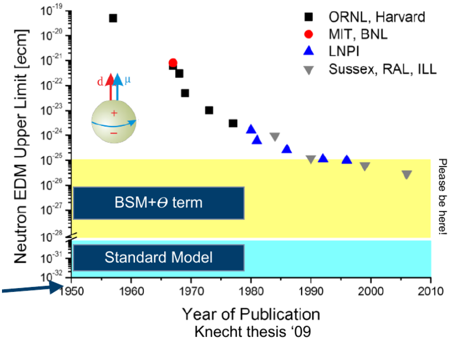
QCD Theta term contribution to nEDM with Stabilized Wilson Fermion on the lattice

Jangho Kim
in collaboration with
Anthony Francis Thomas Luu Giovanni Pederiva Andrea Shindler
Savvas Zafeiropoulos

ECT@Trento
August 5, 2022

CP violation Sources



- Standard Model
  - CKM matrix
  - Strong θ-term
- Beyond the Standard Model
  - quark EDM
  - quark Chromo EDM
  - Weinberg three-gluon operator
  - 4-quark operator

QCD θ-term

- QCD Lagrangian in Minkowski space

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu,\nu} + \bar{q}(i\not{D} - M)q - \bar{\theta}\frac{g^2}{64\pi^2}\epsilon^{\mu\nu\alpha\beta}G^a_{\mu\nu}G^{a,\mu\nu} \tag{1}$$

- $\bar{\theta}$ is the coupling of the CP-odd interaction.
- $\bar{\theta} \ll 1$, evaluate correlation function in a $\bar{\theta}$ vacuum.

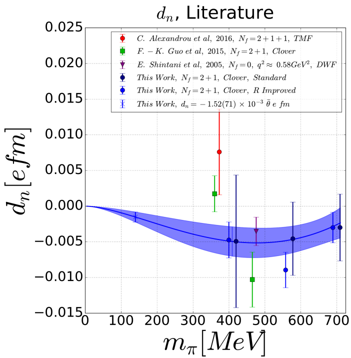
$$\langle O\rangle_{\bar{\theta}} = \langle O\rangle + i\bar{\theta}\langle OQ\rangle + \mathcal{O}(\bar{\theta}^2) \tag{2}$$

- Topological charge Q

$$Q(t) = a^4\sum_x \frac{1}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}G^a_{\mu\nu}(x,t)G^a_{\rho\sigma}(x,t) \tag{3}$$

- t is flow-time.
- Here we study the vector form factor with inserting topological charge.

Previous Result



- Previous lattice calculation [1902.03254v2] show the difficulties to determine the nEDM, especially close to the physical value of the pion mass.
- Here we present first results obtained with Stabilized Wilson Fermions.
- We focus on SU(3) flavor-symmetric OpenLat ensembles at $\sim$ 400 MeV pion mass, and 3 different lattice spacings in the range of 0.065 fm $< a < 0.12$.

Stabilized Wilson ensembles

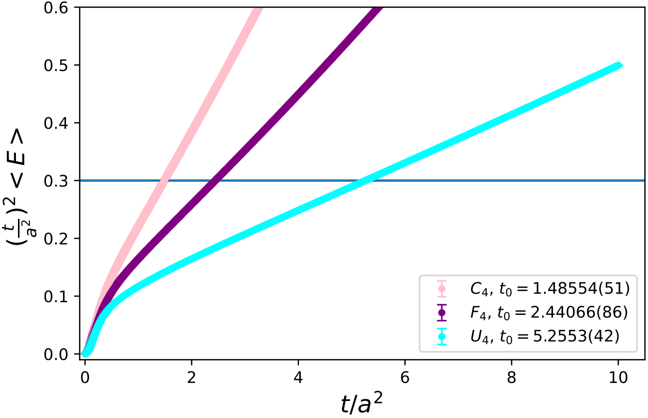
Ens	a [fm]	β	m_π [MeV]	$\frac{L}{a} \times \frac{T}{a}$	N_{conf}	N_{src}
C_4	0.12	3.685	410	$24^3 \times 96$	1189	150
F_4	0.094	3.8	408	$32^3 \times 96$	1001	100
U_4	0.065	4.0	411	$48^3 \times 96$	214	150

- N_{src} is the number of stochastic source locations.
- The first goal is to perform the continuum limit at $m_\pi \simeq 400$ MeV.

