Neutron Electric Dipole Moment from QCD?

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY

With

Yoshifumi Nakamura

RIKEN Center for Computational Science, Kobe

RIKEN Center for Computational Science

Outline

Objective

Gradient Flow

Running Coupling and Confinement

Vacuum Structure at Finite θ

Electric Dipole Moment

Conclusions

Objective

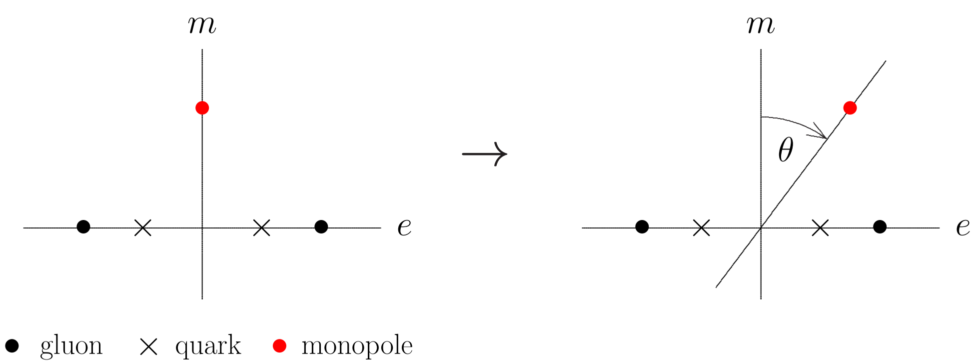
- QCD decribes the strong interactions remarkably well, from the smallest distances probed so far to hadronic scales where quarks and gluons confine to hadrons. Yet it faces a problem. The theory allows for a CP-violating term S_θ in the action. In Euclidean space-time it reads

$$S = S_{\rm QCD} + S_\theta \; : \quad S_\theta = i\,\theta\,Q\,, \quad Q = \frac{1}{32\pi^2}\int d^4x\; F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \;\in\; \mathbb{Z}\,,$$

  where Q is the toplogical charge, and θ is an arbitrary phase with values $-\pi < \theta \leq \pi$. A nonvanishing value of θ would result in an electric dipole moment (EDM) d_n of the neutron. The current experimental upper limit is $|d_n| < 1.8 \times 10^{-13} e\,$fm, which suggests that θ is anomalously small. This feature is referred to as the strong CP problem, which is considered as one of the major unsolved problems in the elementary particles field

- The prevailing paradigm is that QCD is in a single confinement phase for $|\theta| < \pi$. The Peccei-Quinn solution of the strong CP problem, for example, is realized by the shift symmetry $e^{i\,\delta\,Q_5}:\; \theta \to \theta + \delta$, trading the theta term S_θ for the hitherto undetected axion

- However, it is known from the case of the massive Schwinger model that a θ term may change the phase of the system. Callan, Dashen and Gross have claimed that a similar phenomenon will occur in QCD. The statement is that the color fields produced by quarks and gluons will be screened by instantons for $|\theta| > 0$. 't Hooft has argued that the relevant degrees of freedom responsible for confinement are color-magnetic monopoles. Confinement occurs when the monopoles condense in the vacuum, by analogy to superconductivity. In the θ vacuum the monopoles acquire a color-electric charge proportional to θ. Due to the joint presence of gluons and monopoles a rich phase structure is expected to emerge



Idea: Isolate the relevant dynamical variables at the hadronic scale by gauge fixing SU(3) $\rightarrow$ U(1)$\times$U(1)

For $|\theta| > 0$ quarks and gluons will be screened by forming bound states with the monopoles

- In this talk I will investigate the long-distance properties of the theory in the presence of the θ term, S_θ, and show that CP is naturally conserved in the confining phase

Gradient Flow

QCD exhibits a striking change in behavior over different length scales. To reveal the macroscopic properties of the theory, we are faced with a multi-scale problem, involving the passage from the short-distance perturbative regime to the long-distance confining regime. Such multi-scale behavior is typically addressed by renormalization group (RG) techniques bridging the different regimes

A promising framework is provided by the gradient flow (GF), which evolves the gauge field along the gradient of the action. The flow of SU(3) gauge fields is defined by the diffusion equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \qquad B_\mu(t=0,x) = A_\mu(x)$$

The scale is set by $\boxed{\mu = 1/\sqrt{8t}}$ $\sqrt{8t} \mathrel{\widehat{=}}$ smoothing range over which B_μ is averaged Lüscher

Formally, GF is an infinitesimal realization of the coarse-graining step of momentum space RG transformations (à la Wilson, Polchinski, Wetterich) and, as such, keeps the long-distance physics unchanged

Lüscher
Makino, Morikawa, Suzuki
Carosso, Hasenfratz, Neil

GF defines a running coupling α_{GF}

Number one choice for studying physical system over several length scales

The expectation value $\langle E(t) \rangle$ of the energy density

$$E(t,x) = \frac{1}{4} G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$$

has the perturbative expansion

$$\langle E(t) \rangle = \frac{3}{4\pi t^2} \alpha_{\overline{MS}}(\mu) \left[1 + k_1 \alpha_{\overline{MS}}(\mu) + k_2 \alpha_{\overline{MS}}(\mu)^2 + \cdots \right]$$

$t = 1/8\mu^2$

$$\equiv \frac{3}{4\pi t^2} \alpha_{GF}(\mu)$$

Thus

$$\boxed{\alpha_{GF}(\mu) = \frac{4\pi^2}{3} t^2 \langle E(t) \rangle}$$

$$\Lambda_{GF} = \exp\left\{ \frac{2\pi}{11} k_1 \right\} \Lambda_{\overline{MS}}$$

Lüscher

For a start we may restrict our investigations to the Yang-Mills (YM) theory. If the strong CP problem is resolved in the YM theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \left(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\,U_{\mu\nu}(x)\right)$$

