

EDMs from Lattice QCD

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Neutron Electric Dipole Moment: from Theory to Experiment
Aug 1 – 5, 2022, ECT*, Trento, Italy

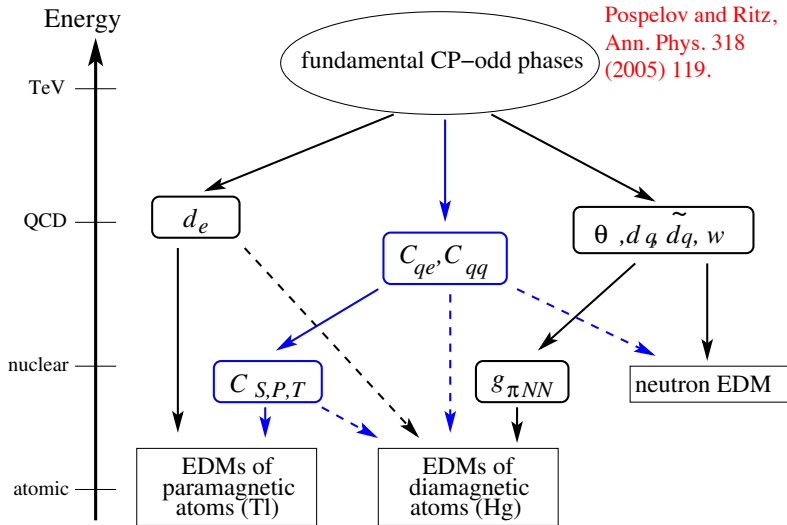
Collaboration Members

- Tanmoy Bhattacharya (LANL)
- Vincenzo Cirigliano (University of Washington)
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Outline

1. Introduction
2. Neutron EDM from QCD θ -term
3. Neutron EDM from Quark EDM
4. Neutron EDM from quark Chromo-EDM
5. Neutron EDM from Weinberg's three-gluon Operator
6. Conclusion

Hierarchy of EDM Scales



Pospelov and Ritz,
Ann. Phys. 318
(2005) 119.

$$\mathcal{L}_{\text{CPV}} = \mathcal{L}_{\text{CKM}} + \mathcal{L}_{\bar{\theta}} + \mathcal{L}_{\text{BSM}} \longrightarrow \mathcal{L}_{\text{CPV}}^{\text{eff}}$$

Effective CPV Lagrangian at Hadronic Scale

$$\begin{aligned}\mathcal{L}_{\text{CPV}}^{d \leq 6} &= -\frac{g_s^2}{32\pi^2} \bar{\theta} G \tilde{G} && \text{dim}=4 \text{ QCD } \theta\text{-term} \\ &- \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F) \gamma_5 q && \text{dim}=5 \text{ Quark EDM (qEDM)} \\ &- \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q} (\sigma \cdot G) \gamma_5 q && \text{dim}=5 \text{ Quark Chromo EDM (CEDM)} \\ &+ d_w \frac{g_s}{6} G \tilde{G} G && \text{dim}=6 \text{ Weinberg's } 3g \text{ operator} \\ &+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim}=6 \text{ Four-quark operators}\end{aligned}$$

- $\bar{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \nu / \Lambda_{BSM}^2$; effectively dim=6
- All terms up to $d = 6$ are leading order

Calculation of Neutron EDM d_n

$$d_n = \bar{\theta} \cdot C_\theta + d_q \cdot C_{q\text{EDM}} + \tilde{d}_q \cdot C_{\text{CEDM}} + \dots$$

- SM and BSM theories
→ Coefficients of the effective CPV Lagrangian $(\bar{\theta}, d_q, \tilde{d}_q, \dots)$
- Lattice QCD
→ Nucleon matrix elements in presence of CPV interactions
 $(C_\theta, C_{q\text{EDM}}, C_{\text{CEDM}}, \dots)$

Physical Results from Unphysical Simulations

- **Finite Lattice Spacing**

- Simulations at finite lattice spacings $a \approx 0.045 - 0.15$ fm

- ⇒ **Extrapolate to continuum limit, $a \rightarrow 0$**

- **Heavy Pion Mass**

- Lattice simulation: **Smaller quark mass** \rightarrow **Larger computational cost**

- Simulations at or heavier than physical pion mass

- **Finite Volume**

- Simulations at finite lattice volume; small effect in most of the EDM calculations

- **Renormalization**

- Lattice results \rightarrow continuum $\overline{\text{MS}}$; **involves complicated/divergent mixing**

- **Excited state contamination**

- Lattice interpolating operators of meson and nucleon **couple to excited states**

Neutron EDM from QCD θ -term

$$\begin{aligned}\mathcal{L}_{\text{CPV}}^{d\leq 6} &= -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim}=4 \text{ QCD } \theta\text{-term} \\ &- \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q}(\sigma \cdot F)\gamma_5 q && \text{dim}=5 \text{ Quark EDM (qEDM)} \\ &- \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma \cdot G)\gamma_5 q && \text{dim}=5 \text{ Quark Chromo EDM (CEDM)} \\ &+ d_w \frac{g_s}{6} G\tilde{G} && \text{dim}=6 \text{ Weinberg's } 3g \text{ operator} \\ &+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim}=6 \text{ Four-quark operators}\end{aligned}$$

QCD θ -term

$$S = S_{QCD} + i\bar{\theta}Q, \quad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

