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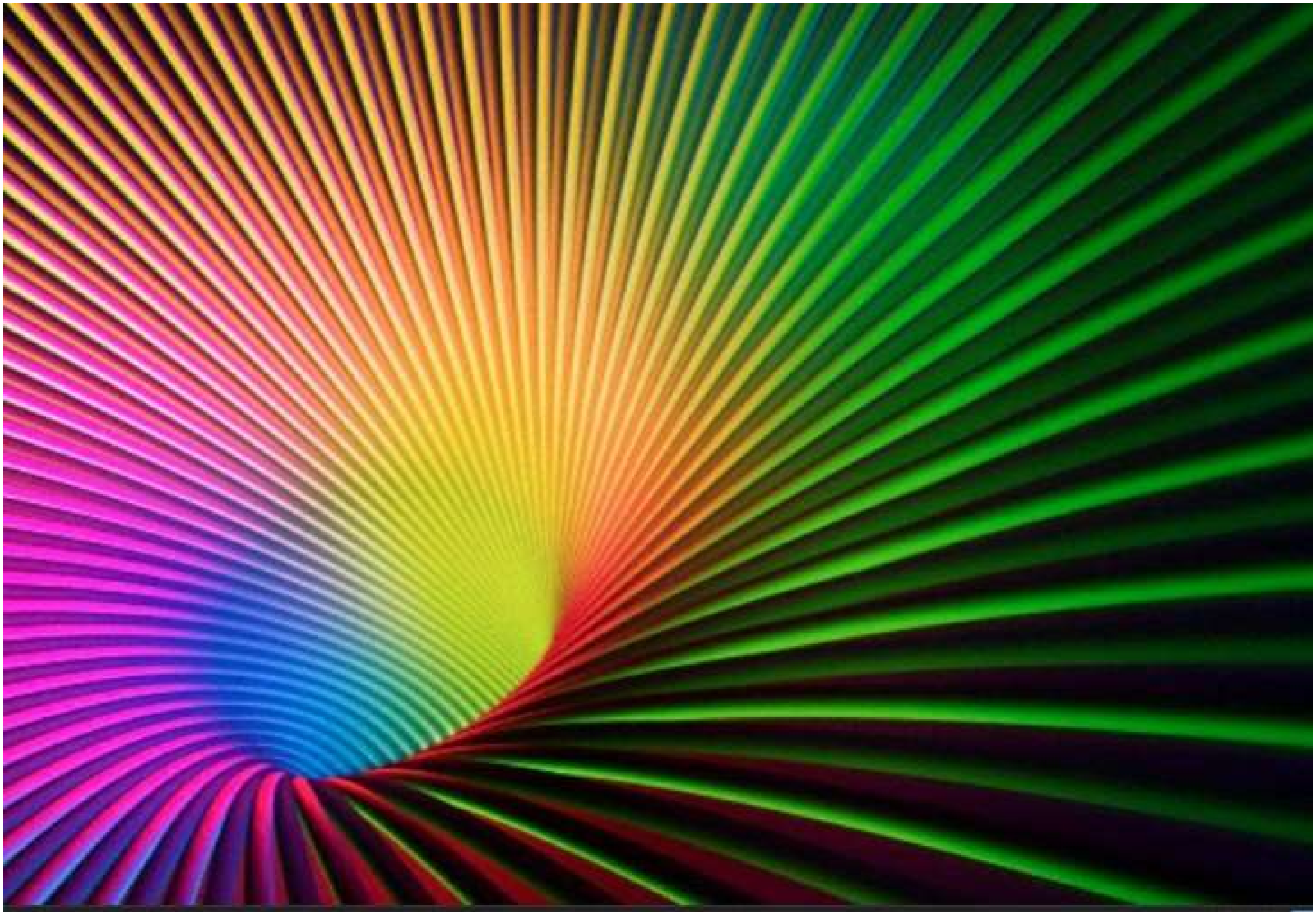
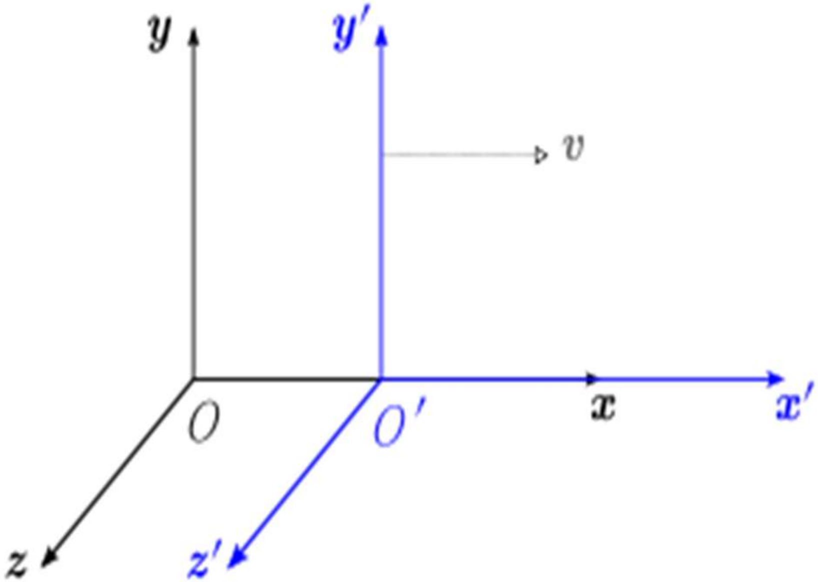