	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

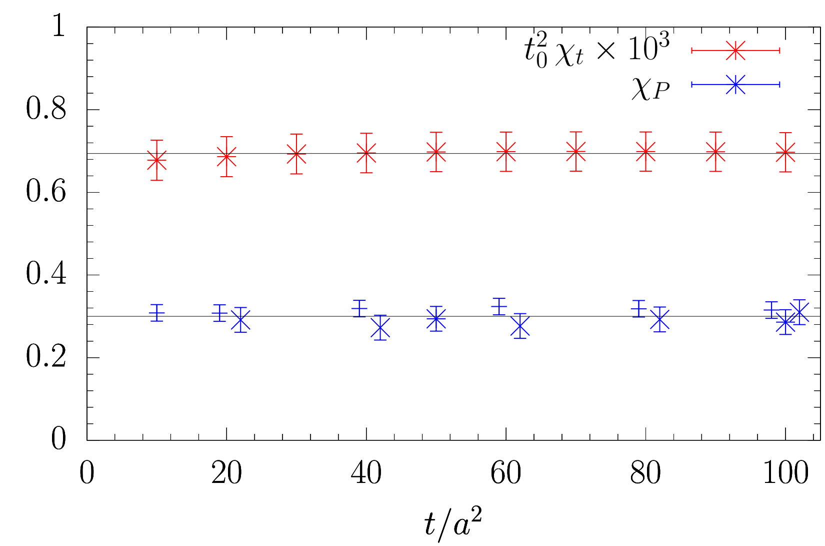
In the YM theory quantities that can be computed precisely are limited. Two examples:

- Topological susceptibility

$$\chi_t = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

- Normalized Polyakov susceptibility

$$\chi_P = \frac{\langle |P|^2\rangle - \langle |P|\rangle^2}{\langle |P|\rangle^2}\,, \quad P = \frac{1}{V_3}\sum_{\mathbf{x}} P(\mathbf{x})$$



Both quantities, χ_t and χ_P, are independent of the flow time t, as expected

The Polyakov loop (nonlocal operator) requires normalization, to be interpreted as free energy of static quarks

$$\sqrt{t_0}\,\chi_t^{\frac{1}{4}} = 0.162(3)$$

$$\chi_P = 0.289(7)$$

Literature:

$$\sqrt{t_0}\,\chi_t^{\frac{1}{4}} = 0.161(4)$$

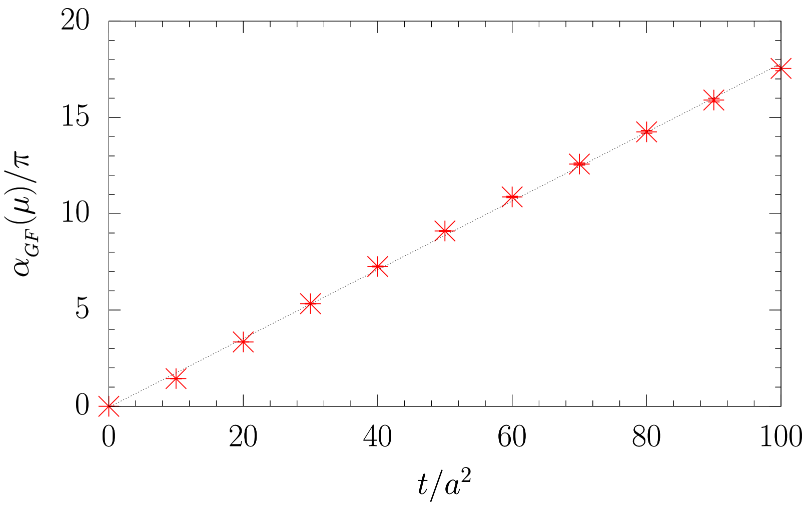
arXiv:1506.06052

2D Gaussian distribution:

$$\chi_P = 4/\pi - 1 = 0.273$$

Running Coupling and Confinement

Confinement is intimately connected with the IR behavior ($\mu \to 0$) of the running coupling $\alpha_{GF}(\mu)$



$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF}) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -\,2\,\alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0\, \alpha_{GF})^{-\frac{b_1}{2\,b_0^2}} \exp\left\{-\frac{1}{8\pi b_0\, \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha\, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0\, \alpha^2} + \frac{b_1}{2\, b_0^2\, \alpha}\right\}$$

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

To make contact with phenomenology, it is desirable to transform the GF coupling α_{GF} to a common scheme. A preferred scheme in the YM theory is the V scheme: $V(q) = -4\pi C_F \alpha_V(\mu)/q^2$

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int_0^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int_0^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

IR behavior universal

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{q}\; e^{i\,\mathbf{q}\mathbf{r}}\,\frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

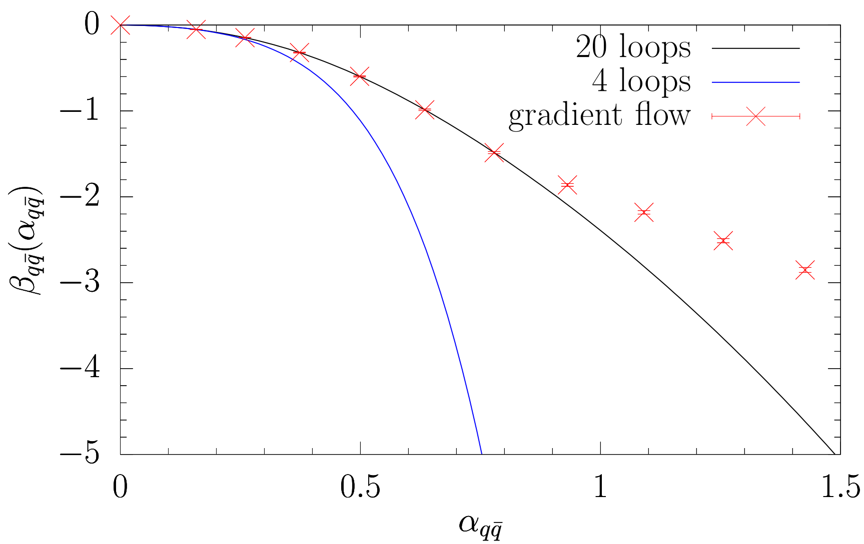
$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

It is interesting to compare the nonperturbative GF beta function with the perturbative beta function known up to twenty loops



