# Radiative Correction for PVDIS at JLab, from 6 to 12 GeV using SoLID

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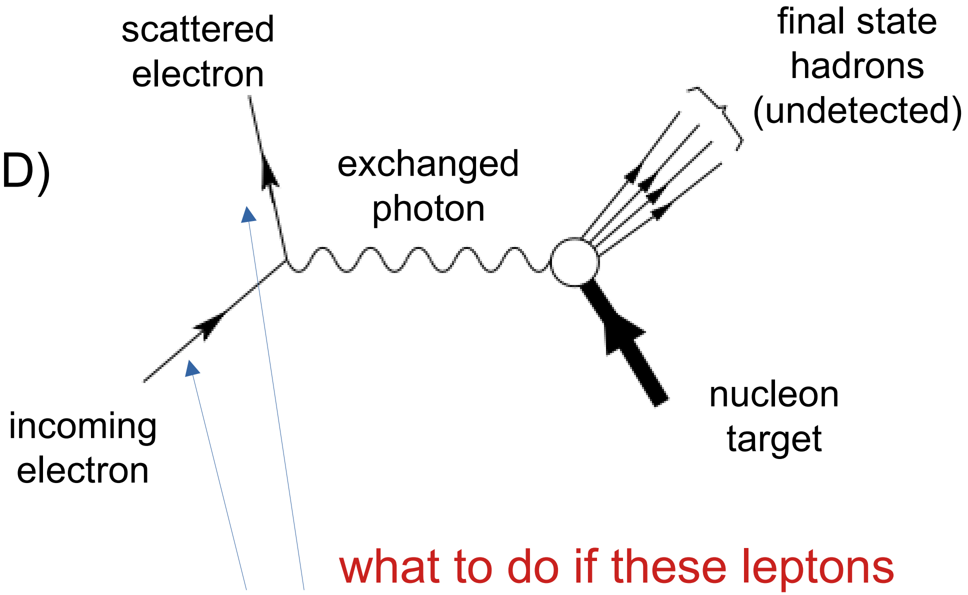
ECT* Workshop “Radiative Corrections from Medium to High Energy Experiments”

# Radiative Corrections

- General approach at JLab
  - apply corrections to the measured cross section
  - full simulation method
- What was done for PVDIS 6 GeV
- What we plan to do for PVDIS 11 GeV (SoLID)

- Note: we do not typically deal with
  - box diagrams
  - weak effects
  - QED effects (quark line)



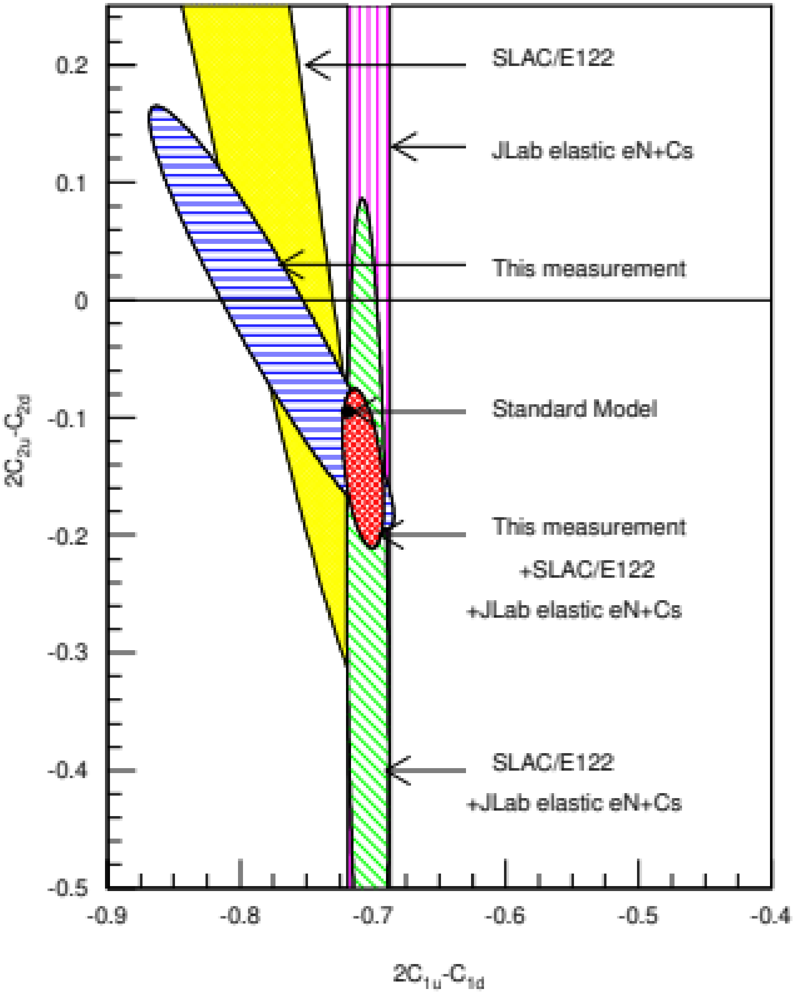
what to do if these leptons
→ ionization loss
→ external bremsstrahlung
→ multiple scattering
→ internal bremsstrahlung?

JLab 6 GeV PVDIS long paper:

https://doi.org/10.1103/PhysRevC.91.045506

e-Print: 1411.3200 [nucl-ex]

# 6 GeV PVDIS at JLab



# PVDIS at JLab 6 GeV

- Experiment (E05-007 → ) E08-011 was done in Hall A of JLab from Oct-Dec. 2009
- Inclusive electrons detected by the two Hall A High Resolution Spectrometers (HRS)
- Measured PVDIS asymmetry at two $Q^2$ settings to (4-5)% relative
- Similar to SLAC E122, counting DIS electrons (not integrating as other modern PVES)

$$A_{RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

$$A_d = |P_b| (108\,ppm)\, Q^2 \left[ (2C_{1u} - C_{1d}) + Y(y)(2C_{2u} - C_{2d}) R_V(x) \right]$$

beam polarization

$$Y(y) = \frac{1 - (1-y)^2}{1 + (1-y)^2}$$

$$R_V(x) = \frac{u_V(x) + d_V(x)}{u(x) + \bar{u}(x) + d(x) + \bar{d}(x)}$$

(indicates spin flip of quarks)

$$E_{\mathrm{beam}} = 6.067\,\mathrm{GeV}$$

DIS Kine #1:

$$\theta = 12.9^o, E' = 3.66\,\mathrm{GeV}$$

$$\langle x \rangle_{\mathrm{data}} = 0.241, \langle Q^2 \rangle_{\mathrm{data}} = 1.085\,\mathrm{GeV}^2$$

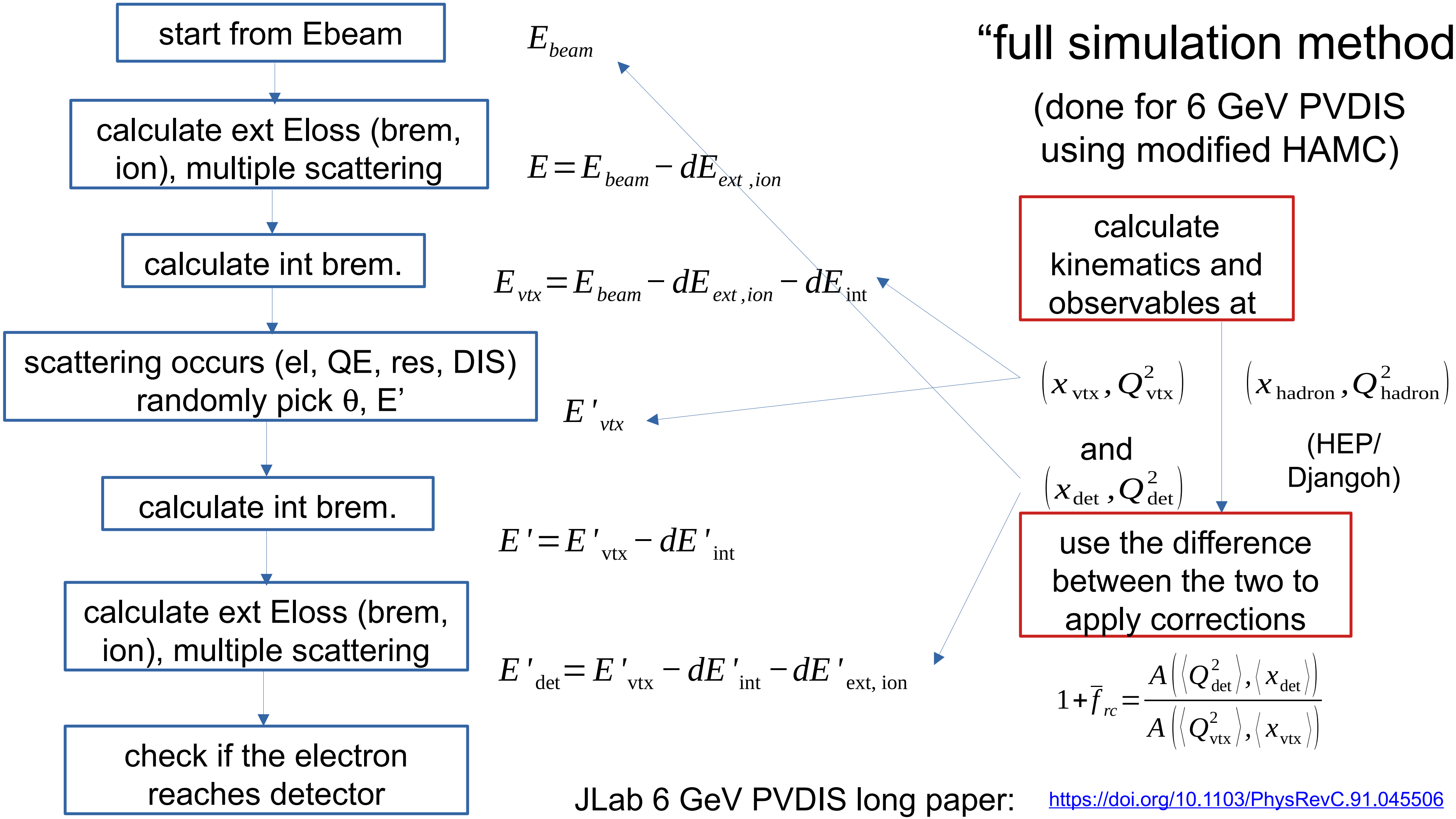
DIS Kine #2:

$$\theta = 20^o, E' = 2.63\,\mathrm{GeV}$$

$$\langle x \rangle_{\mathrm{data}} = 0.295, \langle Q^2 \rangle_{\mathrm{data}} = 1.901\,\mathrm{GeV}^2$$

# “full simulation method”

(done for 6 GeV PVDIS using modified HAMC)



start from Ebeam

calculate ext Eloss (brem, ion), multiple scattering

calculate int brem.

scattering occurs (el, QE, res, DIS) randomly pick θ, E’

calculate int brem.