observers in foundations of relativity



observer in quantum foundations

Planck length as the minimum allowed value for wavelengths:

- **suggested by several indirect arguments combining quantum mechanics and GR**
- **found in some detailed analyses of formalisms in use in the study of the QG problem**

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other results from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided "theoretical evidence" of Planck-scale modifications of the on-shell relation, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

Toward the mid 1990s these observations led several researchers to work at the hypothesis that in order to address the quantum-gravity problem one should give up the relativity of observers (preferred-frame picture)

GAC+Ellis+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia,PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer,PhysRevLett(1999)

This would be "Planck-scale broken Lorentz symmetry"

but from 2000 onwards together with broken Lorentz symmetry there starts to be a literature on the possibility of "Planck-scale deformations of Lorentz symmetry" [jargon: "DSR", for "doubly-special", or "deformed-special", relativity]

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

change the laws of transformation between observers so that the new properties are observer-independent

- **a law of minimum wavelength can be turned into a DSR law**
- **could be used also for properties other than minimum wavelength, such as deformed on-shellness, deformed uncertainty relations…**

The notion of DSR-relativistic theories is best discussed in analogy with the transition from Galileian Relativity to Special Relativity

minimum wavelength from noncommutativity:

the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

evidently not invariant under «classical translations»

$$[\boldsymbol{x_j} + \boldsymbol{a_j}, \boldsymbol{x_0} + \boldsymbol{a_0}] = [\boldsymbol{x_j}, \boldsymbol{x_0}] = \boldsymbol{i}\lambda\boldsymbol{x_j} \neq \boldsymbol{i}\lambda(\boldsymbol{x_j} + \boldsymbol{a_j})$$

but adding commutative numbers to the noncommutative coordinates of kappa-Minkowski is evidently not a reasonable thing….

Adopting in particular noncommutative translation parameters such that

$$[\mathcal{E}_j, \mathcal{E}_0] = \mathbf{0} \qquad [\mathcal{E}_j, \boldsymbol{x}_\mu] = \mathbf{0} \qquad [\mathcal{E}_j, \boldsymbol{x_k}] = \mathbf{0} \qquad [\mathcal{E}_j, \boldsymbol{x_0}] = \boldsymbol{i}\lambda\mathcal{E}_j$$

Sitarz, PhysLettB349(1995)42 Majid+Oeckl, math.QA/9811054

then

$$[\boldsymbol{x_j} + \mathcal{E}_j, \boldsymbol{x_0} + \mathcal{E}_0] = [\boldsymbol{x_j}, \boldsymbol{x_0}] + [\mathcal{E}_j, \boldsymbol{x_0}] = \boldsymbol{i}\lambda(\boldsymbol{x_j} + \mathcal{E}_j)$$

boosts must adapt to these deformed translations, resulting in deformed mass Casimir

deformed boosts are such that there is a maximum momentum (minimum wavelength)