20 loops arXiv:1309.4311

4 loops arXiv:1012.3037

$$\frac{\Lambda_{q\bar{q}}}{\Lambda_V} = 0.655$$

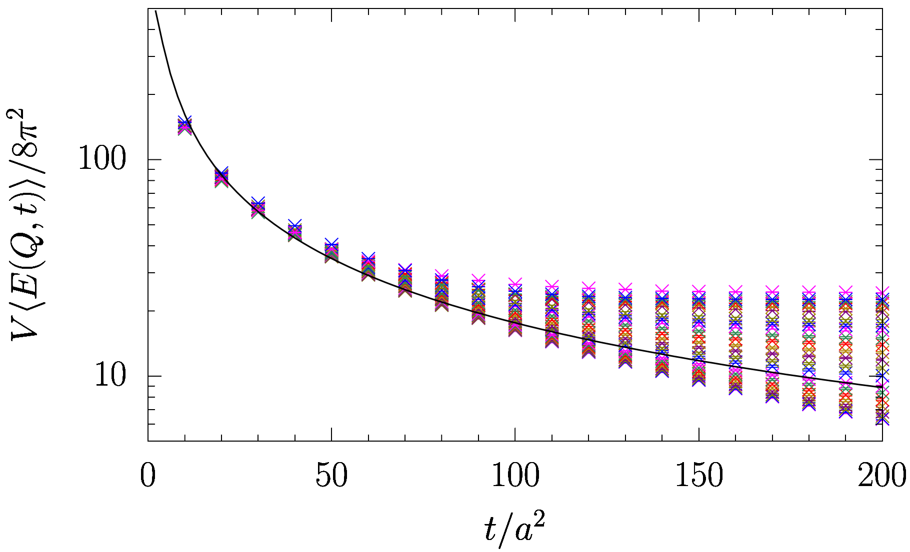
In general

$$\alpha_S \underset{\mu \to 0}{\longrightarrow} \left\{ \begin{array}{c} 0 \\ \infty \end{array} \right. \quad \text{possible}$$

As was to be expected, the perturbative beta function gradually approaches the nonperturbative beta function with increasing order

Vacuum Structure at Finite θ

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased



$$Z(\theta) = \int \mathcal{D}A_\mu \, e^{-S+i\theta Q}$$
$$= \sum_Q e^{i\theta Q} \int_Q \mathcal{D}A_\mu \, e^{-S}$$
$$= \sum_Q e^{i\theta Q} \, P(Q)$$

$V\langle E(Q,t)\rangle/8\pi^2 \equiv S_Q \simeq |Q|$, while the ensemble average vanishes like $1/t$

$$Q = \int d^4x \, \partial_\mu \omega_\mu \,, \quad \partial_t \omega_\mu = (1/8\pi^2) D_\rho G_{\nu\rho} \tilde{G}_{\mu\nu}$$

gauge invariant

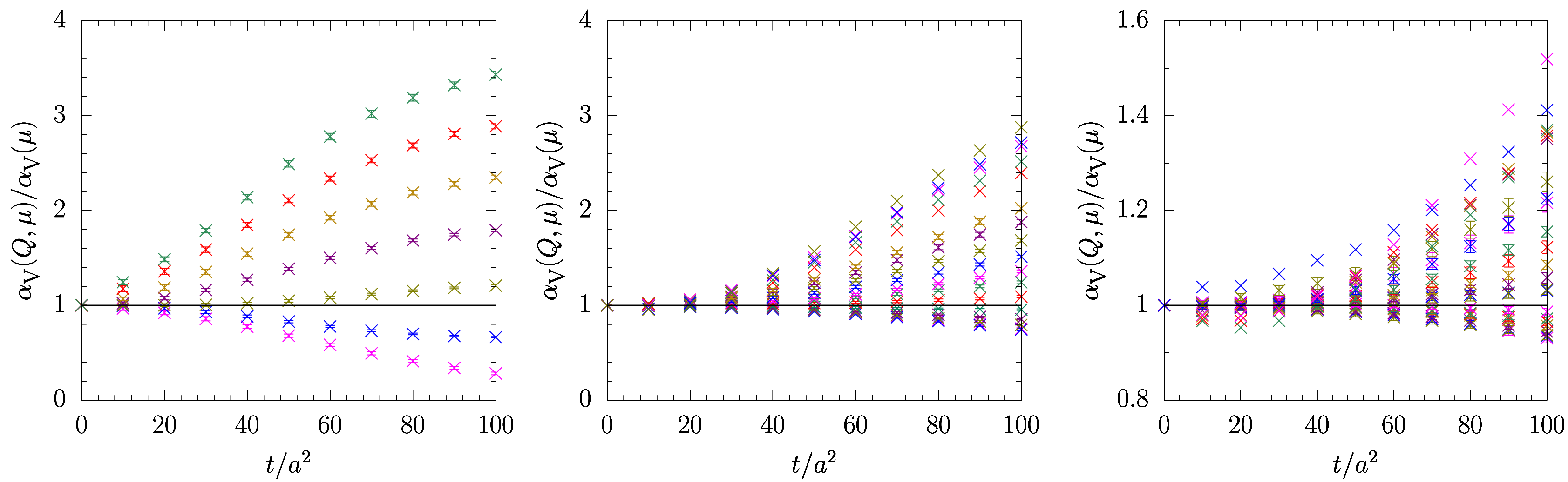
$$\Rightarrow \; \partial_t Q = 0$$

Running coupling α_V

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top

16^4 24^4 32^4



Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

The transformation of $\alpha_V(Q,\mu)$ from the 'Q vacua' to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{\,i\,\theta\,Q} P(Q)\,\alpha_V(Q,\mu)\,, \quad Z(\theta) = \sum_Q e^{\,i\,\theta\,Q} P(Q)$$

