The decay law in the multichannel case
and
for moving particles

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Nuclear and Atomic transitions as laboratories for high precision tests
of Quantum Gravity inspired models

Outline

1. General discussion of the decay law

2. Non-exponential decay: experiments

3. Multichannel Decay
   - Multichannel decay in QM
   - Short digression: double-delta potential
   - Multichannel case in QFT

4. Decay of a moving particle

5. Related topics and conclusions

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General discussion of the decay law

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Exponential decay law

- N_0 : Number of unstable particles at the time t = 0.

$$\boxed{N(t) = N_0 e^{-\Gamma t}} \;, \quad \tau = 1/\Gamma \text{ mean lifetime}$$

  Confirmend in countless cases!

- For a single unstable particle:

$$\boxed{p(t) = e^{-\Gamma t}}$$

  is the survival probability for a single unstable particle created at t=0.
  (Intrinsic probabilty, see Schrödinger´s cat).

  For small times: $\quad p(t) = 1 - \Gamma t + ...$

Let $|S\rangle$ be an unstable state prepared at $t = 0$.

Survival probabilty ammplitude at $t > 0$:

$$a(t) = \langle S | e^{-iHt} | S \rangle$$

Survival probability : $p(t) = |a(t)|^2$

Deviations from the exp. law at short times

Taylor expansion of the amplitude:

$$a(t) = \langle S|e^{-iHt}|S\rangle = 1 - it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + ...$$

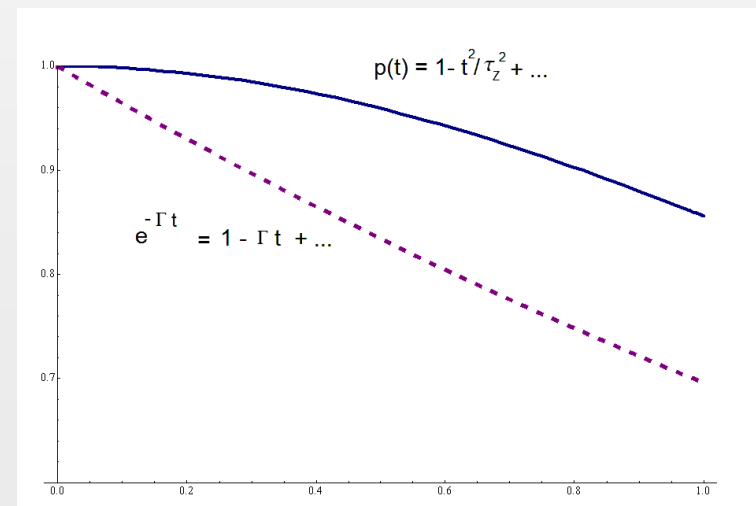$$a^*(t) = \langle S|e^{-iHt}|S\rangle = 1 + it\langle S|H|S\rangle - \frac{t^2}{2}\langle S|H^2|S\rangle + ...$$

It follows:

$$p(t) = |a(t)|^2 = a^*(t)a(t) = 1 - t^2\left(\langle S|H^2|S\rangle - \langle S|H|S\rangle^2\right) + ... = 1 - \frac{t^2}{\tau_Z^2} + ...$$

p(t) decreases quadratically (<u>not linearly</u>);

no exp. decay for short times.

τ_Z is the `Zeno time´.

where $\tau_Z = \dfrac{1}{\sqrt{\langle S|H^2|S\rangle - \langle S|H|S\rangle^2}}$.

$$p(t) = 1 - t^2/\tau_Z^2 + ...$$

$$e^{-\Gamma t} = 1 - \Gamma t + ...$$

Note: the quadratic behavior holds
for any quantum transition, not only for decays.

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Time evolution and energy distribution (1)

The unstable state $|S\rangle$ is not an eigenstate of the Hamiltonian H.

Let $d_S(E)$ be the energy distribution of the unstable state $|S\rangle$.

Normalization holds: $\int_{-\infty}^{+\infty} d_S(E) dE = 1$

$$p(t) = \left| \int_{E_{th,1}}^{\infty} dE d_S(E) e^{-\frac{i}{\hbar} E t} \right|^2$$

In stable limit : $d_S(E) = \delta(E - M) \rightarrow a(t) = e^{-iMt} \rightarrow p(t) = 1$

Time evolution and energy distribution (2)

Breit-Wigner distribution:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4} \rightarrow a(t) = e^{-iM_0 t - \Gamma t/2} \rightarrow p(t) = e^{-\Gamma t}.$$
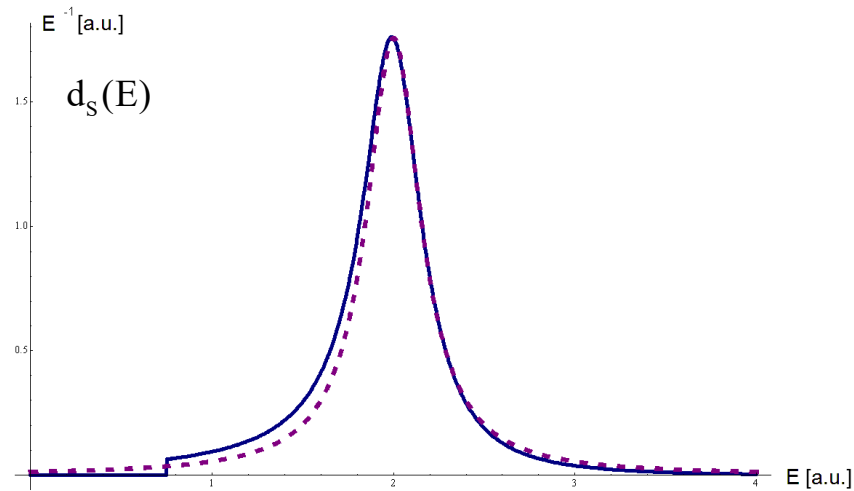
The Breit-Wigner energy distribution cannot be exact.

Two physical conditions for a realistic $d_S(E)$ are:

1) Minimal energy: $\quad d_S(E) = 0$ for $E < E_{min}$

2) Mean energy finite: $\quad \langle E \rangle = \int_{-\infty}^{+\infty} d_S(E) E \, dE = \int_{E_{min}}^{+\infty} d_S(E) E \, dE < \infty$
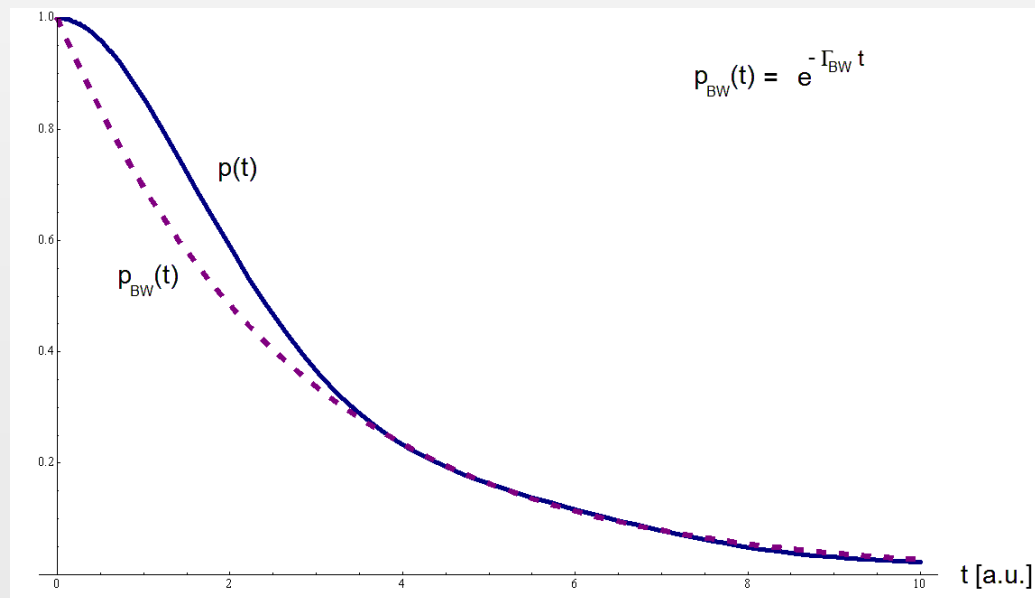
A very simple numerical example

E^{-1} [a.u.]

$d_S(E)$

E [a.u.]

$$M_0 = 2; \quad E_{min} = 0.75; \quad \Gamma = 0.4; \quad \Lambda = 3$$

$$d_S(E) = N_0 \frac{\Gamma}{2\pi} \frac{e^{-(E^2-E_0^2)/\Lambda^2}\,\theta(E-E_{min})}{(E-M_0)^2 + \Gamma^2/4}$$

$$d_{BW}(E) = \frac{\Gamma_{BW}}{2\pi} \frac{1}{(E-M_0)^2 + \Gamma_{BW}^{\ 2}/4}$$

$$\Gamma_{BW}, \text{ such that } d_{BW}(M_0) = d_S(M_0)$$

$p_{BW}(t) = e^{-\Gamma_{BW}\,t}$

$p(t)$

$p_{BW}(t)$

t [a.u.]

$$a(t) = \int_{-\infty}^{+\infty} d_S(E) e^{-iEt}\,dE; \quad p(t) = |a(t)|^2$$

$$p_{BW}(t) = e^{-\Gamma_{BW}t}$$

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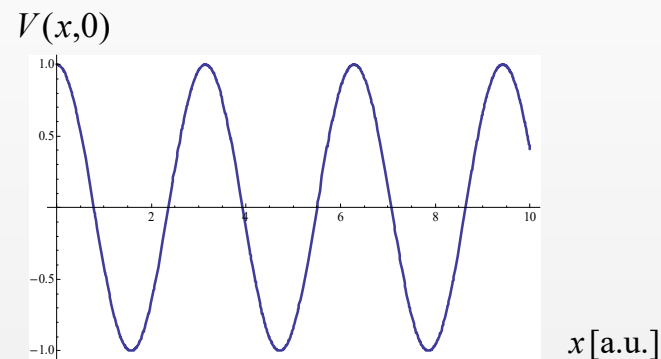# Experimental confirmation of non-exponential decay (1)

Experimental evidence for non-exponential decay in quantum tunnelling

Steven R. Wilkinson, Cyrus F. Bharucha, Martin C. Fischer, Kirk W. Madison, Patrick R. Morrow, Qian Niu, Bala Sundaram* & Mark G. Raizen
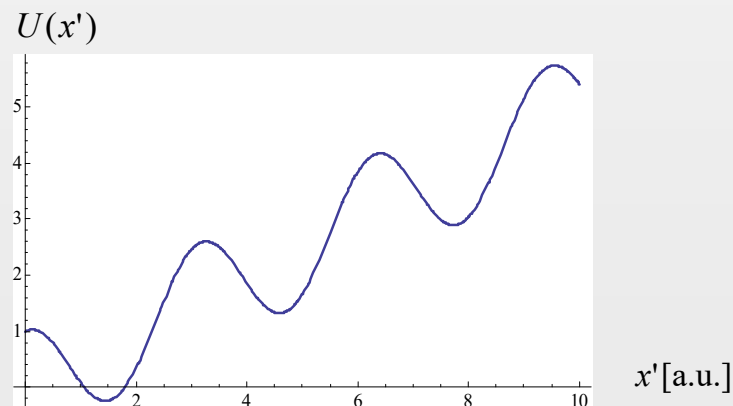
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An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Cold Na atoms in a optical potential

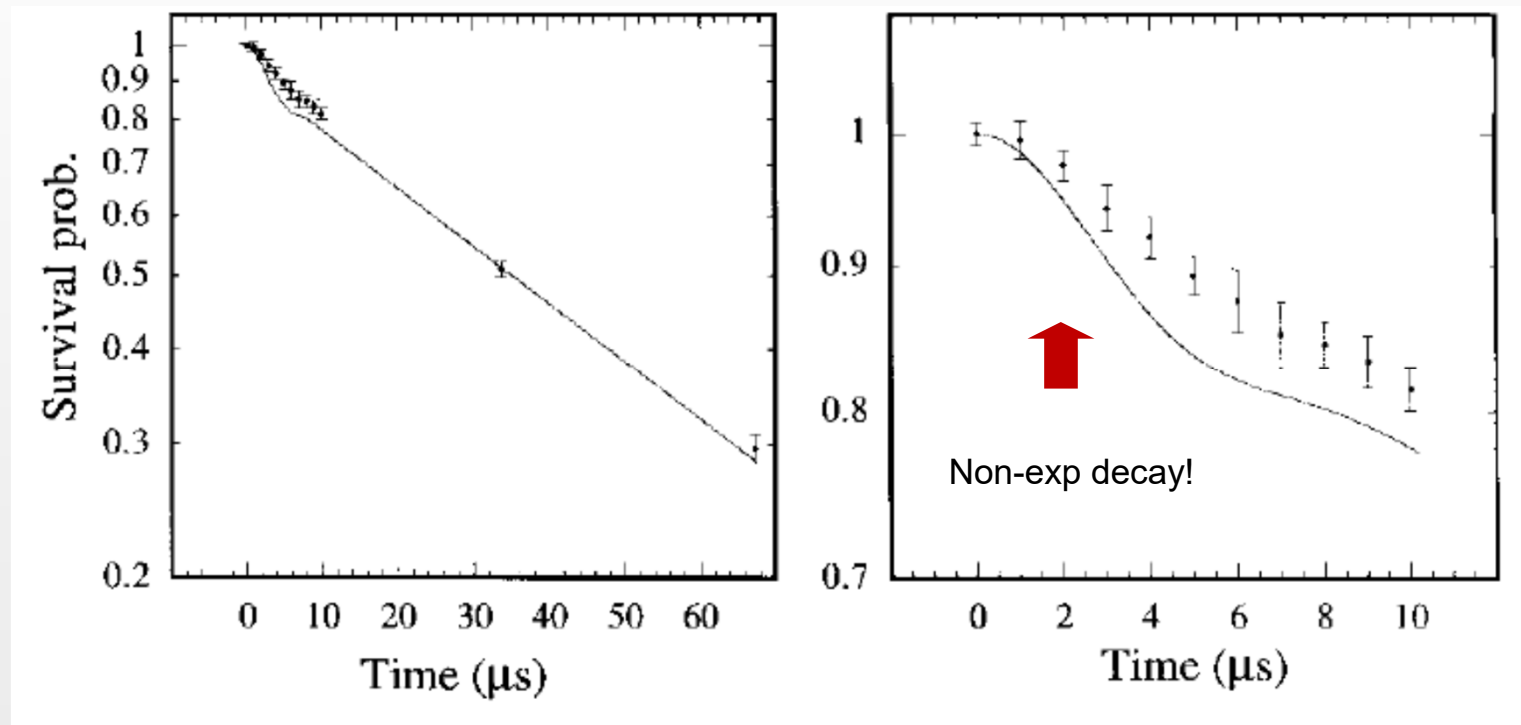$V(x,0)$

x [a.u.]

$$V(x,t) = V_0 \cos\left(2k_L x - k_L a t^2\right)$$

$U(x')$

x' [a.u.]

$$x' = x - \frac{1}{2} a t^2$$

$$U(x') = V_0 \cos\left(2k_L x'\right) + Max'$$

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Experimental confirmation of non-exponential decay (2)

Measured survival probabilty p(t)

Non-exp decay!

Experimental confirmation of non-exponential decay and Zeno /Anti-Zeno effects

Same exp. setup,
but with measurements in between

Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System

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(Received 30 March 2001; published 10 July 2001)

We report the first observation of the quantum Zeno and anti-Zeno effects in an unstable system. Cold sodium atoms are trapped in a far-detuned standing wave of light that is accelerated for a controlled duration. For a large acceleration the atoms can escape the trapping potential via tunneling. Initially the number of trapped atoms shows strong nonexponential decay features, evolving into the characteristic exponential decay behavior. We repeatedly measure the number of atoms remaining trapped during the initial period of nonexponential decay. Depending on the frequency of measurements we observe a decay that is suppressed or enhanced as compared to the unperturbed system.
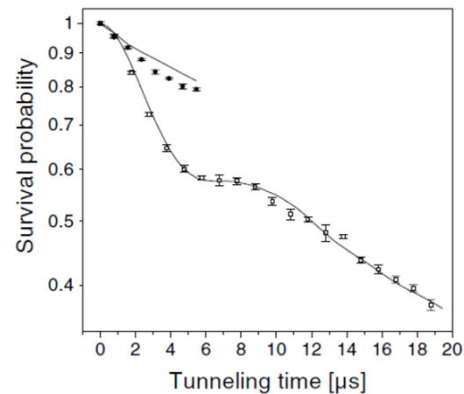
FIG. 3. Probability of survival in the accelerated potential as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 50 μs duration every 1 μs. The error bars denote the error of the mean. The data have been normalized to unity at $t_{tunnel} = 0$ in order to compare with the simulations. The solid lines are quantum mechanical simulations of the experimental sequence with no adjustable parameters. For these data the parameters were $a_{tunnel} = 15\,000$ m/s^2, $a_{interr} = 2000$ m/s^2, $t_{interr} = 50$ μs, and $V_0/h = 91$ kHz, where h is Planck's constant.
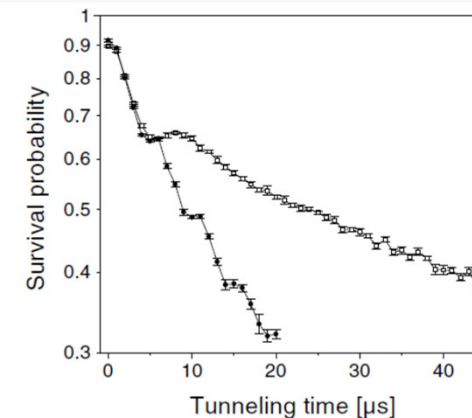
FIG. 4. Survival probability as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 40 μs duration every 5 μs. The error bars denote the error of the mean. The experimental data points have been connected by solid lines for clarity. For these data the parameters were: $a_{tunnel} = 15\,000$ m/s^2, $a_{interr} = 2800$ m/s^2, $t_{interr} = 40$ μs, and $V_0/h = 116$ kHz.

Zeno effekt

Anti-Zeno effect

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Late-time deviations

Violation of the Exponential-Decay Law at Long Times

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Department of Physics, University of Durham, Durham, DH1 3LE, United Kingdom
(Received 4 July 2005; published 26 April 2006)

First-principles quantum mechanical calculations show that the exponential-decay law for any metastable state is only an approximation and predict an asymptotically algebraic contribution to the decay for sufficiently long times. In this Letter, we measure the luminescence decays of many dissolved organic materials after pulsed laser excitation over more than 20 lifetimes and obtain the first experimental proof of the turnover into the nonexponential decay regime. As theoretically expected, the strength of the nonexponential contributions scales with the energetic width of the excited state density distribution whereas the slope indicates the broadening mechanism.
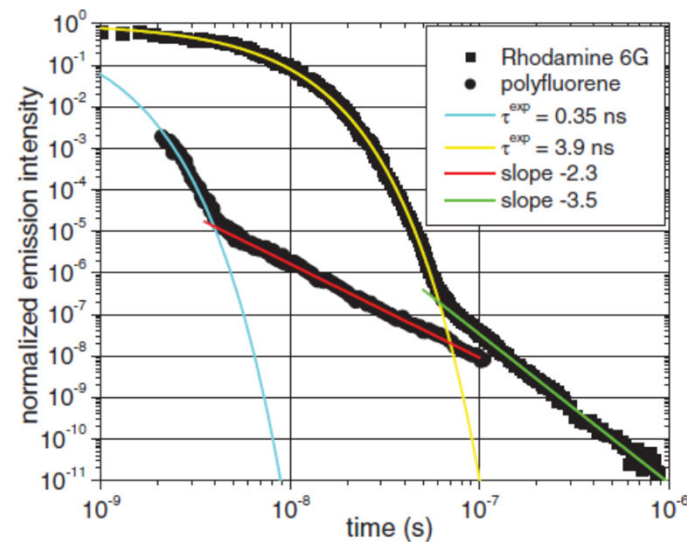
FIG. 2 (color). Corresponding double logarithmic fluorescence decays of the emissions shown in Fig. 1. Exponential and power law regions are indicated by solid lines and the emission intensity at time zero has been normalized.

Confirmation of: L. A. Khalfin. 1957. 1957 (Engl. trans. Zh.Eksp.Teor.Fiz.,33,1371)

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Considerations

- No other short- or long-time deviation from the exp. law was seen in unstable states.

- Verification of the two aforementioned works (Reizen + Rothe) would be needed.

- The measurement of deviations in simple natural systems (elementary particles, nuclei, atoms) would be a great achievement.

Probability of decay: formal argument

$$p(t) = \left| \int_{E_{th,1}}^{\infty} \mathrm{d}E\, d_S(E) e^{-\frac{i}{\hbar}Et} \right|^2$$

$$\Gamma \to \Gamma(E)$$

$$d_S(E) = -\frac{1}{\pi} \mathrm{Im}\left[\frac{1}{E - M + \Pi(E)} \right]$$

with $\Pi(E)$ being the 'loop' wihth

$$\mathrm{Im}\,\Pi(E) = \Gamma(E)/2$$

$\mathrm{Re}\,\Pi(E)$ can be obtained by dispersion relations. (In some cases, one may directly calculate $\Pi(E)$).

Technical remark: the propagator

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M + \Pi(E) + i\varepsilon}$$

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M + \Pi(E) + i\varepsilon}$$

$$= \int_{E_{1,th}}^{+\infty} dE' \frac{d_S(E')}{E - E' + i\varepsilon} .$$

Multichannel decay law

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Multichannel decay law

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A R T I C L E   I N F O

A B S T R A C T

It is well known, both theoretically and experimentally, that the survival probability for an unstable quantum state, formed at $t = 0$, is not a simple exponential function, even if the latter is a good approximation for intermediate times. Typically, unstable quantum states/particles can decay in more than a single decay channel. In this work, the general expression for the probability that an unstable state decays into a certain i-th channel between the initial time $t = 0$ and an arbitrary $t > 0$ is provided, both for nonrelativistic quantum states and for relativistic particles. These partial decay probabilities are also not exponen͏͏͏͏͏ be in the exponential limit. Quit͏͏͏ them potentially interesting in a͏͏͏ unexplored framework to search systems, such as quantum tunneli͏

Non-exponential Decay in Quantum Field Theory and in Quantum Mechanics: The Case of Two (or More) Decay Channels

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A simple question

Next, a simple question might be asked: Which is the probability that the decay of the unstable state occurs between $t = 0$ and the time t? The answer is trivial, since the probability that the decay has actually occurred, denoted as $w(t)$, must be

$$w(t) = 1 - p(t)$$

Similarly, the quantity $h(t) = w'(t) = -p'(t)$ is the probability decay density, with $h(t)dt$ being the probability that the decay occurs between t and $t + dt$.

$$h(t) = w'(t) = -p'(t)$$

Then, a natural, but less easy question is the following: *How to calculate, in a general fashion, the probability, denoted as $w_i(t)$, that the decay occurs in the i-th channel between 0 and t?*

Which are the $w_i(t)$????

N Decay channels: formal aspects

$$\Gamma_i(E)$$

$i = 1, ..., N$ enumerates the decay channels

$$E_{th,1} \leq E_{th,2} \leq ... \leq E_{th,N}$$

$$\Pi(E) = \sum_{i=1}^{N} \Pi_i(E) \, , \, \Gamma_i(E) = 2\,\text{Im}\,\Pi_i(E)$$

partial BW widths are $\Gamma_i = \Gamma_i(M)$

$$\Gamma = \Gamma(M) = \sum_{i=1}^{N} \Gamma_i$$

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The Friedrichs-Lee Hamiltonian

$$H_L = M\left|S\right\rangle\left\langle S\right| + \sum_{i=1}^{N}\int_{E_{i,th}}^{\infty}\mathrm{d}E\,E\left|E,i\right\rangle\left\langle E,i\right|$$

$$+ \sum_{i=1}^{N}\int_{E_{i,th}}^{\infty}\mathrm{d}E\,\sqrt{\frac{\Gamma_i(E)}{2\pi}}\left(\left|E,i\right\rangle\left\langle S\right| + h.c.\right)$$

$$w_i(t) = \int_{E_{th,i}}^{\infty}\mathrm{d}E\left|\left\langle E,i\left|e^{-\frac{i}{\hbar}H_L t}\right|S\right\rangle\right|^2$$

Solution

$$w_i(t) = \int_{E_{th,i}}^{\infty} dE \frac{\Gamma_i(E)}{2\pi} \left| \int_{E_{th,1}}^{+\infty} dE' d_S(E') \frac{e^{-\frac{i}{\hbar}E't} - e^{-\frac{i}{\hbar}Et}}{E' - E} \right|^2$$

$$h_i(t) = w_i'(t)$$

$h_i(t)dt$ is the probability that the unstable particle decays in the i-th channel between t and t+dt

Breit-Wigner limit

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \Gamma^2/4}$$

$$\Gamma = \sum_{i=1}^{N} \Gamma_i$$

$$p(t) = e^{-\frac{\Gamma}{\hbar}t}$$

$$w_i(t) = \frac{\Gamma_i}{\Gamma} w(t) = \frac{\Gamma_i}{\Gamma}\left(1 - e^{-\frac{\Gamma}{\hbar}t}\right)$$

$$h_i(t) = \frac{\Gamma_i}{\Gamma} h(t) = \frac{\Gamma_i}{\Gamma} e^{-\frac{\Gamma}{\hbar}t}. \qquad \frac{w_i(t)}{w_j(t)} = \frac{h_i(t)}{h_j(t)} = \frac{\Gamma_i}{\Gamma_j} = const.$$

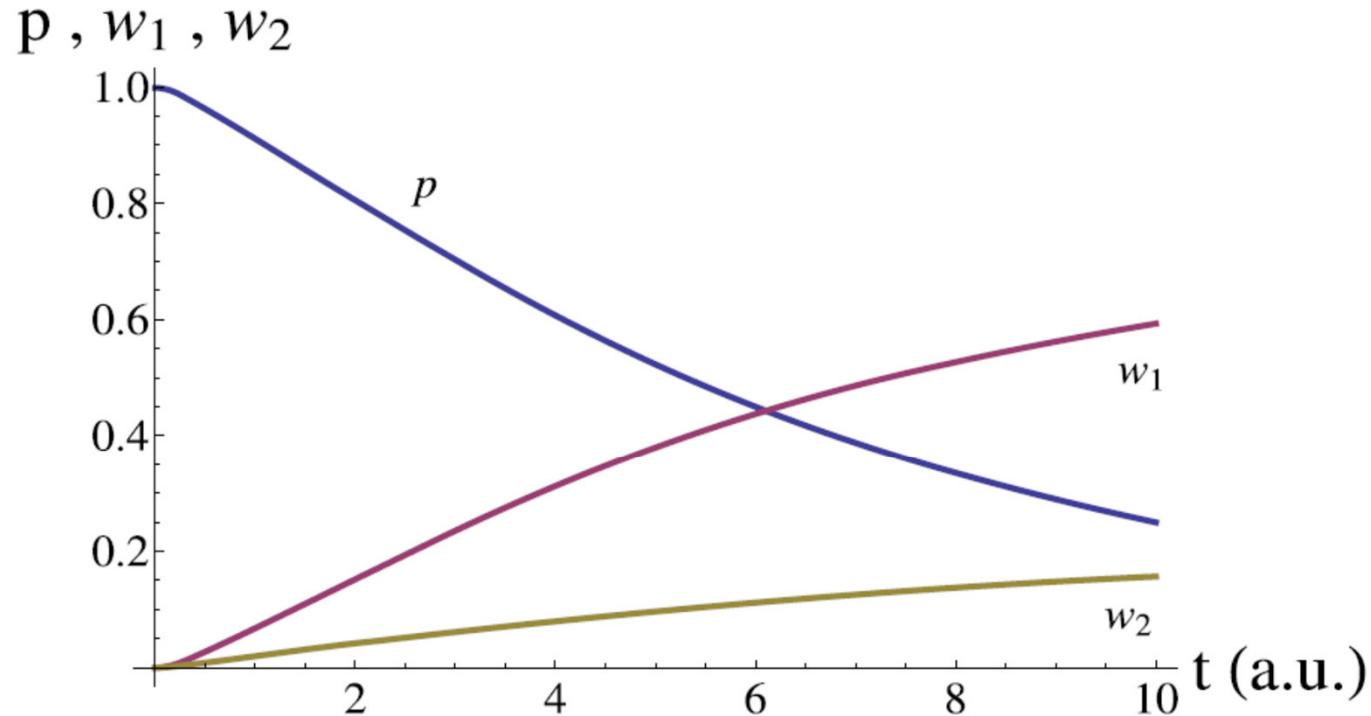# A simple model $\quad \Gamma_i(E) = 2g_i^2 \dfrac{\sqrt{E-E_{th,i}}}{E^2+\Lambda^2}$

Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p + w_1 + w_2 = 1$ holds. Note, t is expressed in a.u. of $[M^{-1}]$.

$g_1/\sqrt{M} = 1, \; g_2/\sqrt{M} = 0.6 \quad E_{th,1}/M = 1/10, \; E_{th,2}/M = 1/2, \; \Lambda/M = 4,$
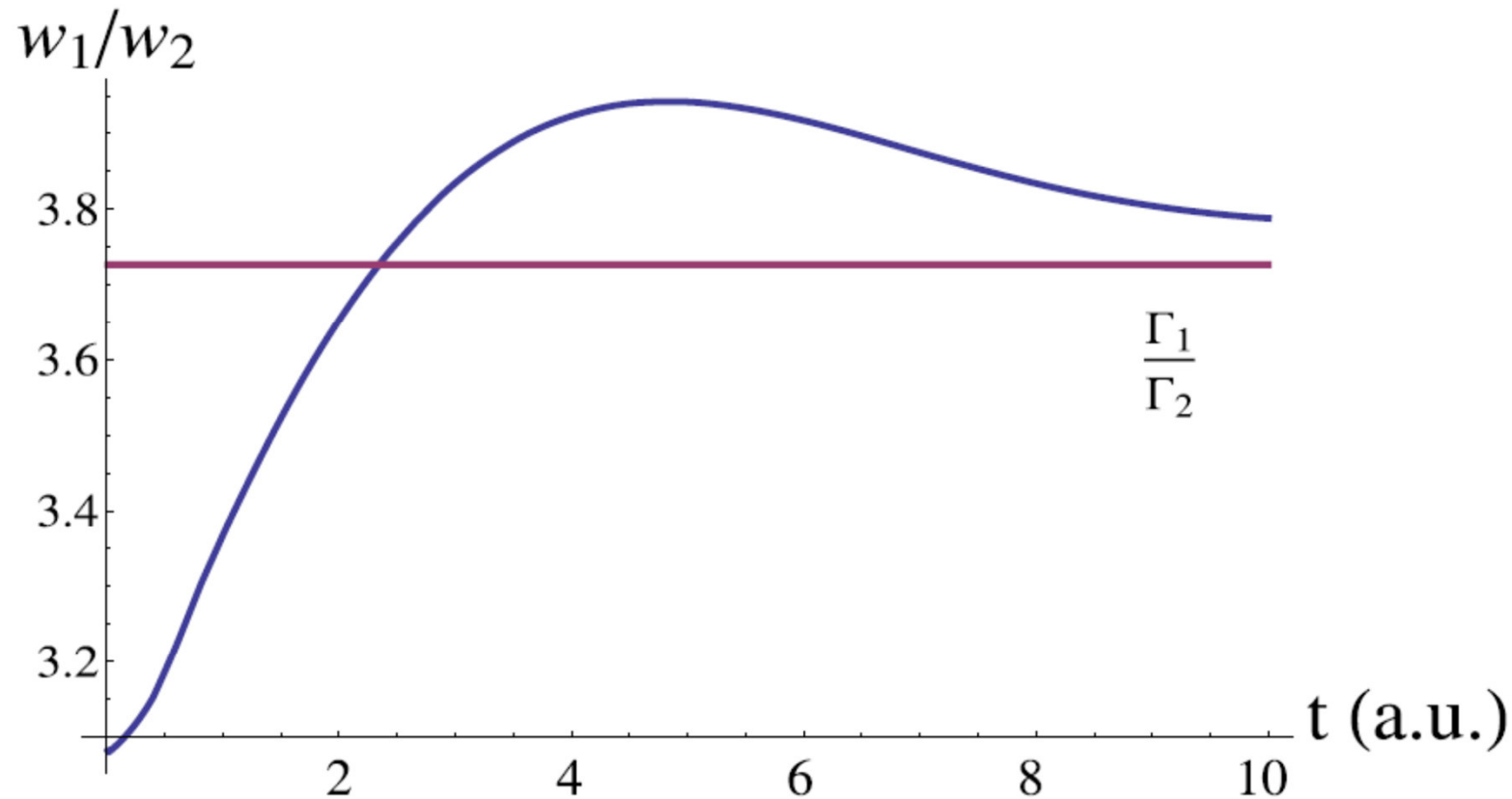
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Fig. 2. The ratio w_1/w_2 is plotted as function of t. The straight line corresponds to the BW limit Γ_1/Γ_2, see Eq. (19).
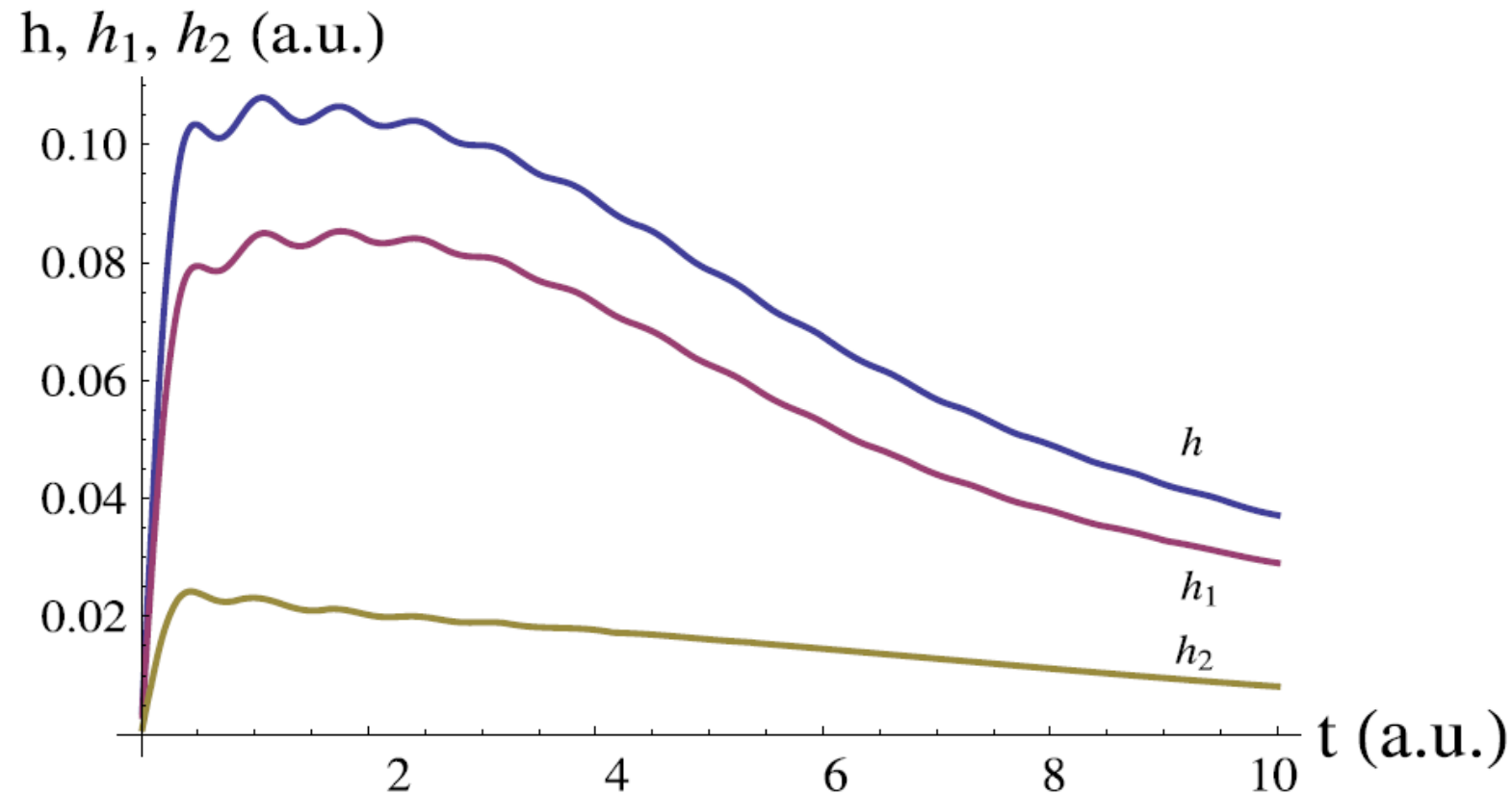
Fig. 3. The quantity $h(t) = w'(t) = -p'(t)$ as well as $h_i(t) = w'_i(t)$ is plotted. The equality $h(t) = h_1(t) + h_2(t)$ holds. Note, h and h_i are in units of $[M]$.
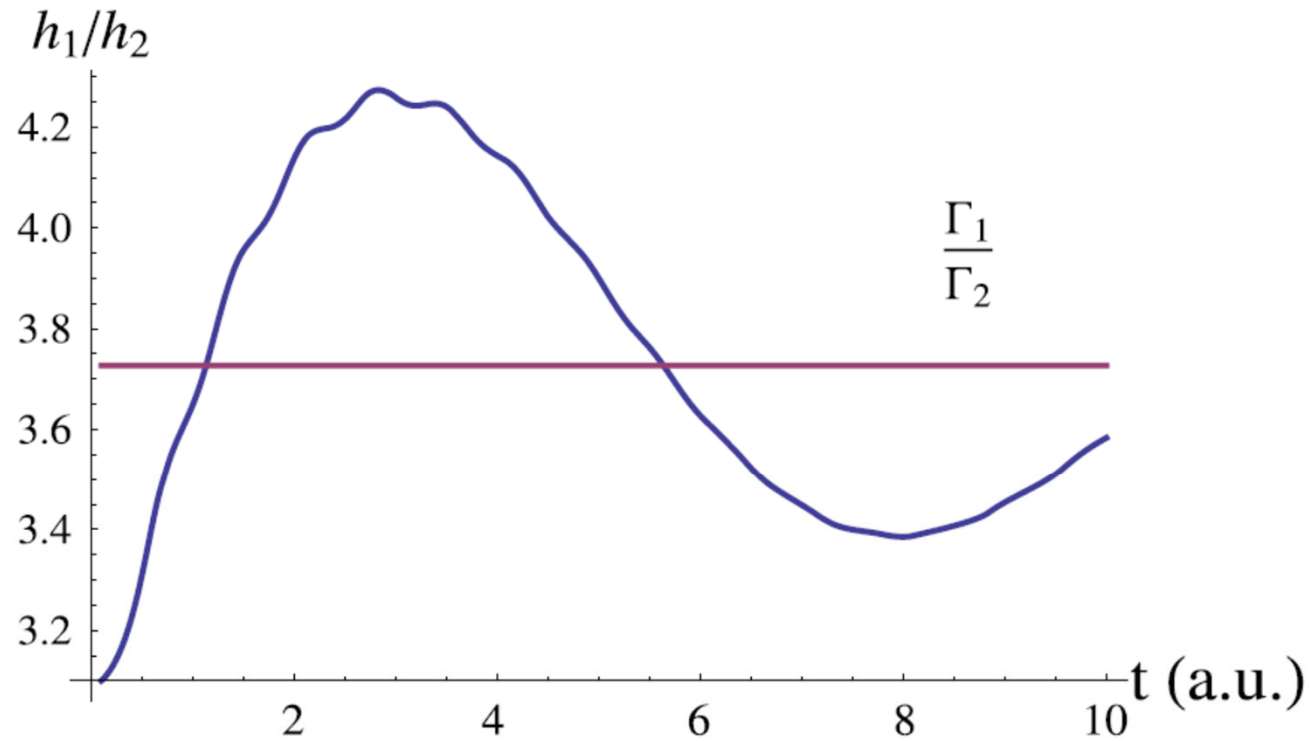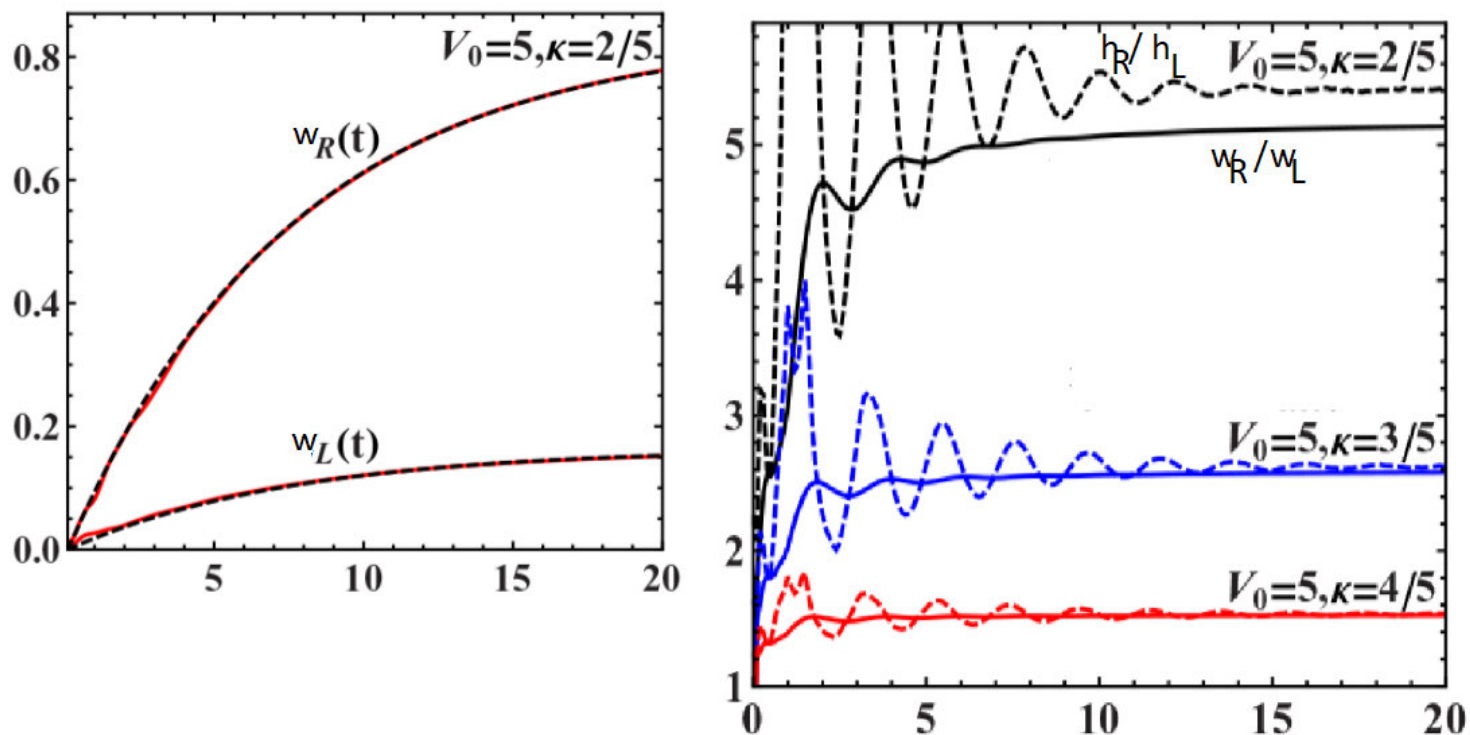
Fig. 4. Ratio h_1/h_2 as function of t. The straight line corresponds to the BW limit Γ_1/Γ_2, see Eq. (19). For the time intervals where $h_1/h_2 > \Gamma_1/\Gamma_2$, the decay in the first channel is enhanced (the opposite is true for $h_1/h_2 < \Gamma_1/\Gamma_2$).

Double-delta:

$$V_0[\delta(x+1) + \kappa\delta(x-1)]$$

Capturing nonexponential dynamics in the presence of two decay channels

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QFT case: result

$$w_i(t) = \int_{E_{th,i}}^{\infty} dE \frac{2E^2 \Gamma_i(E)}{\pi} \left| \int_{E_{th,1}}^{\infty} dE' d_S(E') \frac{e^{-iE't} - e^{-iEt}}{E'^2 - E^2} \right|^2$$

Recalll QM:

$$w_i(t) = \int_{E_{th,i}}^{\infty} dE \frac{\Gamma_i(E)}{2\pi} \left| \int_{E_{th,1}}^{+\infty} dE' d_S(E') \frac{e^{-\frac{i}{\hbar}E't} - e^{-\frac{i}{\hbar}Et}}{E' - E} \right|^2$$

QFT: example

$$\Gamma_i(s) = g_i^2 \sqrt{(s - s_{th,i})/s}$$

A simple alternative to the relativistic Breit–Wigner distribution

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[2] Institute for Theoretical Physics, Johann Wolfgang Goethe—University, Max von Laue-Str. 1, 60438 Frankfurt, Germany

$$d_S^{\text{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + (\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma})^2} \theta(E - E_{th})$$

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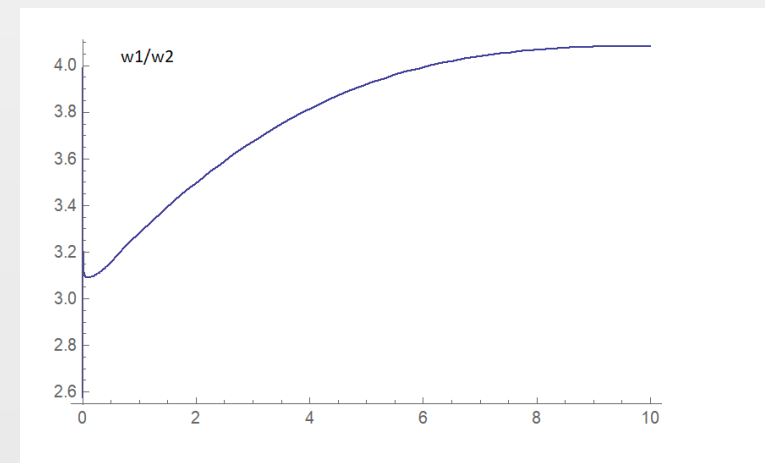
Decay of a moving particle

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DECAY LAW AND TIME DILATATION*

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Boosting Unstable Particles

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[2]*Institute of Physics, Jan-Kochanowski Univ., ul. Uniwersytecka 7, 25-406 Kielce, Poland and*
[3]*Institute for Theoretical Physics, J. W. Goethe University,*
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2206.05125

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Unstable particle with momentum p: decay law

$$a(t,p) = \frac{1}{\delta(p=0)} \langle S,p | e^{-iHt} | S,p \rangle$$

$$= \int_{-\infty}^{\infty} dm\, d_S(m) e^{-i\sqrt{m^2+p^2}\, t}$$

$$P_{nd}(t) = |a(t,p)|^2$$

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Unstable particle with momentum p: exp. limit

M = rest mass; Γ= decay width in the rest frame.

An unstable particle moves with definite momentum p.

Which is its decay width? The standard expression is:

$$\tilde{\Gamma}_p = \frac{\Gamma}{\gamma} \equiv \frac{\Gamma M}{\sqrt{p^2 + M^2}}$$

Important but sublte point:
in QM and QFT a state with definite momentum has not definite velocity.

Finite momentum vs finite velocity

- Subtle but important point: in the long-life limit, a particle with definite momentum has also definite velocity.

- $$p = \frac{M}{\sqrt{1-v^2}} v$$

- In general, however, there is a difference! For an unstable state a boost is not equivalent to a momentum translation.

- Here, we consider (at first) definite momentum

$$|S,p\rangle = \int_0^\infty \mathrm{dm}\, a_S(m)\, |m,p\rangle$$

Unstable particle with momentum p: unexpected result for the nondecay probability

The non-decay probability in the exp. limit:

$$|S,p\rangle = \int_0^\infty dm\, a_S(m)\,|m,p\rangle$$

$$P_{nd}(t) = e^{-\Gamma_p t}$$

$$\Gamma_p = \sqrt{2}\sqrt{\left[\left(M^2 - \frac{\Gamma^2}{4} + p^2\right)^2 + M^2\Gamma^2\right]^{1/2} - \left(M^2 - \frac{\Gamma^2}{4} + p^2\right)}$$
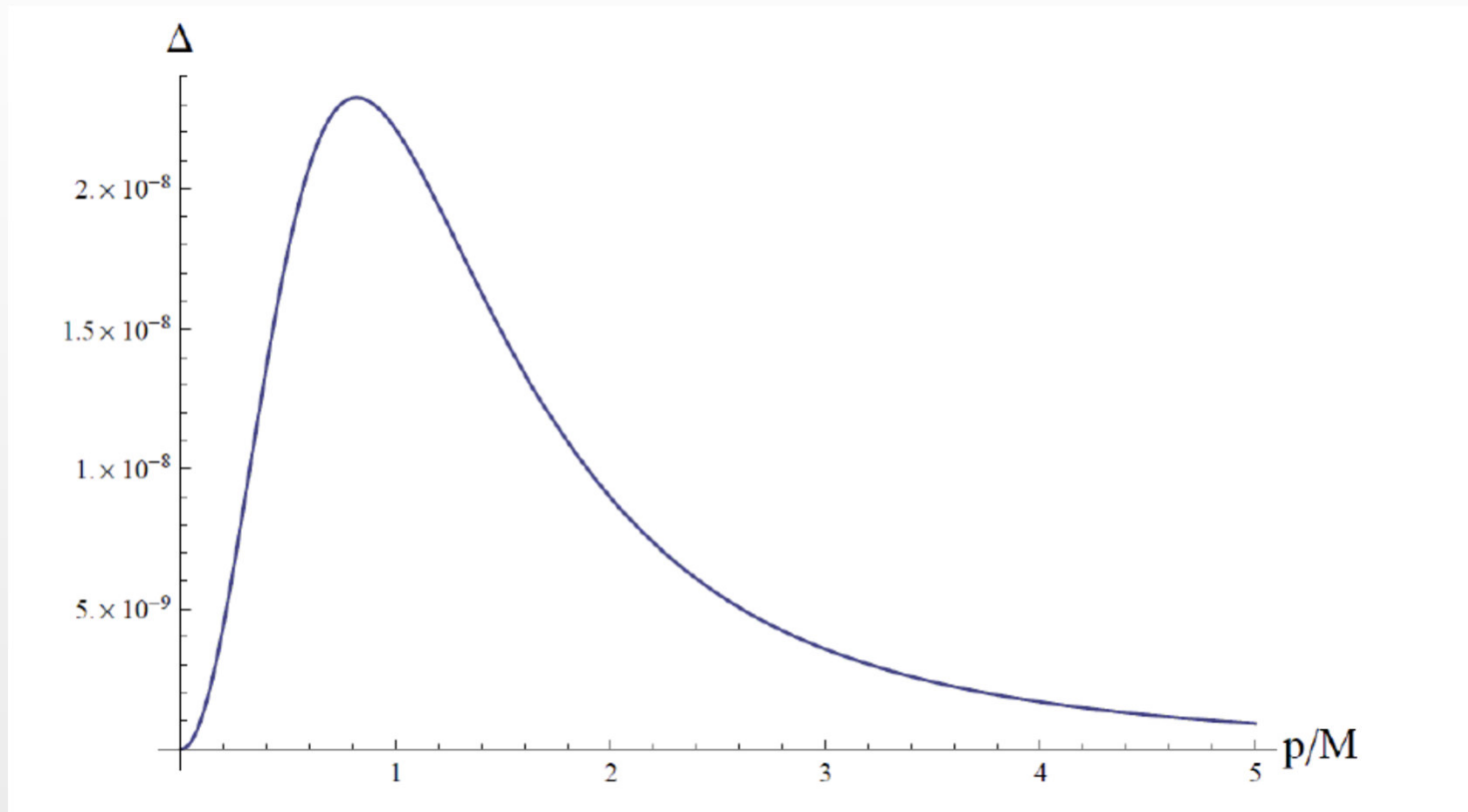
F. G.   arXiv:1512.00232 [hep-ph]

$$\Gamma_p \neq \tilde{\Gamma}_p = \Gamma M / \sqrt{p^2 + M^2}$$

But this is not a breaking of relativity!
It is a different setup.

Unstable particle with momentum p: deviations

$$\Delta = \frac{\Gamma_p - \tilde{\Gamma}_p}{M}$$

$$\frac{p_{\max}}{M} = \sqrt{\frac{2}{3}} \simeq 0.816$$

$$\Delta_{\max} = \frac{\Gamma_{p_{\max}} - \tilde{\Gamma}_{p_{\max}}}{M} \simeq \frac{3}{100}\sqrt{\frac{3}{5}}\left(\frac{\Gamma}{M}\right)^3$$

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Unstable particle with momentum p: some examples of deviations

Muon
M = 105.65 MeV
$\Gamma = 2.99 \cdot 10^{-16}$ MeV

$$\Gamma_{p\text{max}} - \tilde{\Gamma}_{p\text{max}} \simeq 5.598 \cdot 10^{-53} \text{ MeV}$$

Neutral pion
M = 134.98 MeV
$\Gamma = 7.72 \cdot 10^{-6}$ MeV

$$\Gamma_{p\text{max}} - \tilde{\Gamma}_{p\text{max}} \simeq 5.81 \cdot 10^{-22} \text{ MeV}.$$

Rho meson
M = 775.26 MeV
$\Gamma = 147.8$ MeV

$$\Gamma_{p\text{max}} - \tilde{\Gamma}_{p\text{max}} \simeq 0.125 \text{ MeV}$$

Very small deviations!

Boost: state with definite velocity

$$U_v \left| S, 0 \right\rangle \equiv \left| S, v \right\rangle \qquad \left| S, v \right\rangle = \int_0^\infty dm\, a_S(m) \sqrt{m}\, \gamma^{3/2} \left| m, m\gamma v \right\rangle$$

$$P_{nd}(t) = 0$$

A boosted muon consists of an electron and two neutrinos!
A boosted neutron consists of a proton, an electron, and a neutrino
etc etc

FIG. 1. Minkowski diagram of our thought experiment. In Alice's frame, there is $\{1/2, 1/2\}$ probability of having a neutron (green circles) in either of two boxes (grey areas), at $t_A = 0$. After a time T, the neutron decays into $p^+ + e^- + \bar{\nu}_e$ (blue+yellow+red circles) in both boxes. Bob moves with velocity $-v$ with respect to Alice. His line of contemporary events (red dashed line) is oblique, and intersects the two boxes at different Alice's times. As a consequence, in Bob's frame there is $1/2$ probability of having a neutron in the right box, and $1/2$ probability of having a proton, an electron, and a neutrino in the left box. Therefore, the Lorentz boost has entangled two observables: the neutron projector $\mathcal{Q}$, and the position of the center of mass of the system.

$$|\psi\rangle = \frac{|n \text{ in box ``}{-}L\text{''}\rangle + |n \text{ in box ``}0\text{''}\rangle}{\sqrt{2}}$$

$$e^{-iHT}|\psi\rangle \approx \frac{|p, e, \bar{\nu} \text{ in box ``}{-}L\text{''}\rangle + |p, e, \bar{\nu} \text{ in box ``}0\text{''}\rangle}{\sqrt{2}}$$

$$\Lambda = \begin{bmatrix} \gamma & \gamma v \\ \gamma v & \gamma \end{bmatrix}$$

$$U(\Lambda)|\psi\rangle \approx \frac{|p, e, \bar{\nu} \text{ in box ``}{-}L\text{''}\rangle + |n \text{ in box ``}0\text{''}\rangle}{\sqrt{2}}$$
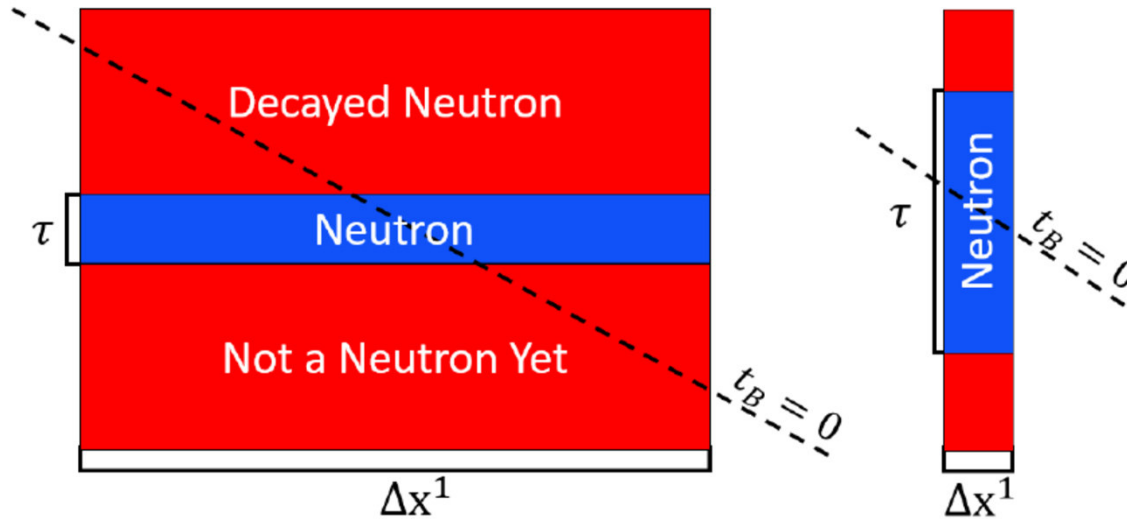
Francesco Giacosa

FIG. 2. Minkowski diagrams of the wavefunction of a neutron for two different values of the ratio $\Delta x^1/\tau$. Left diagram: if $\Delta x^1 \gg \tau$, the hypersurface $\{t_B = 0\}$ of a moving observer (dashed line) covers a large region of spacetime where the particle is not a neutron (in red), so that $\langle \Lambda\alpha | \mathcal{Q} | \Lambda\alpha \rangle \ll 1$. Right diagram: the condition $\Delta x^1 \ll \tau$ restores $\langle \Lambda\alpha | \mathcal{Q} | \Lambda\alpha \rangle \approx 1$, because the hypersurface $\{t_B = 0\}$ intersects only the "neutron region" (in blue).

projector $\mathcal{Q}$

$\langle \alpha | \mathcal{Q} | \alpha \rangle = 1$

$|\alpha\rangle$ models a neutron at rest in the origin

$|\Lambda\alpha\rangle = U(\Lambda)|\alpha\rangle$

$$\langle \Lambda\alpha | \mathcal{Q} | \Lambda\alpha \rangle \approx e^{a^2} \operatorname{erfc}|a| \quad \text{with } a := \frac{v\Delta x^1}{\tau\sqrt{2}}$$

Summary and outlook

- The decay is never exponential! This is a fact.

- Multichannel decay: promising new direction fro the study of decay beyond the exponential limit.
  Formulas presented both for the QM and QFT cases.

Summary and outlook

- Decay of a moving particle: interesting link between relativity and QM and QFT.

- For a particle with definite momentum p (for the measuring observer) there is a different formula.

- A boost is a very subtle operation in QM and QFT.

Other decay-related topics

Pulsed and continuous measurements of exponentially decaying systems

Francesco Giacosa[1,2] and Giuseppe Pagliara[3]

Measurement of the neutron lifetime and inverse quantum Zeno effect

Francesco Giacosa[1,2,*] and Giuseppe Pagliara[3,†]

Leggett-Garg inequalities and decays of unstable systems

Francesco Giacosa[1,2] and Giuseppe Pagliara[3,4]

Thank You

Wave packet in momentum

$$|\Psi\rangle = \int_{-\infty}^{+\infty} dp\, B(p) \,|S, p\rangle$$

the quantity $\langle\Psi|e^{-iHt}|\Psi\rangle$ is *not* what we are looking for.

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\, \left|\langle S, p\, |e^{-iHt}|\, \Psi\rangle\right|^2$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp\, |B(p)|^2\, e^{-\Gamma_p t}$$

Inclusion of spatial wave function is simple.

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Boost: state with definite velocity

Point: a velocity translation (i.e. a boost) is not a momentum translation!!!!

$$U_v \, |S, 0\rangle \equiv |S, v\rangle$$

$$\left| \langle S, v \, |e^{-iHt}| \, S, v \rangle \right|^2 = e^{-i\gamma\Gamma t}$$

The survival probability shows here an absurd Lorentz contraction!

Boost: wave packet in velocity (is qualitatively different!)

$$|\Phi\rangle = \int_{-1}^{+1} dv\, C(v)\, |S, v\rangle$$

$$C(v) = N e^{-(v-v_0)^2/(4\sigma_v^2)}$$

$$P_{nd}(t) = \int_{-\infty}^{+\infty} dp \left| \langle S, p | e^{-iHt} | \Phi \rangle \right|^2$$

Pnd[0]

Pnd[t] (for $\sigma_v = 0.015$)

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