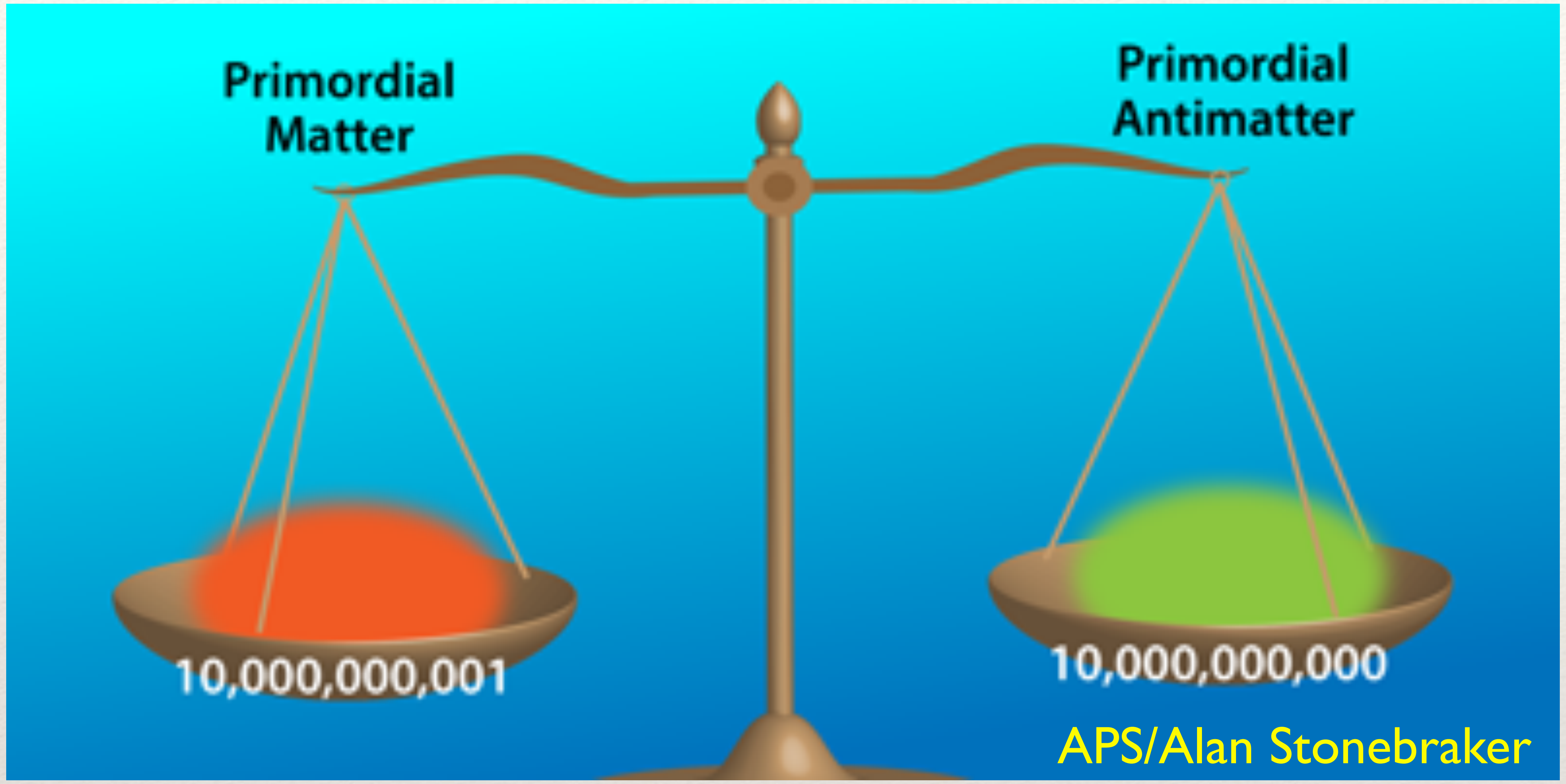
Estimating CP-violating nucleon matrix elements from CP-conserving ones

ECT* Workshop:
Neutron Electric Dipole Moment: from Theory to Experiment
August 1 - 5

André Walker-Loud

1 slide of fluff



The universe is matter dominated at about 1 part-per-billion

$$\eta \equiv \frac{X_N}{X_\gamma} = 6.19(10) \times 10^{-10}$$

While this is a very small number, it is orders of magnitude larger than we understand from known CP-violation within the SM

- Highly suggestive of BSM CP-violating physics
- CPT → CP-violation → T-violation → permanent EDMs
- Significant effort to search for EMDs, e, n, p, D, t, ^{3}He, …, ^{199}Hg, ^{225}Ra, ^{229}Pa, …
- As I'm learning from Michael R-M, even if the SM had sufficient CP-violation, We need BSM physics to explain the phase transition

Sources of CP violation

- Assume CP-violating physics is heavy $\rightarrow$ use SM Effective Lagrangian to encode BSM
  - Ignore SM weak-sector CP-violation in quark-mixing matrix
  - Ignore potential CP-violation in neutrino-mixing matrix (but $0\nu\beta\beta$ is another fun topic)
- 1 dimension-4 operator, the QCD θ-term

$$\mathcal{L}_{\rm CPV}^{\rm QCD} = -\frac{\alpha_S \bar{\theta}}{8\pi}\tilde{G}G$$

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- What we do know - $\bar{\theta}$ seems to be too small to explain the observed matter/anti-matter asymmetry
- We speculate their are additional higher-dimensional operators which can give rise to CP-violation

Sources of CP violation

Operator	[Operator]	No. Operators (for 2-flavors)
$\bar{\theta}$	4	1
quark EDM	6	2
quark chromo-EDM	6	2
Weinberg (GGG)	6	1
4-quark	6	2
4-quark induced	6	1

- These operators will give rise to nucleon EDMs, and hence, nuclear EDMs
- As is well known, the full set of EDM operators mix under renormalization (if you have one, you have them all)
- At very high-energies, the QCD-induced mixing is perturbative, and they can be related to SMEFT parameters
- At nuclear scales, we have effective operators which are linear combinations of the full set of SMEFT sources of CP-violation - it is also an important challenge to relate the SMEFT operators to potential nucleon and nuclear EDMs

Sources of CP violation

- ❑ At nuclear scales - we can relate operators at the quark level to those in a low-energy EFT
- ❑ EDMs of nucleons and light nuclei can then be computed in terms of these EFT operators
- ❑ Focus on just a few operators for simplicity

Operator	[Operator]	No. Operators
$\bar{\theta}$	4	1
quark Chromo-EDM	6	2

$$\mathcal{L}_{CPV} = -\frac{g_s^2 \bar{\theta}}{32\pi^2} \tilde{G}_{\mu\nu} G^{\mu\nu} - \frac{i}{2} \bar{q} \sigma^{\mu\nu} \gamma_5 \left(\tilde{d}_0 + \tilde{d}_3 \tau_3 \right) G_{\mu\nu} q$$

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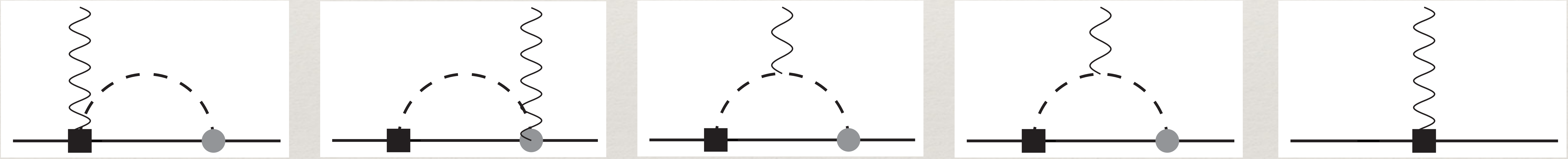
$$\mathcal{L}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N - \frac{\bar{g}_1}{2F_\pi}\bar{N}\pi_3 N - \frac{\bar{g}_2}{2F_\pi}\pi_3\bar{N}\left(\tau_3 - \frac{\pi_3}{F_\pi}\vec{\pi}\cdot\vec{\tau}\right)N$$

- How are these $\bar{g}_i$ couplings related to the quark-level ones?
- For the θ-term, there is the well known relation with chiral symmetry - we can perform a U(1) axial rotation to shift the θ-operator into a phase of the quark mass matrix

θ-term example

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f \left[i\not{D} - m_f \right] \psi_f - \frac{1}{4} G^2 - \bar{\theta} \frac{\alpha_S}{8\pi} \tilde{G} G \xrightarrow[U(1)_A]{} \mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f \left[i\not{D} - m_f e^{i\gamma_5 \bar{\theta}} \right] \psi_f - \frac{1}{4} G^2$$

- Then, since we know how to map QCD to χPT, we do this with a modified quark mass
- The nucleon EDM can then be computed - it arises from a radiative pion correction



$$d_n = \bar{g}_0 \frac{e g_A}{(4\pi F_\pi)^2} \ln \left(\frac{M_\pi^2}{\mu^2} \right) + \bar{\theta} \frac{m_u m_d}{m_u + m_d} \frac{e}{(4\pi F_\pi)^2} c(\mu) \qquad \bar{g}_0 = 2\alpha \frac{2 m_u m_d}{m_u + m_d} \bar{\theta}$$

θ-term example

- Additionally, the $SU(2)_A$ symmetry of QCD can be exploited to relate the CP-odd coupling to the isospin breaking quark mass contribution to M_n-M_p
  Crewther, Vecchia, Veneziano, Witten, Phys.Lett. 91B (1980)

$$\bar{g}_0 = \frac{\delta M_{n-p}^{m_d-m_u}}{m_d - m_u} \frac{2 m_d m_u}{m_d + m_u} \bar{\theta}$$
$$= \frac{\delta M_{n-p}^{m_d-m_u}}{m_d - m_u} \frac{\hat{m}^2 - \delta^2}{\hat{m}} \bar{\theta}$$

$$\delta M_{n-p}^{m_d-m_u} = \lim_{\alpha_{f.s.}=0} M_n - M_p$$

$$\hat{m} = \frac{1}{2}(m_d + m_u) \qquad \delta = \frac{1}{2}(m_d - m_u)$$

- This is a powerful idea - relate phenomenologically interesting CP-violating physics to simple spectroscopic CP-conserving physics

- Exploiting this idea, plus nuclear modeling uncertainty, is how we place a constraint on $\bar{\theta}$ through the limit on the ^{199}Hg EDM measurement

θ-term example

- Jordy, Emanuele and I checked how well this relation holds at N^{2}LO in SU(3) heavy baryon χPT (HBχPT) - de Vries, Mereghetti, Walker-Loud PRC92 (2015) [1506.06247]

- While SU(3) HBχPT can not be used for precision predictions
  It is great for a qualitative assessment - how large are SU(3) breaking corrections?

  The relation between $\bar{g}_0$ and $\delta M_{n-p}^{m_d-m_u}$ is preserved through N^{2}LO, up to different N^{2}LO LEC contributions which are estimated to be a few percent

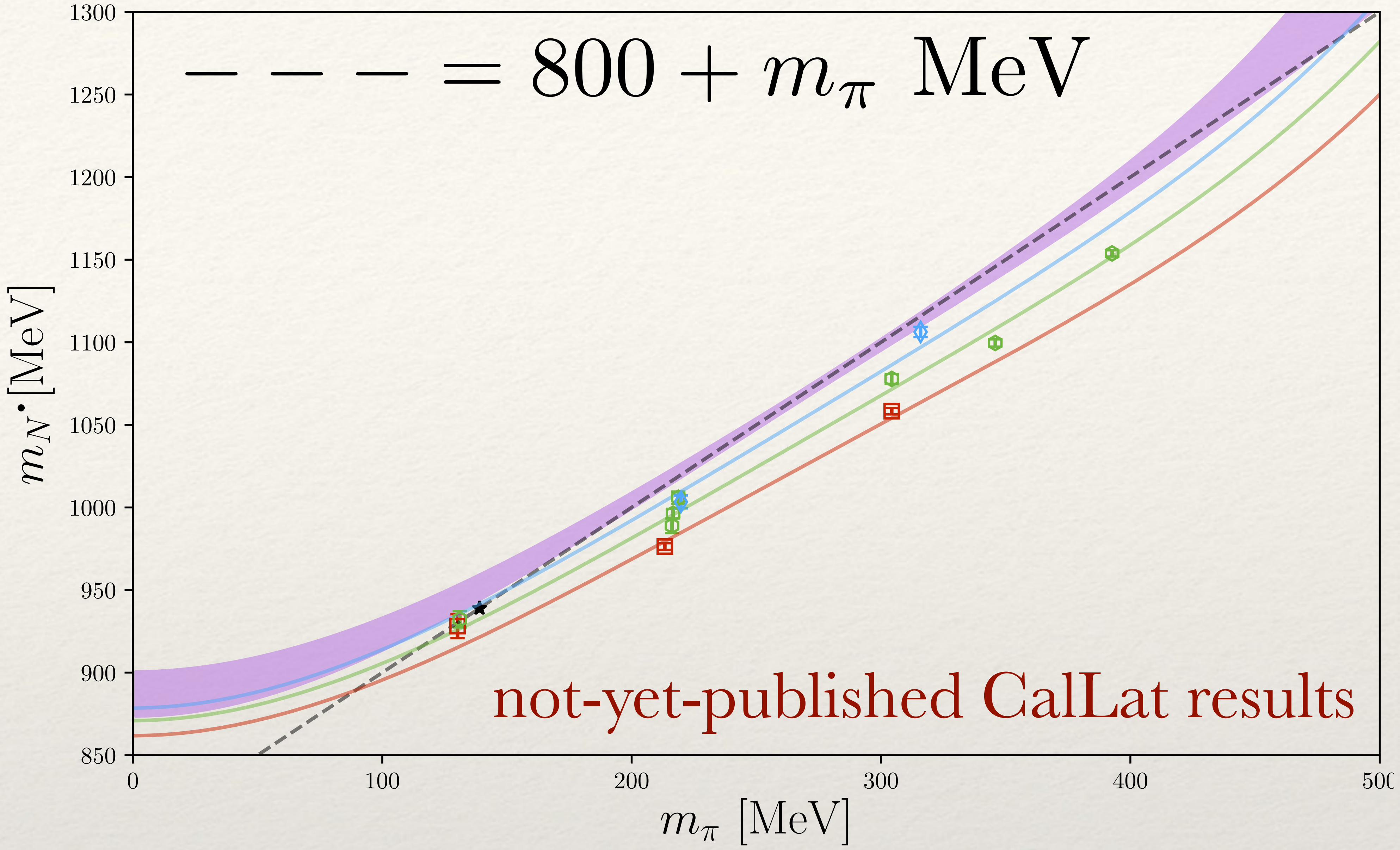
- Can we use the same idea to understand BSM sources of CP-violation?

θ-term example

- Before shifting to BSM - a few important points
- Lattice QCD calculations of the nucleon EDMs are very challenging (see other talks, and limited results in the literature)
- The EDMs can be predicted with results from heavier than physical Mπ by utilizing known chiral extrapolation formula (as others emphasize)
- Given the challenges evident in SU(2) HBχPT from LQCD results
  - It is important to check the convergence of the expansion
  - The strategy should be to perform simultaneous calculations and extrapolations of the nucleon mass splitting combined with EDMs

Nucleon Mass versus Pion Mass

- The expansion of the nucleon mass has difficulties describing lattice QCD results



$$M_N = M_0 + \sigma \frac{M_\pi^2}{4\pi F} - \frac{3\pi g_A^2}{2} \frac{M_\pi^3}{(4\pi F)^2} + \alpha \frac{M_\pi^4}{(4\pi F)^3} + \beta \frac{M_\pi^4}{(4\pi F)^3} \ln\left(\frac{M_\pi^2}{\mu^2}\right) + \cdots$$

- Note: @ NLO, there is a predicted large negative contribution proportional to g_A^2

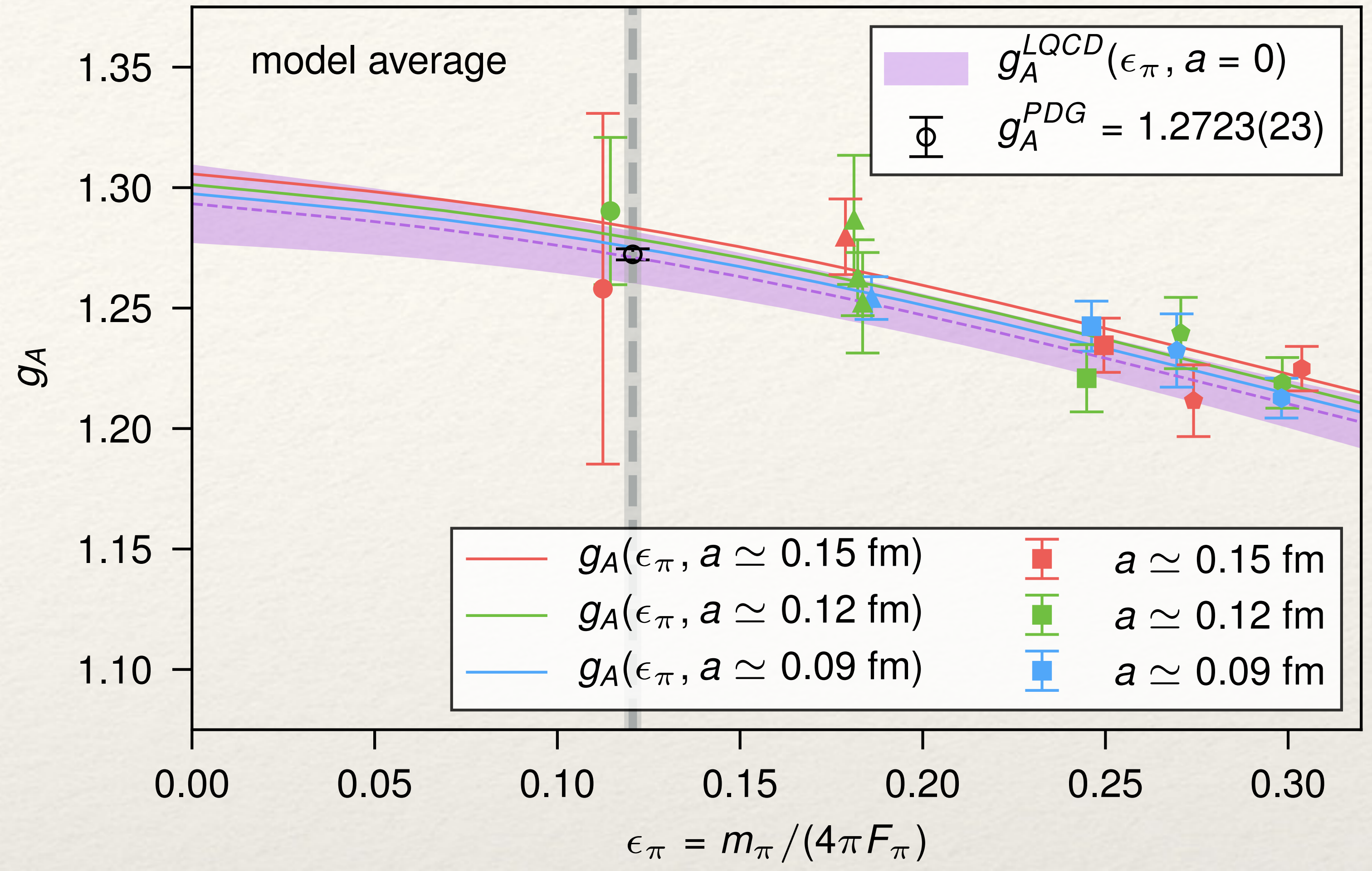
- Given the LQCD results, this is an unnatural contribution
  - Leaving g_A as a free parameter in the extrapolation - the fit returns 0
  - It is not satisfactory to force $g_A \approx 1.27$ (eg. with a prior with a tight width) We would like to see this emerge naturally from a (numerical) data-driven approach
  - Solution: do simultaneous extrapolation of M_N and g_A - this is in the works for us

g_A versus Pion Mass

Nature 558 (2018) no.7708, 91-94
Chang et al.
[arXiv:1805.12130]

