INFN
Istituto Nazionale di Fisica Nucleare

FONDAZIONE
BRUNO KESSLER

ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

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Accurate analysis method to detect rare events in VIP-2

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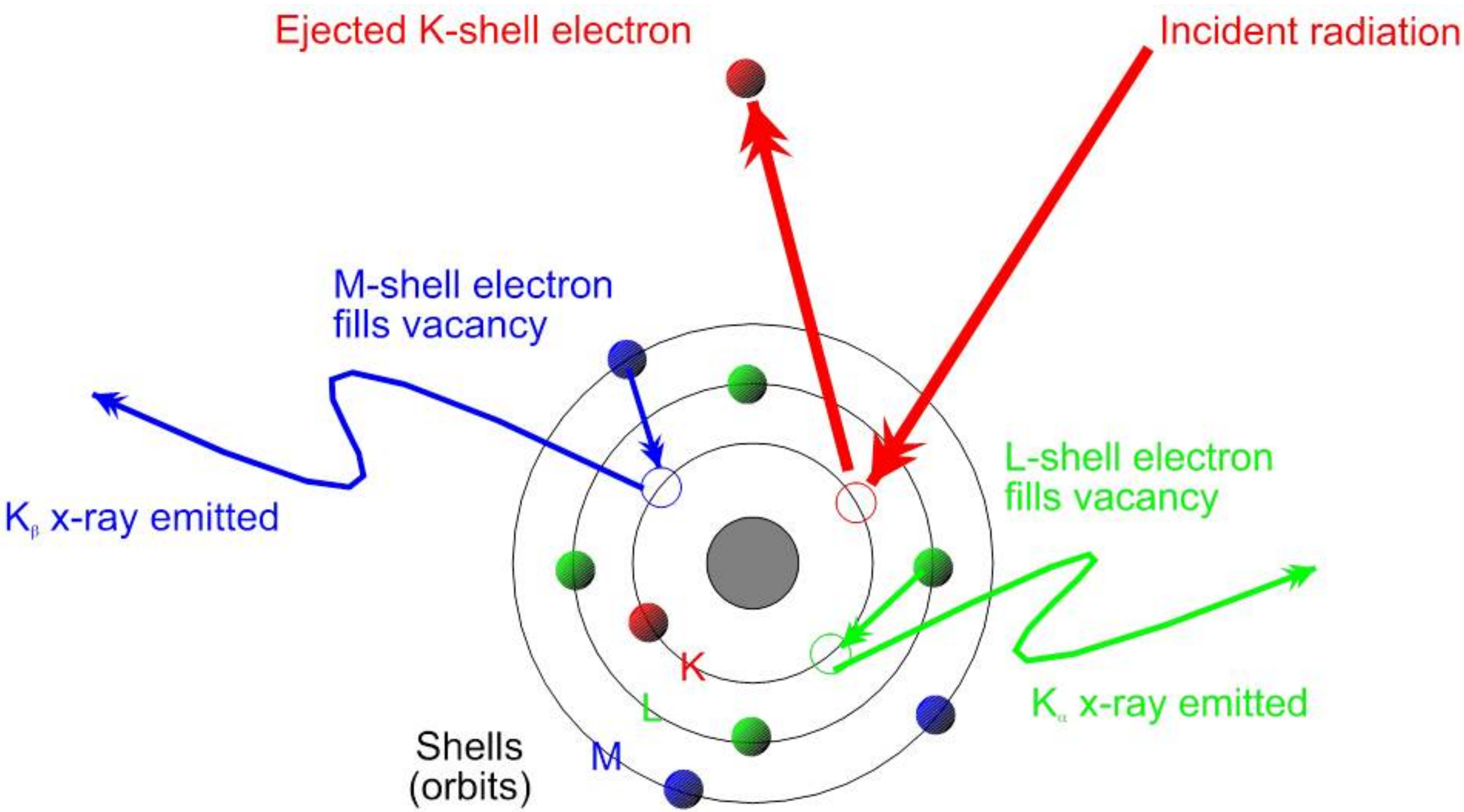
21 September 2022

TRENTINO

X-Rays



Ejected K-shell electron
Incident radiation
M-shell electron fills vacancy
K_β x-ray emitted
L-shell electron fills vacancy
K_α x-ray emitted
K
L
M
Shells (orbits)

Searching for PEP violation

Why Fermi-Dirac and Bose-Einstein are distinct?

- **Green's general quantum field:** paronic particles
  - Order 1: fermionic/bosonic fields
  - Order>1: parafermionic/parabosonic fileds
  - Messiah-Greenberg Super-Selection: no fermion/boson decays into parafermion/paraboson (and vice-versa)
  - **Paronic**: a mixture of fermionic/bosonic and parefermionic/parabosonic states
- **Non-Commutative Quantum Gravity**
  - **θ-Poincaré**: distortion of Lorentz symmetry (visible in a two identical particles system)

Both break the anti-/symmetric commutativity with an amplitude β.
In a system of two fermions (i.e., two electrons),
PEP is violated with a probability of $\beta^2/2$
[See Fabrizio's Talk for more details]