calculate ext Eloss (brem, ion), multiple scattering

check if the electron reaches detector

$$E_{beam}$$

$$E = E_{beam} - dE_{ext,ion}$$

$$E_{vtx} = E_{beam} - dE_{ext,ion} - dE_{\text{int}}$$

$$E'_{vtx}$$

$$E' = E'_{\text{vtx}} - dE'_{\text{int}}$$

$$E'_{\text{det}} = E'_{\text{vtx}} - dE'_{\text{int}} - dE'_{\text{ext, ion}}$$

calculate kinematics and observables at

$$\left(x_{\text{vtx}}, Q^2_{\text{vtx}}\right)$$

and

$$\left(x_{\text{det}}, Q^2_{\text{det}}\right)$$

$$\left(x_{\text{hadron}}, Q^2_{\text{hadron}}\right)$$

(HEP/ Djangoh)

use the difference between the two to apply corrections

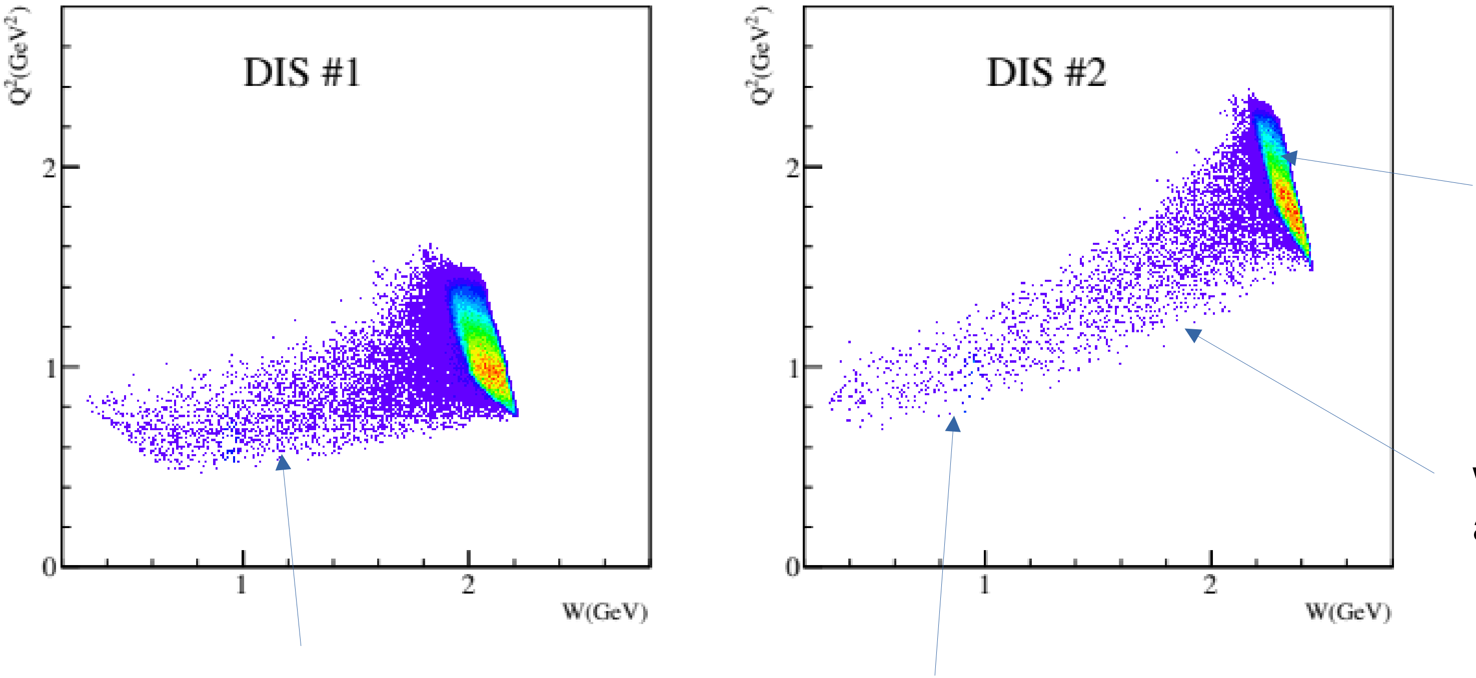
$$1 + \bar{f}_{rc} = \frac{A\left(\left\langle Q^2_{\text{det}}\right\rangle, \left\langle x_{\text{det}}\right\rangle\right)}{A\left(\left\langle Q^2_{\text{vtx}}\right\rangle, \left\langle x_{\text{vtx}}\right\rangle\right)}$$

JLab 6 GeV PVDIS long paper: https://doi.org/10.1103/PhysRevC.91.045506

# Radiative Correction for 6 GeV PVDIS

Q2_vertex vs. W_vertex for 6 GeV that includes both internal and external radiations

internal use Mo&Tsai's effective radiator formula: $t_{\text{equiv}} = \frac{3\alpha}{4\pi}\left[\ln\left(\frac{Q^2}{m^2}\right)-1\right]$ (see HAMC manual)

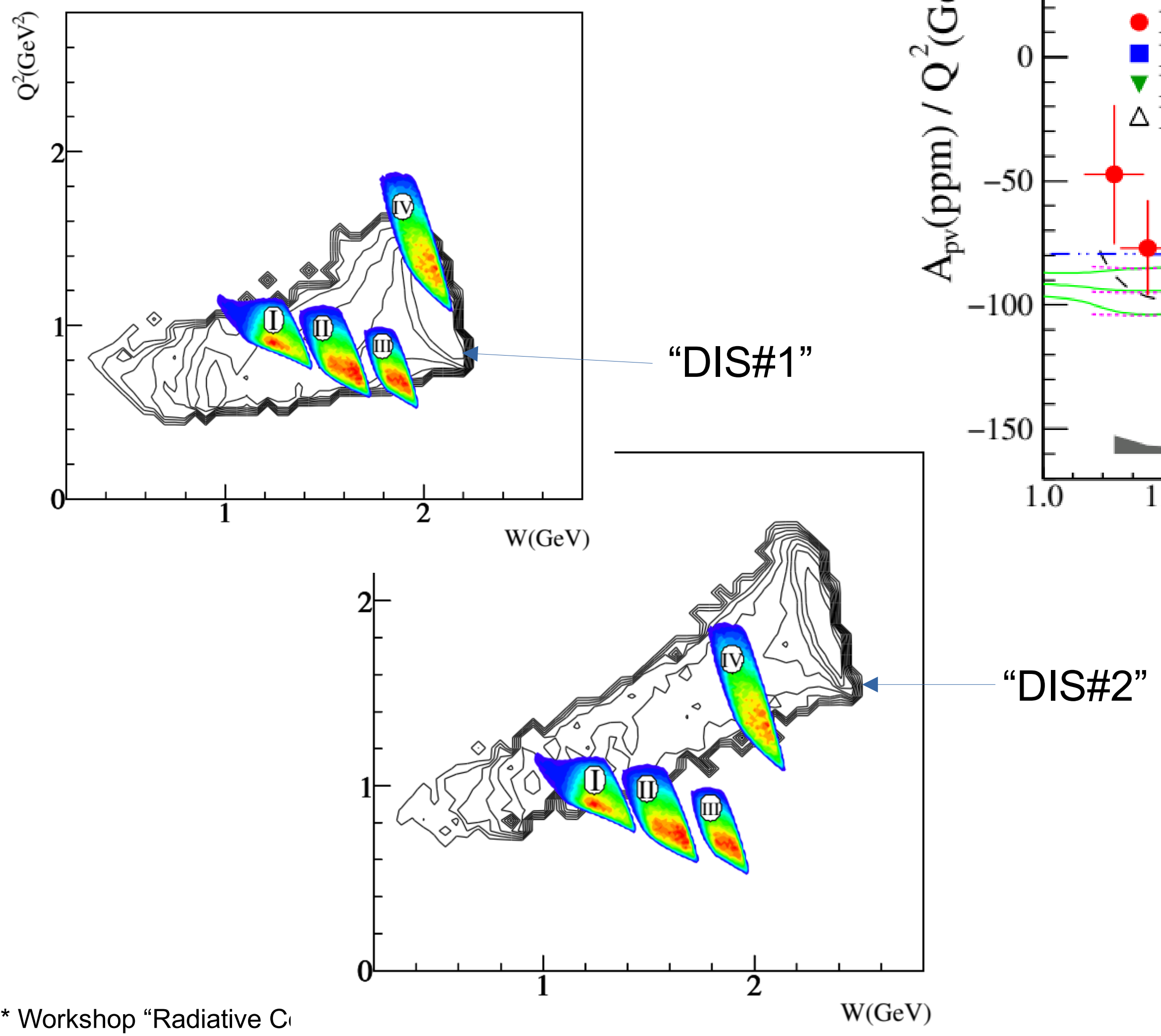


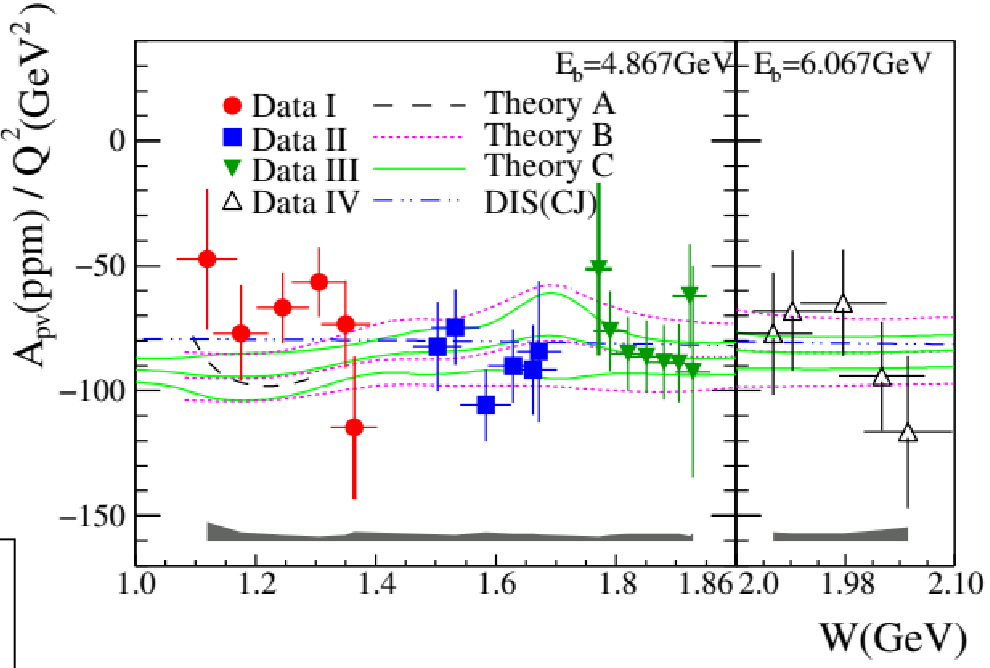
spectrometer sits here (and where we think DIS events occurs)

where scattering actually occurs

resonance Apv: used model, checked with data (next slide)

Resonance data taken during 6 GeV

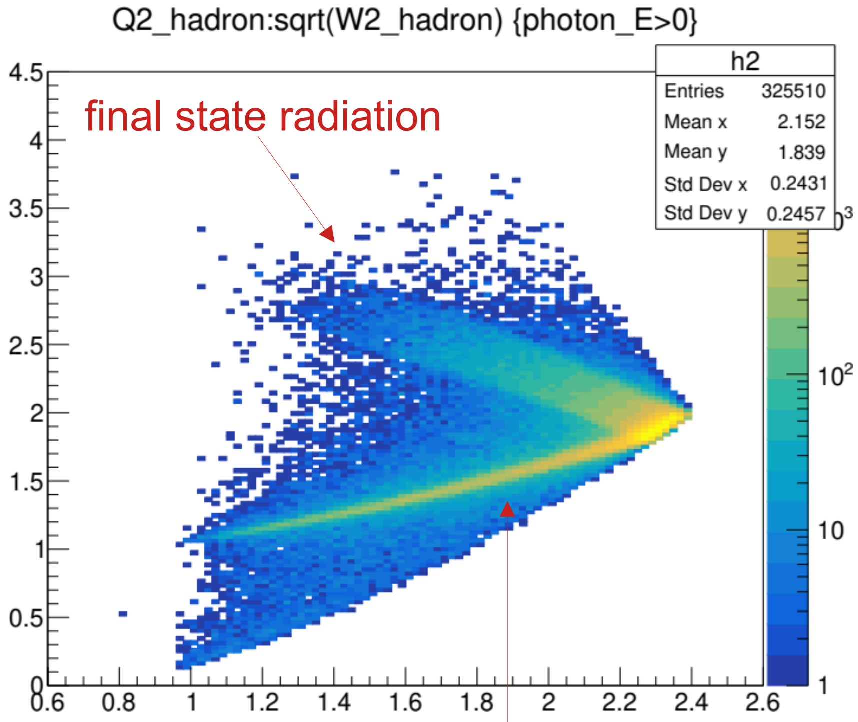




Elastic and QE calculated separately;

Uncertainty in resonance Apv as input to radiative corrections:
- W<1.4 GeV: 25%
- 1.4<W<1.7 GeV: 10%
- 1.7<W<2.0 GeV: 7.7%

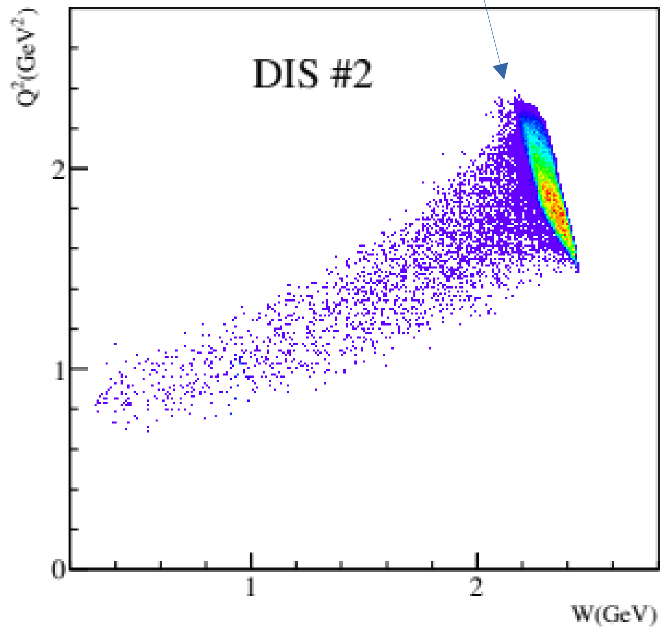
Q2_vertex vs. W_vertex for 6 GeV that includes only internal radiations (Djangoh simulation)



final state radiation

initial state radiation

simulation from 6 GeV (both int and ext radiation), barely any seen for final state radiation (note that this is a linear z plot), or could it be that initial state radiation dominates for fixed-target experiments (due to extended target material)?