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- More precise results at the physical pion mass will improve the three largest uncertainties:
  - statistical (s), extrapolation (χ) and model selection (M) **NOTE**, **a12m130** has 2.3% uncertainty
- Following our existing strategy, we anticipate getting to 0.5% by the end of this year
- Getting below (or maybe to 0.5%) will require a 4th lattice spacing as well (~0.06fm)
- Adding a FV study on additional pion mass points will improve the FV uncertainty
- The isospin uncertainty seems unnecessary… we now know radiative QED corrections can be O(2%)

Cirigliano, de Vries, Hayen, Mereghetti, Walker-Loud [2202.10439]
to be published in PRL

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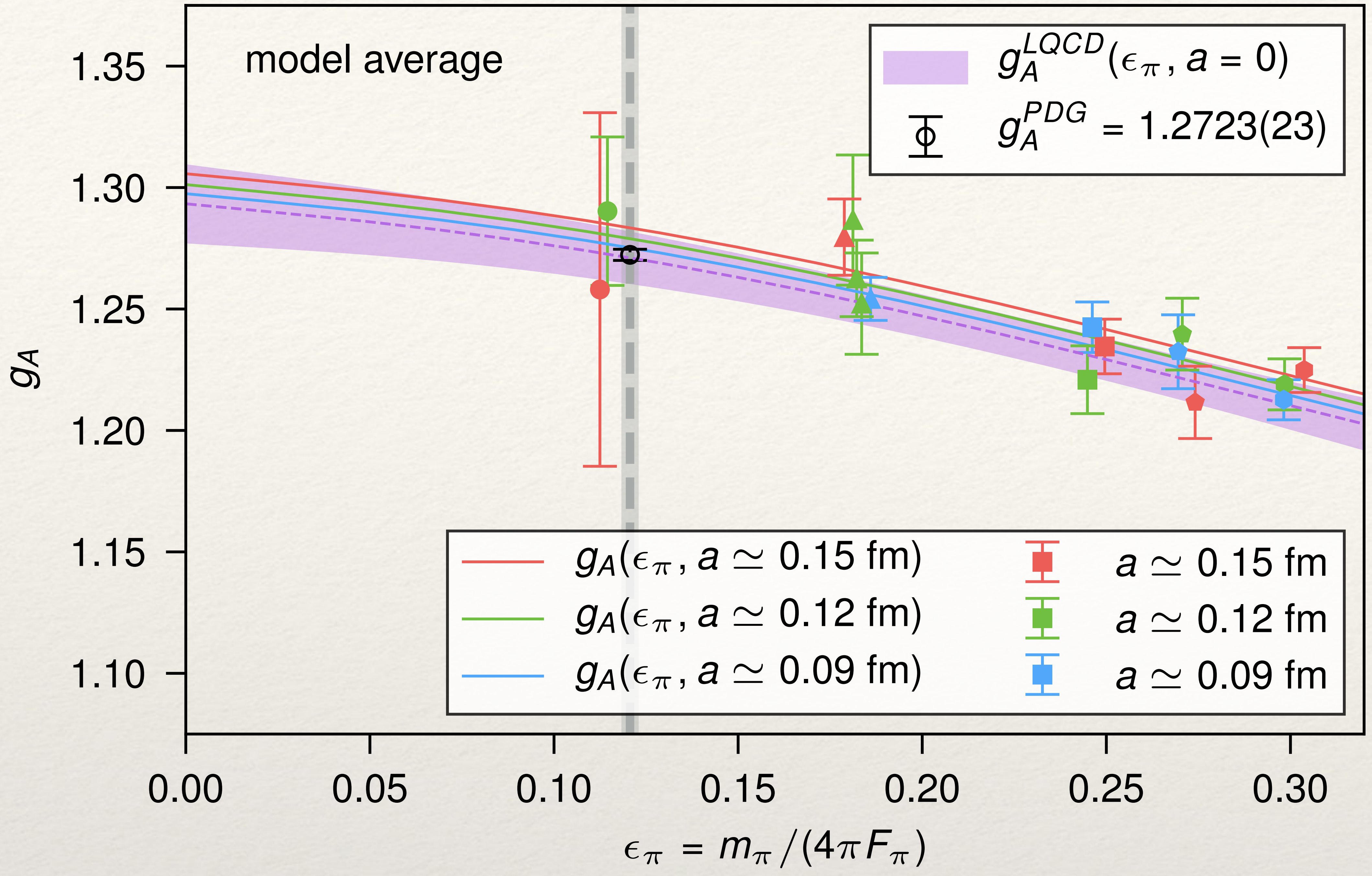
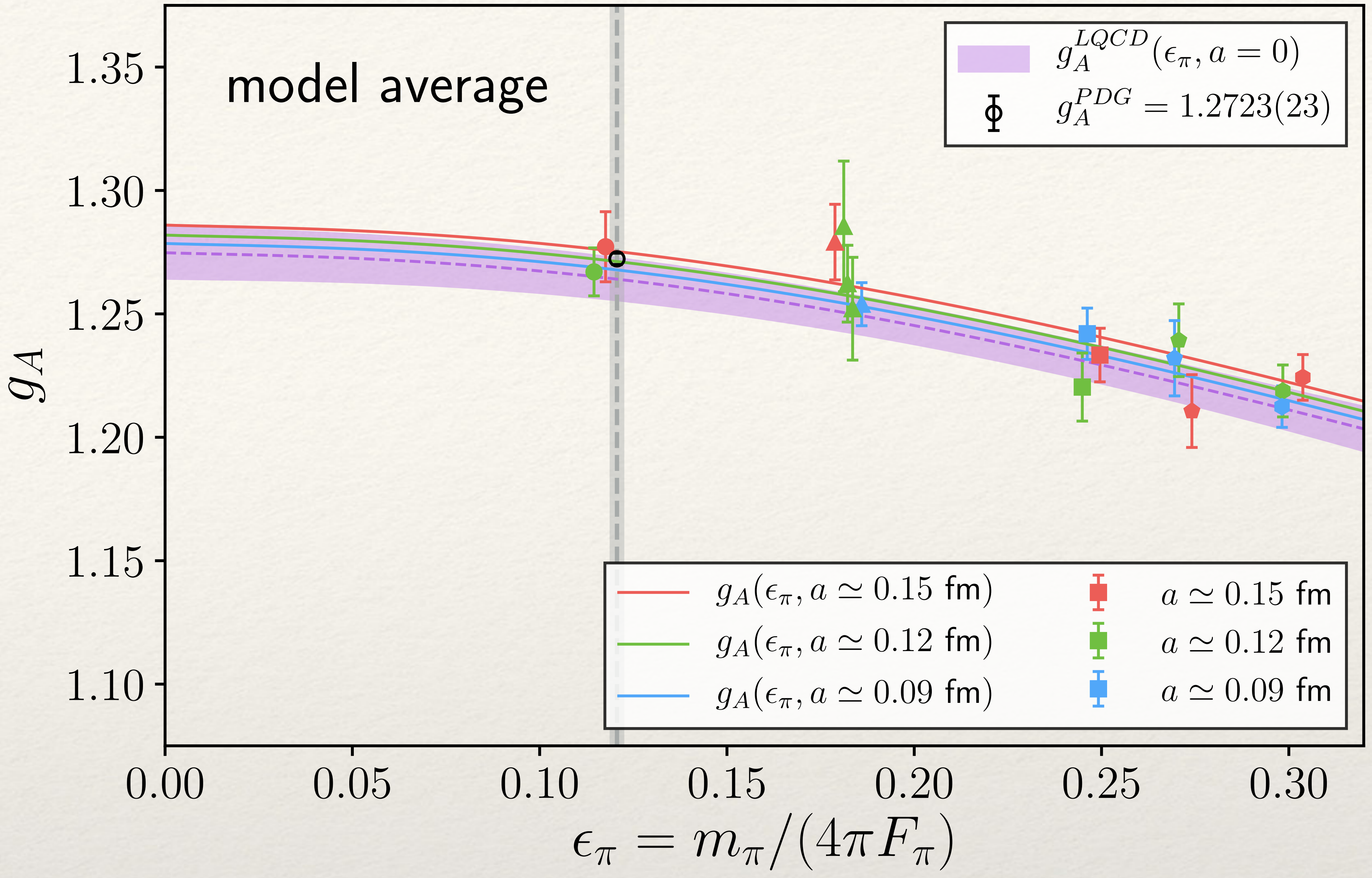
1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



Sierra Early Science

PRELIMINARY



California Lattice Collaboration

- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$

$$g_A = 1.2711(125) \rightarrow 1.2641(93) \quad [0.74\%]$$

- We have 2 additional pion masses (180, 260) and a 4th finer lattice spacing, a≈0.06fm @ $M_\pi \approx 220$, 310 MeV
  - We anticipate improving g_A to ~0.5% — we need to address the radiative QED correction to make this useful

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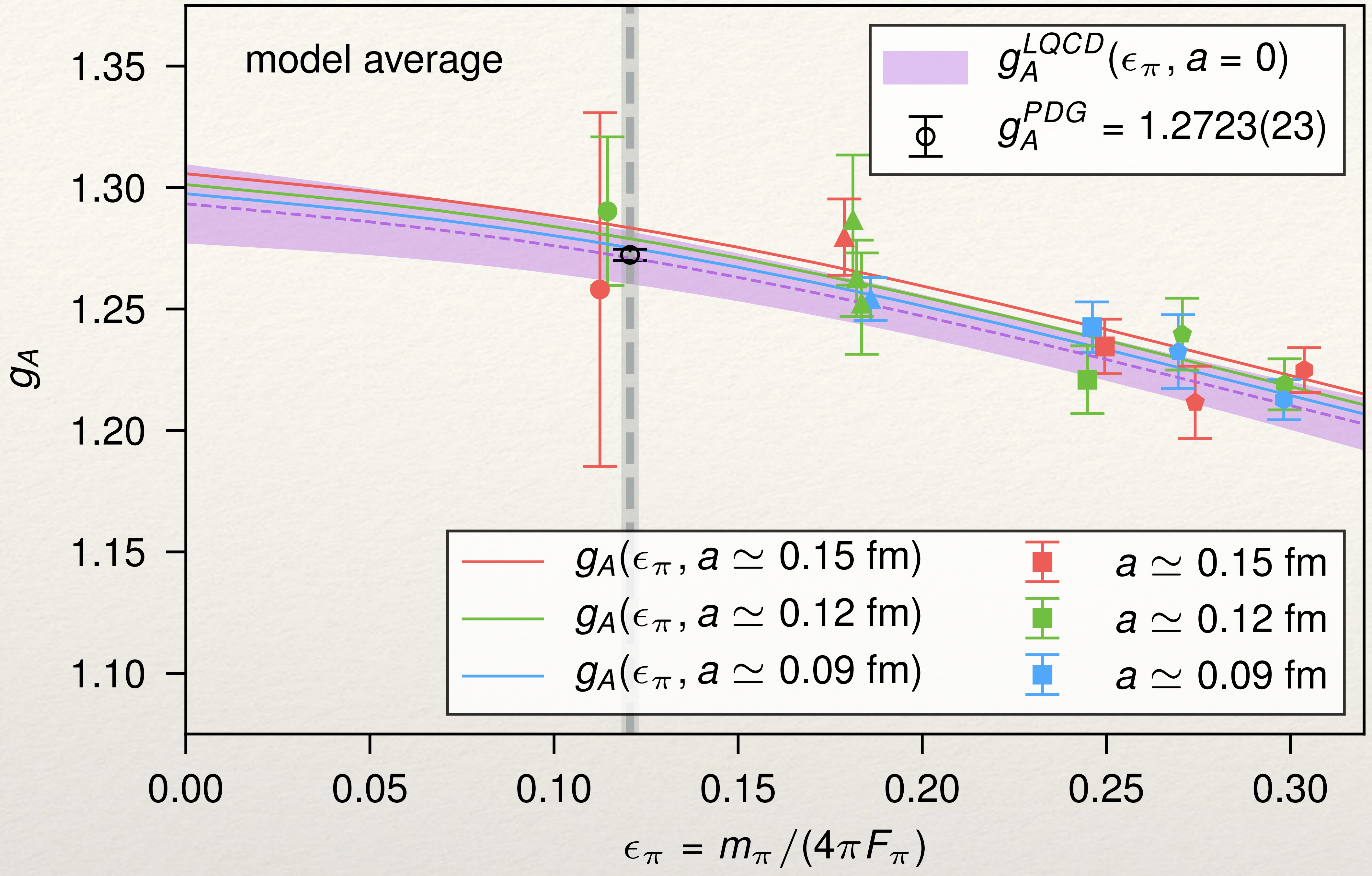

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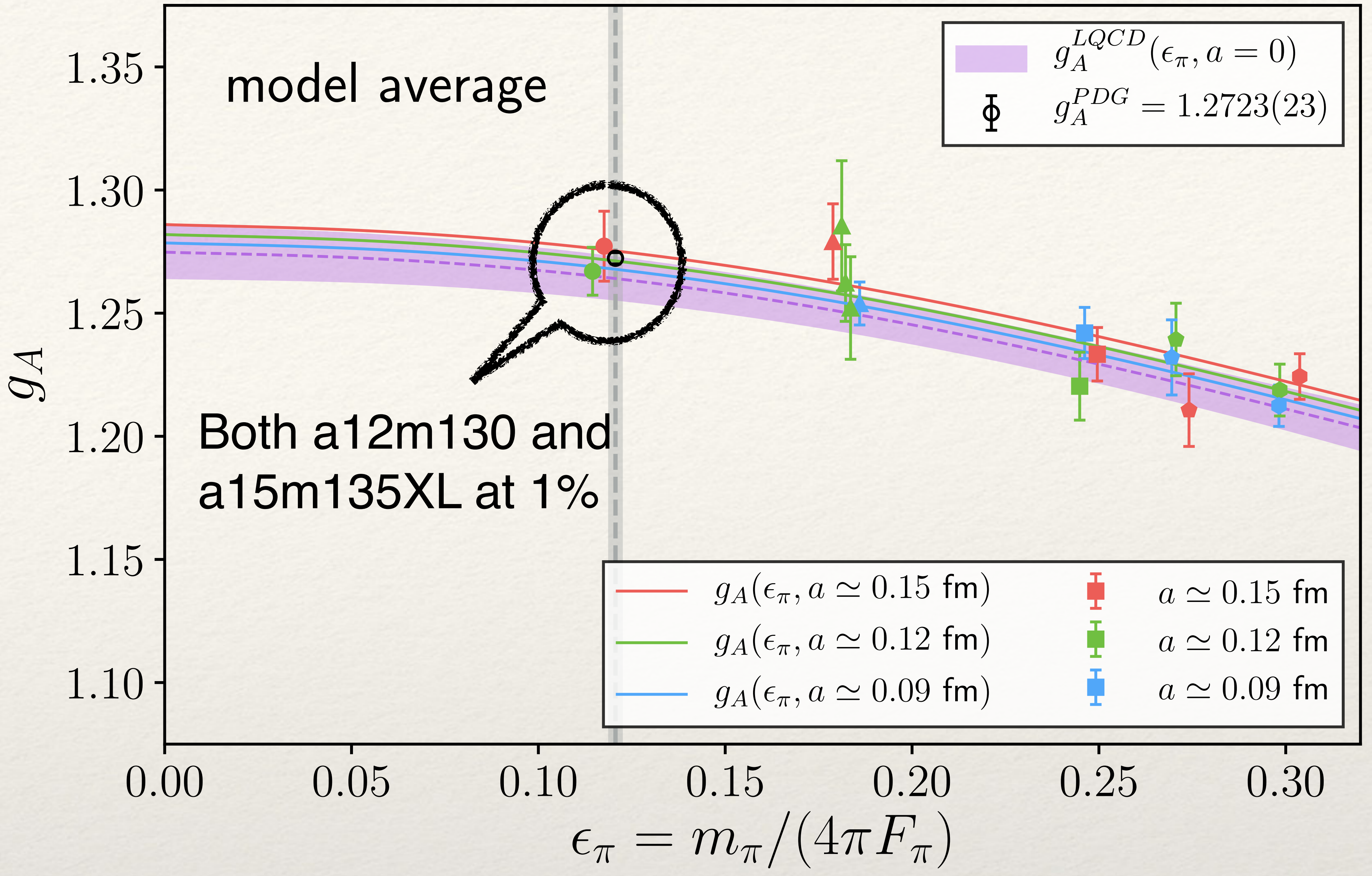

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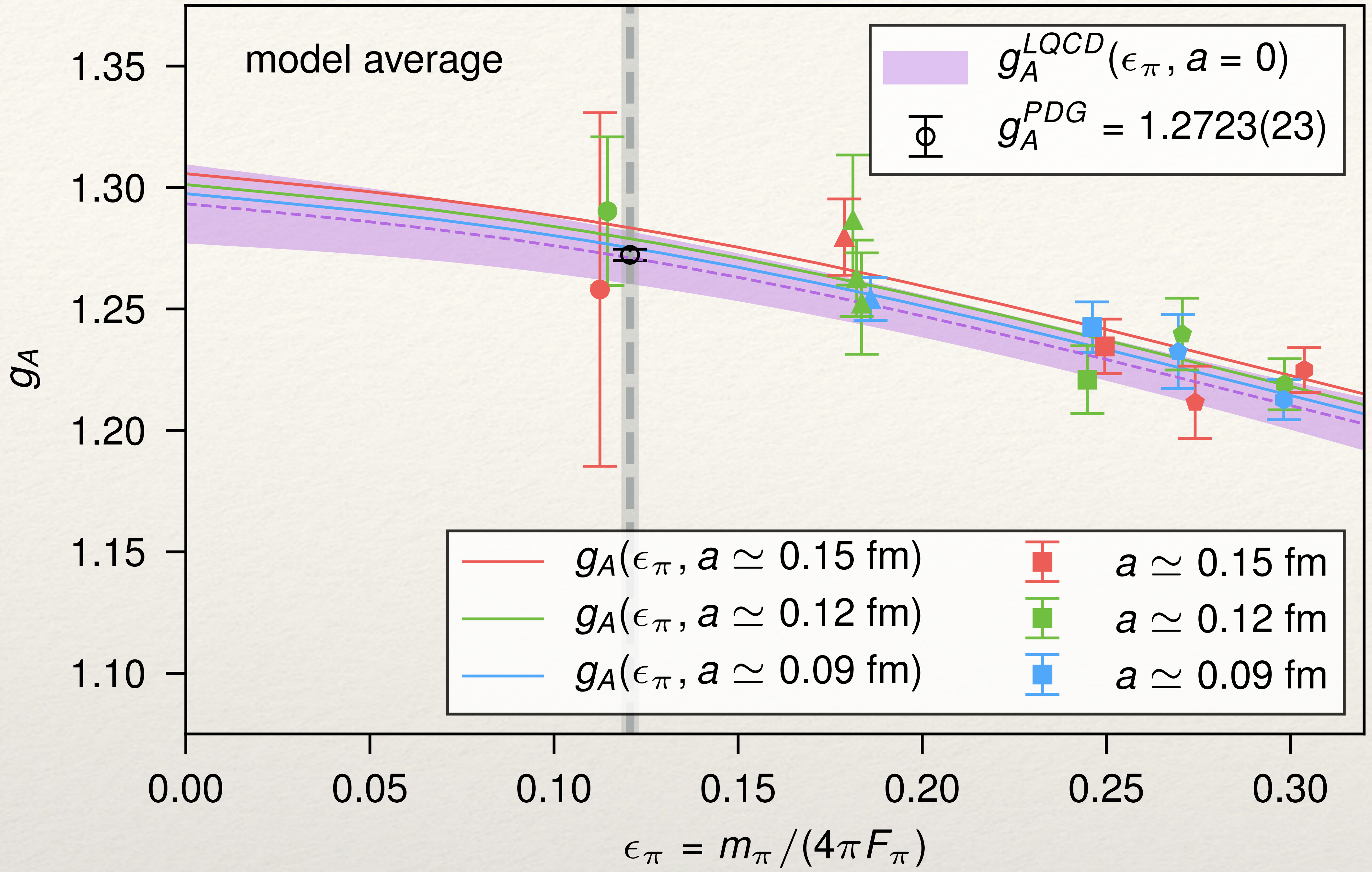