- Three different approaches

- External electric field method: $\langle N\bar{N} \rangle_{\bar{\theta}}(\vec{\mathcal{E}}, t) = \langle N(t)\bar{N}(0)e^{i\bar{\theta}Q} \rangle_{\vec{\mathcal{E}}}$
Aoki and Gocksch (1989), Aoki, Gocksch, Manohar, and Sharpe (1990),
CP-PACS Collaboration (2006), Abramczyk, *et al.* (2017)

- Simulation with imaginary $\bar{\theta}$: $\bar{\theta} = i\tilde{\theta}, \quad S_{\bar{\theta}}^q = \tilde{\theta} \frac{m_l m_s}{2m_s + m_l} \sum_x \bar{q}\gamma_5 q$
Horsley, *et al.*, (2008), Guo, *et al.* (2015)

- Expansion in $\bar{\theta}$:
$$\langle O(x) \rangle_{\bar{\theta}} = \frac{1}{Z_{\bar{\theta}}} \int d[U, q, \bar{q}] O(x) e^{-S_{QCD} - i\bar{\theta}Q}$$
$$= \langle O(x) \rangle_{\bar{\theta}=0} - i\bar{\theta} \langle O(x)Q \rangle_{\bar{\theta}=0} + \mathcal{O}(\bar{\theta}^2)$$

Shintani, *et al.*, (2005), Berruto, Blum, Orginos, and Soni (2006)
Shindler, T. Luu, J. de Vries (2015), Shintani, Blum, Izubuchi, and Soni (2016),
Alexandrou, *et al.*, (2016), Abramczyk, *et al.* (2017), Dragos, *et al.* (2019)
Bhattacharya, *et al.* (2021)

Simulation with QCD θ -term

- Expansion in $\bar{\theta}$, which is expected to be tiny

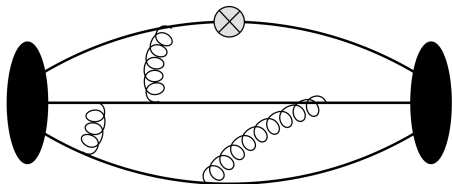
$$\langle O(x) \rangle_{\bar{\theta}} = \langle O(x) \rangle_{\bar{\theta}=0} - i\bar{\theta} \langle O(x)Q \rangle_{\bar{\theta}=0} + \mathcal{O}(\bar{\theta}^2)$$

- Do not need to generate new gauge fields (lattices) at $\bar{\theta} \neq 0$ or $d_w \neq 0$
- Neutron EDM requires calculations of
(1) vector current $O = V_\mu$, and (2) topological charge Q_{top}
- Vector current V_μ and its renormalization
 - Reusing data generated for vector form-factor
- Topological charge Q_{top}
 - $\mathcal{O}(a^4)$ -improved gluon field strength tensor
 - **Gradient flow** for cooling/renormalization

⇒ Obtain CP violating form factor using

$$\langle N|V_\mu(q)|N \rangle_{\text{CPV}} = \langle N|V_\mu(q)|N \rangle - i\bar{\theta} \langle N|V_\mu(q)Q_{\text{top}}|N \rangle + \mathcal{O}(\bar{\theta}^2)$$

CPV Form Factor F_3



- F_3 is extracted from vector current in presence of CPV interactions

$$\langle N|V_\mu(q)|N\rangle_{\text{CPV}} = \bar{u}_N \left[F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu - \frac{F_3(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu\gamma_5 \right] u_N(p)$$

Caveat: CPV interactions introduce phase in neutron mass

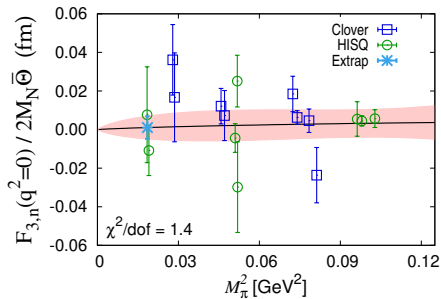
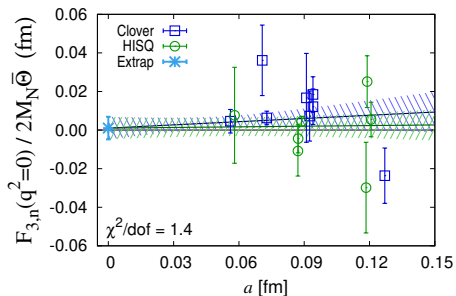
→ γ_4 is not a parity operator of neutron state

[Abramczyk, et al., 2017]

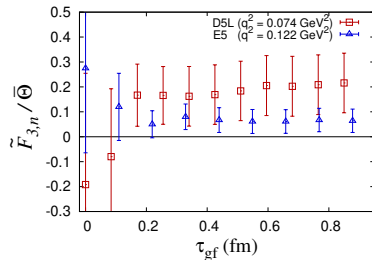
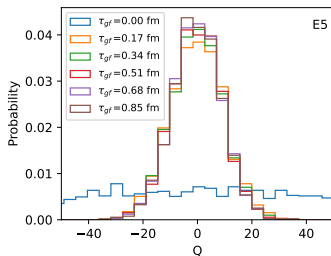
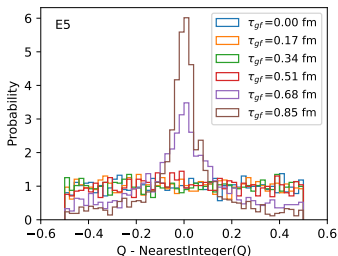
- Neutron EDM $d_n = |e|F_3(Q^2 = 0) / 2M_N$