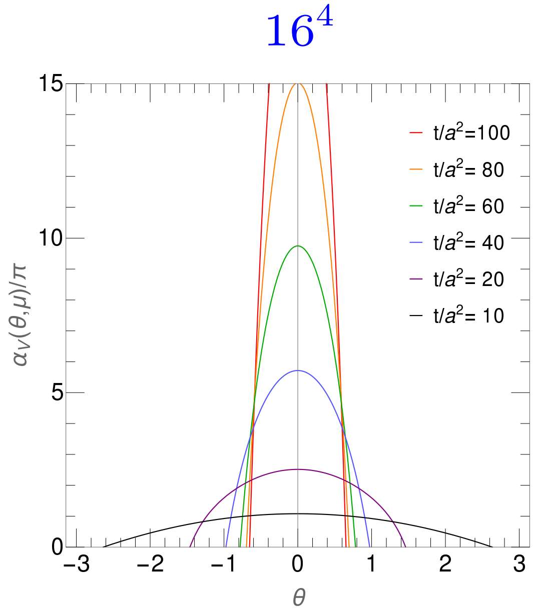
weighted by the charge density $P(Q)$, i.e. the probability of finding a configuration with charge Q

A few remarks are in order

- Here the parameter θ is the bare vacuum angle that labels the superselection sectors. It is the parameter that appears in the (lattice) action and determines the topological properties of the vacuum

- $P(Q)$ is determined by the real part of the action, $S_{\rm QCD}$, which increases proportionally to $|Q|$ and suppresses configurations which hold a large number of (anti-)instantons. It thus becomes increasingly difficult to determine $P(Q)$ precisely for large values of $|Q|$. This circumstance is completely independent of whether we simulate at $\theta = 0$ or any other value $|\theta| > 0$. This is to say, the situation would not improve if we could simulate the complex action

- As we shall see, we need to know the Fourier sum for small values of $|\theta|$ only, which is rather insensitive to fluctuations at large values of $|Q|$



At a first glance: The color charge gets totally screened for $|\theta| > 0$ in the infrared, while it becomes gradually independent of θ as we approach the perturbative regime

Analytically

$$\alpha_V(\theta, \mu) = \alpha_V(\mu)[1 - \alpha_V(\mu)(D/\lambda)\theta^2]^\lambda$$

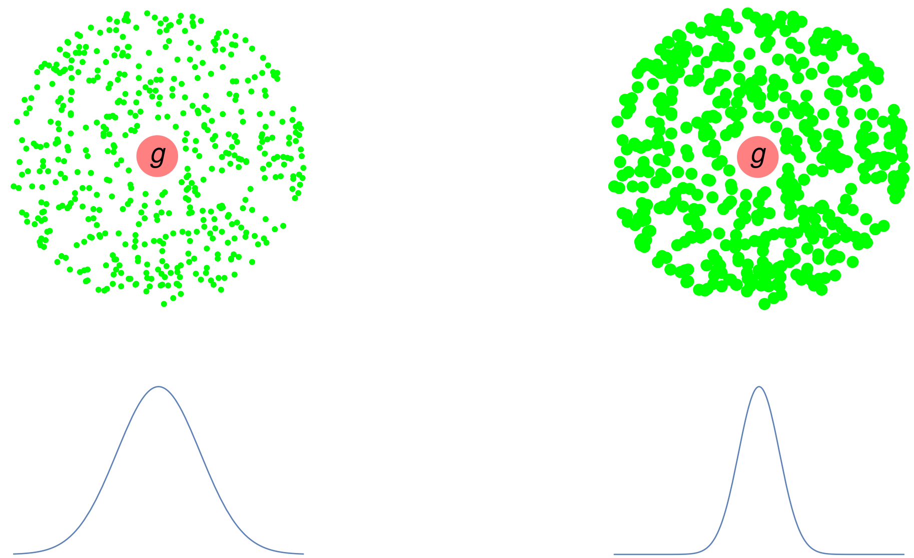
with $D \approx 0.12$ and $\lambda \approx 0.75$, leading to the screening length of the color charge $\boxed{\lambda_c \approx 0.5/\theta\,[\mathrm{fm}]}$

Pictorially ['t Hooft, Witten]

$\theta \quad < \quad \boldsymbol{\theta}$

$$q_m = \frac{\theta}{2\pi}$$

$$\rho_m^{\theta=0} \approx \rho_m^{\theta\neq 0}$$

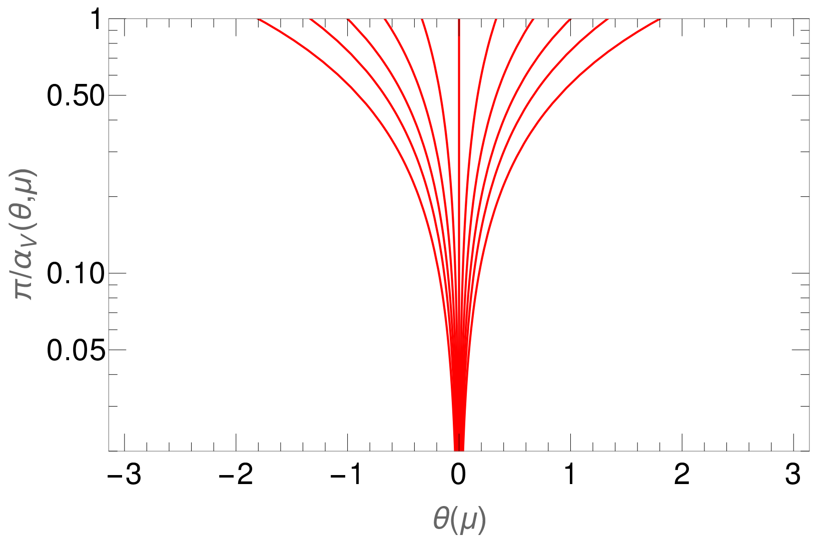


The Debye screening length in a plasma is given by $\lambda_D = \sqrt{E_F/4\rho\, q^2}$, where E_F is the Fermi energy, ρ the density and q the charge. For the model of 't Hooft and Witten this leads to $\lambda_D = \sqrt{(\pi\, E_F/\rho_m)/\theta}$ in perfect agreement with our findings, i.e. $\underline{\lambda_c \propto \lambda_D}$

This implies that $\theta \to \theta(\mu)$ is renormalized. From $\alpha_V(\theta, \mu)$ derive coupled RG equations, which for larger values of t decouple and take the form

$$\frac{\partial(\pi/\alpha_V)}{\partial \ln t} \simeq -\frac{\pi}{\alpha_V} + \pi D\,\theta^2\,, \quad \frac{\partial\theta}{\partial \ln t} \simeq -\frac{1}{2}\,\theta$$

Solution (full) for various initial values of $\theta(\mu)$



IR fixed point (?)