# 6 GeV PVDIS long paper:

$$A^{\text{rad-corrected}} = A^{\text{meas}}\left(1+\bar{f}_{rc}\right)$$

$$1+\bar{f}_{rc} = \frac{A\left(\left\langle Q^2_{\text{det}}\right\rangle, \left\langle x_{\text{det}}\right\rangle\right)}{A\left(\left\langle Q^2_{\text{vtx}}\right\rangle, \left\langle x_{\text{vtx}}\right\rangle\right)}$$

## DIS Kine #1:

$$E_{\text{beam}} = 6.067\,\text{GeV}$$

$$\theta = 12.9^o, E' = 3.66\,\text{GeV}$$

$$\langle x\rangle_{\text{data}} = 0.241, \langle Q^2\rangle_{\text{data}} = 1.085\,\text{GeV}^2$$

$$1+f_{\text{rc}} = 1.015 \pm 0.02$$

$$f_{\gamma\gamma} = -0.002 \pm 0.002$$

$$A_{\text{phys}} = -91.10 \pm 4.30\,\text{ppm}\left(4.7\,\%\right)$$

## DIS Kine #2:

$$E_{\text{beam}} = 6.067\,\text{GeV}$$

$$\theta = 20^o, E' = 2.63\,\text{GeV}$$

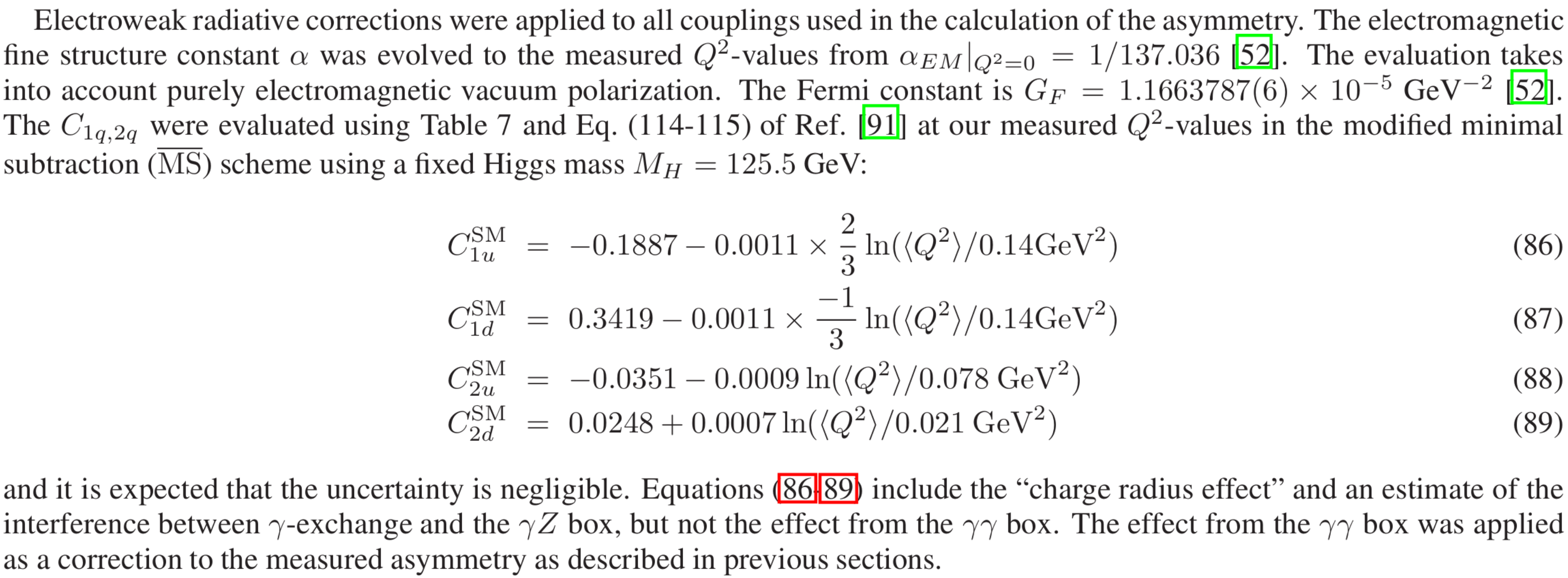
$$\langle x\rangle_{\text{data}} = 0.295, \langle Q^2\rangle_{\text{data}} = 1.901\,\text{GeV}^2$$

$$1+f_{\text{rc}} = 1.019 \pm 0.004$$

$$f_{\gamma\gamma} = -0.003 \pm 0.003$$

$$A_{\text{phys}} = -160.80 \pm 7.12\,\text{ppm}\left(4.4\,\%\right)$$

# γ–Z box



Electroweak radiative corrections were applied to all couplings used in the calculation of the asymmetry. The electromagnetic fine structure constant $\alpha$ was evolved to the measured $Q^2$-values from $\alpha_{EM}|_{Q^2=0} = 1/137.036$ [52]. The evaluation takes into account purely electromagnetic vacuum polarization. The Fermi constant is $G_F = 1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ [52]. The $C_{1q,2q}$ were evaluated using Table 7 and Eq. (114-115) of Ref. [91] at our measured $Q^2$-values in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme using a fixed Higgs mass $M_H = 125.5$ GeV:

$$C_{1u}^{\mathrm{SM}} = -0.1887 - 0.0011 \times \frac{2}{3} \ln(\langle Q^2 \rangle / 0.14\mathrm{GeV}^2) \tag{86}$$

$$C_{1d}^{\mathrm{SM}} = 0.3419 - 0.0011 \times \frac{-1}{3} \ln(\langle Q^2 \rangle / 0.14\mathrm{GeV}^2) \tag{87}$$

$$C_{2u}^{\mathrm{SM}} = -0.0351 - 0.0009 \ln(\langle Q^2 \rangle / 0.078\ \mathrm{GeV}^2) \tag{88}$$

$$C_{2d}^{\mathrm{SM}} = 0.0248 + 0.0007 \ln(\langle Q^2 \rangle / 0.021\ \mathrm{GeV}^2) \tag{89}$$

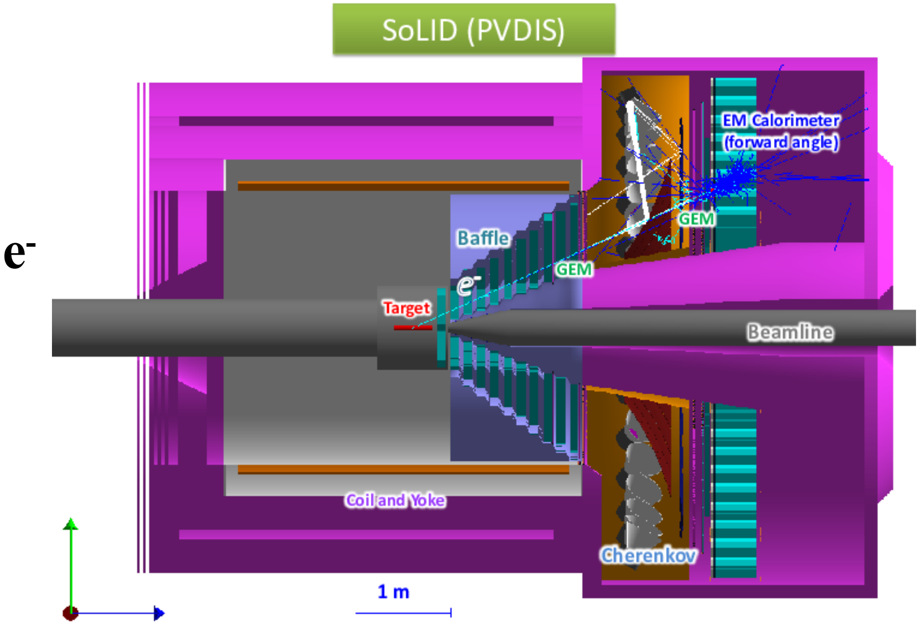
and it is expected that the uncertainty is negligible. Equations (86-89) include the "charge radius effect" and an estimate of the interference between $\gamma$-exchange and the $\gamma Z$ box, but not the effect from the $\gamma\gamma$ box. The effect from the $\gamma\gamma$ box was applied as a correction to the measured asymmetry as described in previous sections.

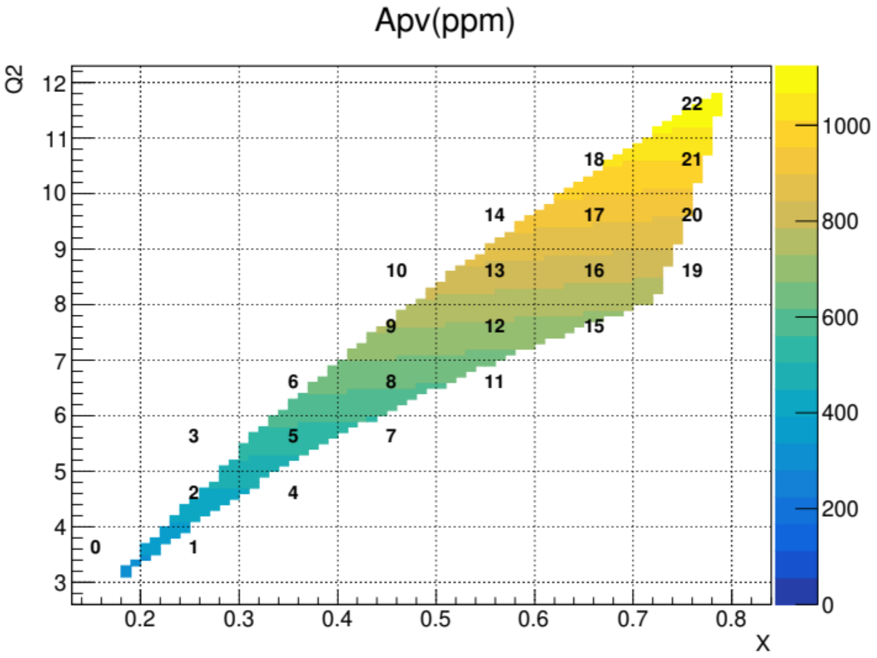
# 12 GeV PVDIS with SoLID
# (11 GeV beam)

# SoLID (Solenoid Large Intensity Device)

- A general purpose, large acceptance spectrometer for Jefferson Lab Hall A
- Three pillars of physics programs started in 2009-2012
  - SIDIS, J/Psi
  - PVDIS (p and D)
  - (GPD/DVCS in progress)

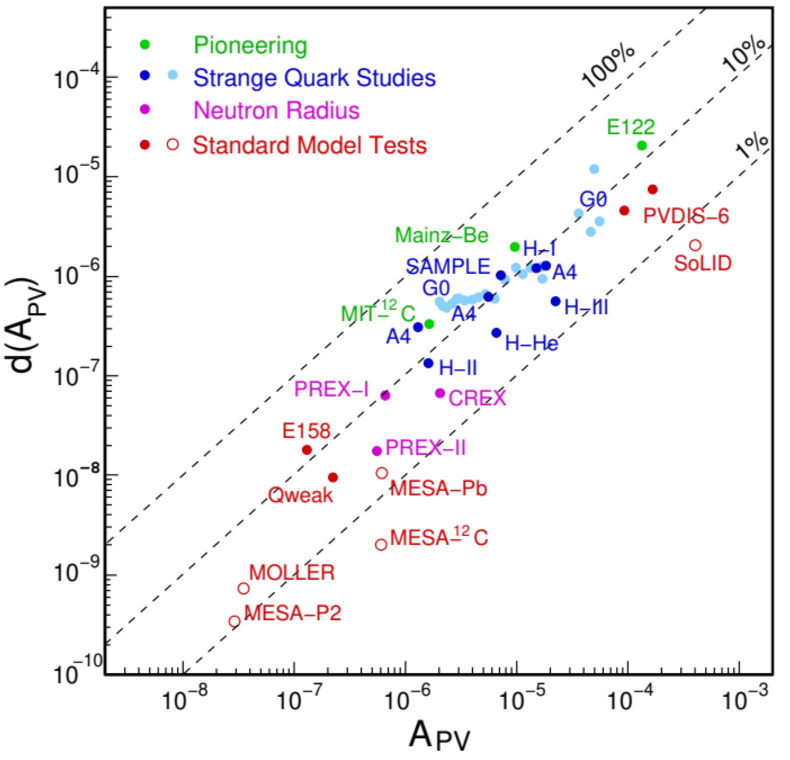
PVDIS-d (11 GeV beam, 22-35 deg DIS)