VIP-2 GOAL

searching **VI**olation of **P**auli Exclusion Principle

Searching for PEP violation

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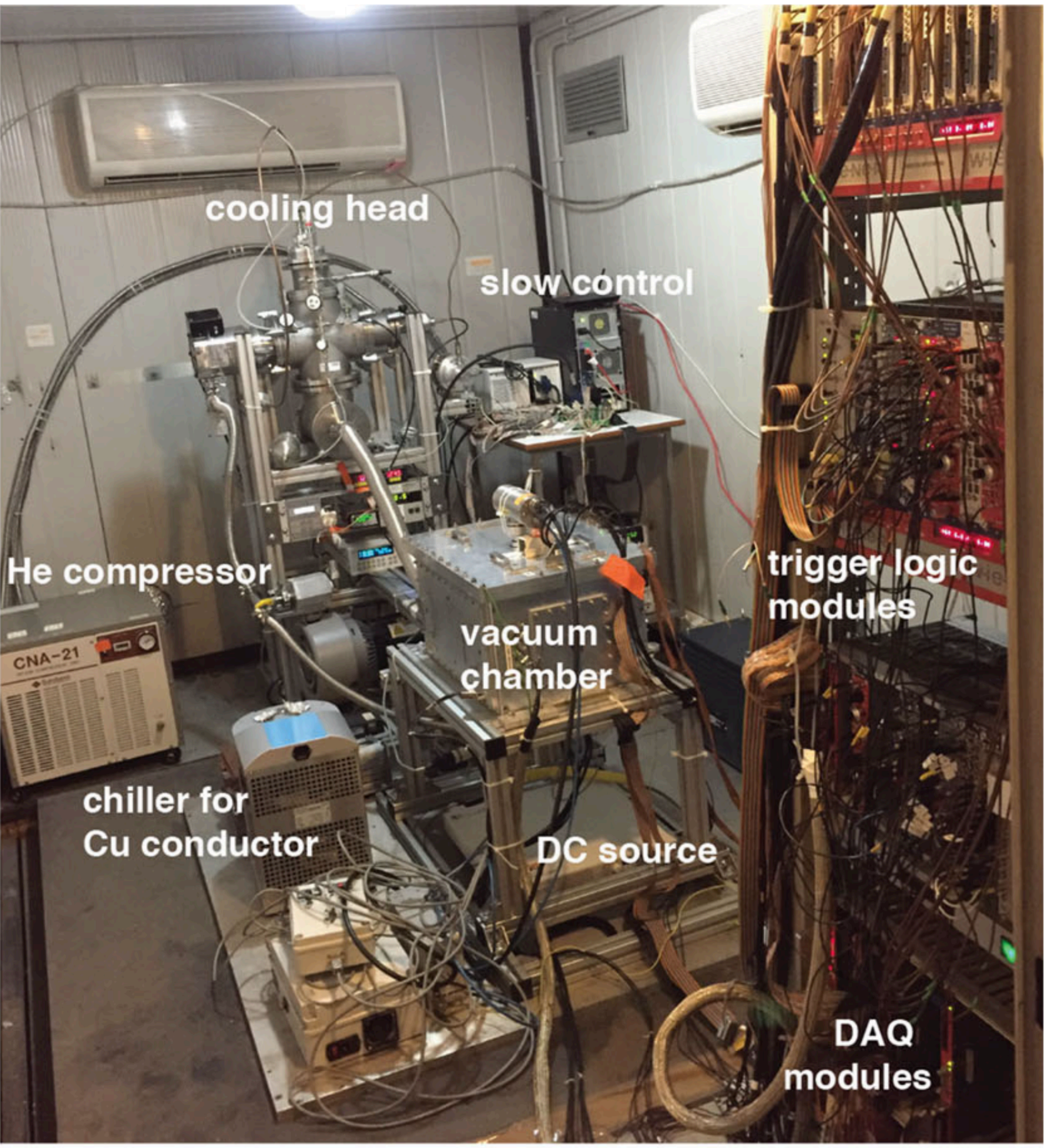
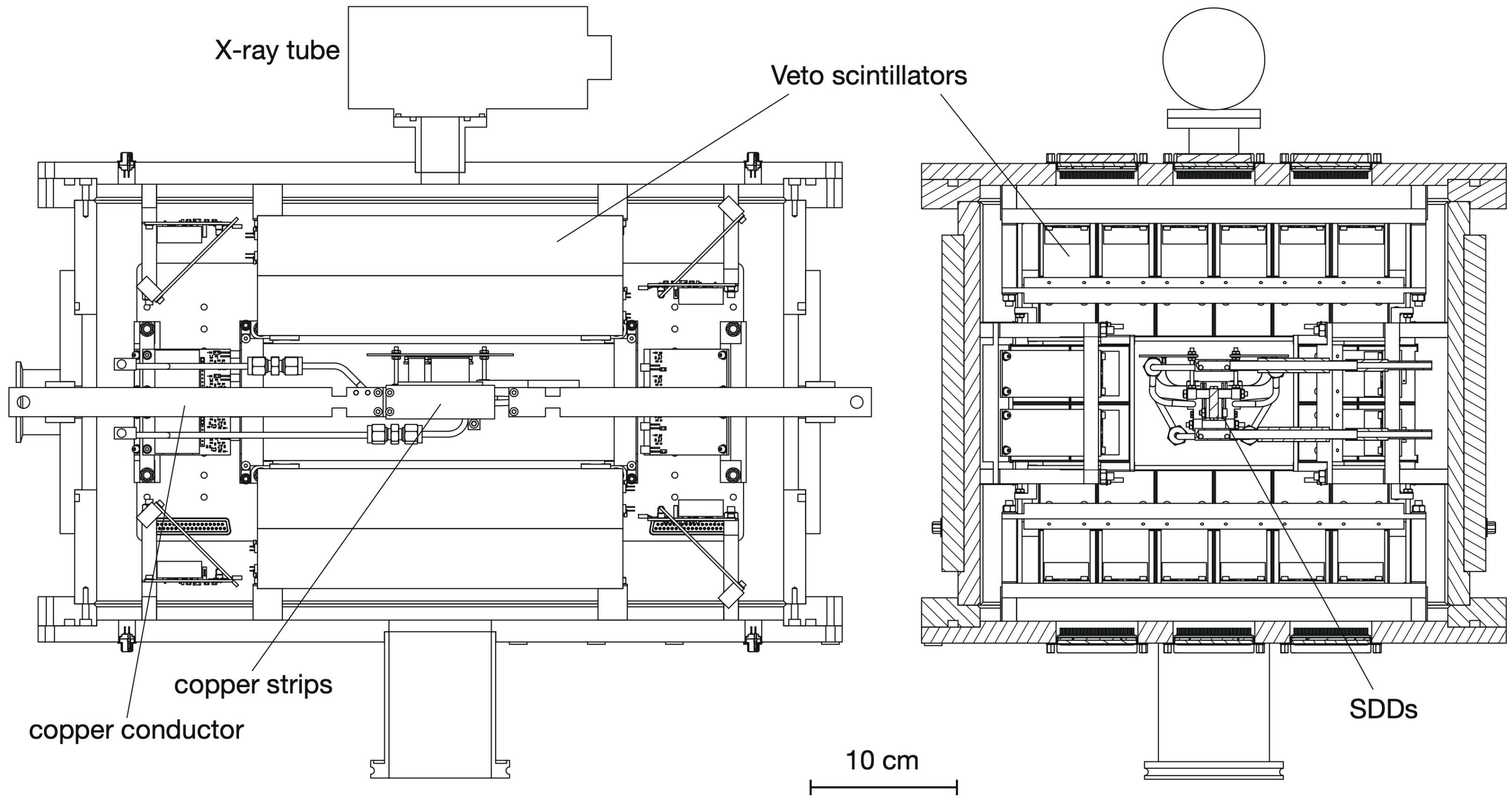
searching **VI**olation of **P**auli Exclusion Principle