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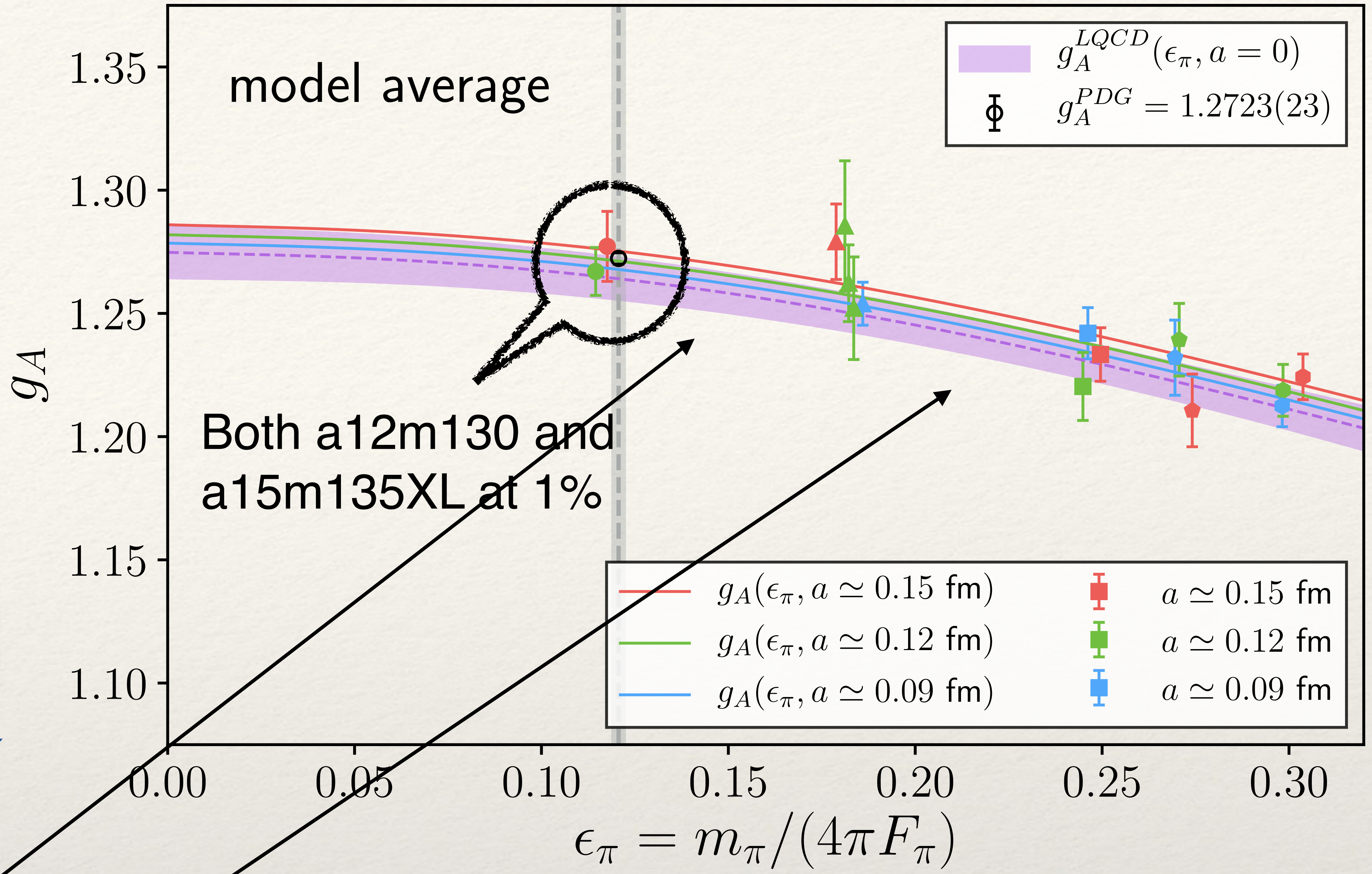

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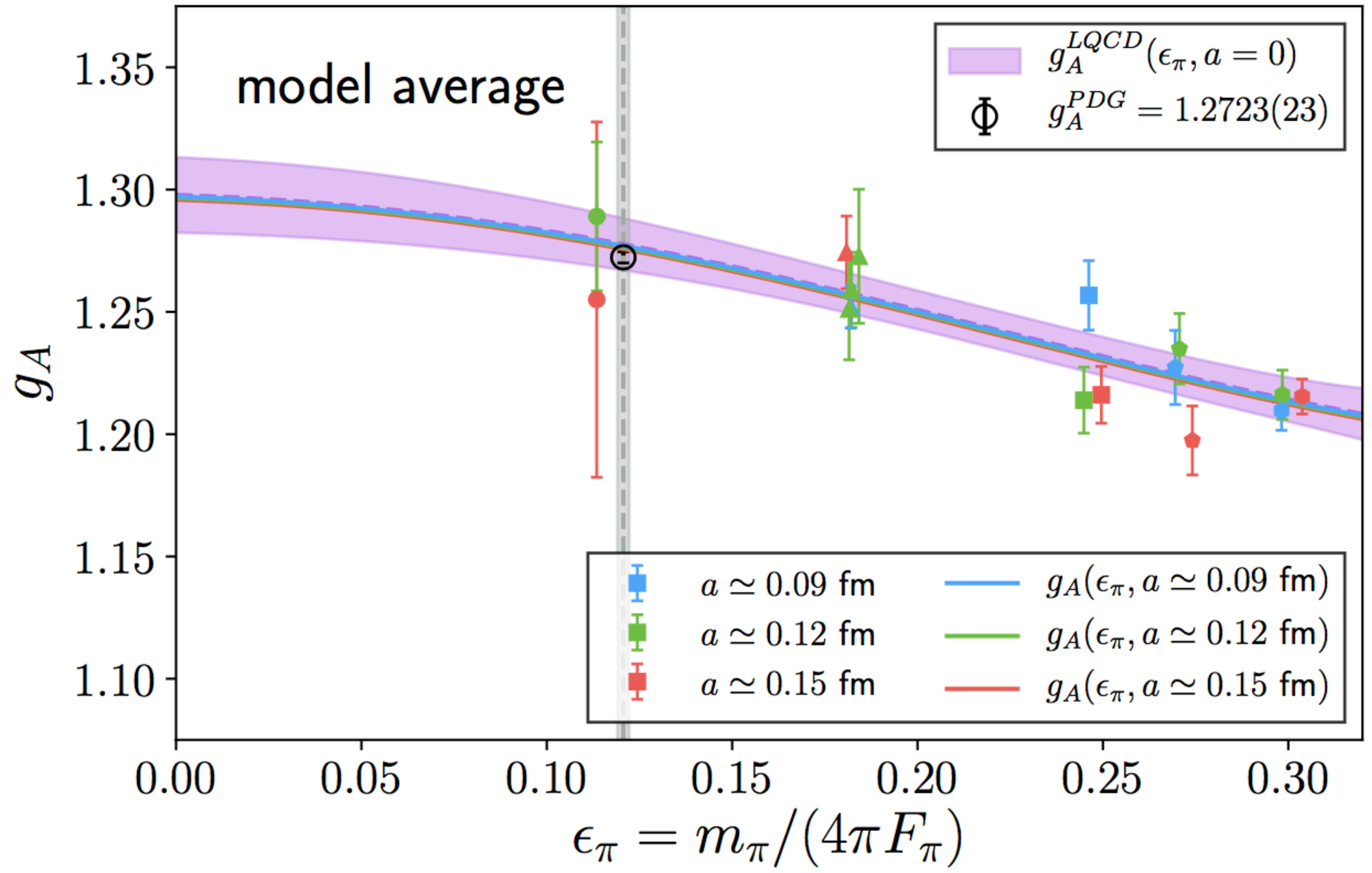
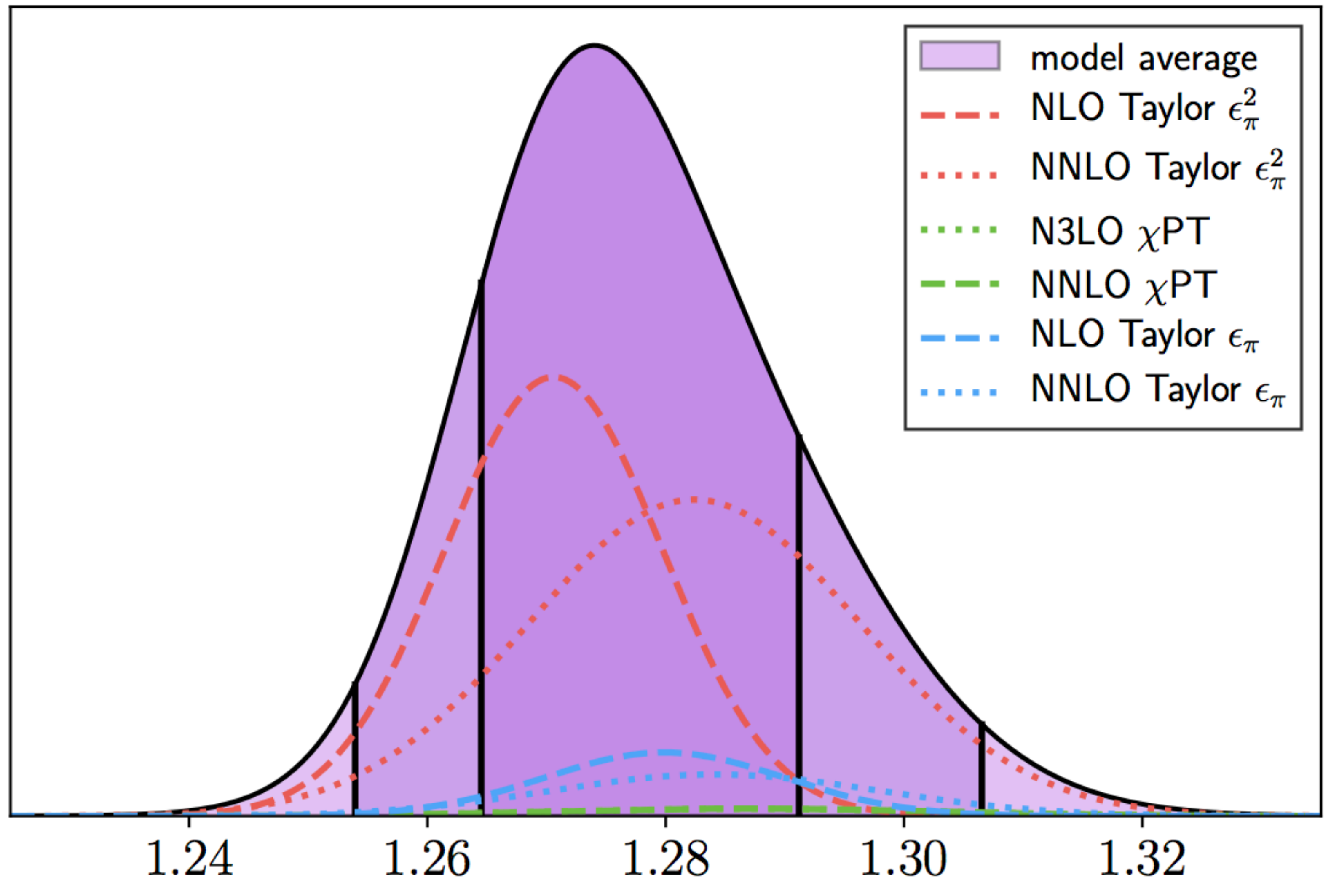
Analysis Details

Model average extrapolation





Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
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NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$

NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

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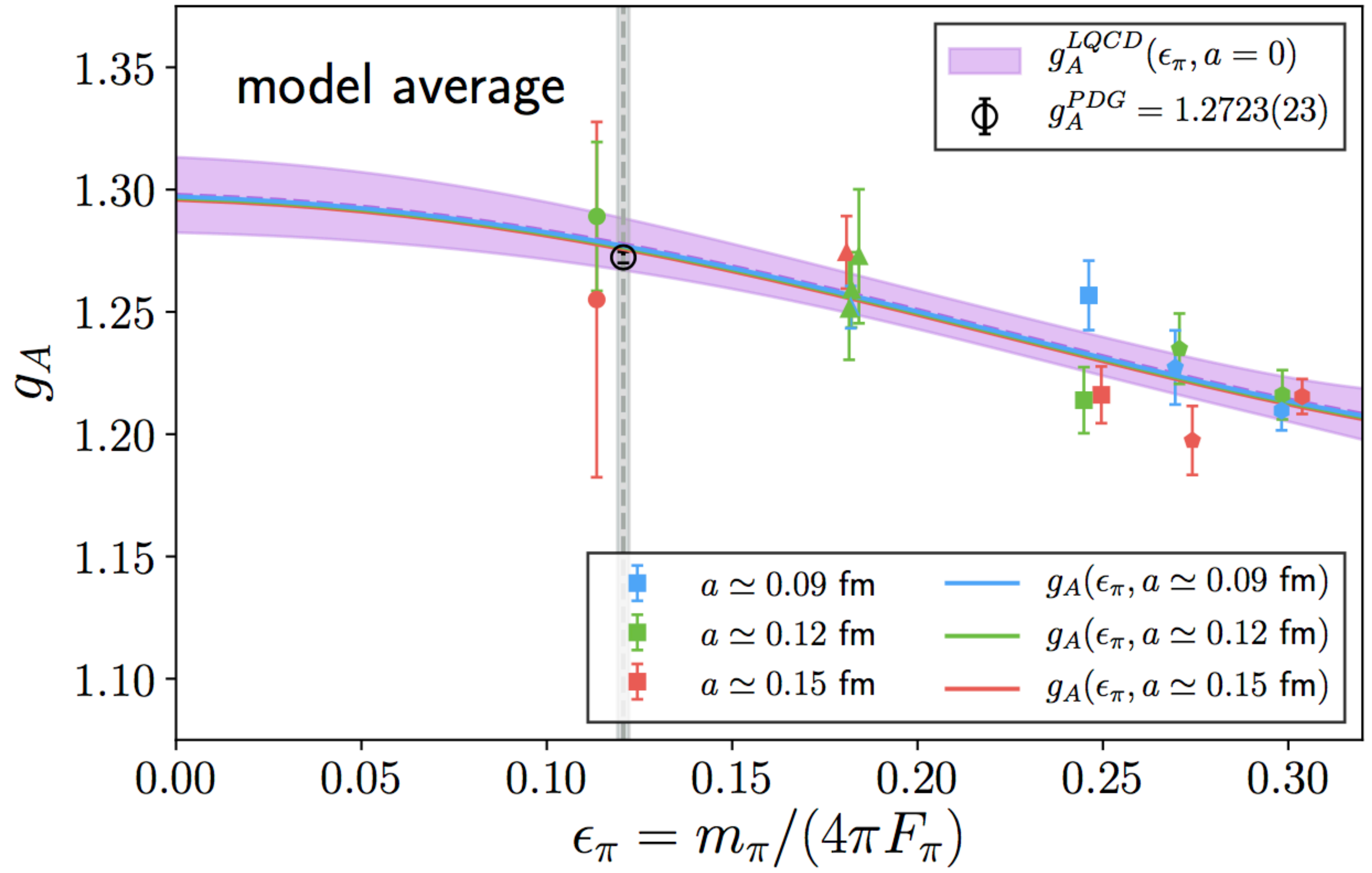