Translation generators in kappa-Minkowski:

$$P_\mu\left(e^{ikx}\,e^{ik_0t}\right)=k_\mu\left(e^{ikx}\,e^{ik_0t}\right)$$

classical action on functions written with a certain ordering convention

$$[x_j,t]=i\lambda x_j$$

$$[x_j,x_m]=0$$

$$e^{ikx}e^{ik_0t}e^{iKx}e^{iK_0t}=e^{i(k+e^{\lambda k_0}K)x}e^{i(k_0+K_0)t}$$

!!

then "non-primitive coproduct"

$$P_\mu\left(e^{ikx}\,e^{ik_0t}e^{iKx}\,e^{iK_0t}\right)=P_\mu\left(e^{i(k+e^{\lambda k_0}K)x}\,e^{i(k_0+K_0)t}\right)$$

$$=\left(k_\mu+e^{-\lambda k_0}K_\mu\right)\left(e^{ikx}\,e^{ik_0t}e^{iKx}\,e^{iK_0t}\right)$$

$$=\left[P_\mu\left(e^{ikx}\,e^{ik_0t}\right)\right]\left(e^{iKx}\,e^{iK_0t}\right)+\left[e^{-\lambda P_0}\left(e^{ikx}\,e^{ik_0t}\right)\right]P_\mu\left(e^{iKx}\,e^{iK_0t}\right)$$

!!

deformation of relativistic symmetries!!

rich phenomenology based on astrophysics (see talk by Ellis)

In ordinary QFT the full (bosonic) **Fock space** is obtained from symmetrized tensor prods of $\mathcal{H}$

In the κ-**deformed case** try to proceed in an analogous way BUT...

$$1/\sqrt{2}\,(|\mathbf{k}_1\rangle \otimes |\mathbf{k}_2\rangle + |\mathbf{k}_2\rangle \otimes |\mathbf{k}_1\rangle)$$

is NOT an eigenstate of P_μ due to the **non-trivial coproduct** of spatial translation generators!!

$$\Delta(P_i) = P_i \otimes 1 + e^{-P_0/\kappa} \otimes P_i$$

Multi-particle states of κ-**Fock-space** are built via a **"momentum dependent" symmetrization**

$$\sigma^\kappa(|\mathbf{k}_1\rangle \otimes |\mathbf{k}_2\rangle) = |(1-\epsilon_1)\,\mathbf{k}_2\rangle \otimes |(1-\epsilon_2)^{-1}\,\mathbf{k}_1\rangle\,, \qquad \epsilon_i = \frac{|\mathbf{k}_i|}{\kappa}$$

E.g. there will be **two** 2-particle states

$$|\mathbf{k}_1\mathbf{k}_2\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_1\rangle \otimes |\,\mathbf{k}_2\rangle + |\,(1-\epsilon_1)\mathbf{k}_2\rangle \otimes |\,(1-\epsilon_2)^{-1}\mathbf{k}_1\rangle\right]$$
$$|\mathbf{k}_2\mathbf{k}_1\rangle_\kappa = \frac{1}{\sqrt{2}}\left[|\,\mathbf{k}_2\rangle \otimes |\,\mathbf{k}_1\rangle + |\,(1-\epsilon_2)\mathbf{k}_1\rangle \otimes |\,(1-\epsilon_1)^{-1}\mathbf{k}_2\rangle\right]$$

with **same energy** and different linear momentum

$$\mathbf{K}_{12} = \mathbf{k}_1 \oplus \mathbf{k}_2 = \mathbf{k}_1 + (1-\epsilon_1)\mathbf{k}_2$$
$$\mathbf{K}_{21} = \mathbf{k}_2 \oplus \mathbf{k}_1 = \mathbf{k}_2 + (1-\epsilon_2)\mathbf{k}_1$$

given n-different modes one has $n!$ different n-particle states, one for each permutation of the n modes $\mathbf{k}_1\,, \mathbf{k}_2 \ldots \mathbf{k}_n$

courtesy Michele Arzano

similar implications for antisymmetrization… in turn there are implications for the Spin-Statistics Theorem….and the Pauli Exclusion Principle….