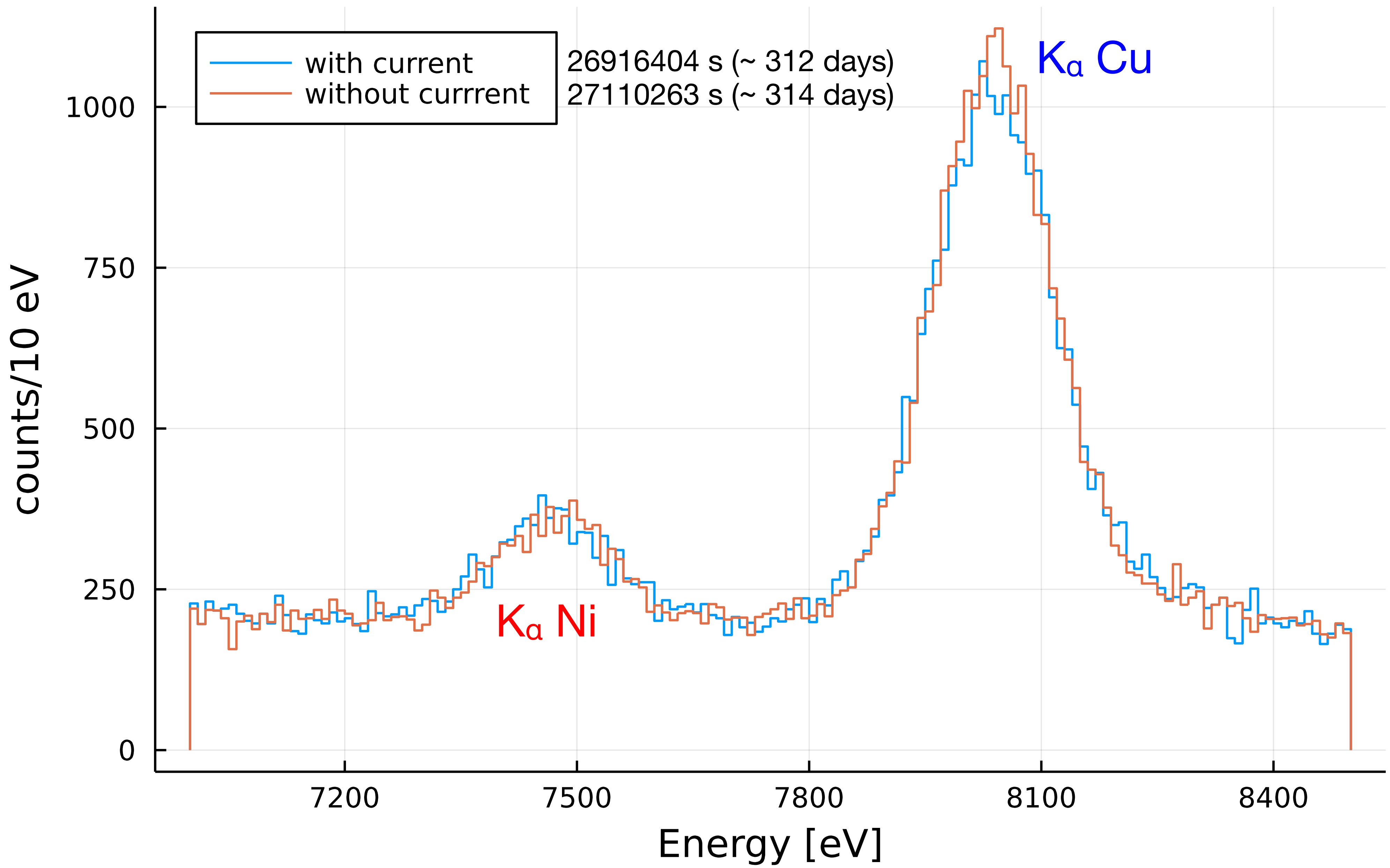
VIP-2





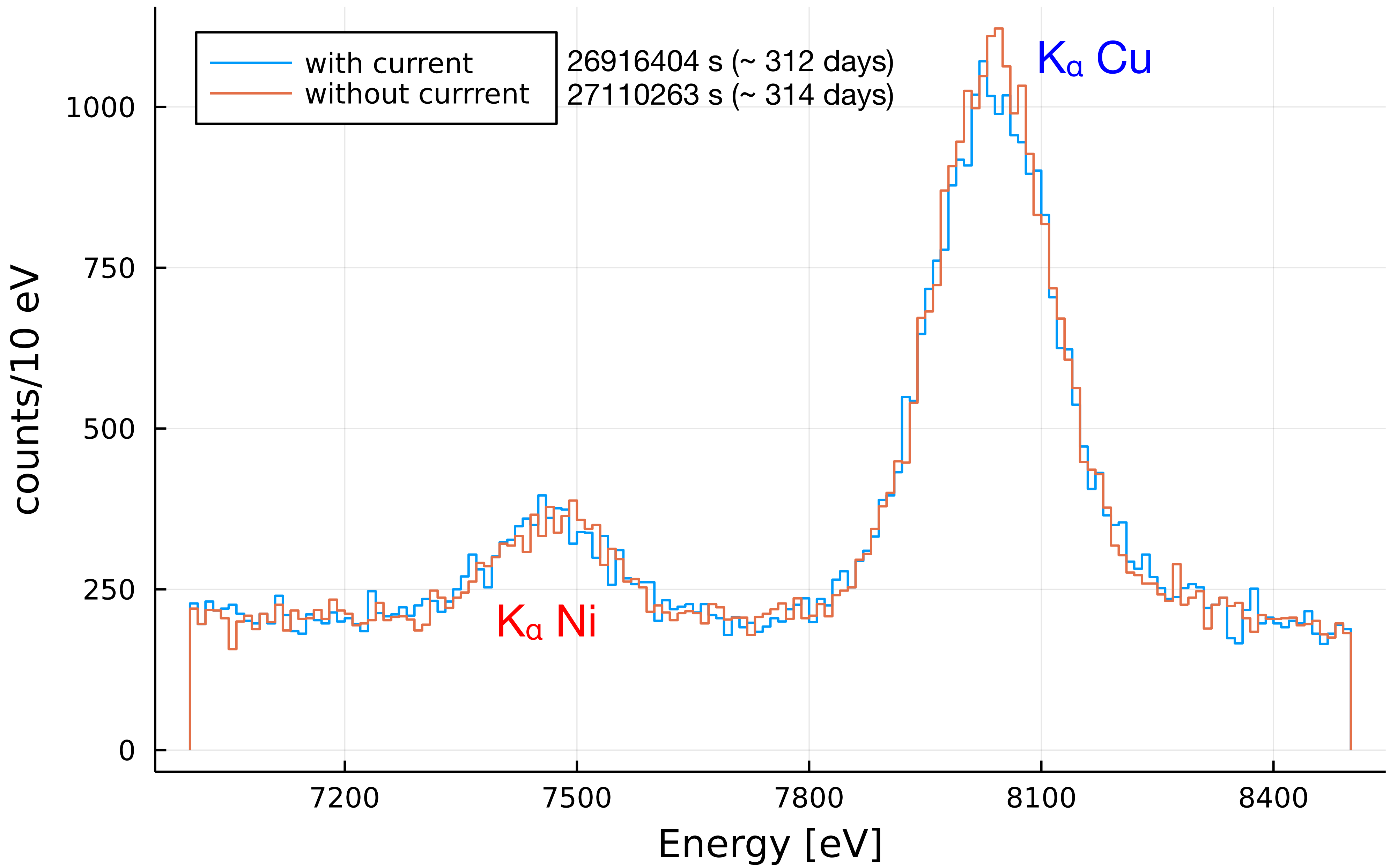
- **Target**: Copper strips
  - WITHOUT CURRENT configuration: regime case (stable states: background)
  - WITH CURRENT configuration (180 A): dynamic case (PEP violation through electron capture)
- **SDD**: 32 detectors by SDDs, stably kept @ -170^{+1}_{-0} °C even with the current in Cu
- **@LNGS Underground** (beneath Gran Sasso Mountain – IT): ~1400 m of rock shielding

Data model


with current
without currrent
26916404 s (~ 312 days)
27110263 s (~ 314 days)
Kα Cu
Kα Ni
counts/10 eV
Energy [eV]
0
250
500
750
1000
7200
7500
7800
8100
8400