Scale Setting

- t_0/a^2 values are determined by

$$t_0^2 \langle E(t_0) \rangle_{\text{lat}} = 0.3 \tag{4}$$

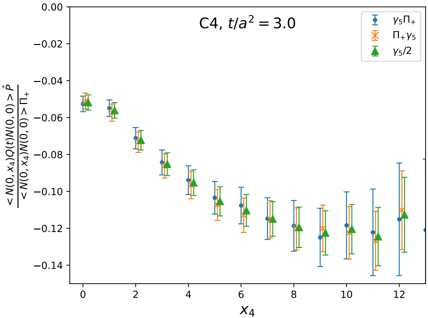


Nucleon mixing angle α_N (Preliminary)

$$G_2(\vec{p'}, x_4, \Pi) = a^3 \sum_{\vec{x}} e^{-i\vec{p'}\cdot\vec{x}} \mathrm{Tr}[\hat{P}\langle N(x)\overline{N}(0)\rangle] \tag{5}$$

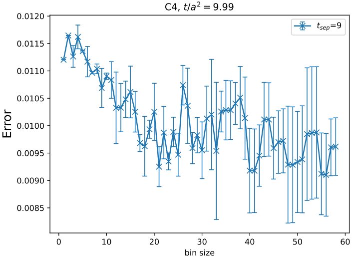
$$G_2^{(Q)}(\vec{p'}, x_4, \Pi) = a^3 \sum_{x} e^{-i\vec{p'}\cdot\vec{x}} \mathrm{Tr}[\hat{P}\langle N(x)\overline{N}(0)Q(t)\rangle] \tag{6}$$

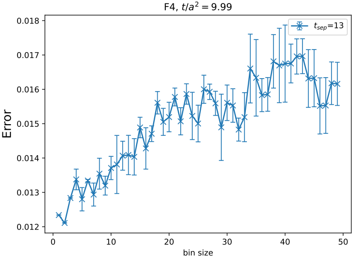
$$\alpha_N(\hat{P}) = \frac{G_2^{(Q)}(\vec{p'}=0, x_4, \hat{P}, t)}{G_2(\vec{p'}=0, x_4, \Pi_+)}, \quad \Pi_+ = \frac{I+\gamma_4}{2} \tag{7}$$

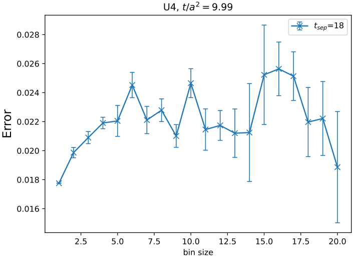


Autocorrelation

- Plot error of α_N versus bin size.
- Error saturates about bin size 20 for C_4, F_4 and 10 for U_4.





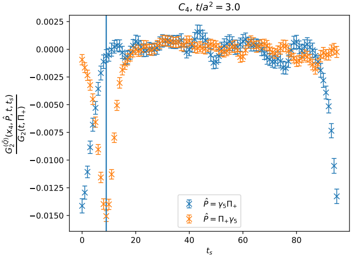


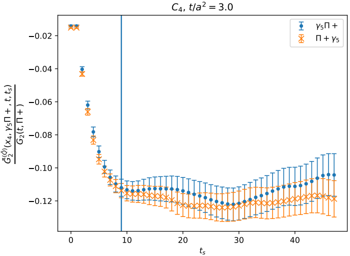
α_N improvement method (Preliminary)

$$\bar{Q}(\tau_Q, t) = a^3 \sum_{\vec{x}} q(\vec{x}, \tau_Q; t)\,, \quad Q(t) = a \sum_{\tau_Q} \bar{Q}(\tau_Q, t) \tag{8}$$

$$\Delta_2^{(Q)}(t, \hat{P}, t, \tau_Q) = a^3 \sum_{\vec{x}} \mathrm{Tr}[\langle N(x)\bar{Q}(\tau_Q, t)\bar{N}(0)\rangle] \tag{9}$$

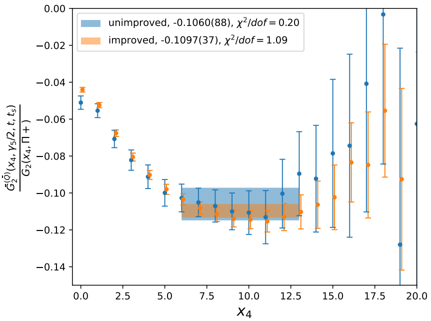
$$\overline{G}_2^{(Q)}(x_4, \hat{P}, t, t_s) = a \sum_{\tau_Q/a=0}^{t_s/a} [\Delta_2^{(Q)}(x_4, \hat{P}, t, \tau_Q) + \Delta_2^{(Q)}(x_4, \hat{P}, t, T - \tau_Q)] \tag{10}$$

