Decoherence and discrete symmetries from Planck-scale deformed relativistic kinematics

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ECT - Nuclear and Atomic transitions as laboratories for high precision tests of Quantum Gravity inspired models

Where it all started from...

1983: pre-history of QG phenomenology

SEARCH FOR VIOLATIONS OF QUANTUM MECHANICS*

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tests of **fundamental decoherence** using **neutral kaons** and **neutron interferometry**, main motivation given by:

PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Breakdown of predictability in gravitational collapse*

S. W. Hawking†
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England and California Institute of Technology, Pasadena, California 91125
(Received 25 August 1975)

suggested **fundamental loss of information** in black hole evaporation

Is purity eternal?

Fundamental decoherence in quantum gravity?

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- BH quantum radiance suggests the possibility that $\rho_{in}(\mathrm{pure}) \rightarrow \rho_{fin}(\mathrm{mixed})$
- Hawking proposed that in quantum gravity (QG) S is replaced by a **"superscattering"** operator $

$$\rho_{fin} = \$\rho_{in} \neq S\rho_{in}S^{\dagger}$$

so that $\mathrm{Tr}\rho_{fin}^2 \leq 1$

The idea of Ellis et al. was to explore the **phenomenology** of such non-unitary evolution as determined by a differential evolution equation for ρ

$$\dot{\rho} = \not{H}\rho \neq -i[H, \rho]$$

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$$\dot{\rho} = -i[H, \rho] - \frac{1}{2} h_{\alpha\beta} \left(Q^\alpha Q^\beta \rho + \rho\, Q^\beta Q^\alpha - 2 Q^\alpha \rho\, Q^\beta \right)$$

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Can such modification of fundamental quantum evolution
be obtained **from a model incorporating quantum gravity effects?**

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- At algebraic level: "deformation" of the action of **translation** and **Lorentz generators** on **states** and **observables** of a relativistic system
- Such deformation affects basic notions in quantum theory leading to
  - potential **fundamental decoherence**
  - **deformed** discrete symmetries and CPT

MA, Phys. Rev. D **90**, 024016 (2014) arXiv:1403.6457
MA and J. Kowalski-Glikman, Phys. Lett. B **760**, 69 (2016) arXiv:1605.01181
MA, J. Kowalski-Glikman, W. Wislicki, Phys. Lett. B **794**, 41 (2019) arXiv:1904.06754
MA, V. D'Esposito and G. Gubitosi, [arXiv:2208.14119 [gr-qc]]

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- The **four-momenta** describing the particle kinematics become **coordinates on a non-abelian Lie group**

Group-valued momenta from $2+1$-dimensional gravity

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- Upon quantization relativistic particles are described by a **non-commutative field theory** with $\mathfrak{sl}(2,\mathbb{R})$ coordinates (Freidel and Livine, Phys.Rev.Lett. **96** (2006))

$$[X_\mu, X_\nu] = \frac{i}{\kappa}\epsilon_{\mu\nu\lambda}X_\lambda$$

(see also 't Hooft, Class. Quant. Grav. **13**, 1023-1040 (1996))

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$\Rightarrow$ use **quantum groups tools** to *deform* symmetries introducing a **UV energy-scale** κ

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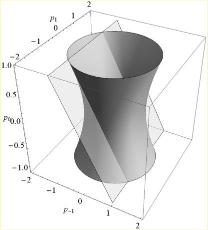
κ-**four-momenta**: coordinates on **Lie group** $AN(3)$ obtained form the **Iwasawa decomposition** of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4

embedding coordinates

$$-p_0^2 + \vec{p}^{\,2} + p_{-1}^2 = \kappa^2 , \quad p_0 + p_{-1} > 0$$



(see e.g. Kowalski-Glikman and Nowak, hep-th/0411154)

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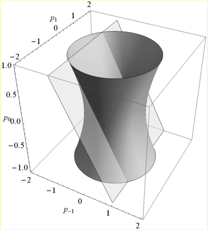

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- $\mathfrak{an}(3)$ Lie algebra: κ-Minkowski **"non-commutative space-time"**

$$[X_0, X_a] = \frac{i}{\kappa} X_a\,, \quad [X_a, X_b] = 0$$

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i.e. just the familiar **adjoint action** $\mathrm{ad}_{P_\mu}\pi_k = [P_\mu, \pi_k]$

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- for the **action on bras** the non-trivial properties of momenta come into play

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Key point: the action on operators will be deformed accordingly