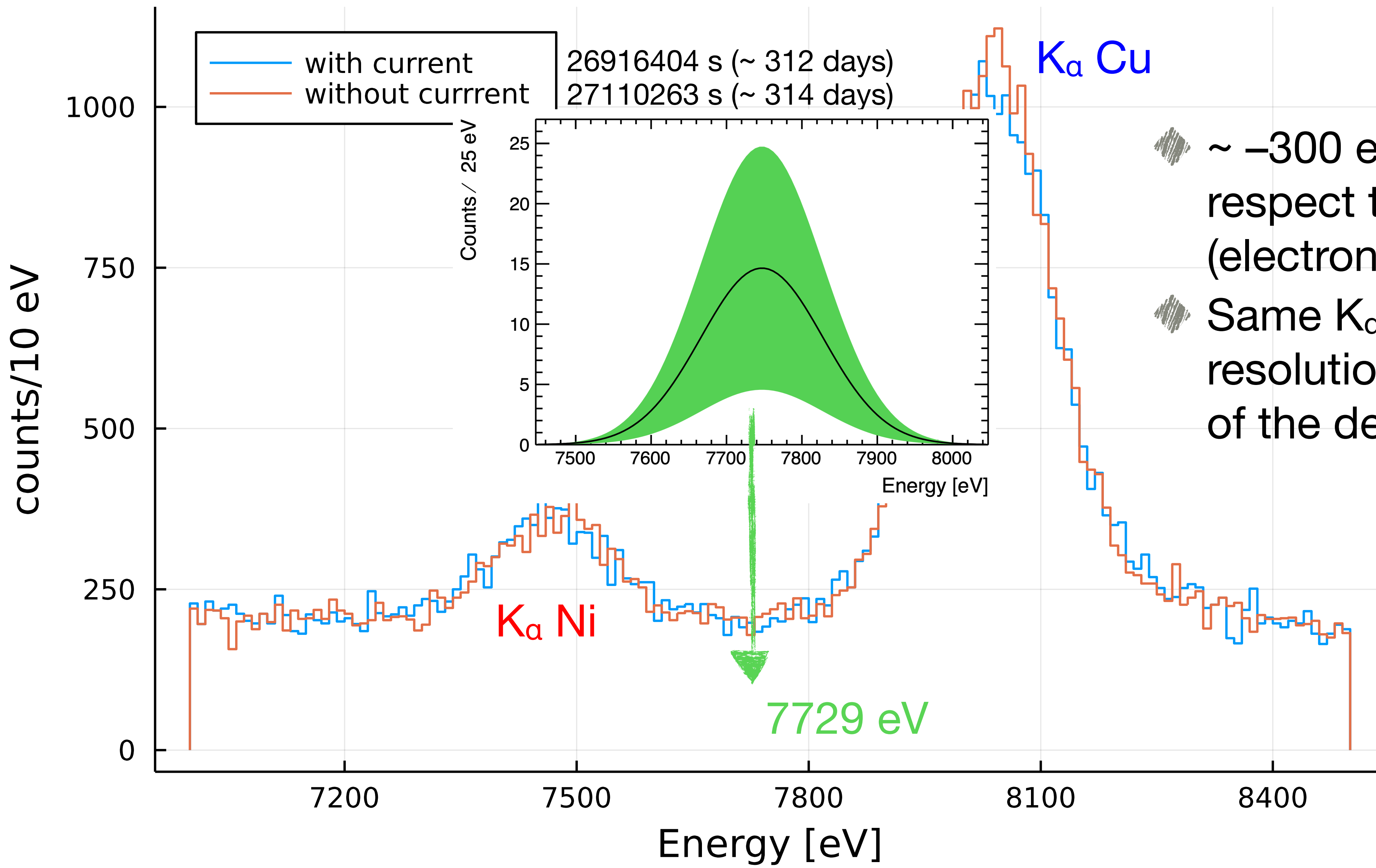
Data model

$$\mathcal{F}^{woc}(\boldsymbol{\theta}, \boldsymbol{y}) = y_1 \times Ni(\theta_1, \theta_2) + y_2 \times Cu(\theta_3, \theta_4) + y_3 \times \mathrm{pol}_1(\theta_5)$$



Data model

$$\mathscr{F}^{wc}(\boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S}) = y_1 \times Ni(\theta_1, \theta_2) + y_2 \times Cu(\theta_3, \theta_4) + y_3 \times \mathrm{pol}_1(\theta_5) + \mathscr{S} \times PEPV(\theta_4)$$



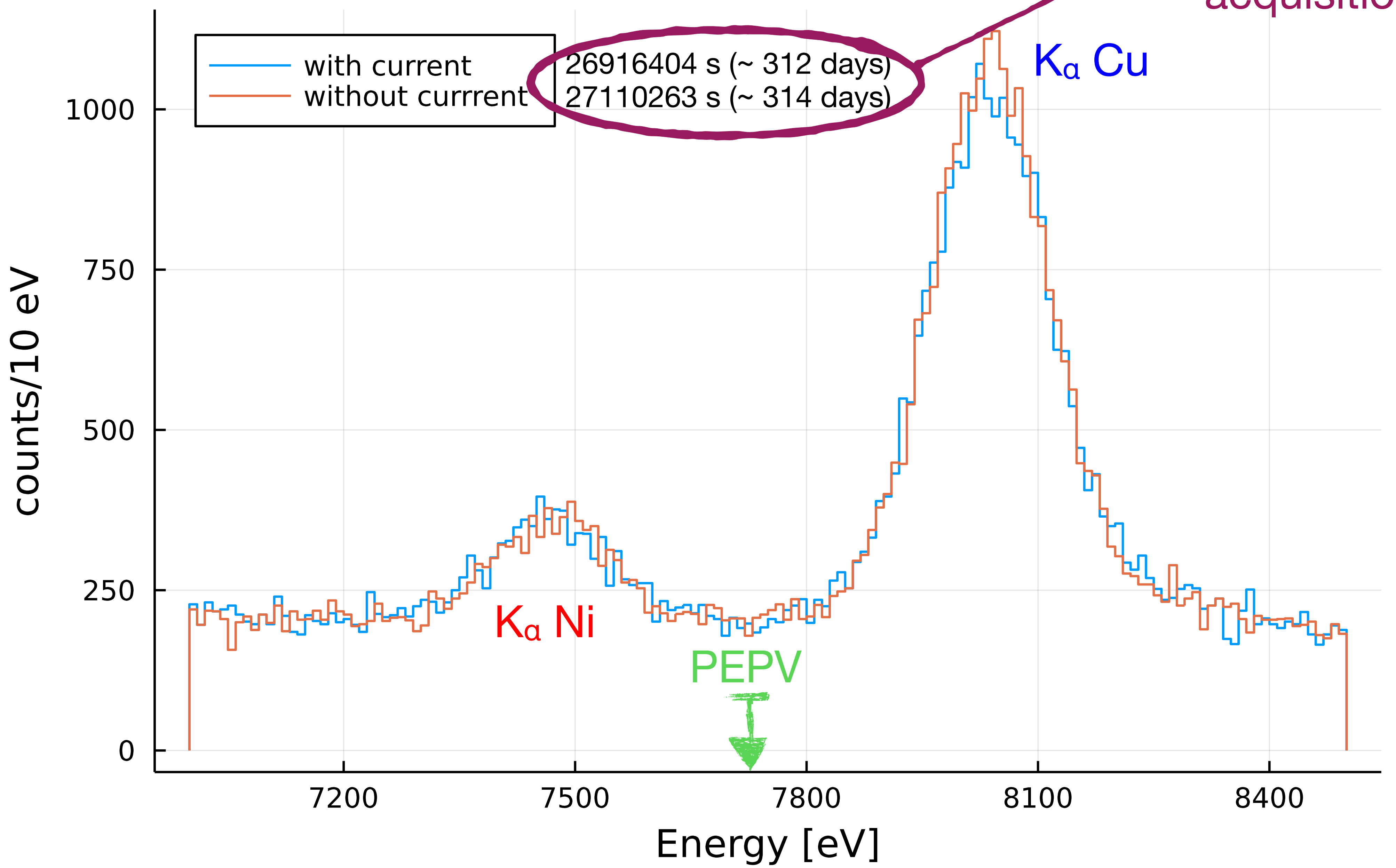
- ~ –300 eV with respect to Kα Cu (electron shielding)
- Same Kα Cu resolution (i.e., σ of the detector)

Data Likelihood

$$\mathscr{L}(\mathscr{D}^{wc}, \mathscr{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S}) = \text{Poiss}(\mathscr{D}^{wc} \,|\, \mathscr{F}^{wc}(\boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S})) \times \text{Poiss}(\mathscr{D}^{woc} \,|\, \mathscr{F}^{woc}(\boldsymbol{\theta}, \boldsymbol{y} \times \mathscr{R}))$$