- $\theta(\mu)$ appears in the effective Lagrangian
- $\theta(\mu) \to 0$ as $\alpha_V(\theta, \mu) \to \infty$

  Confinement $\widehat{=}$ CP Invariance

- Bar any loops, properties can be directly read off from fixed point values (universality class)
- CP trivially conserved at the upper (perturbative) end

Literature

Reuter arXiv:hep-th/9604124

$$\theta(\mu = 0) = 0 \quad \text{for} \quad \alpha_s(\mu = 0) = \infty$$

based on exact RG evolution equation à la Wetterich

Knizhnik & Morozov JETP Lett. 39 (1984) 240

$$\partial(1/g^2)/\partial \ln \mu = C + D\cos\theta\,, \quad \partial\theta/\partial\ln\mu = 8\pi^2 D \sin\theta$$

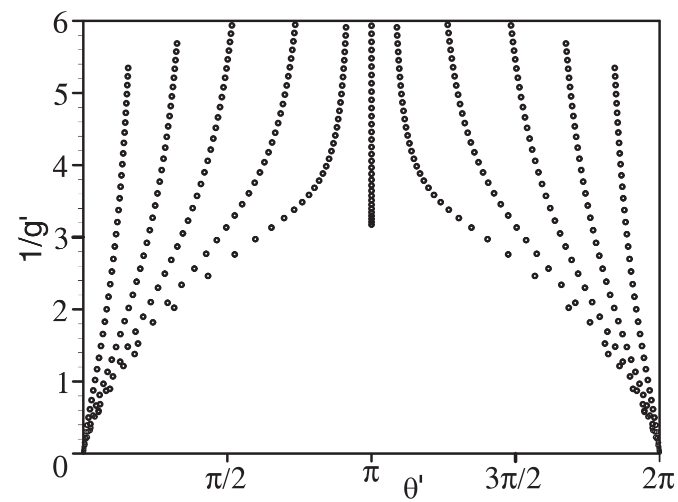
from instanton density. Renormalization of θ appears to be a generic property of instanton fluctuations

Levine, Libby & Pruisken, Apenko Phys. Rev. Lett. 51 (1983) 1915, 52 (1984) 1254

Perhaps the most known example where such renormalizations have proved to be important is the quantum Hall effect, described by a matrix nonlinear σ model

$$\mathbf{J} = \sigma \mathbf{E} \;: \quad \sigma_{xx} \sim 1/g^2\,, \quad \sigma_{xy} \sim \theta$$

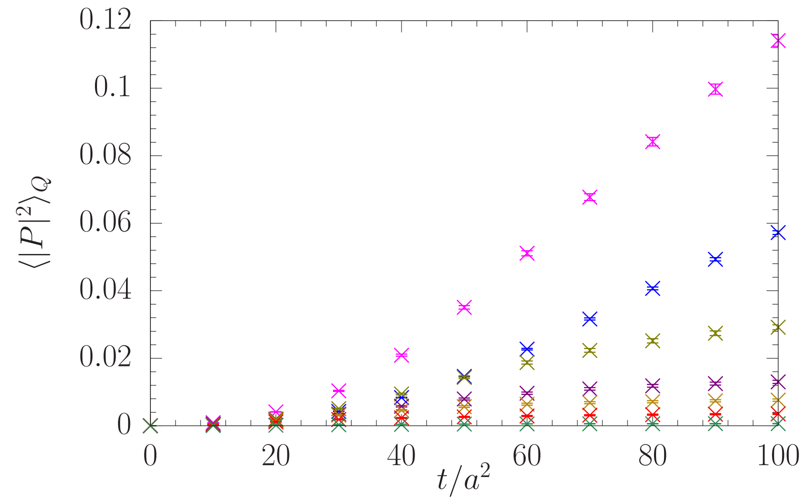
which has served as a model for the solution of the strong CP problem



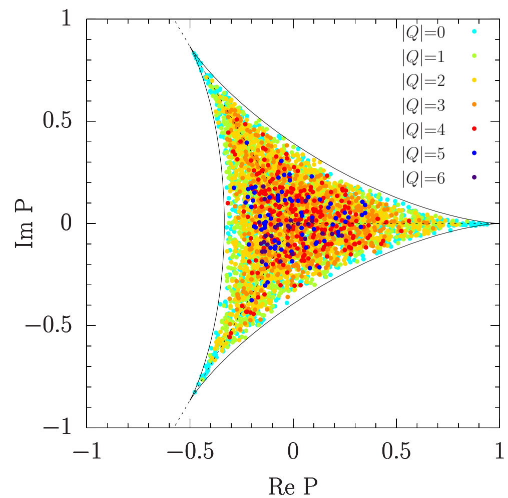
Polyakov loop

The Polyakov loop P describes the propagation of a single static quark travelling around the periodic lattice

16^4



$t/a^2 = 60$



From $Q = 0$ (top) to 6 (bottom)