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Consider **translation generators** P_μ associated to *embedding* coordinates p_μ on dS_4

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In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

Lindblad evolution from κ-translations

Evolution of the density operator ρ = **adjoint action** of H generator of time translations

$$i\,\partial_t \rho = [H, \rho] = \mathrm{ad}_H\, \rho$$

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plugging the κ-**deformed coproduct and antipode** one obtains

$$\boxed{\partial_t \rho = -i\,[P_0, \rho] - \frac{1}{2\kappa}\left(\mathbf{P}^2 \rho + \rho\, \mathbf{P}^2 - 2\, P_i\, \rho P^i\right)}$$

a momentum-dependent **Lindblad equation**

Purity is not eternal in quantum space-time

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- Look at evolution of the **linear entropy**

$$S(t) = 1 - \mathrm{Tr}(\rho^2)$$

one has

$$\frac{d}{dt} S = \frac{1}{2\kappa} \mathrm{Tr}\left(\rho\left(\boldsymbol{P}^2 \rho + \rho \boldsymbol{P}^2 - 2\, P_i \rho\, P^i\right)\right)$$

Free particle in the limit $t \to \infty$

$$S(t) \sim 1 - \left(\frac{\pi\,\kappa}{t}\right)^{\frac{3}{2}} [1 - S(0)]$$

i.e. for long enough time any state becomes a **maximally mixed** one!

Testing deformations via precision measurements of neutral kaons

*Phenomenology of κ-Lindblad evolution? (*Ellis et al.*"Search for Violations of Quantum Mechanics," Nucl. Phys. B* **241***, 381 (1984)); bounds on κ using* **precision measurements of neutral kaon systems** *(KLOE and KLOE-2 experiment)?*

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- **On the agenda:** input from κ-Lindblad to **derive** deformed evolution based on effective Hamiltonian for K^0-$\bar{K}^0$...

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Other experimental windows?

Maybe neutrino oscillations?

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Natural question: do the new structures introduced so far affect **discrete symmetries** ?

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As shown in (MA and J Kowalski-Glikman, Phys. Lett. B **760**, 69 (2016)) the non-trivial **antipode** for κ-**Poincaré generators** plays a prominent role in defining discrete symmetries ultimately leading to a **deformed notion of CPT transformation**

Idea: use basic physical requirements and algebraic consistency to define the action of P, T and C....

κ-deformed P and T

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- **TIME REVERSAL**: require that in the limit $\kappa \to \infty$, $\mathbb{T}$ flips sign of M_i

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  - imposing that in the $\kappa \to \infty$ one recovers usual ordinary $\mathbb{C}$ we obtain

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$$\mathbb{CPT}(P_i) = P_i - \frac{P_0 P_i}{\kappa} + O\left(\frac{1}{\kappa^2}\right), \quad \mathbb{CPT}(P_0) = -S(P)_0 = P_0 - \frac{\mathbf{P}^2}{\kappa} + O\left(\frac{1}{\kappa^2}\right)$$

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MAIN MESSAGE: non-trivial antipode $\Rightarrow$ the action of the $\mathbb{CPT}$ operator **is deformed**

(NOTE: this differs from the usual violation of $\mathbb{CPT}$ expected in presence of decoherence (Wald, 1980))

A bound on κ from muon lifetime

The deformed $\mathbb{CPT}$ map leads to **different lifetimes** between particles and anti-particles

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from experimental uncertainty on μ-lifetime ([PDG], PRD **98**, no. 3, 030001 (2018))

$\kappa \gtrsim 4 \times 10^{14}$ GeV ($\mathbf{p} = 6.5$ TeV (LHC))

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$$\mathcal{P}_{\text{part}} = \frac{\Gamma E}{m} \exp\left(-\Gamma t\, \frac{E}{m}\right), \qquad \mathcal{P}_{\text{apart}} = \Gamma\left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m}\right) e^{-\Gamma t\left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m}\right)}$$

from experimental uncertainty on μ-lifetime ([PDG], PRD **98**, no. 3, 030001 (2018))

$\kappa \gtrsim 4 \times 10^{14}$ GeV ($\mathbf{p} = 6.5$ TeV (LHC)) $\qquad \kappa \gtrsim 2 \times 10^{16}$ GeV ($\mathbf{p} = 50$ TeV (FCC))

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We need the input of experimentalists to take advantage of these possible new windows on the QG world...

Appendix: BH quantum radiance in a nutshell

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Started from the pure state $\rho_0 \longrightarrow$ BH evaporation left us with a *mixed state*