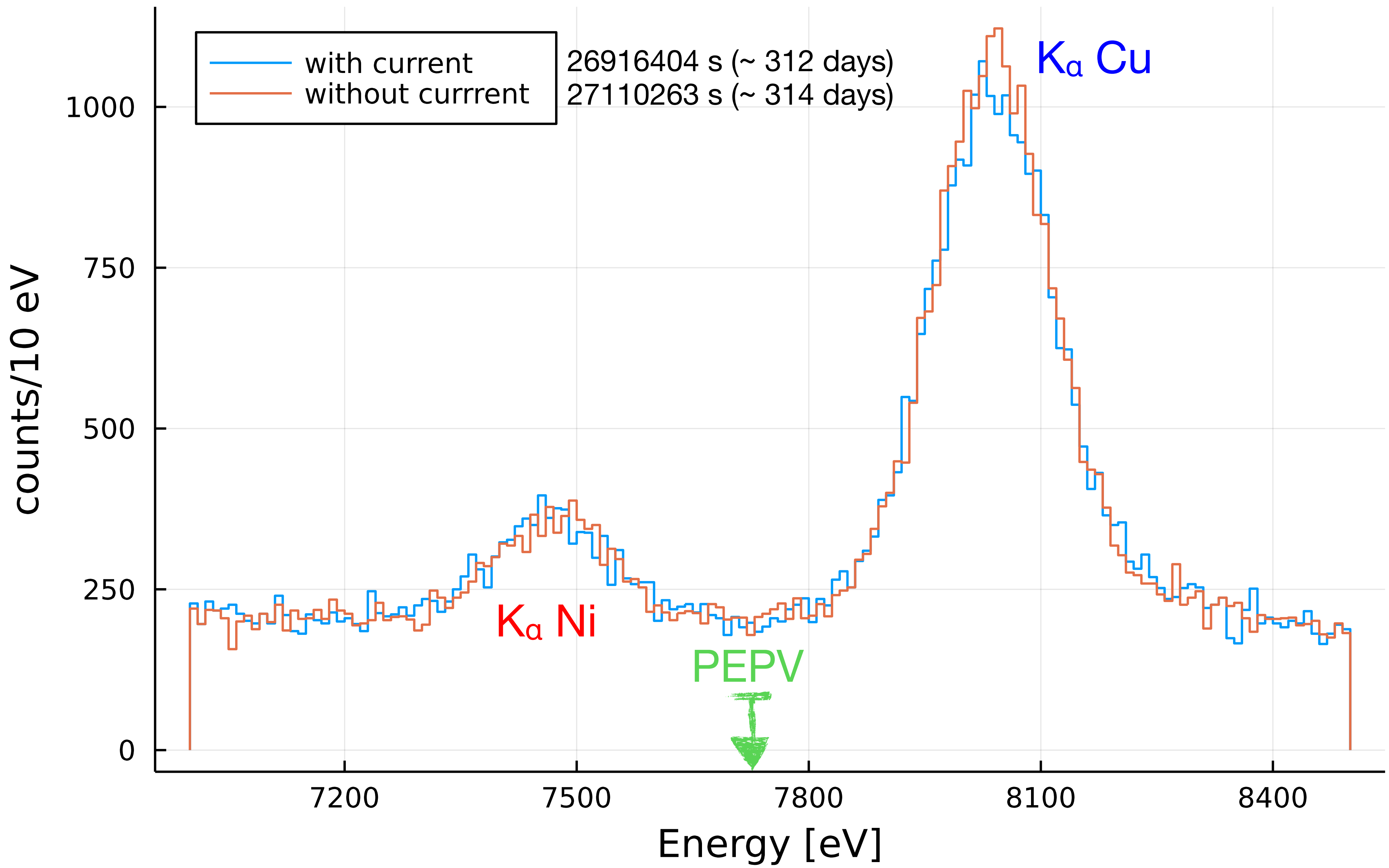
[mind: $\text{Poiss}(\mathscr{D} \,|\, \mathscr{F}) = \frac{\mathscr{F}^{\mathscr{D}}}{\mathscr{D}!} e^{-\mathscr{F}}$, $\mathscr{D}$ are data, $\mathscr{F}$ is the model]

Ratio of data acquisition time



Bayesian approach

$$p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S} \,|\, \mathcal{D}^{wc}, \mathcal{D}^{woc}) = \frac{\mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}{\int d\boldsymbol{\theta} d\boldsymbol{y} \mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}$$

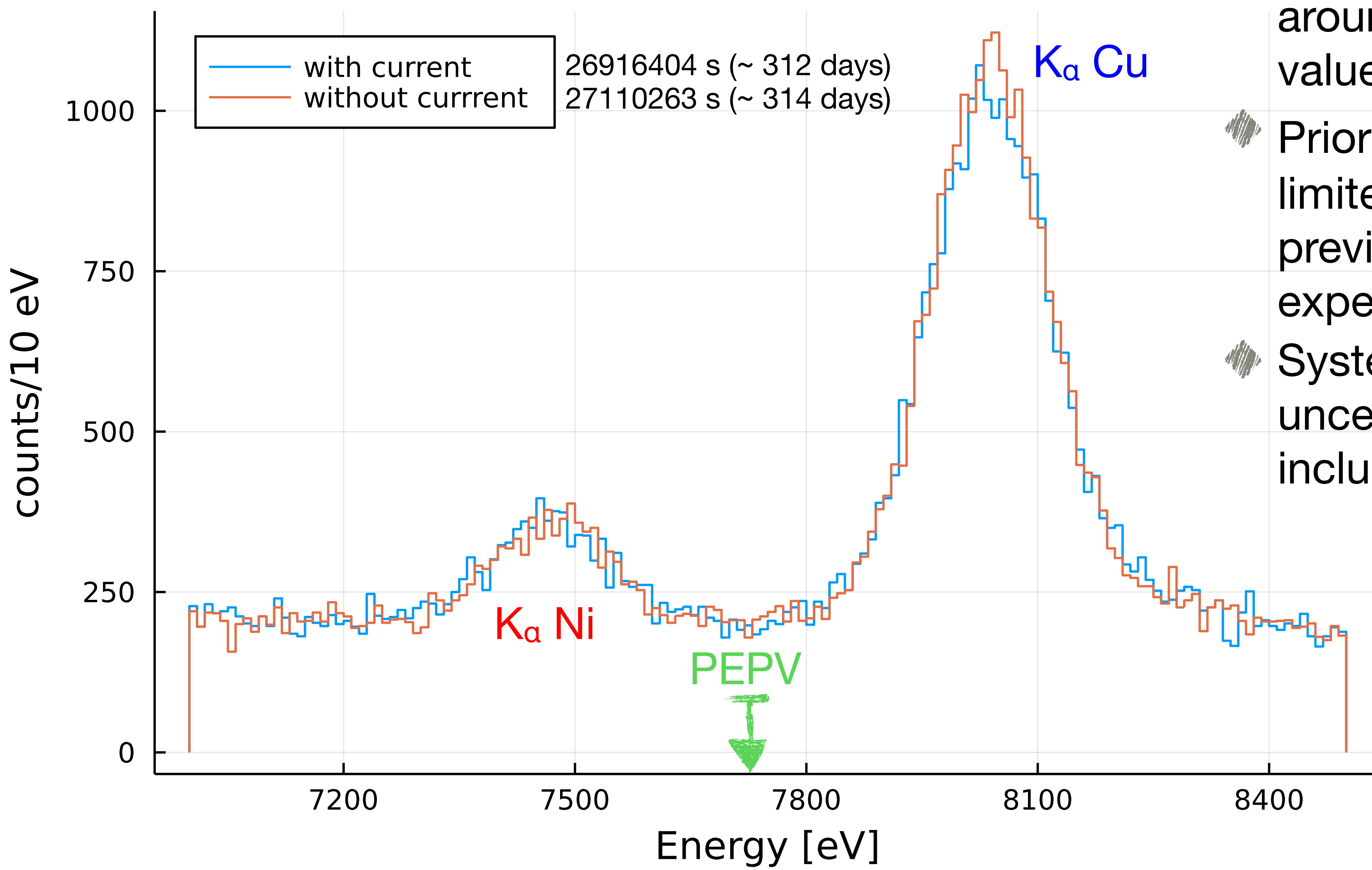


Bayesian approach

Posterior

$$p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S} \mid \mathcal{D}^{wc}, \mathcal{D}^{woc}) = \frac{\mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \mid \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}{\int d\boldsymbol{\theta} d\boldsymbol{y} \mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \mid \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}$$

- Priors of $\boldsymbol{\theta}$ and $\boldsymbol{y}$ are Gaussians: statistical fluctuations around known values
- Prior of $\mathcal{S}$ is flat, limited from previous experiments
- Systematic uncertainties included