Two Lattice Discretizations in Comparison

- Previous calculation [Bhattacharya, *et al.* (2021)]
 - Fermion: Clover (valence) on HISQ (sea)
 - Lattice spacings: 0.057 fm – 0.151 fm
 - Pion mass: 128 MeV – 320 MeV
 - Number of configurations: 550 – 2200
- New calculation
 - Fermion: Clover (valence) on Clover (sea)
 - Lattice spacings: 0.056 fm – 0.127 fm
 - Pion mass: 167 MeV – 285 MeV
 - Number of configurations: 810 – 2100



Calculation of Topological Charges



- Converging to an integer Q_{top} takes long flow time τ_{gf} (longer on coarser lattice)
- But Q_{top} and F_3 converge to a distribution with short τ_{gf}

Topological Susceptibility

- Topological Susceptibility χ_Q provides a good check for the lattice calculation of Q_{top}

$$\chi_Q = \int d^4 \langle Q(x)Q(0) \rangle$$

- Fit to leading M_π - and a -dependence

$$\begin{aligned} \chi_Q &= c_1 M_\pi^2 + c_2 a^2 M_\pi^2 + c_3 a^2 \\ &= [79.5(3.0) \text{ MeV}]^4 \end{aligned}$$

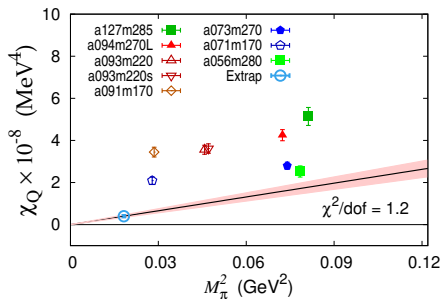
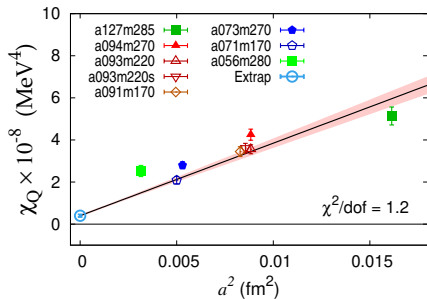
c.f., $\chi_Q = [66(9)(4) \text{ MeV}]^4$ from HISQ

- The chiral behavior precisely agree with χ^{PT}

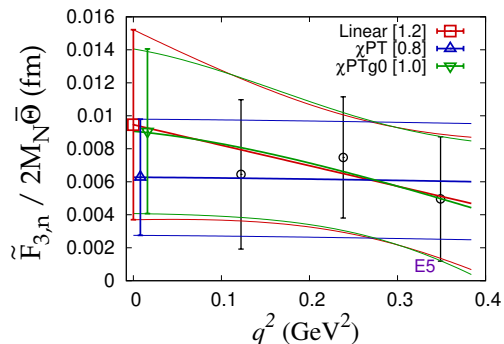
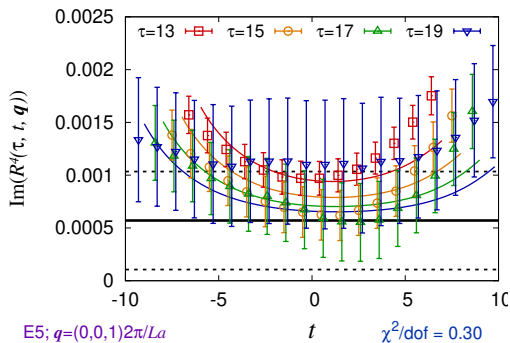
$$\frac{1}{\chi_Q} = \frac{1}{\chi_Q^{\text{quench.}}} + \frac{4}{M_\pi^2 F_\pi^2} \left(1 - \frac{M_\pi^2}{3M_\eta^2} \right)^{-1}$$

$$\chi_Q = [79 \text{ MeV}]^4$$

Bhattacharya, *et al.* (2021)

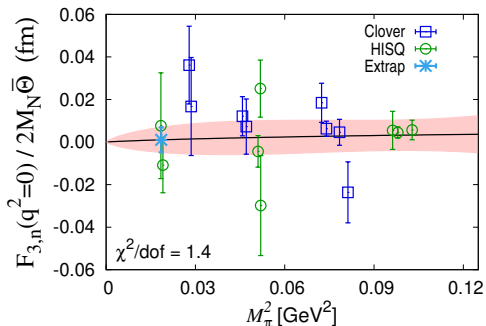
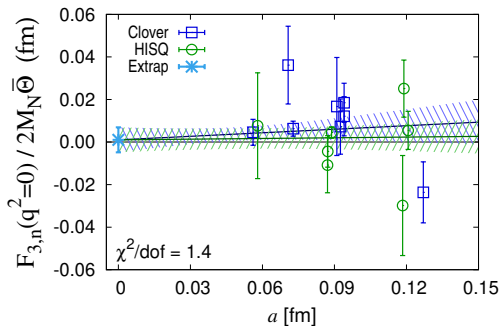


Neutron EDM from QCD θ -term on each Ensemble



- Calculate $\langle N | V_\mu(q) Q_{\text{top}} | N \rangle$ and remove excited-state contamination
- Extract CPV form-factor F_3 and extrapolate to $Q^2 = 0$ limit
- Neutron EDM $d_n = |e| F_3(Q^2 = 0) / 2M_N$

Neutron EDM from QCD θ -term Extrapolated to Continuum



- Extrapolation to continuum $a \rightarrow 0$ and physical pion mass $M_\pi \rightarrow 135\text{MeV}$
- Simultaneous fit to Clover-on-HISQ and Clover-on-Clover results

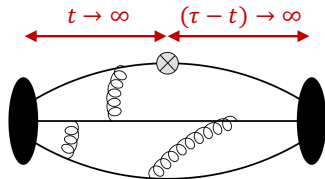
$$d_N = c_1 M_\pi^2 + c_2 M_\pi^2 \log\left(\frac{M_\pi^2}{M_N^2}\right) + c_3^{\text{HISQ}} a + c_3^{\text{Clover}} a = 0.0010(59)$$

PRELIMINARY; statistical error only.