$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

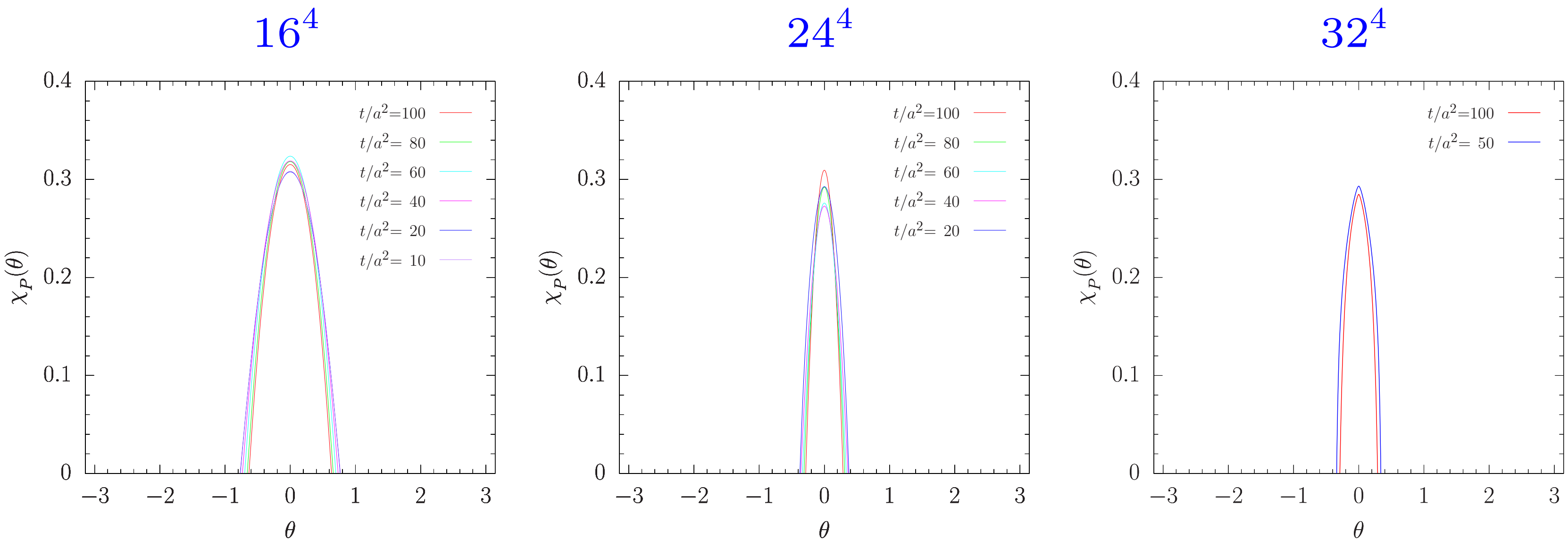
The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The connected part of $\langle |P|^2 \rangle_\theta$ is described by the normalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The normalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

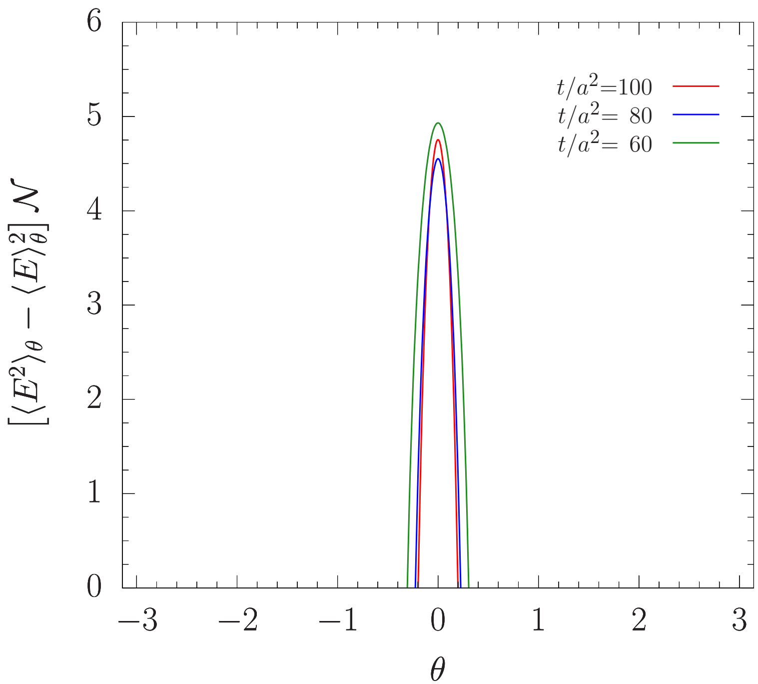
Mass gap

$$\langle E^2 \rangle = \frac{1}{T} \sum_t \langle E(0) E(t) \rangle \qquad [\langle E^2 \rangle - \langle E \rangle^2]\, \mathcal{N} = \sum_{n>0,t} \frac{1}{2m_n} |\langle 0|E|n\rangle|^2 e^{-m_n t}$$

$$E(t) = \frac{1}{V_3} \sum_{\vec{x}} E(\vec{x}, t) \qquad \simeq \frac{1}{m^2_{0^{++}}} |\langle 0|E|0^{++}\rangle|^2 \propto \xi^2$$

24^4

Correlation length



$\langle E^2 \rangle_\theta - \langle E \rangle_\theta^2$ Independent of flow time t

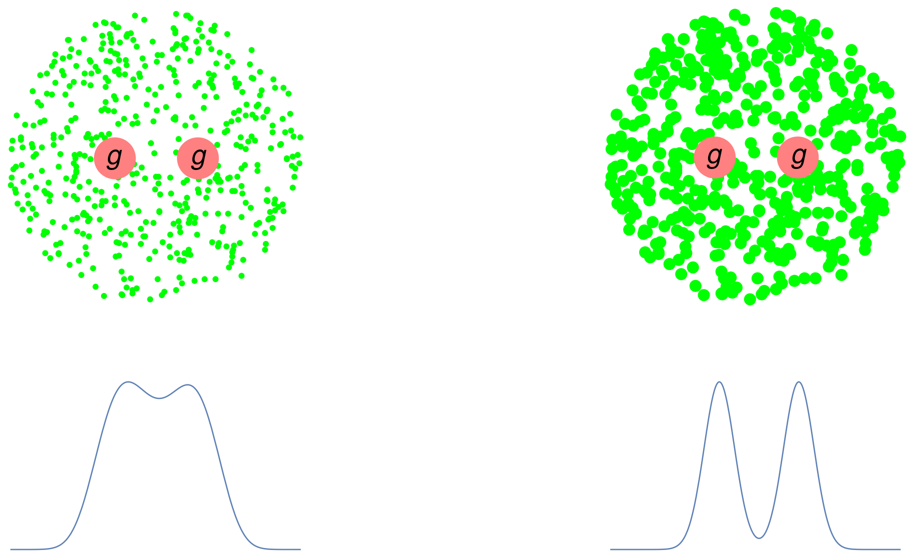
$\xi \simeq 0$ for $|\theta| \gtrsim 0$

No mass gap

The operators P and E overlap in correlators χ_P and $\langle E^2 \rangle$ spatially and temporally