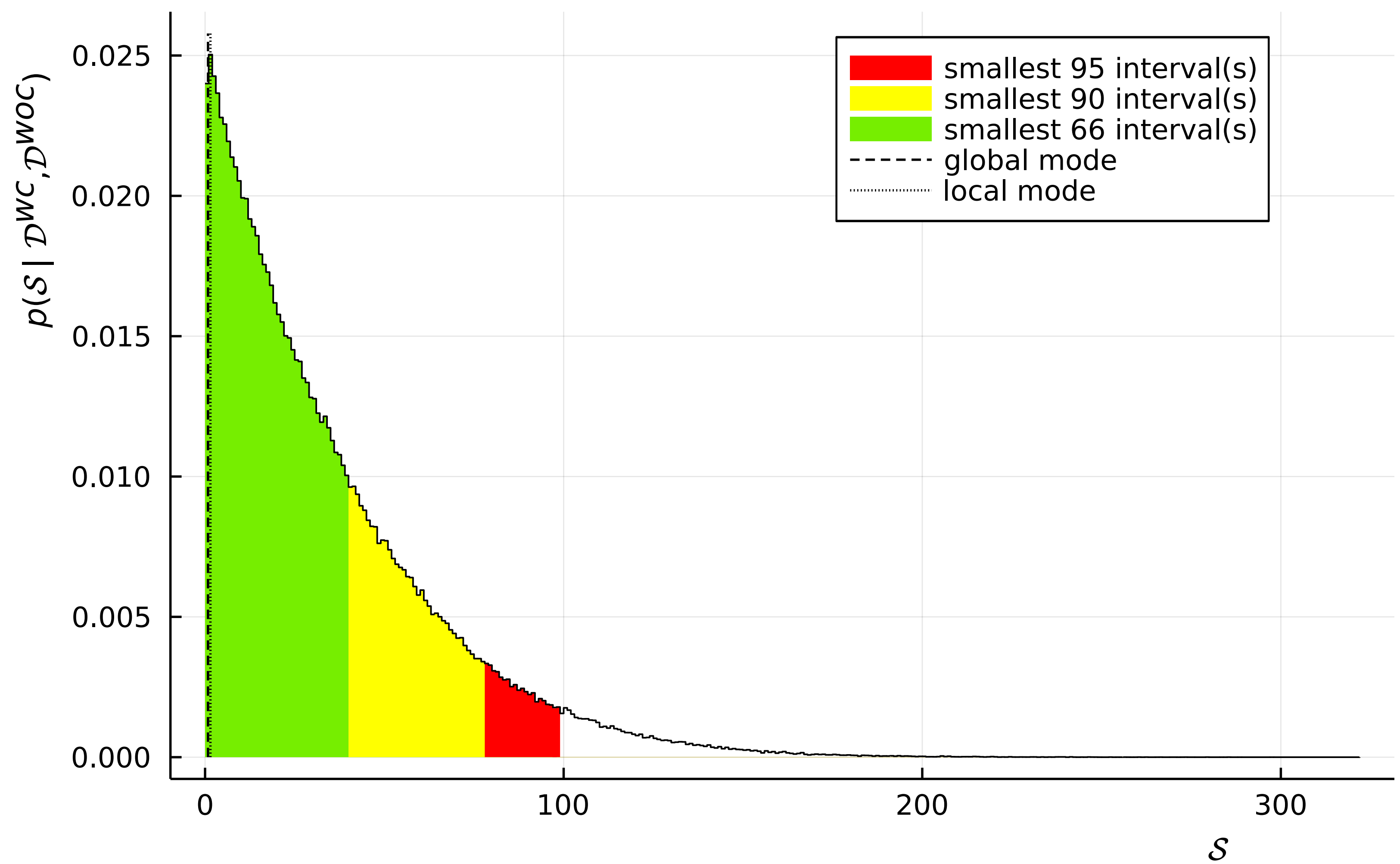


Bayesian result

(marginalized Posterior)

$$p(\mathcal{S} \mid \mathcal{D}^{wc}, \mathcal{D}^{woc}) = \int p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S} \mid \mathcal{D}^{wc}, \mathcal{D}^{woc}) d\boldsymbol{\theta} d\boldsymbol{y}$$

Integrals with Markov Chain Monte Carlo method



Modified frequentist CL$_s$

one-sided test statistic

$$t_{\mathscr{S}} = -2\ln\Lambda(\mathscr{S}) = -2\ln\frac{\mathscr{L}(\hat{\hat{\boldsymbol{\theta}}}, \hat{\hat{\mathbf{y}}}, \mathscr{S})}{\mathscr{L}(\hat{\boldsymbol{\theta}}, \hat{\mathbf{y}}, \hat{\mathscr{S}})}$$

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Profile Likelihood;
$\mathscr{L}$ now includes multiplicative penalties given by experimental uncertainties: i.e., the priors in the Bayesian

$$\mathscr{L}(\boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S}) = \mathscr{L}(\mathscr{D}^{wc}, \mathscr{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S})\, p(\boldsymbol{\theta}, \boldsymbol{y}, \mathscr{S})$$

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$\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{y}}, \hat{\mathscr{S}}$ are the values that maximize the Likelihood;
i.e., the denominator is the standard maximum Likelihood

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$\hat{\hat{\boldsymbol{\theta}}}, \hat{\hat{\boldsymbol{y}}}$ are the values that maximize the Likelihood with a given $\mathscr{S}$;
i.e., a set of parameters for each test-value $\mathscr{S}$

Modified frequentist CLs

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$t_{\mathscr{S}}$ distribution, given $\mathscr{S}$

$$p_{\mathscr{S}} = \int_{t_{obs}}^{\infty} f(t_{\mathscr{S}} \,|\, \mathscr{S})dt_{\mathscr{S}}$$

$t_{\mathscr{S}}$ of observed $\mathscr{S}$

$$\text{CLs} = \frac{p_{\mathscr{S}}}{1 - p_0} < 1 - \text{C.L.} \quad (\text{i.e., 90\% C.L.} \Rightarrow \text{CLs} < 0.1)$$

background case (i.e., $\mathscr{S} = 0$)

CLs result

$$p_{\mathcal{S}} = \int_{t_{obs}}^{\infty} f(t_{\mathcal{S}} \,|\, \mathcal{S}) dt_{\mathcal{S}}$$

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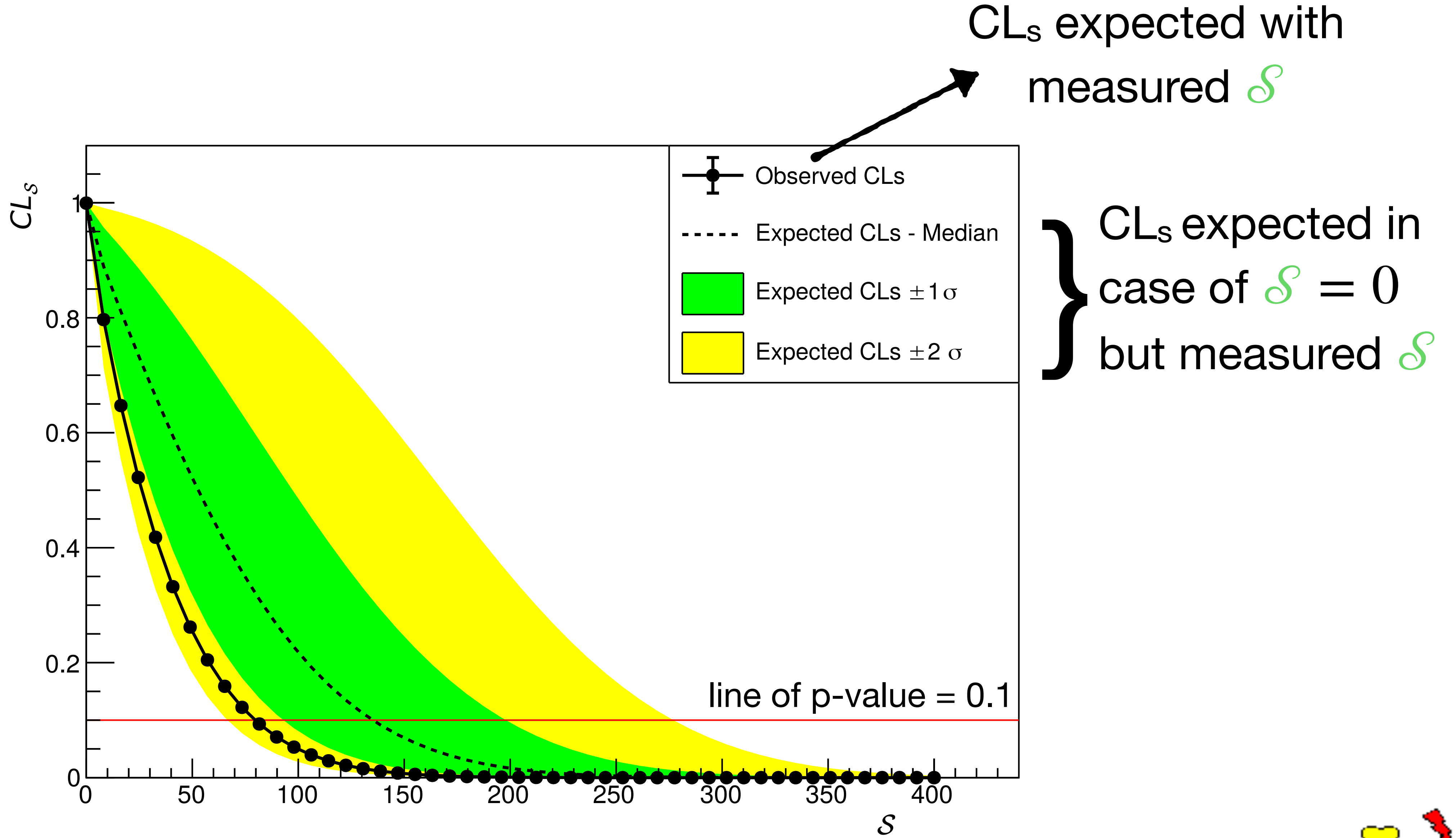
Computation with RooFit



