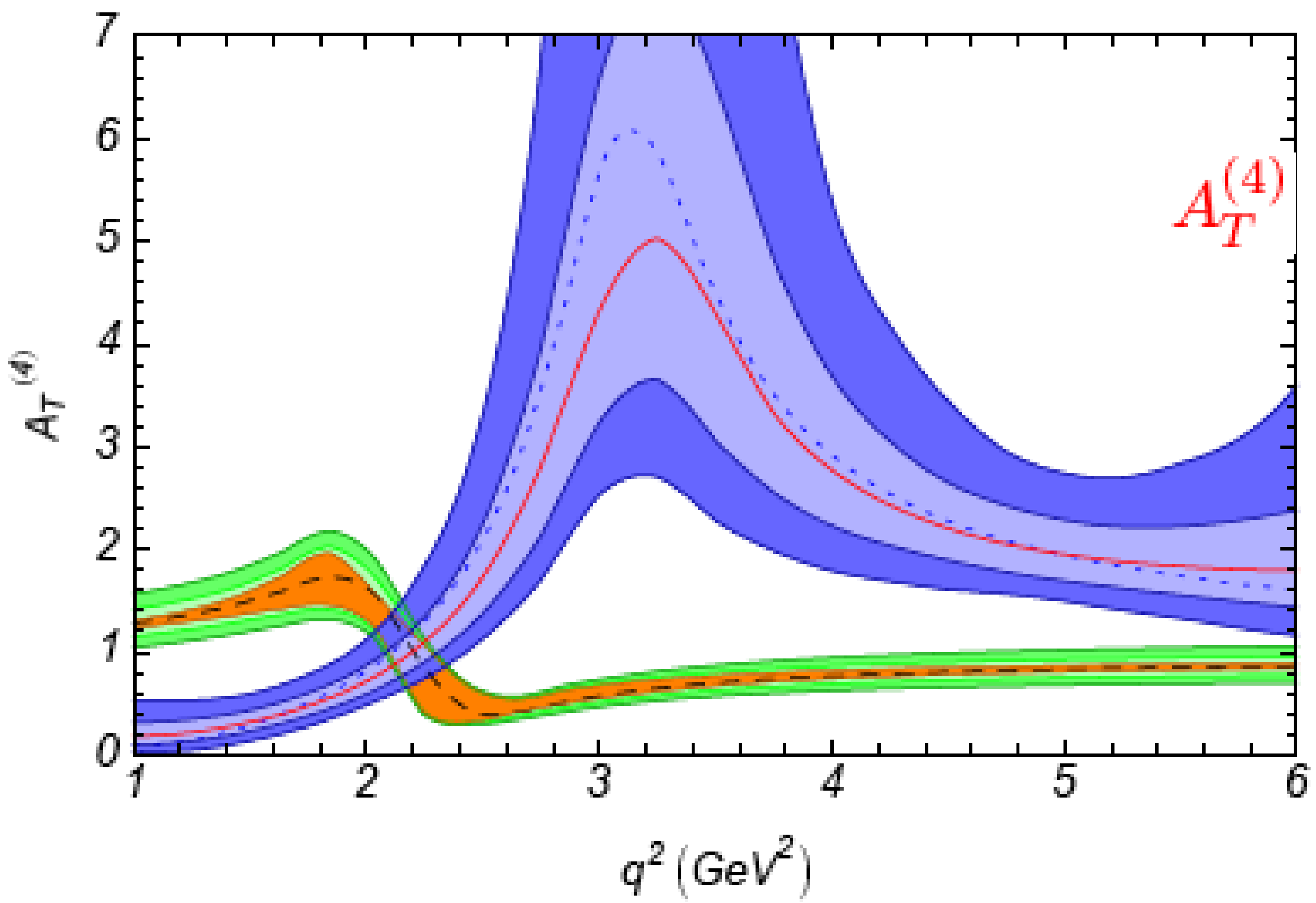
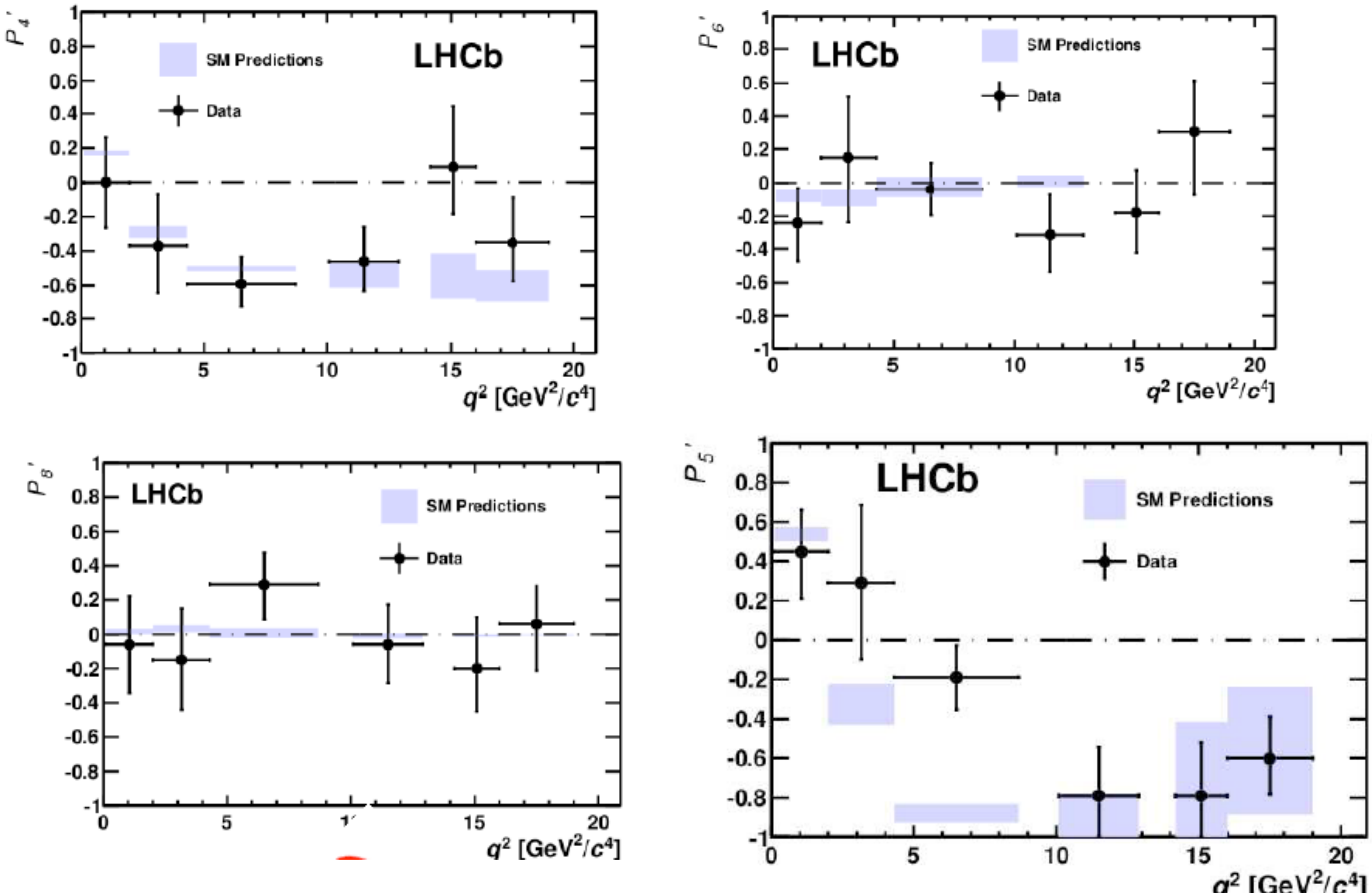




This was the dream in 2008

First measurements of new angular observables LHCb arXiv:1308.1707

SM predictions Descotes-Genon, Hurth, Matias, Virto arXiv:1303.5794



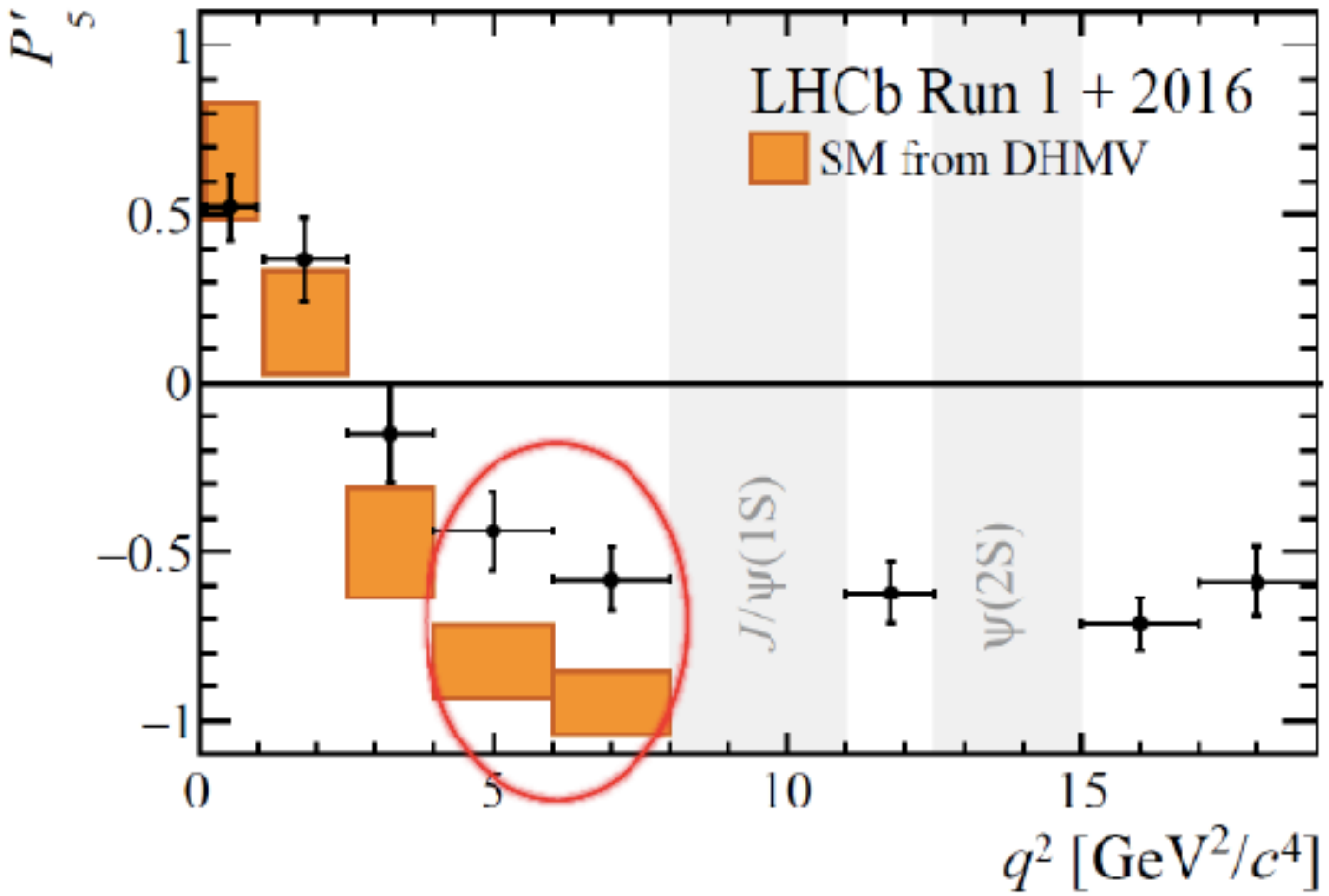
Good agreement with SM in P_4', P_6' and P_8',

but a 3.7σ deviation in the third bin in P_5'

Anomalies in $B \to K^*\mu^+\mu^-$ angular observables, in particular P_5' ; S_5

Long standing anomaly in the $B \to K^*\mu^+\mu^-$ angular observable P_5' / S_5 $\left(= P_5' \times \sqrt{F_L(1-F_L)}\right)$

- 2013 LHCb (1 fb^{-1})
- 2016 LHCb (3 fb^{-1})
- 2020 LHCb (4.7 fb^{-1})



[E. Smith CERN Seminar '20
LHCb 2003.04831]

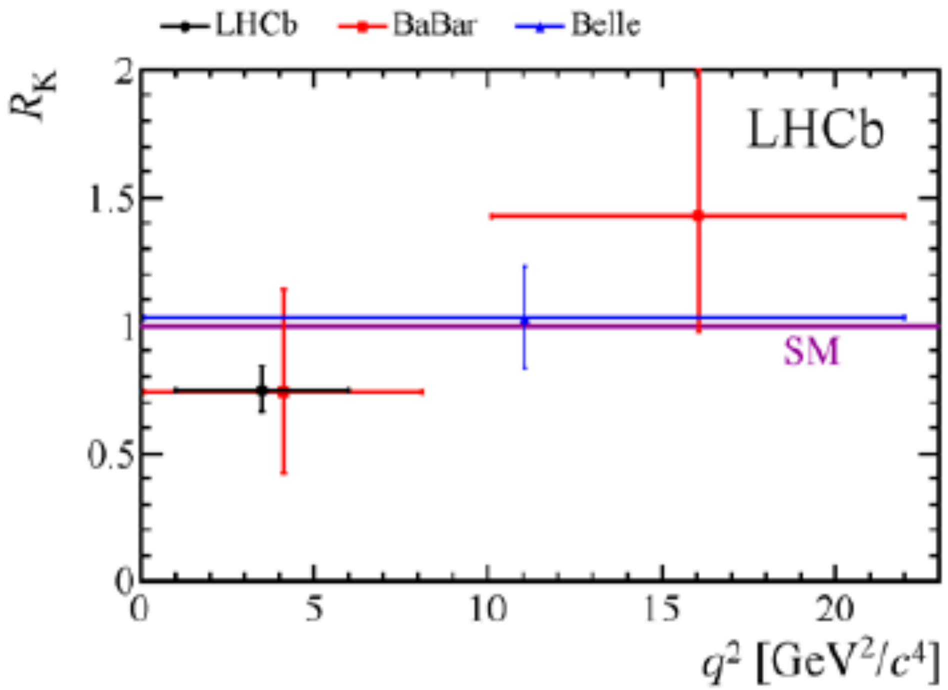
"$\approx 3\sigma$" local tension in P_5' with the respect SM predictions (DHMV)

Also deviations in other angular observables/bins and other decay modes

New Physics or underestimated hadronic uncertainties (form factors, power corrections) ?

Lepton flavour universality in $B^+ \to K^+\ell^+\ell^-$

- June 2014 (3 fb^{-1}): measurement of R_K in the [1-6] GeV^2 bin (PRL 113, 151601 (2014)): **2.6σ** tension in [1-6] GeV^2 bin
- SM prediction very accurate (leading corrections from QED, giving rise to large logarithms involving the ratio $m_B/m_{\mu,e}$)



BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

$$R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$

$$R_K^{\rm exp} = 0.745^{+0.090}_{-0.074}(\rm stat) \pm 0.036(\rm syst)$$

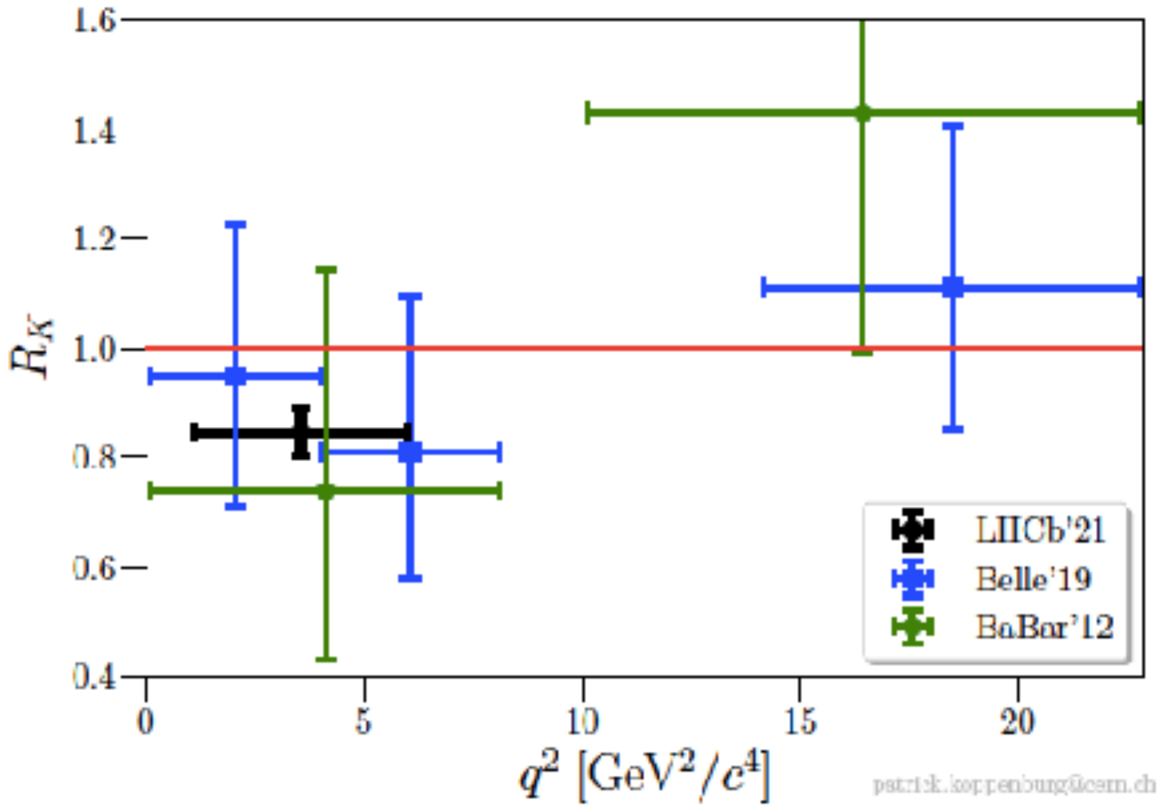
$$R_K^{\rm SM} = 1.0006 \pm 0.0004$$

Bordone, Isidori, Pattori, arXiv:1605.07633

Would be a spectacular fall of the SM !