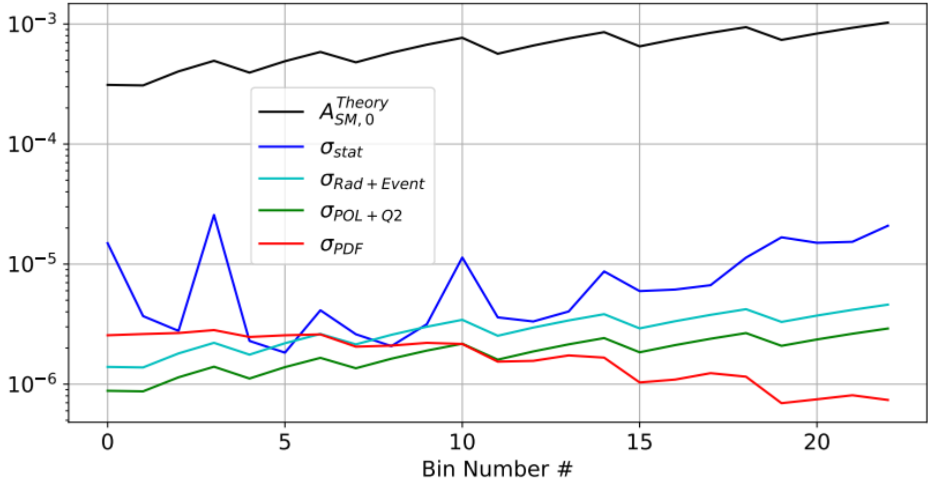


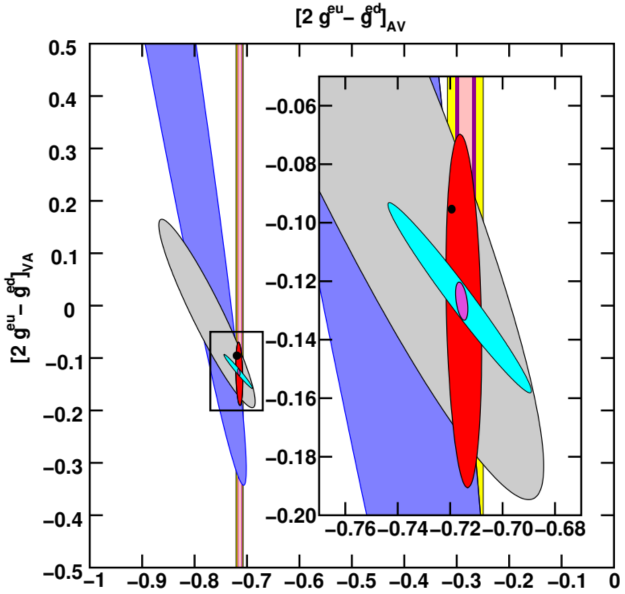
# SoLID Recent Activities

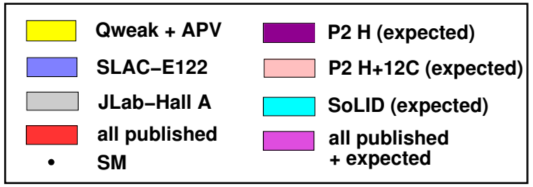
- DOE Science Review (3/2021): waiting for report (and CD0)
- Magnet Cold Test
- Pre-R&D activities: LGC, DAQ, FTBF ECal, current beam test
- Software and Simulation Development
  - Synergy with EIC software development
  - Continue AI/ML collaboration with JLab data science group
  - Update event generators
  - PID and Tracking developments
- INT22-81W "PVDIS at JLab 12 GeV and Beyond"
- JLab PAC50: All 5 approved SoLID experiments successfully passed jeopardy review
  - J/Psi scientific rating A- → A
  - PV-EMC proposal C2 approval
  - BNSSA (A_n) proposal approved for 38 PAC days, A-
- Next: White paper, NSAC LRP

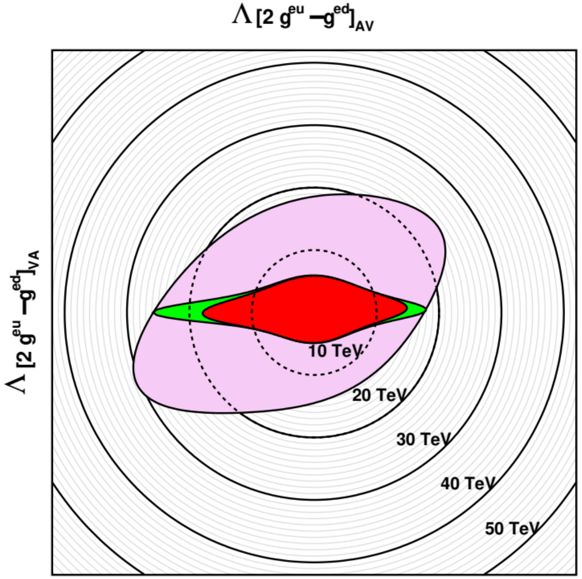


# PVDIS-d with SoLID

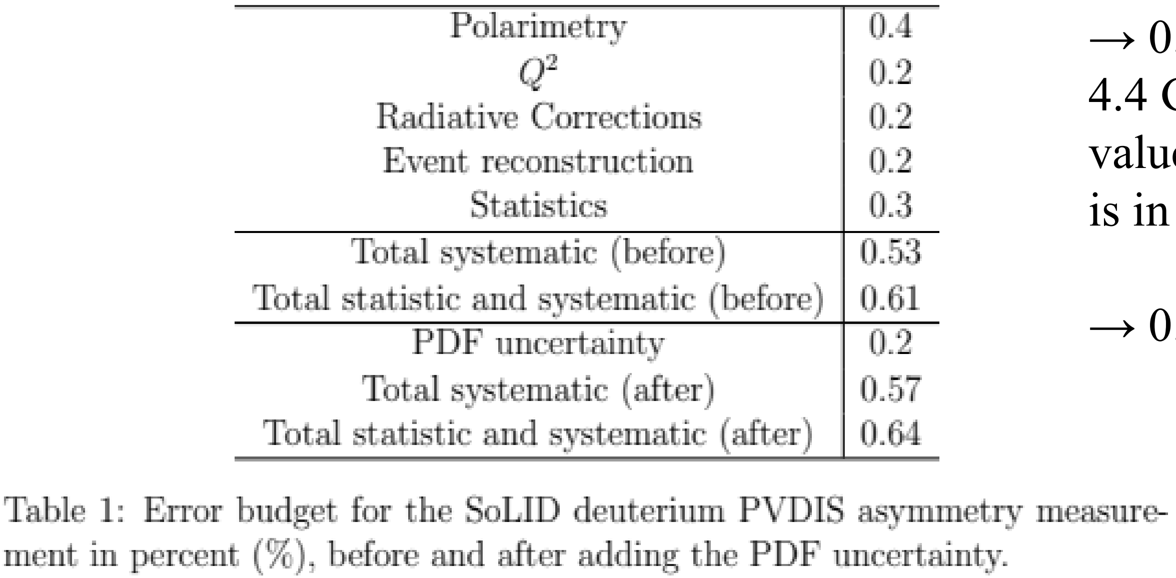








# Uncertainty Budget



| | |
|---|---|
| Polarimetry | 0.4 |
| $Q^2$ | 0.2 |
| Radiative Corrections | 0.2 |
| Event reconstruction | 0.2 |
| Statistics | 0.3 |
| Total systematic (before) | 0.53 |
| Total statistic and systematic (before) | 0.61 |
| PDF uncertainty | 0.2 |
| Total systematic (after) | 0.57 |
| Total statistic and systematic (after) | 0.64 |

Table 1: Error budget for the SoLID deuterium PVDIS asymmetry measurement in percent (%), before and after adding the PDF uncertainty.

0.2% Radiative Correction

→ 0.1% external, take data with 6.6 and 4.4 GeV beam for low W, low $Q^2$ Apv values (a PV-resonance run-group proposal is in progress, for PAC51)

→ 0.1% on internal, higher-order effects

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$A_{\text{phys}} = -91.10 \pm 4.30\,\text{ppm}\,(4.7\,\%)$

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$f_{\gamma\gamma} = -0.003 \pm 0.003$

$A_{\text{phys}} = -160.80 \pm 7.12\,\text{ppm}\,(4.4\,\%)$

Recent calculation using stand-alone Mo&Tsai equivalent radiator:
internal: -0.7% (original HAMC -0.33%) -1.2% (original HAMC -0.7%)

Djangoh:
internal with lepton radiation: -0.75% -1.23%
internal with both lepton and quark radiation: -0.3% -0.7%
γγ and γZ boxes: 0.026% 0.03%

# 6 GeV PVDIS long paper: $A^{\text{rad-corrected}}=A^{\text{meas}}\left(1+\bar{f}_{rc}\right)$

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Recent calculation using
internal: -0.7% (ori

Djangoh:
internal with lepton radiat
internal with both lepton a
γγ and γZ boxes:

2012 vs. now:

- size of internal Bremsstrahlung seems to be consistent/comparable;
- slight difference between SM prediction quoted in 2014 paper and Djangoh output, could be due to RC of $C_{1,2}$;
- all are “small” compared with precision of 6 GeV measurement, but non-trivial now for SoLID.

# Radiative Correction for SoLID PVDIS – some ideas

Internal: Mo&Tsai does not deal with weak, box, etc → switch to Djangoh or another modern tool

Djangoh generator:

- specify Ebeam, specify $(x,Q^2)$ range
- parton-model based physics
- custom input $F_{1,2}$ possible
- can run in 3 modes:
  - generate full events (lepton, hadron)
  - generate just final-state lepton
  - do not generate events, calculate cross section only:
    - unpolarized (also for event-gen mode)
    - R-L (PV) or LC difference
- can turn on/off leptonic radiation, quark (QED) radiation, and interference
- can turn on/off pure-weak box diagrams

External: using GEANT-based SoLID simulation

technicality:

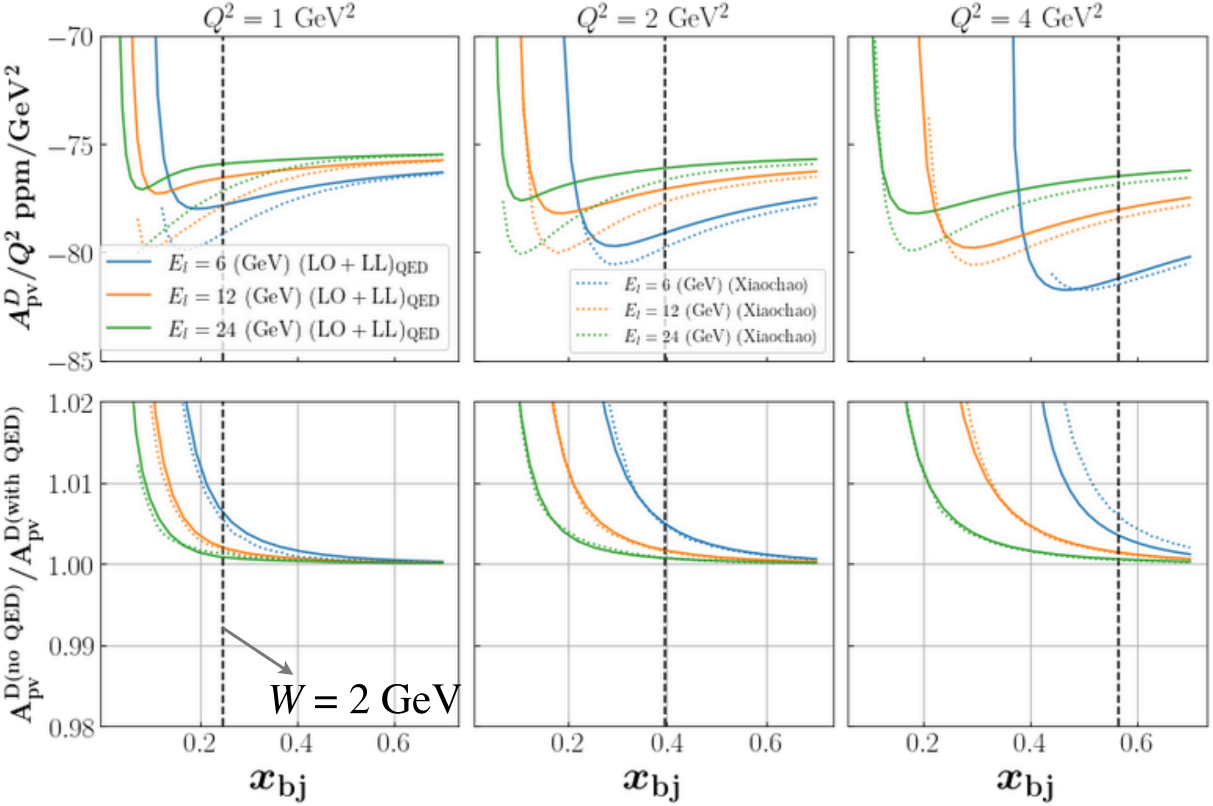
- beam energy loss in target needs implemented (ref. COMPASS)
- what about low W, low $Q^2$?
- custom-input of $F^{gZ}$ would be helpful, for R-L (PV) cross section calculation
- could be useful for background study (?)
- can combine with SoLID sim for external energy loss correction in the final state

- can these corrections be separated from int/ext radiative corrections?