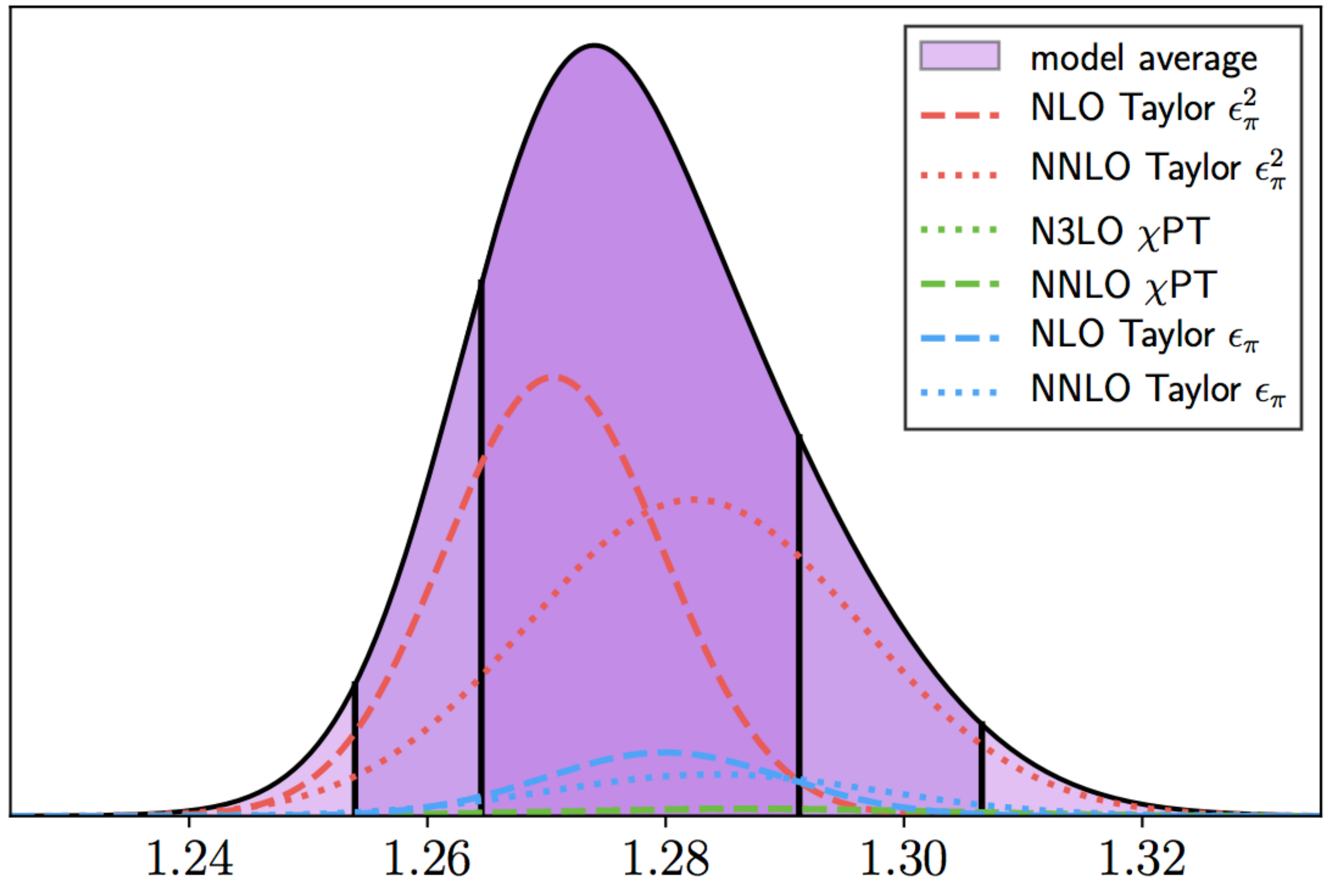

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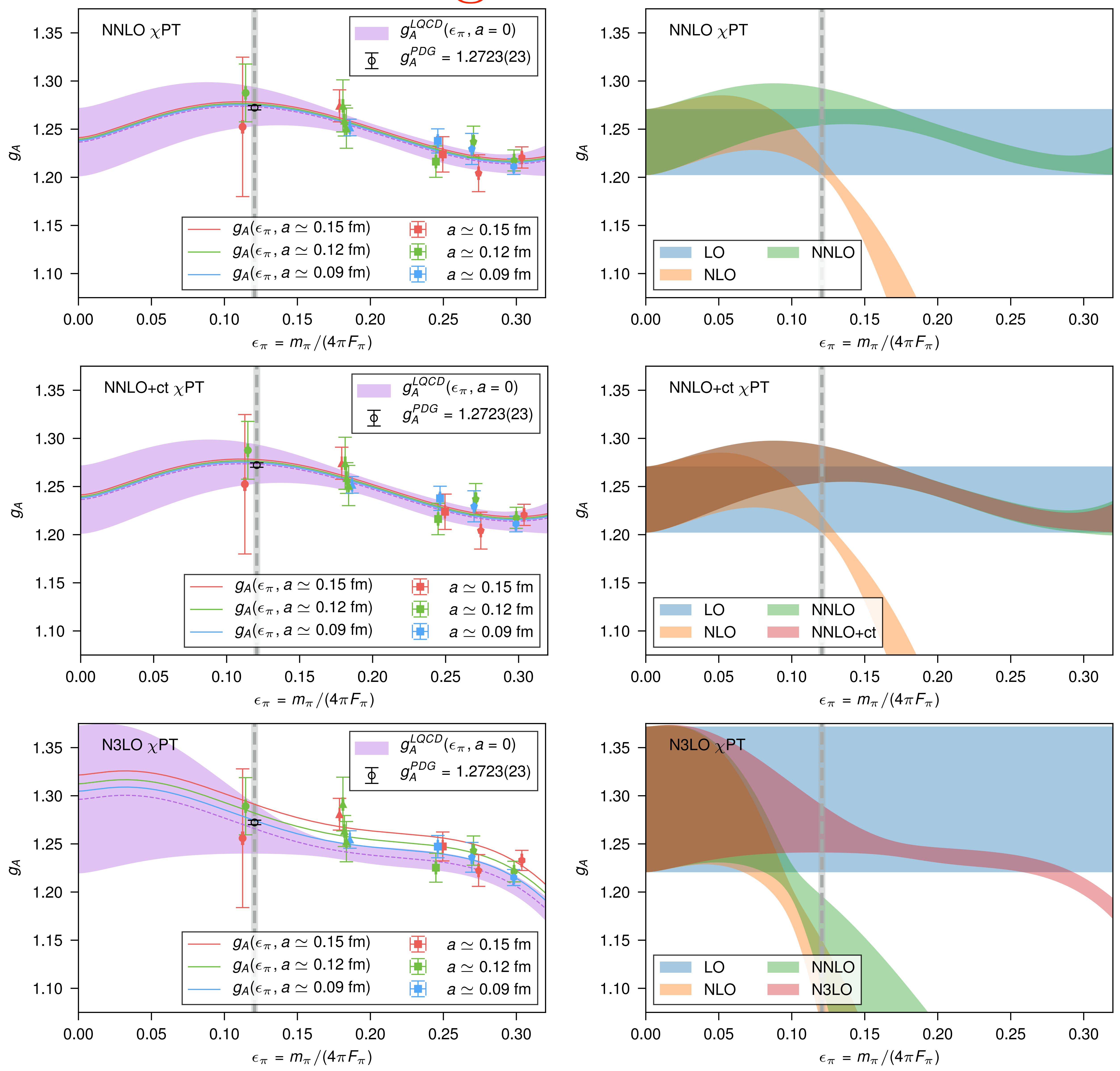
The numerical results “do not like χPT” 😱

convergence of the chiral expansion…



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

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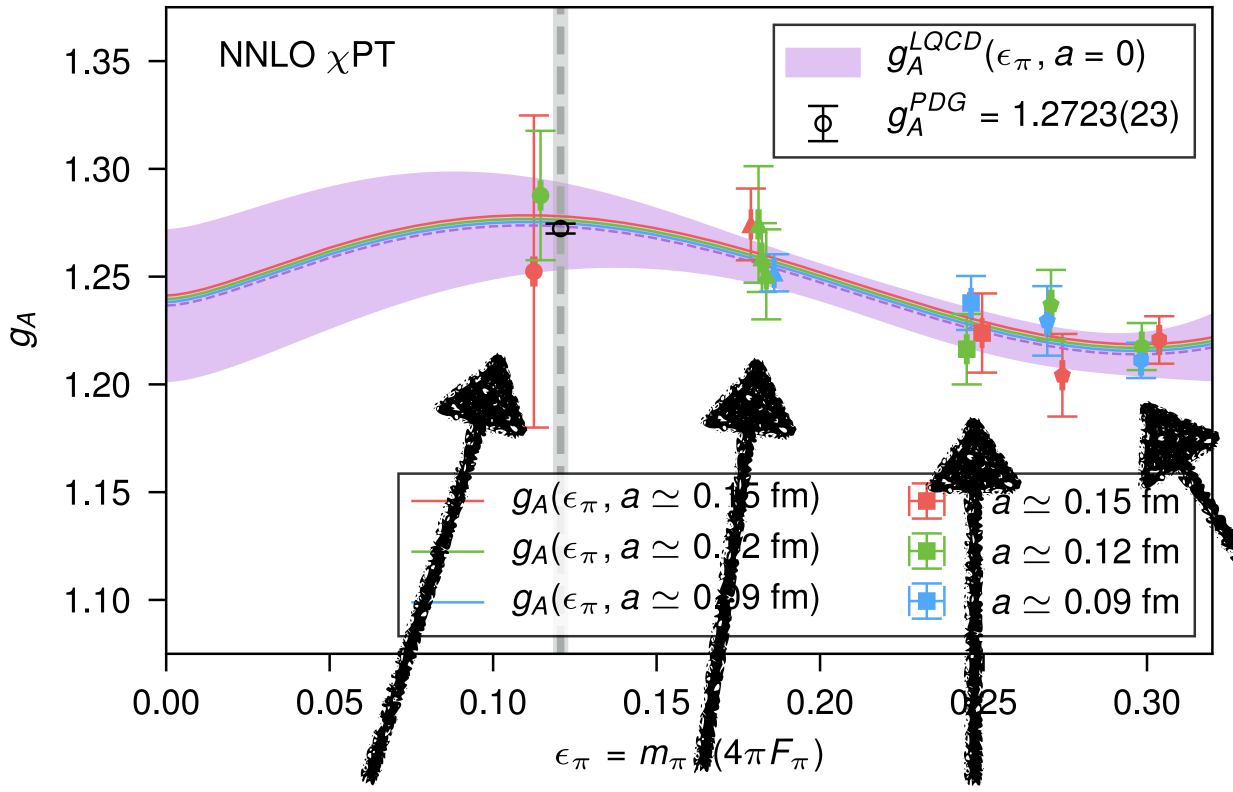
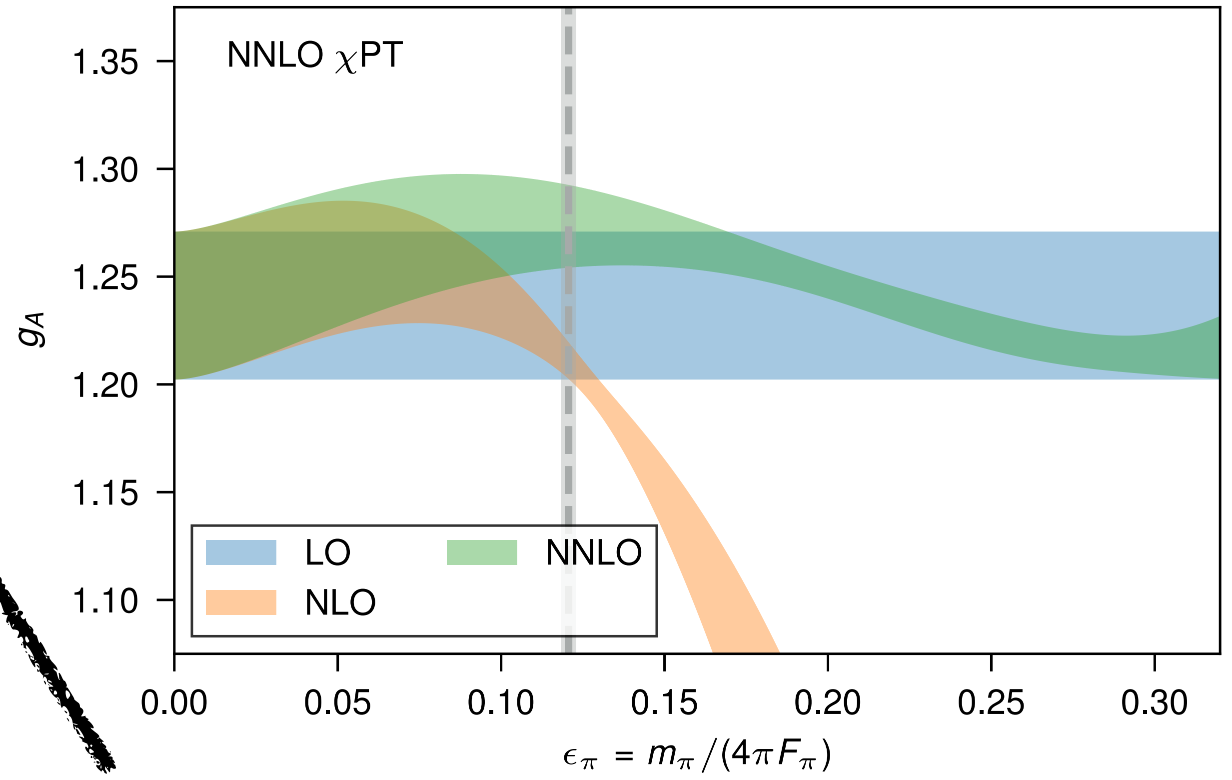
Bernard and Meissner (CD06)

Phys.Lett.B639 [hep-lat/0605010]

F —> F_π

convergence of the chiral expansion…





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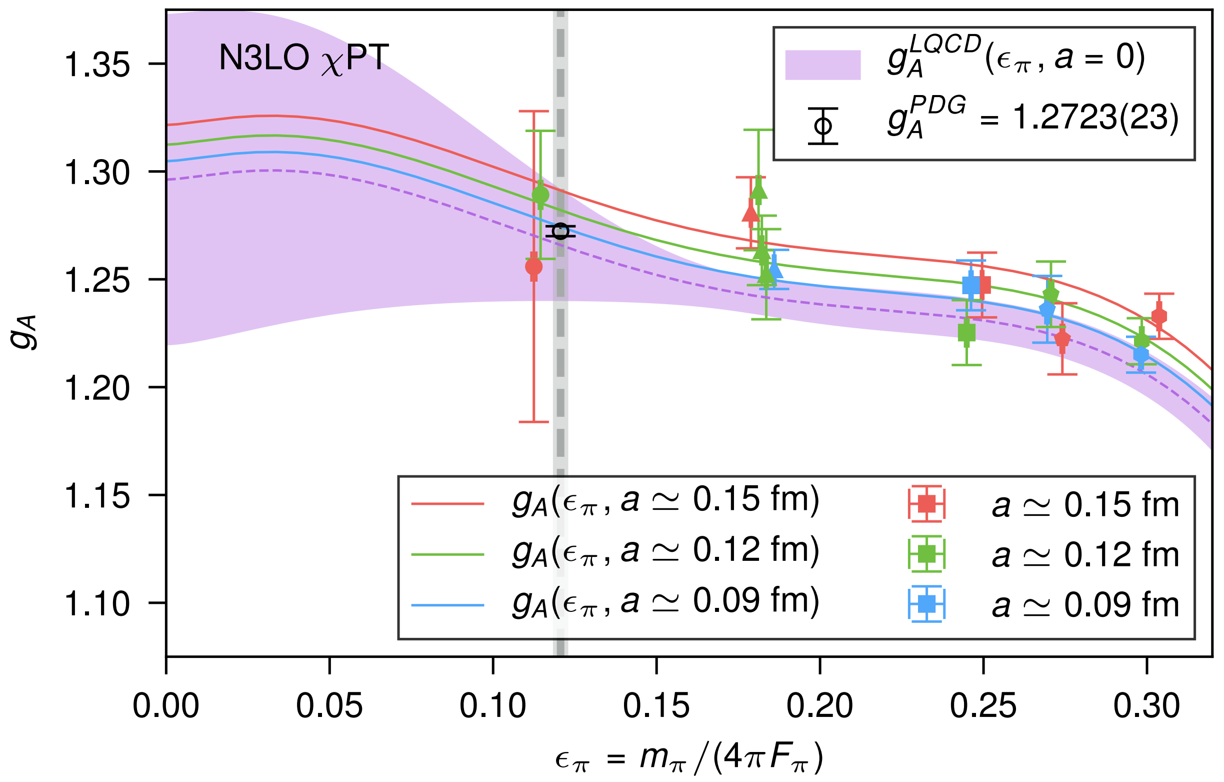
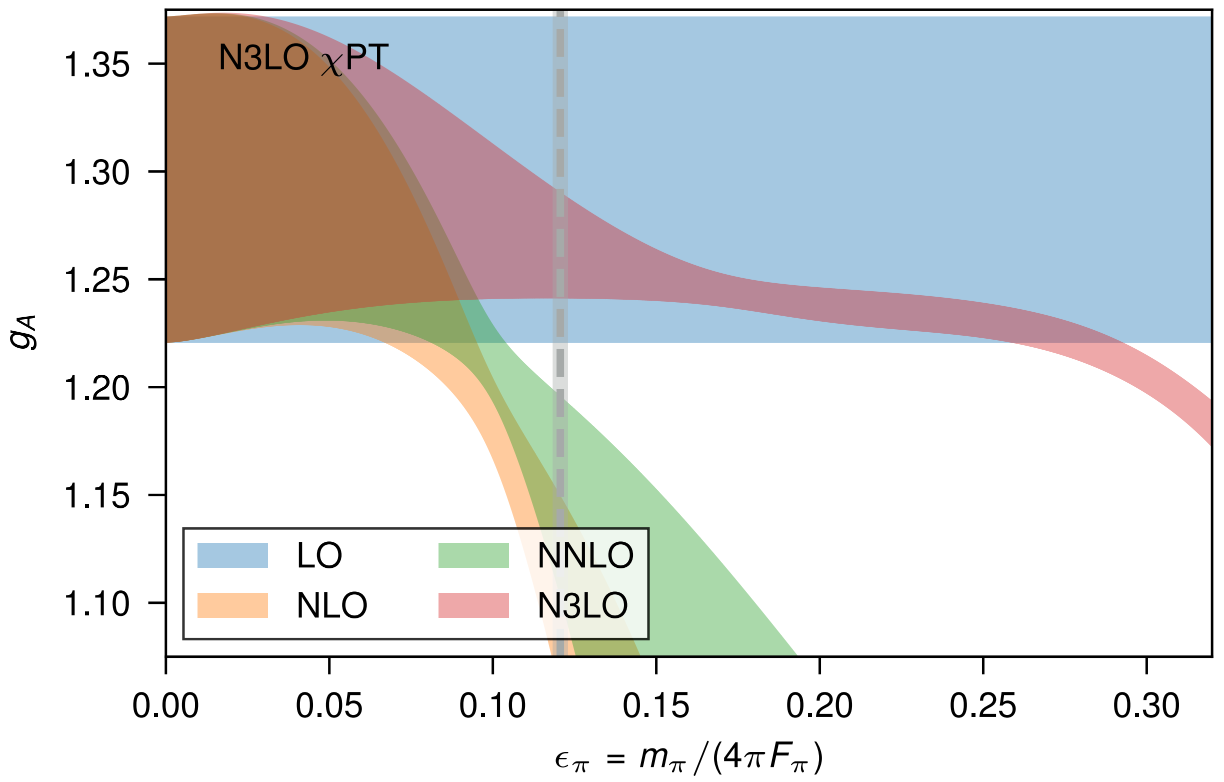
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130$ MeV 220 310 400

- g_A is ubiquitous in all nucleon properties (and nuclear physics)
- if g_A does not have a controlled extrapolation, can we trust convergence for other quantities?