New results: Lepton flavour universality in $B^+ \to K^+\ell^+\ell^-$

$$R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^+e^+e^-)$$



Run 1 (PRL 113, 151601 (2014)):
$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.717^{+0.083+0.017}_{-0.071-0.016}$

Run 2 (arXiv:1903.09252):
$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.928^{+0.089+0.020}_{-0.076-0.017}$

(Combined) (arXiv:1903.09252):
$R_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.846^{+0.060+0.016}_{-0.054-0.014}$

$R_K^{\mathrm{SM}} = 1.0006 \pm 0.0004$
Bordone, Isidori, Pattori, Eur.Phys.J. C76 (2016) 8, 440

New result (9 fb^{-1})(arXiv : 2103.11769) :

$$\mathrm{R}_K([1.1, 6.0]\,\mathrm{GeV}^2) = 0.846^{+0.042+0.013}_{-0.039-0.012}$$

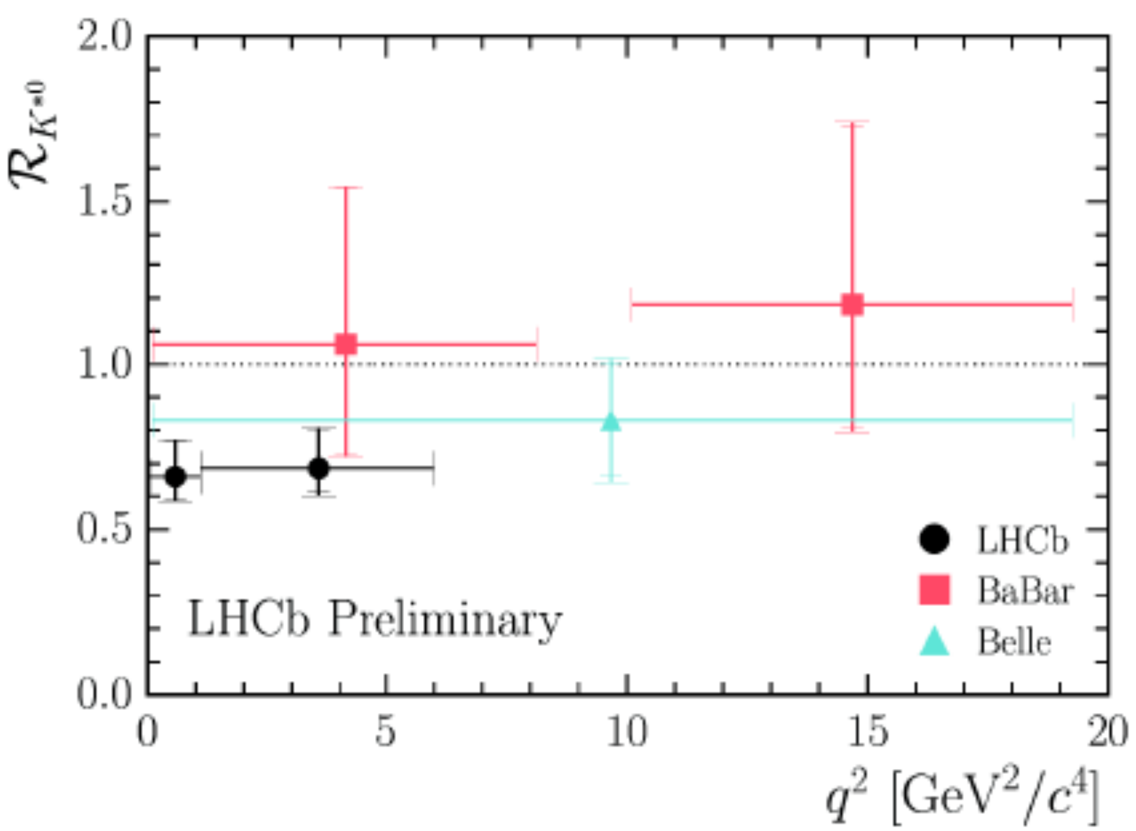
Central value is exactly the same, but the tension increases to 3.1σ due to the smaller uncertainty of the new measurement.

Lepton flavour universality in $B^0 \to K^{*0}\ell^+\ell^-$

- LHCb measurement (April 2017): **JHEP 1602, 104 (2016)**

$$R_{K^*} = BR(B^0 \to K^{*0}\mu^+\mu^-)/BR(B^0 \to K^{*0}e^+e^-)$$

- Two q^2 regions: [0.045-1.1] and [1.1-6.0] GeV^2



$$R_{K^*}^{\rm exp,bin1} = 0.660^{+0.110}_{-0.070}(\rm stat) \pm 0.024(\rm syst)$$

$$R_{K^*}^{\rm exp,bin2} = 0.685^{+0.113}_{-0.069}(\rm stat) \pm 0.047(\rm syst)$$

$$R_{K^*}^{\rm SM,bin1} = 0.906 \pm 0.020_{\rm QED} \pm 0.020_{\rm FF}$$

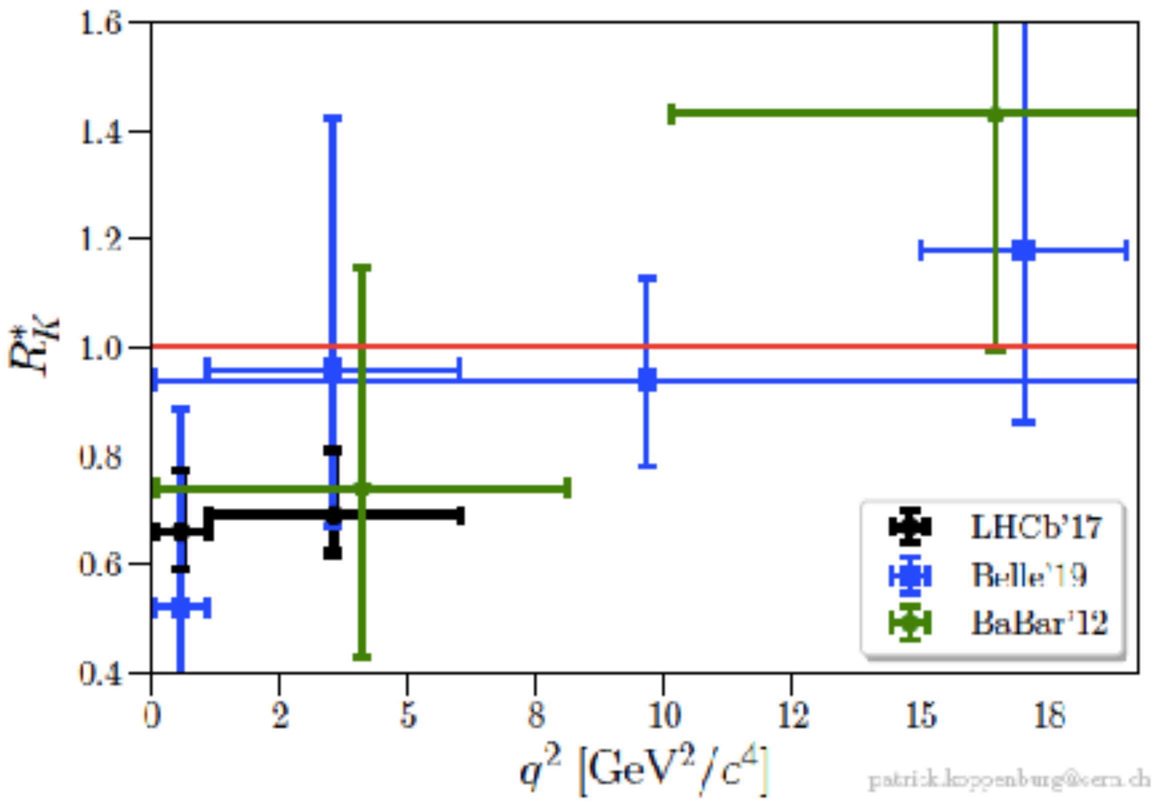
$$R_{K^*}^{\rm SM,bin2} = 1.000 \pm 0.010_{\rm QED}$$

Bordone, Isidori, Pattori, arXiv:1605.07633

BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

2.2-2.5σ tension with the SM predictions in each bin

New results: Lepton flavour universality in $B^0 \to K^{*0}\ell^+\ell^-$



LHCb (JHEP 08 (2017) 055):

$$R_{K^*}([0.045, 1.1]\,\mathrm{GeV}^2) = 0.660^{+0.110}_{-0.070} \pm 0.024$$

$$R_{K^*}([1.1, 6]\,\mathrm{GeV}^2) = 0.685^{+0.113}_{-0.069} \pm 0.047$$

Belle (arXiv:1904.02440):

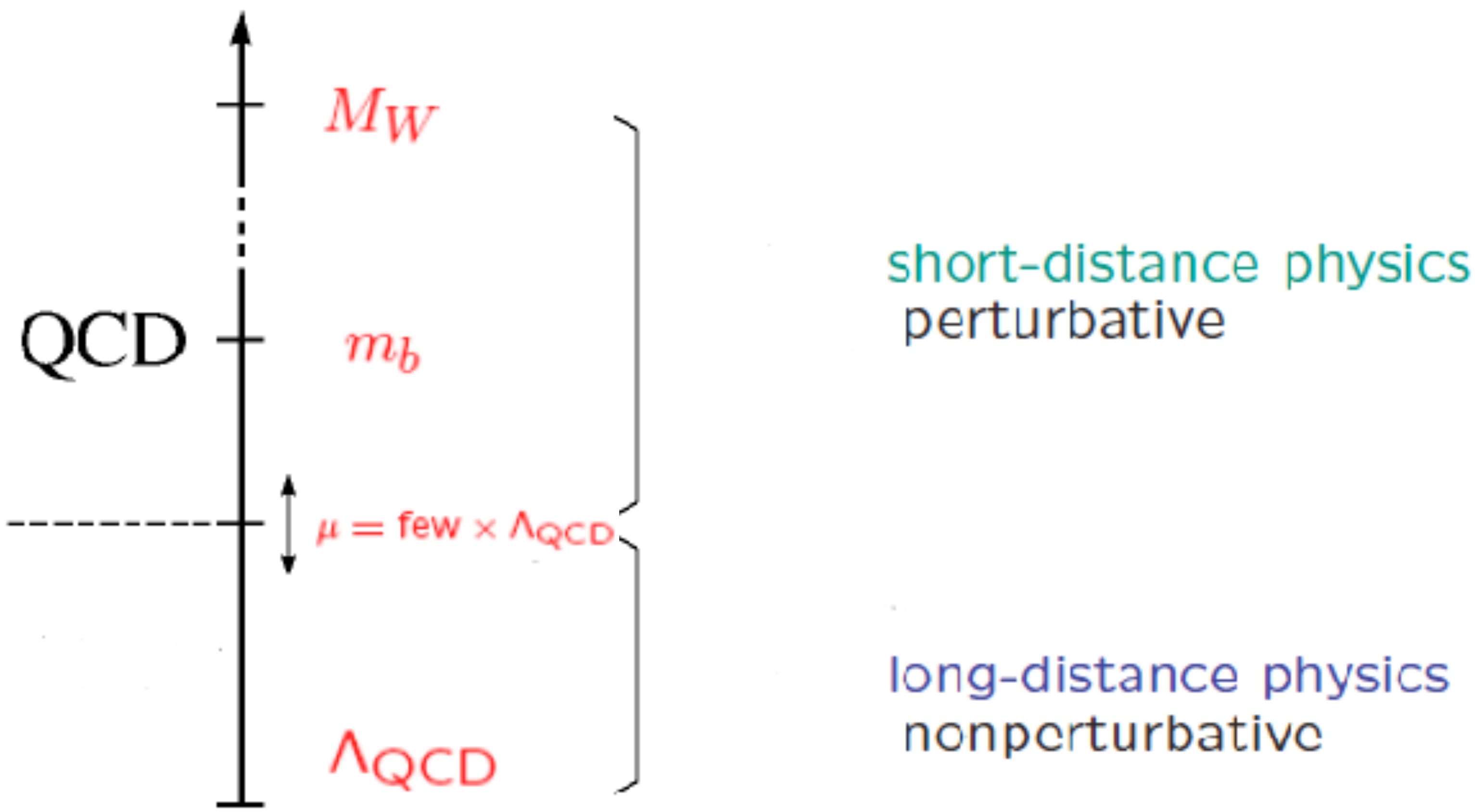
$$R_{K^*}([0.045, 1.1]\,\mathrm{GeV}^2) = 0.52^{+0.36}_{-0.26} \pm 0.05, \quad R_{K^*}([1.1, 6.0]\,\mathrm{GeV}^2) = 0.96^{+0.45}_{-0.29} \pm 0.11,$$

$$R_{K^*}([0.1, 8]\,\mathrm{GeV}^2) = 0.90^{+0.27}_{-0.21} \pm 0.10, \quad R_{K^*}([15, 19]\,\mathrm{GeV}^2) = 1.18^{+0.52}_{-0.32} \pm 0.10.$$

The very low-q^2 bin has a tension with the SM prediction slightly more than 1σ, while the other bins are all well in agreement with the SM at the 1σ-level.

Theoretical Tools

Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy})\ \mathcal{O}_i(\mu)$

- $\mu^2 \approx M_{New}^2 >> M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

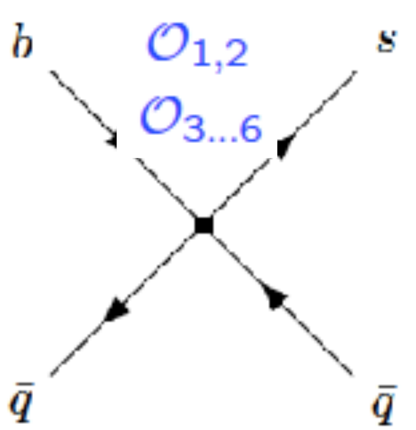
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

HQET, SCET, ...

Effective Weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Bigg(\sum_{i=1\cdots 10,S,P} \Big(C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu) \Big) \Bigg)$$

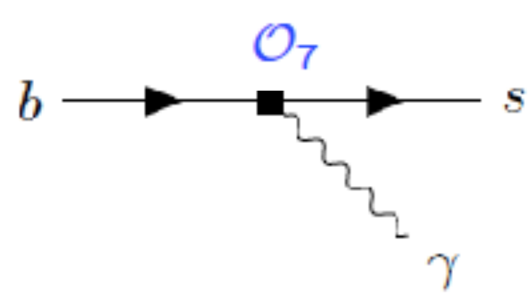
4-quark operators



$\mathcal{O}_{1,2} \propto (\bar{s}\Gamma_\mu c)(\bar{c}\Gamma^\mu b)$

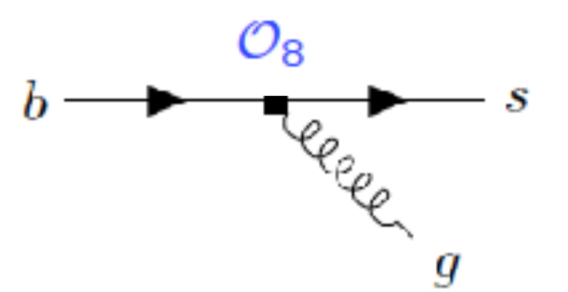
$\mathcal{O}_{3...6} \propto (\bar{s}\Gamma_\mu b)\sum_q(\bar{q}\Gamma^\mu q)$

electromagnetic dipole operator



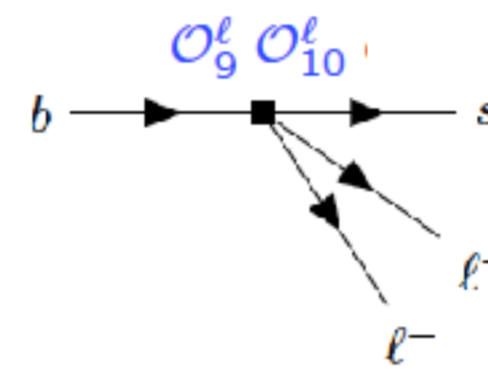
$\mathcal{O}_7 \propto (\bar{s}\sigma^{\mu\nu}P_R)F^a_{\mu\nu}$

chromomagnetic dipole operator



$\mathcal{O}_8 \propto (\bar{s}\sigma^{\mu\nu}T^a P_R)G^a_{\mu\nu}$

semileptonic operators



$\mathcal{O}^\ell_9 \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell)$

$\mathcal{O}^\ell_{10} \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$

In the SM: $C_7 = -0.29$ $\quad C_9 = 4.20$ $\quad C_{10} = -4.01$

New physics:

- Corrections to the Wilson coefficients: $C_i \to C_i^{\text{SM}} + \delta C_i^{\text{NP}}$
- Additional operators: Chirally flipped ($\mathcal{O}_i'$), (pseudo)scalar ($\mathcal{O}_S$ and $\mathcal{O}_P$)

Exclusive modes $B \to K^{(*)}\ell\ell$

Soft-collinear effective theory

$$\mathcal{T}_a^{(i)} = C_a^{(i)}\xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

– Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

– Relations between formfactors in large-energy limit

– Limitation: insufficient information on power-suppressed Λ/m_b terms (breakdown of factorization: 'endpoint divergences')

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

(This does not affect R_K and R_K^* of course, but does affect combined fits!)

Problem of nonfactorizable power corrections in angular observables

Crosscheck with $R_{\mu,e}$ ratios:

NP in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables (if there is a coherent picture)

Ongoing efforts: Estimate of power corrections based on analyticity

van Dyk et al.: arXiv:2011.09813, 2206.03797

In the long run: Solution with refactorization techniques

New developments in the SCET community

Neubert et al., arXiv:2009.06779

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

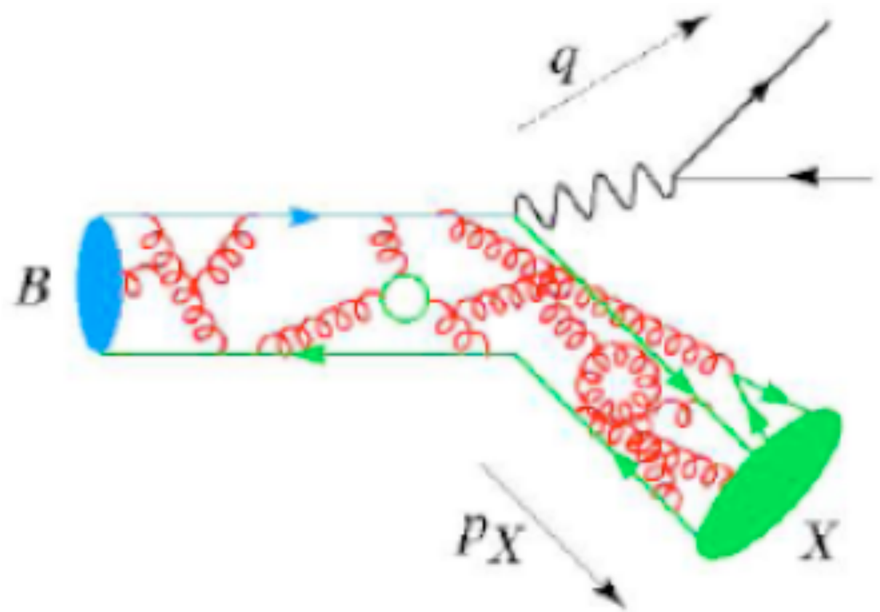
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

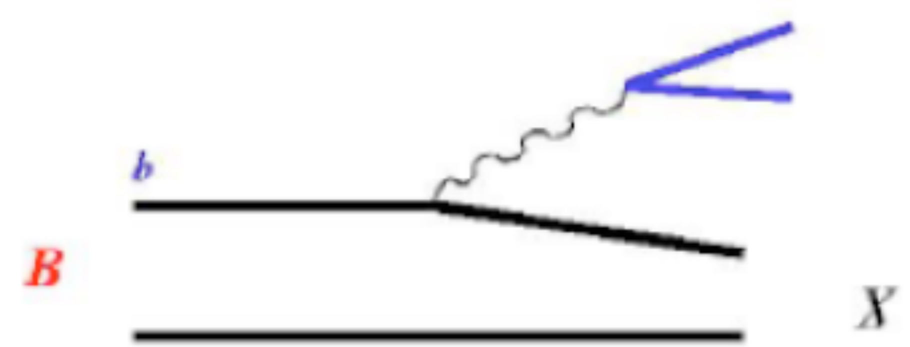
- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b \to \infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda_{QCD}^2/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990





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No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

An old story:

– If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012 [2]

Analysis in $B \to X_s\ell\ell$ in this talk; Benzke,Fickinger,Hurth,Turczyk

Model independent Analysis of Anomalies

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Update 2022

Separate NP fits with a single operator

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Update 2022

Comparison of fits to clean and other $b \to s$ observables

Only R_K, R_{K^*} and $B_{s,d} \to \ell^+\ell^-$ 2022 data ($\chi^2_{\rm SM} = 30.63$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
δC_9	-2.00 ± 5.00	30.5	0.4σ
δC_9^e	0.83 ± 0.21	10.8	4.4σ
δC_9^μ	-0.80 ± 0.21	11.8	4.3σ
δC_{10}	0.03 ± 0.20	30.6	0.1σ
δC_{10}^e	-0.81 ± 0.19	8.7	4.7σ
δC_{10}^μ	0.50 ± 0.14	16.2	3.8σ
$\delta C_{\rm LL}^e$	0.43 ± 0.11	9.7	4.6σ
$\delta C_{\rm LL}^\mu$	-0.33 ± 0.08	12.4	4.3σ

All observables except LFUV ratios and $B_{s,d} \to \ell^+\ell^-$ 2022 data ($\chi^2_{\rm SM} = 221.8$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
δC_9	-0.95 ± 0.13	185.1	6.1σ
δC_9^e	0.70 ± 0.60	220.5	1.1σ
δC_9^μ	-0.96 ± 0.13	182.8	6.2σ
δC_{10}	0.29 ± 0.21	219.8	1.4σ
δC_{10}^e	-0.60 ± 0.50	220.6	1.1σ
δC_{10}^μ	0.35 ± 0.20	218.7	1.8σ
$\delta C_{\rm LL}^e$	0.34 ± 0.29	220.6	1.1σ
$\delta C_{\rm LL}^\mu$	-0.64 ± 0.13	195.0	5.2σ

Clean observables
$R_{K^{(*)}}$ AND $B_s^0 \to \mu^+\mu^-$

Dependent on the assumptions on the nonfactorizable power corrections

Our guesstimate is 10% power corrections

Update 2022

New measurement of BR($B_s \to \mu\mu$) from CMS update at ICHEP!

Old Combination: $\mathrm{BR}(B_s \to \mu\mu)^{\mathrm{comb}}_{\mathrm{exp}} = \left(2.85^{+0.34}_{-0.31}\right) \times 10^{-9}$

New Combination: $\mathrm{BR}(B_s \to \mu\mu)^{\mathrm{comb}}_{\mathrm{exp}} = \left(3.52^{+0.32}_{-0.30}\right) \times 10^{-9}$

SM Prediction: $\mathrm{BR}(B_s \to \mu\mu)_{\mathrm{SM}} = (3.58 \pm 0.17) \times 10^{-9}$

Separate NP fits with two operators

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

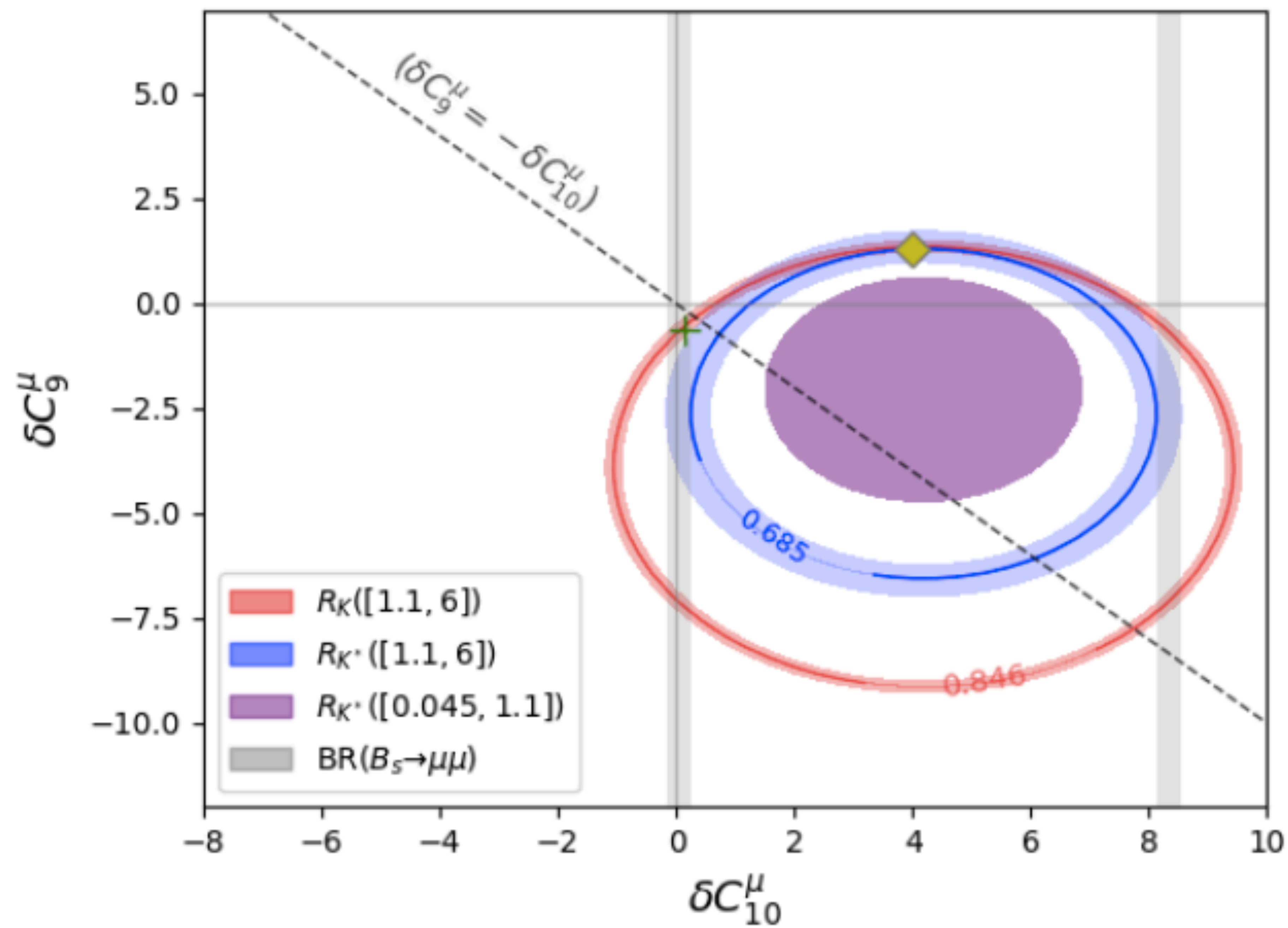
Role of $B_s \to \mu\mu$

Red (blue) solid line: central value of $R_{K^{(*)}}^{\rm exp}$

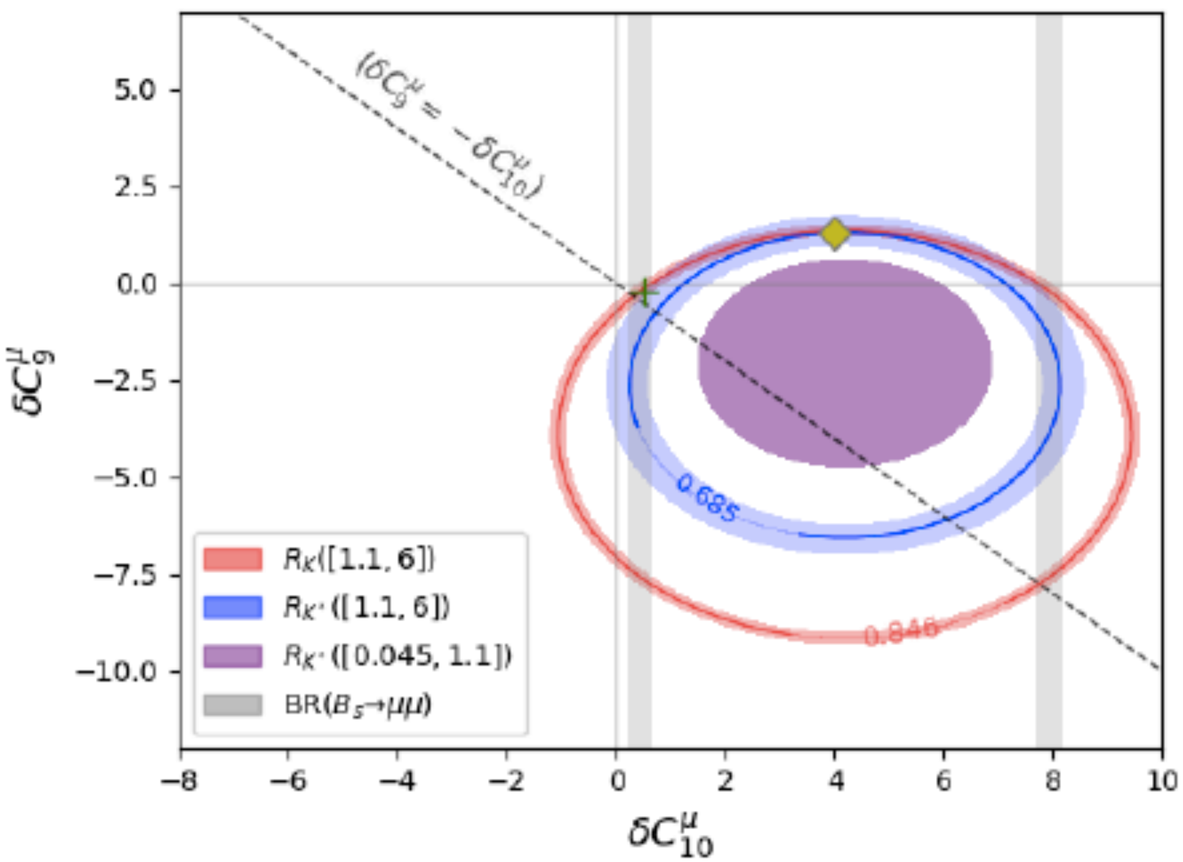
Coloured regions: 1σ range (th+exp) with the experimental central value

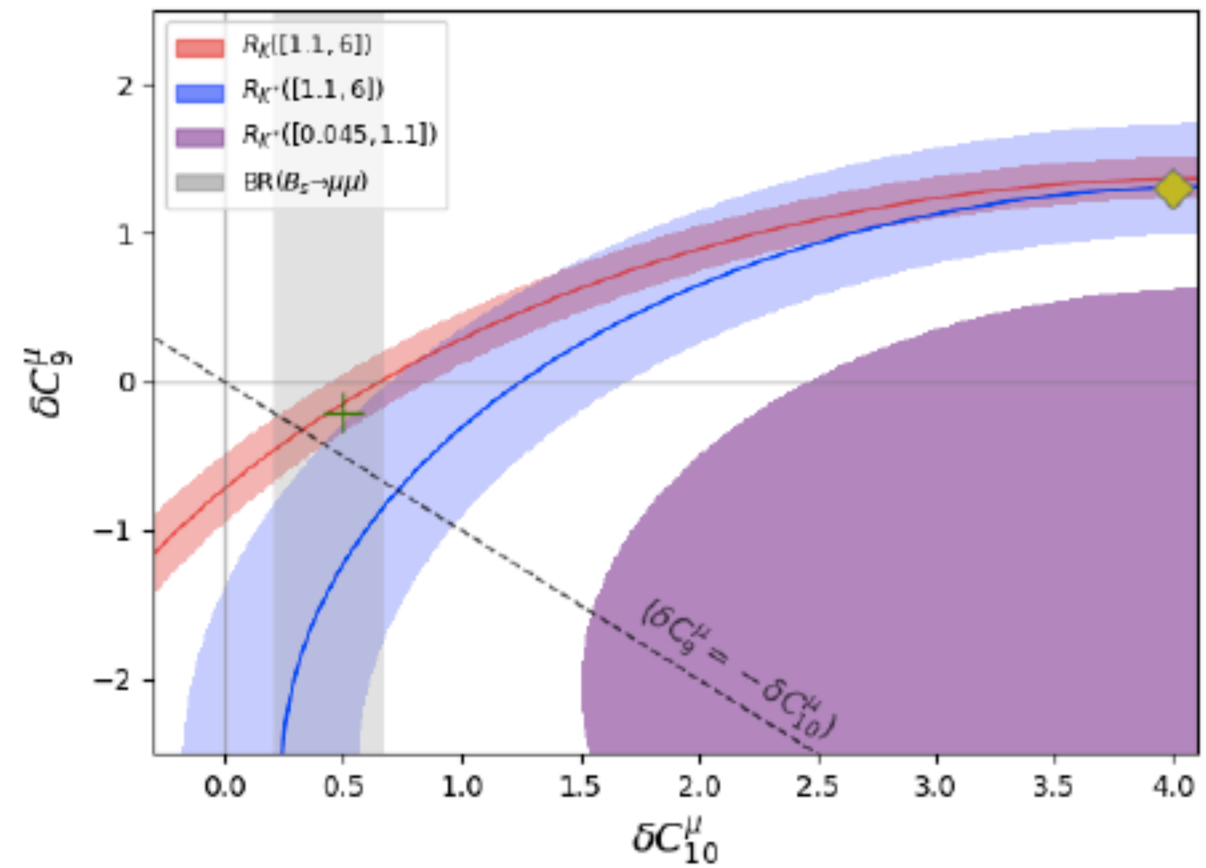
Yellow diamond ◇: best fit point of (C_9^μ, C_{10}^μ) of the fit to $R_{K^{(*)}}$

Green cross: best fit point of (C_9^μ, C_{10}^μ) of the fit to $R_{K^{(*)}}$ and $B_s^0 \to \mu^+\mu^-$

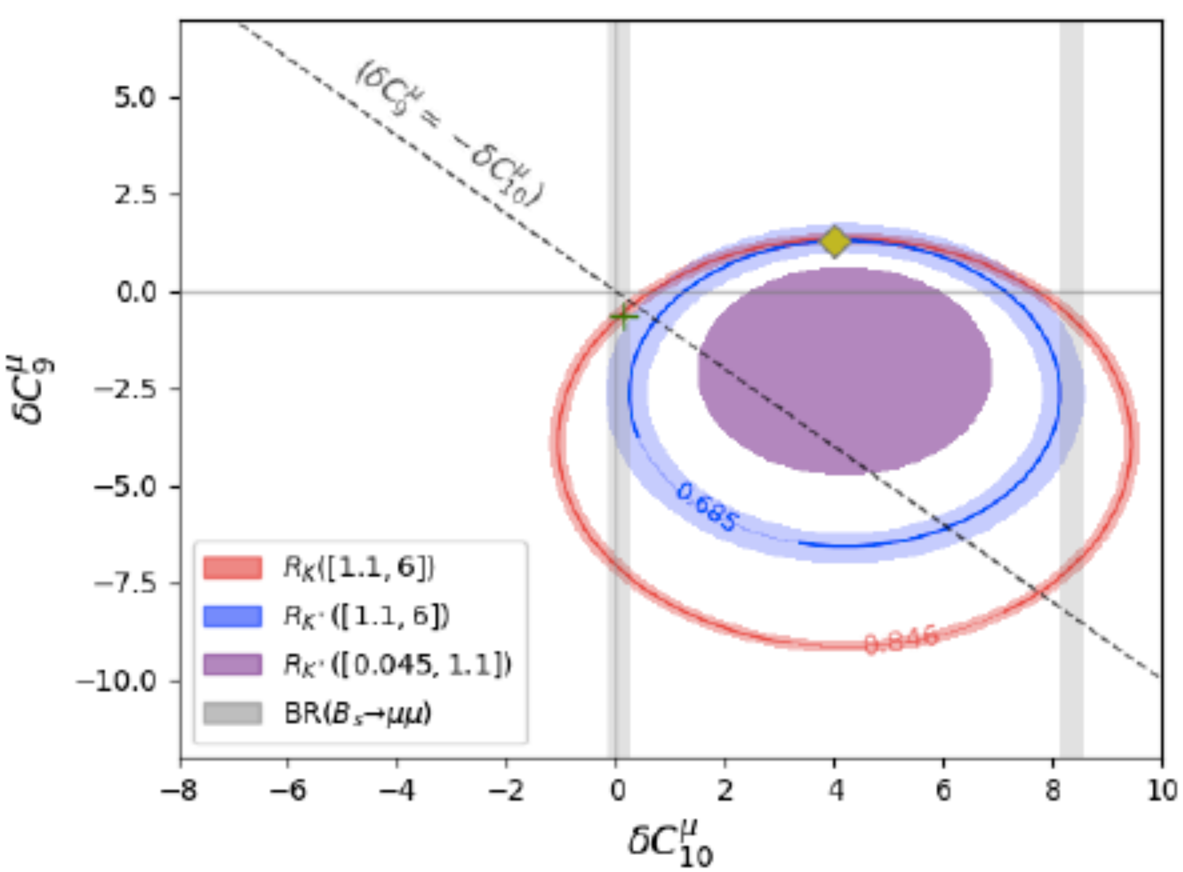


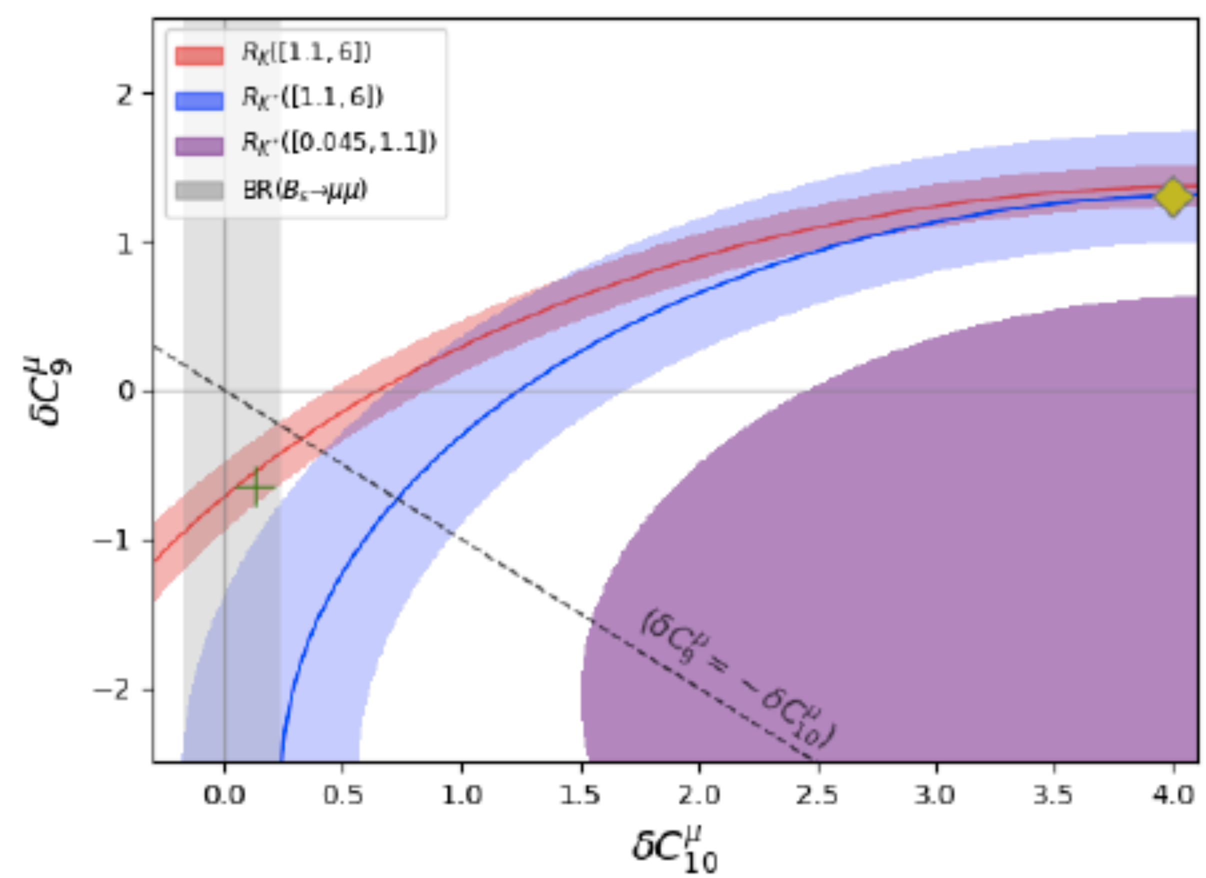
2021





Update 2022





Separate NP fits with a single operator

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Fit to clean observables $R_K, R_{K^*}, B_s \to \mu\mu$

Only R_K, R_{K^*} and $B_{s,d} \to \ell^+\ell^-$ **2021 data** ($\chi^2_{\rm SM} = 28.19$)			
	b.f. value	$\chi^2_{\rm min}$	${\rm Pull}_{\rm SM}$
δC_9	-1.00 ± 6.00	28.1	0.2σ
δC_9^e	0.80 ± 0.21	11.2	4.1σ
δC_9^μ	-0.77 ± 0.21	11.9	4.0σ
δC_{10}	0.43 ± 0.24	24.6	1.9σ
δC_{10}^e	-0.78 ± 0.20	9.5	4.3σ
δC_{10}^μ	0.64 ± 0.15	7.3	4.6σ
$\delta C_{\rm LL}^e$	0.41 ± 0.11	10.3	4.2σ
$\delta C_{\rm LL}^\mu$	-0.38 ± 0.09	7.1	4.6σ

Only R_K, R_{K^*} and $B_{s,d} \to \ell^+\ell^-$ **2022 data** ($\chi^2_{\rm SM} = 30.63$)			
	b.f. value	$\chi^2_{\rm min}$	${\rm Pull}_{\rm SM}$
δC_9	-2.00 ± 5.00	30.5	0.4σ
δC_9^e	0.83 ± 0.21	10.8	4.4σ
δC_9^μ	-0.80 ± 0.21	11.8	4.3σ
δC_{10}	0.03 ± 0.20	30.6	0.1σ
δC_{10}^e	-0.81 ± 0.19	8.7	4.7σ
δC_{10}^μ	0.50 ± 0.14	16.2	3.8σ
$\delta C_{\rm LL}^e$	0.43 ± 0.11	9.7	4.6σ
$\delta C_{\rm LL}^\mu$	-0.33 ± 0.08	12.4	4.3σ

Global fit to 108 $b \to s$ observable with 20 operators

Hurth,Mahmoudi,Martinez-Santos,Neshatpour: arXiv:2104.10058

(see also 1806.02791,1812.07602)

New approach see Isidori et al.: arXiv: 2104.05631

Considering only one or two Wilson coefficients may not give the full picture!

Issues:

LEE and method for eliminating insensitive parameters and flat directions (Use profile of likehoods and correlation matrix....)

A generic set of Wilson coefficients:

$$\text{complex } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

The available observables are mainly insensitive to the imaginary parts, one can limit the set to

$$\text{real } C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell + \text{primed coefficients}$$

corresponding to 20 degrees of freedom.

Some of the coefficients may have only weak effects on the observables, and affect the number of dof without affecting the χ^2, acting as *spurious* degrees of freedom.

Effective degrees of freedom (e-dof): degrees of freedom minus the parameters δC_i only weakly affecting the χ^2, defined such as

$$|\chi^2(\delta C_i = 1) - \chi^2(\delta C_i = 0)| < 1$$

Inclusive semi-leptonic penguins

- Results competitive with LHCb expected with $5ab^{-1}$

Observables	Belle $0.71\,\mathrm{ab}^{-1}$	Belle II $5\,\mathrm{ab}^{-1}$	Belle II $50\,\mathrm{ab}^{-1}$
R_K ($[1.0, 6.0]\,\mathrm{GeV}^2$)	28%	11%	3.6%
R_K ($> 14.4\,\mathrm{GeV}^2$)	30%	12%	3.6%
R_{K^*} ($[1.0, 6.0]\,\mathrm{GeV}^2$)	26%	10%	3.2%
R_{K^*} ($> 14.4\,\mathrm{GeV}^2$)	24%	9.2%	2.8%
R_{X_s} ($[1.0, 6.0]\,\mathrm{GeV}^2$)	32%	12%	4.0%
R_{X_s} ($> 14.4\,\mathrm{GeV}^2$)	28%	11%	3.4%

The Belle II Physics Book, Prog Theor Exp Phys (2019)

Complete angular analysis of inclusive $B \to X_s \ell\ell$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z \quad \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad\qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$

- Dependence on Wilson coefficients
  Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}\,C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}\,C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\rm em}\,\log(m_b^2/m_\ell^2)$$

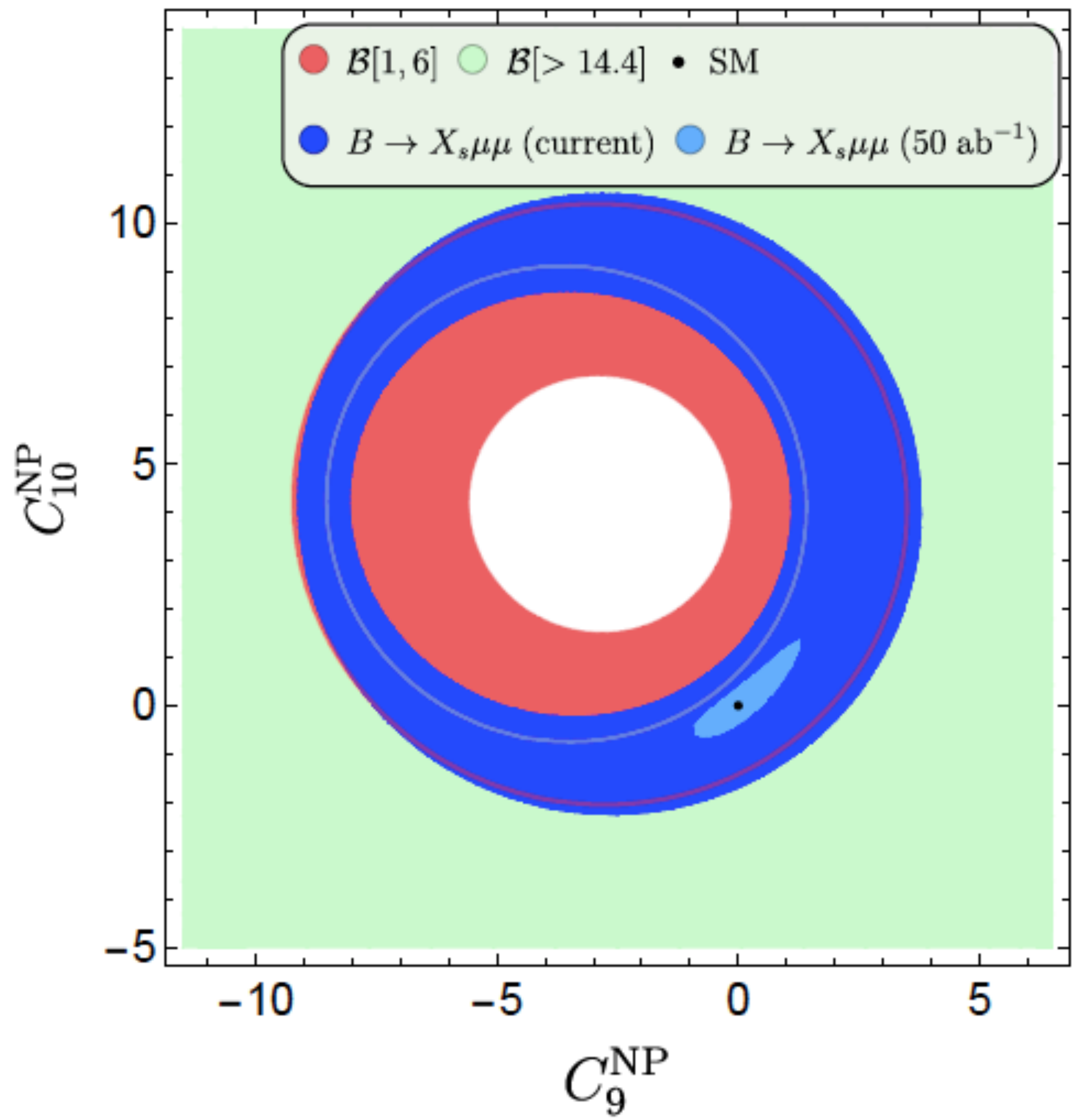
New physics sensitivity

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Constraints on Wilson coefficients C_9^{NP} and C_{10}^{NP}

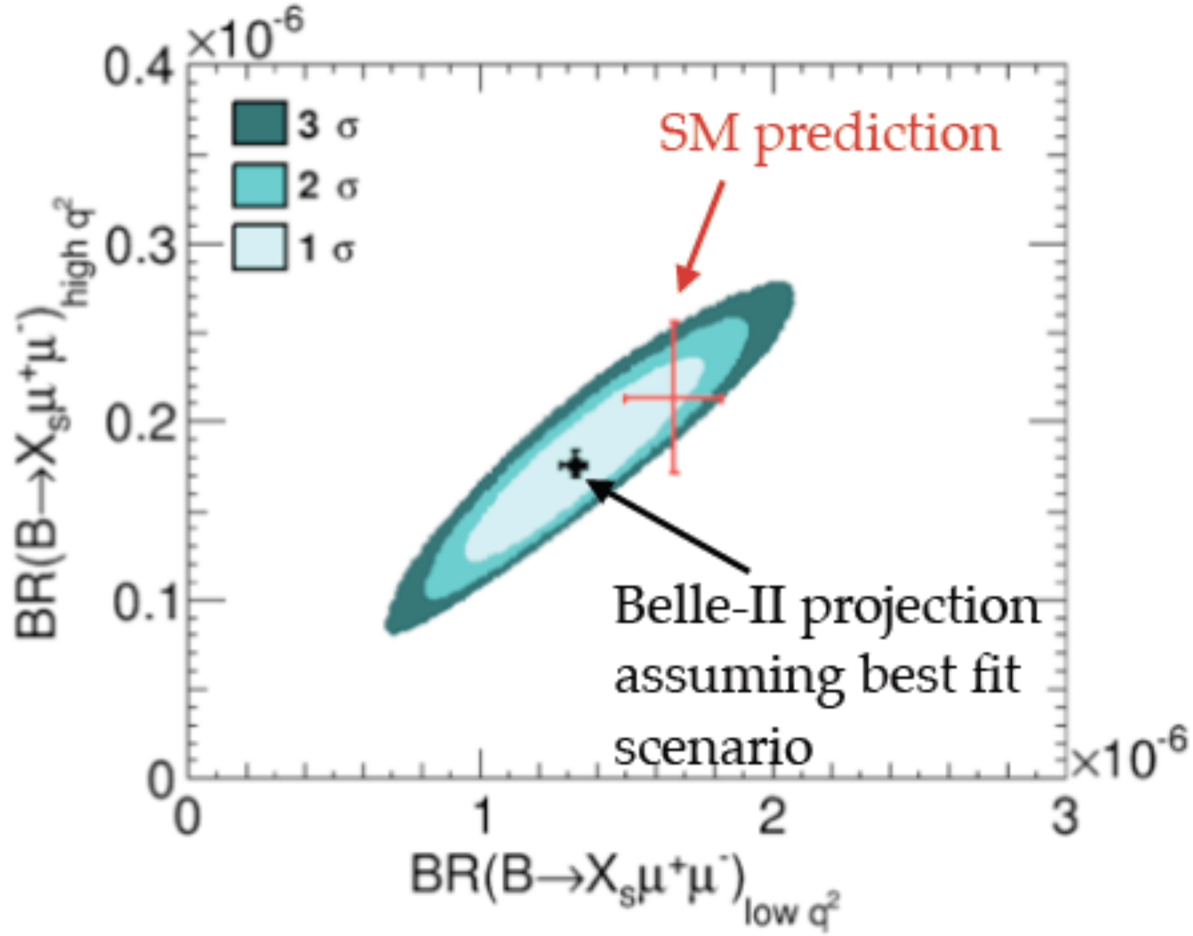
that we obtain at 95% C.L. from present experimental data
(red low q^2, green high q^2)

that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II
(light blue)



Crosscheck of LHCb anomalies with inclusive modes

Hurth,Mahmoudi,Neshatpour,arXiv:1410.4545

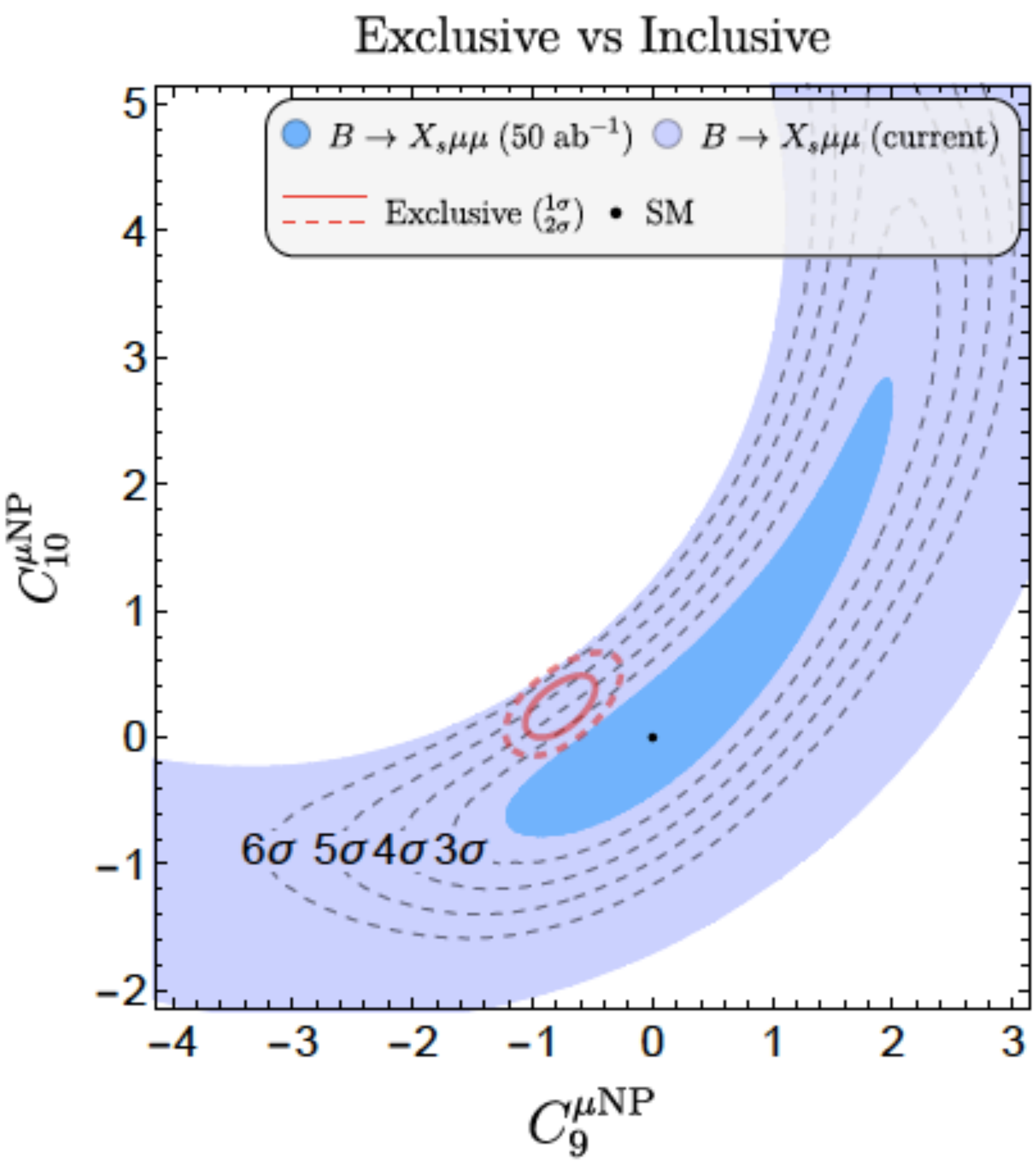


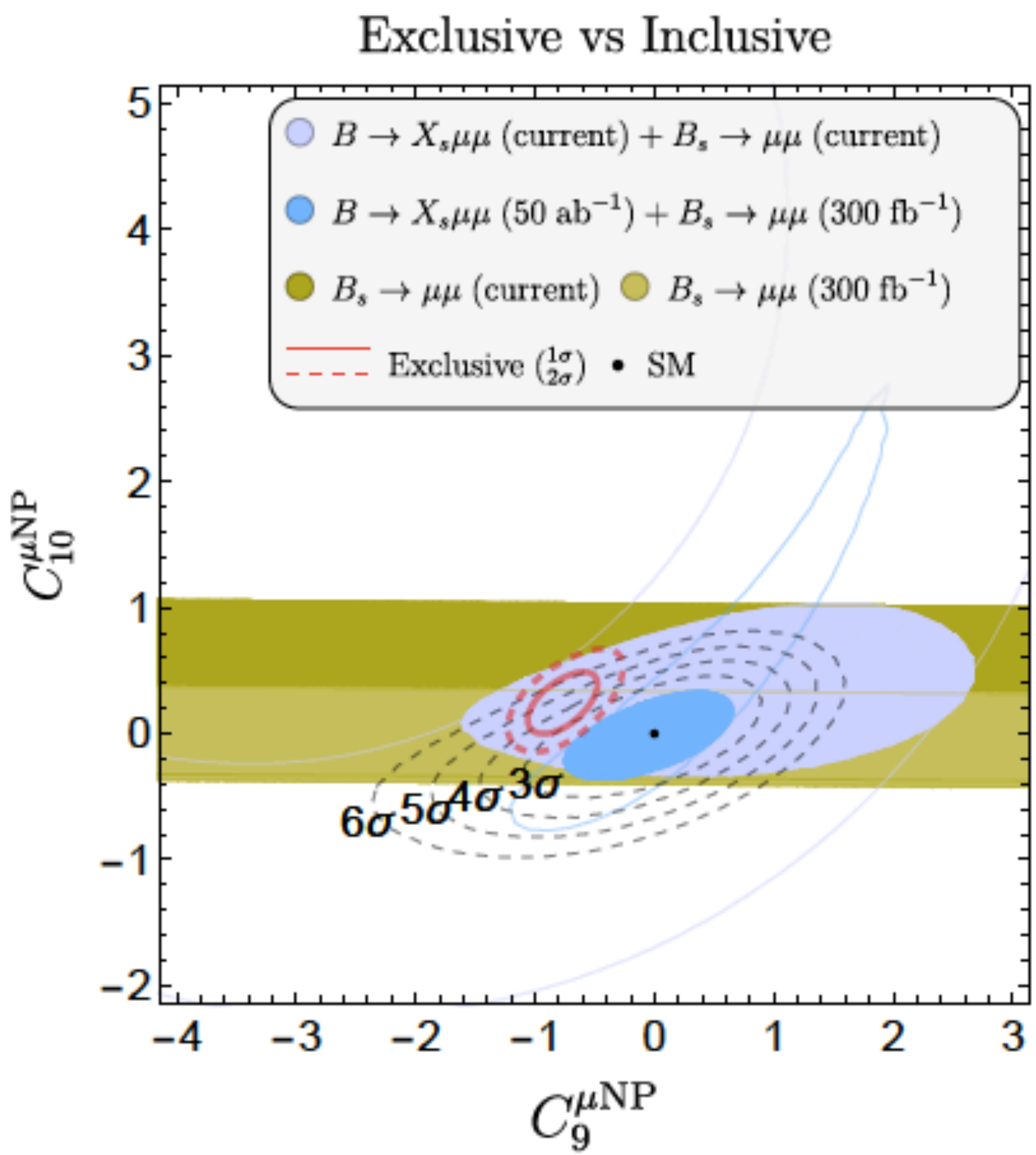
If NP then the effect of C_9 and C_9' are large enough to be checked at Belle-II with theoretically clean modes.

Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

Assuming Belle II measures SM values

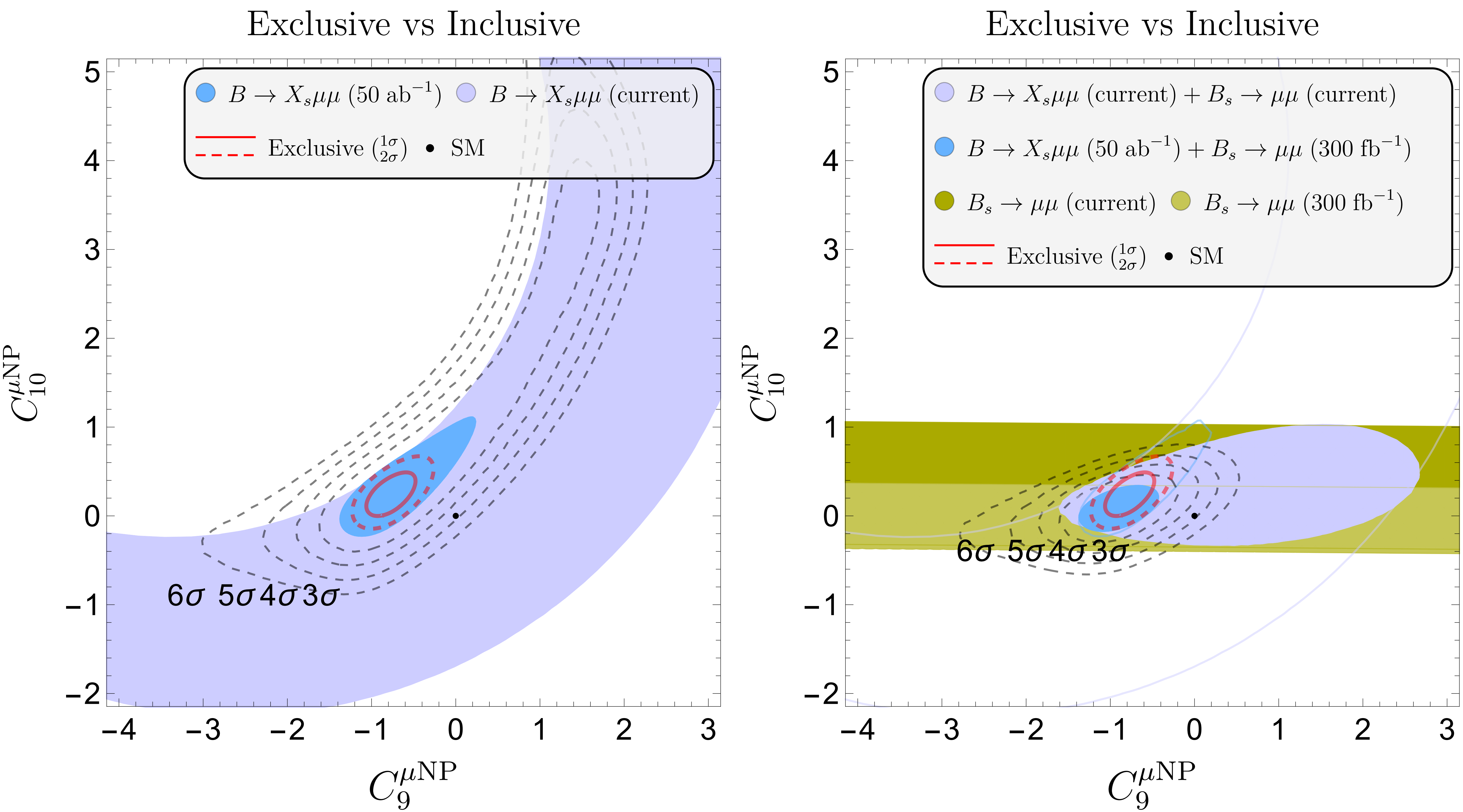
Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191





Assuming Belle II measures best fit point of exclusive fit

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191



Cuts in the dilepton and hadronic mass spectra

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
  1GeV$^2 < q^2 <$ 6GeV^2 and 14.4GeV$^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \to c\,(\to se^+\nu)e^-\bar{\nu} = b \to se^+e^- +$ missing energy
  * Babar,Belle: $m_X < 1.8 \,\text{or}\, 2.0 GeV$
  * high-q^2 region not affected by this cut
  * kinematics: X_s is jetlike and $m_X^2 \leq m_b \Lambda_{QCD}$ $\Rightarrow$ shape function region
  * SCET analysis: universality of jet and shape functions found:
    the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \to X_s\gamma$ shape function
    5% additional uncertainty for $2.0 GeV$ cut due to subleading shape functions

Lee,Stewart hep-ph/0511334
Lee,Ligeti,Stewart,Tackmann hep-ph/0512191
Lee,Tackmann arXiv:0812.0001 (effect of subleading shape functions)
Bell,Beneke,Huber,Li arXiv:1007.3758 (NNLO matching QCD $\to$ SCET)

Nonlocal subleading contributions

Benzke,Hurth,Turczyk,arXiv:1705.10366; Benzke,Hurth,arXiv:2006.00624

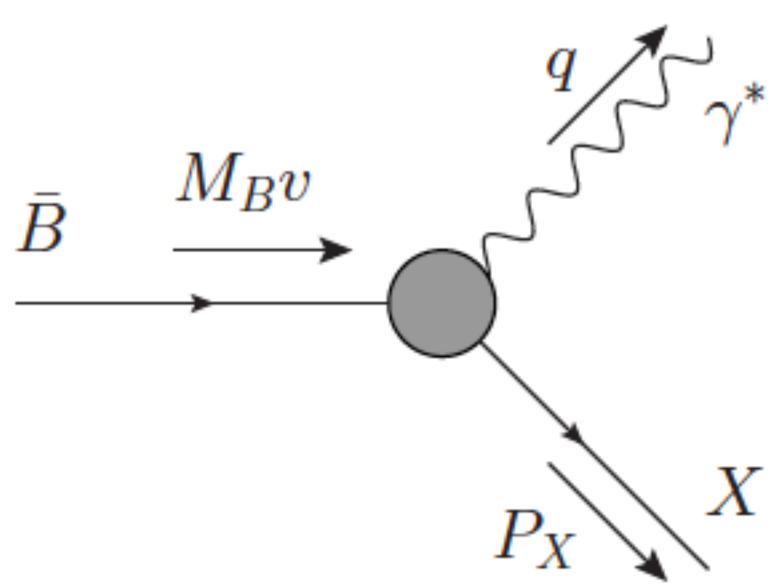
Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Hadronic cut

Additional cut in X_s necessary to reduce background
affects only low-q^2 region.

Hadronic invariant $m_X^2 < 1.8(2.0) GeV^2$

Multiscale problem $\to$ SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD} m_b \gg \Lambda_{\rm QCD}^2$$

Scaling $\lambda = \Lambda_{\rm QCD}/m_b$

Kinematics

B meson rest frame

$$q = p_B - p_X \qquad 2\, m_B\, E_X = m_B^2 + M_X^2 - q^2$$

X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

two light-cone components $\qquad p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$
$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\rm QCD})$$

$$q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n} q = m_B - p_X^-$$

Kinematics

B meson rest frame

$$q = p_B - p_X \qquad 2\, m_B\, E_X = m_B^2 + M_X^2 - q^2$$

X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

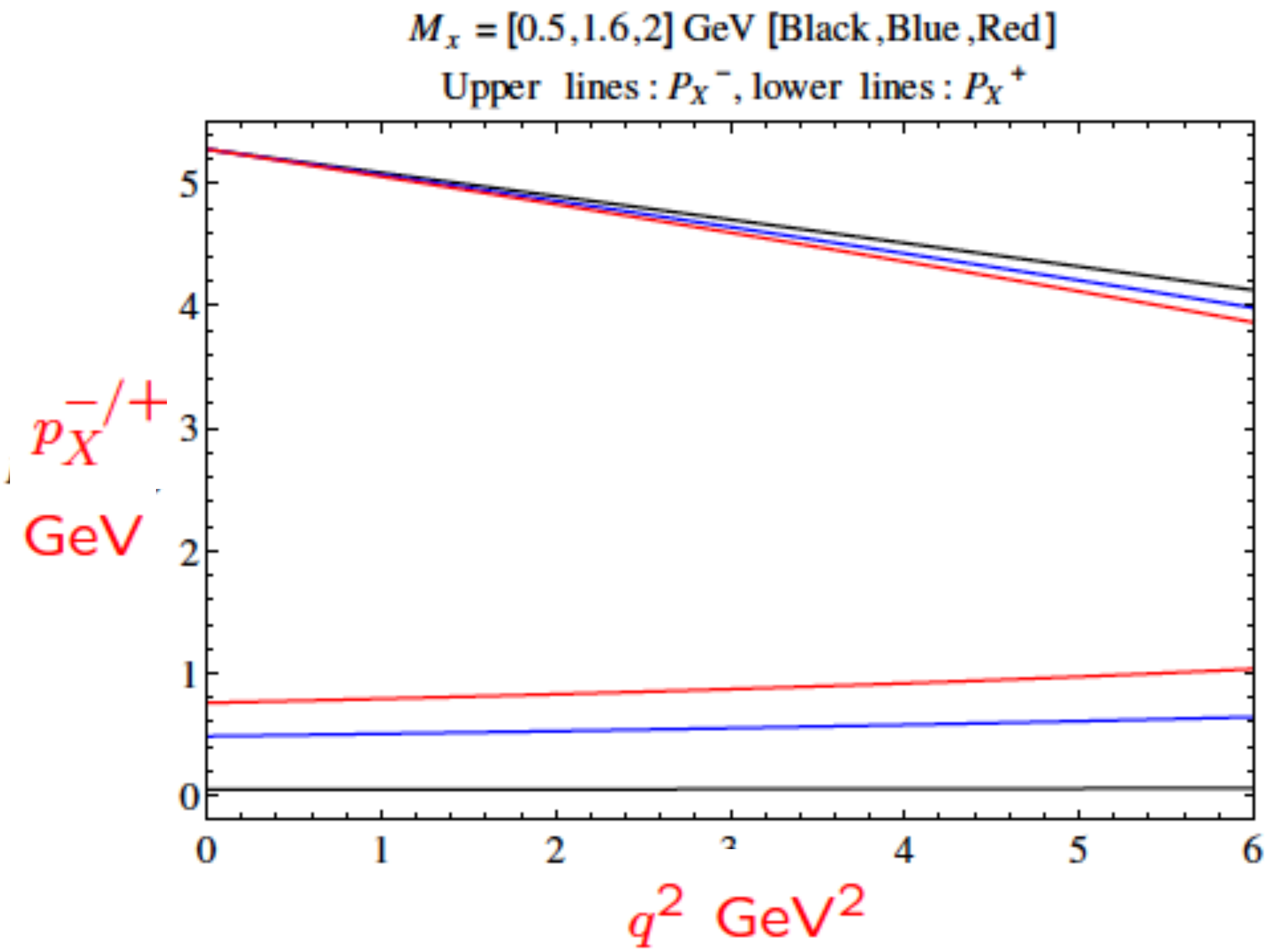
two light-cone components $\qquad p_X^- p_X^+ = m_X^2$

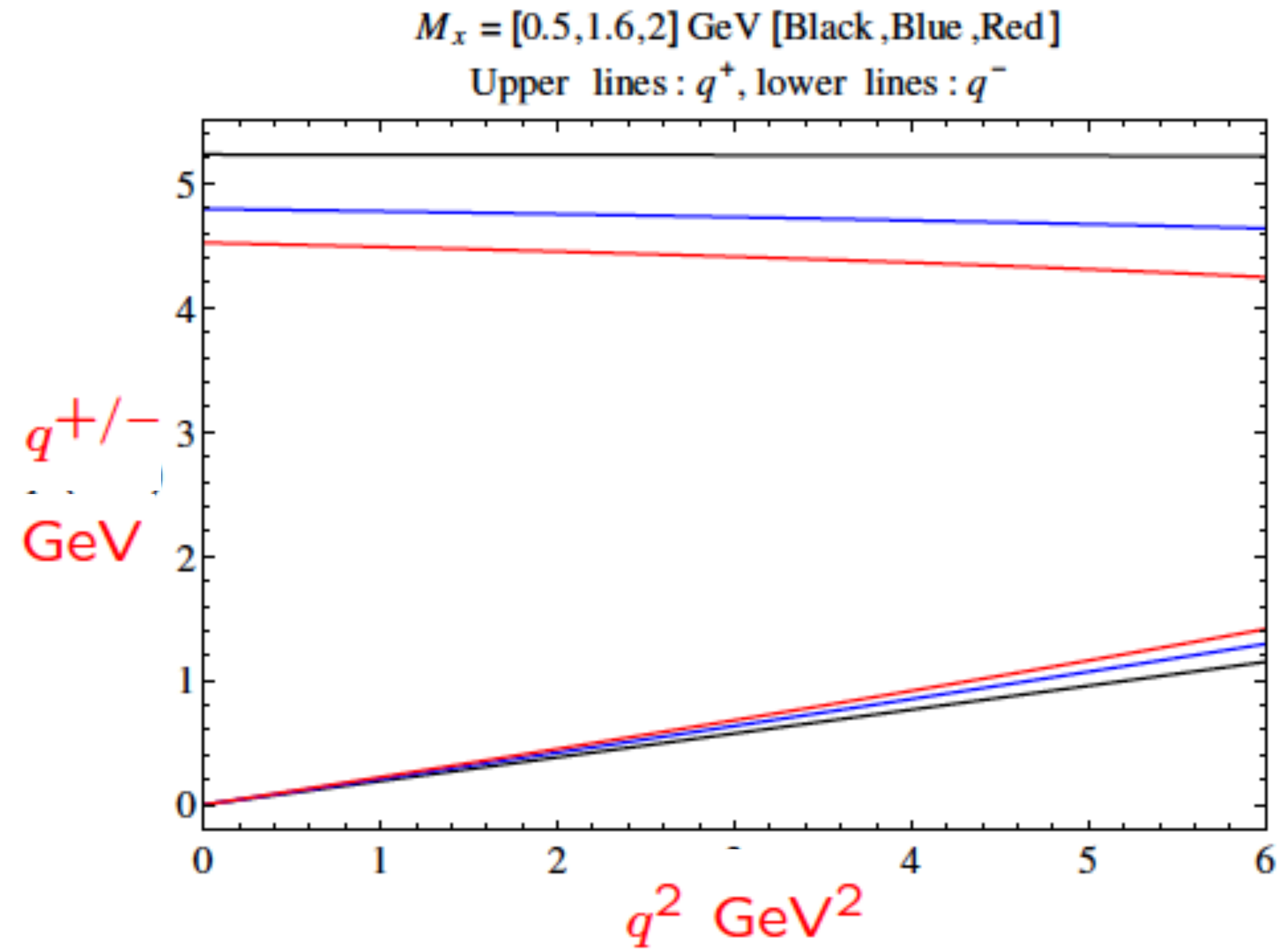
$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$
$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

$$q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n}q = m_B - p_X^-$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

Scaling $\lambda = \Lambda_{\rm QCD}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n\cdot q \sim \lambda$





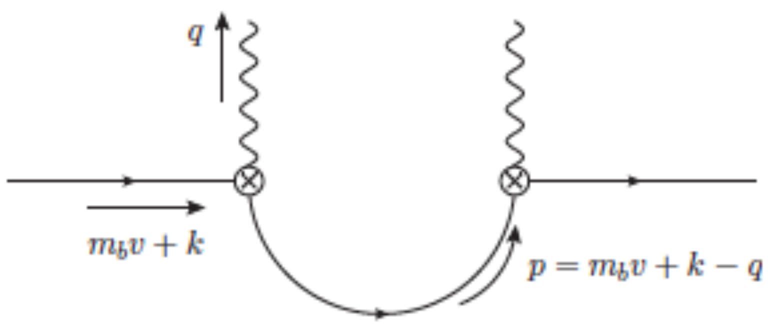
For $q^2 < 6GeV^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ, means q anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.

This problematic assumption implies a different matching of SCET/QCD.

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



$$\frac{1}{(m_b v+k-q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \dots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

Factorization theorem $d\Gamma \sim H \cdot J \otimes S$

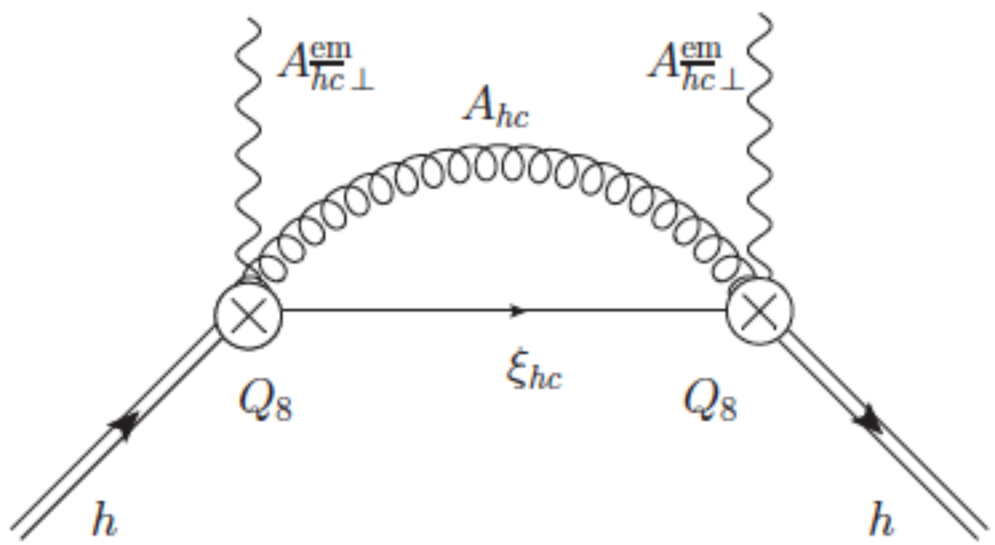
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

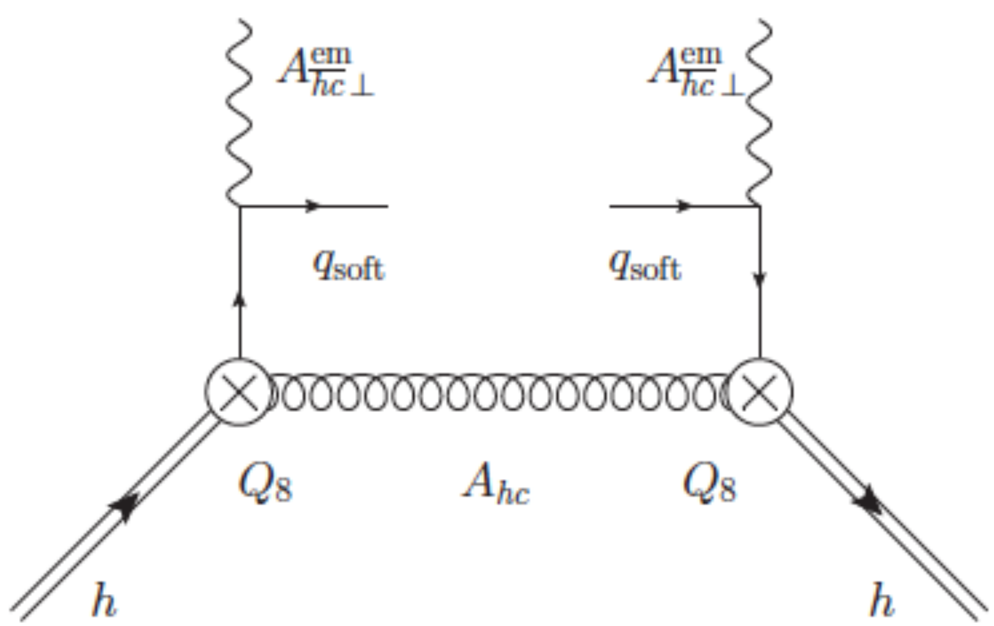
Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

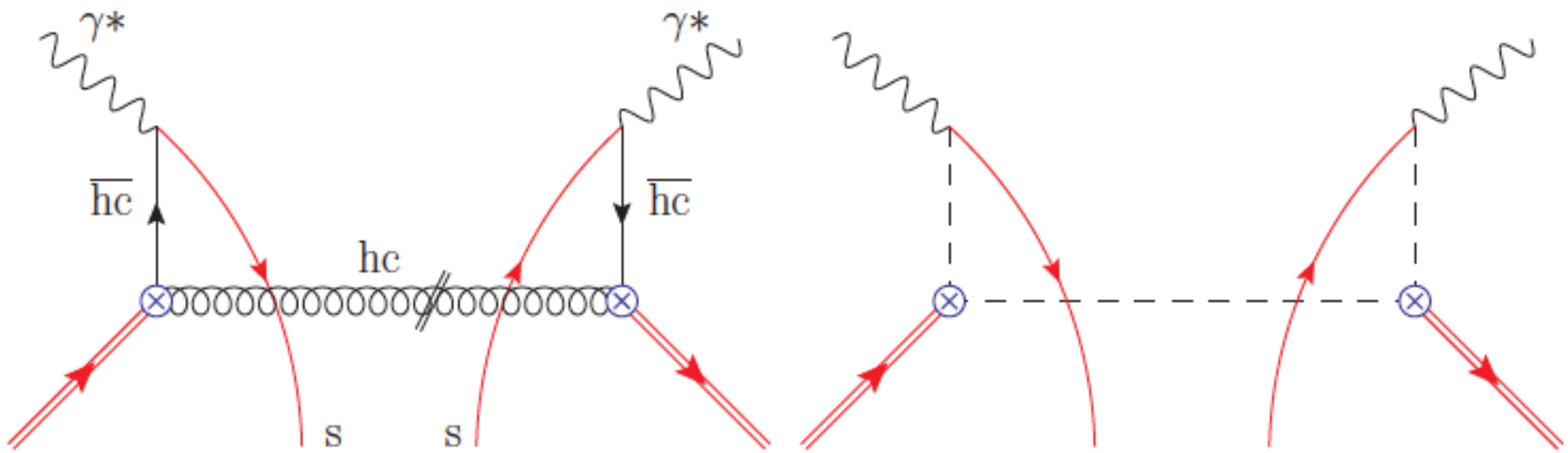
$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_8 and Q_8



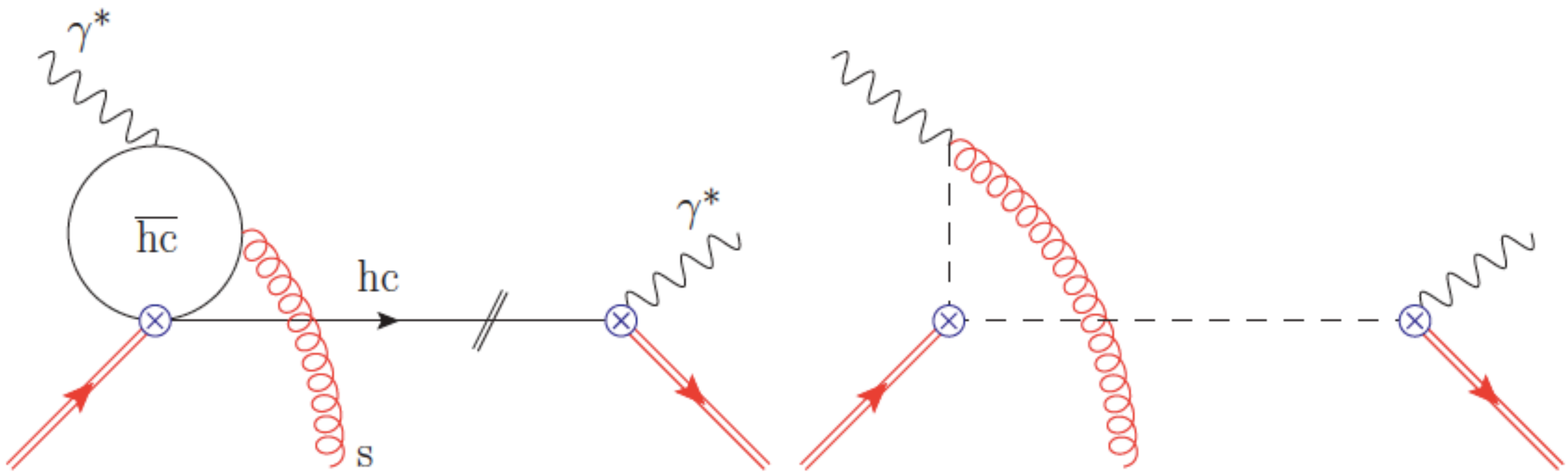
$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\,\delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int\frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon}\, g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

- Convolution of jet function and shape function

- No resolved contribution if the photon is assumed to be hard !

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}$$

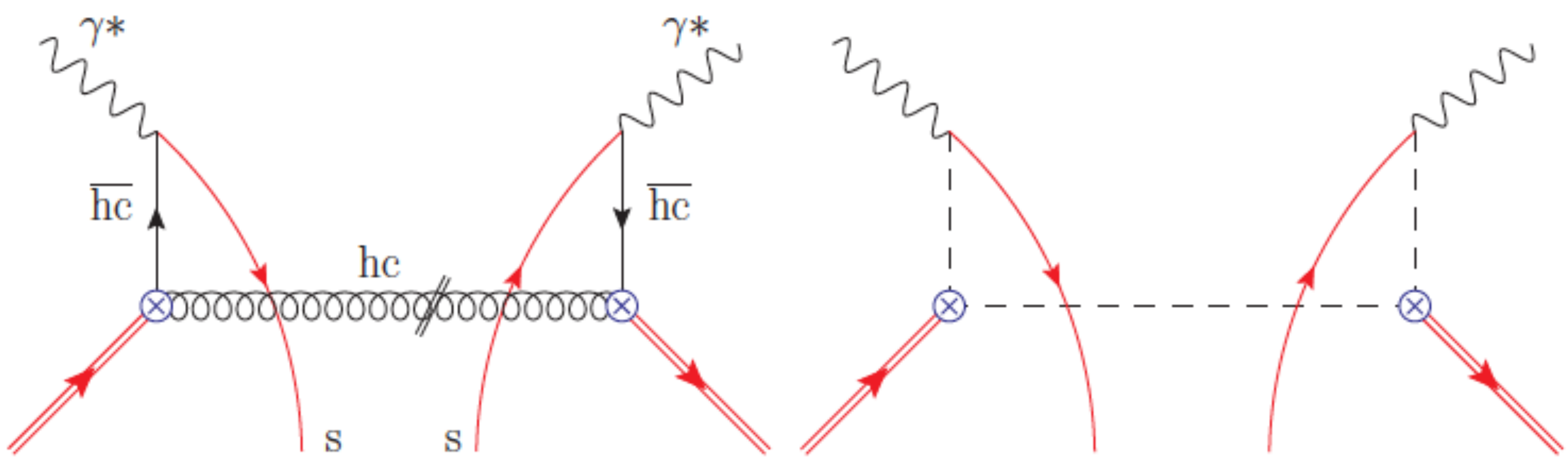
$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - 1\right) - (\bar{n}\cdot q + \omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q + \omega_1)}\right) - 1\right)\right.$$

$$\left. + \bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q + \omega_1)}\right)\right)\right] g_{17}(\omega, \omega_1)$$

$$g_{17}(\omega,\omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B}|\bar{h}(\mathbf{tn}) \ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}}) \ldots h(0)|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\text{res}}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\,\delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int\frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon}g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2)=\frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\text{F.T.}}$$

- Subtlety in the Q_8-Q_8 contribution: convolution integral is UV divergent
  - This implies that there is no complete proof of the factorization formula yet.
  - Nevertheless one shows that scale dependence of direct and resolved contribution cancel.
  - Refactorization methods allow to resolve the problem and reestablish factorization formula.

Numerical evaluation

- Subleading shape functions of resolved contributions similar to $b \to s\gamma$
- Use explicit defintion to determine properties:
  * PT invariance: soft functions are real
  * Moments of g_{17} related to HQET parameters
  * Vacuum insertion approximation relates g_{78} to the B meson LCDA
- Perform convolution integrals with model functions

Numerical evaluation $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$

$$\mathcal{F}_{17}^q = \frac{1}{m_b} \frac{C_1(\mu) C_{7\gamma}(\mu)}{C_{\rm OPE}} e_c \,\mathrm{Re}\left[\frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2}\right] \int_{-\infty}^{+\infty} d\omega_1 \, J_{17}(q^2_{\rm min}, q^2_{\rm max}, \omega_1) \, h_{17}(\omega_1, \mu)$$

$$J_{17}(q^2_{\rm min}, q^2_{\rm max}, \omega_1) = \mathrm{Re} \frac{1}{\omega_1 + \imath\epsilon} \int_{\frac{q^2_{\rm min}}{M_B}}^{\frac{q^2_{\rm max}}{M_B}} \frac{d\overline{n}\cdot q}{\overline{n}\cdot q} \frac{1}{\omega_1}$$

$$h(\omega_1, \mu) := \int_{-\infty}^{\bar\Lambda} d\omega \, g_{17}(\omega, \omega_1, \mu)$$

$$\left[(\overline{n}\cdot q + \omega_1)\left(1 - F\left(\frac{m_c^2}{m_b(\overline{n}\cdot q + \omega_1)}\right)\right) - \overline{n}\cdot q \left(1 - F\left(\frac{m_c^2}{m_b \overline{n}\cdot q}\right)\right) - \overline{n}\cdot q \left(G\left(\frac{m_c^2}{m_b(\overline{n}\cdot q + \omega_1)}\right) - G\left(\frac{m_c^2}{m_b \overline{n}\cdot q}\right)\right)\right].$$

- One derives normalization of soft function: $\int_{-\infty}^{\infty} d\omega_1 h_{17}(\omega_1, \mu) = 2\,\lambda_2$
- h_{17} should not have any significant structure (maxima or zeros) outside the hadronic range
- Values of h_{17} should be within the hadronic range

Additional constraints on shape function h_{17}

Paz et al. arXiv:1908.02812

Systematic analysis with the Hermite polynomials:

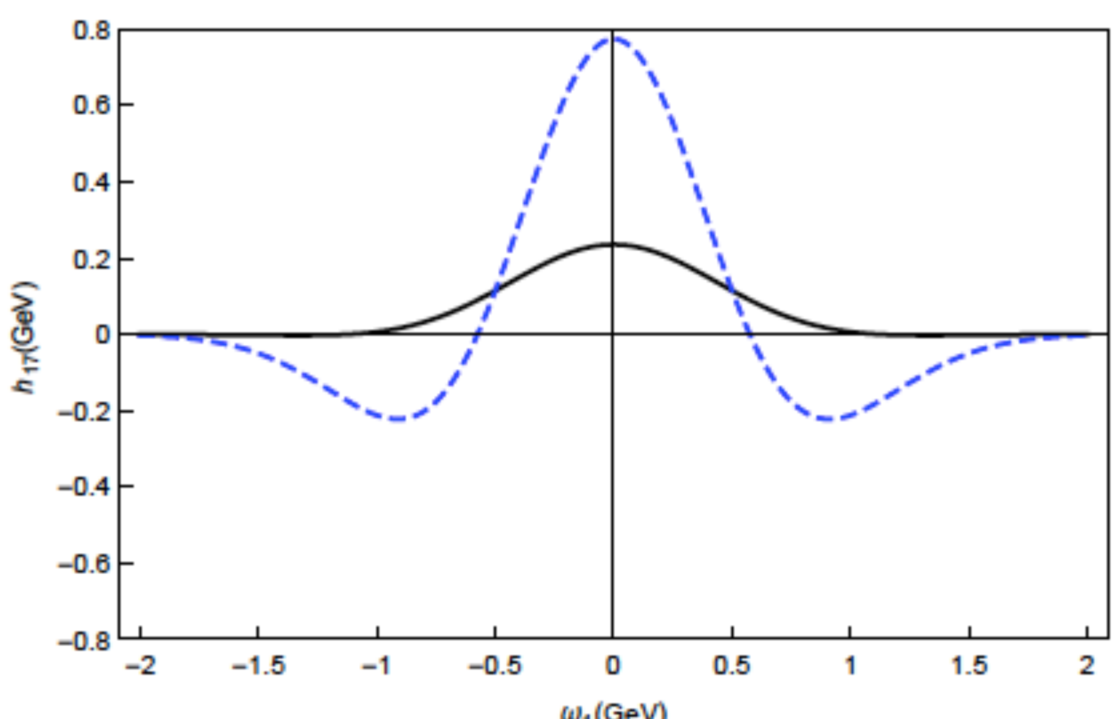
$$h_{17}(\omega_1, \mu) = \sum_n a_{2n} H_{2n} \left(\frac{\omega_1}{\sqrt{2}\sigma} \right) e^{-\frac{\omega_1^2}{2\sigma^2}}$$

Further constraints from higher moments of soft function:

$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^0 \, h_{17}(\omega_1, \mu) = 0.237 \pm 0.040 \, \mathrm{GeV}^2$$

New input:

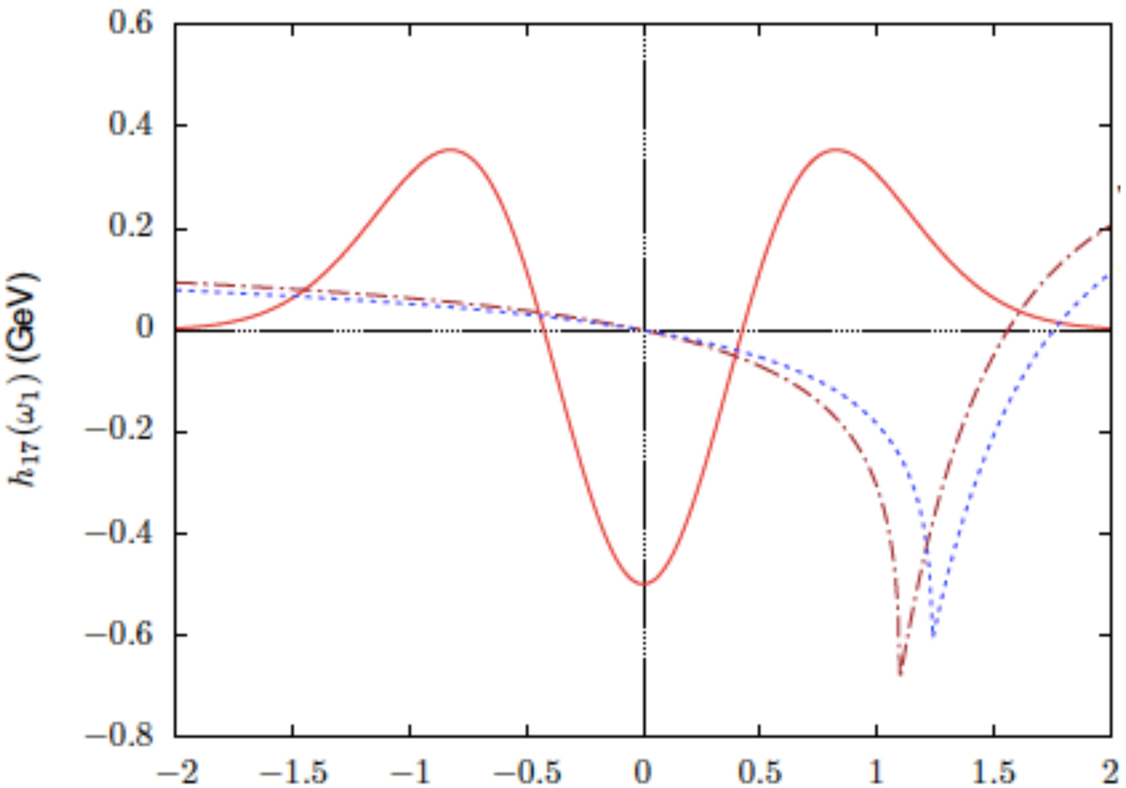
$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^2 \, h_{17}(\omega_1, \mu) = 0.15 \pm 0.12 \, \mathrm{GeV}^4$$

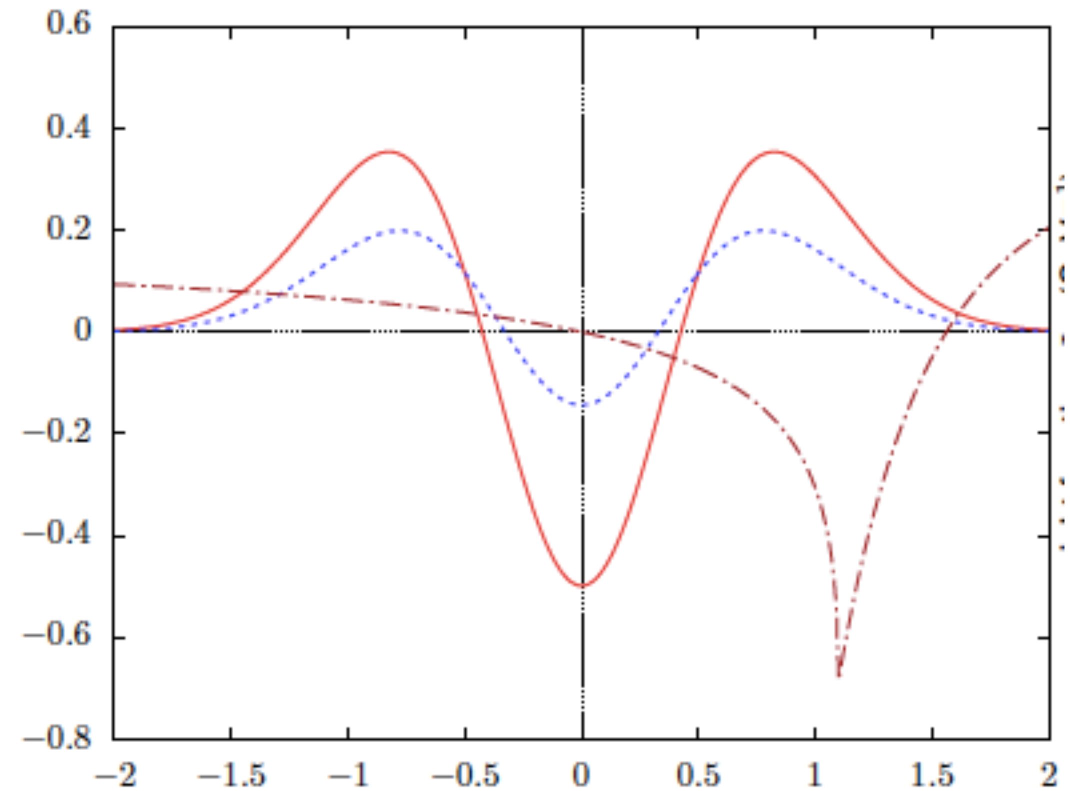


Updated result for $\bar{B} \to X_s\gamma$

Benzke,Hurth,arXiv:2006.00624

Charm dependence of jet function: Constraint on shape function:





Benzke,Hurth,arXiv:2006.00624

Neubert et al., arXiv: 1003.5012

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\ 4.7\%]$$

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-1.9\%,\ 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-3.7\%,\ 6.5\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-5.2\%,\ 6.5\%]$$

(In addition: large scale dependence)

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \to X_s\gamma$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO.

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

- Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%]$$

  Reason for significantly smaller error is twofold:

  - For charm dependence only the parametric uncertainty was used

$$1.17\,\mathrm{GeV} \leq m_c \leq 1.23\,\mathrm{GeV}$$

    We use scale variation of the hard-collinear scale and get

$$\mu_{\mathrm{hc}} \sim \sqrt{m_b\,\Lambda_{\mathrm{QCD}}} \ \text{ from } \ 1.3\,\mathrm{GeV} \text{ to } 1.7\,\mathrm{GeV} \qquad 1.14\,\mathrm{GeV} \leq m_c \leq 1.26\,\mathrm{GeV}$$

  - Numerically large $1/m_b^2$ term due to kinematic factors was dropped compared to the original analysis in Neubert et al., arXiv: 1003.5012
    Other $1/m_b^2$ terms due to operator interferences are shown to be small.

Underestimation of the uncertainty due to the resolved contribution.