JLab theory group factorization approach (see T. Liu’s talk at INT22-81W)

# Comparison with Traditional Method

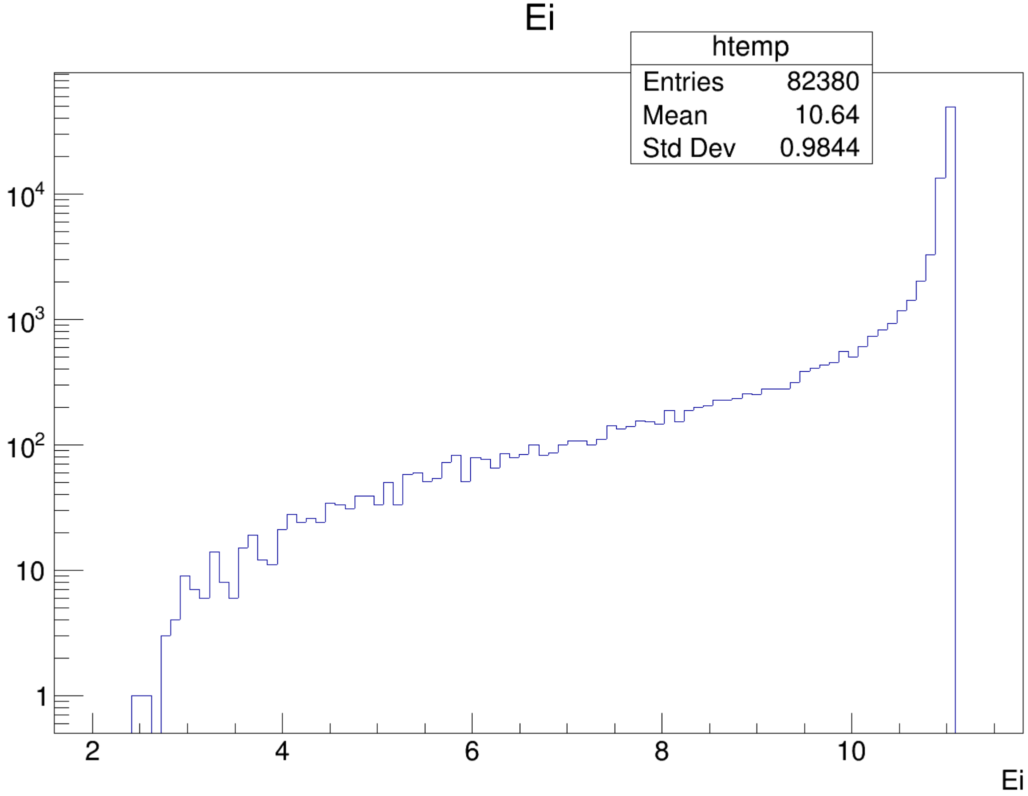
QED impact on $A_{pv}$



dotted curves from X. Zheng, generated using “Mo&Tsai” approach.

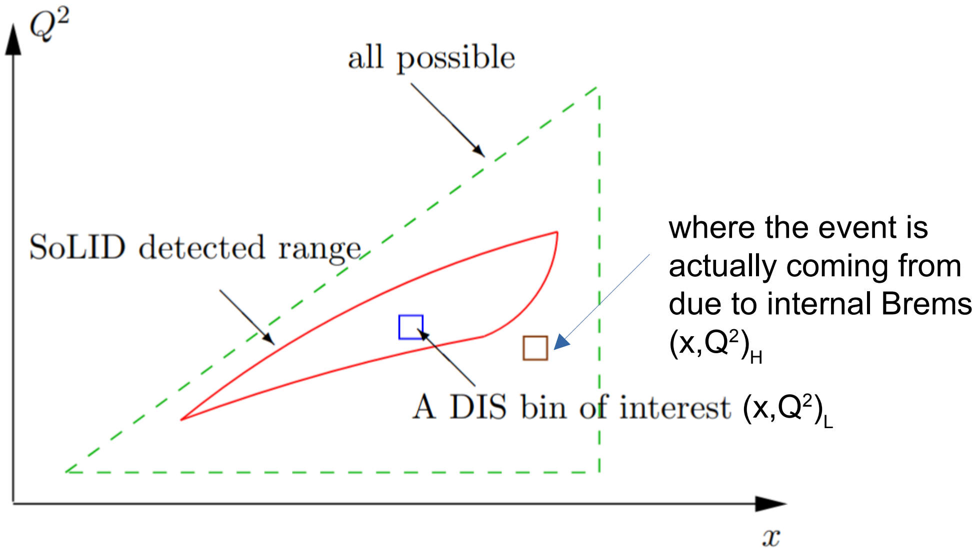
# General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different `EELE`, sampled from the spectrum above



# General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
- Choose 100(?) different `EELE`, sampled from the spectrum above
- Use Djangoh to generate events for the full allowed region for each `EELE` value, generate 1M (?) events (lepton only)
  - with internal Bremsstrahlung turned on



where the event is actually coming from due to internal Brems $(x,Q^2)_H$

$(x,Q^2)_L$

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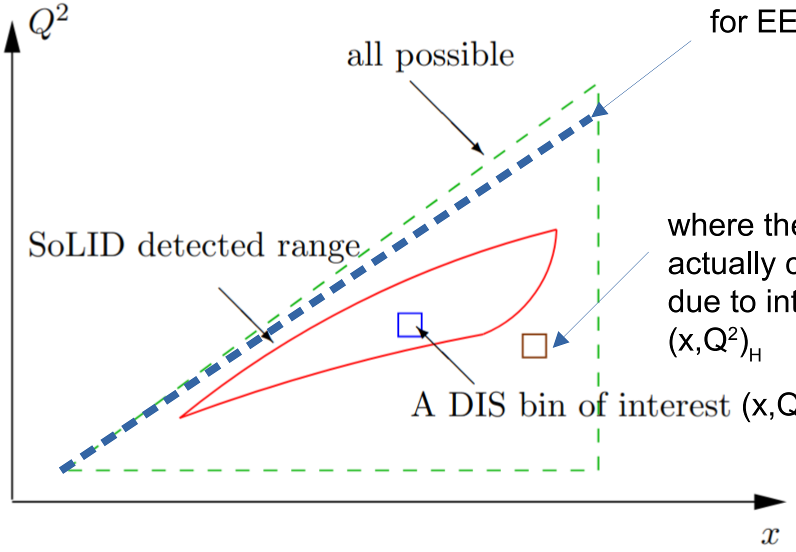

for EELE < 11 GeV

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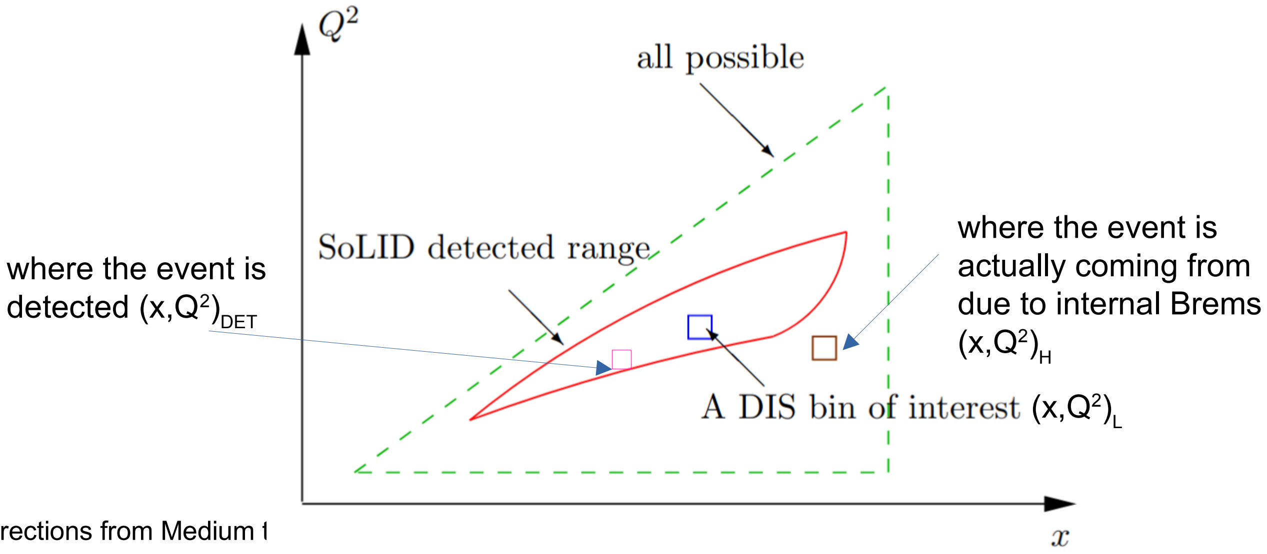
  - output final-states in LUND format that can be passed onto SoLID sim



where the event is detected $(x,Q^2)_{DET}$

where the event is actually coming from due to internal Brems $(x,Q^2)_H$

# General Ideas for SoLID PVDIS Radiative Corrections

- Generate energy spectrum for electron beam in 40-cm LD2 target
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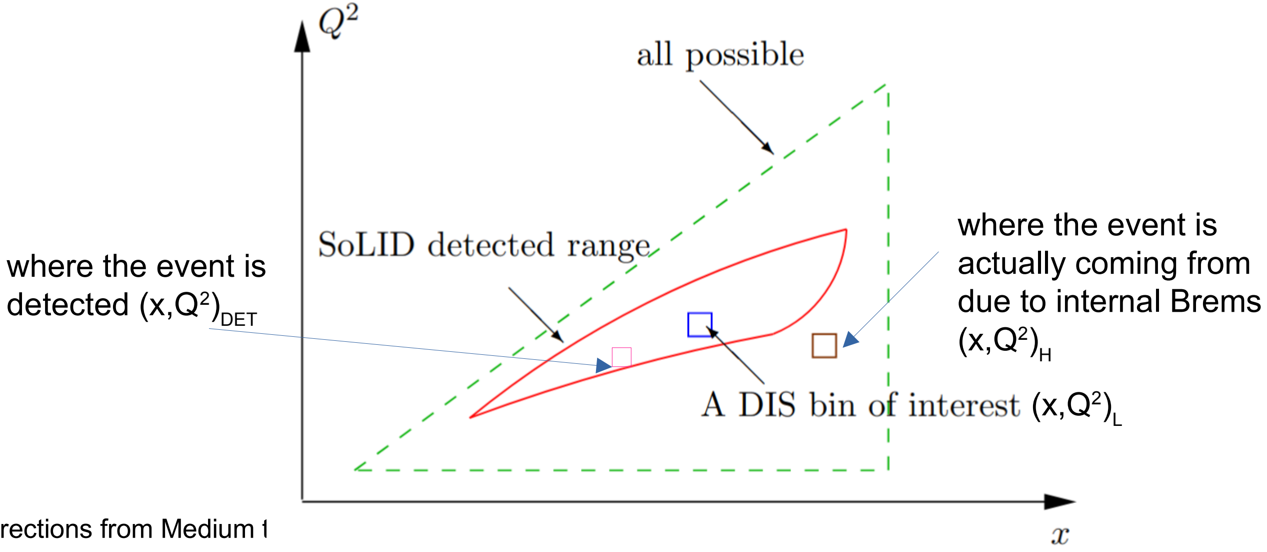
  - with internal Bremsstrahlung turned on
  - output final-states in LUND format that can be passed onto SoLID sim
- Use cross-section only mode of Djangoh, calculate $\sigma_0$ and Apv for 100x(~9000) bins
  - If there is an alternative method, calculate/generate the same “grid”



where the event is detected $(x,Q^2)_{DET}$

where the event is actually coming from due to internal Brems $(x,Q^2)_H$

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- Generate energy spectrum for electron beam in 40-cm LD2 target
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  - output final-states in LUND format that can be passed onto SoLID sim
- Use cross-section only mode of Djangoh, calculate $\sigma_0$ and Apv for 100x(~9000) bins
  - If there is an alternative method, calculate the same “grid”

- Generate 1000(?) MC events along target length
- look for closest(?) `EELE` Djangoh simulation and input all 1M events
- pass 1G final-state electrons to SoLID simulation for evaluating final-state electrons
- for each detected events, look for Apv at the interaction vertex $(x,Q^2)_H$
- apply proper normalization=
- evaluate Apv_detected vs. Apv_true(H), the difference would be the RC factor

# Summary

For SoLID 11 GeV PVDIS (note: statistical goal 0.4% on Apv, need RC uncertainty at 0.2% or smaller)

- Can external, internal EM effects be determined to <0.1% precision?
  - Can we do a data-driven approach (like 6 GeV) for low W, low $Q^2$?
  - Three methods now exist for internal: 6 GeV approach, JLab's factorization approach, and Djangoh/SoLID MC. Is any of these tools working for the precision needed? What is the difference among three and what if there is a large difference?
- Can ext/int EM effects be separated from all box diagram corrections (as in 6 GeV)?
- Do we need 2-loop corrections? Can we have two parallel methods for these higher-order corrections and constrain them to <<(?)0.1% precision?
- What about QCD, HT? → factorization approach (global constraint provide consistency in HT fitting, one single experiment cannot be used to determine both HT and EW parameters)

- PVDIS (or any asymmetry measurement) at EIC: unfolding uncertainty?