Pictorially ['t Hooft, Witten]

$$\theta \quad < \quad \boldsymbol{\theta}$$

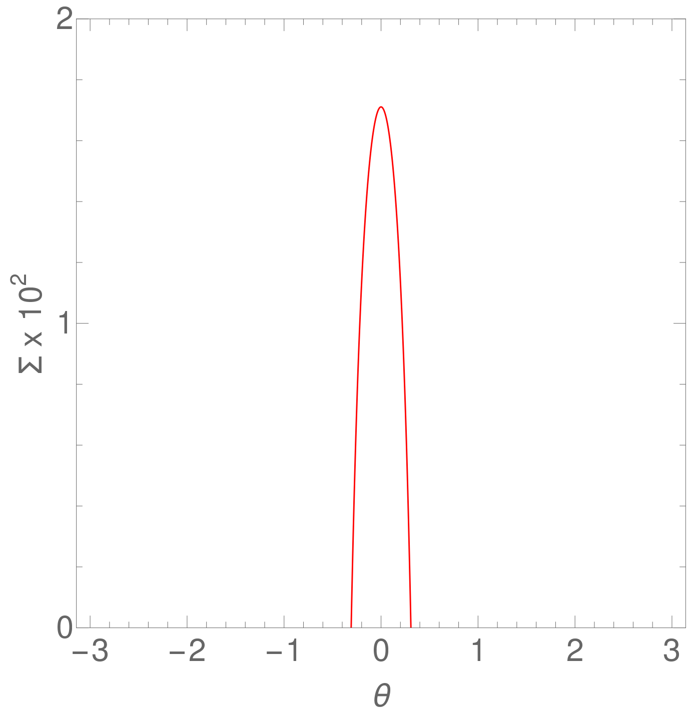


For the operators P and E to be totally screened, the screening length must be smaller than the hadron radius. This appears to be the case for $|\theta| \gtrsim 0.4$. At this value $\lambda_c \approx 1 \text{ fm}$

Hadron masses are obtained from the exponential decay at large Euclidean separations. Thus, there is <u>no hadron spectrum for $|\theta| > 0$</u>

$\mathbf{\Sigma}(\theta)$ Chiral condensate Preliminary [with Hinnerk Stüben]

24^4



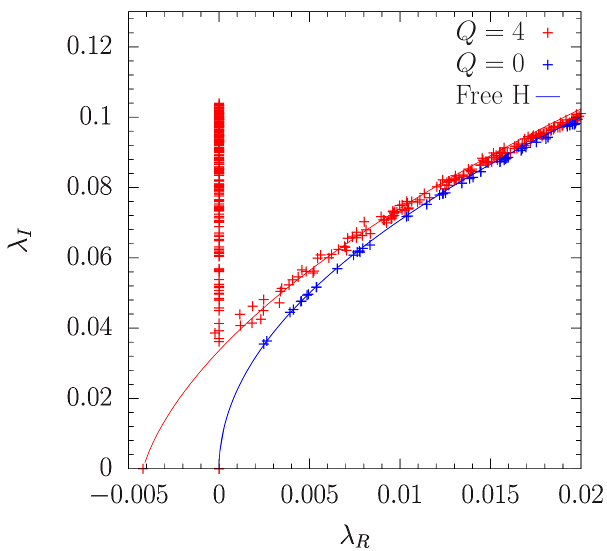
$$\Sigma(Q) \propto \sqrt{|Q|}$$

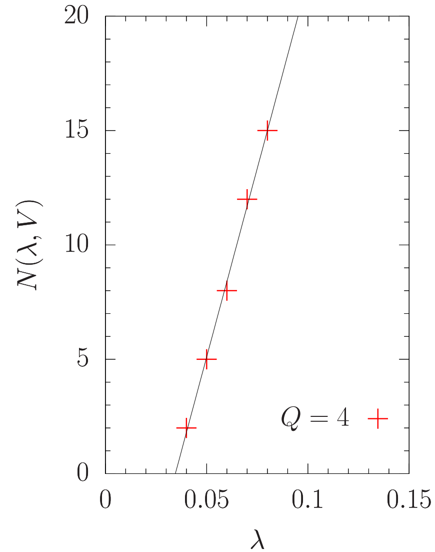
Diakonov
Schäfer & Shuryak
Follana et al

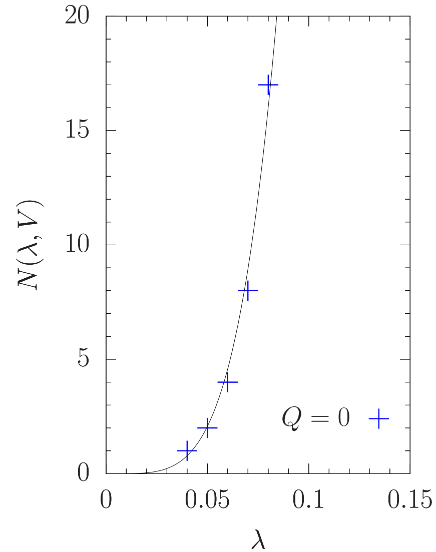
$$\Sigma(\theta) = \frac{1}{Z(\theta)} \sum_Q e^{\,i\,\theta\,Q}\, P(Q)\, \Sigma(Q)$$

Chiral symmetry breaking closely linked with confinement

Thus, predictions of nonvanishing electric dipole moment d_n for $|\theta| > 0$ from ChPT not valid







$$D_N = \rho \left(1 + \frac{D_W(\rho)}{|D_W(\rho)|}\right)$$

$$D^{\text{imp}} = D_N \left(1 - \frac{1}{2\rho} D_N\right)^{-1}$$

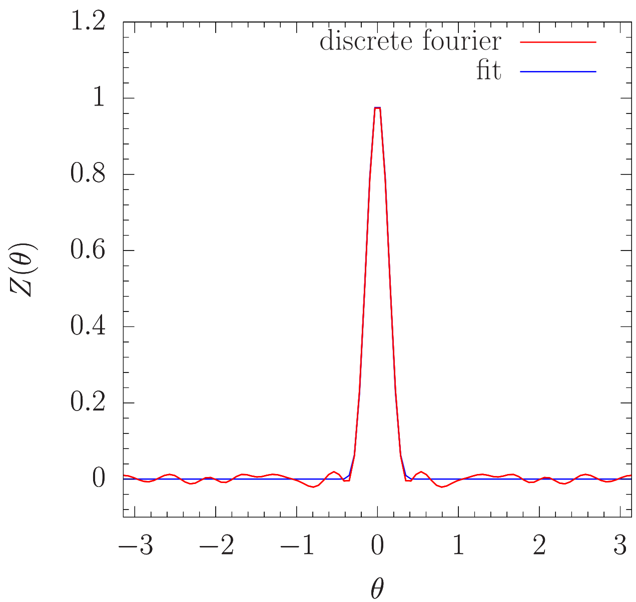
$$N = \left(\frac{\lambda}{\pi}\Sigma - \frac{1}{V}\right) \Theta \left(\lambda V \Sigma - \frac{1}{4}\pi\right) + O(\lambda^2)$$

[Leutwyler, Göckeler et al.]

Errors

Source of errors

- Convergence of the (discrete) Fourier series $\sum_Q \exp\{i\theta Q\}\, P(Q)\, \cdots$
- Statistics
- Topological charge generally limited to $|Q| \leq |Q|_{\max}$, $|Q|_{\max} \propto \sqrt{V}$



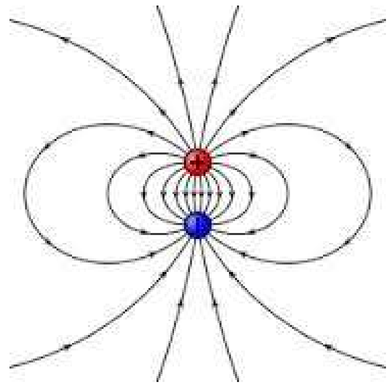
$Z(\theta), \alpha_V(\theta), \chi_P(\theta), \cdots$ are positive functions of θ

After the quantities I showed have dropped to 'zero' at $|\theta| \gtrsim 0$, they start to oscillate around zero with frequency $\nu \approx |Q|_{\max}$ due to the truncated Fourier series

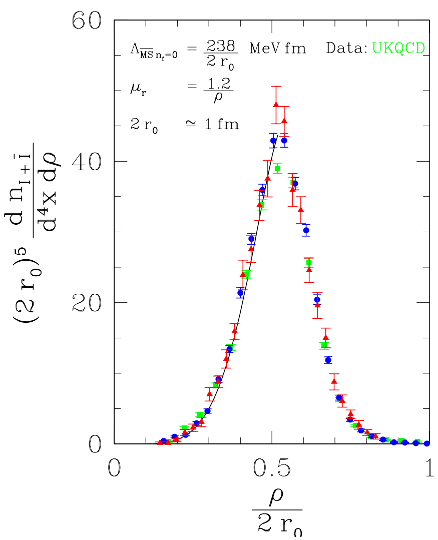
Various techniques to filter unphysical high-frequency modes are discussed in the literature. We fit the tail of the distributions to a smooth function. Alternatively, one can employ a low-pass filter, which practically gives the same result

Electric Dipole Moment

$|d_n| < 1.8 \times 10^{-13} e\, \mathrm{fm}$



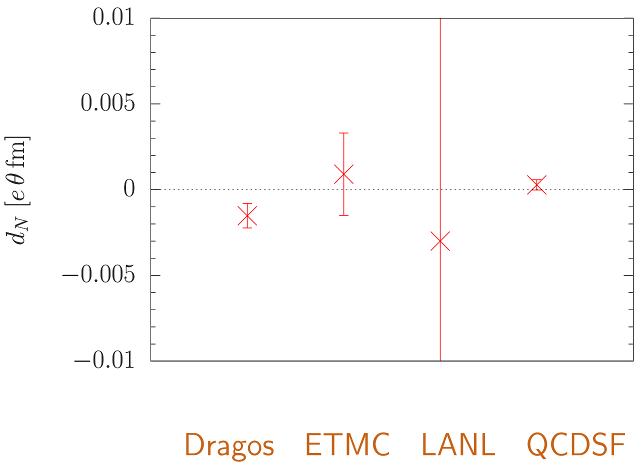
$\} \quad < 10^{-13}\, \mathrm{fm}$



- It is an important and fundamental question to ask why d_n is so small

- If $d_n \neq 0$ is found eventually, it is rather naive though to believe that it has any relation to instantons and the topological structure of the vacuum

Lattice



- With one exception (QCDSF), the calculations have been done on background gauge field configurations with $\theta = 0$, taking the θ term into the observable. Valid perhaps for $\theta^2\langle Q^2\rangle \ll 1$. At imaginary values of θ (QCDSF) the vacuum confines as well
- Not surprising to find a stable nucleon. At small θ will need ultra-long separations to see an effect. Similar to finite temperature phase transition. The results are compatible with $d_n = 0$

Message

Continuum: CP is conserved in the strong interactions, i.e. the world in which we live. For $|\theta| > 0$, however small, quarks will eventually get liberated

Lattice: Verification might be difficult for very small values of θ, which requires to follow the decay of the correlation functions over very large distances

A lesson might be learned from the finite temperature transition, where the screening length is proportional to $1/(T - T_c)$

Conclusions

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. A key point is that the path integral splits into disconnected topological sectors for $t \gtrsim 0$, which is expected to occur at ever smaller flow times with decreasing lattice spacing. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The novel result is that color charges are screened, and confinement is lost, for $|\theta| > 0$ due to nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level

★ Screening process is in qualitative agreement with the dual superconductor model of confinement. A full understanding goes hand in hand with the understanding of the confinement mechanism

★ To 'solve' the mystery of the electric dipole moment, we suggest to compute the relevant correlation functions on flowed gauge field configurations, separately for each topological sector, and Fourier-sum the result to the theta vacuum. The EFT calculations of the dipole moment need to be revised, with help from the lattice

It is surprising that the seminal work of Schwinger, Coleman, Callan, Dashen, Gross, 't Hooft and on QHE, etc. has been completely ignored