

Wave function collapse models tests in the cosmic silence

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**Centro Ricerche Enrico Fermi
LNF (INFN)**

on behalf of the VIP-2 collaboration

**Nuclear and Atomic transitions as laboratories for
high precision tests of Quantum Gravity inspired
models**

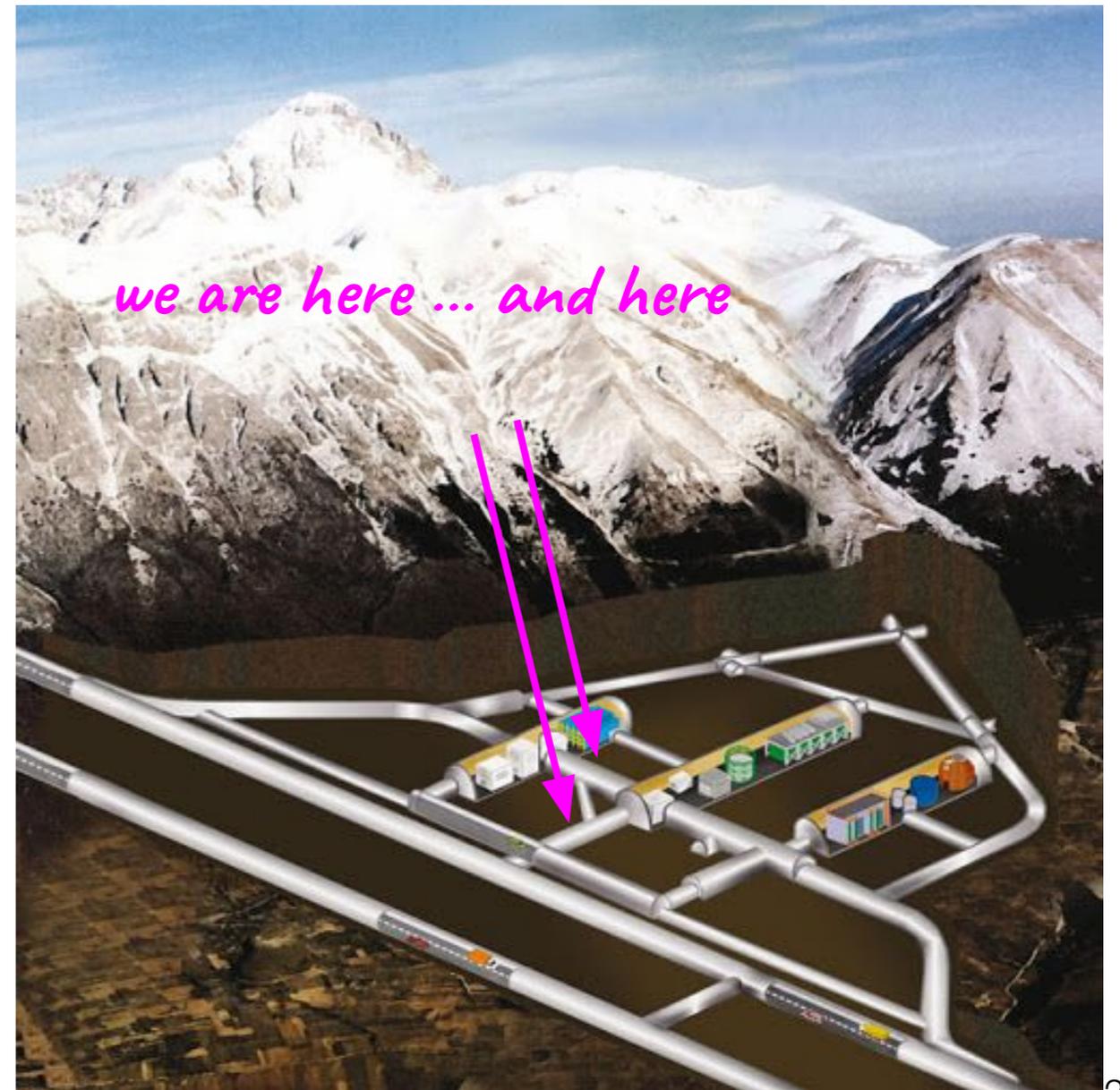
19-23 September 2022, ECT*

**We acknowledge the John Templeton Foundation for the project QUBO
(Exploring the QUantum Boundaries of many-body systems – an Odyssey into
the gravity related collapse models), Grant 62099**

The LNGS laboratories environment

The experiments are performed in the low-background environment of the underground Gran Sasso National Laboratory of INFN:

- overburden corresponding to a minimum thickness of 3100 m w.e.
- the muon flux is reduced by almost six orders of magnitude, to a flux of three oom.
- the main background source consists of γ -radiation produced by long-lived γ -emitting primordial isotopes and their decay products.



Models of w.f. dynamical reduction

Why the quantum properties of microscopic systems, most notably, the possibility of being in the superposition of different states at once, do not seem to carry over to larger objects? A debate which is as old as the quantum theory itself.

Even perfectly isolating a quantum system, regardless its size, will the linear and deterministic Schroedinger evolution manifest forever? -> direct impact on Quantum Technologies

Superposition principle may progressively break down when atoms glue together to form larger systems [Károlyhazi, Diósi, Lukács, Penrose, Ghirardi, Rimini, Weber, Pearle, Adler, Milburn, Bassi ...]

But what triggers the w.f. Collapse?

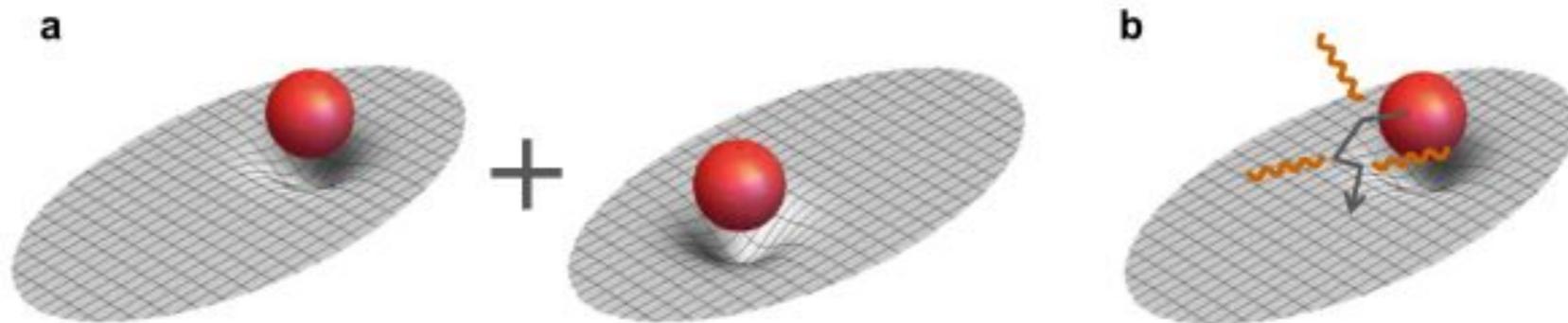
Feynman in lectures on gravitation: breakdown of the quantum superposition at macroscopic scale, possibility that gravity might not be quantized.

Gravity induced collapse: the Diosi-Penrose model

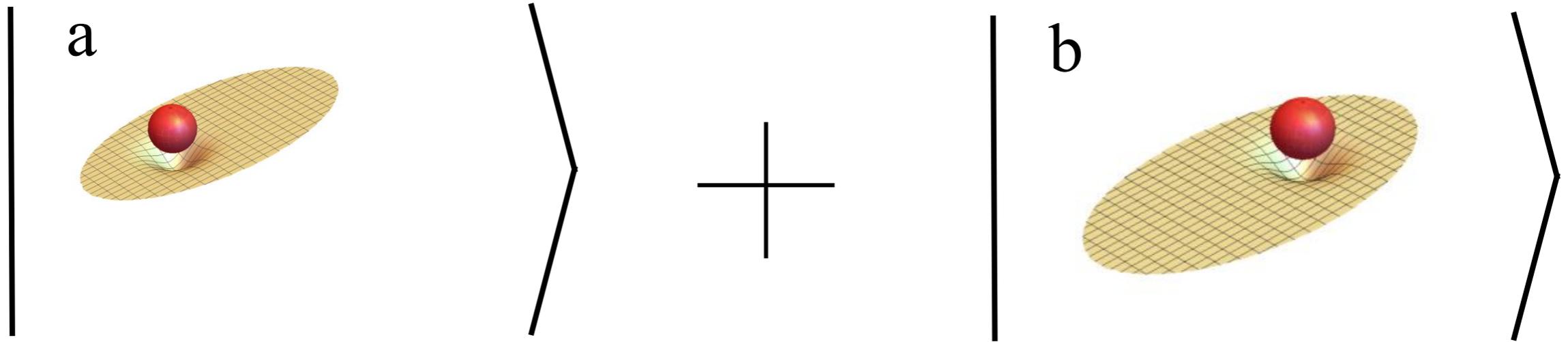
Diósi: QT requires an absolute indeterminacy of the gravitational field, -> the local gravitational potential should be regarded as a stochastic variable, whose mean value coincides with the Newton potential, and the correlation function is:

$$\langle \phi(\mathbf{r}, t) \phi(\mathbf{r}', t') \rangle - \langle \phi(\mathbf{r}, t) \rangle \langle \phi(\mathbf{r}', t') \rangle \sim \frac{\hbar G}{|\mathbf{r} - \mathbf{r}'|} \delta(t - t')$$

Penrose: When a system is in a spatial quantum superposition, a corresponding superposition of two different space-times is generated. The superposition is unstable and decays in time. The more massive the system in the superposition, the larger the difference in the two space-times and the faster the wave-function collapse.



Gravity induced collapse



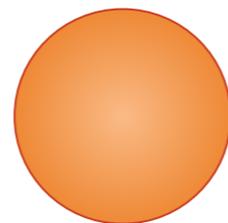
$$\Delta E_{\text{DP}} = 4\pi G \int d\mathbf{r} \int d\mathbf{r}' \frac{[\mu_a(\mathbf{r}) - \mu_b(\mathbf{r})] [\mu_a(\mathbf{r}') - \mu_b(\mathbf{r}')] }{|\mathbf{r} - \mathbf{r}'|}.$$

$$\tau_{\text{DP}} = \frac{\hbar}{\Delta E_{\text{DP}}}$$



Proton: $m \approx 10^{-27}$ Kg, $R \approx 10^{-15}$ m

$\tau_{\text{DP}} \approx 10^6$ years



Dust grain: $m \approx 10^{-12}$ Kg, $R \approx 10^{-5}$ m

$\tau_{\text{DP}} \approx 10^{-8}$ s

Gravity induced collapse

The DP theory is parameter-free, but the gravitational self energy difference diverges for point-like particles \rightarrow a short-length cutoff R_0 is introduced to regularize the theory.

Diósi: minimum length R_0 limits the spatial resolution of the mass density, a short-length cutoff to regularize the mass density.

Penrose: solution of the stationary Schrödinger-Newton equation, with R_0 the size of the particle mass density. $\mu(\mathbf{r}) = m|\psi(\mathbf{r}, t)|^2$

EG becomes a function of R_0 , the larger R_0 , the longer the collapse time, the fainter the radiation

Direct tests: creating a large superposition of a massive system, to guarantee that decay time is short enough for the collapse to become effective before any kind of external noise disrupts the measurement, matter-wave interferometry with macromolecules, phononic states, experiments in space: no gravity \rightarrow more time (MAQRO, CAL, etc..).

Testing collapse models by means of Gamma ray spectroscopy

Indirect tests of collapse models exploit an *unavoidable side effect of the collapse*: a *Brownian-like diffusion of the system in space*.

Collapse probability is Poissonian in t \rightarrow Lindblad dynamics for the statistical operator \rightarrow free particle average square momentum increases in time. *For a general result see Donadi's talk !!!*

Then *charged particles emit spontaneous radiation*. We search for spontaneous radiation emission from a germanium crystal and the surrounding materials in the experimental apparatus.

Strategy: simulate the background from all the known emission processes \rightarrow perform a Bayesian comparison of the residual spectrum with the theoretical prediction \rightarrow extract the pdf of the model parameters \rightarrow bound the parameters.

Theoretical prediction

GAMMA RAYS spontaneous emission $E > 0.5 \text{ MeV}$

- CSL - s. e. photons rate:

$$\frac{d\Gamma}{dE} = N_{atoms} \times (N_A^2 + N_A) \times \frac{\lambda \hbar e^2}{4\pi^2 \epsilon_0 m_0^2 r_C^2 c^3 E}$$

- DP - s. e. photons rate:

$$\frac{d\Gamma_t}{d\omega_k} = \frac{2}{3} \frac{Ge^2 N^2 N_a}{\pi^{3/2} \epsilon_0 c^3 R_0^3 \omega_k}$$

Bassi - Donadi

In range $\Delta E = (1 - 4) \text{ MeV}$
electrons are relativistic, only the
contribution of protons (N) is
considered.

λ - collapse strength

r_c - correlation length

see e. g. S. L. Adler, JPA 40, (2007) 2935, Adler, S.L.; Bassi, A.; Donadi, S., JPA 46, (2013) 245304.

R_0 - size of the particle mass density. See e.g. Diósi, L. J. Phys. Conf. Ser. 442, 012001, (2013)., Penrose, R. Found. Phys. 44, 557-575 (2014).