another quantum group: $SU_q(2)$

with D'Esposito, Fabiano,Frattulillo,Hoehn, Mercati, on arXiv this week

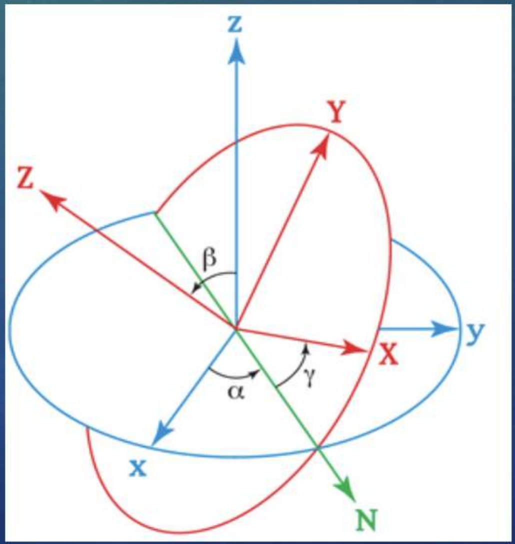
- In classical and quantum mechanics, rotation transformations are governed by the group SU(2)

$$SU(2) \ni U = \begin{pmatrix} a & -c^* \\ c & a^* \end{pmatrix} \quad a, c \in \mathbb{C} : \quad |a|^2 + |c|^2 = 1$$

$$a = e^{i\chi} \sin\left(\frac{\theta}{2}\right) \quad c = e^{i\phi} \cos\left(\frac{\theta}{2}\right)$$

- SU(2) parameters and Euler Angles

$$\begin{cases} \theta = \beta \\ \chi = \frac{\alpha+\gamma}{2} \\ \phi = \frac{\pi}{2} - \frac{\alpha-\gamma}{2} \end{cases}$$



- The connection between SU(2) and classical rotations is established via the canonical homomorphism with SO(3).

$$R = \begin{pmatrix} \frac{1}{2}(a^2 - c^2 + (a^*)^2 - (c^*)^2) & \frac{i}{2}(-a^2 + c^2 + (a^*)^2 - (c^*)^2) & a^*c + c^*a \\ \frac{i}{2}(a^2 + c^2 - (a^*)^2 - (c^*)^2) & \frac{1}{2}(a^2 + c^2 + (a^*)^2 + (c^*)^2) & -i\,(a^*c - c^*a) \\ -(ac + c^*a^*) & i(ac - c^*a^*) & 1 - 2cc^* \end{pmatrix}$$

$SU_q(2)$ (''q = 1- ε'')

- Parameters become the generators of $C_q\big(SU(2)\big)$, the algebra of complex functions on $SU(2)$

$$\begin{pmatrix} a & -c^* \\ c & a^* \end{pmatrix} \Rightarrow \begin{pmatrix} a & -qc^* \\ c & a^* \end{pmatrix} \qquad a, c \in C_q\big(SU(2)\big)$$