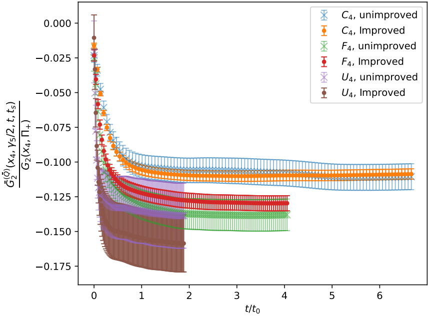




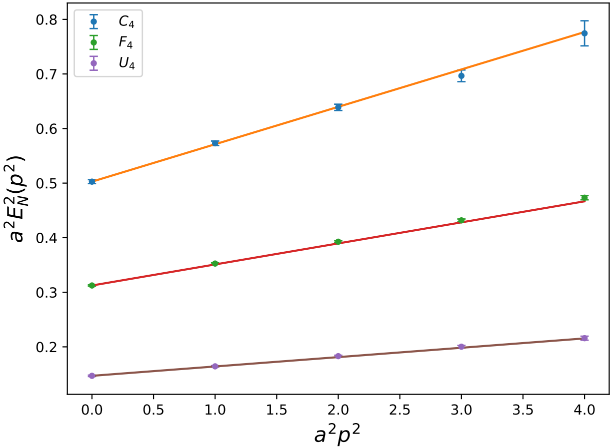
Improved α_N (Preliminary)

- summation window $t_s = x_4$



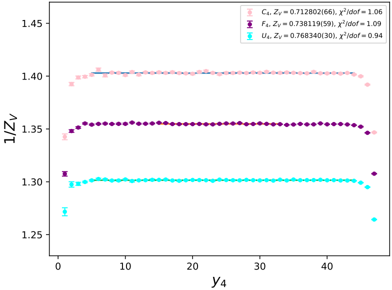


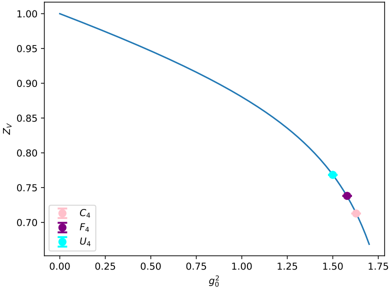
Energy dispersion relation



- Nucleon energy squared at difference values of $|p|^2$ compare with the continuum dispersion relation $E_N^2 = m^2 + |p|^2$

Z_V using vector Ward identity





- Vector Ward identity

$$\frac{1}{Z_V} = \frac{\langle P(T/2)V_4(y_4)P(0)\rangle - \langle P(T/2)V_4(T-y_4)P(0)\rangle}{\langle P(T/2)P(0)\rangle} \tag{11}$$

- fitting with one-loop coefficient $f(g_0^2) = 1 - 0.10057 g_0^2 \frac{1+ag_0^2}{1+bg_0^2}$

Electric Dipole Moment

- CP-odd electric dipole form factor

$$\frac{F_3(q^2)}{2m} \xrightarrow{q^2 \ll m_\pi^2} d_n - S_n q^2 + \mathcal{O}(q^2) \tag{12}$$

- q^2 is momentum transfer, m is nucleon mass.
- The three-point function with topological charge

$$G_3^{(Q)}(\vec{p'}, x_4, \vec{q}, \tau, \hat{P}, \gamma_\mu, t) \tag{13}$$

$$= a^6 \sum_{\vec{x},\vec{y}} e^{-i(\vec{p'}\cdot\vec{x} - \vec{q}\cdot\vec{y})} \mathrm{Tr}[\langle N(\vec{x}, x_4) J_\mu(\vec{y}, \tau) \overline{N}(\vec{0}, 0) Q(t) \rangle] \tag{14}$$

$$J_\mu(\vec{y}, \tau) = \frac{2}{3}\bar{u}(\vec{y}, \tau)\gamma_\mu u(\vec{y}, \tau) - \frac{2}{3}\bar{d}(\vec{y}, \tau)\gamma_\mu d(\vec{y}, \tau) \tag{15}$$

- The ratio for removing the leading Euclidean time dependence and nucleon-to-vacuum amplitude contributions.