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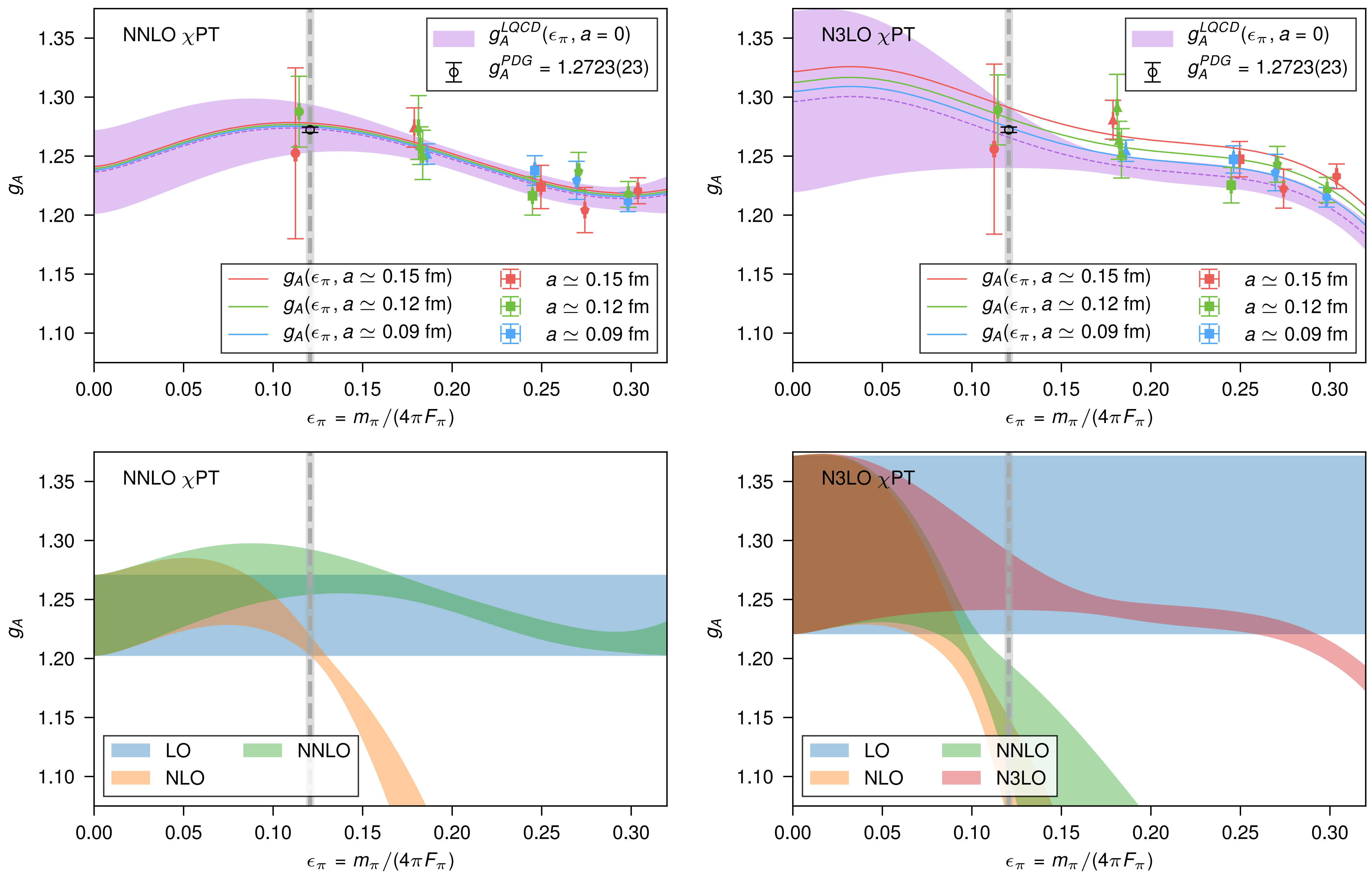
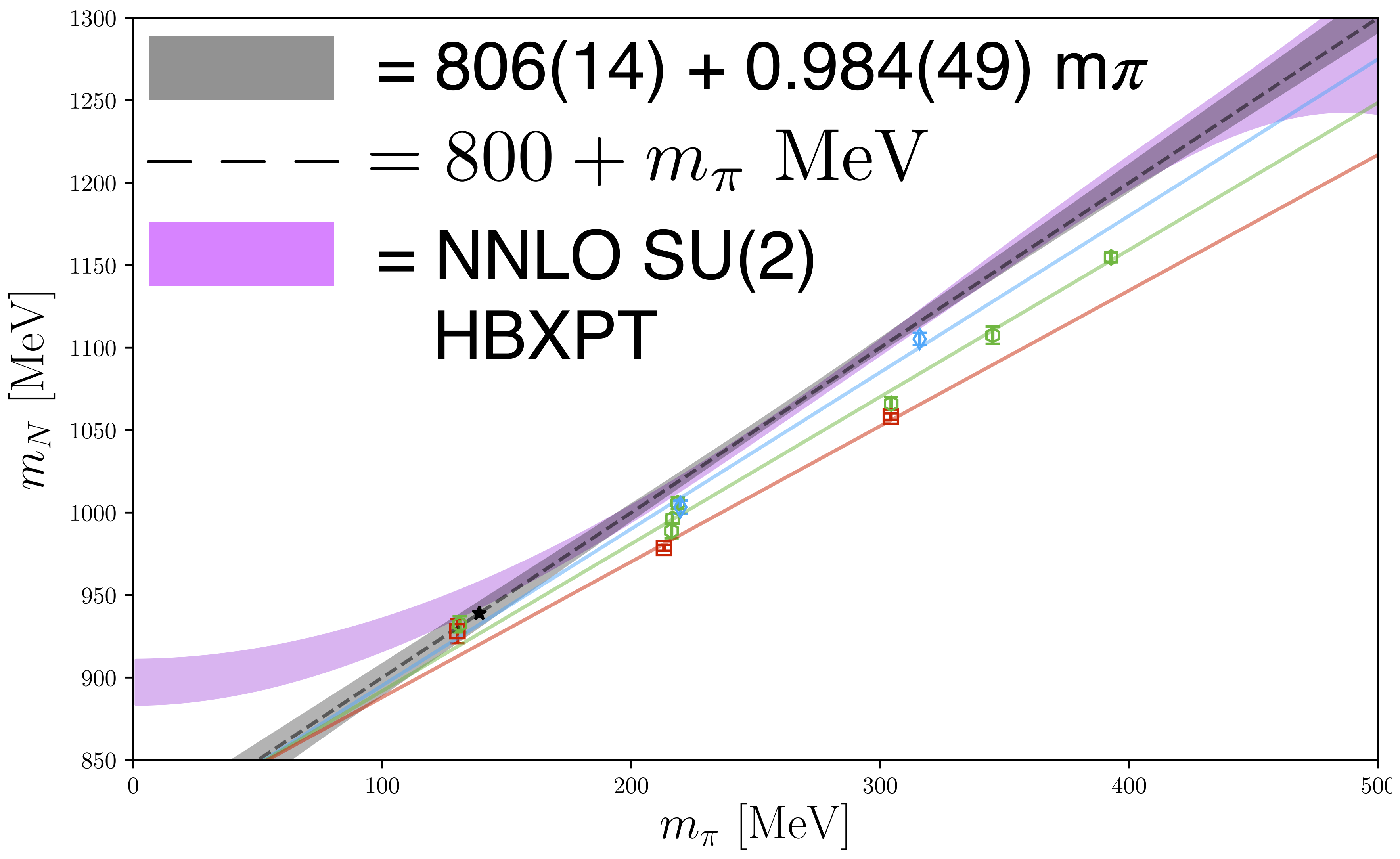
Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

convergence of the chiral expansion…



PRELIMINARY 2019

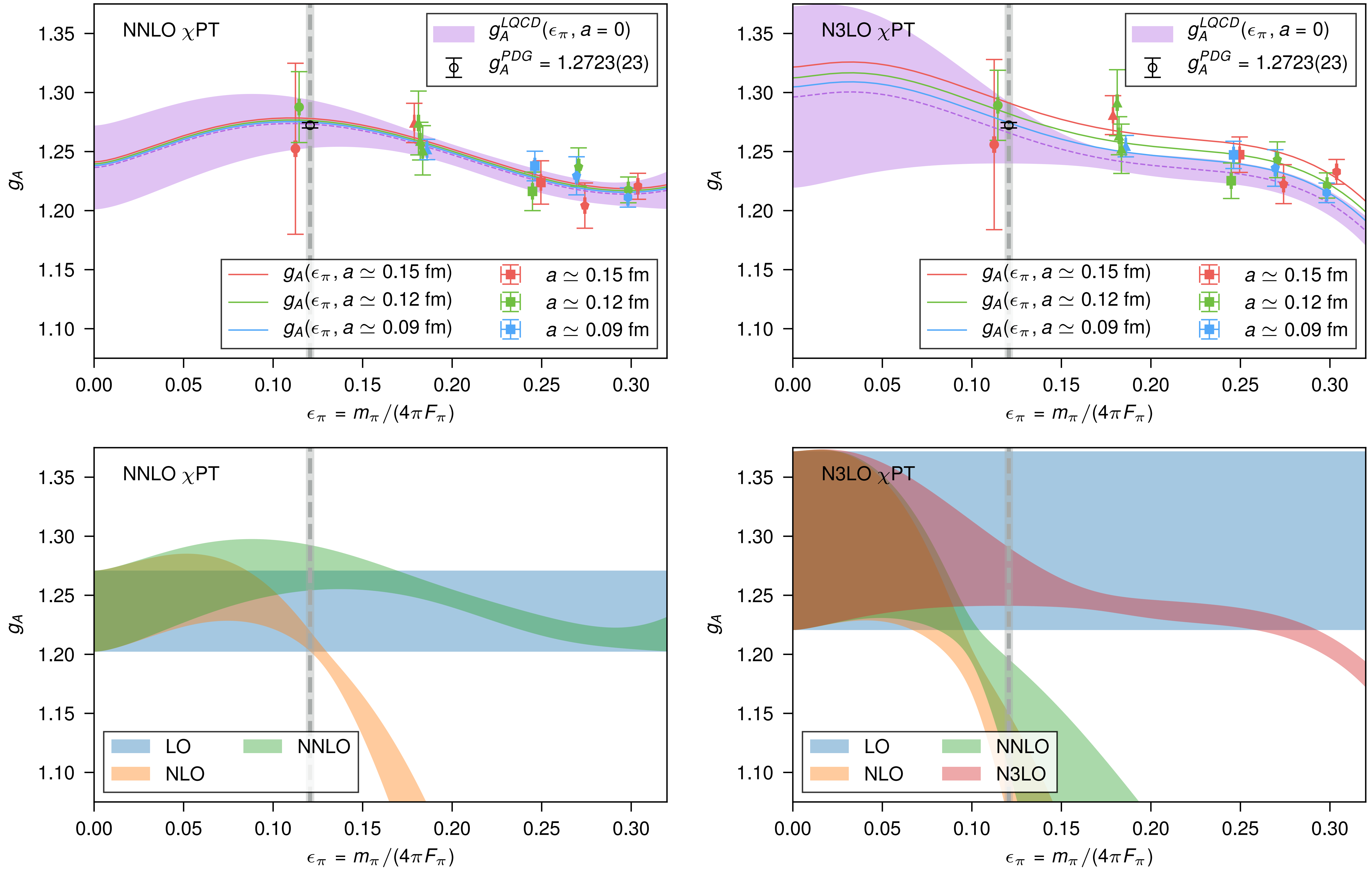


- Chiral corrections to g_A from $SU(2)$ HBχPT(Δ̸) at the physical pion mass

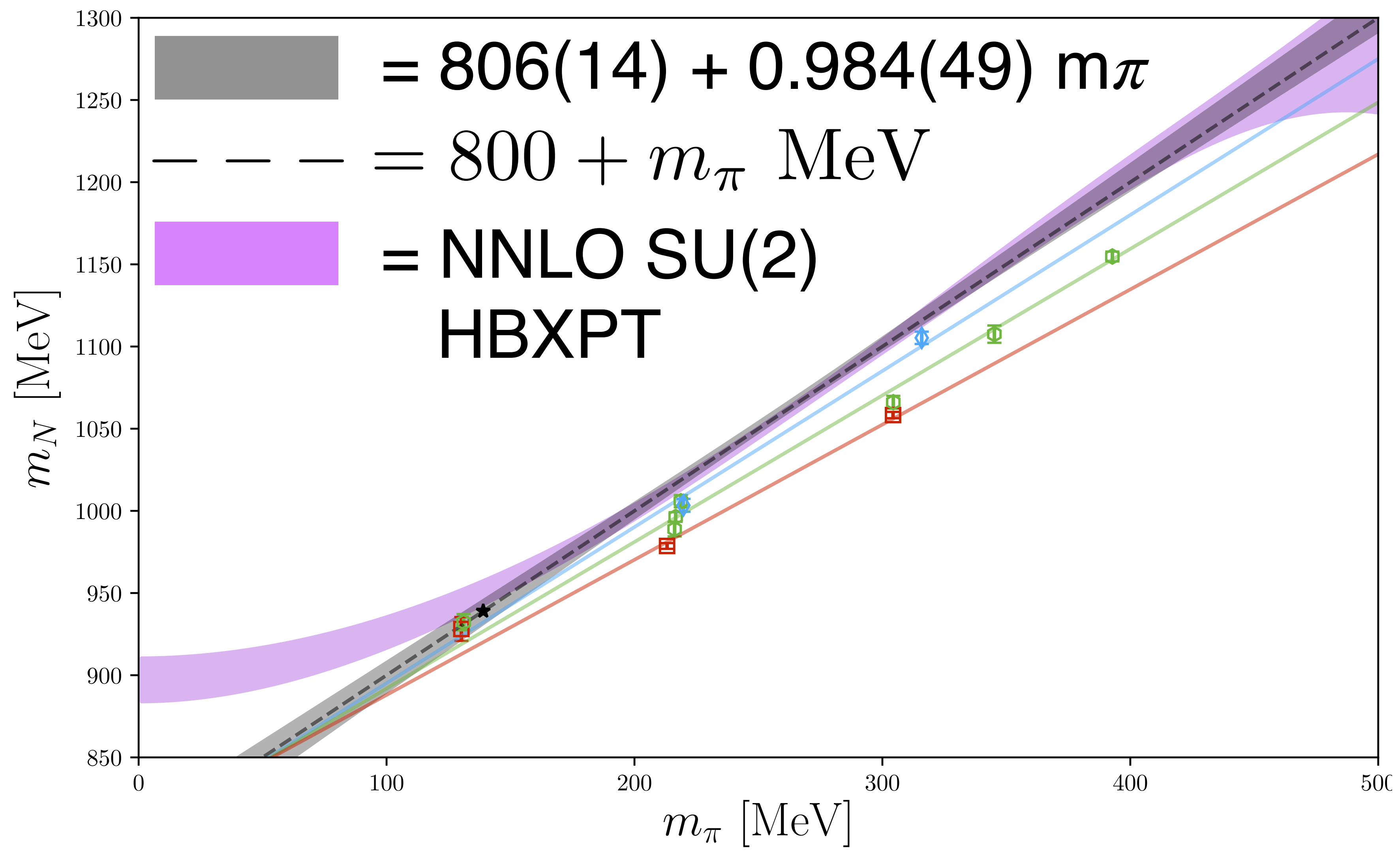
N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
- $SU(2)$ HBχPT(Δ) has a chance of being a converging expansion - but it won't be pretty

convergence of the chiral expansion…



PRELIMINARY 2019



- Chiral corrections to g_A from $SU(2)$ HBχPT($\not\Delta$) at the physical pion mass

N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

- $SU(2)$ HBχPT($\not\Delta$) is a failed expansion…

- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
- $SU(2)$ HBχPT(Δ) has a chance of being a converging expansion - but it won't be pretty

Returning to the isovector nucleon mass

- Isospin breaking scheme that preserves separability of QED and m_d-m_u at LO
  - Use M_{π^0} to define m_d+m_u in LQCD calculations with and without QED
    - $M_{\pi^\pm}^2 - M_{\pi^0}^2 \propto (m_d - m_u)^2$
    - The leading QED correction to M_{π^0} is suppressed in the chiral expansion and tiny
      Bijnens, Prades [hep-ph/9610360]
  - Use $M_{\Sigma^-} - M_{\Sigma^+} = 1197.45(4) - 1189.37(7) = 8.08(8)$ MeV to define m_d-m_u
    - The charged Sigma baryons should have the same elastic QED self-energy
      A Cottingham analysis finds the structure corrections to be small of O(0.1MeV)
      Erben, Shanahan, Thomas, Young [1408.6628]
    - Tune m_d-m_u to reproduce the splitting
- If all has gone well, and one uses a chiraly symmetric lattice regulator - turning on QED, one should find the need for small re-tunings of O($\alpha_{f.s.}$ x m_l)

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M_n-M_p in χPT

- "Symmetric breaking of isospin symmetry"

$$m_{u,d}^{\text{sea}} = \hat{m} \qquad m_d^{\text{val}} = \hat{m} + \delta \qquad m_u^{\text{val}} = \hat{m} - \delta$$

AWL [0904.2404]
de Divitis et al JHEP 1204 (2012)
de Divitis et al PRD87 (2013)

$$\begin{aligned} \mathcal{Z}_{u,d} &= \int DU_\mu \, \text{Det}(D + m_l - \delta\tau_3) \; e^{-S[U_\mu]} \\ &= \int DU_\mu \, \text{Det}(D + m_l) \; \det\left(1 - \frac{\delta^2}{(D + m_l)^2}\right) \; e^{-S[U_\mu]} \end{aligned}$$

- One can run calculations with a slight miss-match in valence (val) and sea quark masses
- de Divitis et al insert md-mu operator (3pt function)

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Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$
Isospin violating quantities: error $\mathcal{O}(\delta^3)$

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Pion Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8}\,\text{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right) - \frac{f^2}{8}\,\text{tr}\left(\chi'^\dagger\Sigma + \Sigma^\dagger\chi'\right) - \frac{l_1}{4}\left[\text{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right)\right]^2 - \frac{l_2}{4}\,\text{tr}\left(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger\right)\text{tr}\left(\partial_\mu\Sigma\partial_\nu\Sigma^\dagger\right)$$
$$- \frac{l_3 + l_4}{16}\left[\text{tr}\left(\chi'^\dagger\Sigma + \Sigma^\dagger\chi'\right)\right]^2 + \frac{l_4}{8}\,\text{tr}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right)\text{tr}\left(\chi'^\dagger\Sigma + \Sigma^\dagger\chi'\right) - \frac{l_7}{16}\left[\text{tr}\left(\chi'^\dagger\Sigma - \Sigma^\dagger\chi'\right)\right]^2$$

$$m_{\pi^\pm}^2 = 2Bm_l\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{4m_\pi^2}{f_\pi^2}l_4^r(\mu)\right\} - \frac{\Delta_{PQ}^4}{2(4\pi f_\pi)^2}$$

$$m_{\pi^0}^2 = m_{\pi^\pm}^2 + \frac{16B^2\delta^2}{f_\pi^2}l_7$$

$$\Delta_{PQ}^2 = 2B\delta$$

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Can also construct the partially quenched baryon chiral Lagrangian

$$M_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$M_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

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$$M_n - M_p = \alpha(m_d - m_u) + \mathcal{O}(\delta^2, m_\pi^2 \delta)$$

$$(2\delta = m_d - m_u)$$

Problematic terms exactly drop out of expansion for mass difference!
This only works for this symmetric choice of partial quenching

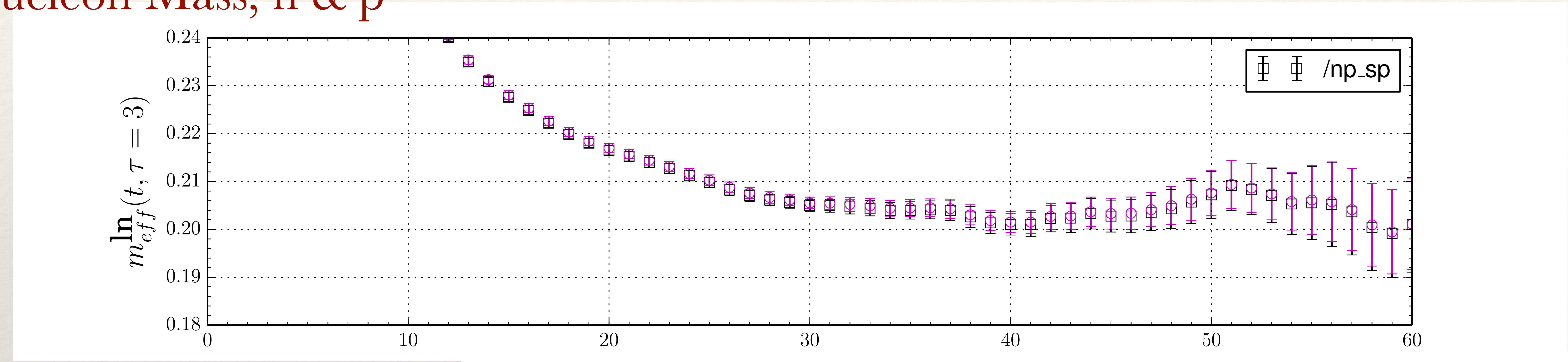
M_n-M_p in LQCD

- "Symmetric breaking of isospin symmetry"

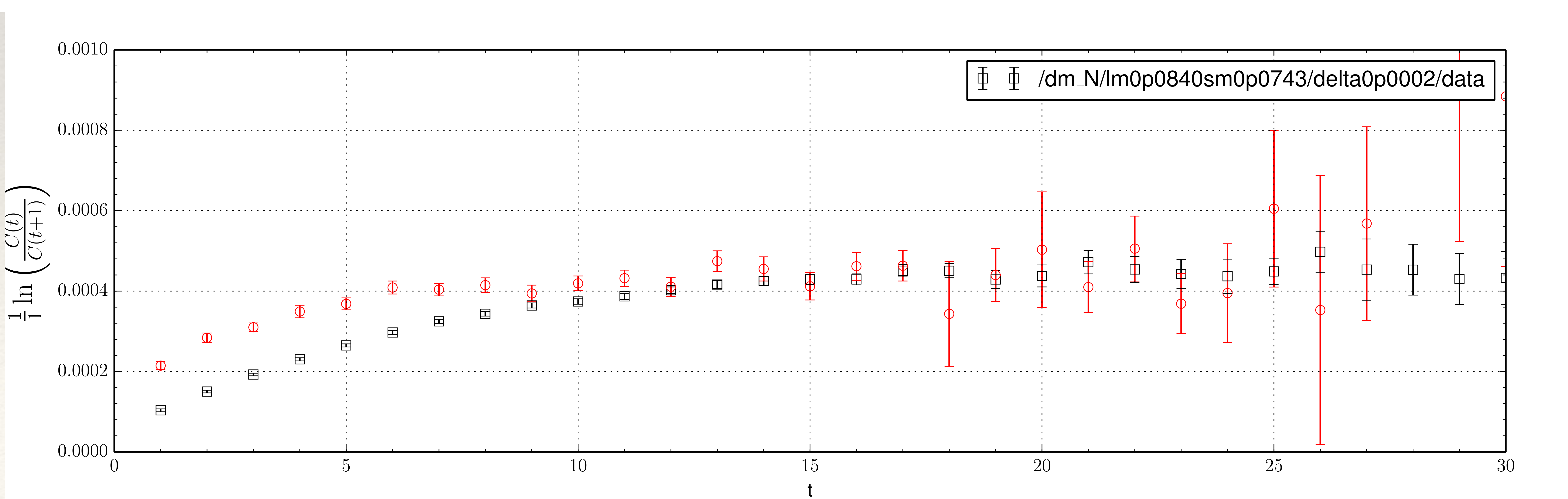
Brantley, Joó, Mastropas, Mereghetti, Monge-Camacho, Tiburzi, Walker-Loud [1612.07733]

$$m_{u,d}^{\text{sea}} = \hat{m} \qquad m_d^{\text{val}} = \hat{m} + \delta \qquad m_u^{\text{val}} = \hat{m} - \delta$$