# Backup Slides

## using SF, “full” expression (include F_L, TMC)

$$\eta_{\gamma Z}=\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{M_Z^2}{M_Z^2+Q^2}$$

$$A_{RL}^{e^-}=\frac{|\lambda|\eta_{\gamma Z}\left[g_A^e 2yF_1^{\gamma Z}+g_A^e\left(\frac{2}{xy}-\frac{2}{x}-\frac{2M^2xy}{Q^2}\right)F_2^{\gamma Z}+g_V^e(2-y)F_3^{\gamma Z}\right]}{2yF_1^{\gamma}+\left(\frac{2}{xy}-\frac{2}{x}-\frac{2M^2xy}{Q^2}\right)F_2^{\gamma}-\eta_{\gamma Z}\left[g_V^e 2yF_1^{\gamma Z}+g_V^e\left(\frac{2}{xy}-\frac{2}{x}-\frac{2M^2xy}{Q^2}\right)F_2^{\gamma Z}+g_A^e(2-y)F_3^{\gamma Z}\right]}$$

*ignore TMC and assuming F2=2x * F1*

## using SF, with approximations

$$A_{RL}^{e^-}=|\lambda|\left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right)\left[g_A^e\frac{F_1^{\gamma Z}}{F_1^{\gamma}}+\frac{g_V^e}{2}Y(y)\frac{F_3^{\gamma Z}}{F_1^{\gamma}}\right]$$

$$C_{1u}=2g_A^e g_V^u=-\frac{1}{2}+\frac{4}{3}\sin^2(\theta_W)$$

$$C_{1d}=2g_A^e g_V^d=\frac{1}{2}-\frac{2}{3}\sin^2(\theta_W)$$

$$C_{2u}=2g_V^e g_A^u=-\frac{1}{2}+2\sin^2(\theta_W)$$

$$C_{2d}=2g_V^e g_A^d=\frac{1}{2}-2\sin^2(\theta_W)$$

## using parton-model interpretation of F1,3 and PDF:

$$A_{RL}^{e^-}=|\lambda|\left(\frac{3G_F Q^2}{2\sqrt{2}\pi\alpha}\right)\frac{\left[2(1+R_C)C_{1u}-(1+R_S)C_{1d}\right]+Y(y)(2C_{2u}-C_{2d})R_V(x)}{5+4R_C+R_S}$$

# Measurement of the Beam-Normal Single Spin Asymmetry in Deep Inelastic Scattering using the SoLID Spectrometer

On behalf of
William Henry[1], Michael Nycz[2], Ye Tian[3], Weizhi Xiong[4], Xiaochao Zheng[2]
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[2]University of Virginia
[3]Syracuse University
[4]ShandongUniversity

PAC 50
07/12/2022

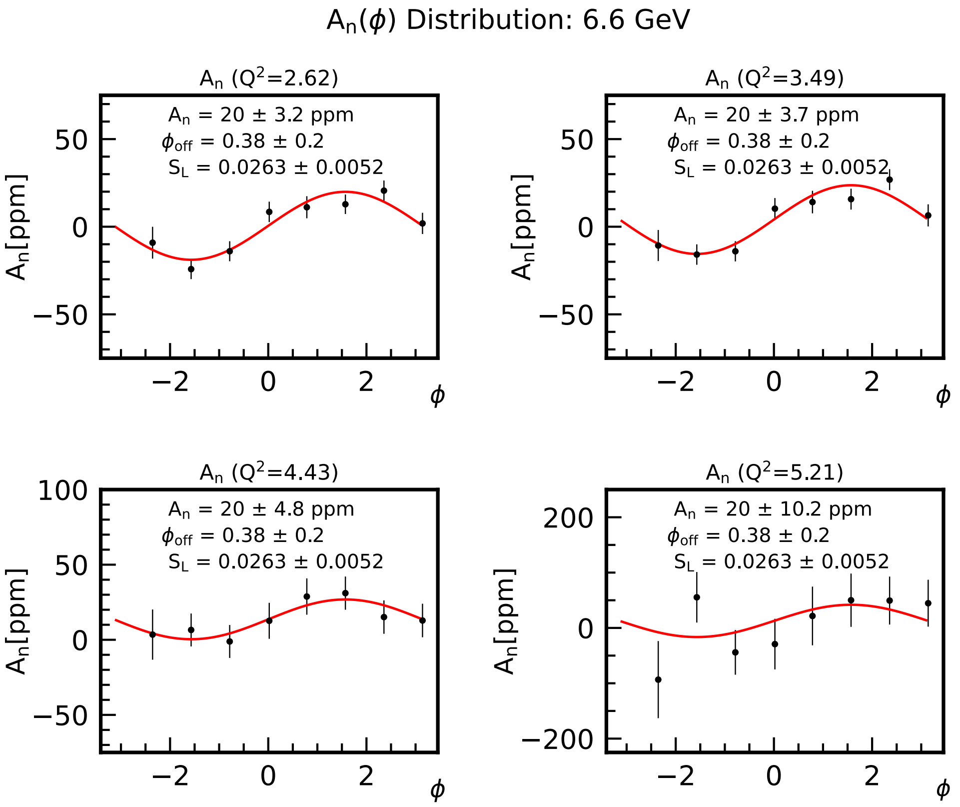
**A Hall A and SoLID proposal**

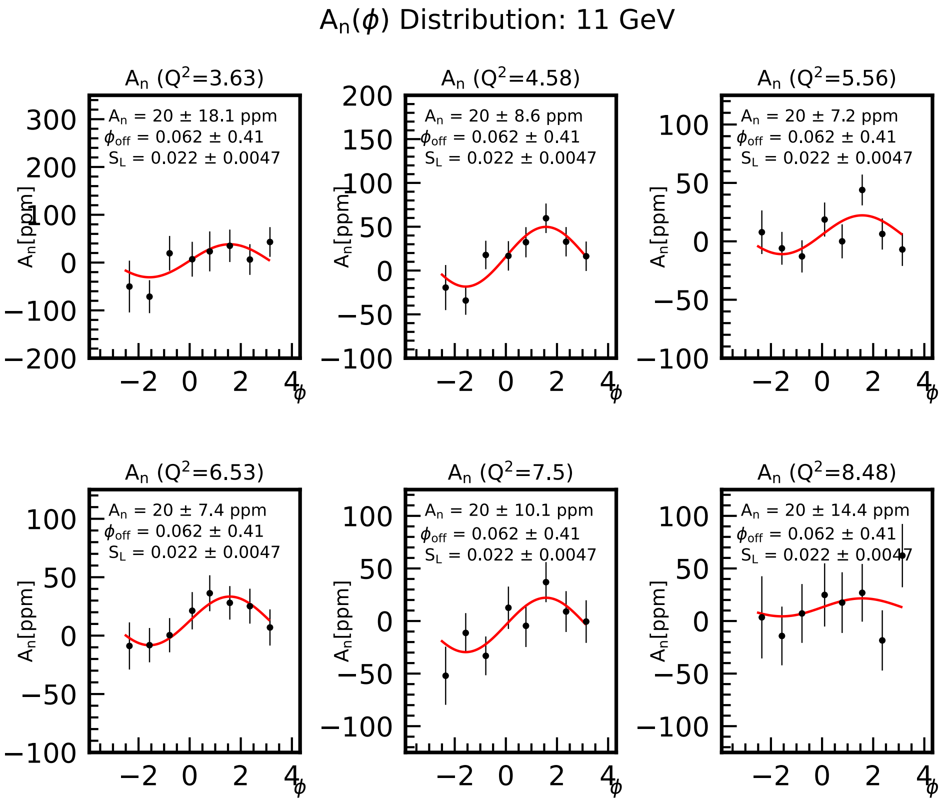
# Projected Results

SoLID simulation based on PVDIS configuration

- 40-cm $LH_2$ target
- W >2, 22°< θ < 35 °, $P_b$=85%

Beam-normal SSA extracted from multi-parameter fit: $C_n \sin(\varphi + \varphi_{\text{offset}}) + S_L A_{\text{PVDIS}}$