The experimental setup

The experimental apparatus is based on a coaxial p-type high purity germanium detector (HPGe):

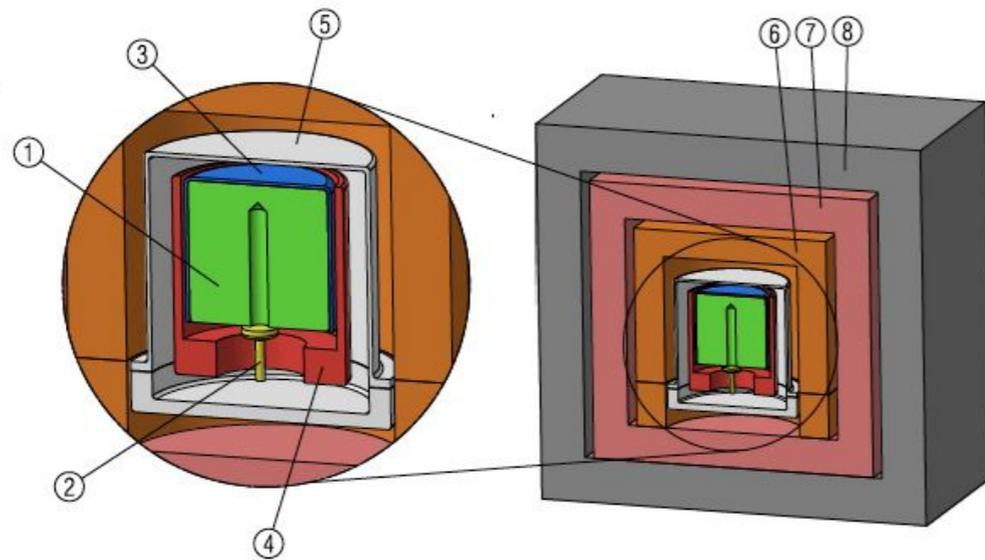
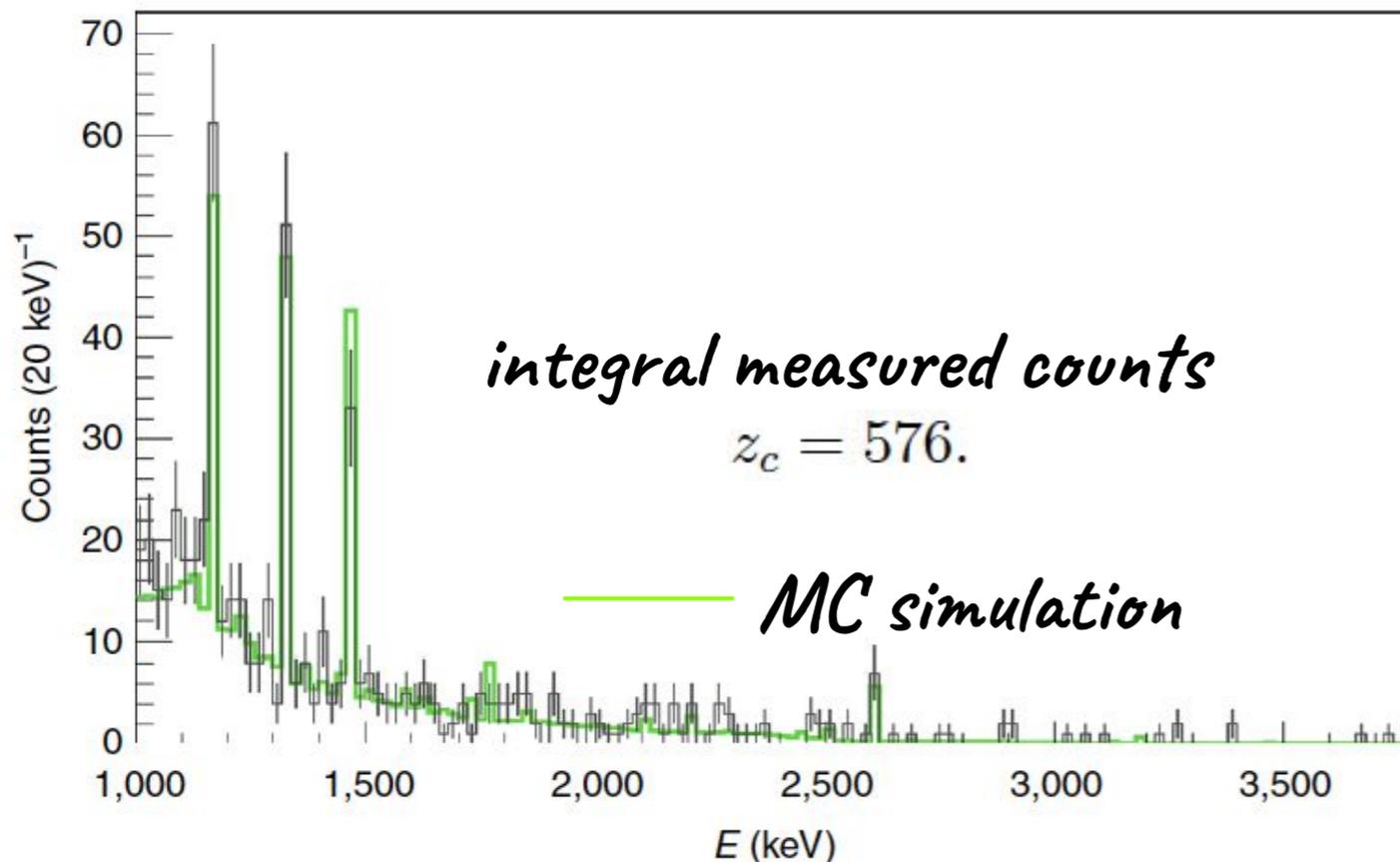


Figure 1: Schematic representation of the experimental setup: 1 - Ge crystal, 2 - Electric contact, 3 - Plastic insulator, 4 - Copper cup, 5 - Copper end-cup, 6 - Copper block and plate, 7 - Inner Copper shield, 8 - Lead shield.

- Exposure $124 \text{ kg} \cdot \text{day}$, $m_{\text{Ge}} \sim 2 \text{ kg}$
- passive shielding: inner - electrolytic copper, outer - lead
- on the bottom and on the sides 5 cm thick borated polyethylene plates give a partial reduction of the neutron flux
- an airtight steel housing encloses the shield and the cryostat, flushed with boil-off nitrogen to minimize the presence of radon.

Measured spectrum and background simulation

The experimental apparatus is characterised, through a validated MC code, based on the GEANT-4 software library. The background is due to emission of residual radio-nuclides:



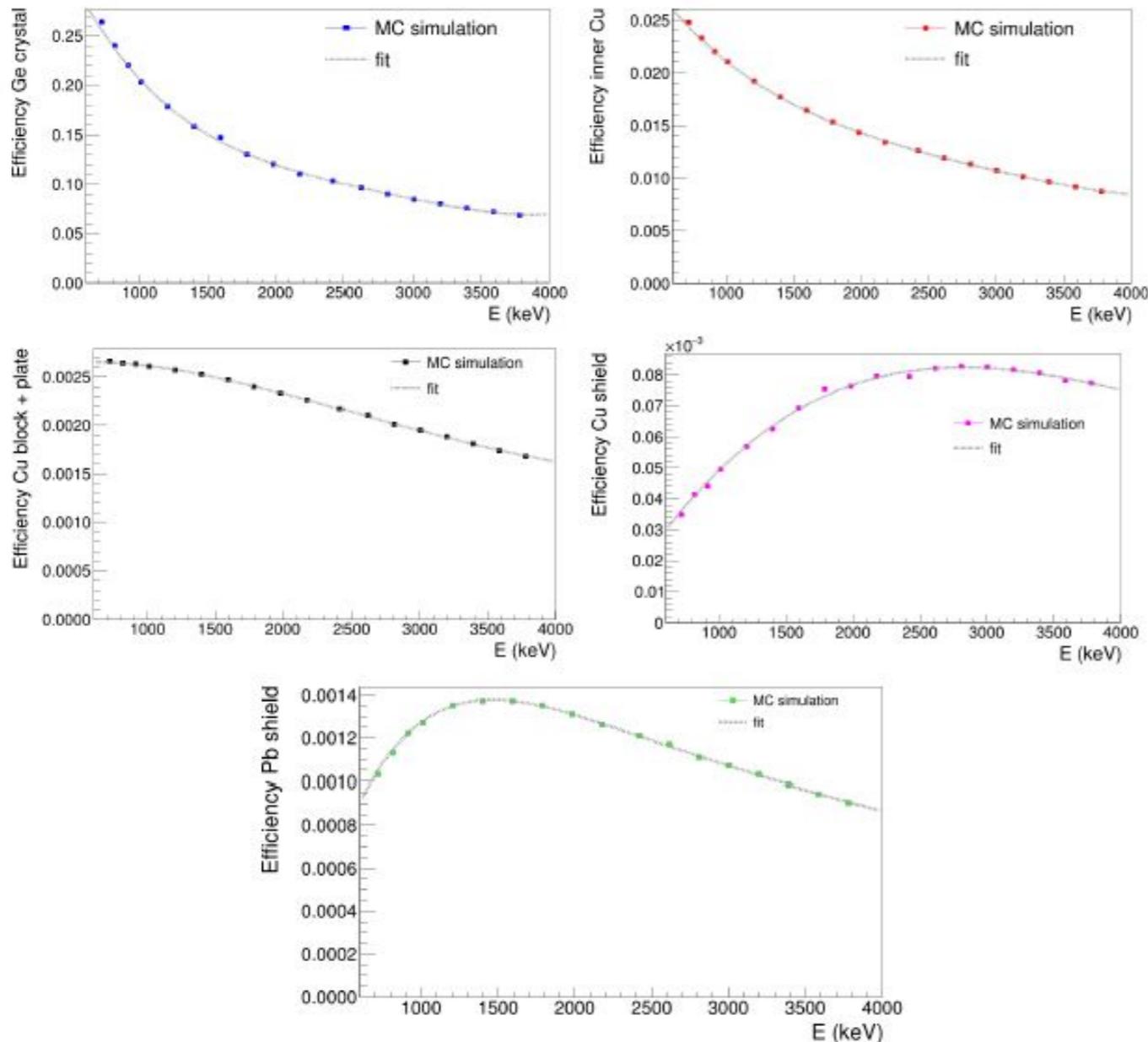
- the activities are measured for each component
- the MC simulation accounts for:
 1. emission probabilities and decay schemes
for each radio-nuclide in each material
 1. photons propagation and interactions
 2. detection efficiencies.

The simulation describes 88% of the integral counts:

$$z_{b,ij} = \frac{m_i A_{ij} T N_{rec,ij}}{N_{ij}}, \quad z_b = \sum_{i,j} z_{b,ij} = 506.$$

Lower bound on R_0 expected signal contribution

The expected number of photons spontaneously emitted by the nuclei of all the materials of the detector are obtained weighting the theoretical rate for the detection efficiencies:



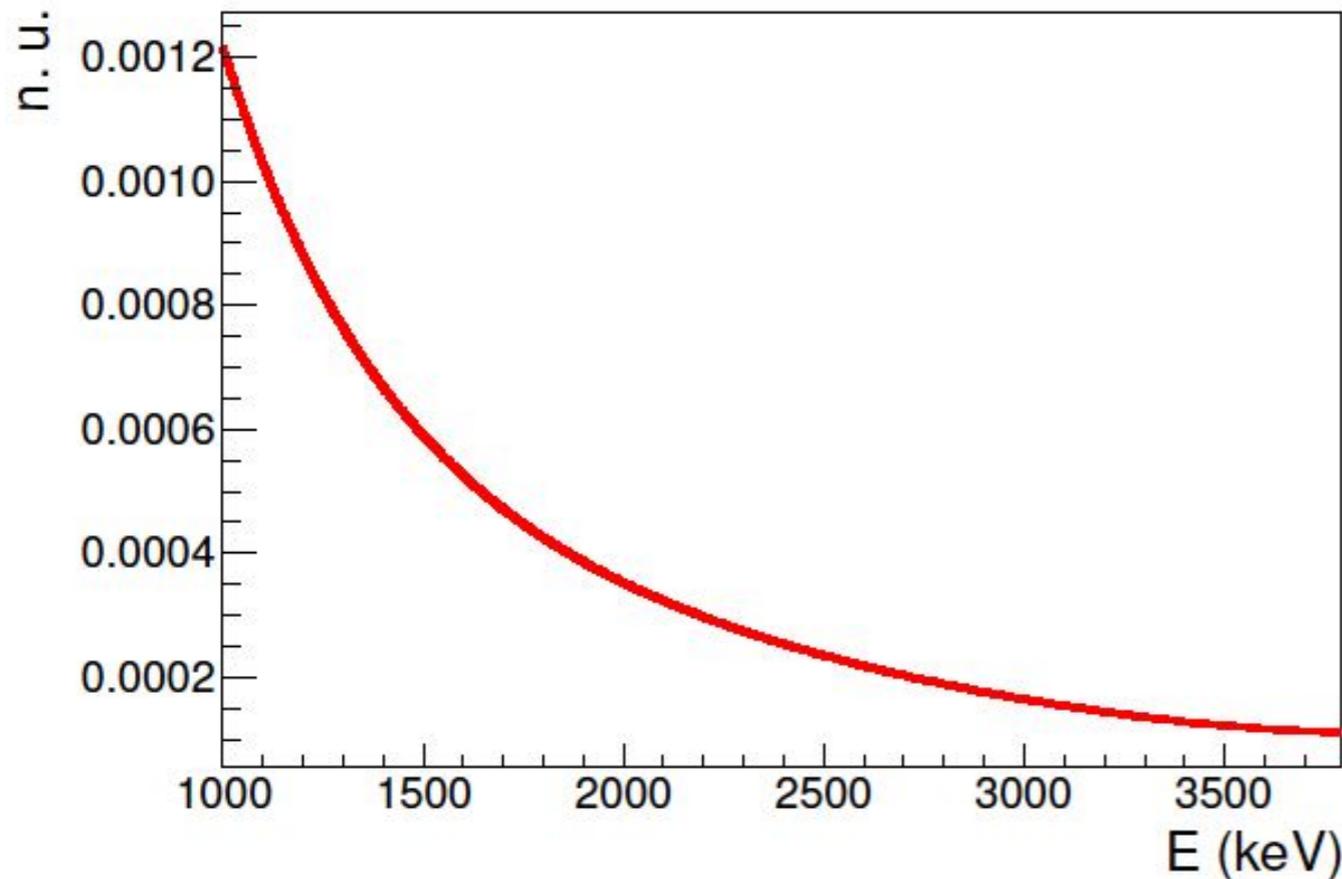
- 10^8 photons generated for each energy for each material
- efficiency functions are obtained by polynomial fits $\epsilon_i(E) = \sum_{j=0}^{c_i} \xi_{ij} E^j$
- the expected signal contribution is:

$$z_s(R_0) = \sum_i \int_{\Delta E} \left. \frac{d\Gamma_t}{dE} \right|_i T \epsilon_i(E) dE = \frac{a}{R_0^3}$$

with $a = 1.8 \cdot 10^{-29} \text{ m}^3$

Lower bound on R_0 expected signal contribution

The expected signal of spontaneously emitted photons by the nuclei of all the materials of the detector is obtained weighting the theoretical rate for the detection efficiencies:



Energy distribution of the expected signal, resulting from the sum of the emission rates of all the materials, weighted for the efficiency functions.