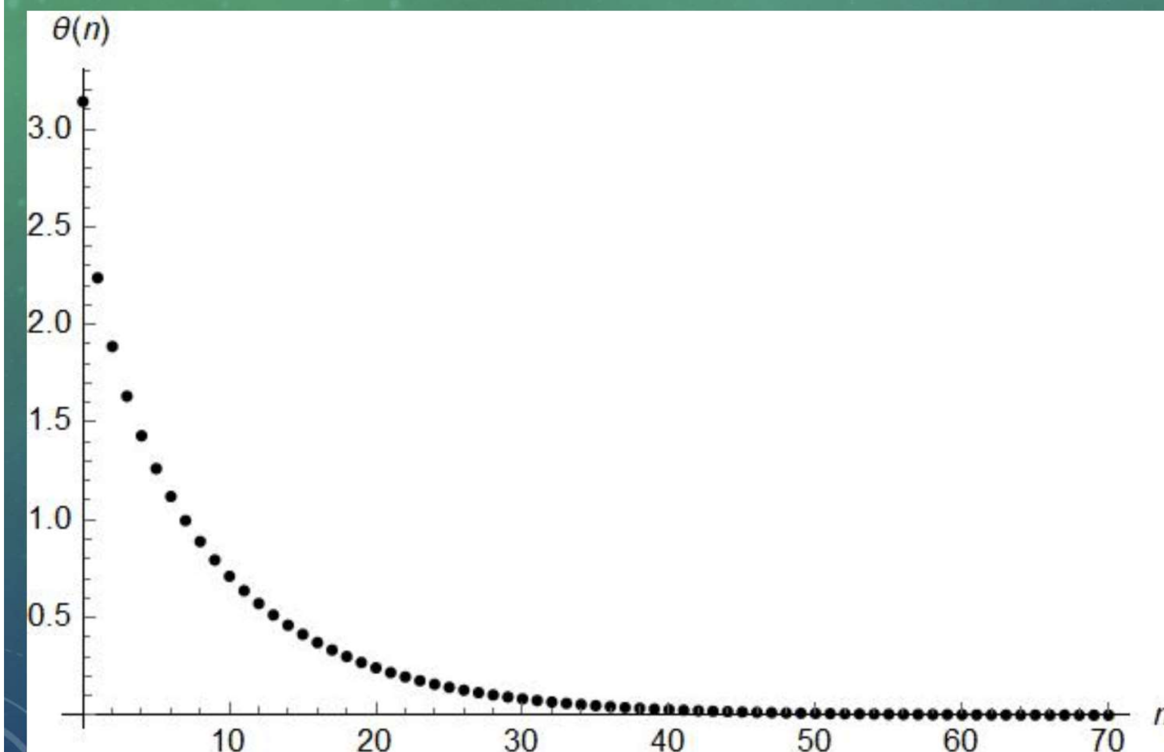
endowed with a non-commutative product realized by

$$ac = qca \qquad ac^* = qc^*a \qquad cc^* = c^*c$$

$$c^*c + a^*a = 1 \qquad aa^* - a^*a = (1 - q^2)c^*c$$

- We promote the SU(2)-Euler Angles relations to the quantum case.
- Comparing the phases of a and c to their classical analogues, we identify ϵ with χ and δ with ϕ. They are continuous and play the same role as before.
- Exploiting the fact that c is a diagonal operator

$$q^n = Sin\left(\frac{\theta(n)}{2}\right) \leftrightarrow \theta(n) = 2Arcin(q^n)$$



$$\theta(n) = 2Arcin(q^n)$$

q=0.99

Physical interpretation and Quantum rotations

- A state $|\psi\rangle \in H$ is representative of the relative orientation between two reference frames, A and B.

- Our interpretation is that the mean value of R_q on $|\psi\rangle$ will give an estimate of the entries of the rotation matrix that connects A and B

$$\langle\psi|R_q|\psi\rangle_{\mathrm{ij}}$$

- However, due to non-commutatitvity, we will have a non vanishing variance for the matrix elements, in general:

$$\Delta_{ij} = \sqrt{\langle\psi|R_q^2|\psi\rangle_{\mathrm{ij}} - \langle\psi|R_q|\psi\rangle_{\mathrm{ij}}^2}$$

- The choice of the z-axis is “special”. Rotations around it are not affected by uncertainties.
- A rotation of this z-axis described by the state $\left|0; \frac{\pi}{2}; 0\right\rangle$ is affected by uncertainty.
- An observer A who identifies a sharp object along its z-axis, will identify a “fuzzy” object along the z-axis of an observer B whose relative orientation with respect to A is described by $\left|0; \frac{\pi}{2}; 0\right\rangle$ (and viceversa).
- Therefore, the space we infer depends on the choice of the z-axis…in this sense we say that space is agency dependent