$$R^{(Q)}(\vec{p'}, x_4, \vec{q}, \tau, \hat{P}, \gamma_\mu, t) = \frac{G_3^{(Q)}(\vec{p'}, x_4, \vec{q}, \tau, \hat{P}, \gamma_\mu)}{G_2(\vec{p'}, x_4, \Pi_+)} K(\vec{p'}, x_4, \vec{q}, \tau) \tag{16}$$

$$K(\vec{p'}, x_4, \vec{q}, \tau) = \sqrt{\frac{G_2(\vec{p'}, \tau, \Pi_+) G_2(\vec{p'}, x_4, \Pi_+) G_2(\vec{p}, x_4 - \tau, \Pi_+)}{G_2(\vec{p}, \tau, \Pi_+) G_2(\vec{p}, x_4, \Pi_+) G_2(\vec{p'}, x_4 - \tau, \Pi_+)}} \tag{17}$$

- Here we use the fixed sink method, $\vec{p'} = 0$, which implies $\vec{q} = -\vec{p}$.
- The spectral decomposition of the ratio at leading order in $\bar{\theta}$

$$R^{(Q)}(0, x_4, \vec{q}, \tau, \hat{P}, \gamma_\mu, t) = 2mA(m, E_p)\Big[\alpha_N 2m \mathrm{Tr}\{\hat{P}\Pi_+ \widetilde{\Gamma}_\mu(Q^2)\gamma_5\} + \alpha_N \mathrm{Tr}\{\hat{P}\gamma_5 \widetilde{\Gamma}_\mu(q^2)(-i\not{p} + m)\} + \mathrm{Tr}\{\hat{P}\Pi_+ \frac{\sigma_{\mu\nu}\gamma_5 q_\nu}{2M_N} \widetilde{F}_3(q^2)(-i\not{p} + m)\}\Big] \quad (18)$$

- With the fixed sink method $p' = 0$, $E_p = m$

$$A(m, E_p) = \frac{1}{4\sqrt{mE_p(2m)(E_p + m)}} \quad (19)$$

$$\widetilde{\Gamma}_\mu(q^2) = \gamma_\mu F_1(q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2M_N} \widetilde{F}_2(q^2) \quad (20)$$

$$F_3(q^2) = \cos(2\alpha_N)\widetilde{F}_3(q^2) + \sin(2\alpha_N)\widetilde{F}_2(q^2) \quad (21)$$

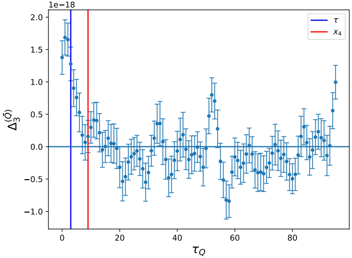
$R^{(Q)}$ improvement (Preliminary)

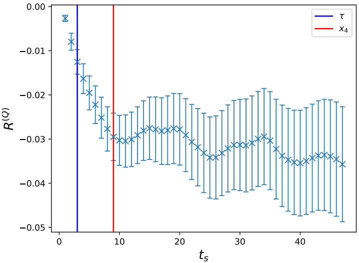
- As well as α_N improvement, the same method is applied to $R^{(Q)}$.

$$\Delta_3^{(\bar{Q})}(\vec{p'}, x_4, \vec{q}, \tau, \tau_Q, \hat{P}, \gamma_\mu, t) = a^6 \sum_{\vec{x},\vec{y}} e^{-i\vec{p'}\cdot\vec{x}+i\vec{q}\cdot\vec{y}} \mathrm{Tr}\{\hat{P}\langle N(\vec{x}, x_4) J_\mu(\vec{y}, \tau) \bar{Q}(\tau_Q, t) \bar{N}(\vec{0}, 0)\rangle \quad (22)$$

$$\bar{G}_3^{(\bar{Q})}(t_s) = a \sum_{\tau_Q/a=0}^{t_s/a} [\Delta^{(\bar{Q})}(\tau_Q) + \Delta^{(\bar{Q})}(T - \tau_Q)] \quad (23)$$

- x_4 is source-sink separation of nucleon. We set $t_s = x_4$ for our analysis.

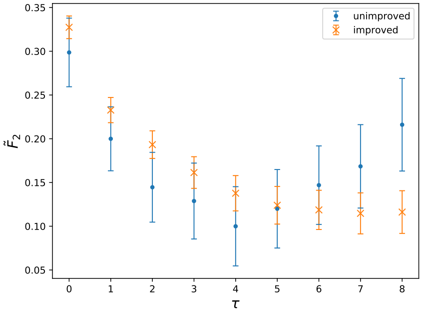


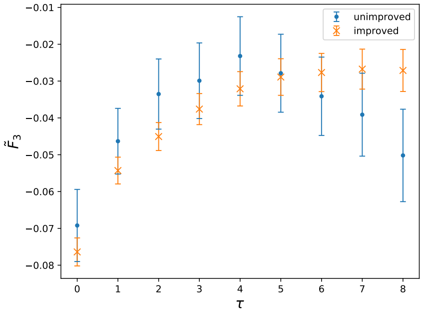


Improved $\tilde{F}_2$ and $\tilde{F}_3$ (Preliminary)

$$\widetilde{F}_2 = \frac{R^{(Q)}}{2m\alpha_N A(m,E_q)\left(2m\mathrm{Tr}\{\hat{P}\Pi_+\frac{\sigma_{\mu\nu}q_\nu}{2m}\gamma_5\} + \mathrm{Tr}\{\hat{P}\gamma_5\frac{\sigma_{\mu\nu}q_\nu}{2m}(-i\not{p}+m)\}\right)}$$

$$\widetilde{F}_3 = \frac{R^{(Q)}}{2mA(m,E_q)\left(\mathrm{Tr}\{\hat{P}\Pi_+\frac{\sigma_{\mu\nu}q_\nu}{2m}\gamma_5(-i\not{p}+m)\}\right)}, \quad \hat{P} = i\Pi_+\gamma_5\gamma_3 \qquad (24)$$



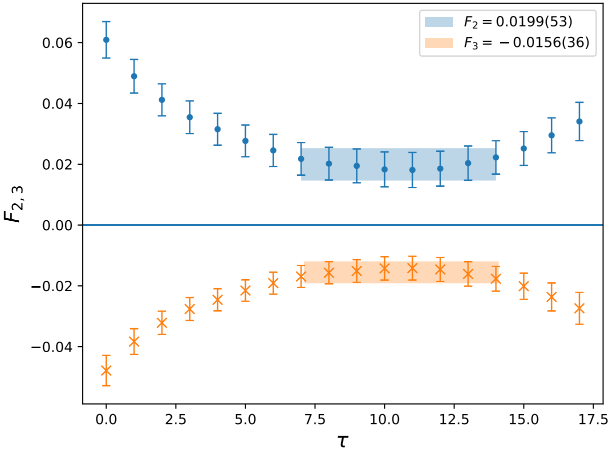


F_2 and F_3 (Preliminary)

- Extract F_2 and F_3 by plateau fitting.

$$F_2(q^2) = -\sin(2\alpha_N)\widetilde{F}_3(q^2) + \cos(2\alpha_N)\widetilde{F}_2(q^2) \quad (25)$$

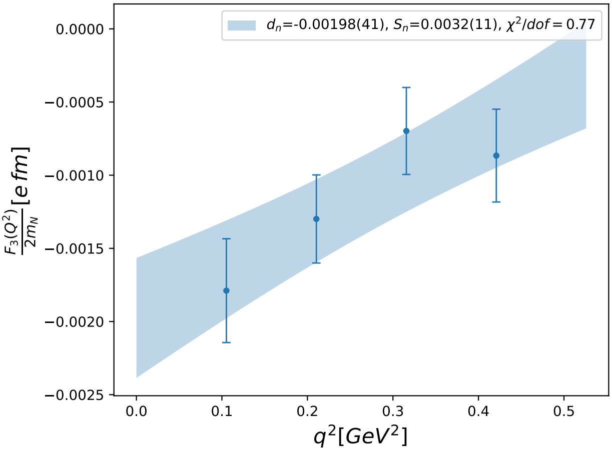
$$F_3(q^2) = \cos(2\alpha_N)\widetilde{F}_3(q^2) + \sin(2\alpha_N)\widetilde{F}_2(q^2) \quad (26)$$



Transfer momentum fit (Preliminary)

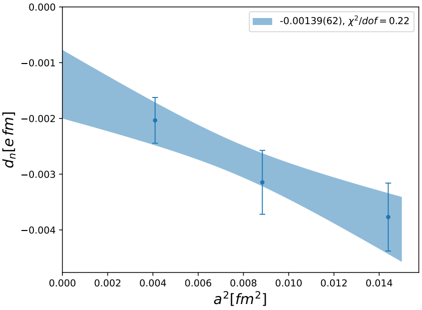
- Obtain d_n and S_n by linear fit.

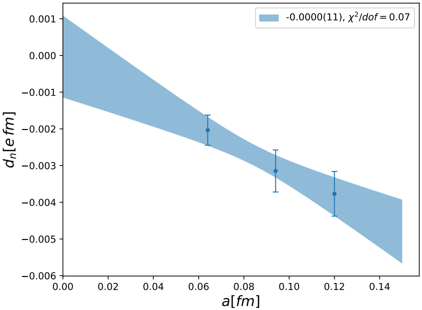
$$\frac{F_3(q^2)}{2m} = d_n - S_n q^2 \tag{27}$$



Continuum limit of d_n (Preliminary)

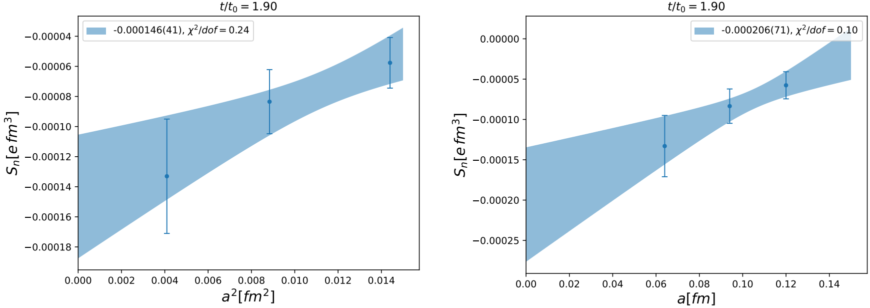
- Neutron EDM d_n at 400 MeV pion mass.
- In order to take continuum limit with $\mathcal{O}(a^2)$, the current improvment coefficient c_V is required.
- Scale is fixed to $t/t_0 = 1.9$ for these ensemble.





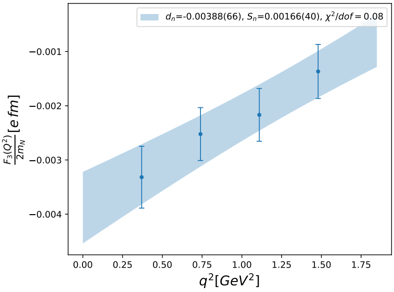
Continuum limit of Schiff Moment S_n (Preliminary)

- Schiff Moment S_n at 400 MeV pion mass.
- Chiral perturbation theory: $S_n = -1.7(3) \times 10^{-4}\bar{\theta} e fm^3$

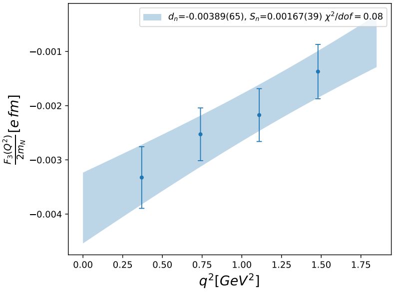


Effect of reweighting factor (Preliminary)

- Stabilized Wilson ensembles are generated using reweighting method.
- Reweighting factor should be multiplied $\langle O \rangle = \frac{\langle rO \rangle}{\langle r \rangle}$.
- Reweighting factors are not completely computed yet, except C_4 ensemble.
- The effect of reweighting factor on C_4 ensemble is less than 0.7%.



(a) Without reweighting factor



(b) With reweighting factor

Conclusion and future plan

- We compute neutron EDM at 400 MeV pion mass.
- To obatin neutron EDM at the physical pion mass, we are preparing 200 MeV and 300 MeV pion mass ensembles.
- Reweighting factor are not ready yet for all ensembles. Its effect is less than 1%.

Ens	a [fm]	β	m_π [MeV]	$\frac{L}{a} \times \frac{T}{a}$	N_{conf}	N_{src}
C_4	0.12	3.685	410	$24^3 \times 96$	1189	150
F_4	0.094	3.8	408	$32^3 \times 96$	1001	100
F_3	0.094	3.8	293	$32^3 \times 96$	200	
F_2	0.094	3.8	215	$32^3 \times 96$	200	
S_4	0.077	3.9	~ 400	$48^3 \times 96$	928	
U_4	0.065	4.0	411	$48^3 \times 96$	214/580	150
U_3	0.065	4.0	~ 300	$48^3 \times 96$	200	