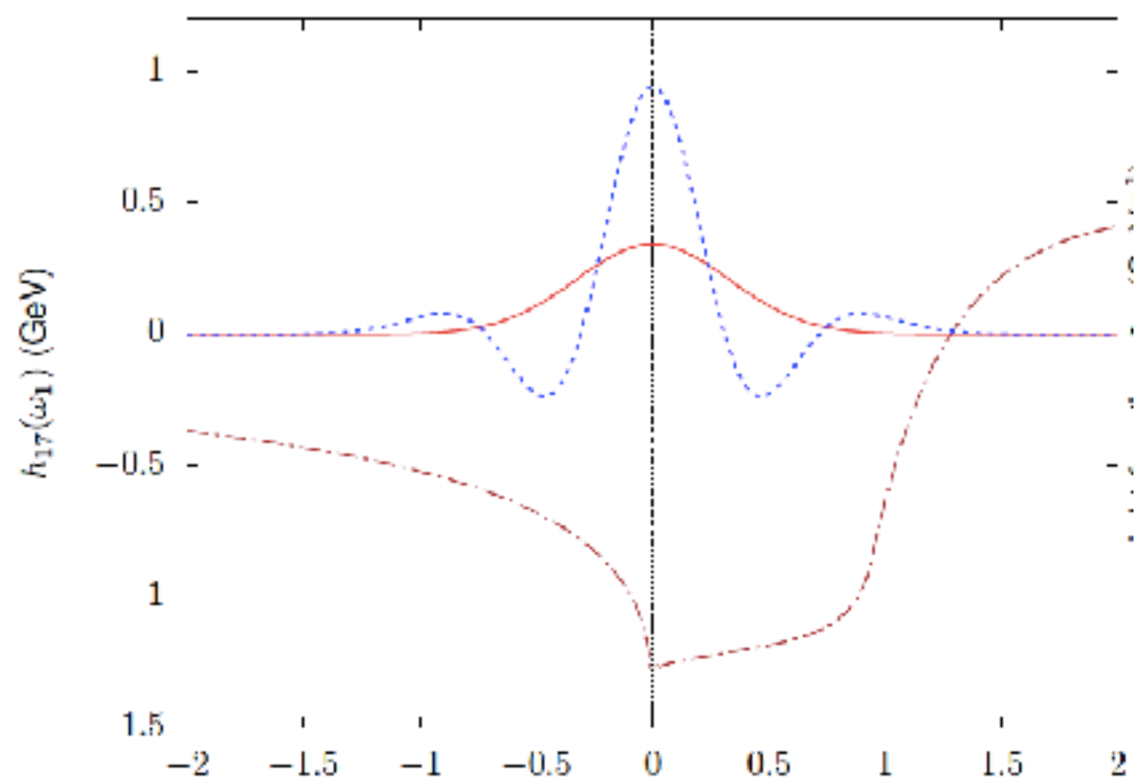
But used in recent $b \to s\gamma$ analysis. Misiak, Rehman, Steinhauser, arXiv:2002.01548v2

Updated result for $\bar{B} \to X_s \ell\ell$

Benzke,Hurth,arXiv:2006.00624

Rather symmetric jet function $\to$

Various shape functions lead to very similar values of the convolution



arXiv:2006.00624

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\ell\ell} \in [+0.2\%,\ +2.6\%]$$

$$\mathcal{F}^{1/m_b}_{\mathrm{b}\to\mathrm{s}\ell\ell} \in [0.2\%,\ 3.2\%]$$

arXiv:1705.10366

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\ell\ell}|_{1/m_b} \in\ [-0.5\%,\ +3.4\%]$$

$$\mathcal{F}_{1/m_b} \in [-0.7, +3.8]$$

We find large scale dependence of the results in both penguins

$\Rightarrow$ α_s corrections desirable

Numerical relevant contributions to $O(1/m_b^2)$

Benzke,Hurth,work in progress

$\mathcal{F}_{1/m_b^2}$

$$\mathcal{F}_{19}: \quad O(1/m_b^2) \text{ but } |C_{9/10}| \sim 13|C_{7\gamma}|$$

- Interference of Q_1 and Q_9: Subleading power correction to BR

Indications that additional suppression in all terms are within the jet function!
$\rightarrow$ Q_1 and Q_7 and Q_1 and Q_9 terms could have the same shape function

- Interference of Q_1 and Q_{10}: First contribution to A_{FB}

Resolved contributions to inclusive penguin modes

- For q anti-hard-collinear we have a new type of subleading power corrections.
- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.
- They constitute an irreducible uncertainty because they survive the $M_X \to 1$ limit.
- If q was hard then these resolved contributions would not exist
- Information on higher moments of the shape functions will reduce the error in the future

Nonlocal power corrections of $O(1/m_b^2)$ numerically relevant

Calculation of α_s corrections desirable due to large scale dependence

M_X cut effects in the low-q^2 region with q^2 anti-hard-collinear

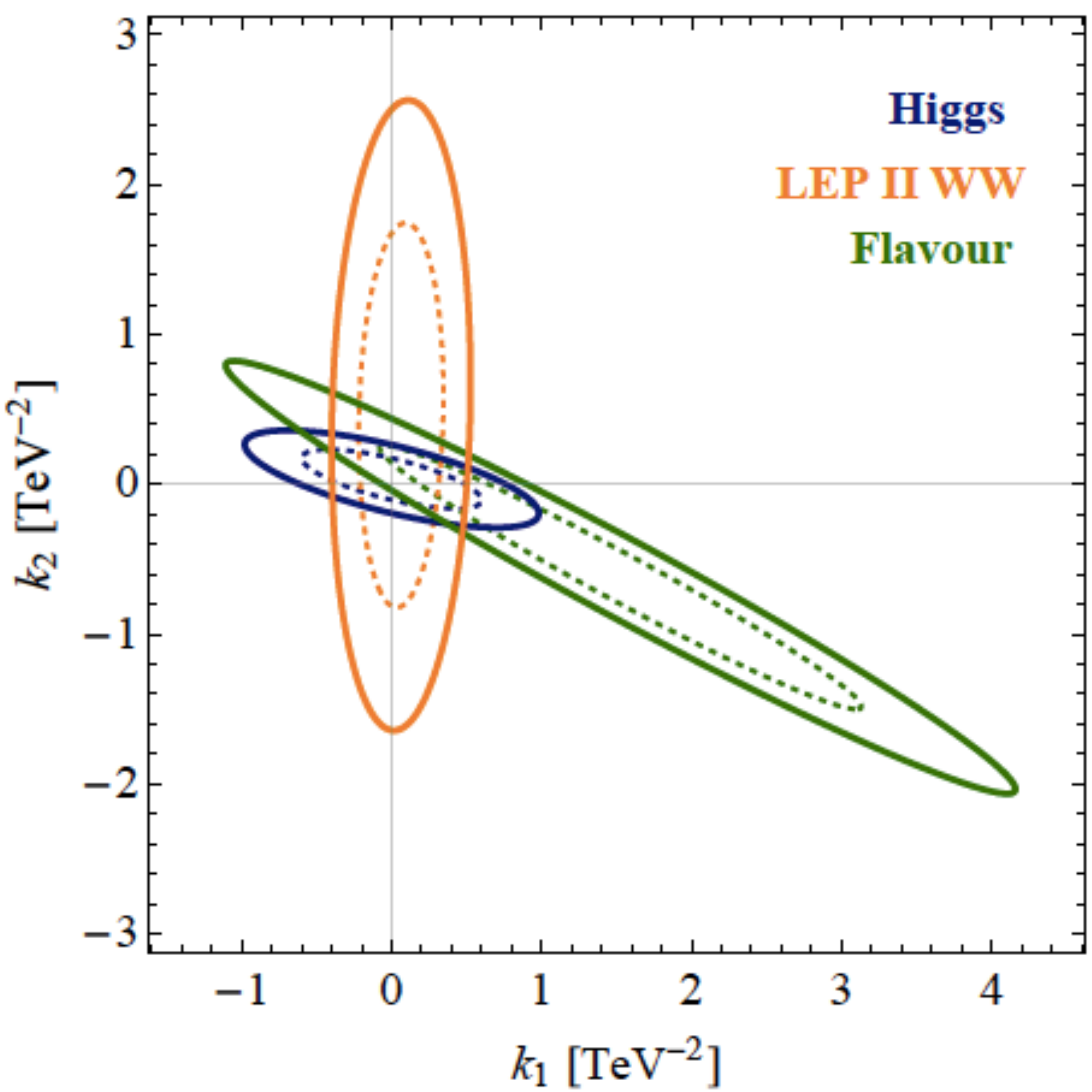
Work in progress

Epilogue

Flavour matters

Aoude, Hurth, Renner, Shepherd arXiv:1903.00500 and arXiv2003.5432

Role of flavour data in global SMEFT fits using the leading term in spurionic Yukawa expansion at the new physics scale as initial conditions (there are no FCNC at the tree level at the NP scale) "*leading MFV*"



Example: 2 flat directions when fitting Z-pole data

Flavour data is competitive with existing constraints.

- The days of "guaranteed" discoveries or no-lose theorems in particle physics are over, at least for the time being .....
- but the big questions of our field remain open (hierarchy problem. flavour, neutrinos, dark matter, baryogenesis,...)
- This simply implies that, more than for the past 30 years, future HEP's progress is to be driven by experimental exploration, possibly renouncing/reviewing deeply rooted theoretical bias.

Experimental flavour opportunities

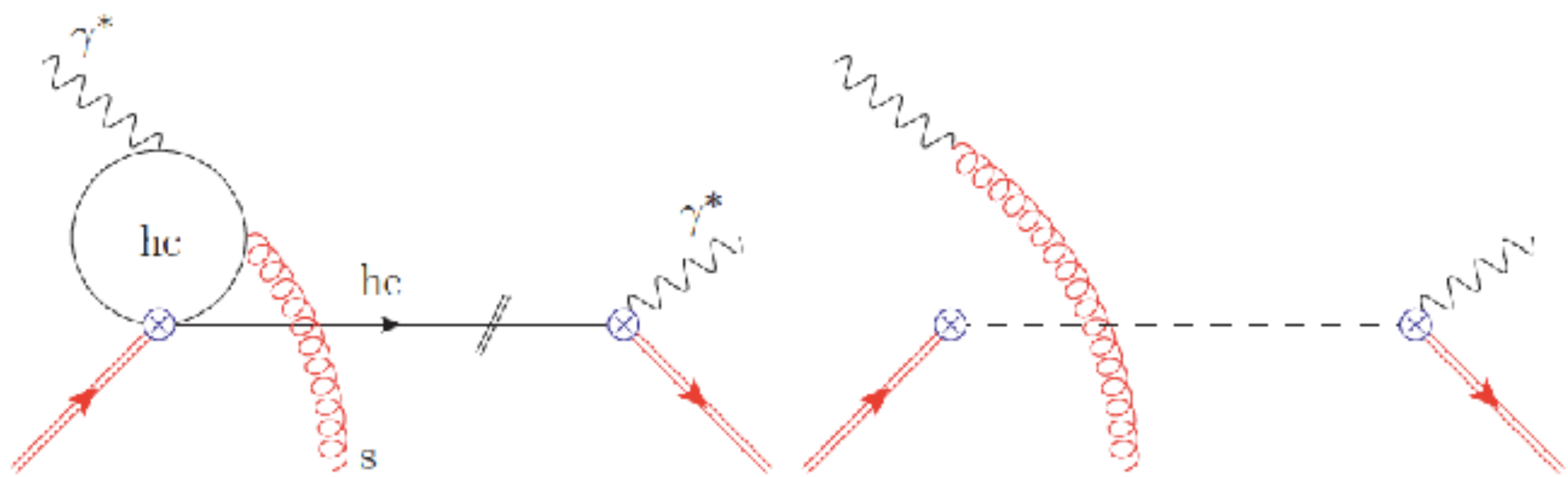
- LHCb: allows for wide range of analyses, highlights: B_s mixing phase, angle γ, $B \to K^* \mu\mu$, $B_s \to \mu\mu$, $B_s \to \phi\phi$ then upgrades to 50 and 300fb^{-1}

- Dedicated kaon experiments J-PARC E14 and CERN P-326/NA62: rare kaon decays $K_L^0 \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$

- Super-B factory Belle-II at KEK (50ab^{-1})
  Belle-II is a Super Flavour factory: besides precise B measurements CP violation in charm, lepton flavour violating modes $\tau \to \mu\gamma$, ...

Extra

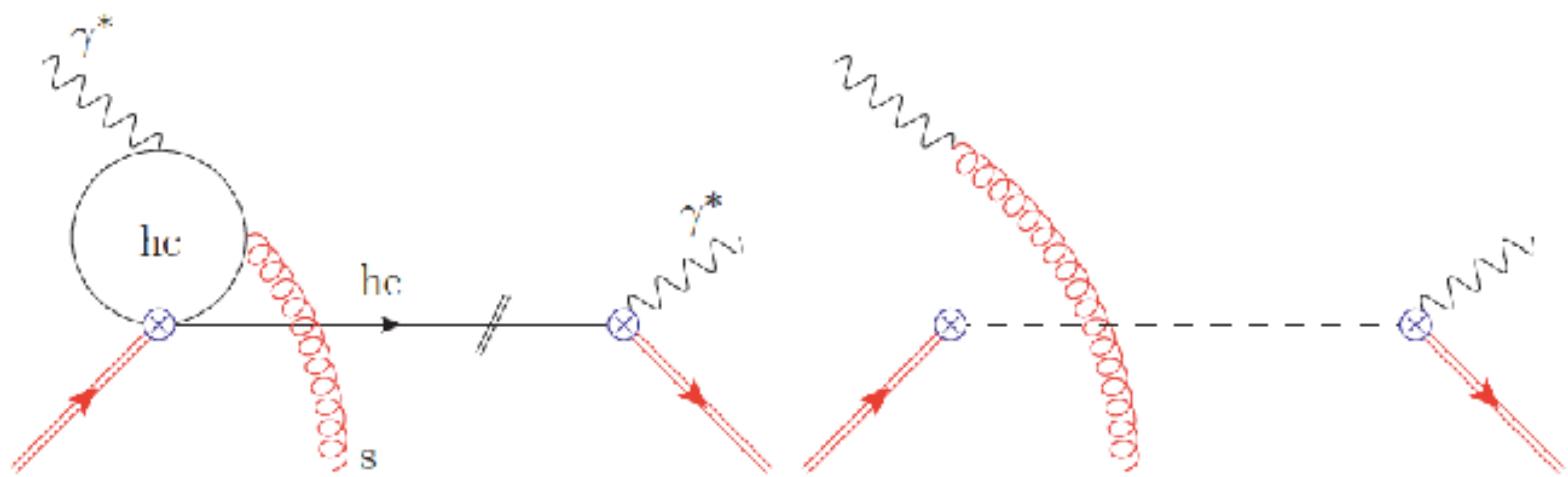
Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat{\Gamma}_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q d\bar{n}\cdot q\,\frac{(n\cdot q)^3}{\bar{n}\cdot q}$$
$$\times\mathrm{Re}\int d\omega\;\delta(\omega+m_b-n\cdot q)\int d\omega_1\frac{1}{\omega_1+i\epsilon}$$
$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$
$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right]g_{17}(\omega,\omega_1,\mu)\,,$$

$$g_{17}(\omega,\omega_1,\mu)=\int\frac{dr}{2\pi}\,e^{-i\omega_1 r}\int\frac{dt}{2\pi}\,e^{-i\omega t}$$
$$\times\frac{\langle\bar{B}|\left(\bar{h}S_n\right)(tn)\,\bar{\not n}(1+\gamma_5)\left(S_n^\dagger S_{\bar{n}}\right)(0)\,i\gamma_\alpha^\perp\bar{n}_\beta\left(S_{\bar{n}}^\dagger\,gG_s^{\alpha\beta}S_{\bar{n}}\right)(r\bar{n})\left(S_{\bar{n}}^\dagger h\right)(0)|\bar{B}\rangle}{2M_B}$$

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat{\Gamma}_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q d\bar{n}\cdot q\,\frac{(n\cdot q)^3}{\bar{n}\cdot q}$$
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$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right]\,g_{17}(\omega,\omega_1,\mu)\,,$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c \to m_u = 0$

$$\times\frac{1}{\omega_1}\,[\;\omega_1\;]\;g_{17}(\omega,\omega_1,\mu)$$

$$g_{17}(\omega,\omega_1,\mu) = \int\frac{dr}{2\pi}\,e^{-i\omega_1 r}\int\frac{dt}{2\pi}\,e^{-i\omega t}$$
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Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Trace formalism of HQET:** $\int_{-\infty}^{\bar\Lambda} d\omega\, g_{17}(\omega,\omega_1,\mu) = \int_{-\infty}^{\bar\Lambda} d\omega\, (g_{17}(\omega,-\omega_1,\mu))^*$

$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat\Gamma_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\, dn\cdot q d\bar n\cdot q\,\frac{(n\cdot q)^3}{\bar n\cdot q}$$
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- **Limit** $m_c \to m_u = 0$

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- **PT invariance:** g_{17} is real

$$
\begin{aligned}
d\Gamma_{17} &= \frac{1}{m_b}\,\mathrm{Re}\left[\hat\Gamma_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q d\bar n\cdot q\,\frac{(n\cdot q)^3}{\bar n\cdot q} \\
&\times \mathrm{Re}\int d\omega\ \delta(\omega + m_b - n\cdot q)\int d\omega_1 \frac{1}{\omega_1 + i\epsilon} \\
&\times \frac{1}{\omega_1}\left[(\overline{n}\cdot q + \omega_1)\left(1 - F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q + \omega_1)}\right)\right) - \overline{n}\cdot q\left(1 - F\left(\frac{m_c^2}{n\cdot q \overline{n}\cdot q}\right)\right)\right. \\
&\left. - \overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q + \omega_1)}\right) - G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right] g_{17}(\omega, \omega_1, \mu)\,,
\end{aligned}
$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c \to m_u = 0$

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$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right]g_{17}(\omega,\omega_1,\mu)\,,$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c\to m_u=0$

$$\times\frac{1}{\omega_1}\,[\;\omega_1\;]\;g_{17}(\omega,\omega_1,\mu)$$

- **Integration of ω_1:**

Interference term $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$ vanishes within the integrated rate

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

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- **Integration of** ω_1:

Interference term $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$ vanishes within the integrated rate

Crucial result for all CP averaged inclusive $b\to d\ell^+\ell^-$ quantities

(previously no estimate for this up-quark loop of order Λ/m_b was available)

Minimal flavour violation hypoyhesis

- SM gauge interactions are universal in quark flavour space:

  flavour symmetry $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

- Symmetry is only broken by the Yukawa couplings Y_U and Y_D responsible for the quark masses

- Any new physics model in which all flavour- and CP-violating interactions can be linked to the known Yukawa couplings is MFV

  d'Ambrosio,Giudice,Isidori,Strumia,heo-ph/0207036

- MFV implies model-independent relations between FCNC processes

  - usual CKM relations between $[b \to s] \leftrightarrow [b \to d] \leftrightarrow [s \to d]$ transitions:
    -we need high-precision $b \to s$, but also $s \to d$ measurements
    -$\mathcal{B}(\bar{B} \to X_d\gamma) \leftrightarrow \mathcal{B}(\bar{B} \to X_s\gamma)$, $\mathcal{B}(\bar{B} \to X_s\nu\bar{\nu}) \leftrightarrow \mathcal{B}(K \to \pi^+\nu\bar{\nu})$

  - CKM phase only source of CP violation:
    -phase measurements in $B \to \phi K_s$ or $\Delta M_{B_{(s/d)}}$ are not sensitive to new physics

- The usefulness of MFV-bounds/relations is obvious; any measurement beyond those bounds indicate the existence of new flavour structures

  Hurth,Isidori,Kamenik,Mescia,arXiv:0807.5039
  Hurth,Mahmoudi,arXiv1207.0688

MFV hypothesis is far from beeing verified