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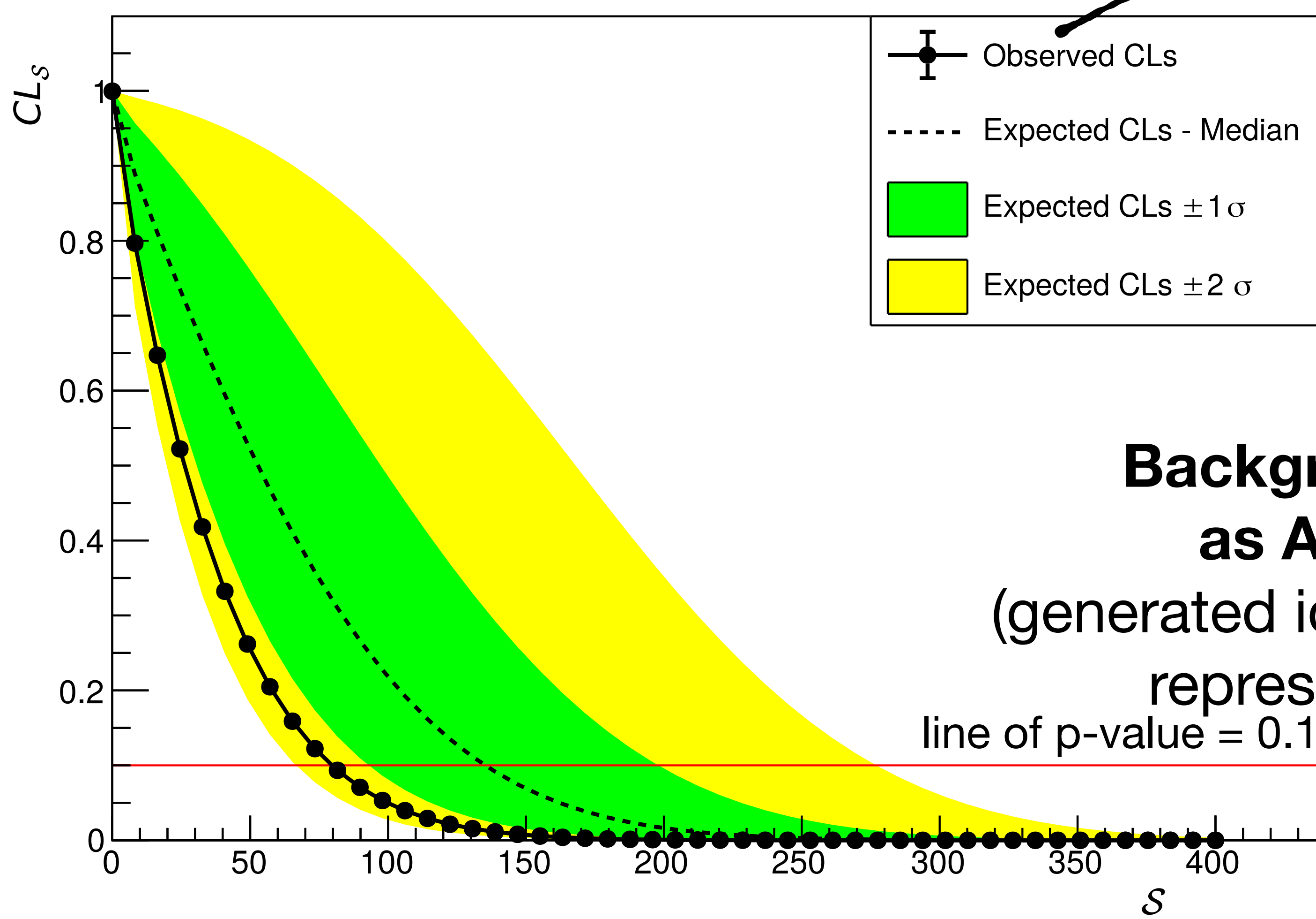
Computation with RooFit

CLs expected with measured $\mathcal{S}$



CLs expected in case of $\mathcal{S} = 0$ but measured $\mathcal{S}$

Background Hypothesis as Asimov Dataset

(generated ideal dataset most likely representing the model)

From $\mathcal{S}$ to $\beta^2/2$

$$N_x \simeq \frac{\beta^2}{2} \cdot N_{\mathrm{new}} \cdot \frac{N_{\mathrm{int}}}{10} \cdot 7.25 \times 10^{-2}$$

From $\mathcal{S}$ to $\beta^2/2$

This is our $\mathcal{S}$!

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Newly injected electrons!

$$\sum_i^{\text{runs}} I_i \Delta t_i / e \; (= I\Delta t / e \;\; \text{for simplicity})$$

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every ~10 interactions, 1 cascade

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$$\frac{\beta^2}{2} \simeq \mathcal{S} \cdot \frac{10}{N_{\text{int}}} \cdot \frac{e}{I\Delta t} \cdot \frac{1}{7.25 \times 10^{-2}}$$