- See **Tanmoy Bhattacharya's** talk at **LATTICE2022** for details

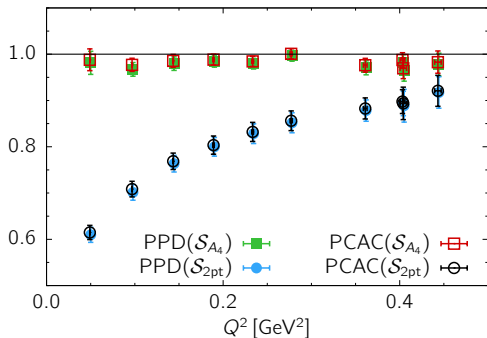
Potential Challenges: Difficult Physical Pion Mass Simulation

- On lattice, nucleon states are generated using an interpolating operator that couples to the ground-state of the nucleon but also to the excited states
- Excited-states (ES) effect exponentially diminishes for large Euclidean time
 - Separating proton sources far from each other leaves only the ground-state
 - **But signal-to-noise ratio drops exponentially for baryons** ($R \sim e^{(M_N - \frac{3}{2}m_\pi)\tau}$)
- For reasonably small source-sink separation, fit correlators to a function including ES
 - $C_{2pt}(\tau) = \sum_i |A_i|^2 e^{-M_i \tau}$
 - $C_{3pt}^\Gamma(\tau, t) = \sum_{i,j} A_i A_j^* \langle i | O_\Gamma | j \rangle e^{-M_i t - M_j (\tau - t)}$
 - $C_{3pt}^\Gamma / C_{2pt}(\tau) \rightarrow g_\Gamma^{u-d}$ as $t \rightarrow \infty, (\tau - t) \rightarrow \infty$
- **Tower of possible excited states**
 - Radial excitations: $N(1440), N(1710), \dots$
 - Multi-hadron states: $N(\mathbf{p})\pi(-\mathbf{p}), N(\mathbf{0})\pi(\mathbf{0})\pi(\mathbf{0}), \dots$
 - **But, which states contribute significantly?**

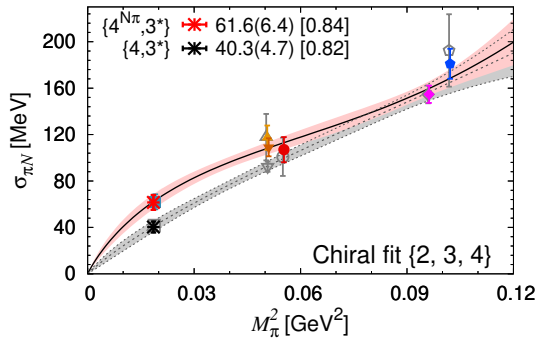


Potential Challenges: Difficult Physical Pion Mass Simulation

- Excited state fits to C_{2pt} gives large first ES mass $M_1 \gtrsim M_{N(1440)}$
- But various evidences advocate $M_1 \ll M_{N(1440)}$
 - PCAC relation between G_A, \tilde{G}_P , and G_P is much better satisfied with smaller M_1
 - Nucleon σ -term results are consistent with χ PT when $M_1 \sim M_{N\pi, N\pi\pi}$

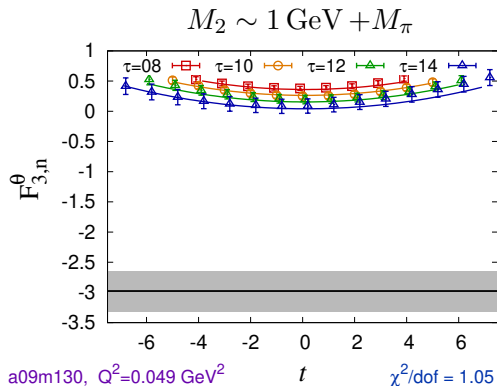
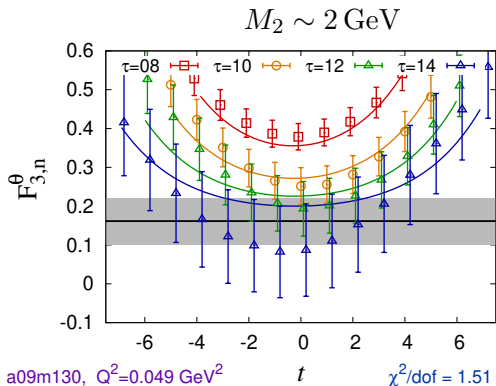


[Jang, *et al.*, (2020)]



[Gupta, *et al.*, (2021)]