The area of the distribution is normalised to the unity (n. u.)

Lower bound on R_0

pdf of R_0

z_c is distributed according to a Poissonian $p(z_c|\Lambda_c) = \frac{\Lambda_c^{z_c} e^{-\Lambda_c}}{z_c!}$ with $\Lambda_c(R_0) = \Lambda_b + \Lambda_s(R_0)$

The pdf of R_0 is then given by probability inversion:

$$\tilde{p}(\Lambda_c(R_0)|p(z_c|\Lambda_c(R_0))) = \frac{p(z_c|\Lambda_c(R_0)) \cdot \tilde{p}_0(\Lambda_c(R_0))}{\int_D p(z_c|\Lambda_c(R_0)) \cdot \tilde{p}_0(\Lambda_c(R_0)) d[\Lambda_c(R_0)]}$$

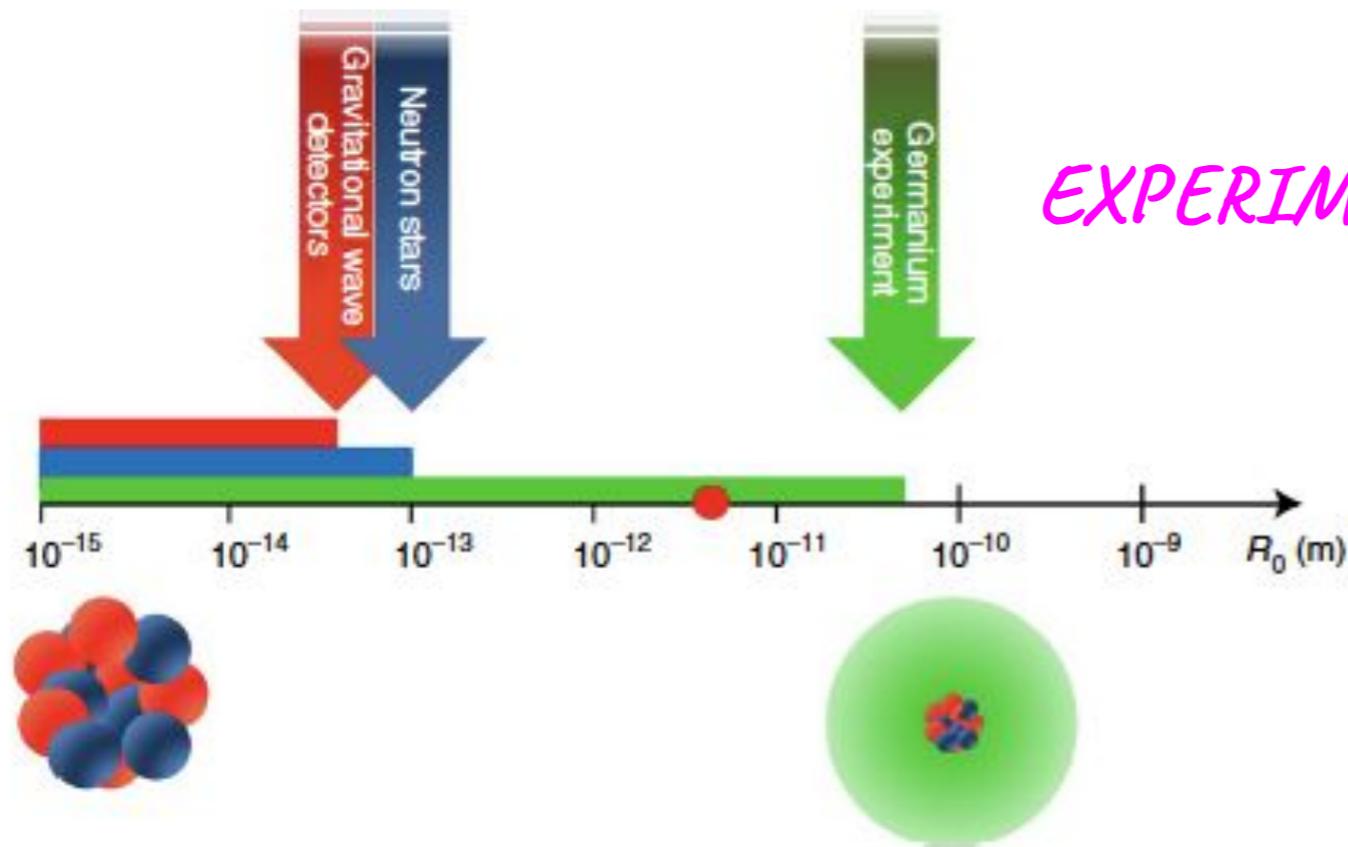
The prior $\tilde{p}_0(\Lambda_c(R_0)) = \theta(\Lambda_c^{\max} - \Lambda_c(R_0))$ accounts for previous limits from gravitational wave detectors and neutron stars data analyses [Phys. Rev. D 95, 084054 (2017), Phys. Rev. Lett. 123, 080402 (2019)].

$$\tilde{P}(\bar{\Lambda}_c) = \frac{\gamma(z_c + 1, \bar{\Lambda}_c)}{\gamma(z_c + 1, \Lambda_c^{\max})} = 0.95$$

A bound on R_0 is obtained from the cumulative pdf:

$$R_0 > 0.54 \cdot 10^{10} \text{ m}$$

Lower bound on R_0



EXPERIMENTAL: $R_0 > 0.54 \cdot 10^{10} \text{ m}$

If R_0 is the size of the nucleus's wave function as suggested by Penrose, we have to compare the limit with the properties of nuclei in matter.

In a crystal $R_0^2 = \langle u^2 \rangle$ is the mean square displacement of a nucleus in the lattice, which, for the germanium crystal, cooled to liquid nitrogen temperature amounts to:

THEORETICAL EXPECTATION $R_0 = 0.05 \cdot 10^{10} \text{ m}$

“Underground test of gravity-related wave function collapse”. *Nature Physics* 17, pages 74–78 (2021)

The future of Gravity-related collapse

DP is ruled out in present formulation!

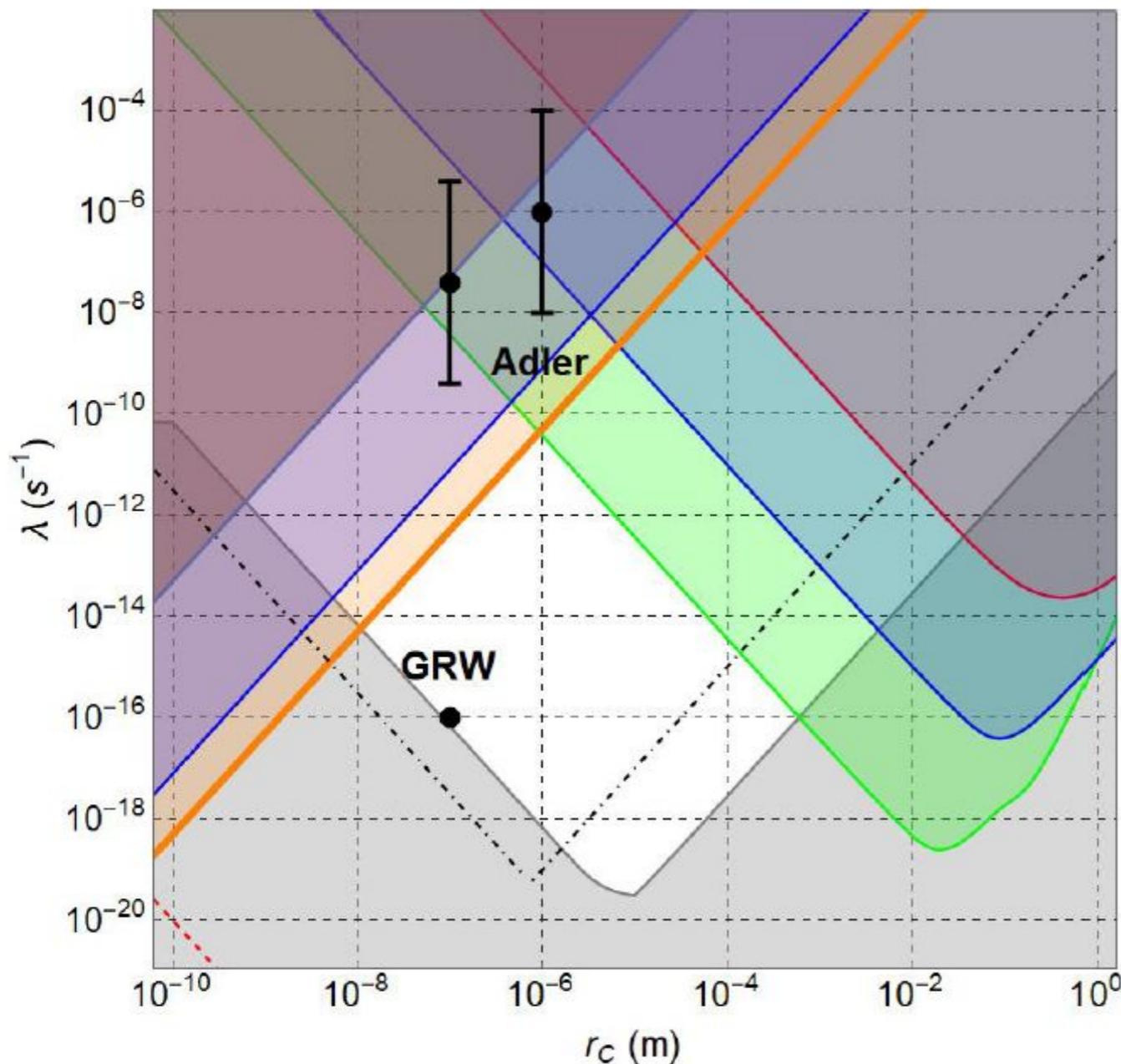
collaboration with Bassi, Donadi, Diosi ... for the development of generalized models e.g. :

- add dissipation terms to the master equation and stochastic nonlinear Schroedinger equation of the DP theory, to counteract the runaway energy increase,
- non-Markovian correlation function.

generalized models lead to *dramatic dependence on the S. E. energy in relation to the atomic structure!* -> discussed in my last slides

Constraints on the CSL

Similar analysis leads to bounds on the strength and correlation length of the CSL
(*Eur. Phys. J. C* (2021) 81: 773)



$$\lambda/r_c^2 < 52 \text{ m}^{-2} \text{ s}^{-1}$$

Fig. 4 Mapping of the $\lambda - r_c$ CSL parameters: the proposed theoretical values (GRW [6], Adler [24,25]) are shown as black points. The region excluded by theoretical requirements is represented in gray, and it is obtained by imposing that a graphene disk with the radius of $10 \mu\text{m}$ (about the smallest possible size detectable by human eye) collapses in less than 0.01 s (about the time resolution of human eye) [31]. Contrary to the bounds set by experiments, the theoretical bound has a subjective component, since it depends on which systems are considered as “macroscopic”. For example, it was previously suggested that the collapse should be strong enough to guarantee that a carbon sphere with the diameter of 4000 \AA should collapse in less than 0.01 s , in which case the theoretical bound is given by the dash-dotted black line [36]. A much weaker theoretical bound was proposed by Feldmann and Tumulka, by requiring the ink molecules corresponding to a digit in a printout to collapse in less than 0.5 s (red line in the bottom left part of the exclusion plot, the rest of the bound is not visible as it involves much smaller values of λ than those plotted here) [37]. The right part of the parameter space is excluded by the bounds coming from the study of gravitational waves detectors: Auriga (red), Ligo (Blue) and Lisa-Pathfinder (Green) [30]. On the left part of the parameter space there is the bound from the study of the expansion of a Bose–Einstein condensate (red) [28] and the most recent from the study of radiation emission from Germanium (purple) [22]. This bound is improved by a factor 13 by this analysis performed here, with a confidence level of 0.95, and it is shown in orange

Novel analysis with upgraded Setup

High purity Ge detector measurement:

- **high purity co-axial p-type germanium detector (HPGe), diameter of 8.0 cm, length of 8.0 cm, surrounded by an inactive layer of lithium-doped germanium of 0.075 mm.**
- **additional target material: three cylindrical sections of radio-pure Roman lead completely surrounding the detector**

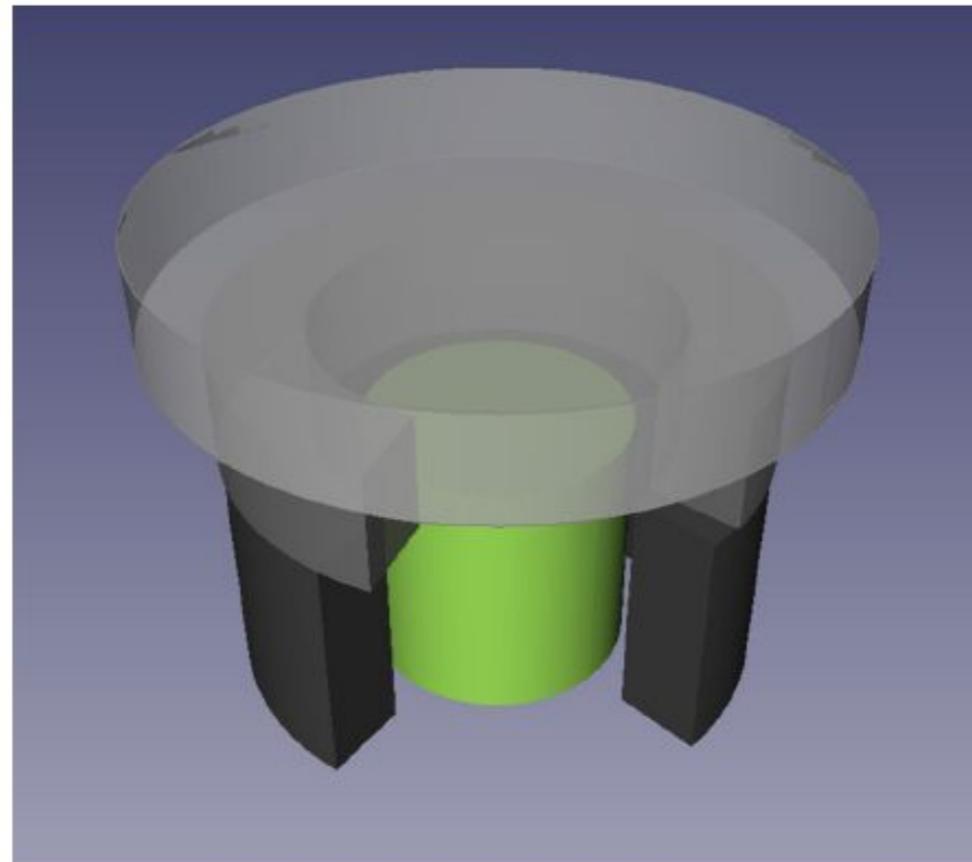


Fig. 1 Schematic representation of the Ge crystal (in green) and the surrounding lead target cylindrical sections (in grey)

Novel analysis with upgraded Setup

- **Passive shielding: inner - electrolytic copper, outer - lead**
- **10B-polyethylene plates reduce the neutron flux towards the detector**
- **shield + cryostat enclosed in air tight steel housing flushed with nitrogen to avoid contact with external air (thus radon).**

K. P. et al., Eur. Phys. J. C (2020) 80: 508

<https://doi.org/10.1140/epjc/s10052-020-8040-5>



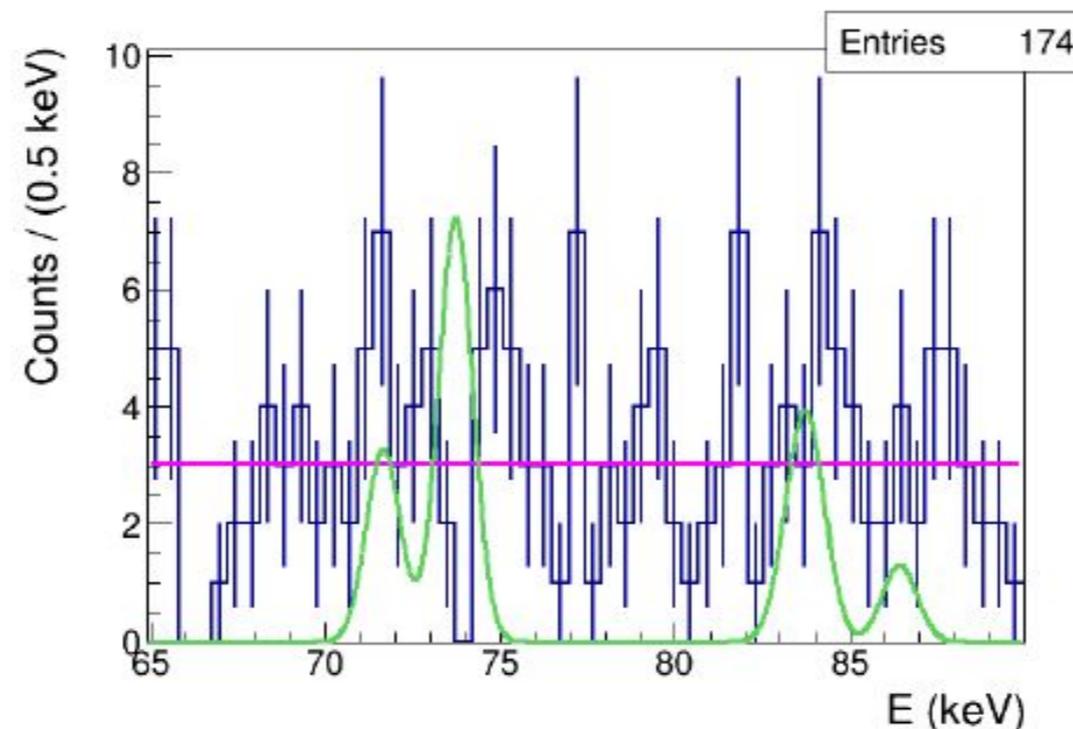
Novel analysis with upgraded Setup

Total acquisition time $3.6 \cdot 10^6$ s, which we now aim to analyze in the energy range:

$\Delta E = (65 - 90)$ keV low energy to comply with the non-white cutoff

considering the detector energy resolution (better than $\sigma = 0.5$ keV) a detailed study of the materials of the apparatus revealed that the only detectable transitions in ΔE would correspond to the K complex in Pb.

Indeed extreme purity + thickness -> “total” suppression of Bremsstrahlung:



Novel analysis with upgraded Setup

Strategy: disentangle, by means of a Bayesian analysis, the shape of the expected S.E. from the flat background, accounting for the cancellation induced by the (photon w.l. - atomic structure) interplay.

The analysis is aimed to obtain the probability density function (*pdf*) of the characteristic parameters of the considered collapse models, namely λ/r_C^2 and R_0 , which will be denoted in general by p

$$P(p|data) = \int_{\mathcal{D}_{\mathbf{b}}} P(p, \mathbf{b}|data) d\mathbf{b}. \quad (13)$$

In Eq. (13) \mathbf{b} is the vector of parameters which characterize the shape of the background; the integration is performed over the domain $\mathcal{D}_{\mathbf{b}}$ of \mathbf{b} . The conditional joint *pdf* of p and \mathbf{b} , given the measured distribution - called *data* - is expressed as:

$$\begin{aligned} P(p, \mathbf{b}|data) &= \\ &= \frac{P(data|p, \mathbf{b}) \cdot P_0(p) \cdot P_0(\mathbf{b})}{\int P(data|p, \mathbf{b}) \cdot P_0(p) \cdot P_0(\mathbf{b}) d(p) d\mathbf{b}} \end{aligned}$$

Novel analysis with upgraded Setup

$$\begin{aligned} P(p, \mathbf{b}|data) &= \\ &= \frac{P(data|p, \mathbf{b}) \cdot P_0(p) \cdot P_0(\mathbf{b})}{\int P(data|p, \mathbf{b}) \cdot P_0(p) \cdot P_0(\mathbf{b}) d(p) d\mathbf{b}}, \end{aligned} \quad (14)$$

according to the Bayes theorem. The likelihood is taken as a product of Poissonian distributions:

$$P(data|p, \mathbf{b}) = \prod_{i=1}^N \frac{\lambda_i(p, \mathbf{b})^{n_i} \cdot e^{-\lambda_i(p, \mathbf{b})}}{n_i!}, \quad (15)$$

where n_i are the measured bin contents, and the expectation values are parametrized as:

$$\begin{aligned} \lambda_i(p, \mathbf{b}) &= \int_{\Delta E_i} f_B(E, \mathbf{b}) dE + \\ &+ \int_{\Delta E_i} f_S(E, p) dE. \end{aligned} \quad (16)$$

Where ΔE_i is the energy range corresponding to the i -th bin, $f_S(E, p)$ is the expected shape of the signal distribution and $f_B(E, \mathbf{b})$ is the shape of the background.

Novel analysis with upgraded Setup

The shape of the signal distribution is obtained by weighting the theoretical expected emission rate (Eqs. (1) and (2)) with the detection efficiency functions, for the various materials of the experimental setup (labelled by j), and summing over j :

$$f_S(E, p) = T \beta^{CSL, DP} p E^{-1} \sum_j \alpha_j \epsilon_j(E) = T \beta^{CSL, DP} p E^{-1} \sum_j \alpha_j \sum_{k=0}^{d_j} \xi_{jk} E^k . \quad (17)$$

Where T is the total acquisition time, the factor β is characteristic of the predicted spontaneous radiation emission rate and is given by:

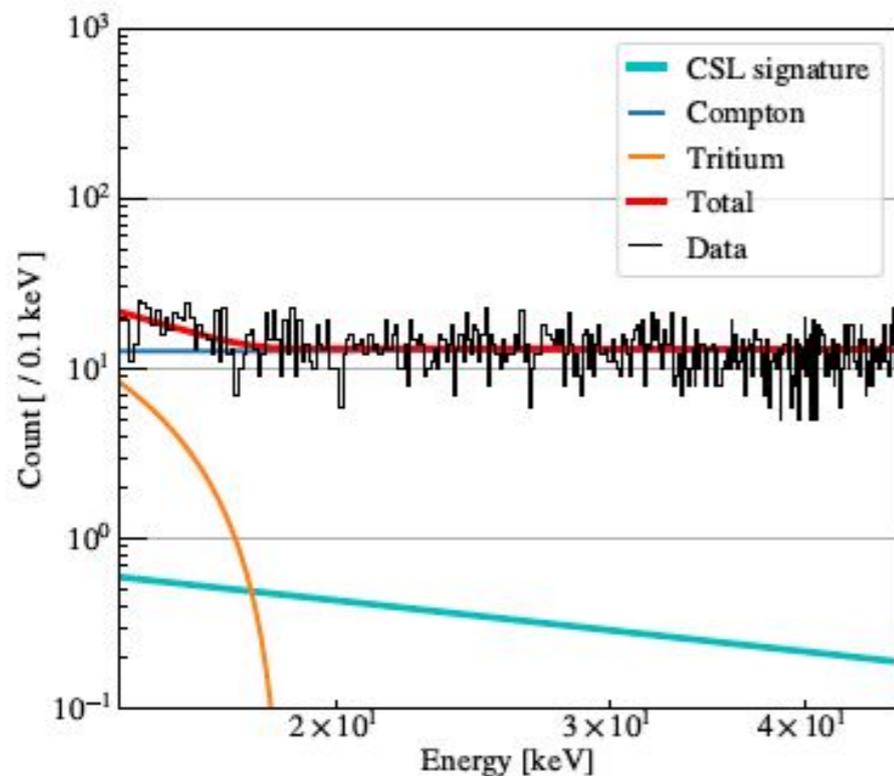
$$\beta^{CSL} = \frac{\hbar e^2}{4 \pi^2 \epsilon_0 c^3 m_0^2} \quad ; \quad \beta^{DP} = \frac{2 G e^2}{3 \pi^{3/2} \epsilon_0 c^3} , \quad (18)$$

with α manifesting a complex dependence on the S.E. photon energy - atomic structure ...

The future of spontaneous radiation as an evidence of w.f. collapse

The good news: the interest of the community is strong!

e.g. MAJORANA DEMONSTRATOR - PHYS. REV. LETT. 129, 080401 (2022)



Non-Markovian extension



cutoff frequency



low-energy range
is relevant

BUT

In this range
S.E. from protons
and valence
electrons cancels !!

applying the same S.E. rate above it is obtained

$$\lambda/r_c^2 < 0.24 \text{ m}^{-2} \text{ s}^{-1}$$



X-rays spontaneous radiation the CSL

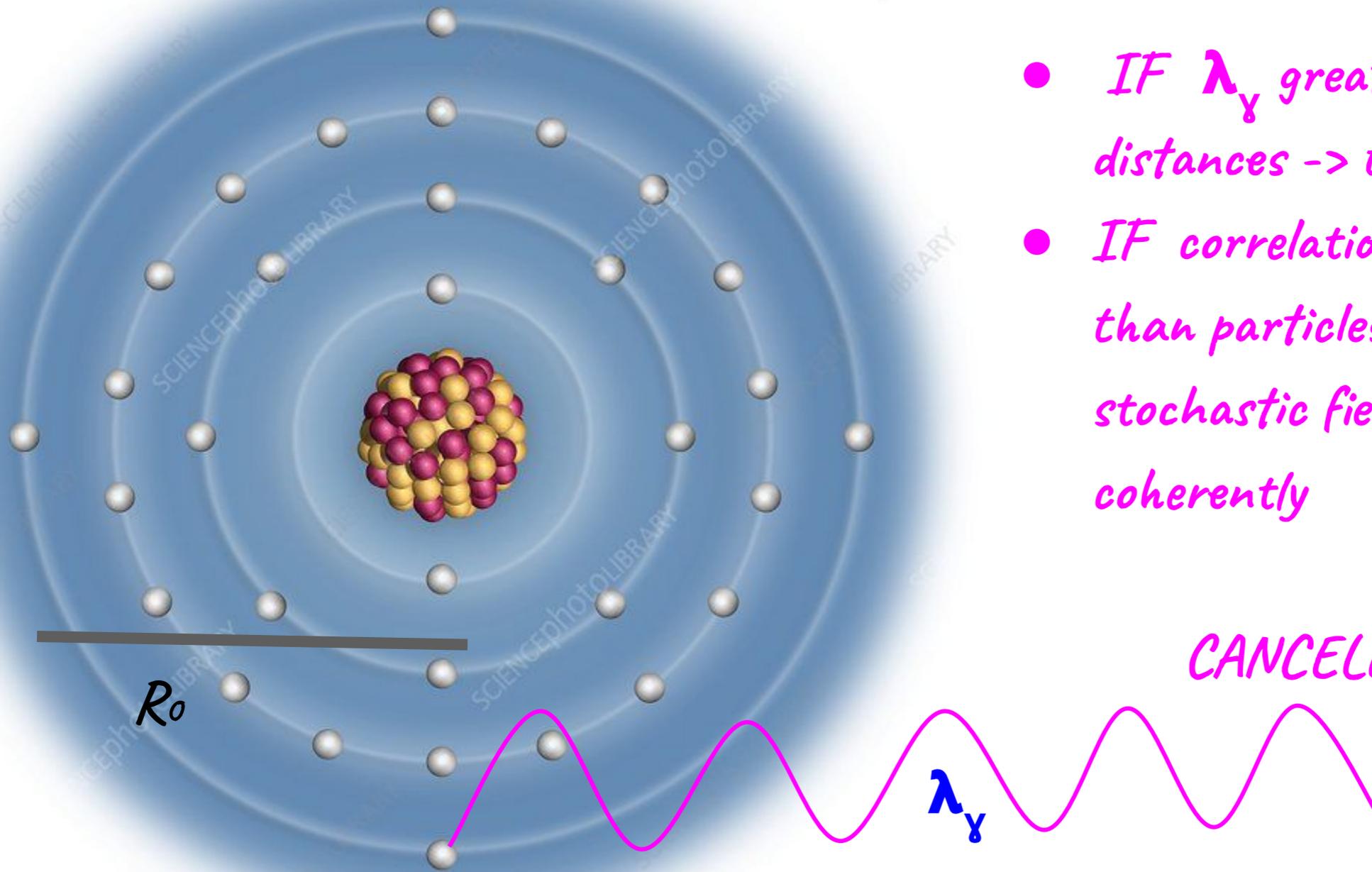
In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions

e.g. $\lambda_{dB}(E=15 \text{ keV}) = 0.8 \text{ \AA}$

$Q_{1s} = 0.025 \text{ \AA}; Q_{4p} = 1.5 \text{ \AA}$

- IF λ_{γ} greater than particles distances \rightarrow they emit coherently
- IF correlation length greater than particles distances \rightarrow the stochastic field vibrates them coherently

CANCELLATION



X-rays spontaneous radiation

the CSL

In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions



e.g. $\lambda_{dB}(E=15 \text{ keV}) = 0.8 \text{ \AA}$

$Q_{1s} = 0.025 \text{ \AA}; Q_{4p} = 1.5 \text{ \AA}$

general expression for the rate applies:

$$\left. \frac{d\Gamma}{dE} \right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar \lambda}{6 \pi^2 \epsilon_0 c^3 m_0^2 E} \sum_{i,j} \frac{q_i q_j}{m_i m_j} \cdot f_{ij}(\mu) \cdot \frac{\sin(b_{ij})}{b_{ij}}$$

$$f_{ij}^k(\mu) := \int ds \int ds' e^{-\frac{(\bar{r}_i - \bar{r}_j + s' - s)^2}{4r_C^2}} \left(\frac{\partial \mu_i(s)}{\partial s^k} \right) \left(\frac{\partial \mu_j(s')}{\partial s'^k} \right)$$

non-Markovian CSL is simpler:

$$r_C \gg |\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \longrightarrow f_{ij}(\mu) = 3 \frac{m_i m_j}{r_C^2}$$

the stochastic fluctuations **ALWAYS**
vibrate electrons and protons coherently

$$b_{i,j} = \frac{2\pi}{c} \frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow \text{if } \lambda_{dB} \ll Q_{1s} \quad \frac{d\Gamma}{dE} = N_{atoms} \times (N_A^2 + N_A) \times \frac{\lambda \hbar e^2}{4\pi^2 \epsilon_0 m_0^2 r_C^2 c^3 E}$$

X-rays spontaneous radiation

the CSL

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the stochastic fluctuations ALWAYS vibrate electrons and protons coherently

$$b_{i,j} = \frac{2\pi}{c} \frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda_\gamma} \longrightarrow$$

if $\lambda_{dB} > Q_{1s}$

electrons and protons emit coherently

X-rays spontaneous radiation the CSL

both electrons and protons contribute:

$$\left. \frac{d\Gamma}{dE} \right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} \left[\sum_{ip,jp} q_{ip} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right.$$

nuclear emission

$$+ \sum_{ip,je} q_{ip} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} +$$

$$+ \sum_{ie,jp} q_{ie} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} +$$

$$\left. + \sum_{ie,je} q_{ie} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} \right]$$

X-rays spontaneous radiation the CSL

both electrons and protons contribute:

$$\begin{aligned}
 \left. \frac{d\Gamma}{dE} \right|_t^{CSL} = & N_{atoms} \cdot \frac{\hbar \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} \left[\sum_{ip,jp} q_{ip} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right. \\
 & + \sum_{ip,je} q_{ip} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} + \\
 & + \sum_{ie,jp} q_{ie} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} + \\
 & \left. + \sum_{ie,je} q_{ie} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} \right] \text{electronic emission}
 \end{aligned}$$

X-rays spontaneous radiation the CSL

both electrons and protons contribute:

$$\frac{d\Gamma}{dE} \Big|_t^{CSL} = N_{atoms} \cdot \frac{\hbar \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} \left[\sum_{ip,jp} q_{ip} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right.$$

$$+ \sum_{ip,je} q_{ip} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} +$$

$$+ \sum_{ie,jp} q_{ie} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} +$$

$$\left. + \sum_{ie,je} q_{ie} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} \right]$$

electrons-protons
coupled emission

X-rays spontaneous radiation the CSL

both electrons and protons contribute:

$$\begin{aligned}
 \left. \frac{d\Gamma}{dE} \right|_t^{CSL} &= N_{atoms} \cdot \frac{\hbar \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} \left[\sum_{ip,jp} q_{ip} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right. \\
 &+ \sum_{ip,je} q_{ip} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} + \\
 &+ \sum_{ie,jp} q_{ie} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} + \\
 &\left. + \sum_{ie,je} q_{ie} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} \right]
 \end{aligned}$$

in the limit $\lambda_{dB} \gg Q_{4p}$

X-rays spontaneous radiation the CSL

both electrons and protons contribute:

$$\begin{aligned} \left. \frac{d\Gamma}{dE} \right|_t^{CSL} = & N_{atoms} \cdot \frac{\hbar \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} \left[\sum_{ip,jp} q_{ip} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j|/\lambda_\gamma} + \right. \\ & + \sum_{ip,je} q_{ip} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ip} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} + \\ & + \sum_{ie,jp} q_{ie} q_{jp} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{jp}|/\lambda_\gamma} + \\ & \left. + \sum_{ie,je} q_{ie} q_{je} \cdot \frac{\sin(2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma)}{2\pi|\bar{\mathbf{r}}_{ie} - \bar{\mathbf{r}}_{je}|/\lambda_\gamma} \right] \end{aligned}$$

In the limit $\lambda_{dB} \gg Q_{ip}$

$$\left. \frac{d\Gamma}{dE} \right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar e^2 \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E} [N_p^2 - 2 \cdot N_p N_e + N_e^2]$$

In neutral matter
complete cancellation!

X-rays spontaneous radiation the CSL

In the general case:

$$\left. \frac{d\Gamma}{dE} \right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar e^2 \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E}$$

X-rays spontaneous radiation the CSL

In the general case:

$$\left. \frac{d\Gamma}{dE} \right|_t^{CSL} = N_{atoms} \cdot \frac{\hbar e^2 \lambda}{4 \pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E}$$

$$\cdot \left\{ N_p^2 + N_e + 2 \cdot \sum_{o o' \text{ pairs}} N_{eo} N_{eo'} \frac{\sin \left[\frac{(\rho_o - \rho_{o'}) E}{\hbar c} \right]}{\left[\frac{(\rho_o - \rho_{o'}) E}{\hbar c} \right]} + \sum_o N_{eo} \frac{\sin \left(\frac{\rho_o E}{\hbar c} \right)}{\left(\frac{\rho_o E}{\hbar c} \right)} \cdot \left[\cos \left(\frac{\rho_o E}{\hbar c} \right) - 2 N_p \right] \right\}$$

at each energy the atomic structure influences the shape
of the expected S.E. spectrum

X-rays spontaneous radiation the DP

The DP is more complex! both $|r_i - r_j|$ vs λ_x and $|r_i - r_j|$ vs R_0 are to be considered.

Notice: no cancellation occurs if gravitational stochastic fluctuations do not vibrate e & p coherently, but we brought R_0 in the domain of the atomic structure

$$R_0 > 0.5 A$$

Formal expression for S.E. rate obtained in analogy to (Eur. Phys. J. C (2021) 81: 773):

$$\left. \frac{d\Gamma}{dE} \right|_t^{DP} = N_{atoms} \cdot \frac{G}{6 \pi^2 \epsilon_0 c^3 E} \sum_{i,j} \frac{q_i q_j}{m_i m_j} \cdot f_{ij}(\mu) \cdot \frac{\sin(b_{ij})}{b_{ij}}$$

$|r_i - r_j|$ vs R_0 $|r_i - r_j|$ vs λ_x
 interplay interplay

X-rays spontaneous radiation the DP

For a non-white time correlation function, e.g. assuming exponential correlation in time:

$$E[\phi(\mathbf{r}, t) \phi(\mathbf{r}', t')] = \frac{\hbar G}{|\mathbf{r} - \mathbf{r}'|} \frac{\Omega}{2} e^{-\Omega|t-t'|}$$

The expected spontaneous emission rate becomes:

$$\left. \frac{d\Gamma}{dE} \right|_t^{DP} = N_{atoms} \cdot \frac{G}{6 \pi^2 \epsilon_0 c^3 E} \sum_{i,j} \frac{q_i q_j}{m_i m_j} \cdot f_{ij}(\mu) \cdot \frac{\sin(b_{ij})}{b_{ij}} \frac{\Omega^2}{\Omega^2 + \omega^2}$$

X-rays spontaneous radiation the DP

$$f_{ij}(\mu) = \sum_{k=x,y,z} \int d^3 r' \left[\int d^3 r \frac{1}{|\mathbf{r} - \mathbf{r}'|} \frac{\partial \mu_j(\bar{\mathbf{r}}_j - \mathbf{r})}{\partial r_k} \right] \frac{\partial \mu_i(\bar{\mathbf{r}}_i - \mathbf{r}')}{\partial r'_k}$$

by integrating by parts and using: $\int d^3 r \mu_j(\bar{\mathbf{r}}_j - \mathbf{r}) \frac{r_k - r'_k}{|\mathbf{r} - \mathbf{r}'|^3} = \frac{F_k(\mathbf{r}')}{G}$

$$f_{ij}(\mu) = \frac{1}{G} \sum_{k=x,y,z} \int d^3 r' F_k(\mathbf{r}') \frac{\partial \mu_i(\bar{\mathbf{r}}_i - \mathbf{r}')}{\partial r'_k} =$$

$$\frac{1}{G} \sum_{k=x,y,z} \int d^3 r' \left[-\mu_i(\bar{\mathbf{r}}_i - \mathbf{r}') \frac{\partial F_k(\mathbf{r}')}{\partial r'_k} \right] =$$

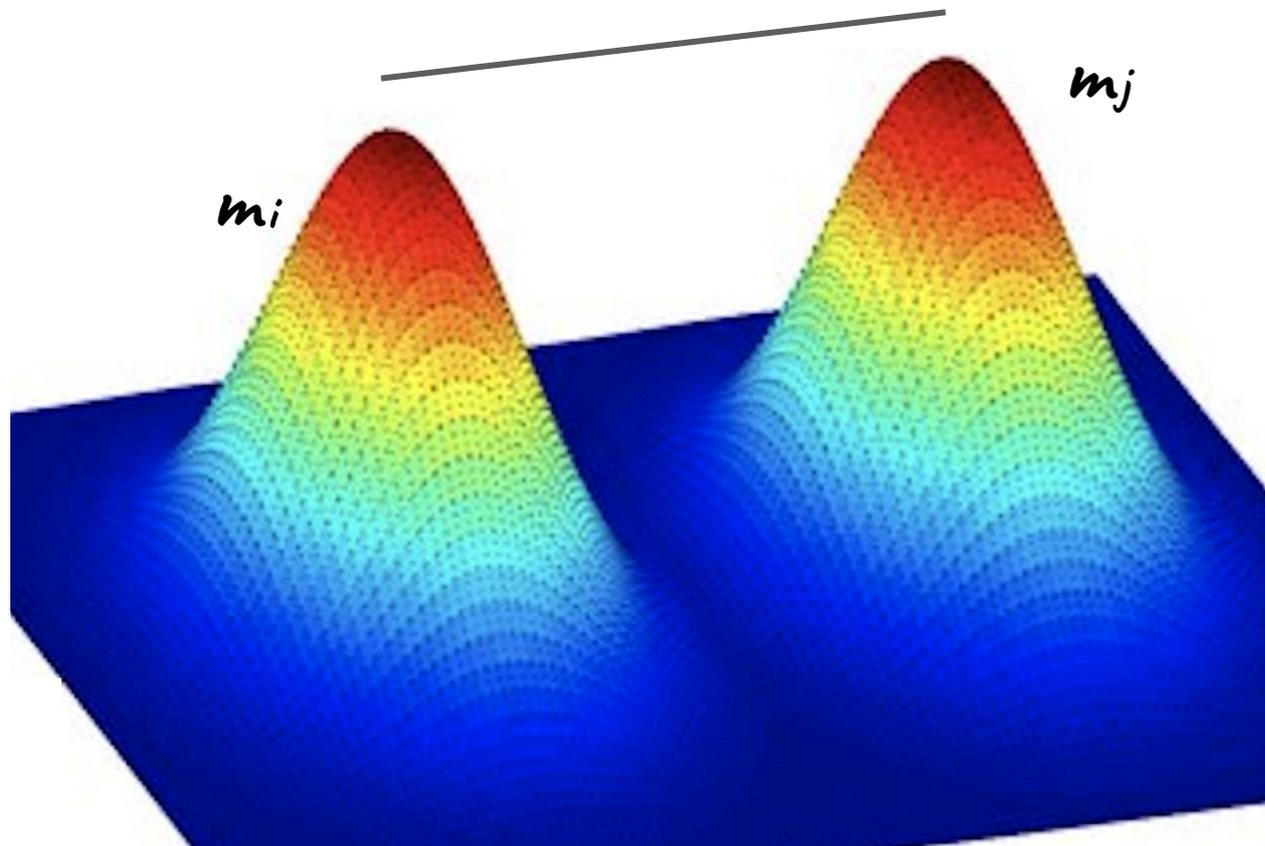
$$-\frac{1}{G} \int d^3 r' \mu_i(\bar{\mathbf{r}}_i - \mathbf{r}') \nabla \bar{F}(\mathbf{r}')$$

finally applying the Poisson equation:

$$f_{ij}(\mu) = 4\pi \int d^3 r' \mu_i(\bar{\mathbf{r}}_i - \mathbf{r}') \cdot \mu_j(\bar{\mathbf{r}}_j - \mathbf{r}')$$

X-rays spontaneous radiation the DP

$$|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \gg R_0$$



$$f_{ij}(\mu) = 4\pi \int d^3r' \mu_i(\bar{\mathbf{r}}_i - \mathbf{r}') \cdot \mu_j(\bar{\mathbf{r}}_j - \mathbf{r}')$$

If the mass distributions (around r_i and r_j) are narrow with respect to R_0 and $|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \gg R_0$ their contribution to the spontaneous radiation is negligible.

On contrary if $|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \ll R_0$

it can be shown that :

$$f_{ij}(\mu) = \frac{m_i^2}{2\pi^{1/2}R_0^3}$$

if the particles are vibrated coherently

and also $|\bar{\mathbf{r}}_i - \bar{\mathbf{r}}_j| \ll \lambda_\gamma$

they emit coherently -> **CANCELLATION**

The Hameroff-Penrose scheme for the emergence of a conscious moment: Orch OR theory

Physics of Life Reviews Volume 11, Issue 1, March 2014, Pages 39-78

- Moments of conscious awareness (choice) depend on biologically ‘orchestrated’ coherent quantum processes in collections of microtubules within brain neurons,
- these quantum processes correlate with, and regulate, neuronal synaptic and membrane activity,
- continuous Schrödinger evolution of such process terminates in accordance with the specific DP scheme of objective reduction!

For tubulins in superposition of size of the C nucleus radius, considering our limit on R_0 , the required number of neurons in coherent superposition would be:

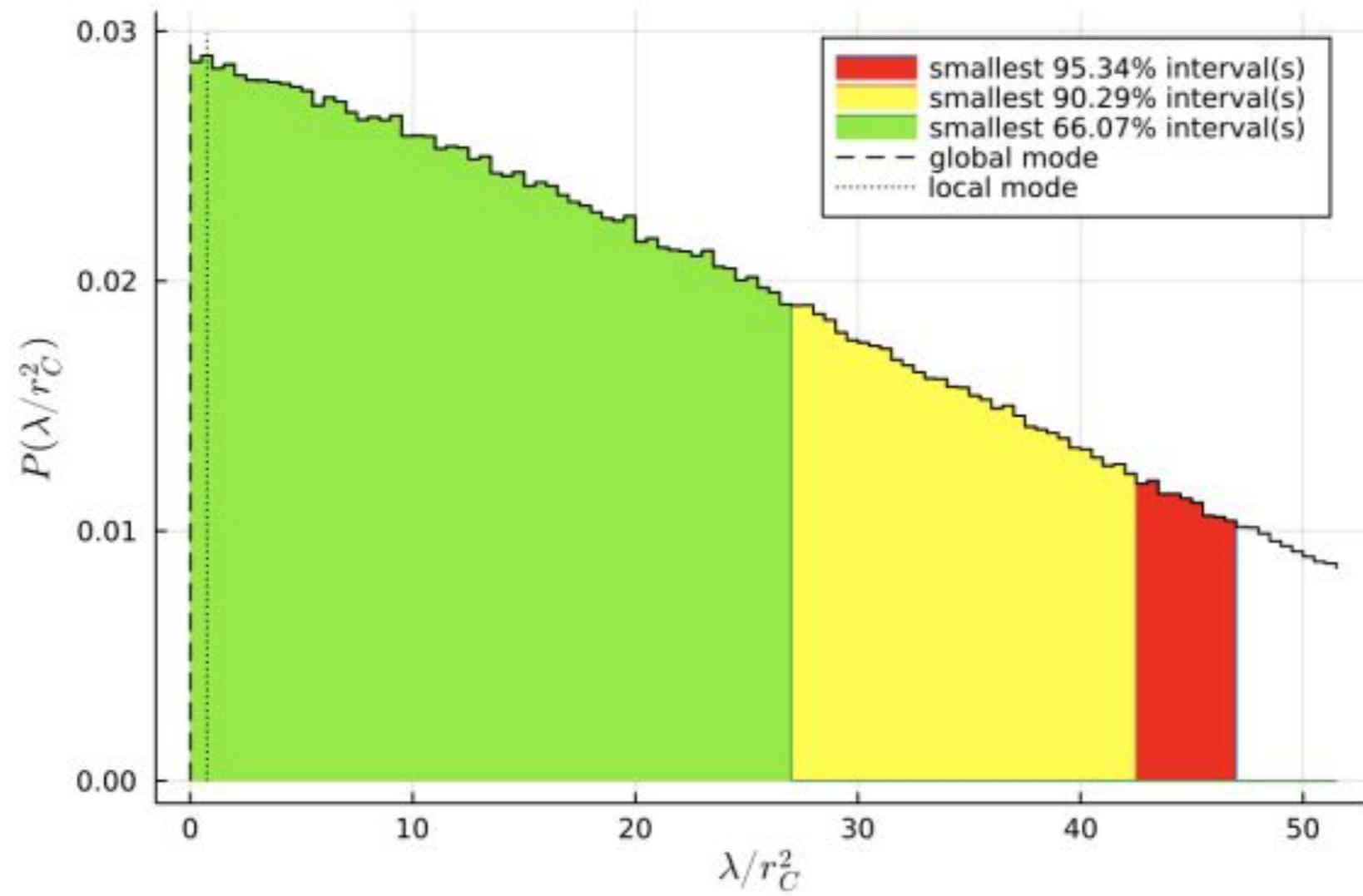
$$N_{neur}^{25ms} = \frac{(4 \times 10^{23})}{(.001)(10^9)} = 4 \times 10^{17}$$

Physics of Life Reviews Volume 42, September 2022, Pages 8-14

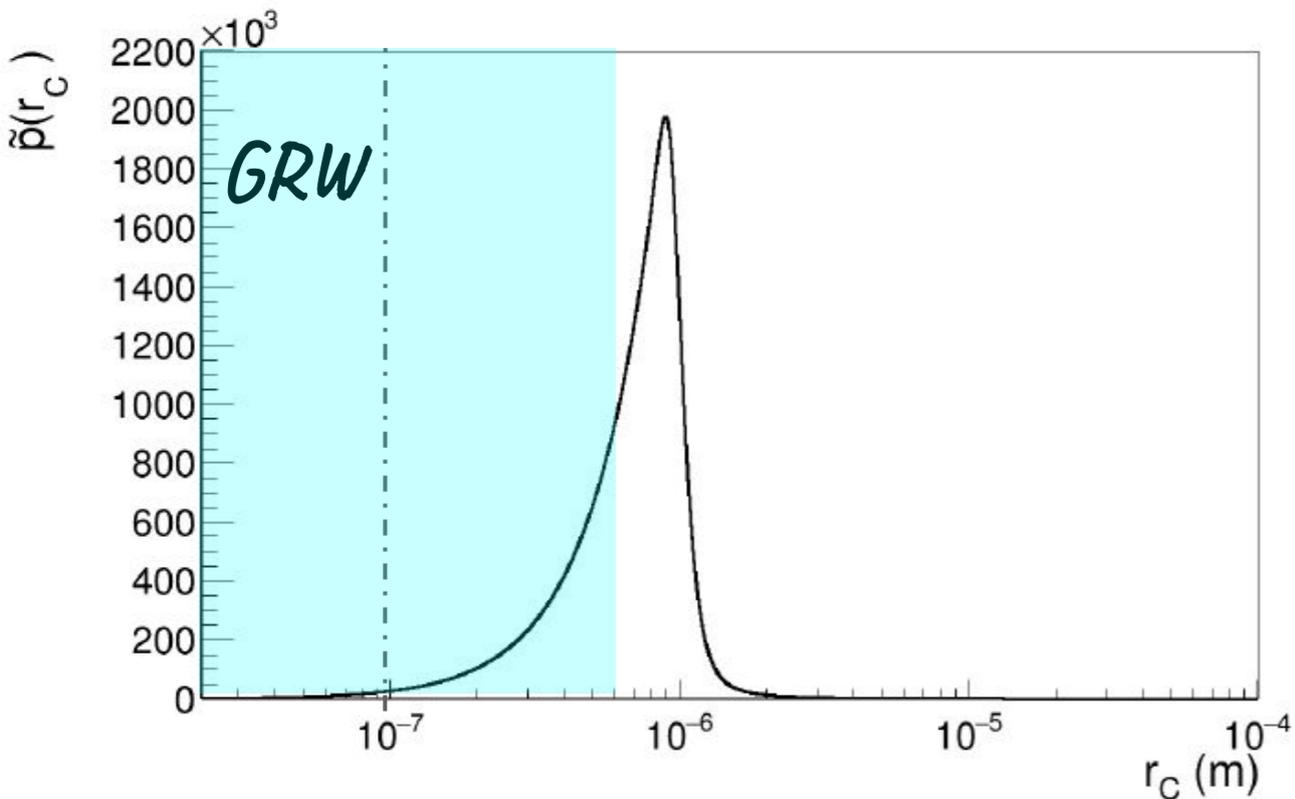
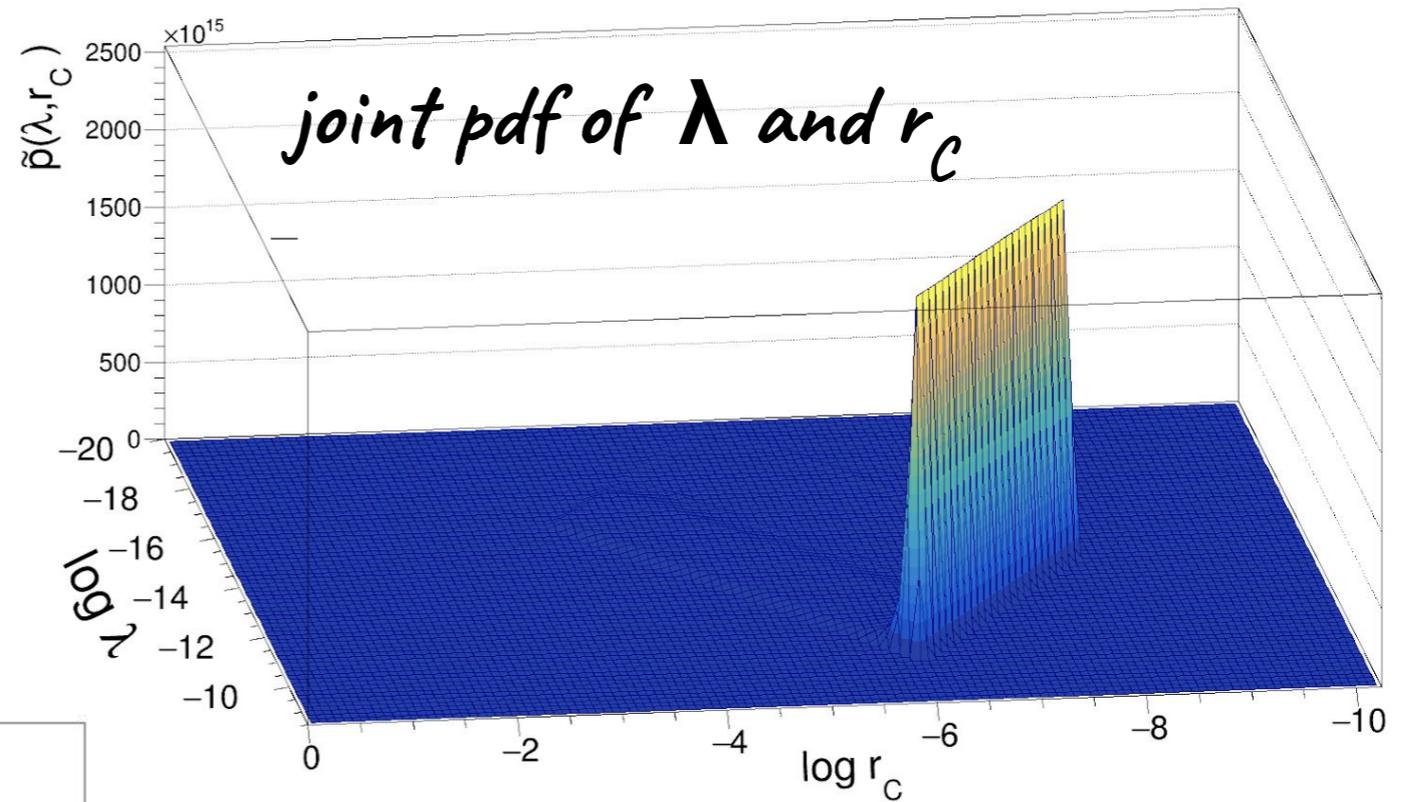
Now generalization of this result to non-Markovian/dissipative DP will be interesting!

Thank you

Spare slides



NEW Bounds on Λ and r_c parameters of the CSL model



$$\Lambda < 3.5 \cdot 10^{11} \text{ s}^{-1}$$

$r_c > 4.9 \cdot 10^7 \text{ m}$ which exceeds the value proposed by the GRW.

paper under finalization

Global time uncertainty and decoherence

Diosi, L. (2005), *Braz. J. Phys.* 35, 260, Diosi, L., and B. Lukacs (1987), *Annalen der Physik* 44, 488, Diosi, L. (1987), *Physics Letters A* 120, 377, A. Bassi et al., *Rev. Mod. Phys.* 85, 471

Initial state of a quantum system is a superposition of two eigenstates of total Hamiltonian

$$|\Psi\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle$$

time evolution

$$|\Psi(t)\rangle = c_1 \exp(-i\hbar^{-1}E_1t)|\varphi_1\rangle + c_2 \exp(i\hbar^{-1}E_2t)|\varphi_2\rangle$$

Let us add an uncertainty to the time

$$t \rightarrow t + \delta t$$

and assume that is distributed Gaussian, with zero mean, and dispersion which is proportional to the mean time, $\mathbf{M}[(\delta t)^2] = \tau t$ then the density matrix evolves as:

$$\begin{aligned} \rho(t) &\equiv \mathbf{M}[|\Psi(t)\rangle\langle\Psi(t)|] = \\ &= |c_1|^2|\varphi_1\rangle\langle\varphi_1| + |c_2|^2|\varphi_2\rangle\langle\varphi_2| + \\ &+ \{c_1^*c_2 \exp(i\hbar^{-1}\Delta Et)\mathbf{M}[\exp(i\hbar^{-1}\Delta E\delta t)]|\varphi_2\rangle\langle\varphi_1| + \\ &+ \text{h.c.} \} . \end{aligned}$$

Global time uncertainty and decoherence

Initial state of a quantum system is a superposition of two eigenstates of total Hamiltonian

$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle$$

time evolution

$$|\Psi(t)\rangle = c_1 \exp(-i\hbar^{-1}E_1t)|\Phi_1\rangle + c_2 \exp(i\hbar^{-1}E_2t)|\Phi_2\rangle$$

If we add an uncertainty to the time

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Let's assume that is distributed Gaussian, with zero mean, and dispersion which is proportional to the mean time, then the density matrix evolves as:

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~~$$\begin{aligned} \rho(t) &\equiv \mathbf{M}[|\Psi(t)\rangle\langle\Psi(t)|] = \\ &= |c_1|^2|\Phi_1\rangle\langle\Phi_1| + |c_2|^2|\Phi_2\rangle\langle\Phi_2| + \\ &+ \{c_1^*c_2 \exp(i\hbar^{-1}\Delta Et) \mathbf{M}[\exp(i\hbar^{-1}\Delta E\delta t)] |\Phi_2\rangle\langle\Phi_1| + \\ &+ \text{h.c.} \}. \end{aligned}$$~~

$$\mathbf{M}[\exp(i\hbar^{-1}\Delta E\delta t)] = e^{-t/t_D}$$

$$t_D = \frac{\hbar^2}{\tau} \frac{1}{(\Delta E)^2}$$

Global time uncertainty and decoherence

The time evolution for the density matrix

$$\hat{\rho}(t+\tau) = \exp\left[\frac{-i\hat{H}\tau}{\hbar}\right] \hat{\rho}(t) \exp\left[\frac{i\hat{H}\tau}{\hbar}\right]$$

Described by the von Neumann equation $\frac{d\rho}{dt} = -i\hbar^{-1}[H, \rho]$

turns to

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H, \rho] - \frac{1}{2}\tau\hbar^{-2}[H, [H, \rho]]$$

G. J. Milburn *Phys. Rev. A* 44 5401 (1991)

Local time uncertainty and decoherence

To generalize the concept for a local time $t_{\Gamma} \rightarrow t + \delta t_{\Gamma}$

one defines the correlation $M[\delta t_{\Gamma} \delta t_{\Gamma'}] = \tau_{\Gamma\Gamma'} t$

Galileo invariant spatial correlation
function

If the total Hamiltonian is decomposed in the sum of the local ones

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H, \rho] - \frac{1}{2}\hbar^{-2} \sum_{\Gamma, \Gamma'} \tau_{\Gamma\Gamma'} [H_{\Gamma}, [H_{\Gamma'}, \rho]]$$

The master equation suppresses superpositions of eigenstates of local energy

Reminder .. proper time interval

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In special relativity the Minkowski metric is

the coordinates of the arbitrary Lorentz frame are $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$

the infinitesimal time-like interval is $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

due to invariance of the interval, if we consider the coordinates of an instantaneous rest frame

$$ds^2 = c^2 d\tau^2 - dx_\tau^2 - dy_\tau^2 - dz_\tau^2 = c^2 d\tau^2$$

Reminder .. proper time interval

The proper time interval is then the integral on the world-line

$$\Delta\tau = \int_P d\tau = \int \frac{ds}{c} \longrightarrow \Delta\tau = \int_P \frac{1}{c} \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}$$

In general relativity the analogous expression for the generic metric tensor yields

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$$

and when constant coordinates are chosen

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c} \sqrt{g_{00}} dx^0$$

local time uncertainty and gravity

In the Newtonian limit $g_{00} = 1 + \frac{2\phi}{c^2}$

Here then comes the crucial point ... it is assumed that the gravitational potential should not be quantized

BUT that QM requires an absolute indeterminacy of the gravitational field.

I.E. the gravitational potential is a c-number stochastic variable, whose mean value is to be identified with the classical Newtonian potential.

Then local time fluctuation is related to a fluctuation of the local gravitational potential

$$\delta t_{\mathbf{r}} \equiv \delta \int_0^t dt' g_{00}^{1/2}(\mathbf{r}, t') \approx -c^{-2} \int_0^t dt' \Phi(\mathbf{r}, t')$$

.. so correlations of local uncertainties of Newtonian gravity can lead to correlation of local time uncertainties.

Can the gravitational field be measured with unlimited precision?

Diosi and Lukacs [Ann. Phys. 44, 488 (1987)] apply the arguments of [N. Bohr and L. Rosenfeld, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 12, 1 (1933)]:

The apparatus, obeying QM, is characterized by parameters m , R , T . In realistic measurements only a time-space averaged gravitational field is meaningful

$$\Delta\phi(\mathbf{r}, t) = -4\pi G\rho(\mathbf{r}, t) \quad \mathbf{g}(\mathbf{r}, t) = -\nabla\phi$$

$$\tilde{\mathbf{g}}(\mathbf{r}, t) = \frac{1}{VT} \int \mathbf{g}(\mathbf{r}', t') d^3r' dt \quad \text{with} \quad |\mathbf{r} - \mathbf{r}'| < R, \quad |t - t'| < T/2$$

The target is a point-like particle (of mass m) at rest at time $t=0$, immersed in the field \mathbf{g} . Detector measures momentum changes. In the time T the momentum gain is $p = m\tilde{\mathbf{g}}T$

$$\delta p = \hbar/R \quad \longrightarrow \quad \sigma(\tilde{\mathbf{g}}) \sim \frac{\hbar}{mRT}$$

Can the gravitational field be measured with unlimited precision?

It's useless to increase R and T , since this would decrease the error on average field, not on the instantaneous local field of the Newtonian theory. m can be increased, till its own field does not perturb g , i.e. till:

$$\sigma(\tilde{g}) \sim \frac{\hbar}{m R T} \qquad \delta\tilde{g}_m \sim \frac{G m}{R^2}$$

Given the optimal mass choice then: $m_{\text{opt}} \sim \left(\frac{\hbar R}{G T}\right)^{1/2}$ $\sigma(\tilde{g}) \sim \sim \left(\frac{\hbar G}{V T}\right)^{1/2}$

$$g(\mathbf{r}, t) = g_N(\mathbf{r}, t) + g_S(\mathbf{r}, t)$$

If the limitation is universal then the actual gravitational field is:

solution of Poisson Eq.

stochastic fluctuation

Uncorrelated gravitational field fluctuations

It's useless to increase R and T , since this would decrease the error on average field, not on the instantaneous local field of the Newtonian theory. m can be increased, till its own field does not perturb g , i.e. till:

$$\sigma(\tilde{g}) \sim \frac{\hbar}{m R T} \quad \delta \tilde{g}_m \sim \frac{G m}{R^2}$$

Given the optimal mass choice then: $m_{\text{opt}} \sim \left(\frac{\hbar R}{G T}\right)^{1/2}$ $\sigma(\tilde{g}) \sim \sim \left(\frac{\hbar G}{V T}\right)^{1/2}$

If the limitation is universal then the actual gravitational field is: $g(\mathbf{r}, t) = g_N(\mathbf{r}, t) + g_S(\mathbf{r}, t)$

$$\langle \tilde{g}_S \rangle = 0 \quad ; \quad \langle \tilde{g}_S^2 \rangle = \frac{\hbar G}{V T}$$

The squared dispersion of the averaged g_S is inversely proportional to the space-time cell volume ->
hence g_S is uncorrelated in time and space

$$\langle g_S(\mathbf{r}, t) g_S(\mathbf{r}', t') \rangle = \hbar G \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

Gravitational potential as a stochastic variable

In terms of the potential, this can be regarded as a stochastic variable, with moments:

$$\langle \phi(\mathbf{r}, t) \rangle = \phi_N(\mathbf{r}, t)$$

$$\langle \phi(\mathbf{r}, t) \phi(\mathbf{r}', t') \rangle - \langle \phi(\mathbf{r}, t) \rangle \langle \phi(\mathbf{r}', t') \rangle \sim \frac{\hbar G}{|\mathbf{r} - \mathbf{r}'|} \delta(t - t')$$

The covariance function for the gravitational potential is not dependent on the parameters of the gedanken apparatus (m, T, R), which may suggest universality of the potential intrinsic fluctuation.

Going back to the searched correlation of the local time fluctuation $\mathbf{M}[\delta t_{\mathbf{r}} \delta t_{\mathbf{r}'}] = \tau_{\mathbf{r}\mathbf{r}'t}$

$$\delta t_{\mathbf{r}} \equiv \delta \int_0^t dt' g_{00}^{1/2}(\mathbf{r}, t') \approx -c^{-2} \int_0^t dt' \Phi(\mathbf{r}, t') \longrightarrow \tau_{\mathbf{r}\mathbf{r}'} = \text{const} \times \frac{G\hbar}{|\mathbf{r} - \mathbf{r}'|} c^{-4}$$

Master equation

$$\tau_{\mathbf{r}\mathbf{r}'} = \text{const} \times \frac{G\hbar}{|\mathbf{r} - \mathbf{r}'|} c^{-4}$$

the local time correlation
is extremely small

substituted in the master equation

$$\frac{d\rho}{dt} = -i\hbar^{-1}[H, \rho] - \frac{1}{2}\hbar^{-2} \sum_{\mathbf{r}, \mathbf{r}'} \tau_{\mathbf{r}\mathbf{r}'} [H_{\mathbf{r}}, [H_{\mathbf{r}'}, \rho]]$$

yields

$$\begin{aligned} \frac{d\rho}{dt} = & - i\hbar^{-1}[H, \rho] \\ & - \frac{G}{2}\hbar^{-1} \int \int \frac{d\mathbf{r}d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} [f(\mathbf{r}), [f(\mathbf{r}'), \rho]] \end{aligned}$$

Master equation

Denote the configuration coordinates (classical and spin) of the dynamical system by X . The corresponding mass density at the point r is $f(\mathbf{r}|X)$

Given the coordinate eigenstate $|X\rangle$ we have $f(\mathbf{r}|X)\delta(X' - X) \equiv \langle X' | \hat{f}(\mathbf{r}) | X \rangle$

So if one introduces the damping time:

$$[\tau_d(X, X')]^{-1} = \frac{G}{2\hbar} \int \int d^3r d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r} - \mathbf{r}'|}$$

the master equation becomes

$$\begin{aligned} \langle X | \dot{\hat{\rho}}(t) | X' \rangle &= (-i/\hbar) \langle X | [\hat{H}_0, \hat{\rho}(t)] | X' \rangle \\ &\quad - [\tau_d(X, X')]^{-1} \langle X | \hat{\rho}(t) | X' \rangle . \end{aligned}$$

Energy decoherence

$$\langle X | \dot{\hat{\rho}}(t) | X' \rangle = (-i/\hbar) \langle X | [\hat{H}_0, \hat{\rho}(t)] | X' \rangle$$

$$- [\tau_d(X, X')]^{-1} \langle X | \hat{\rho}(t) | X' \rangle .$$

$$[\tau_d(X, X')]^{-1} = \frac{G}{2\hbar} \int \int d^3r d^3r' \times \frac{[f(\mathbf{r}|X) - f(\mathbf{r}|X')][f(\mathbf{r}'|X) - f(\mathbf{r}'|X')]}{|\mathbf{r} - \mathbf{r}'|}$$

If the difference between the mass distributions of two states $|X\rangle$ and $|X'\rangle$ in superposition becomes big

the corresponding damping time becomes short

the corresponding off-diagonal terms of the density operator vanish

*this QM violating phenomenon is **ENERGY DECOHERENCE***

in Diosi approach.

Other theories of space-time uncertainty induced decoherence ..

an incomplete list

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Other theories of space-time uncertainty induced decoherence

- Milburn assumes that Planck-time is the smallest time,
- Adler derives quantum theory in the special limit of a hypothetical fundamental dynamics, they share the same master Eq.
- Penrose focuses on the conceptual uncertainty of location in space-time, Penrose and Diosi model share the same "decay time"

The theories have different mathematical apparatuses, interpretations, metaphysics, e.t.c., but have common divisors. "The fact that they are similar but not identical suggests that the involvement of gravity in

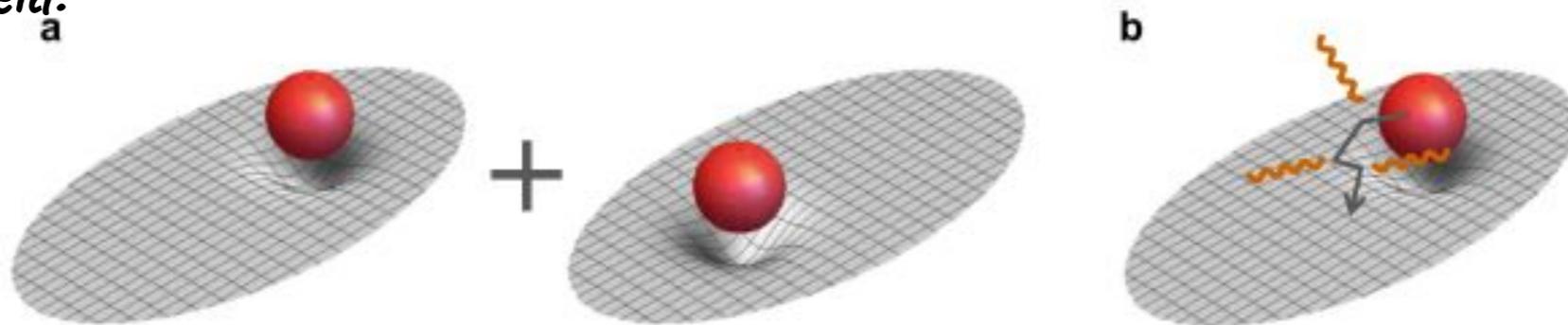
wave-vector reduction is strongly indicated, but the exact mathematical treatment remains to be found." A. Bassi (referred to Gravity-related collapse)

The model of Penrose

Consider a quantum system which consists of a linear superposition of two well-defined stationary states having the same energy E $|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$

If gravitation is ignored, as is done in standard quantum theory, the superposition is also stationary, with the same energy E $i\hbar\frac{\partial|\psi\rangle}{\partial t} = E|\psi\rangle$

BUT when gravitation is introduced in the play, there will be a nearly classical spacetime associated with the state $|\alpha\rangle$ and a Killing vector associated with it which represents the time displacement of stationarity, and the same for $|\beta\rangle$. The two Killing vectors can be identified with each other only if the two space-times can be identified point by point. BUT general covariance forbids that, since the matter distributions associated with the two states are different, in the presence of a background gravitational field.



The model of Penrose

On the other hand, unitary evolution in quantum theory requires and assumes the existence of a Schrödinger operator which applies to the superposition in the same way that it applies to the individual states.

Its action on the superposition is the superposition of its action on individual states.



Conflict between the demands of QM and of General Relativity.

Imagine to make an approximate point-wise identification between the two spacetimes \rightarrow slight error in the identification of the Schrödinger operators for the two space-times \rightarrow slight uncertainty in the energy of the superposition. In the Newtonian approximation of the order of the gravitational self-energy of the mass distribution \hbar/E_G are two superposed states.

Lifetime:

(the same as for Diosi model)

beyond which time the superposition will decay.

hypotheses:

- wave function collapse takes place in an average time τ_{DP} given by Planck's reduced constant divided by ΔE_{DP}

For a superposition for which each mass distribution is a rigid translation of the other, *the gravitational self-energy difference* is the energy it would cost to displace one component of the superposition in the gravitational field of the other, in moving it from coincidence to the quantum-displaced location.

- the quantum superposition has to be 1) Orchestrated (capable of integration and computation) 2) isolated from non-Orchestrated environmental decoherence:

That is to say there would be needed to be coherent superpositions of sufficient amounts of microtubule material such that ΔE_{DP} , undisturbed by environmental decoherence, results in a collapse on a timescale of the general order for a conscious experience $\tau = 0.5s - 10^{-2}s$, such as particular frequencies of EEG, visual gestalts, and reported conscious moments

- If the system is entangled with the environment reduction is random
- If we require that consciousness is triggered by a non-random (non-computable) phenomenon, than entanglement with the environment inducing collapse before the DP OR is effective is to be avoided within τ

HP in their paper Phys. of life reviews (2014) review several studies reporting how Quantum-coherent behavior is relevant, in biological systems, at surprisingly warm temperatures in wet and noisy environment. We didn't deepen the item of environmental decoherence.

In quantum computers information is represented not just as bits of either 1 or 0, but during the deterministic process also as quantum superposition of both 1 *and* 0 together (qubits). Moreover large-scale entanglements among many qubits enable complex parallel processing. At some point a quantum state reduction occurs -> the *output* is a definite state classical bit ->

In a pretty same fashion non-computable DP reduction would induce consciousness

And according to decennial studies of Hameroff the perfect actors of the coherent superposition would be

microtubules within neurons, suitable candidate sites for quantum processing.

- a moment of conscious experience emerges from (or is identical to) a collapse event that destroys coherence in a previously deterministically evolving coherent quantum state of tubulins in neurons.
- coherent quantum processes correlate with, and regulate, neuronal synaptic and membrane activity
- So ΔE_{DP} *is to be calculated* from the difference between the mass distributions between two states of tubulin in coherent superposition
- but the use of an average density is not adequate since the mass is concentrated in the nuclei
- So they calculate ΔE_{DP} for tubulin separated from itself at three possible levels of separation: (a) the entire smoothed-out protein (what they call “partial separation”), (b) its atomic nuclei, and (c) its nucleons (protons and neutrons). They say that the dominant effect is likely to be (b), i.e., separation at the level of atomic nuclei, or 2.5 Fermi for carbon nuclei
ORDER OF ONE MILLIONTH OF ONE BILLIONTH OF m

WHY CARBON NUCLEI:

- carbon is a substantial component of the chemical composition of tubulin.
- certain physical mechanisms in tubulin may be able to dynamically prepare Carbon nuclei into coherent spatial superpositions on the order of a Fermi

separation at the level of atomic nuclei (2.5 Fermi length for carbon nuclei) is the same as that predicted to be caused by electron charge separations of one nanometer, e.g. London force dipoles within aromatic amino acid rings

$$\tau \approx \hbar/E_G \quad \text{choose } \tau \text{ as 25 ms for '40 Hz' gamma synchrony conscious moments}$$

Since the carbon nucleus displacement is greater than its radius, the gravitational self-energy for superposition separation of one carbon atom is

$$E_c = Gm^2/a_c$$

With m_c the carbon mass and $a_c = 2.5$ fm

To obtain the required number of tubulins in superposition we then have to divide by the number of carbon atoms in one tubulin (10^4) and by the number of searched tubulins in coherent superposition.

- HP find 2×10^{10} tubulins, for bigger values of tau we would we would find a smaller N_{tub}

Neurons contain $\sim 10^9$ tubulins, but only a fraction per neuron are likely to be involved in consciousness (e.g., a fraction of those in dendrites and soma). If 0.1% of tubulins within a given set of neurons were coherent for 25 ms, they compute that 20,000 such neurons would be required to elicit OR.

Tibetan monks have found to have 80 Hz gamma synchrony, than E_g requires twice as much brain involved for such intense conscious experience! FASCINATING

Assuming that microtubule quantum states occur in a specific brain neuron, how could it involve microtubules in other neurons throughout the brain?

OrchOR proposes that quantum states can extend by entanglement between adjacent neurons through gap junctions

- given the currently available (simplest) dynamics in DP theory, and the available experimental constraints on it, we have the occasion to examine and constrain a variant of Orch OR in which the collapse time for coherent superpositions of microtubule material (ignoring environmental decoherence effects) is determined by the DP equations and parameters. In the present formulation this is one parameter R_0

Now the crucial point is that the three levels of (spatial) *separation* contemplated by HP, correspond to the levels of (spatial) *resolution represented by R_0* , and the collapse time depends on R_0 . That is to say:

partial separation level (a), atomic nuclei separation (b), and nucleon separation (c) correspond respectively to internuclear (or larger), nuclear, and subnuclear levels of R_0 . In particular the results of HP for option (b), summarized before, require mass density resolution as fine as $R_0 \approx 2.5\text{Fermi}$

But we put a limit on the lower possible value of R_0

$$R_0 > 5.4 \times 10^{-11} \text{ m}$$

that is of the order of 10000 times bigger than the carbon nuclear radius!

The larger R_0 the longer the collapse time

For a superposition state of size a_c , due to $R_0 \gg a_c$, the contribution of mass m_c of a carbon nucleus to ΔE_{DP} is concentrated no longer in spheres of radius a_c but in spheres of radius $\sim R_0$. Since the separation $|X-X'| = a_c$ is kept small, the potential $U(X-X')$ starts quadratically to grow with $X-X'$. So the collapse rate becomes very small:

$$\lambda_c^{a_c \ll R_0} = \frac{Gm_n^2}{\hbar R_0} \left(\frac{a_c}{R_0} \right)^2 \approx 10^{-26} \text{ s}^{-1}$$

Which means that the collapse time for one tubulin is huge:

$$\tau_{tub}^{a_c \ll R_0} := \frac{1}{N_{c/tub}} \frac{1}{\lambda_c^{a_c \ll R_0}} \approx (10^{-4}) (10^{26} \text{ s}) = 10^{22} \text{ s}$$

i.e. the number of tubulins required to be in coherent superposition for a collapse time of 25ms is:

$$N_{tub}^{25ms} = \frac{\tau_{tub}^{a_c \ll R_0}}{.025s} = \frac{10^{22} \text{ s}}{.025s} = 4 \times 10^{23}$$

Now recall that there are $\sim 10^9$ tubulins/neuron and $\sim 10^{11}$ neurons/brain, if 0.1% of tubulins per neuron are involved in consciousness we would need

$$N_{neur}^{25ms} = \frac{(4 \times 10^{23})}{(.001)(10^9)} = 4 \times 10^{17}$$

Even if we assume that all tubulins are involved in coherent superposition, we would need 10^{14} neurons !

These considerations seem to rule out tubulin separation at the level of the atomic nuclei (and it certainly also rules out separation at the level of the nucleons in which case the collapse time would be even larger).

Finally having in mind our limit $|X-X'| = R_0 = 5.4 \cdot 10^{-11}$ m, we approximated the entire smoothed-out protein as a homogeneous bulk of size L and we examined the two cases of the entire smoothed-out protein (partial separation):

L for the smallest tubulin structure is 3×10^{-9} m (actin filament)

- $L \gg |X-X'|$ - roughly 10% of the neurons comprising the brain would have to be involved (for collapse time 25ms)
- $L \sim |X-X'|$ - requires $4 \cdot 10^6$ neurons (for collapse time 25ms) or about
 10^5 neurons (for collapse time 500ms).

despite second case vastly exceeds any of the coherent superposition states achieved with state-of-the-art optomechanics or macromolecular interference experiments, biological matter might find some different way for long term superpositions to develop (Hameroff S., Penrose R. Consciousness in the universe - a review of the 'orch or' theory. Phys Life Rev 2014;11:39–78.)

Did we rule out Orch Or in general? NO!

We analyzed the predictions of a variant of Orch OR in the light of the simplest (currently the only) dynamical DP theory of gravity-related collapse.

If a spontaneous radiation free gravity-related collapse will be developed by Penrose, Diosi or Others, such a theory would represent a significant breakthrough in our understanding of Nature, and would make the tubulin superposition scenarios considered by Hameroff and Penrose, and by our analysis, far more plausible.

Not only! Even the current DP dynamics is being improved in order to include dissipation and non-Markovianity, we are presently analyzing such variants, and re-examining the Orch Or in this light.

CSL (Continuous Spontaneous Localization)

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H dt + \sqrt{\lambda} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\lambda}{2} \int d^3x (N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle_t)^2 dt \right] |\psi_t\rangle$$

System's Hamiltonian

NEW COLLAPSE TERMS



New Physics

$$N(\mathbf{x}) = a^\dagger(\mathbf{x})a(\mathbf{x}) \quad \text{particle density operator}$$

choice of the preferred basis

$$\langle N(\mathbf{x}) \rangle_t = \langle \psi_t | N(\mathbf{x}) | \psi_t \rangle$$

nonlinearity

$$W_t(\mathbf{x}) = \text{noise} \quad \mathbb{E}[W_t(\mathbf{x})] = 0, \quad \mathbb{E}[W_t(\mathbf{x})W_s(\mathbf{y})] = \delta(t-s)e^{-(\alpha/4)(\mathbf{x}-\mathbf{y})^2}$$

stochasticity

$$\lambda = \text{collapse strength} \quad r_C = 1/\sqrt{\alpha} = \text{correlation length}$$

two parameters

the only possible modification of the Schrödinger equation, compatible with the non-faster-than-light signaling condition!