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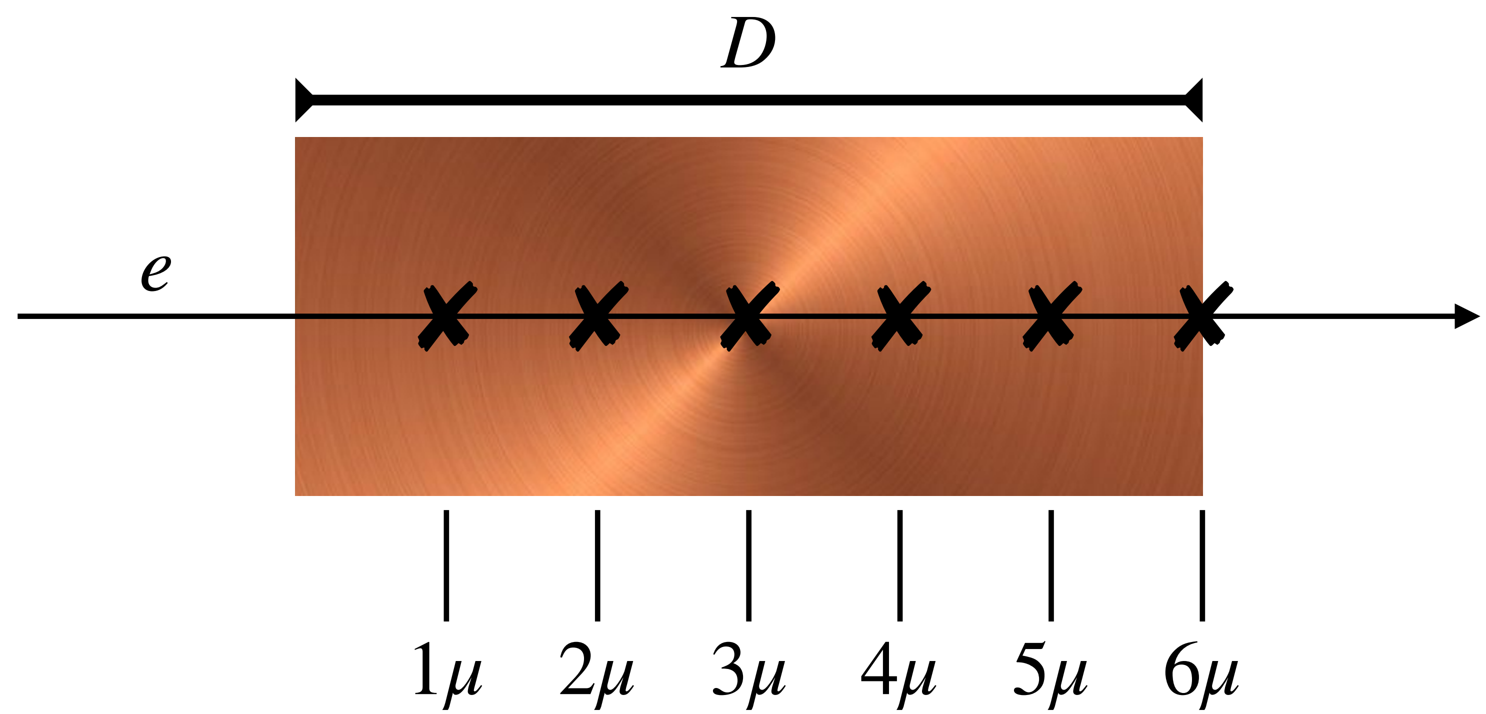
N_{int} is the normalization that decides the order of magnitude of $\beta^2/2$

Let's discuss e–atoms interaction Models!

N_{int} by Linear Scattering



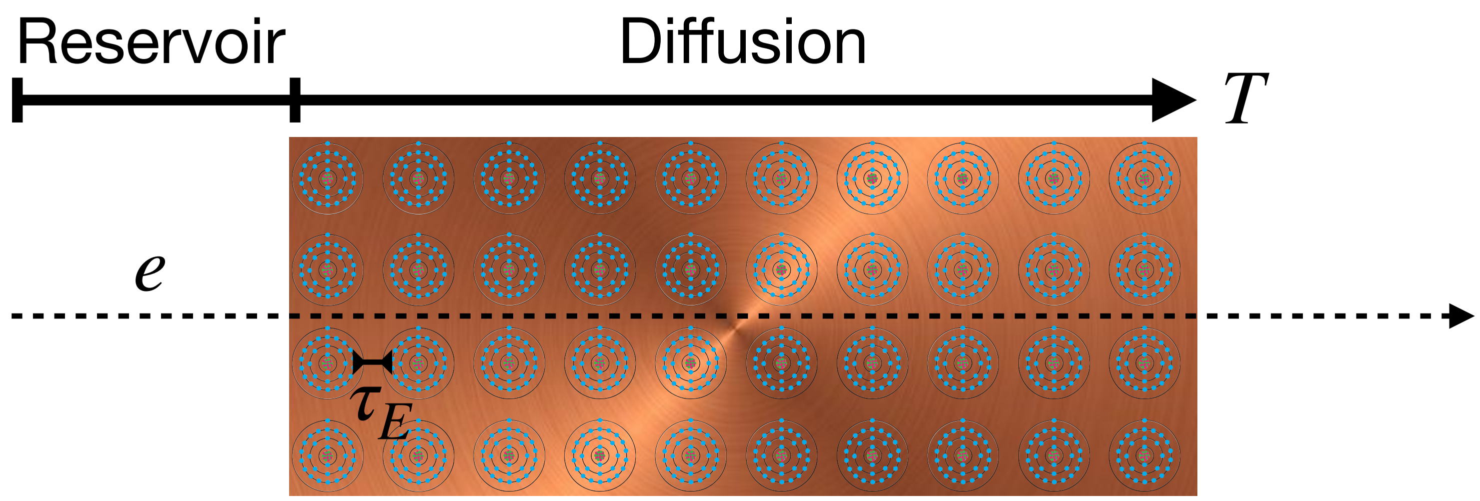
Through Copper Resistance,

we know the average interaction length μ

$$N_{\text{int}} = D/\mu \simeq 1.95 \times 10^6$$

$$\Rightarrow \frac{\beta^2}{2} \lessapprox 10^{-31}$$

N_{int} by Close Encounters



Through Diffusion-Transport theory and Copper atomic density, we know:

- the average time τ_E on atomic encounter for a diffused electron
- the average time T of target crossing by an electron

$$N_{\text{int}} = T/\tau_E \simeq 4.29 \times 10^{17}$$

$$\Rightarrow \frac{\beta^2}{2} \lessapprox 10^{-43}$$

Outlook

Bayesian

- Well established: excellent for low statistical signals
- Systematic uncertainty is the combination of different priors for the various factors

CL_s

- Models with little or no sensitivity to the null hypothesis, e.g., if the data fluctuate very low relative to the expectation of the background-only hypothesis: the lower/upper limit might be anomalously low; more robust compared to the classic p-value
- Sensible to small parameter fluctuations

N_{int}

- **Linear Scattering**: due to phonons and lattice irregularities
  - ☑ Safest hypothesis
  - ☒ Largely underestimation of how many interactions an electron does
- **Close Encounters**: a more realistic model of e-atom encounters, but still approximated
  - ☑ 12 order of magnitudes larger than Linear Scattering!
- **This is the key element to improve the measurement!**

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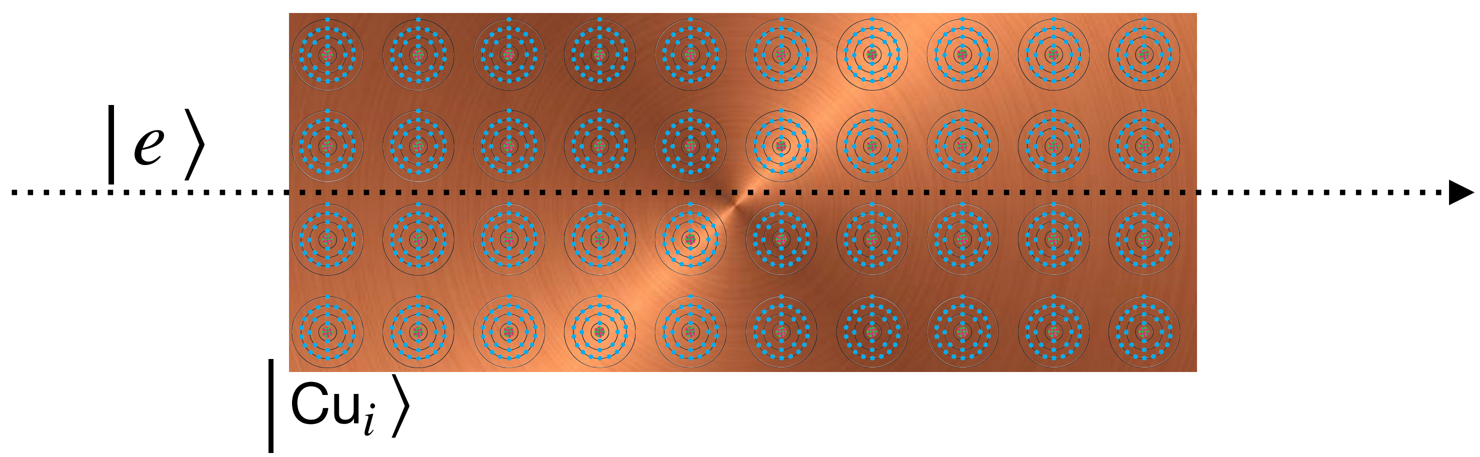
THANK YOU

BACKUPS

TO DO: a quantum N_{int}?



How many interactions between Cu atomic and electron fields occur?