Nucleon Mass, n & p



Ratio $C_n(t)$ / $C_p(t)$



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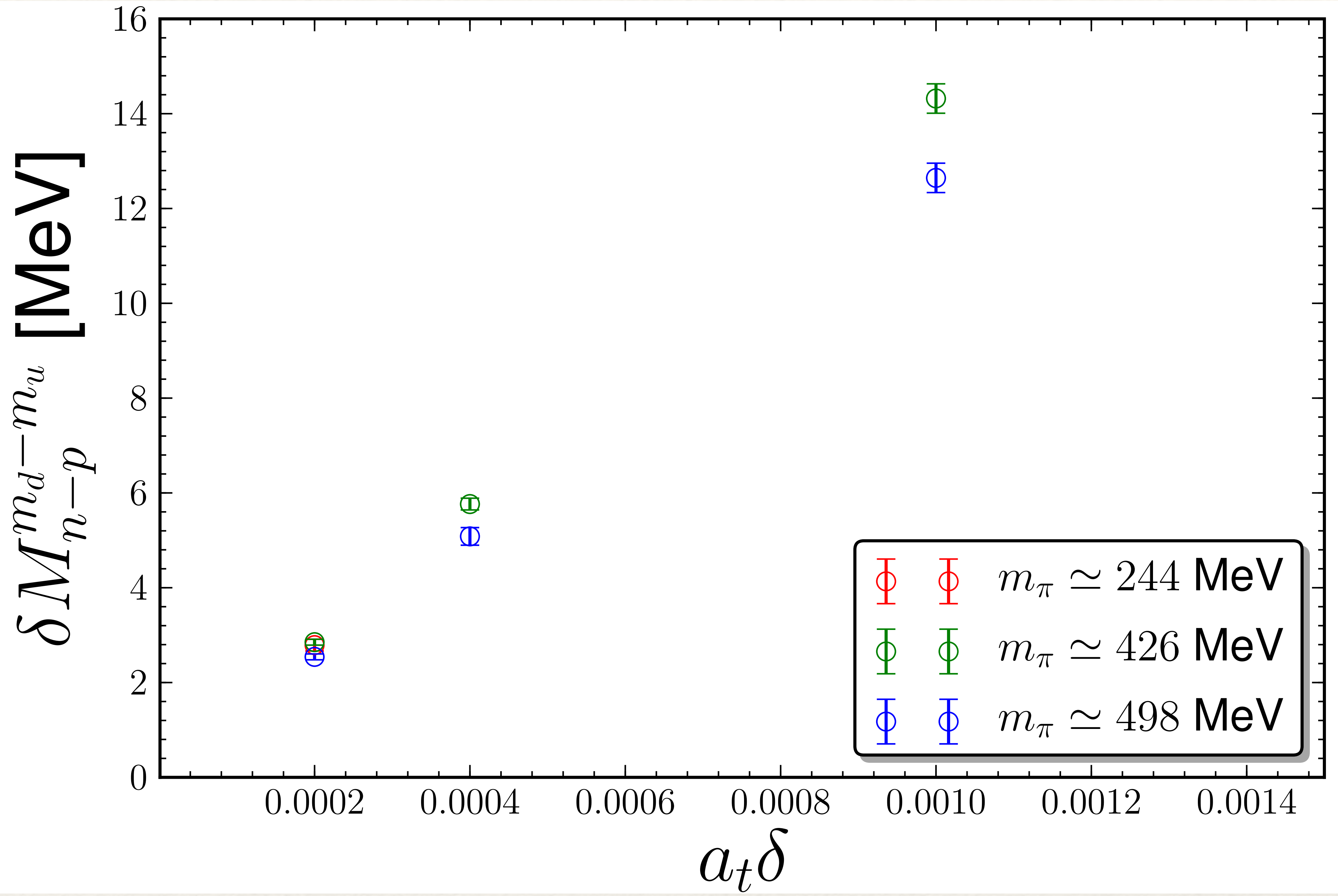

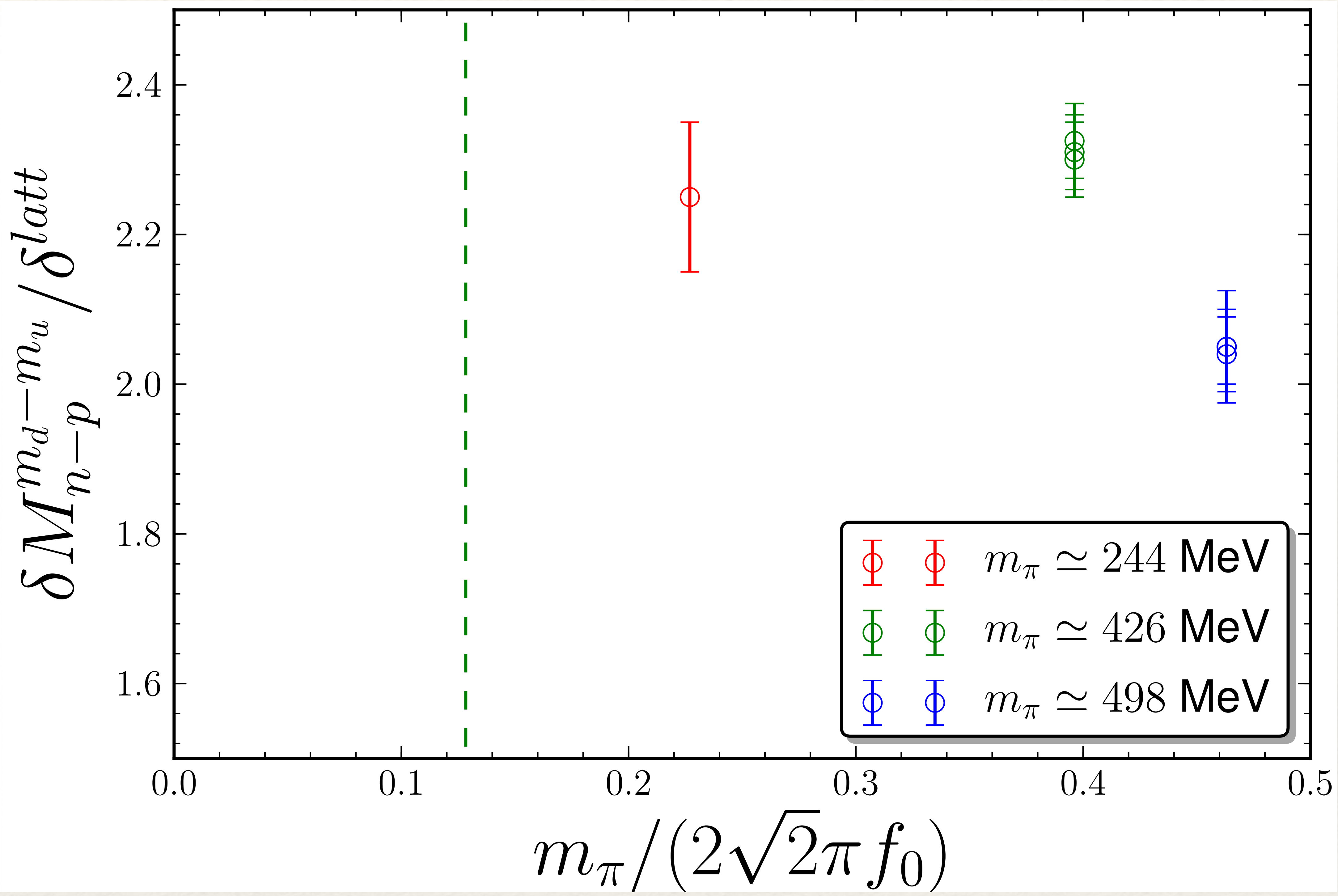

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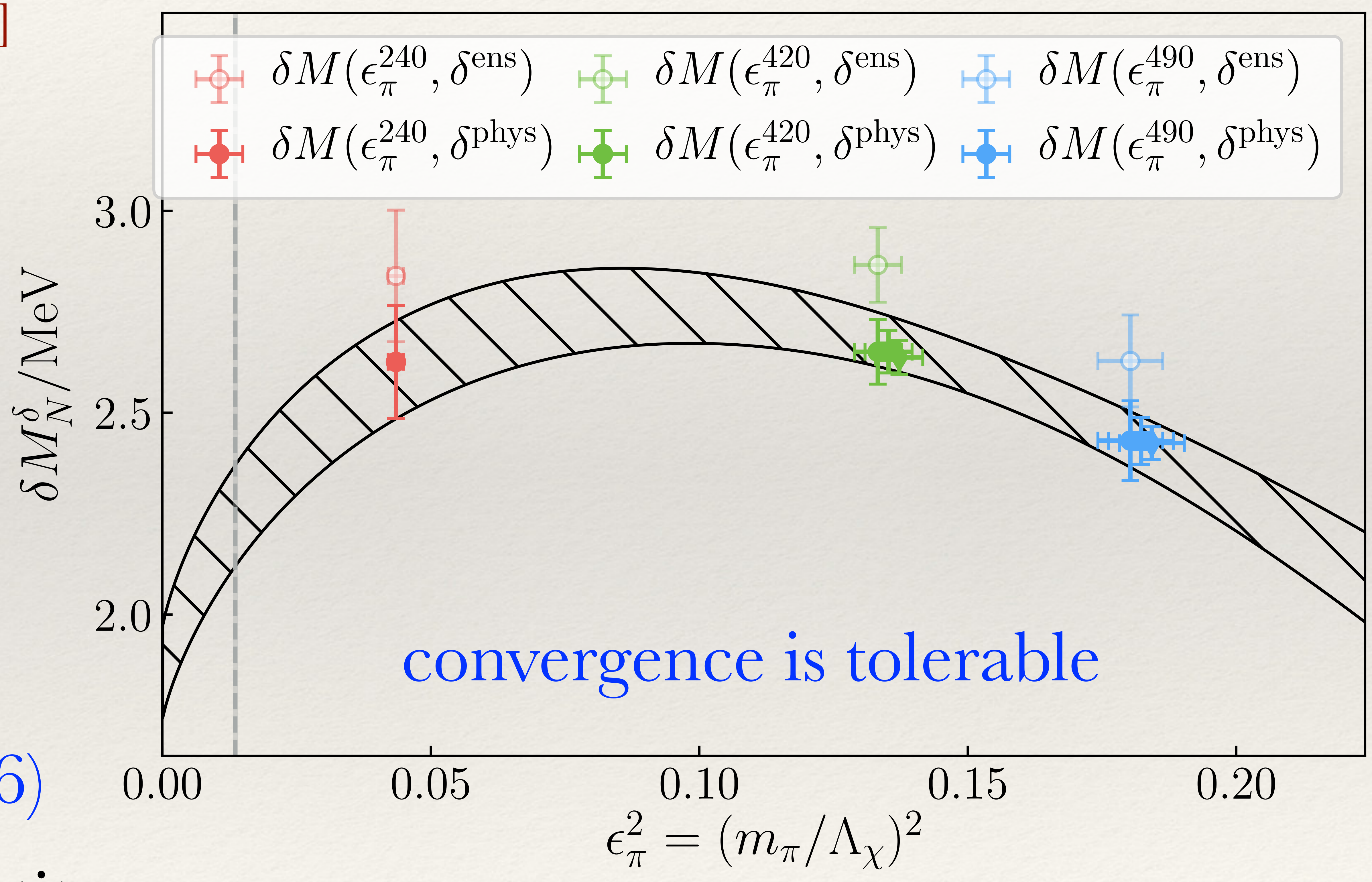
$$m_{u,d}^{\text{sea}} = \hat{m} \qquad m_d^{\text{val}} = \hat{m} + \delta \qquad m_u^{\text{val}} = \hat{m} - \delta$$

- Extrapolation formula at NNLO AWL [0904.2404]

$$\delta M_N^\delta = \delta \left\{ \alpha \left[1 - \frac{M_\pi^2}{(4\pi F_\pi)^2} \frac{6g_A^2+1}{2} \ln\left(\frac{M_\pi^2}{\mu^2}\right) \right] + \beta(\mu) \frac{M_\pi^2}{(4\pi F_\pi)^2} \right\}$$



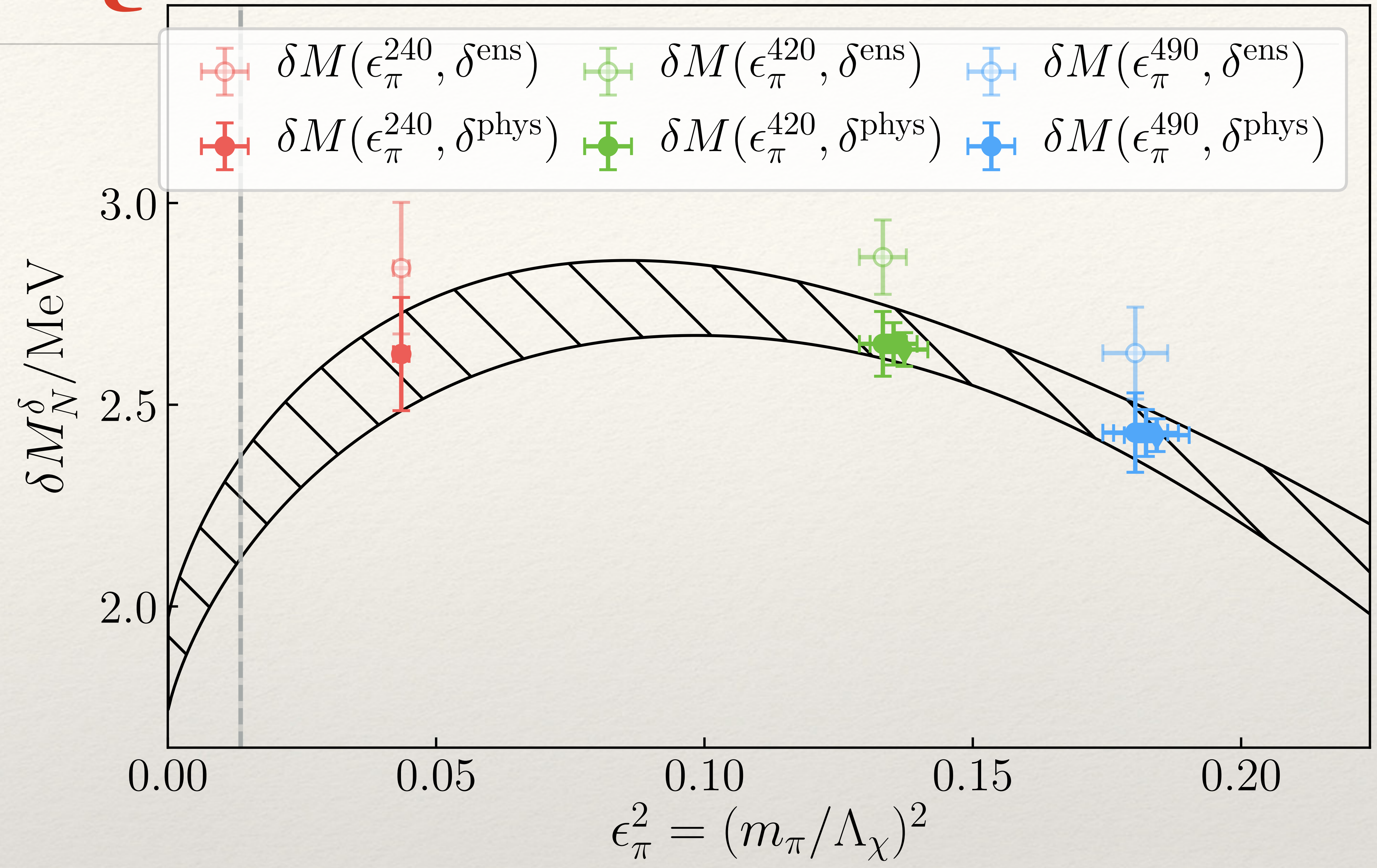
- Fixing g_A=1.27 yields χ^2/dof = 2.73/5
- Fitting g_A yields χ^2/dof = 2.72/4, g_A=1.24(56)
- The chiral log is very prominent for this quantity need a lighter pion mass to be conclusive confirmation of χ-log

M_n-M_p in LQCD

- Given the relation between the mass splitting and the coupling

$$\bar{g}_0 = \frac{\delta M_{n-p}^{m_d-m_u}}{m_d - m_u} \frac{2m_d m_u}{m_d + m_u} \bar{\theta}$$

- And the notable pion-mass dependence of the mass splitting



- Using this "renormalized" coupling in the extrapolation formula for the EDM may provide important corrections when comparing with heavier than physical pion mass
further emphasizing also the importance of a simultaneous extrapolation

$$d_n = \bar{g}_0 \frac{e g_A}{(4\pi F_\pi)^2} \ln\left(\frac{M_\pi^2}{\mu^2}\right) + \bar{\theta} \frac{m_u m_d}{m_u + m_d} \frac{e}{(4\pi F_\pi)^2} c(\mu)$$

BSM CP-violating operators - quark chromo-EDM

- There are two quark chromo-EDM operators (with 2 flavors) and 2 accompanying CP-conserving operators

$$\mathcal{L} = -\frac{i}{2}\bar{q}\sigma^{\mu\nu}\gamma_5(\tilde{d}_0 + \tilde{d}_1\tau_3)G_{\mu\nu}q - \frac{1}{2}\bar{q}\sigma^{\mu\nu}(\tilde{c}_0 + \tilde{c}_1\tau_3)G_{\mu\nu}q$$

- These operators transform into each other under the SU(2)$_A$ rotations that relate the θ-term coupling to the isovector mass

$$\bar{g}_0 = \delta_q M_N \frac{\tilde{d}_0}{\tilde{c}_1} + \delta M_N \frac{\Delta_q M_\pi^2}{M_\pi^2}\frac{\tilde{d}_1}{\tilde{c}_0}$$

$$\bar{g}_1 = -2\sigma_{\pi N}\left(\frac{\Delta_q M_N}{\sigma_{\pi N}} - \frac{\Delta_q M_\pi^2}{M_\pi^2}\right)\frac{\tilde{d}_1}{\tilde{c}_0}$$

$$\mathcal{L}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N - \frac{\bar{g}_1}{2F_\pi}\bar{N}\pi_3 N - \frac{\bar{g}_2}{2F_\pi}\pi_3\bar{N}\left(\tau_3 - \frac{\pi_3}{F_\pi}\vec{\pi}\cdot\vec{\tau}\right)N$$

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- These operators transform into each other under the $SU(2)_A$ rotations that relate the θ-term coupling to the isovector mass

$$\bar{g}_0 = \delta_q M_N \frac{\tilde{d}_0}{\tilde{c}_1} + \delta M_N \frac{\Delta_q M_\pi^2}{M_\pi^2}\frac{\tilde{d}_1}{\tilde{c}_0}$$

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Again, all that is needed are simple spectroscopic quantities

$$\begin{aligned}
\delta M_N &= \text{nucleon mass splitting induced by } \mathcal{O} = \delta\,\bar{q}\,\tau_3\,q\,,\\
\sigma_{\pi N} &= \text{nucleon sigma-term induced by } \mathcal{O} = -\bar{m}\bar{q}q\,,\\
\delta_q M_N &= \text{nucleon mass splitting induced by } \mathcal{O} = -(\tilde{c}_3/2)\,\bar{q}\sigma^{\mu\nu}\tau_3 G_{\mu\nu}q\,,\\
\Delta_q M_N &= \text{nucleon sigma-term induced by } \mathcal{O} = -(\tilde{c}_0/2)\,\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\,,\\
\Delta_q m_\pi^2 &= \text{pion sigma-term induced by } \mathcal{O} = -(\tilde{c}_0/2)\,\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q\,,
\end{aligned}$$

$$\mathcal{L}_{CPV} = -\frac{\bar{g}_0}{2F_\pi}\bar{N}\vec{\pi}\cdot\vec{\tau}N - \frac{\bar{g}_1}{2F_\pi}\bar{N}\pi_3 N - \frac{\bar{g}_2}{2F_\pi}\pi_3\bar{N}\left(\tau_3 - \frac{\pi_3}{F_\pi}\vec{\pi}\cdot\vec{\tau}\right)N$$

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$$\mathcal{L} = -\frac{i}{2}\bar{q}\sigma^{\mu\nu}\gamma_5(\tilde{d}_0 + \tilde{d}_1\tau_3)G_{\mu\nu}q - \frac{1}{2}\bar{q}\sigma^{\mu\nu}(\tilde{c}_0 + \tilde{c}_1\tau_3)G_{\mu\nu}q$$

- To maintain these simple relations, one must work around subtle difficulties
  Seng, Ramsey-Musolf, PRC96 (2017) [1611.08063]
  which one can do with modified relations at N²LO
  de Vries, Mereghetti, Seng, Walker-Loud, PLB 766 (2017) [1612.01567]

$$\bar{g}_0 = \delta_q M_N \frac{\tilde{d}_0}{\tilde{c}_1} + \delta M_N \frac{\Delta_q M_\pi^2}{M_\pi^2}\frac{\tilde{d}_1}{\tilde{c}_0}$$

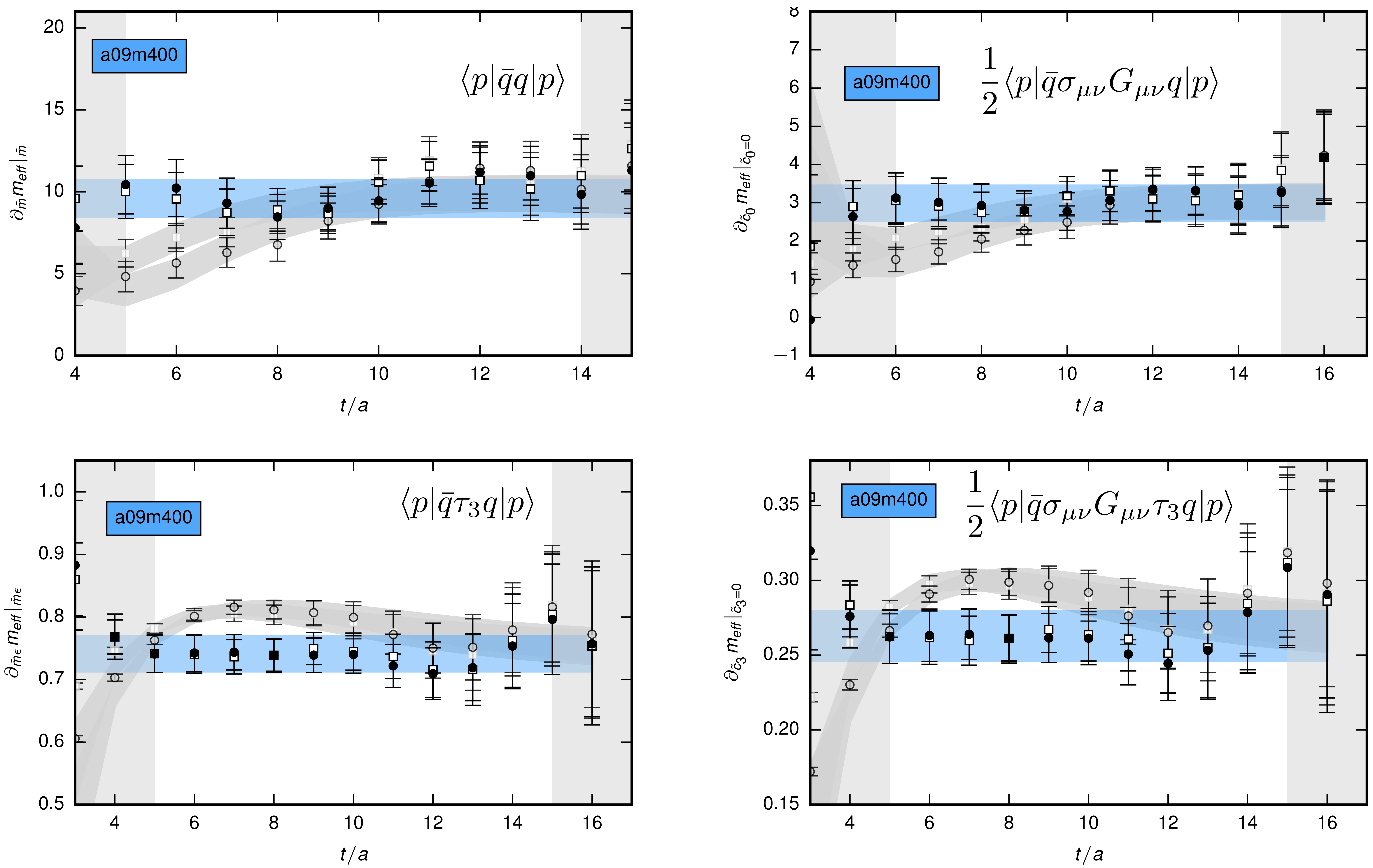
$$\bar{g}_1 = -2\sigma_{\pi N}\left(\frac{\Delta_q M_N}{\sigma_{\pi N}} - \frac{\Delta_q M_\pi^2}{M_\pi^2}\right)\frac{\tilde{d}_1}{\tilde{c}_0}$$

$$\bar{g}_0 = \tilde{d}_0\left(\partial_{\tilde{c}_1} + r\partial_{\hat{m}\epsilon}\right)\delta M_N + \delta M_{N,QCD}\frac{1-\epsilon^2}{2\epsilon}(\bar{\theta} - \bar{\theta}_{\rm ind})$$

$$\bar{g}_1 = -2\tilde{d}_1\left(\partial_{\tilde{c}_0} - r\partial_{\hat{m}}\right)\Delta M_N$$

$$\epsilon = \frac{m_d - m_u}{2\hat{m}} \qquad r = \frac{g_s}{2}\frac{\langle 0|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle}{\langle 0|\bar{q}q|0\rangle}$$

Results: Preliminary



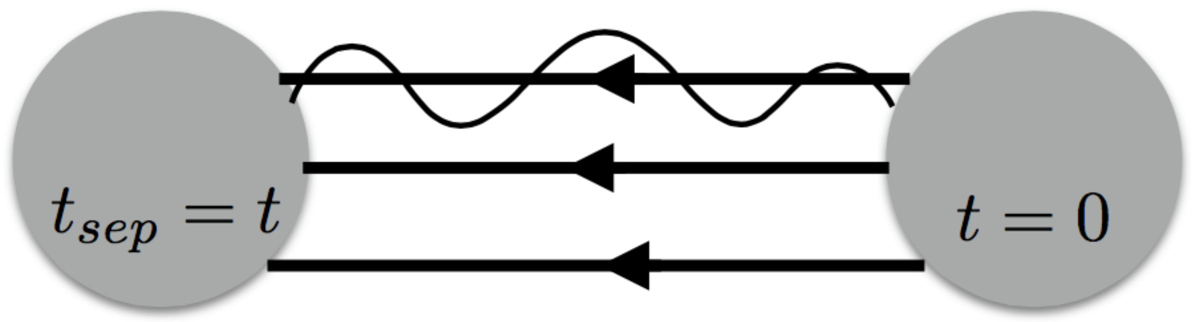
Disconnected Diagrams

Iso-vector CPV pion-nucleon coupling dependent on iso-scalar quantities.

$$\bar{g}_1 = -2\tilde{d}_3 \left(\frac{d}{d\tilde{c}_0} - r\frac{d}{d\bar{m}} \right) \Delta m_N$$

$$\frac{\partial}{\partial \tilde{c}_0} C_{\tilde{c}_0}(t)\Big|_{\tilde{c}_0=0} = \int dt' \langle \Omega | T\{\mathcal{O}(t) \left(\frac{1}{2}\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q(t') \right) \mathcal{O}^\dagger(0)\} | \Omega \rangle$$

$=$



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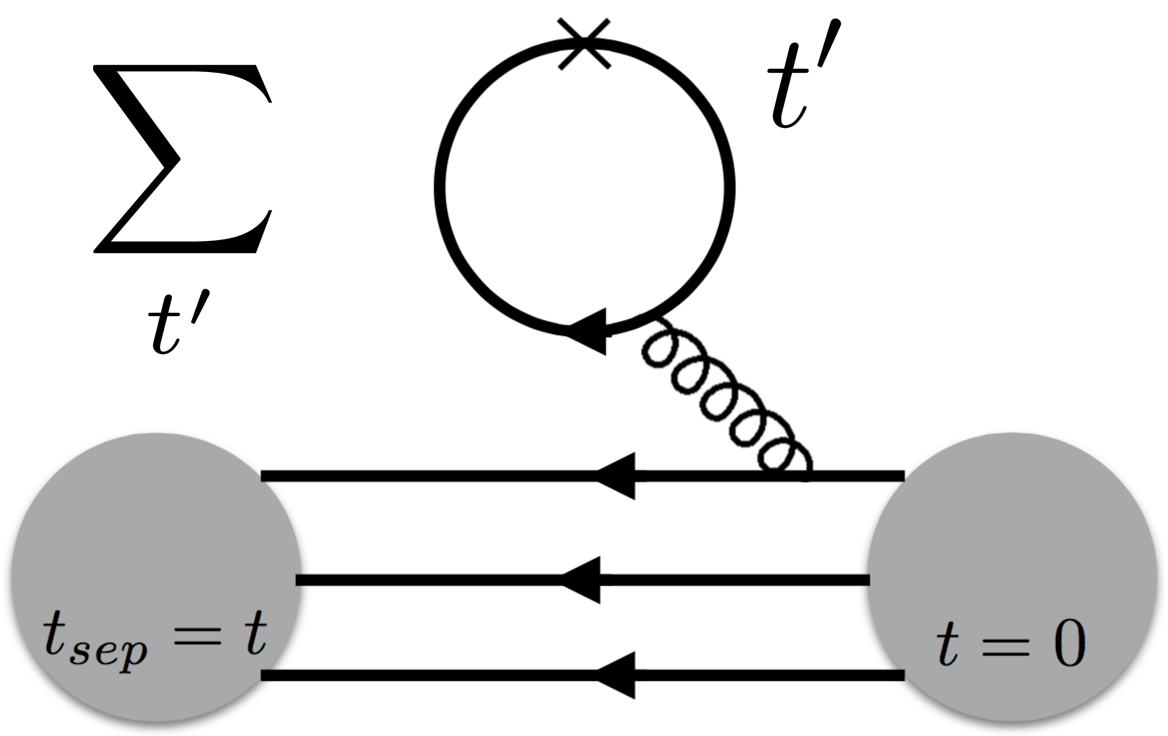
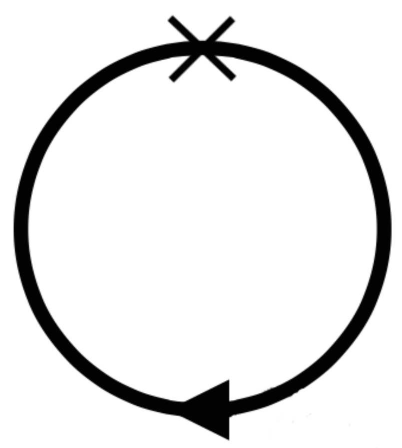
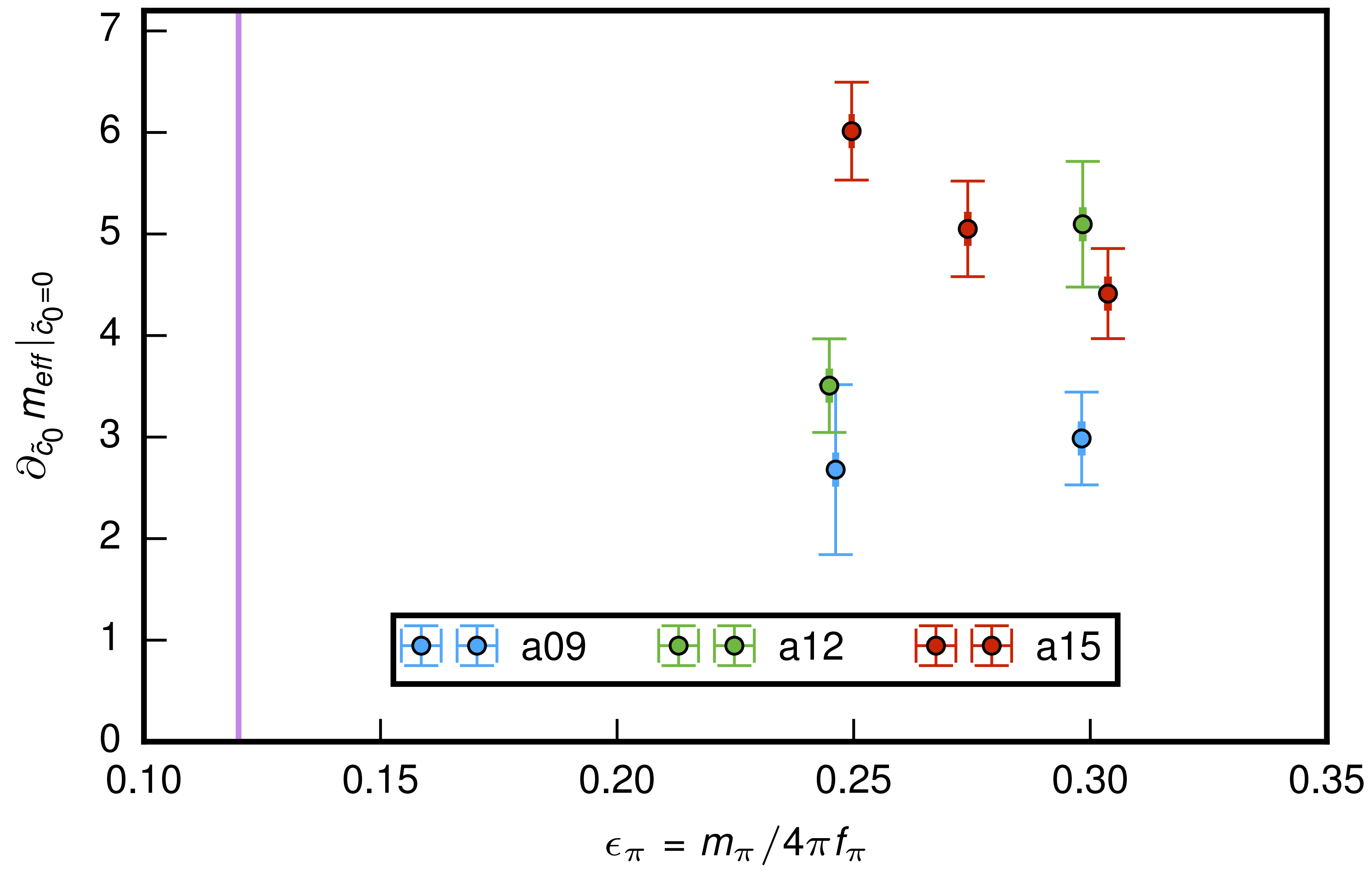




$$= -\text{tr}[\sigma_{\mu\nu}G^{\mu\nu}S(x|x)]$$

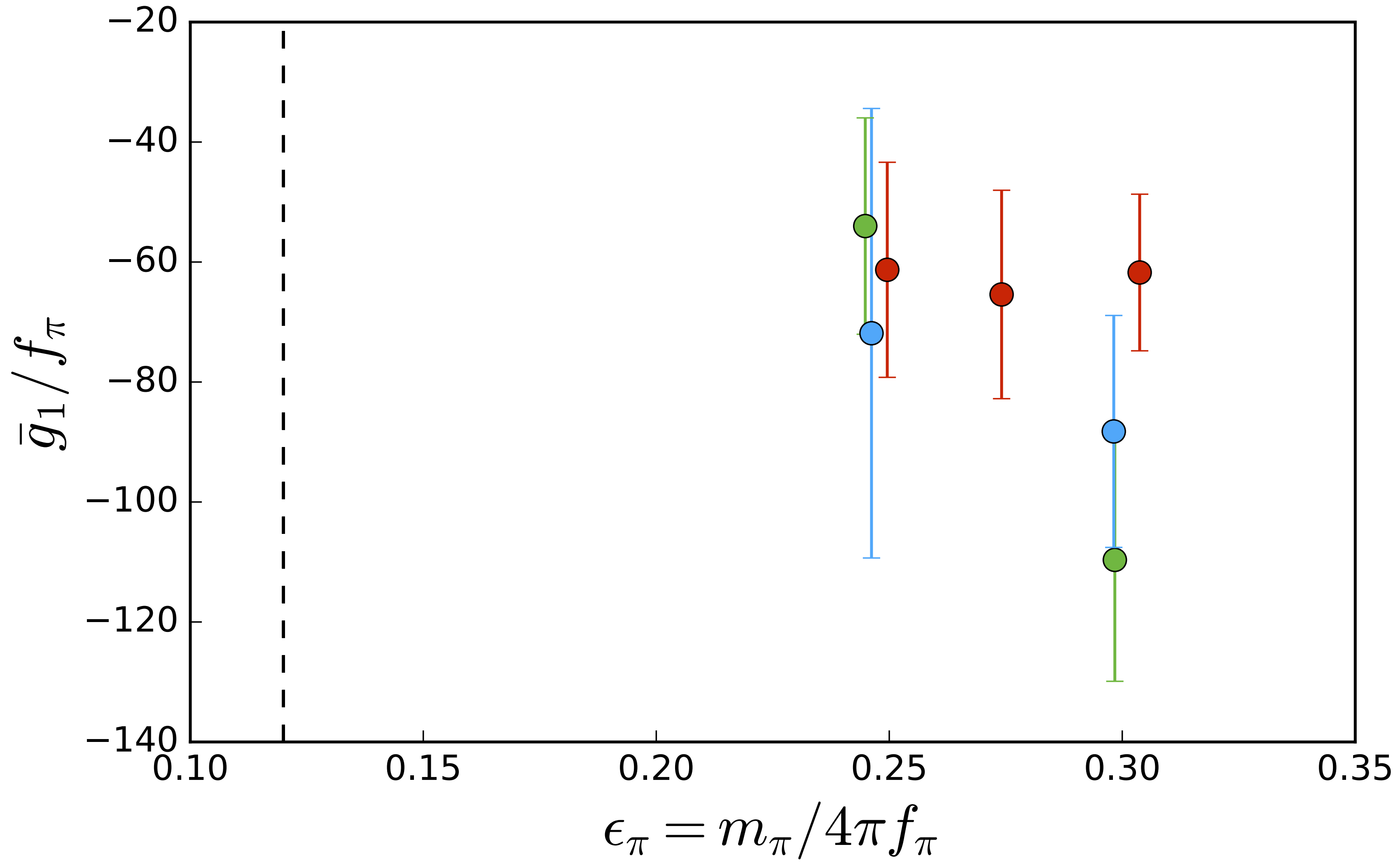
Requires information of all-to-all

Results: Preliminary



Bare matrix element

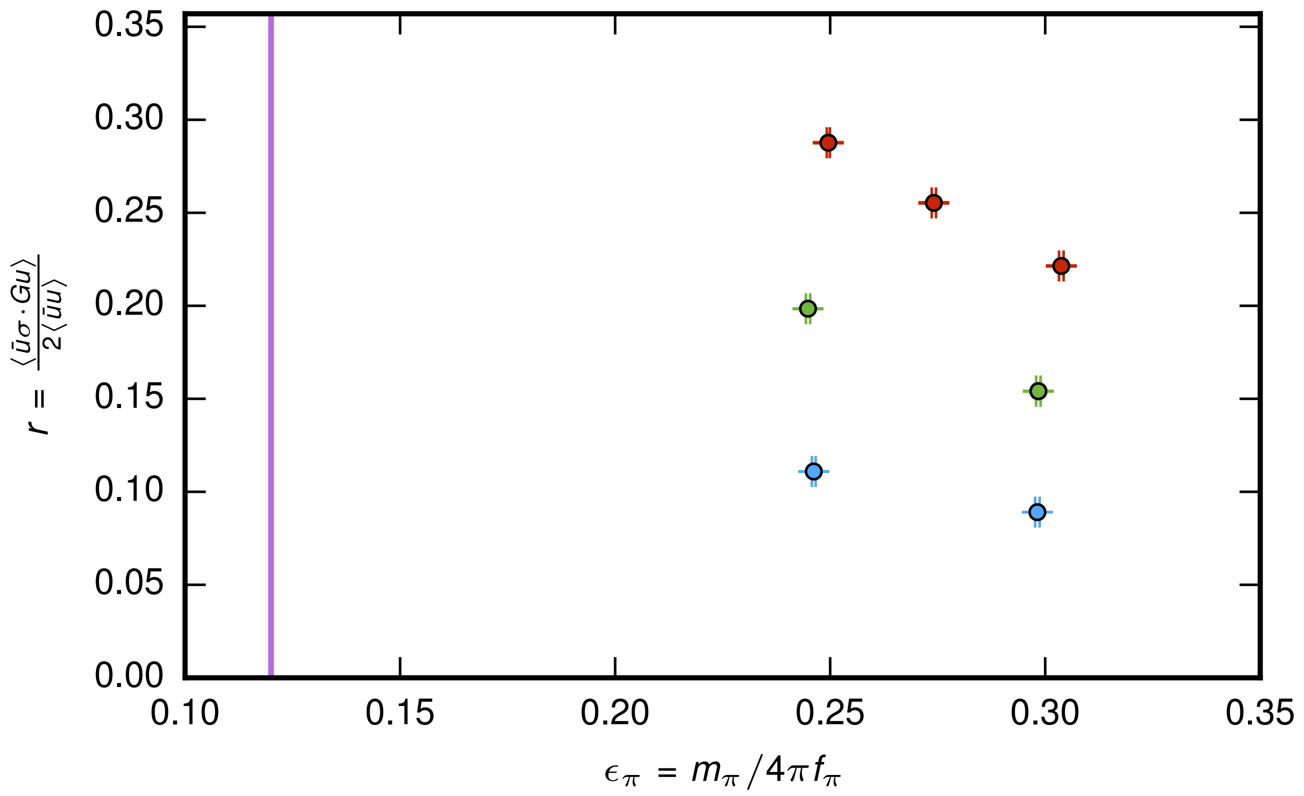
$$\langle p|\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q|p\rangle_{latt}$$



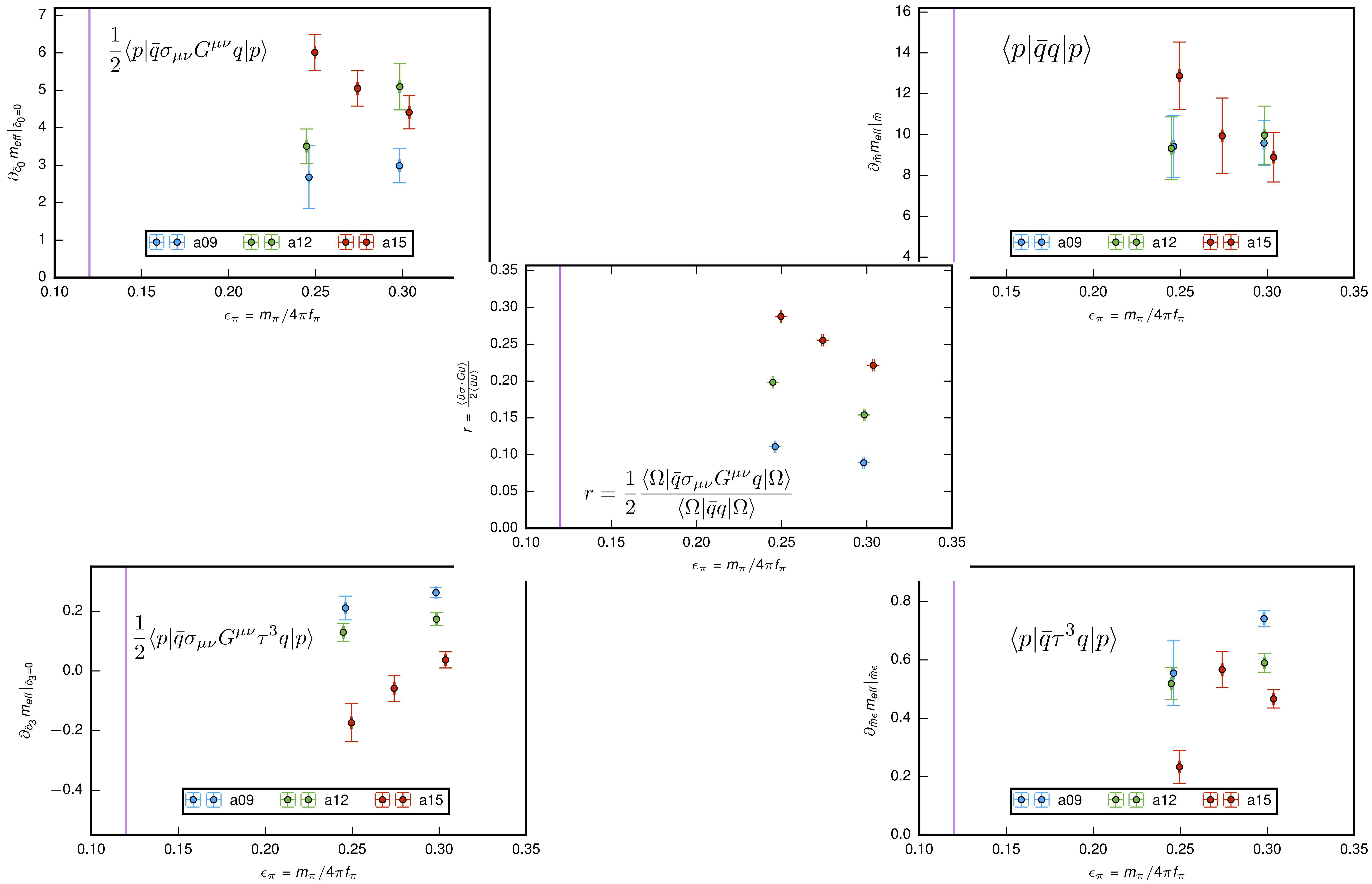
r-subtracted matrix element

$$-\frac{2}{f_\pi}\left(\langle p|\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q|p\rangle_{latt} - r\langle p|\bar{q}q|p\rangle\right)$$

$$r = \frac{\langle\Omega|\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q|\Omega\rangle}{2\langle\Omega|\bar{q}q|\Omega\rangle}$$



Putting it all Together



Renormalization

RI/SMOM Renormalization: MANY extra operators needed for complete renormalization

$$
\begin{aligned}
\mathcal{O}_1 &\equiv C = \bar\psi\sigma^{\mu\nu} g G_{\mu\nu} t^a \psi, \\
\mathcal{O}_2 &\equiv \partial^2 S = \partial^2(\bar\psi t^a \psi), \\
\mathcal{O}_3 &\equiv E = \frac{e}{2}\bar\psi\sigma^{\mu\nu} F_{\mu\nu}\{Q, t^a\}\psi, \\
\mathcal{O}_4 &\equiv mFF = \mathrm{Tr}[\mathcal{M}Q^2 t^a] F_{\mu\nu}F^{\mu\nu}, \\
\mathcal{O}_5 &\equiv mGG = \mathrm{Tr}[\mathcal{M}t^a] G^b_{\mu\nu} G^{b\,\mu\nu}, \\
\mathcal{O}_8 &\equiv (m^2 S)_1 = \frac{1}{2}\bar\psi\left\{\mathcal{M}^2, t^a\right\}\psi, \\
\mathcal{O}_9 &\equiv (m^2 S)_2 = \mathrm{Tr}[\mathcal{M}^2]\,\bar\psi t^a \psi, \\
\mathcal{O}_{10} &\equiv (m^2 S)_3 = \mathrm{Tr}[\mathcal{M}t^a]\bar\psi\mathcal{M}\psi, \\
\mathcal{O}_{11} &\equiv S_{EE} = \bar\psi_E t^a \psi_E, \\
\mathcal{O}_{12} &\equiv (\partial\cdot V)_E = i\partial_\mu\left(\bar\psi\gamma^\mu t^a\psi_E - \bar\psi_E t^a\gamma^\mu\psi\right), \\
\mathcal{O}_{13} &\equiv V_\partial = \bar\psi t^a (i\overrightarrow{\not\partial})\psi_E + \bar\psi_E(-i\overleftarrow{\not\partial})t^a\psi, \\
\mathcal{O}_{14} &\equiv V_{A_\gamma} = \frac{e}{2}\bar\psi\left\{Q, t^a\right\}\not{A}_\gamma\psi_E + \frac{e}{2}\bar\psi_E\left\{Q, t^a\right\}\not{A}_\gamma\psi, \\
\mathcal{O}_{15} &\equiv (mS_E)_1 = \frac{1}{2}\left(\bar\psi\left\{\mathcal{M}, t^a\right\}\psi_E + \bar\psi_E\left\{\mathcal{M}, t^a\right\}\psi_E\right), \\
\mathcal{O}_{16} &\equiv (mS_E)_2 = \mathrm{Tr}[\mathcal{M}t^a]\left(\bar\psi\psi_E + \bar\psi_E\psi\right), \\
\mathcal{O}_{17} &\equiv (mDG) = \mathrm{Tr}[\mathcal{M}t^a]\left(D^{bc}_\mu G^b_{\mu\nu}\right)A^{\nu\,c}.
\end{aligned}
$$

$$
\begin{aligned}
\psi_E &\equiv (iD^\mu\gamma_\mu - \mathcal{M})\psi\ , \\
\bar\psi_E &\equiv -\bar\psi\,(i\overleftarrow{D}^\mu\gamma_\mu + \mathcal{M})\ ,
\end{aligned}
$$

$$
\begin{aligned}
D_\mu &= \partial_\mu - igA^a_\mu T^a - ieQA^{(\gamma)}_\mu \\
\overleftarrow{D}_\mu &= \overleftarrow{\partial}_\mu + igA^a_\mu T^a + ieQA^{(\gamma)}_\mu
\end{aligned}
$$

- And - we basically stalled at this point
- The numerical calculation of the CP-conserving operators is simpler than getting EDMs with the CP-odd ones - but, the renormalization is just as complicated…
- and the grad student working on this graduated and moved on to other things…

Conclusions

- I've presented some conflicting statements
  - Symmetry is great! It shows you how to construct the chiral Lagrangian and tease out CP-violating physics from CP-conserving physics
  - Baryon Chiral Perturbation Theory does not work! (at least the convergence pattern is not healthy) Where is any of the non-analytic quark-mass dependence in nucleon quantities?
  - Look at this beautiful chiral-log in Mn-Mp!

- If we need precision, and we need extrapolation, $SU(2)$ HBχPT($\not\Delta$) can not be relied upon
  - For a given quantity, the precision and accuracy might be OK, but extrapolating to unconstrained parameter space, or unconstrained observables is likely to have issues
  - Adding explicit Δ dof might restore convergence - this requires more LQCD calculations
  - For current LQCD EDM calculations
    - we do not need precision
    - but we do need extrapolation - combining observables in global extrapolations should be done

- The preliminary LQCD results using CP-conserving matrix elements to get couplings looks promising, but the renormalization is still quite complex - need a new strategy perhaps

Thank You

Pion-induced radiative corrections to neutron beta-decay

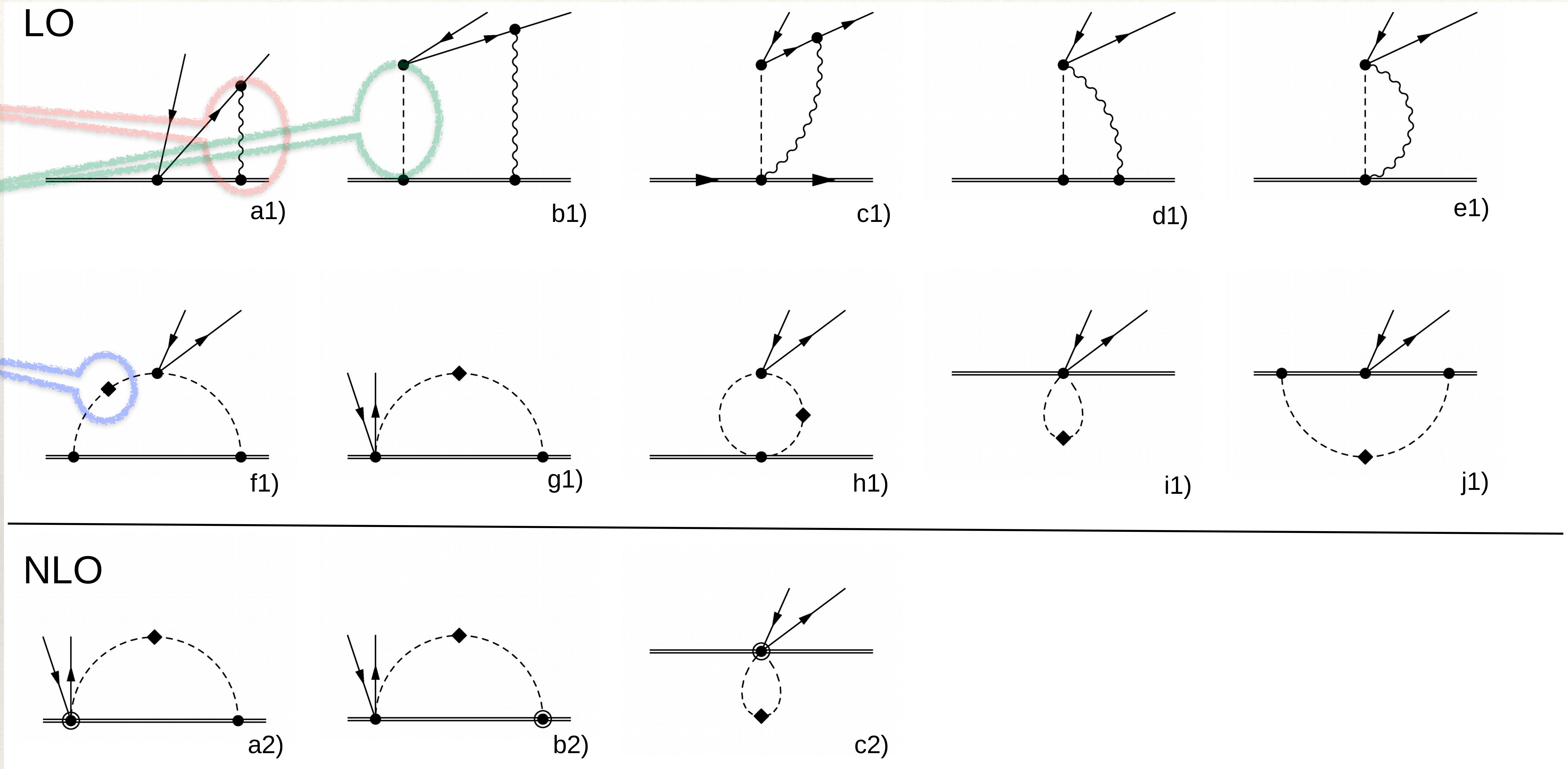
- Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, arXiv:2202.10439
  Sub-set of O(50) diagrams

photons

pions

pion electromagnetic mass splitting

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$



NOTE: at this order, we also include QED, m_d-m_u corrections to M_n-M_p

- iso-vector contributions to M_n-M_p vanish from symmetry constraints for τ^+ current
- iso-scalar contributions do not vanish - but the sum of all of them does vanish through NLO

Pion-induced radiative corrections to neutron beta-decay

- Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, arXiv:2202.10439

Matching $\lambda = g_A^{\rm QCD}\left(1+\delta_{\rm RC}^{(\lambda)} - 2{\rm Re}(\epsilon_R)\right)$ $\quad \delta_{\rm RC}^{(\lambda)} = \frac{\alpha}{2\pi}\left(\Delta_{A,{\rm em}}^{(0)} + \Delta_{A,{\rm em}}^{(1)} - \Delta_{V{\rm em}}^{(0)}\right)$

$$g_{V/A} = g_{V/A}^{(0)}\left[1+\sum_{n=2}^{\infty}\Delta_{V/A,\chi}^{(n)} + \frac{\alpha}{2\pi}\sum_{n=0}^{\infty}\Delta_{V/A,{\rm em}}^{(n)} + \left(\frac{m_u-m_d}{\Lambda_\chi}\right)^{n_{V/A}}\sum_{n=0}^{\infty}\Delta_{V/A,\delta m}^{(n)}\right]$$

$n_V = 2$ $\quad n_A = 1$

$g_V^{(0)} = 1$ $\quad \Delta_{\chi,{\rm em},\delta m}^{(n)} \sim O(\epsilon_\chi^n)$

CVC

explicit calculation:

$$\Delta_{A,\delta m}^{(0),(1)} = 0$$
$$\Delta_{V,\delta m}^{(0)} = 0$$

$$\Delta_{A,{\rm em}}^{(0)} = Z_\pi\left[\frac{1+3g_A^{(0)2}}{2}\left(\log\frac{\mu^2}{m_\pi^2}-1\right) - g_A^{(0)2}\right] + \hat{C}_A(\mu)$$

Low-Energy-Constants (LECs)

$$\Delta_{A,{\rm em}}^{(1)} = Z_\pi\, 4\pi m_\pi\left[c_4 - c_3 + \frac{3}{8m_N} + \frac{9}{16m_N}g_A^{(0)2}\right]$$

$C_A(\mu)$ - completely unknown

c_3 & c_4 are estimated from literature

Using Naive Dimensional Analysis (NDA) to estimate $C_A(\mu)$ and $c_{3,4}$ from the literature

$\delta_{\rm RC}^{(\lambda)} \in \{1.4,\ 2.6\}\cdot 10^{-2}$ an order of magnitude larger than previous estimates

Pion-induced radiative corrections to neutron beta-decay

- Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, arXiv:2202.10439
  Comments:
  - This does not affect neutron lifetime/g_A measurements - it enters both experiments in the same way
  - It impacts our ability to compare lattice QCD results to measured values of
  - We need lattice QCD + QED to determine the unknown LECs and compare with estimate (in progress now)
  - This has led us to wonder if there are similarly sized corrections to two-nucleon matrix elements, such as pp-fusion
  - If so - they likely do not "factorize" in such a clean way as with g_A where they can be ignored except in searches for new physics - many more diagram topologies than neutron decay