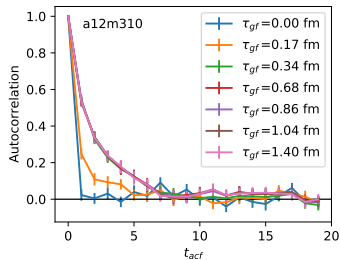
Potential Challenges: Difficult Physical Pion Mass Simulation



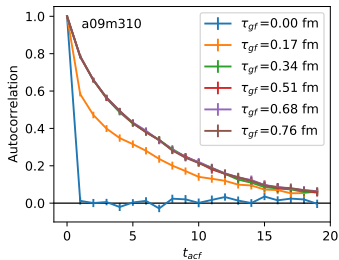
- Calculations at $m_\pi < 200 \text{ MeV}$ have poor signal
- Possibly significant contamination from the $N\pi$ -first-excited-state at near m_π^{phys} . Calculation with high statistics is needed to resolve [Bhattacharya, *et al.*, (2020)]

Potential Challenges: Long Autocorrelation at Fine Lattice

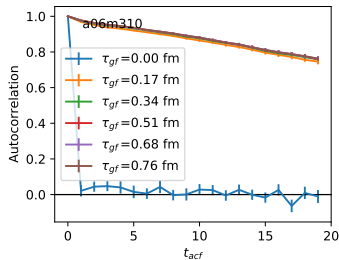
$a \approx 0.12$ fm



$a \approx 0.09$ fm

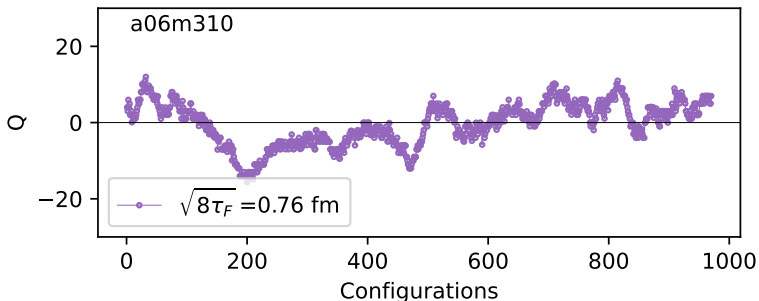


$a \approx 0.06$ fm



- Simulations on small a is required to reduce discretization artifact
- Topological charge Q shows longer autocorrelation on smaller a
- Clover lattices have much weaker autocorrelation, but starts to show autocorrelation at $a \approx 0.06$ fm

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Neutron EDM from Quark EDM term

$$\begin{aligned}\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim}=4 \text{ QCD } \theta\text{-term} \\ & -\frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5q && \text{dim}=5 \text{ Quark EDM (qEDM)} \\ & -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\bar{q}(\sigma\cdot G)\gamma_5q && \text{dim}=5 \text{ Quark Chromo EDM (CEDM)} \\ & +d_w\frac{g_s}{6}G\tilde{G} && \text{dim}=6 \text{ Weinberg's } 3g \text{ operator} \\ & +\sum_i C_i^{(4q)}O_i^{(4q)} && \text{dim}=6 \text{ Four-quark operators}\end{aligned}$$

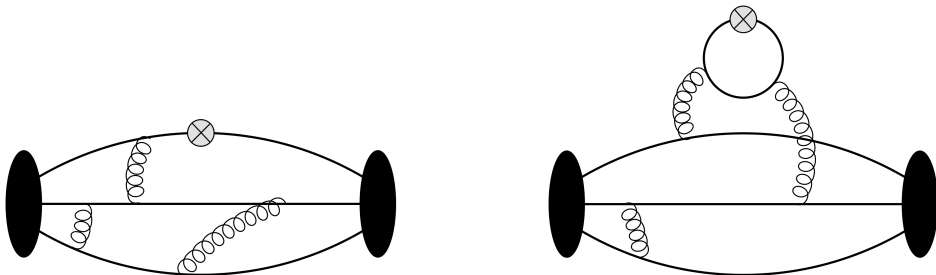
Quark EDM and Tensor Charge

- Neutron EDM (d_N) from Quark EDMs can be written in tensor charges g_T

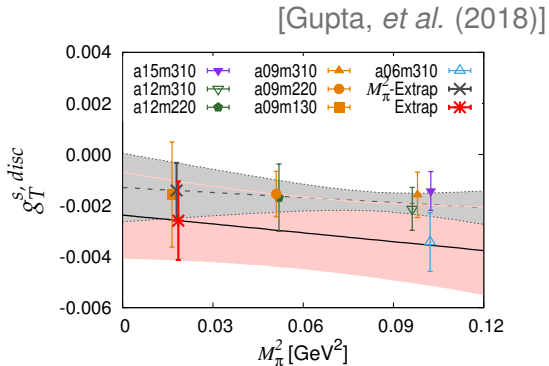
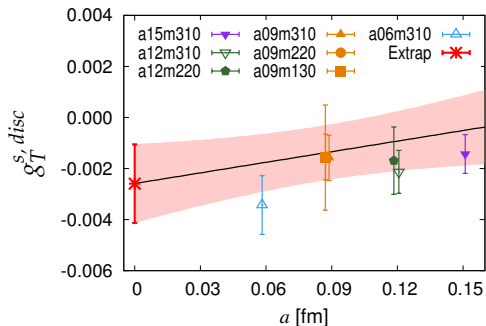
$$-\frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F) \gamma_5 q \quad \longrightarrow \quad d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

$$\langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle = g_T^q \bar{u}_N \sigma_{\mu\nu} u_N$$