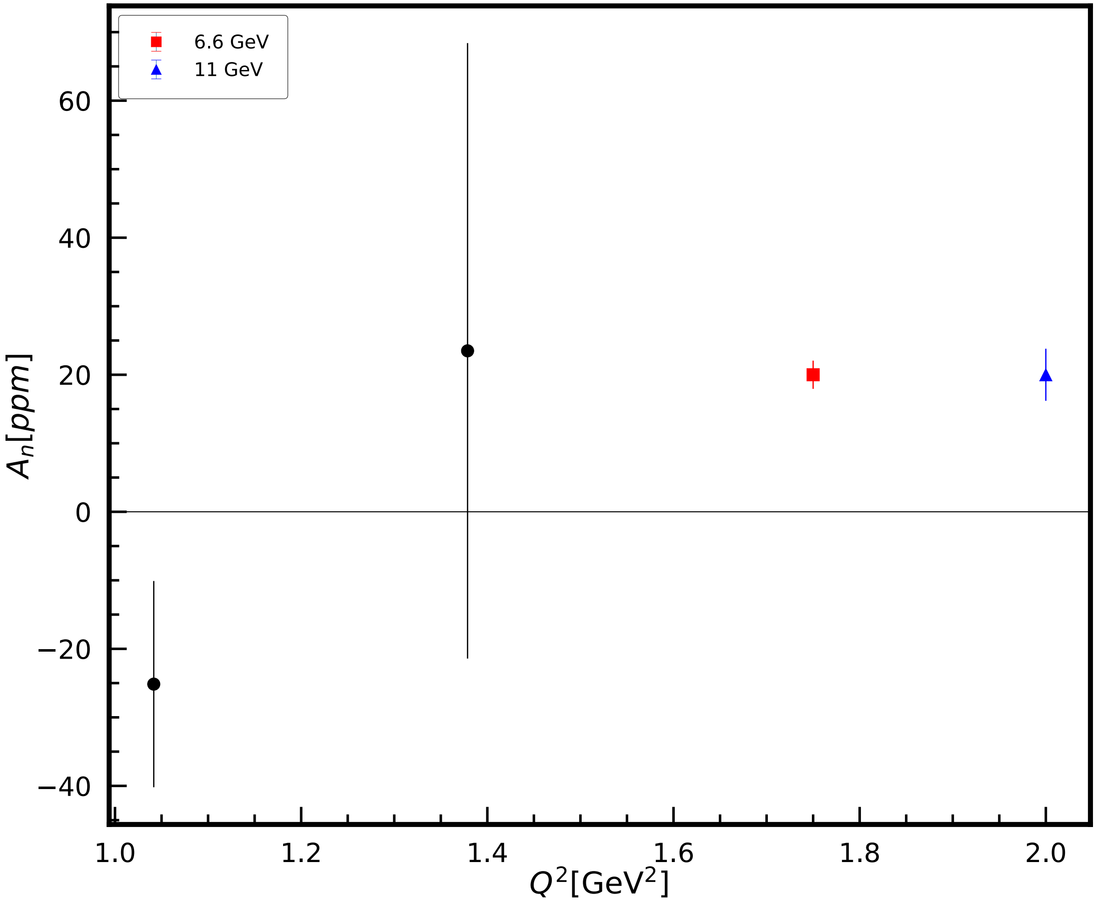
# Projections: Result and Uncertainty

**Combining $Q^2$ bins in each energy setting**

$A_n$= $A_{measured}$ ± 2.06 ppm : 6.6 GeV

$A_n$= $A_{measured}$ ± 3.80 ppm : 11 GeV

**Opportunity to measure non-zero BNSSA**



Projected results not at expected $Q^2$ values

# Projections: Result and Uncertainty

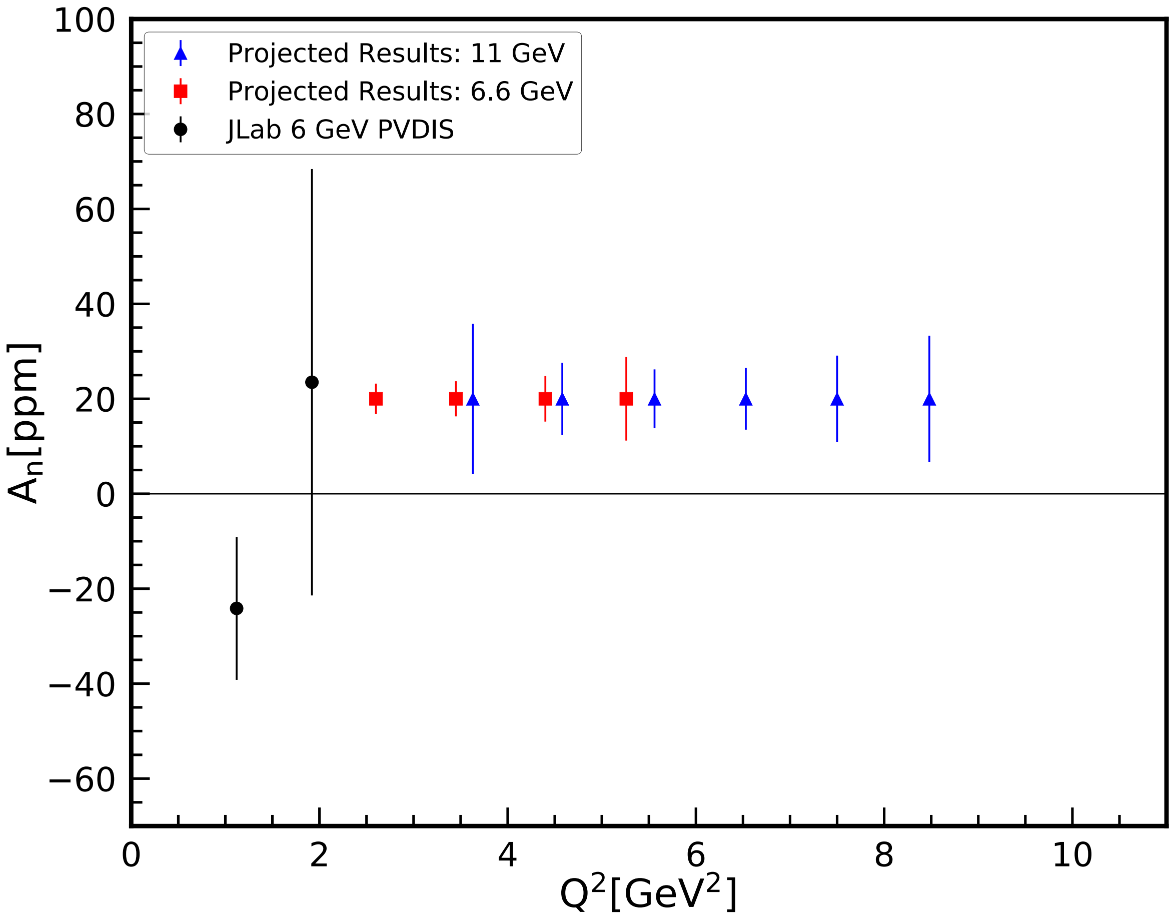
**Combining $Q^2$ bins in each energy setting**

$A_n = A_{measured} \pm 2.06$ ppm : 6.6 GeV
$A_n = A_{measured} \pm 3.80$ ppm : 11 GeV

**Opportunity to measure non-zero BNSSA**

*TAC Theory: "…it will be interesting to observe the $Q^2$ dependence of the asymmetry empirically, to test whether the scattering takes place on a single quark or involved more complicated nonperturbative multi-parton interactions."*

**The opportunity to observe a $Q^2$ dependence with sufficient precision is primarily due to the design SoLID**

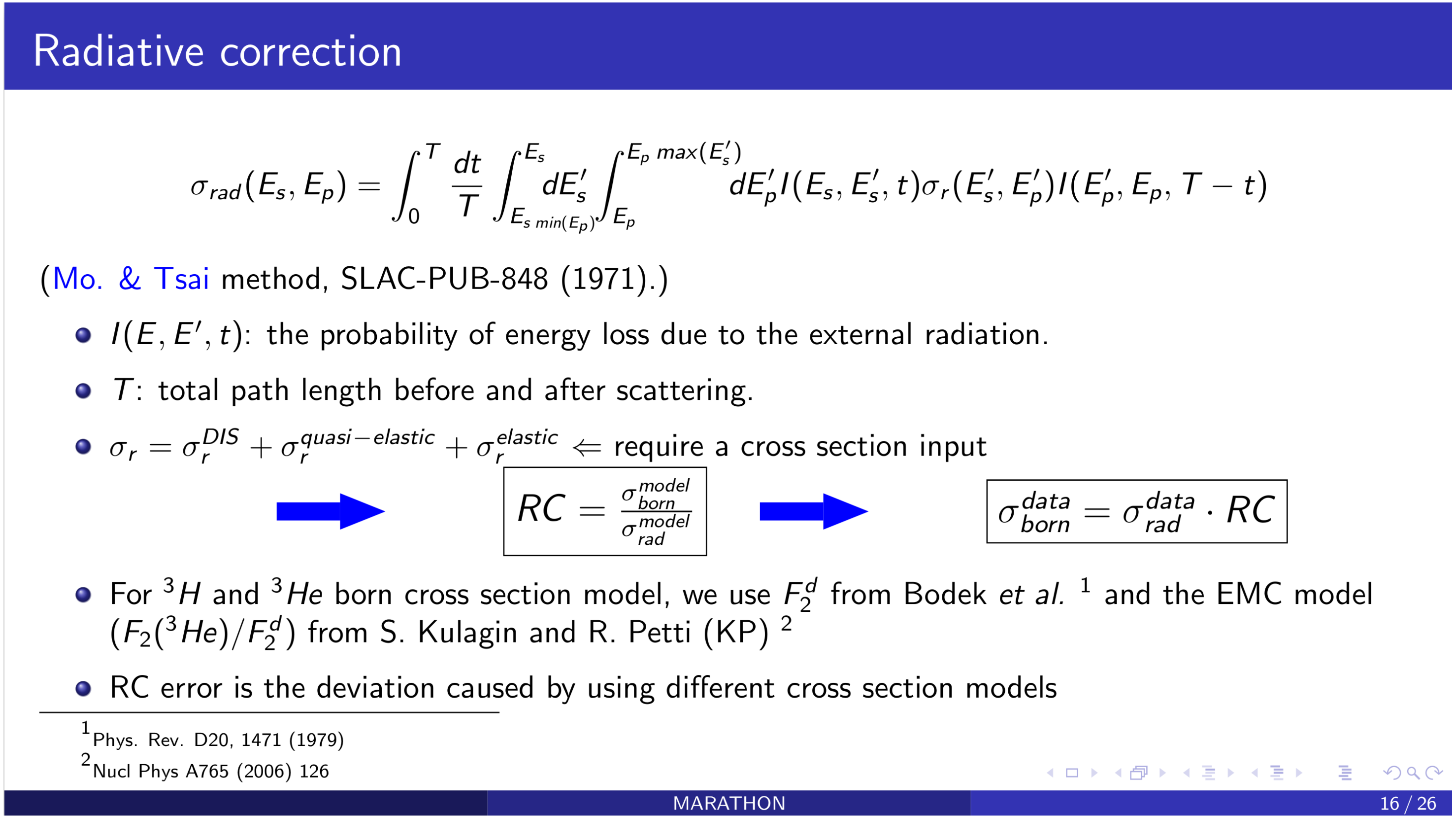


# Summary

- The proposed experiment would be the **first high-precision measurement** of the beam-normal single spin asymmetry in deep inelastic scattering
- No other planned measurements
- The beam-normal single spin asymmetry provides an essential tool for studying the two-photon exchange effect
  - Will add a new observable to the TPE landscape
- Important for theoretical models
  - Further insight into dominant mechanism (single or two quark)
  - Complementary to the already approved target-normal single spin asymmetry
- Theoretical support
  - INT: PVDIS at JLab 12 GeV and beyond
  - Radiative Corrections from medium to high energy experiments
  - Drive interest of both experimental and theoretical communities
- **Jefferson Lab and SoLID make this a realistic measurement**

# First method

- apply correction directly to measured cross sections
- more suitable for small-acceptance spectrometers
- (“RC_external” code calculates both Born and radiated cross sections)



## Radiative correction

$$\sigma_{rad}(E_s, E_p) = \int_0^T \frac{dt}{T} \int_{E_{s\,min}(E_p)}^{E_s} dE'_s \int_{E_p}^{E_{p\,max}(E'_s)} dE'_p I(E_s, E'_s, t)\sigma_r(E'_s, E'_p) I(E'_p, E_p, T-t)$$

(Mo. & Tsai method, SLAC-PUB-848 (1971).)

- $I(E, E', t)$: the probability of energy loss due to the external radiation.
- $T$: total path length before and after scattering.
- $\sigma_r = \sigma_r^{DIS} + \sigma_r^{quasi-elastic} + \sigma_r^{elastic} \Leftarrow$ require a cross section input

→ $\boxed{RC = \frac{\sigma_{born}^{model}}{\sigma_{rad}^{model}}}$ → $\boxed{\sigma_{born}^{data} = \sigma_{rad}^{data} \cdot RC}$

- For $^3H$ and $^3He$ born cross section model, we use $F_2^d$ from Bodek *et al.* [1] and the EMC model ($F_2(^3He)/F_2^d$) from S. Kulagin and R. Petti (KP) [2]
- RC error is the deviation caused by using different cross section models

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[1] Phys. Rev. D20, 1471 (1979)

[2] Nucl Phys A765 (2006) 126

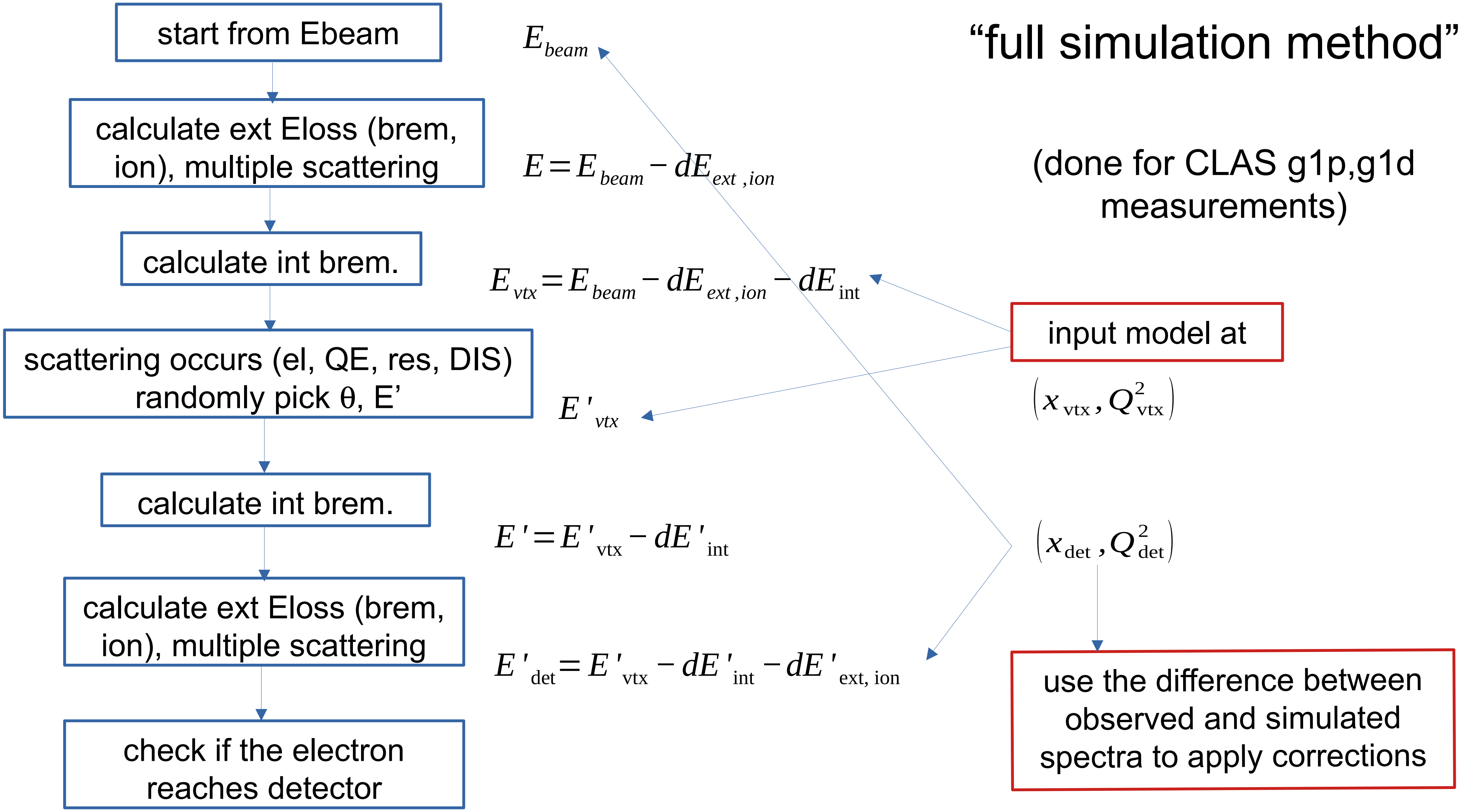
(from H. Liu’s talk at INT workshop)