- $d_q \propto m_q$ in many models \Rightarrow Precision determination of $g_T^{s,N}$ is important
- Requires computationally very expensive quark-line disconnected diagrams

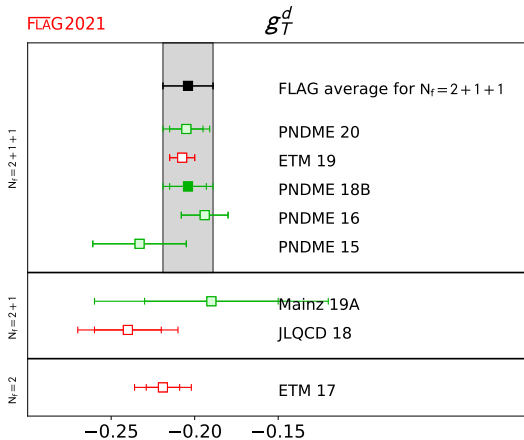
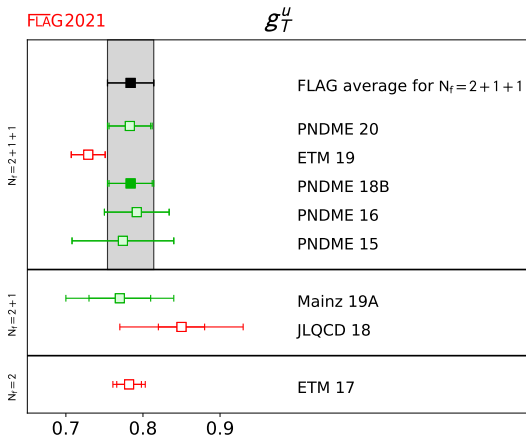


Calculation of Tensor Charge



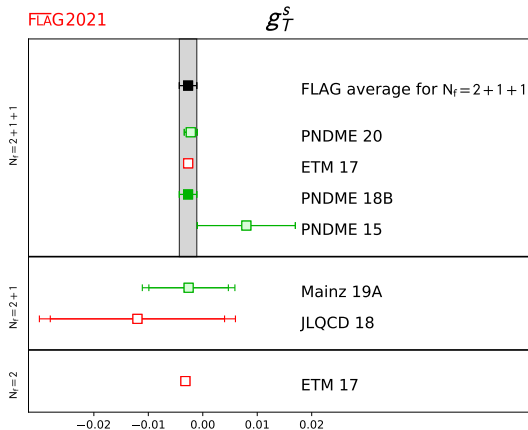
- Calculate matrix element $\langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle$ and remove ESC on each ensemble
- Renormalize $g_T^{u,d,s}$ to $\overline{\text{MS}}$ at 2 GeV and extrapolate the results to continuum limit
- Three-point functions that are easier to calculate than other EDM correlators
- Contributions from the disconnected diagrams, especially for g_T^s , are expensive

qEDM: Current Status



- $g_T^u = 0.784(28)(10)$, $g_T^d = -0.204(11)(10)$, $g_T^s = -0.0027(16)$ [PNDME 18B]
- See **Sungwoo Park's** talk at **LATTICE2022** for updated PNDME results

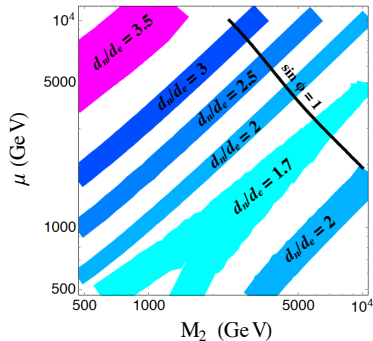
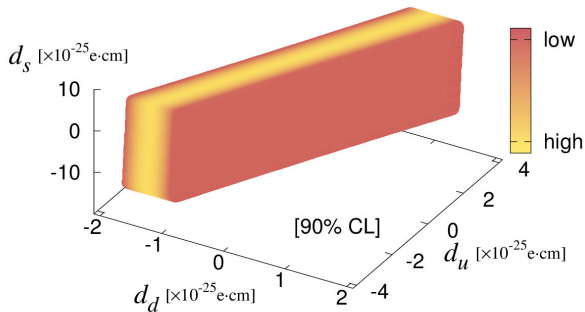
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BSM Constraints from qEDM

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$



[Bhattacharya, *et al.* (2015), Gupta, *et al.* (2018)]

- (L) Constraints on the BSM couplings $d_{u,d,s}$, assuming that these couplings give dominant contribution to nEDM
- (R) Regions in gaugino (M_2) and Higgsino (μ) mass plane corresponding to various values of d_n/d_e in split SUSY

Neutron EDM from quark Chromo-EDM (CEDM)

$$\begin{aligned}\mathcal{L}_{\text{CPV}}^{d\leq 6} &= -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G} && \text{dim}=4 \text{ QCD } \theta\text{-term} \\ &- \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q}(\sigma \cdot F)\gamma_5 q && \text{dim}=5 \text{ Quark EDM (qEDM)} \\ &- \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma \cdot G)\gamma_5 q && \text{dim}=5 \text{ Quark Chromo EDM (CEDM)} \\ &+ d_w \frac{g_s}{6} G\tilde{G}G && \text{dim}=6 \text{ Weinberg's } 3g \text{ operator} \\ &+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim}=6 \text{ Four-quark operators}\end{aligned}$$

Lattice QCD approaches for CEDM

$$S = S_{QCD} + S_{CEDM}; \quad S_{CEDM} = \frac{g_s}{2} \sum_{q=u,d,s} \tilde{d}_q \int d^4x \bar{q} (\sigma \cdot G) \gamma_5 q$$

- Three different approaches developed
 - [Schwinger source method](#) [Bhattacharya, *et al.* (2016)]:

$$D_{clov} \rightarrow D_{clov} + \frac{i}{2} \varepsilon \sigma^{\mu\nu} \gamma_5 G_{\mu\nu}$$

- Expansion in \tilde{d}_q [Abramczyk, *et al.* (2017)]:

$$\langle NV_\mu \bar{N} \rangle_{CEDM} = \langle NV_\mu \bar{N} \rangle + \tilde{d}_q \langle NV_\mu \bar{N} \sum_x O_{CEDM} \rangle + \mathcal{O}(\tilde{d}_q^2)$$

- External electric field method [Abramczyk, *et al.* (2017)]:

$$\langle N \bar{N} \rangle_{CEDM}(\vec{\mathcal{E}}, t) = \langle N(t) \bar{N}(0) O_{CEDM} \rangle_{\vec{\mathcal{E}}}$$

Power-divergent Mixing

- CEDM operator has **power-divergent mixing with the pseudoscalar operator**
[Bhattacharya, *et al.* (2015), Constantinou, *et al.* (2015)]
 - Gradient-flow technique can remove the power-divergent mixing
 - Alternative approach: **define a new operator to subtract power-divergence**

$$\tilde{C} \equiv i\bar{\psi}\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}T^a\psi - \frac{iA}{a^2}\bar{\psi}\gamma_5 T^a\psi$$

[Bhattacharya, *et al.* (2022)]

- The coefficient A should be set to cancel the ultraviolet divergence
- Determine A so that \tilde{C} does not create pion $\langle\Omega|\tilde{C}|\pi(\vec{p}=0)\rangle = 0$
- At $p = 0$, isovector P can be rotated away using non-anomalous Ward identity

$$Z_A(1 + b_A m a)\partial \cdot A + i a Z_{AC} \partial^2 P + 2 m i P - i a K \tilde{C} \approx 0$$

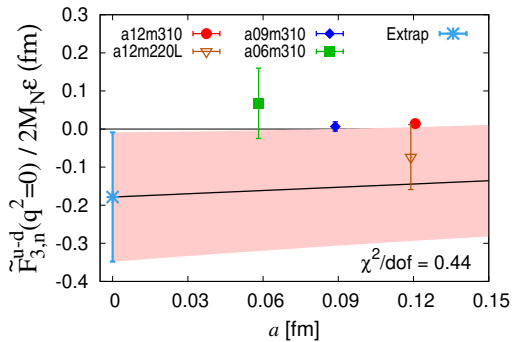
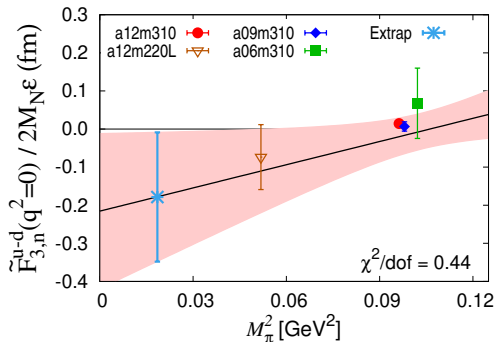
- All below three combinations describe the same physics

$$\left(aC - A\frac{P}{a}\right), \quad \left(\frac{\bar{y}}{\bar{x}} - A\right)\frac{P}{a}, \quad \left(1 - \frac{A\bar{x}}{\bar{y}}\right)aC$$

where A , \bar{x} , and \bar{y} are the coefficients that can be determined from pion correlators

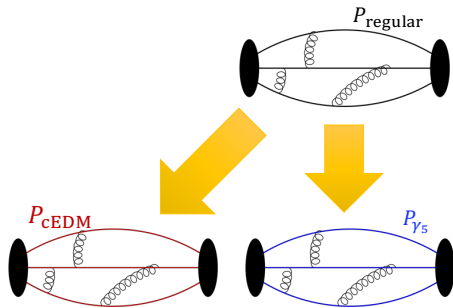
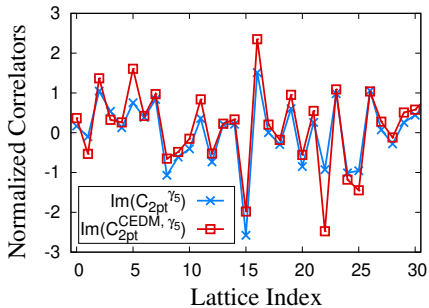
Results

[Bhattacharya, *et al.* (2022)]



- Preliminary data of bare (power-divergent-subtracted) operator \tilde{C}
- Results seem to have better signal than QCD θ -term for $M_\pi \approx 300$ MeV

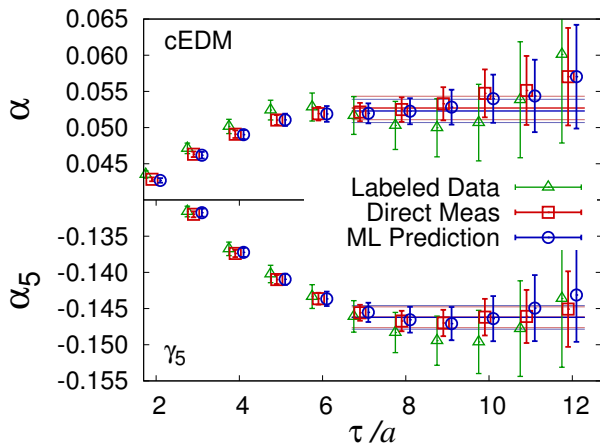
Machine Learning Predictions for CEDM Observables



- Calculation of quark propagators including CEDM interactions are expensive
- Plain C_{2pt} is highly correlated with the C_{2pt}^{CEDM}
- Machine learning can learn the correlation pattern and predict C_{2pt}^{CEDM} from C_{2pt}
- ML prediction error can be estimated statistically – unbiased estimators

[Yoon, *et al.* (2019)]

Prediction of C_{2pt}^{CEDM} from C_{2pt}



- CP violating phase
 $(ip_\mu\gamma_\mu + me^{-2i\alpha\gamma_5})\tilde{u} = 0$
- Training on 70 confs,
 bias correction on 50 confs,
 prediction for 400 confs
- ML algorithm is able to learn the
 computation of CPV observables
- Prediction errors are statistically
 included in the errorbars
 (unbiased estimates)

Neutron EDM from Weinberg's ggg

$$\begin{aligned}\mathcal{L}_{\overline{\text{CPV}}}^{d \leq 6} &= -\frac{g_s^2}{32\pi^2} \bar{\theta} G \tilde{G} && \text{dim}=4 \text{ QCD } \theta\text{-term} \\ &- \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F) \gamma_5 q && \text{dim}=5 \text{ Quark EDM (qEDM)} \\ &- \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q} (\sigma \cdot G) \gamma_5 q && \text{dim}=5 \text{ Quark Chromo EDM (CEDM)} \\ &+ d_w \frac{g_s}{6} G \tilde{G} G && \text{dim}=6 \text{ Weinberg's 3g operator} \\ &+ \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim}=6 \text{ Four-quark operators}\end{aligned}$$

Weinberg's 3g Op: Current Status and Future Prospects

$$\mathcal{L}_{W_{ggg}} = \frac{1}{6} d_w g_s G \tilde{G} G$$

- No publications yet, except a few preliminary studies
[Yoon, *et al.* (2019), Bhattacharya, *et al.* (2022)]
- Calculation is almost the same as the QCD θ -term, but signal seems to be noisier
- Requires renormalization
 - RI-MOM scheme and its perturbative conversion to $\overline{\text{MS}}$ is available
[Cirigliano, Mereghetti, and Stoffer (2020)]
 - Gradient flow scheme is a favorable option due to the complex mixing structure
[Rizik, Monahan, and Shindler (2020)]

Renormalization with Gradient Flow

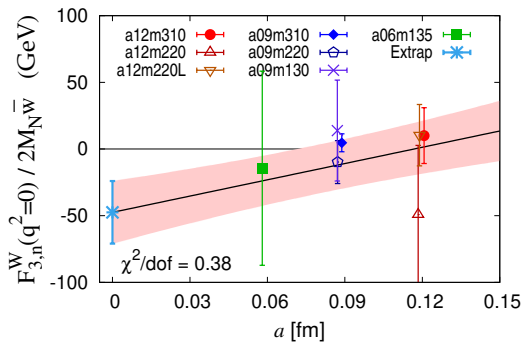
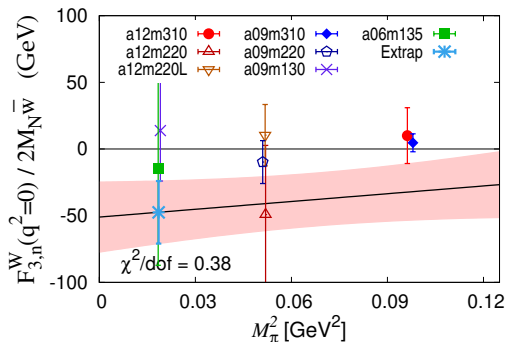
Gradient flow [Lüscher and Weisz (2011)]:

$$\begin{aligned}\partial_t B_\mu(t) &= D_\nu G_{\nu\mu}, & B_\mu(x, t=0) &= A_\mu(x), \\ \partial_t \chi(t) &= \Delta^2 \chi, & \chi(x, t=0) &= \psi(x)\end{aligned}$$

- Smear (flow) gluon and quark fields along the gradient of an action **to a fixed physical size τ_{gf} (sets ultraviolet cutoff of the theory)**
- The flowed operators have finite matrix elements except for an universal Z_ψ
→ Allow us to take continuum limit without power-divergent subtractions
- **Mixing and connection to the $\overline{\text{MS}}$: simpler continuum perturbation calculation**
- Related studies for CPV ops are recently started [Rizik, Monahan, and Shindler (2020)]

Results

[Bhattacharya, *et al.* (2022)]



- Preliminary data of bare operator in Gradient Flow scheme ($\tau_{gf} \approx 0.34$ fm)
- W_{ggg} in Gradient flow scheme has $\mathcal{O}(1/\tau_{gf})$ mixing with topological charge [Rizik, Monahan, and Shindler (2020)]
- Results sensitive to τ_{gf}

Conclusion

- We are calculating matrix elements for neutron EDM from SM and BSM
- The matrix elements will play a crucial role in connecting experiments and BSM
- Significant progress has been made in formulating problems and carrying out preliminary studies
- Large computational resources will be needed
- Inputs from phenomenologists will help us formulating the problems and guiding the continuum/chiral/excited-state extrapolations