# Second method (fully forward simulation method)

- use a full simulation method to calculate “Born” and to simulate “measured” observables using model inputs
- if simulated “measured” observables do not agree with real data, adjustment is made to the model inputs
- more suitable for large-acceptance spectrometers
- can be added to any existing, experimental full simulation packages
- technical complications:
  - tails from elastic scattering may need to be subtracted first
  - positive and negative cross section (difference) regions need to be done separately

# “full simulation method”

(done for CLAS g1p,g1d measurements)



start from Ebeam

$$E_{beam}$$

calculate ext Eloss (brem, ion), multiple scattering

$$E = E_{beam} - dE_{ext,ion}$$

calculate int brem.

$$E_{vtx} = E_{beam} - dE_{ext,ion} - dE_{\text{int}}$$

scattering occurs (el, QE, res, DIS) randomly pick θ, E’

$$E'_{vtx}$$

calculate int brem.

$$E' = E'_{\text{vtx}} - dE'_{\text{int}}$$

calculate ext Eloss (brem, ion), multiple scattering

$$E'_{\text{det}} = E'_{\text{vtx}} - dE'_{\text{int}} - dE'_{\text{ext, ion}}$$

check if the electron reaches detector

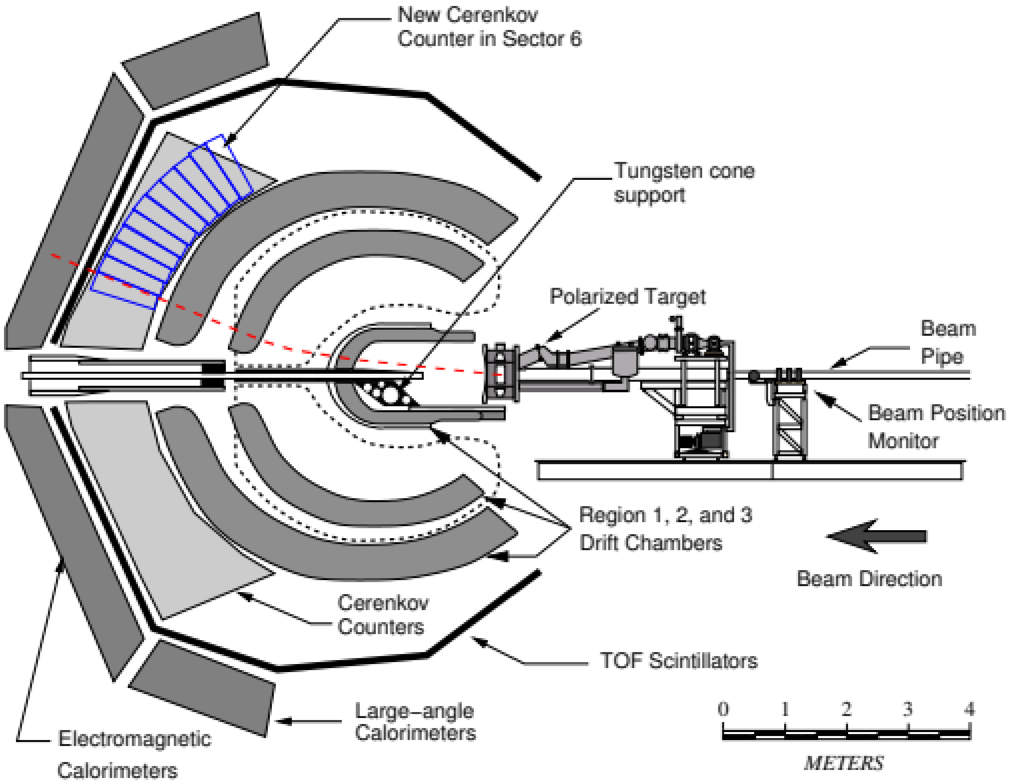
input model at

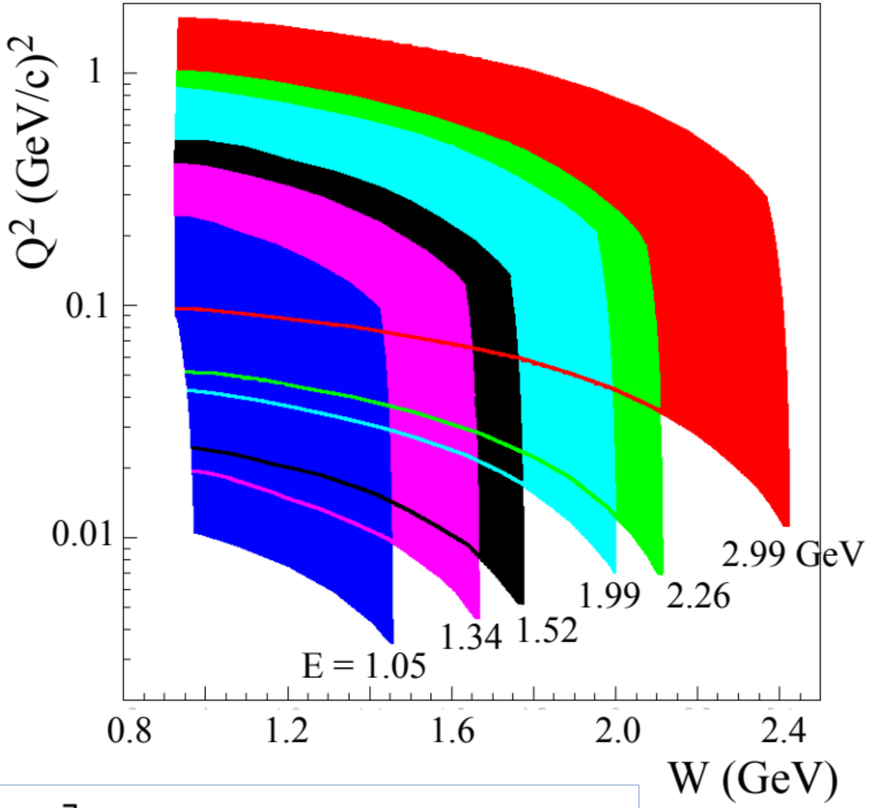
$$\left(x_{\text{vtx}}, Q^2_{\text{vtx}}\right)$$

$$\left(x_{\text{det}}, Q^2_{\text{det}}\right)$$

use the difference between observed and simulated spectra to apply corrections

# Radiative Corrections for CLAS EG4

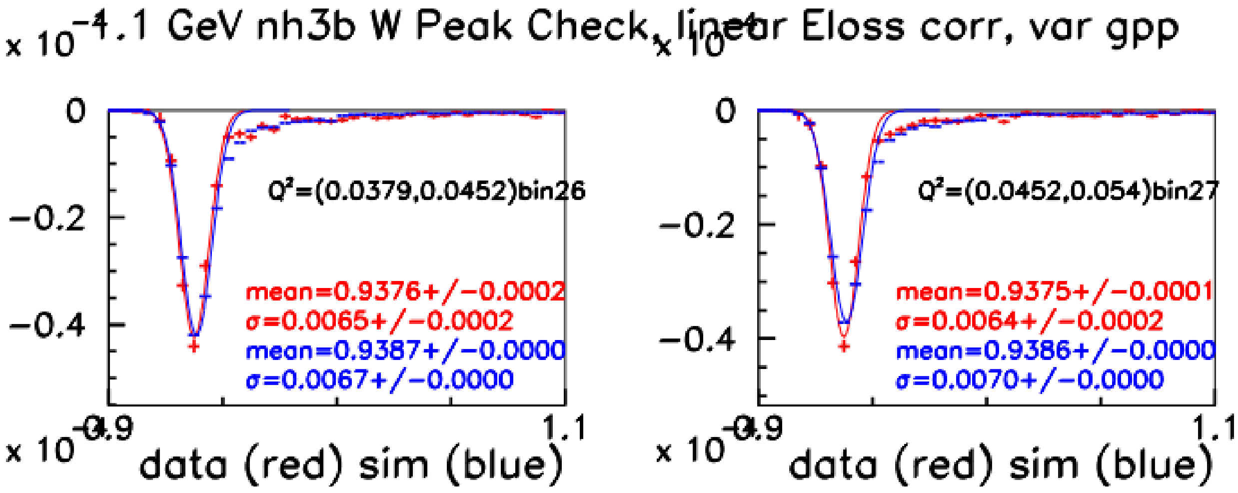




$$\Delta\sigma_{||} = \frac{\mathrm{d}^2\sigma_{\uparrow\Uparrow}}{\mathrm{d}\Omega\mathrm{d}E'} - \frac{\mathrm{d}^2\sigma_{\uparrow\Downarrow}}{\mathrm{d}\Omega\mathrm{d}E'} = \left[\frac{N^+}{N_e^+} - \frac{N^-}{N_e^-}\right]\frac{1}{N_{targ}}\frac{1}{P_bP_t}\frac{1}{\Delta\Omega}\frac{1}{\eta_{detector}}$$

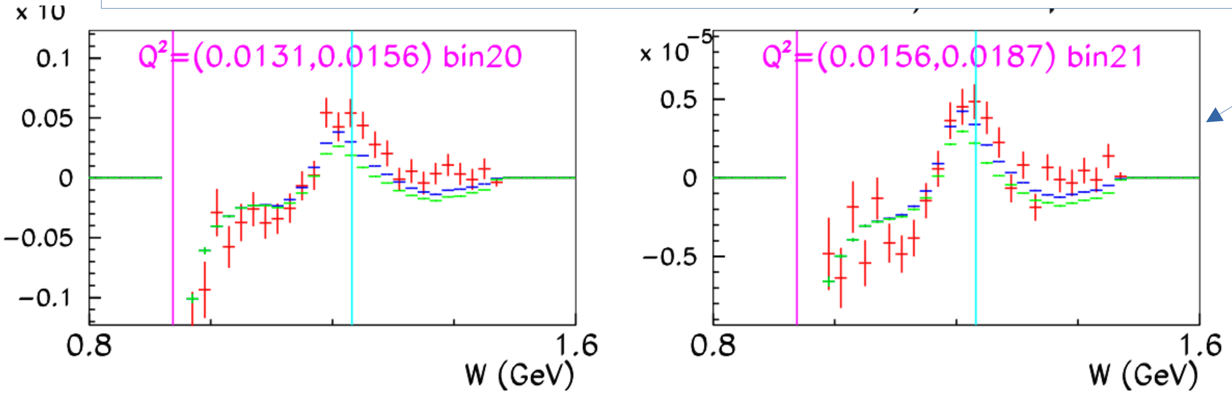
# Simulation of EG4 Proton Elastic Peak

- simulation reproduces measured double-polarized yield (N/Ne) difference
- cross-checking PbPt measurement, tuning detector smearing, material thickness, etc.
- Radiative tail from elastic peak can be determined and subtracted from inelastic data



# Simulation of EG4 Proton Resonance Region

$$\Delta\sigma_{||} = \frac{\mathrm{d}^2\sigma_{\uparrow\Uparrow}}{\mathrm{d}\Omega\mathrm{d}E'} - \frac{\mathrm{d}^2\sigma_{\uparrow\Downarrow}}{\mathrm{d}\Omega\mathrm{d}E'} = \left[\frac{N^+}{N_e^+} - \frac{N^-}{N_e^-}\right]\frac{1}{N_{targ}}\frac{1}{P_bP_t}\frac{1}{\Delta\Omega}\frac{1}{\eta_{detector}}$$



Comparison of polarized yield difference $N^+$ - $N^-$

red: data

blue: simulation with “best” A1 model

green: simulation with “best” A1 model shifted by +0.1

$$g_1^{\mathrm{data}} = g_1^{\mathrm{sim0}} + (g_1^{\mathrm{sim1}} - g_1^{\mathrm{sim0}})\frac{\Delta n^{\mathrm{data}} - \Delta n^{\mathrm{sim0}}}{\Delta n^{\mathrm{sim1}} - \Delta n^{\mathrm{sim0}}}$$

Extracted g1 structure function: