

ECT* Workshop 2022

Radiative corrections from medium
to high energy experiments

21 July, 2022

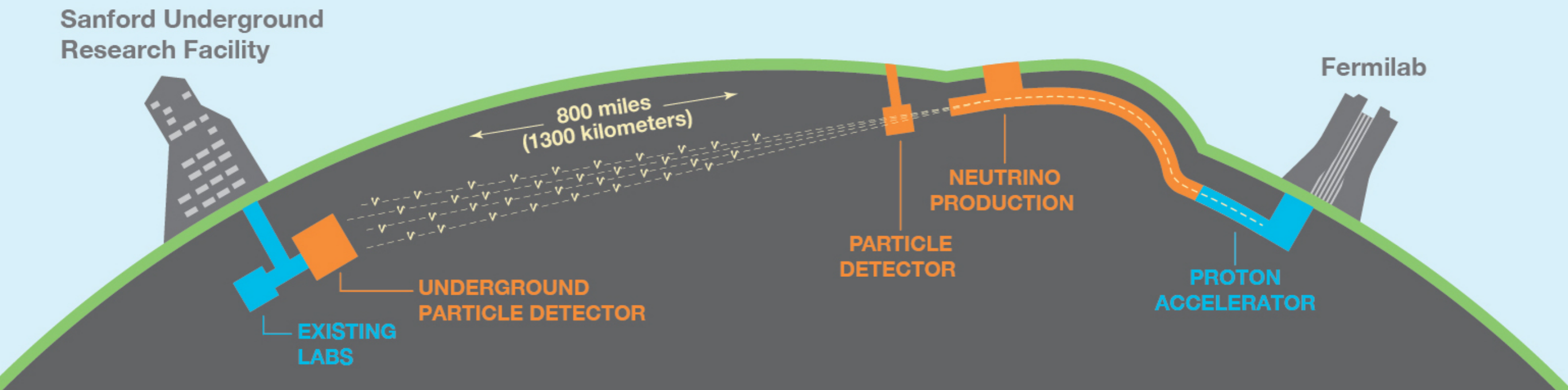
Radiative corrections to charged-current
neutrino scattering at GeV energies



Oleksandr (Sasha) Tomalak
LA-UR-22-25962

Neutrino experiments

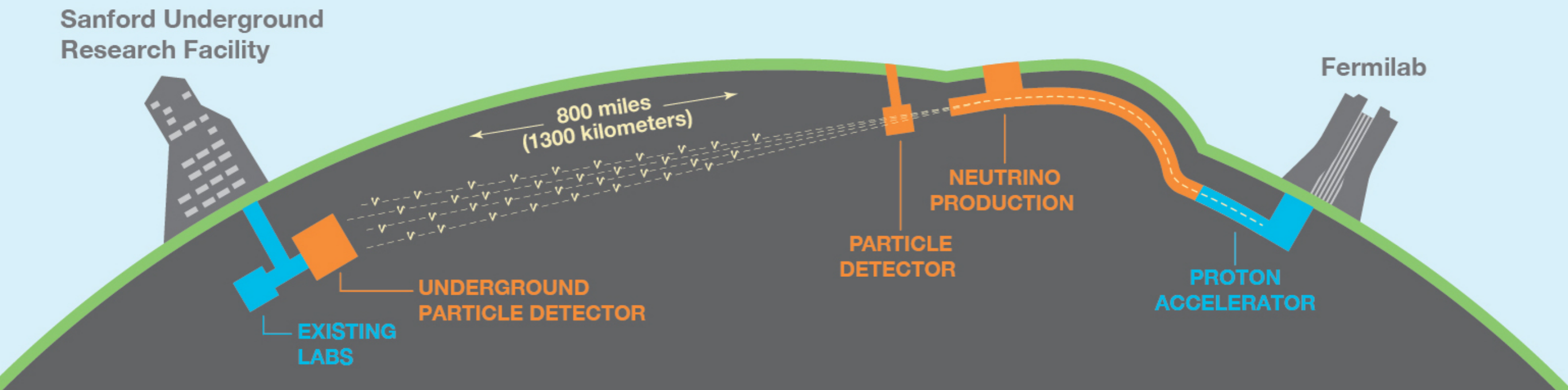
- **DUNE** and Hyper-K: leading-edge ν science experiments



- origin of matter-antimatter asymmetry δ_{CP}
- mass hierarchy and oscillation parameters PMNS matrix, Δm_{31}^2
- Grand Unified Theories proton decay
- dynamics of supernova explosion wait for one;)

Neutrino experiments

- **DUNE** and Hyper-K: leading-edge ν science experiments



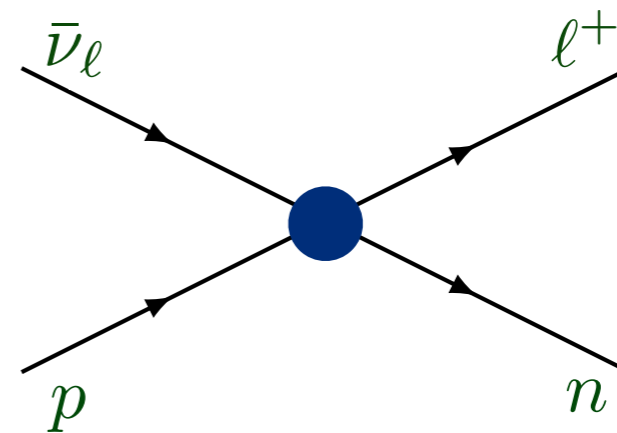
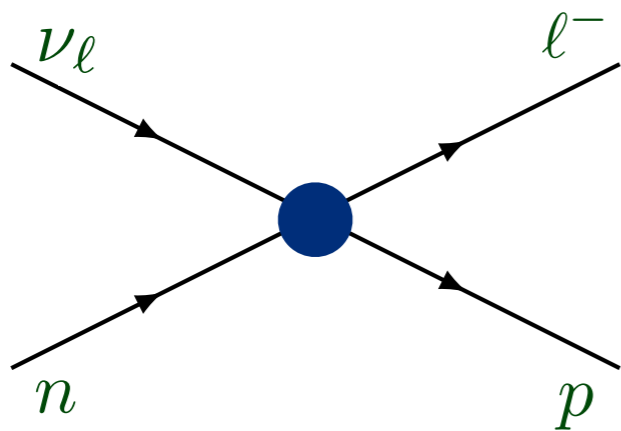
- measurement of ν_μ disappearance and ν_e appearance

$$N_\nu \sim \int dE_\nu \Phi_\nu (E_\nu) \times \sigma (E_\nu) \times R (E_\nu, E_\nu^{\text{rec}})$$

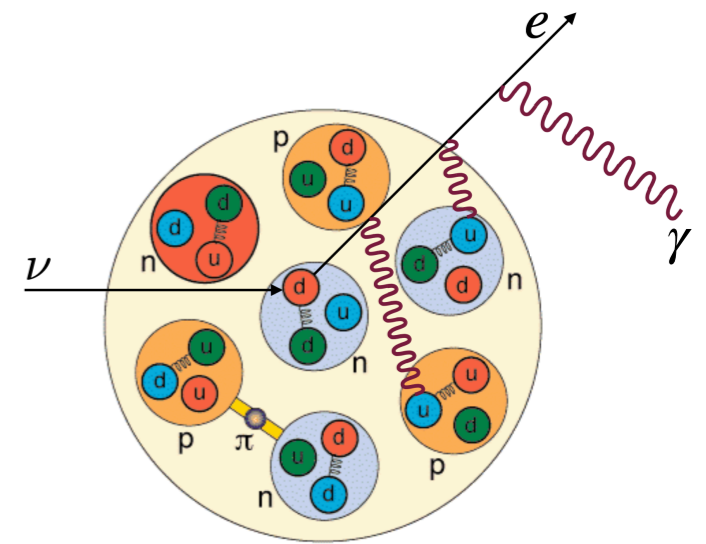
- near detector: determine flux and cross sections

Outline

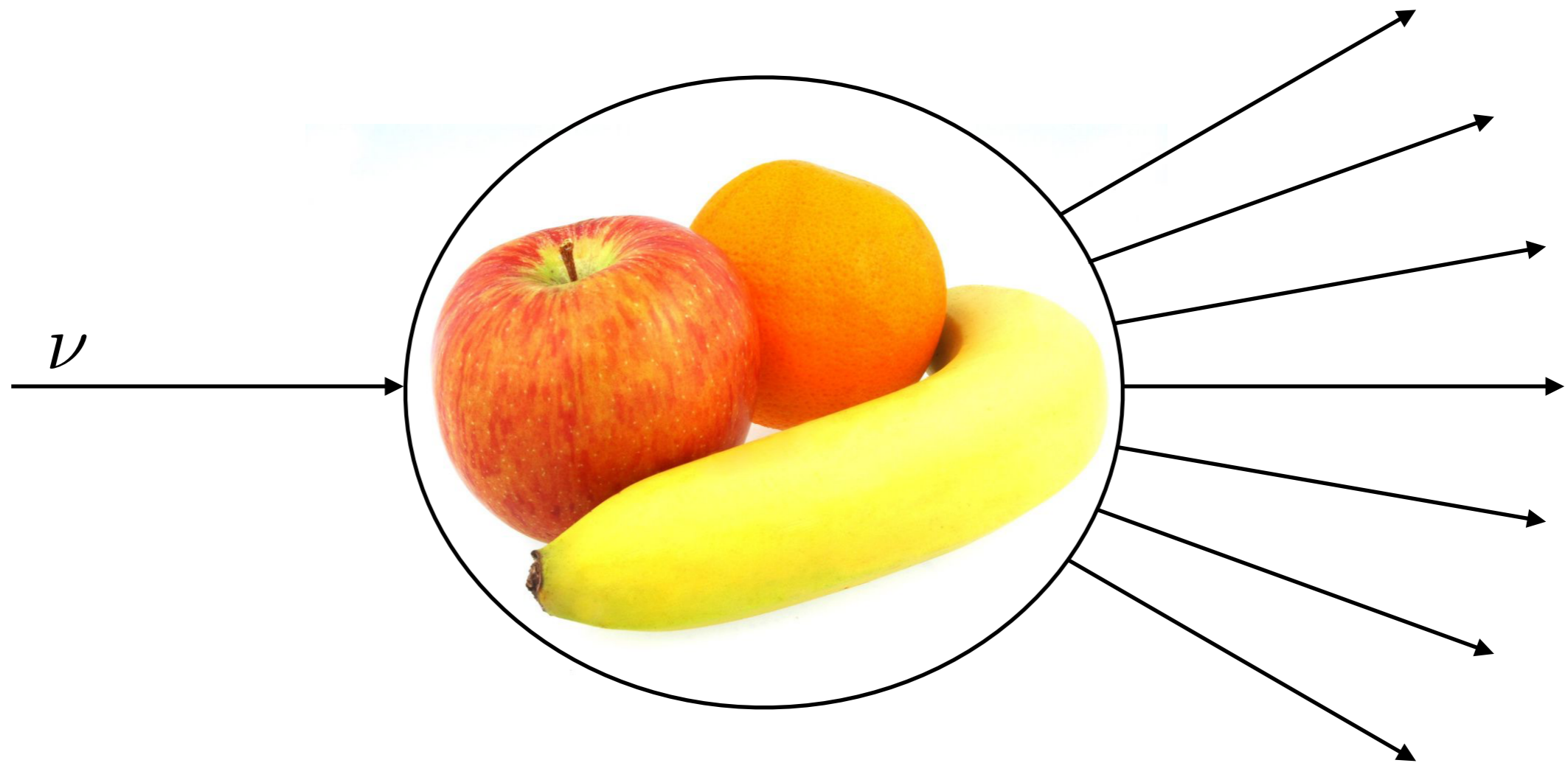
- 1) radiative corrections to
charged-current elastic scattering on nucleons



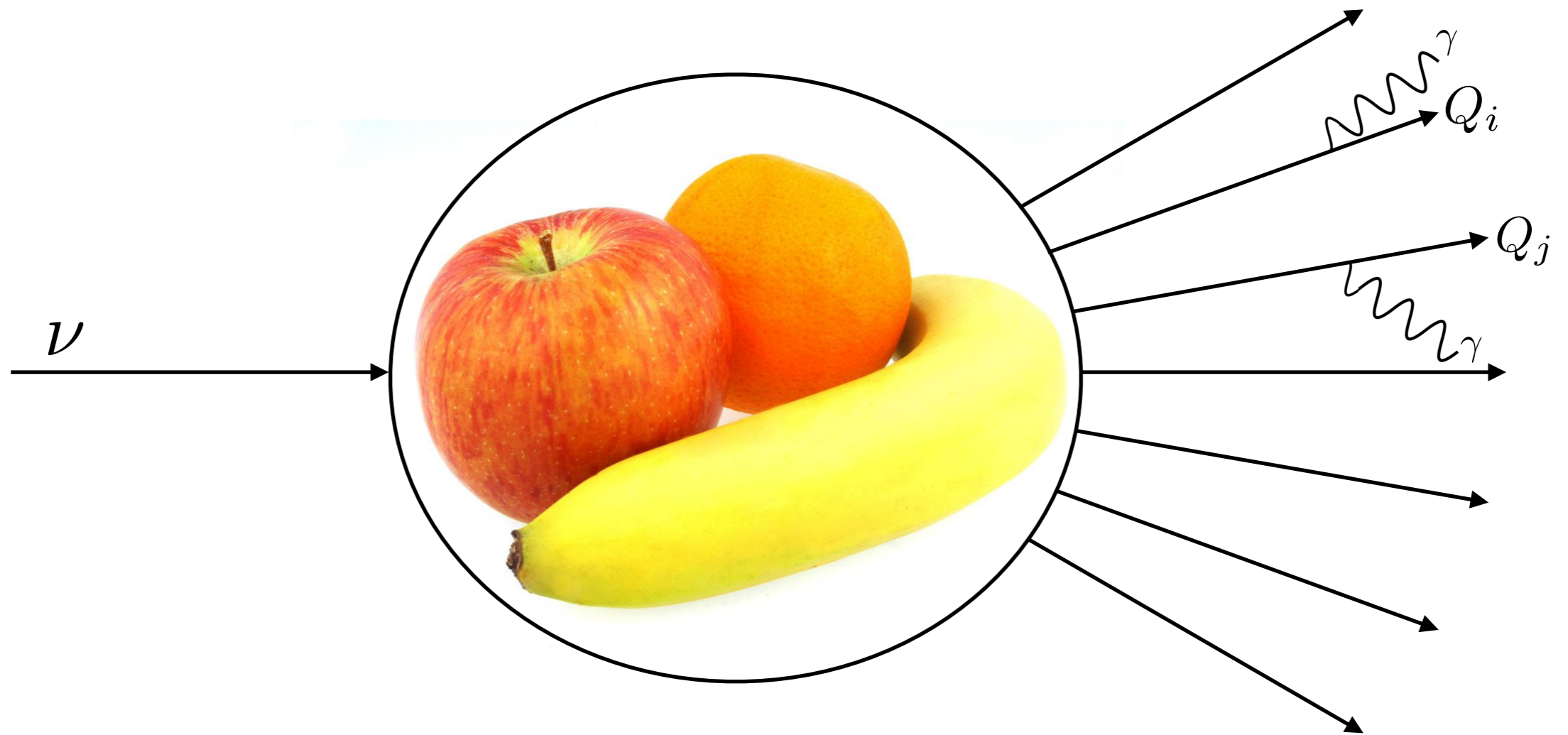
- 2) QED nuclear medium effects in
(anti)neutrino-nucleus and
electron-nucleus scattering



Neutrino interactions

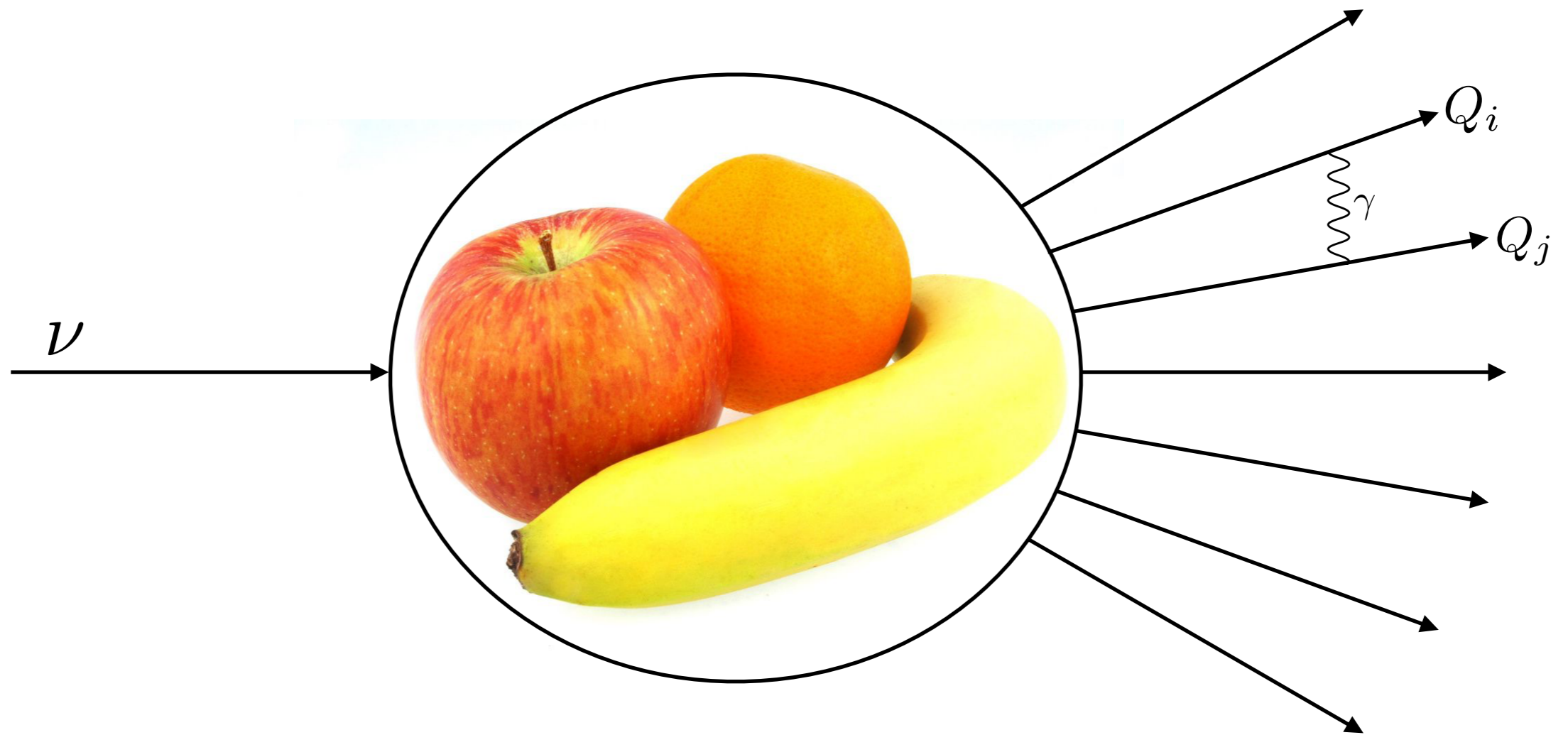


QED corrections



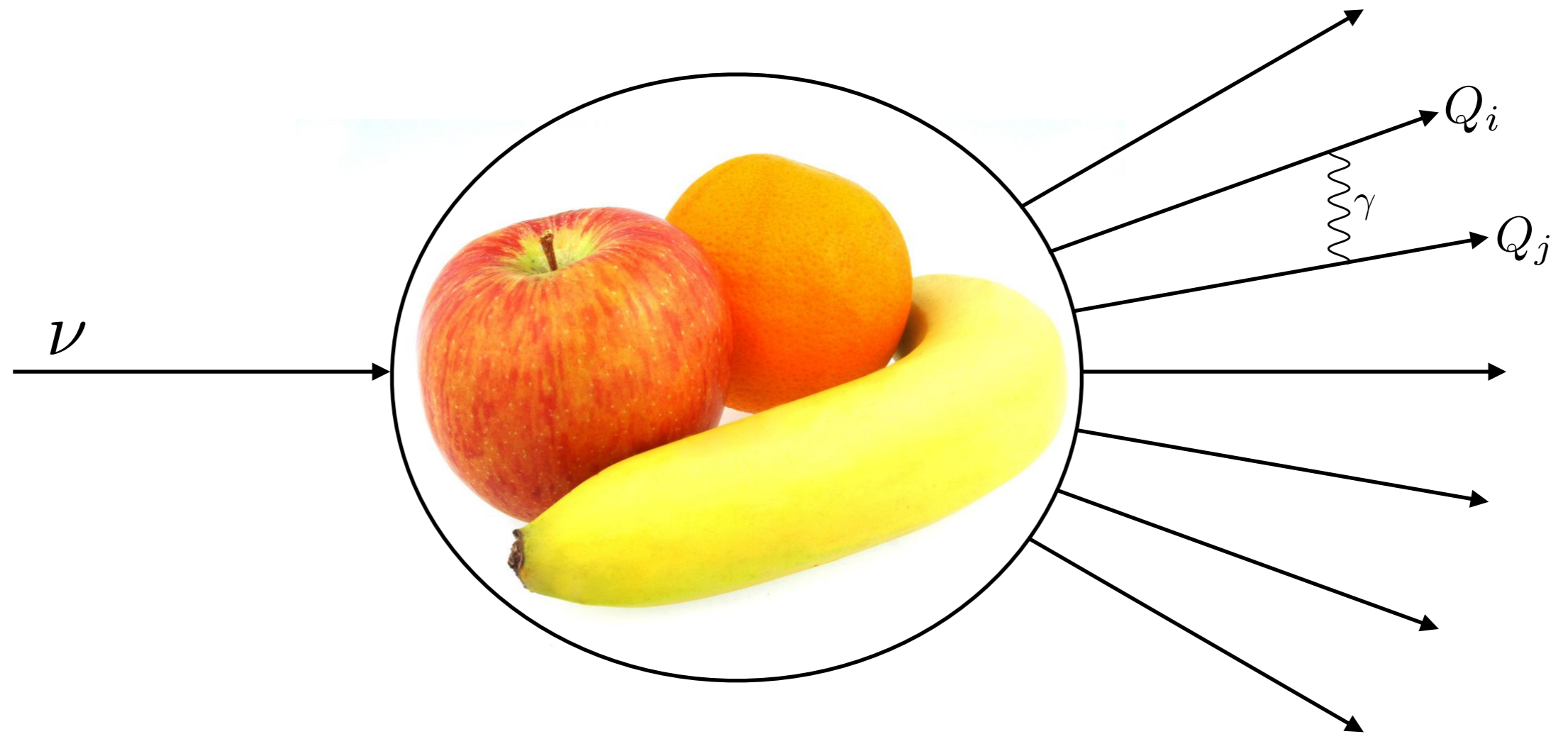
- all charged particles couple to real and virtual photons

QED corrections



- all charged particles couple to real and virtual photons

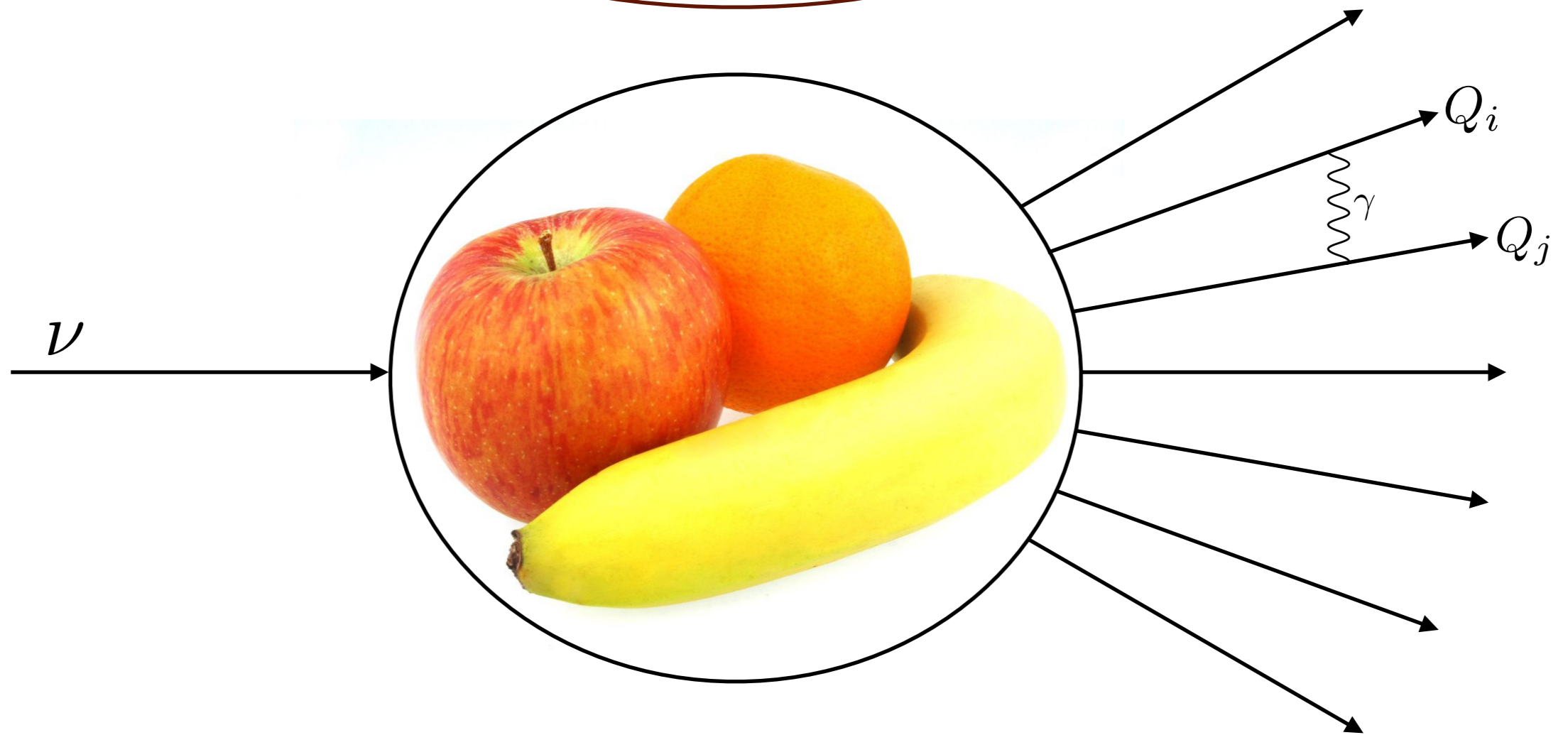
QED corrections



- $\frac{\alpha}{\pi} \sim 0.2\%$ suppression by electromagnetic coupling constant

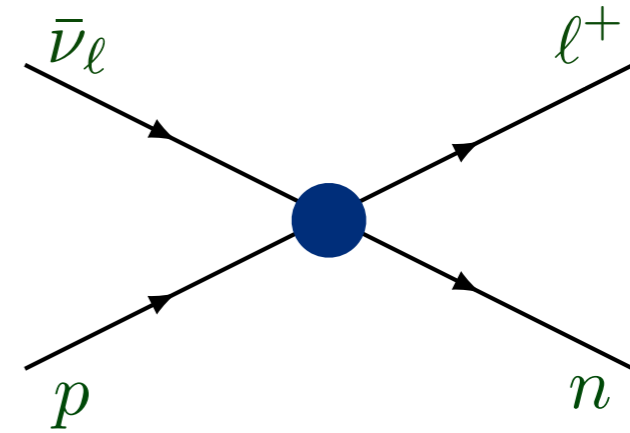
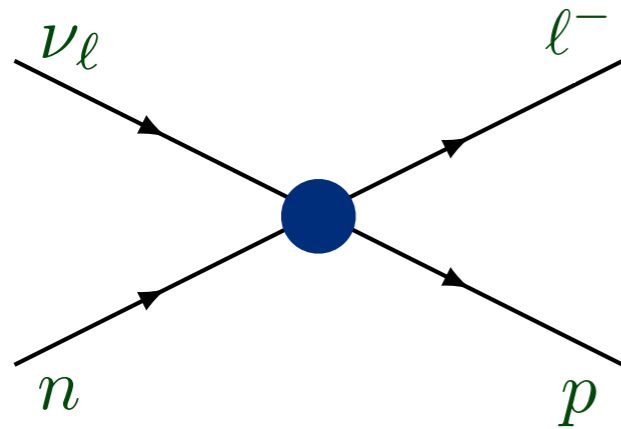
QED corrections

$$m_e \ll m_\mu \ll E_\nu$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms



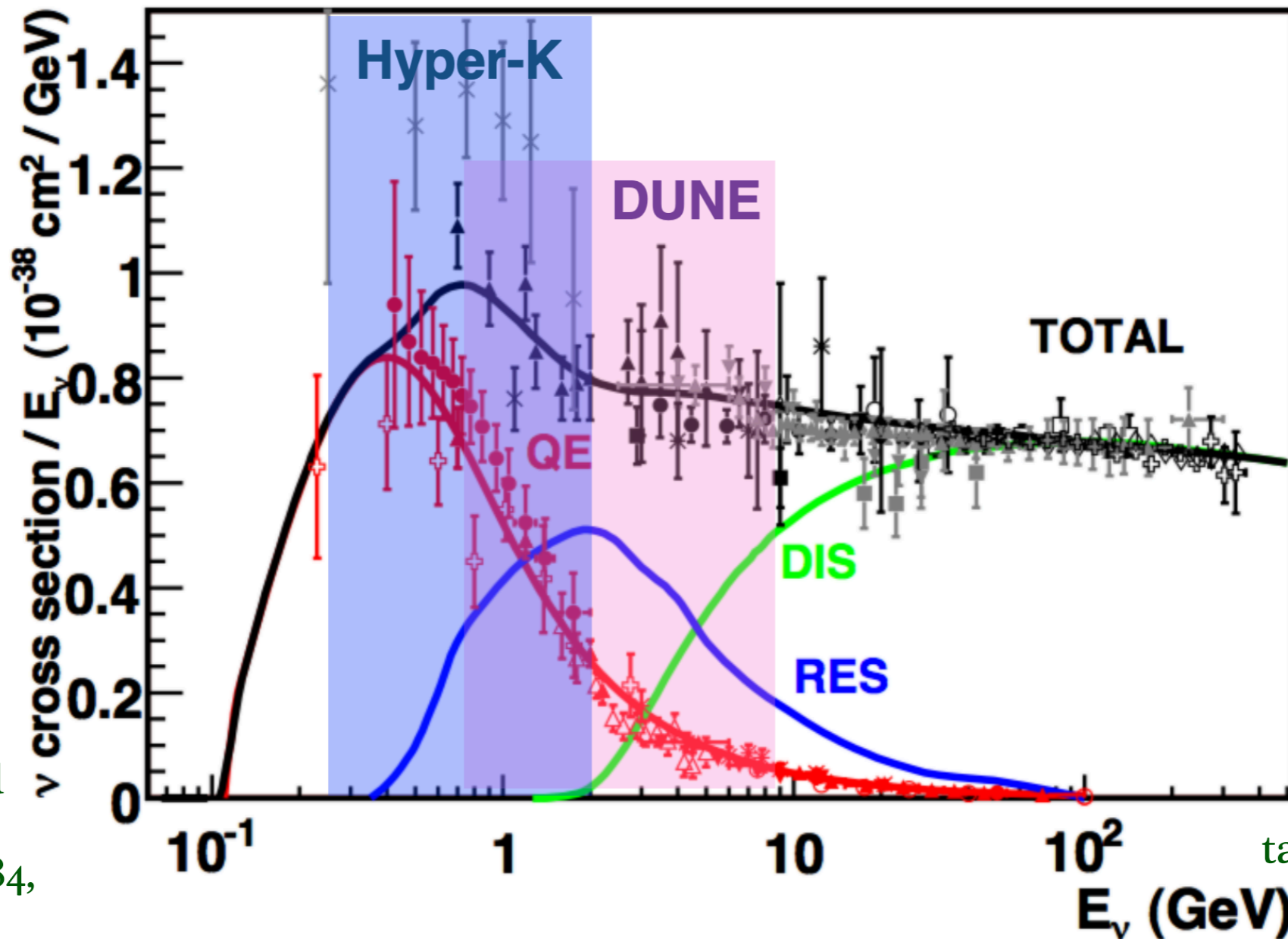
Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, arXiv: 2105.07939

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret, arXiv: 2204.10637

CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes

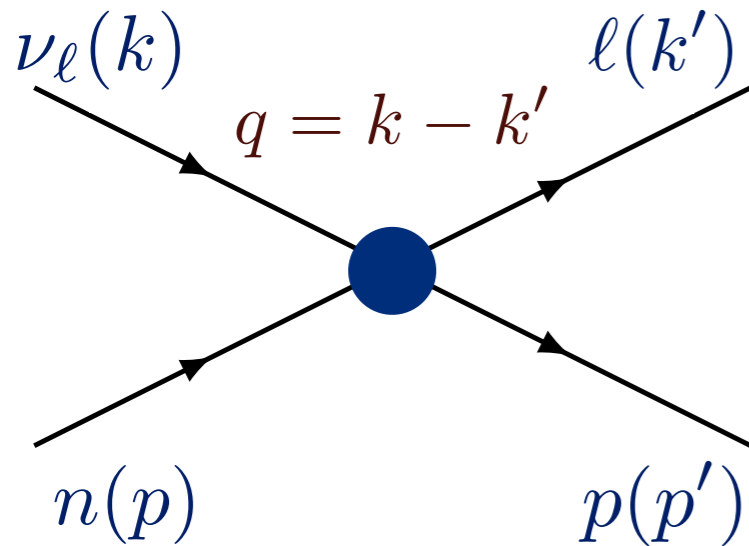


Formaggio and
Zeller
Rev. Mod. Phys. 84,
1307-1341 (2013)

Noemi Rocco
talk at Neutrino 2020

- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

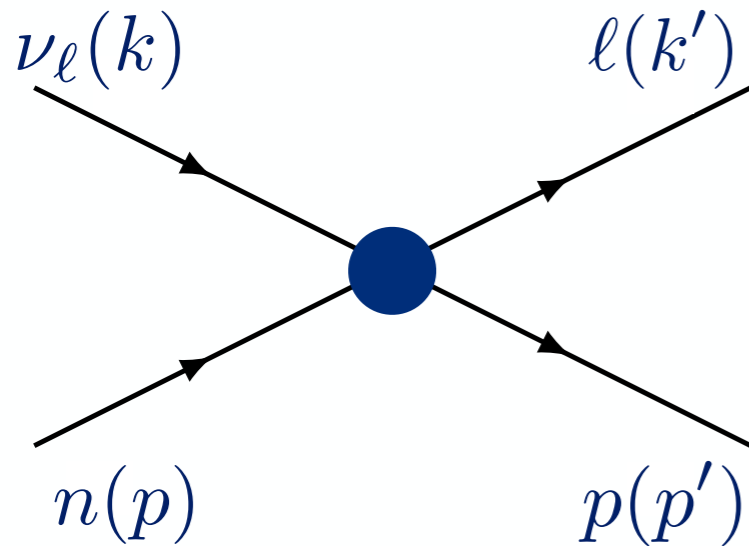
form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

CCQE scattering on free nucleon



$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \quad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left((\tau + r^2) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1 + \tau} C(Q^2) \right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau (G_M^V)^2 - (G_E^V)^2 + (1 + \tau) F_A^2 - r^2 \left((G_M^V)^2 + F_A^2 - \underline{4\tau F_P^2 + 4F_A F_P} \right)$$

$$B = \pm 4\tau F_A G_M^V$$

$$C = \tau (G_M^V)^2 + (G_E^V)^2 + (1 + \tau) F_A^2$$

- **pseudoscalar** form factor contribution is suppressed by lepton mass
- cross section is sensitive to both **vector** and **axial** contributions

Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber
- direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events



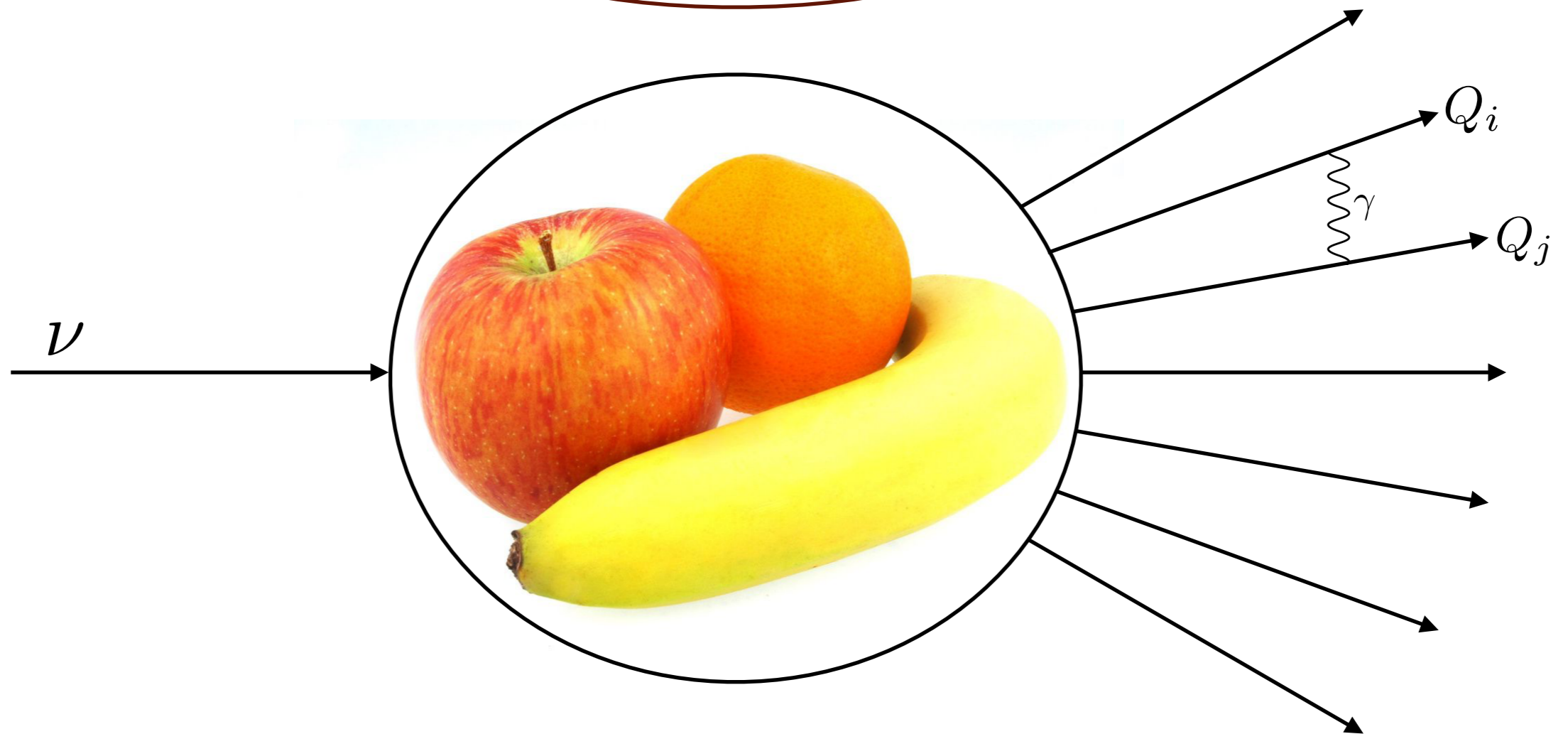
Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill, Phys. Rev. D 93 11, 11305 (2016)

QED corrections

$$m_e \ll m_\mu \ll E_\nu$$

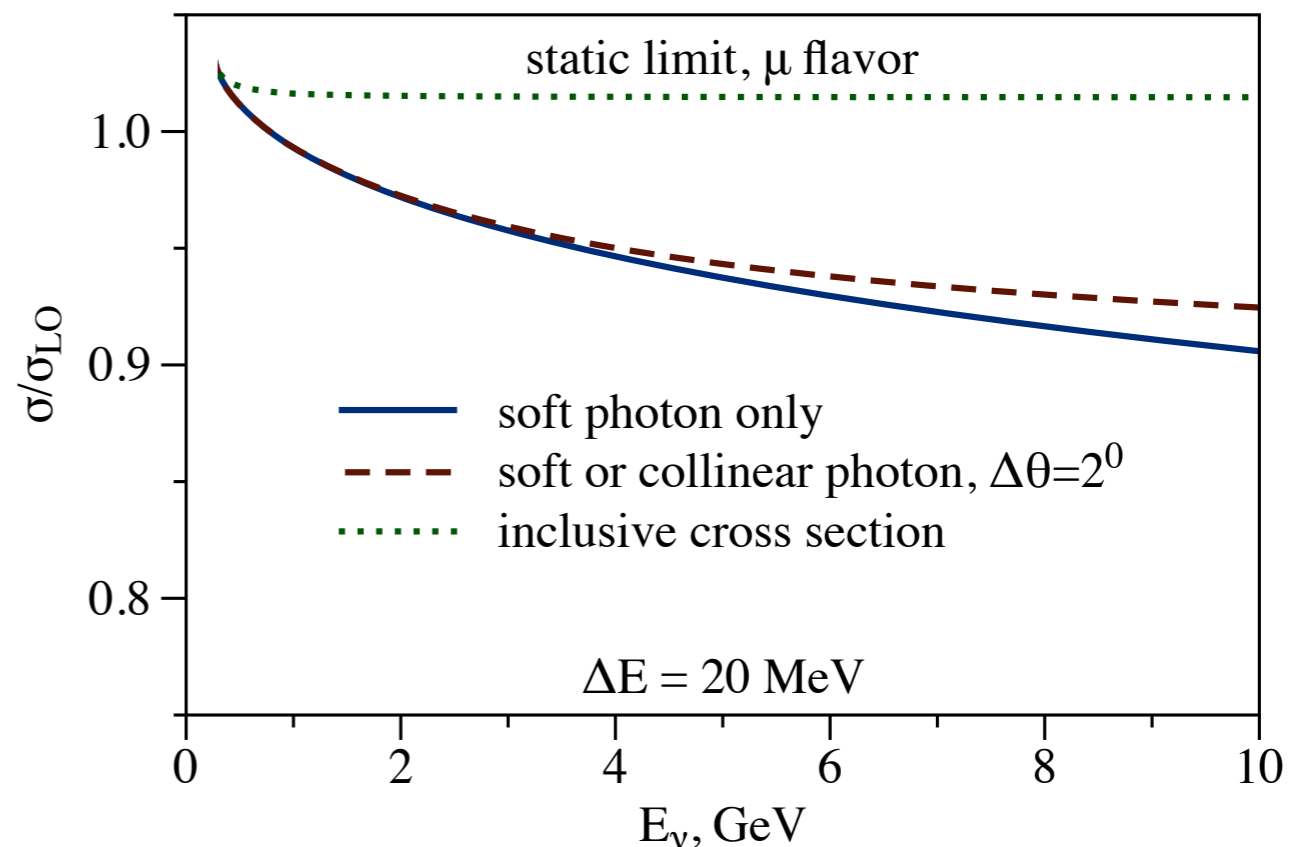
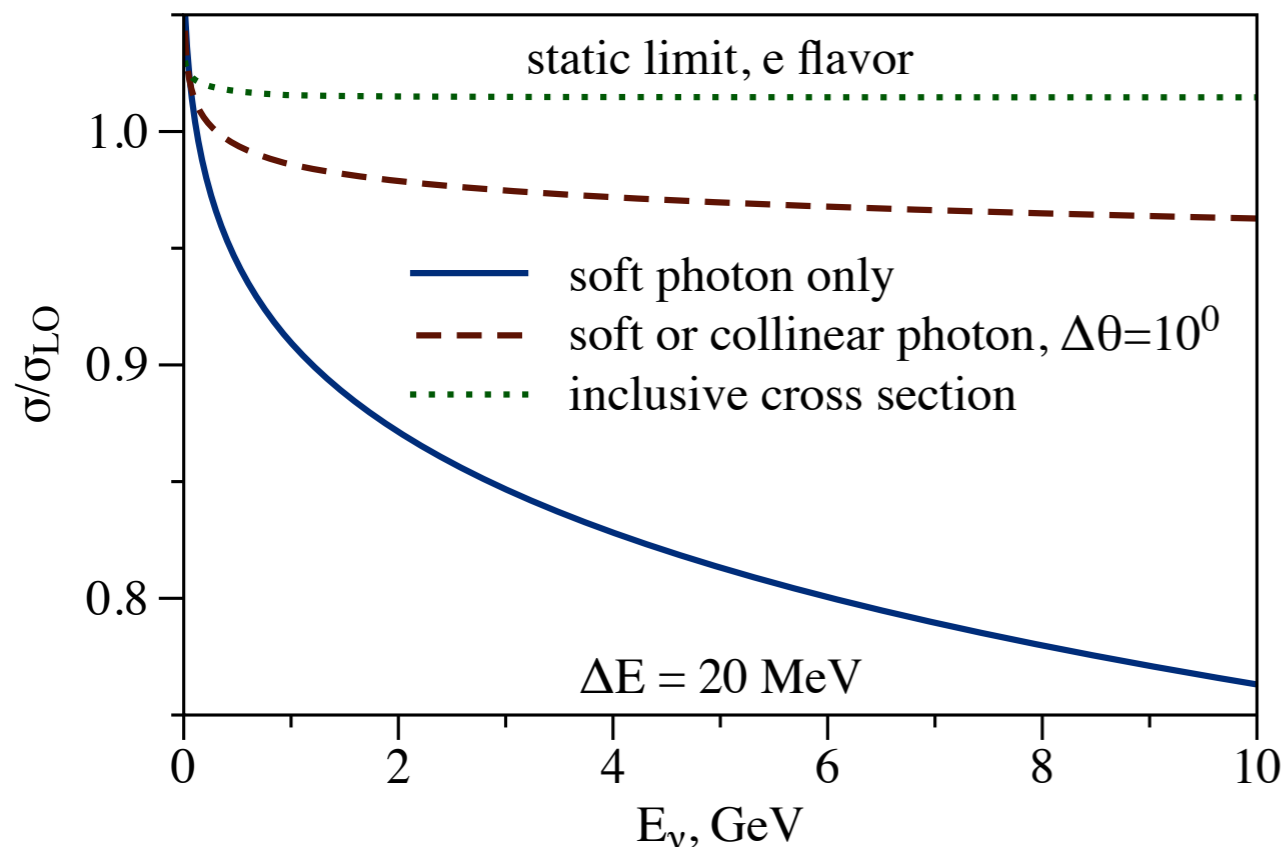


$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms

Static nucleon limit

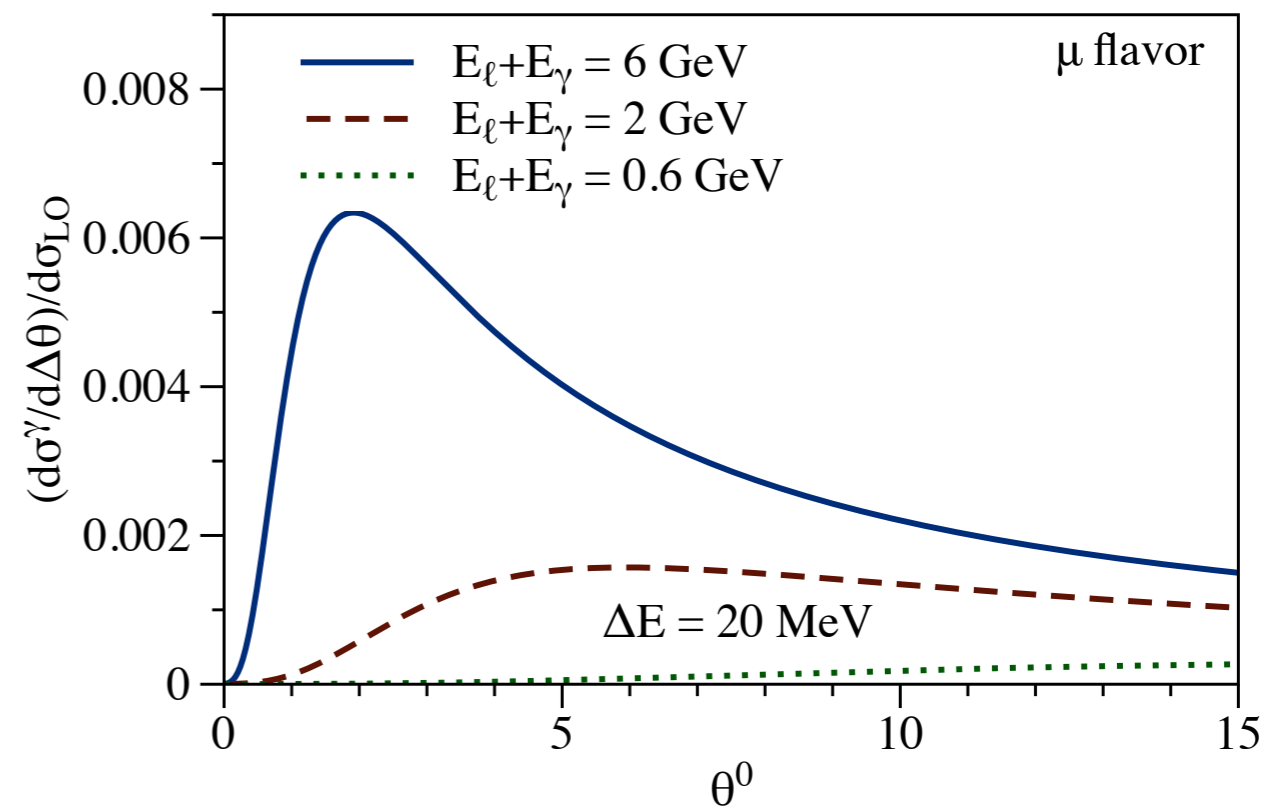
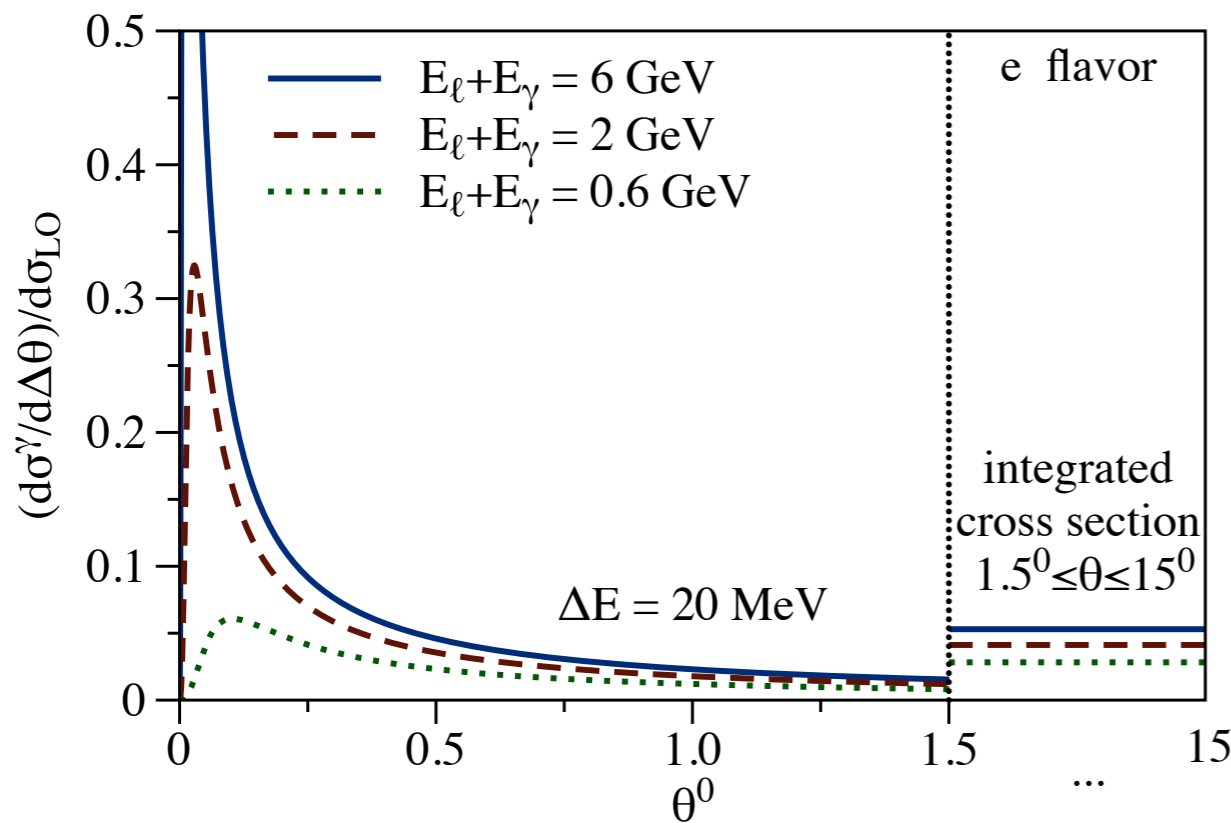
- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+ γ): few % level, flavor independent

Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

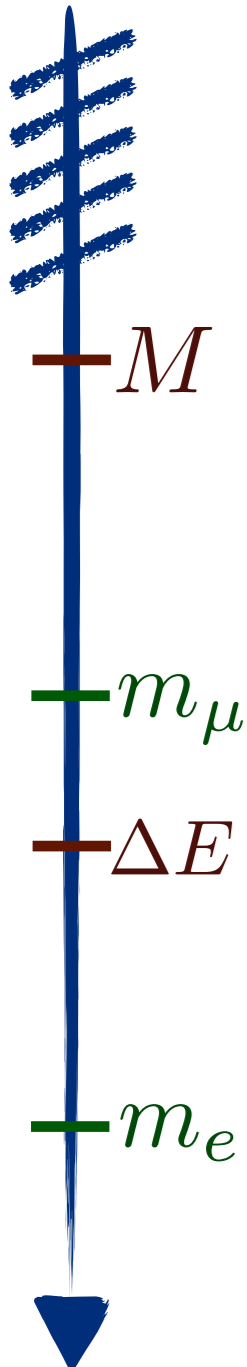
Factorization approach

- cross section is given by **factorization formula**

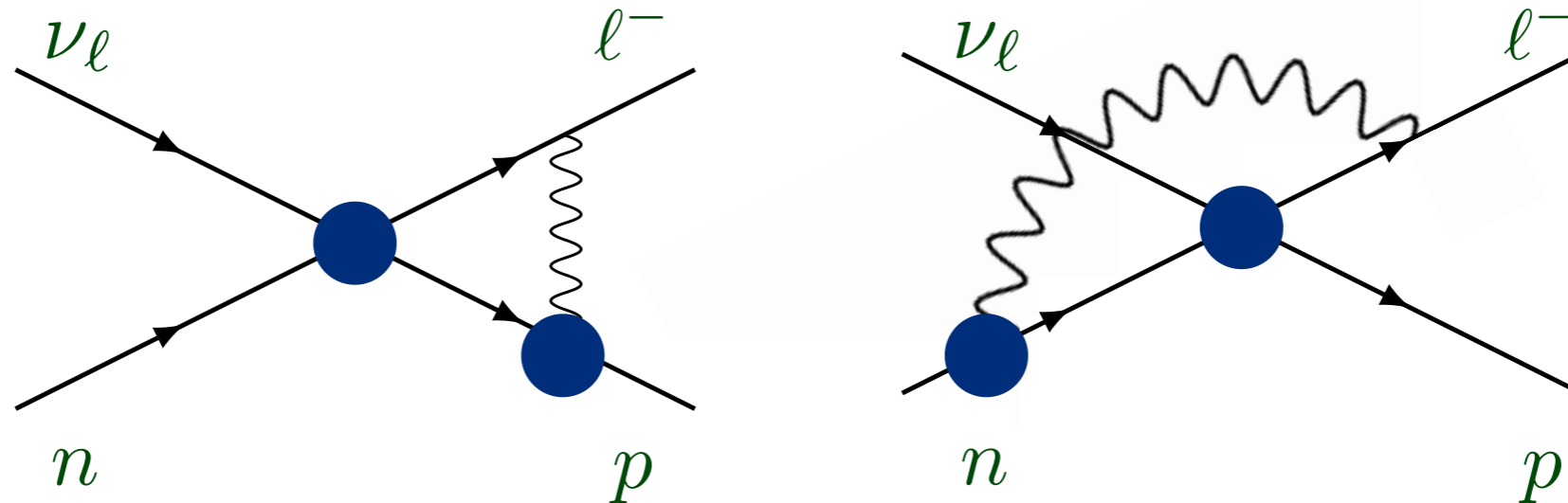
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated **perturbatively**



Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors
discussed for neutrino-nucleon scattering: Graczyk, Phys. Lett. B 732, 315-319 (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach

- cross section is given by **factorization formula**

$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- determine **hard function** at hard scale by matching experiment or **hadronic model** to the theory with heavy nucleon

- **RGE evolution** of the hard function to scales $\Delta E, m_\ell$

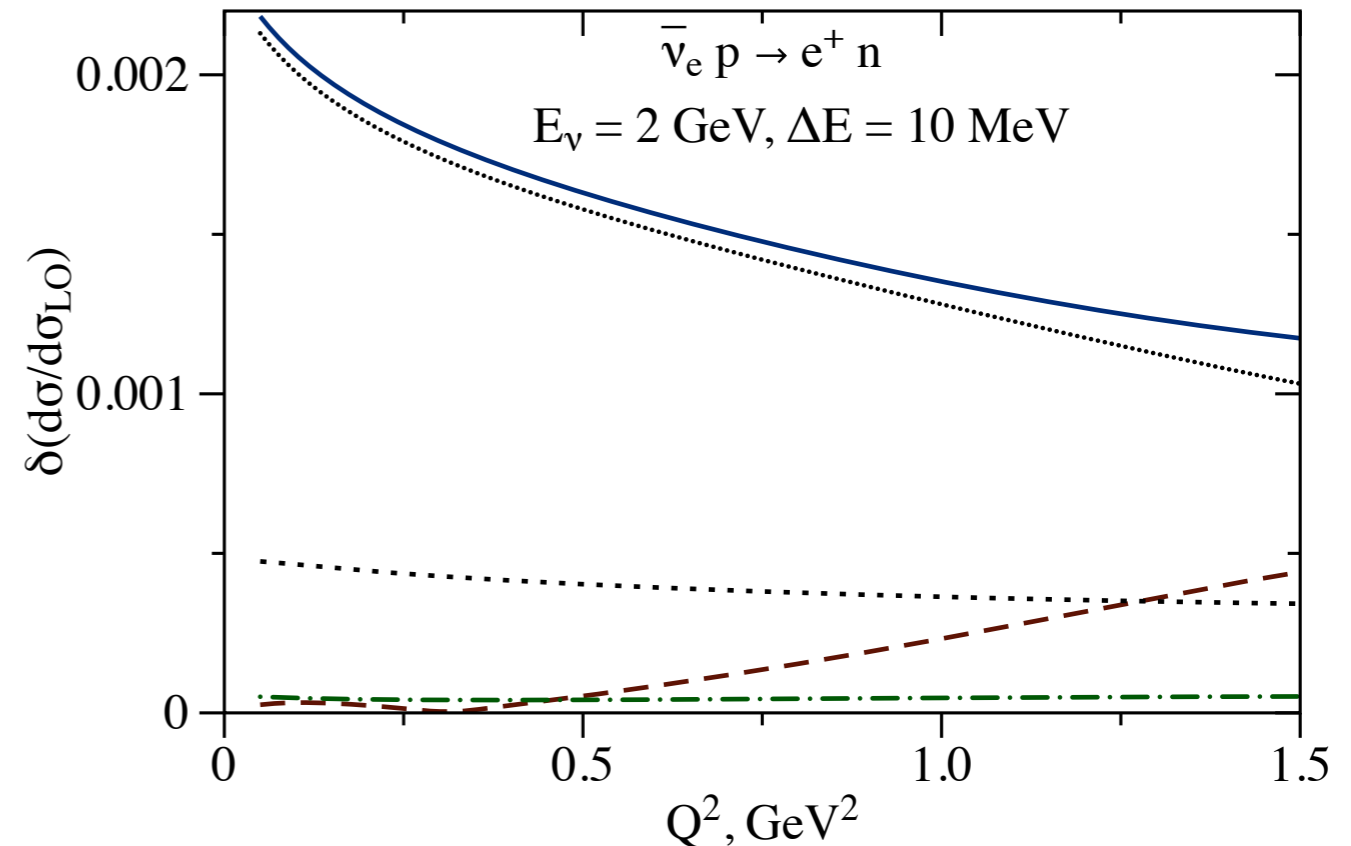
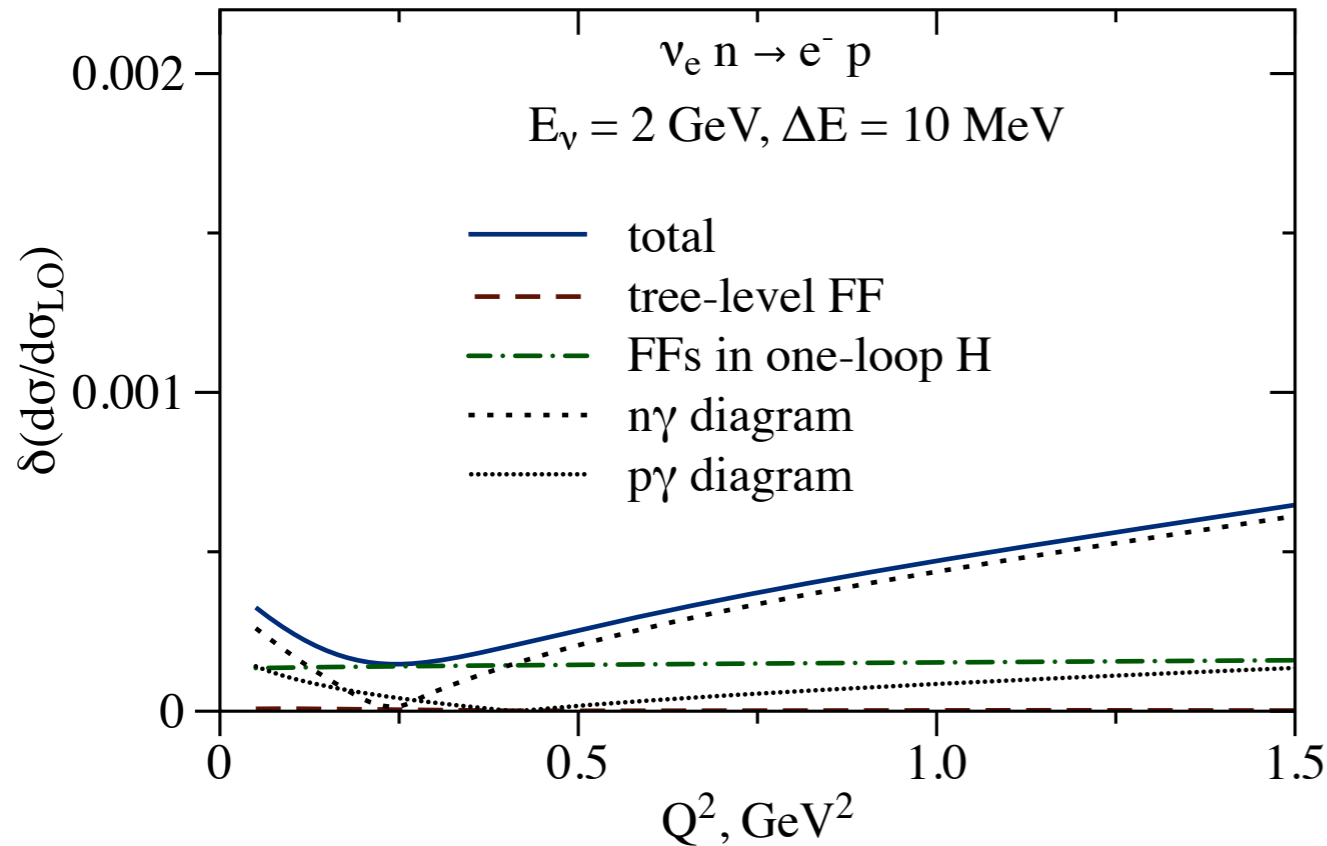
- **soft and collinear functions** are evaluated **perturbatively**

- calculate cross section at low energies accounting for **all large logs**
ep scattering with soft radiation only: Richard J. Hill, Phys. Rev. D 95 1, 013001 (2016)

- **soft and collinear functions** determined **analytically**
- **hard function** describes physics at GeV energies

Error budget

- uncertainties from hard function



A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill, Phys. Rev. D 93 11, 11305 (2016)

- nucleon form factors

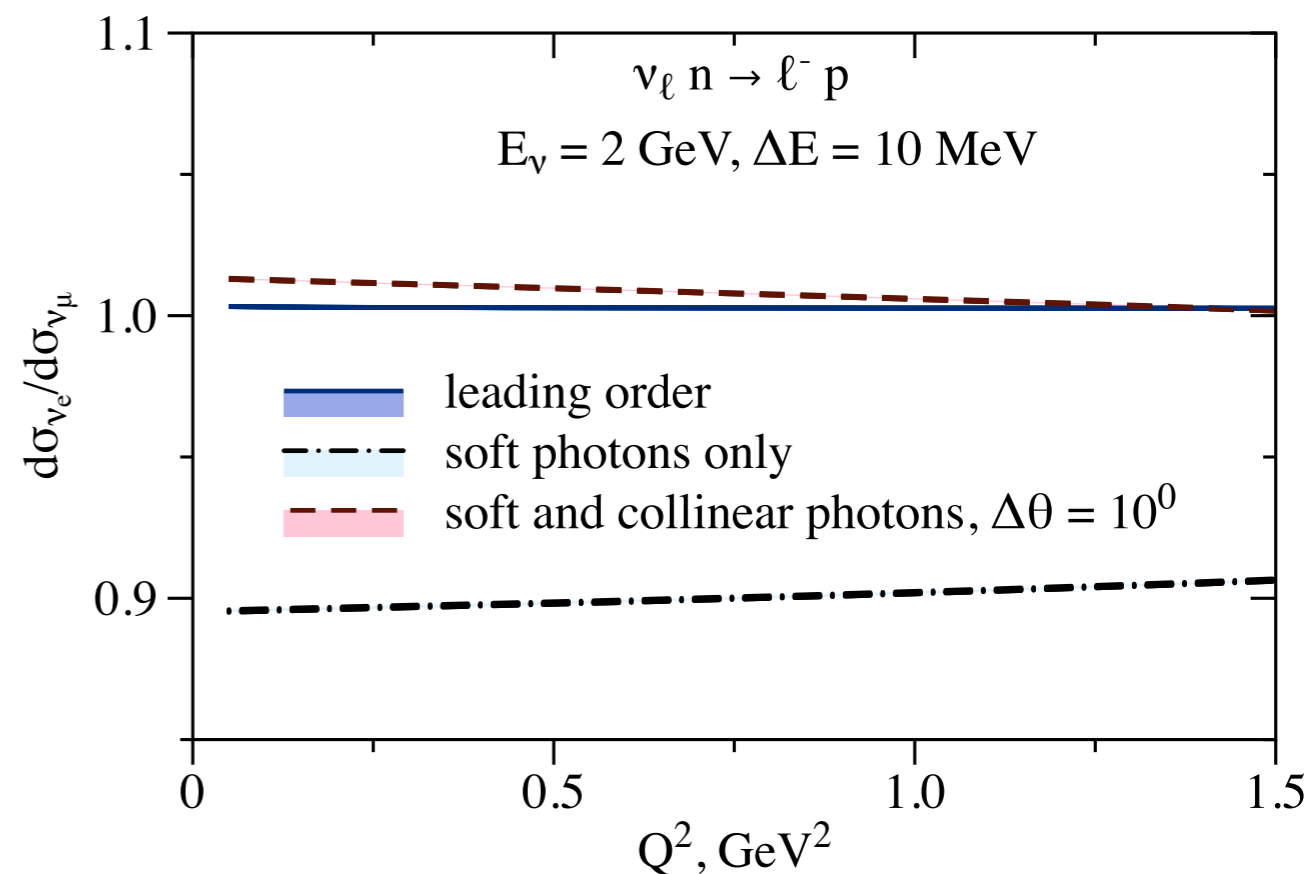
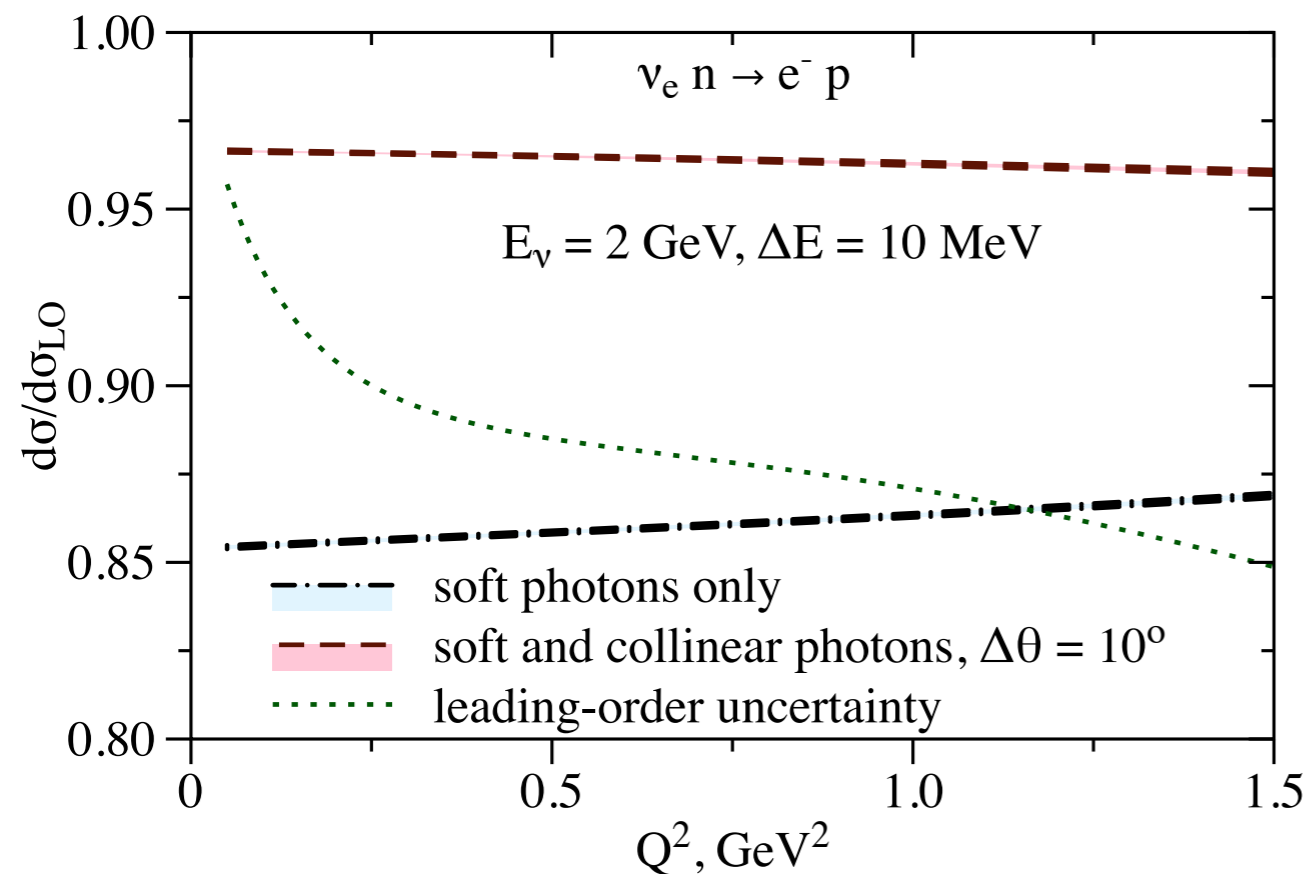
Kaushik Borah, Gabriel Lee, Richard J. Hill and O.T., Phys. Rev. D 102 7, 074012 (2020)

- add perturbative uncertainty by variation of scale

- uncertainty of permille level for the ratio to LO result

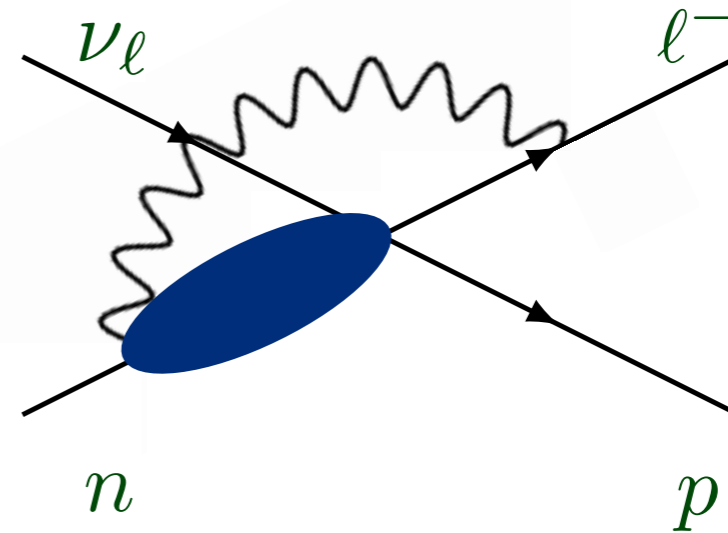
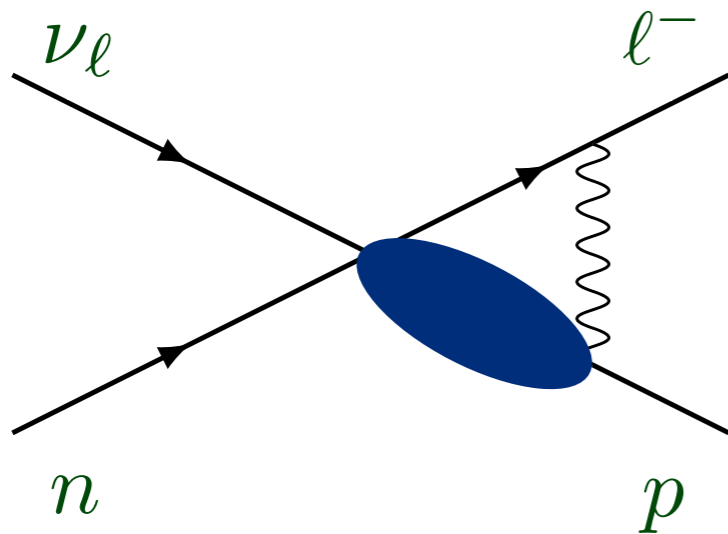
Exclusive observables

- cancellation of uncertainties from hard function for e/μ and ratio to LO

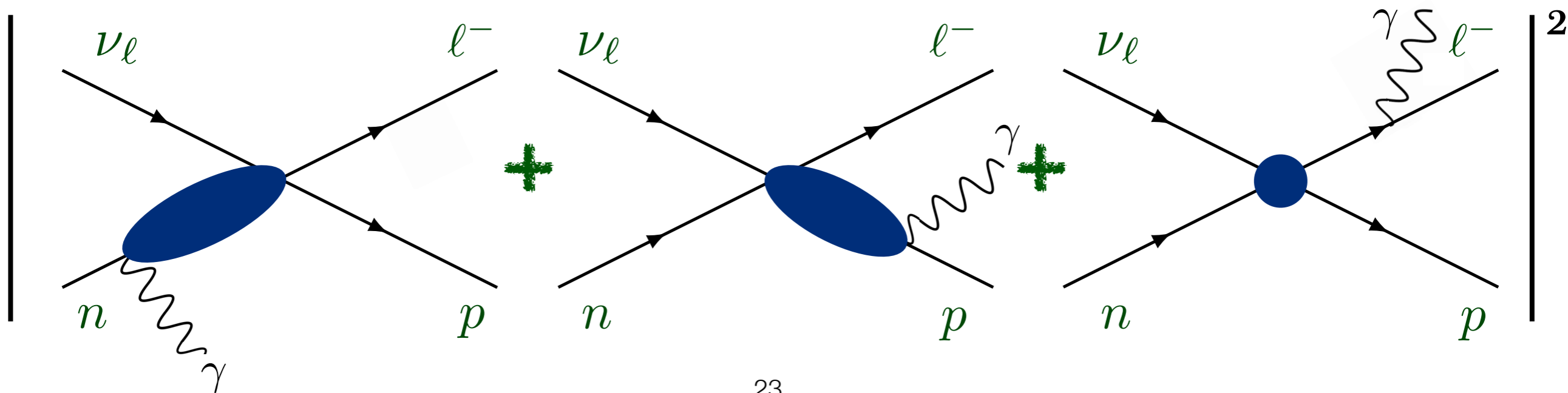


- ratios: cancellation of uncertainty from hard function

Inclusive observables

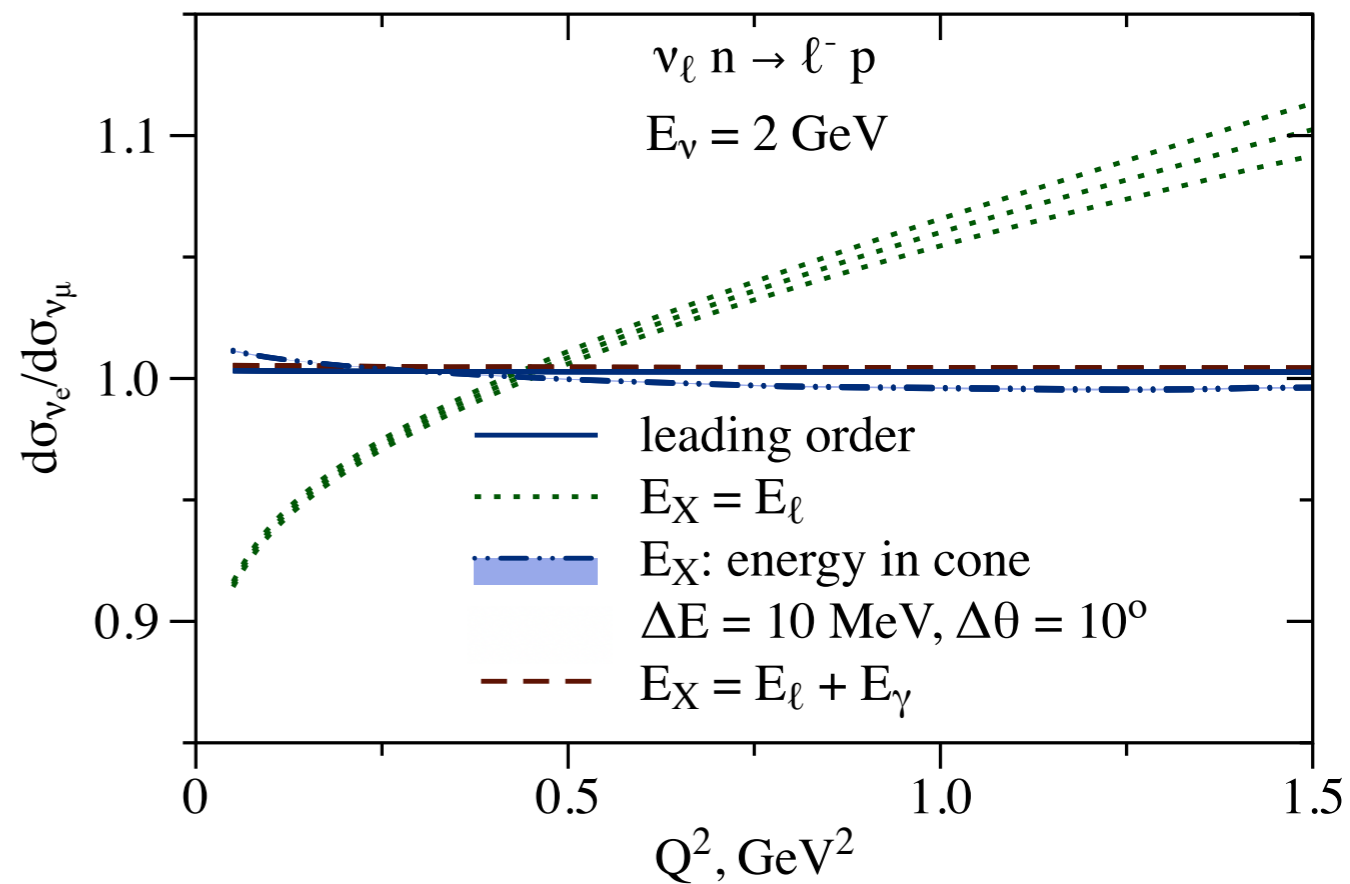
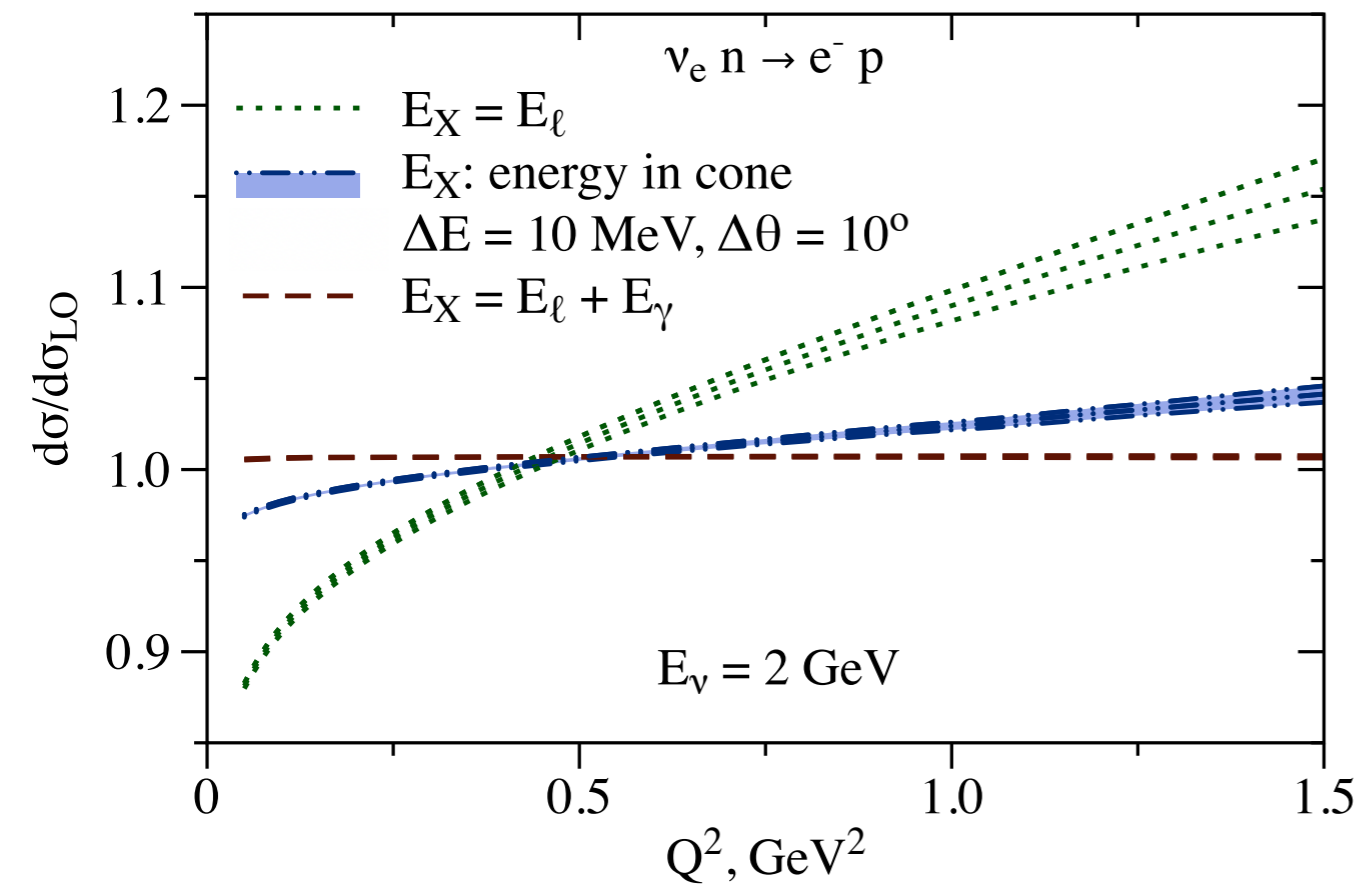


- the same gauge-invariant model for the real radiation
- arbitrary hard photons are part of the observable



Inclusive observables

- kinematics $Q^2 = 2M(E_\nu - E_X)$ is reconstructed with 3 different E_X



- dependence on reconstruction of kinematics and cuts
- predict σ_{ν_e} from σ_{ν_μ} measurements with neutrino beam



Electron/muon ratio

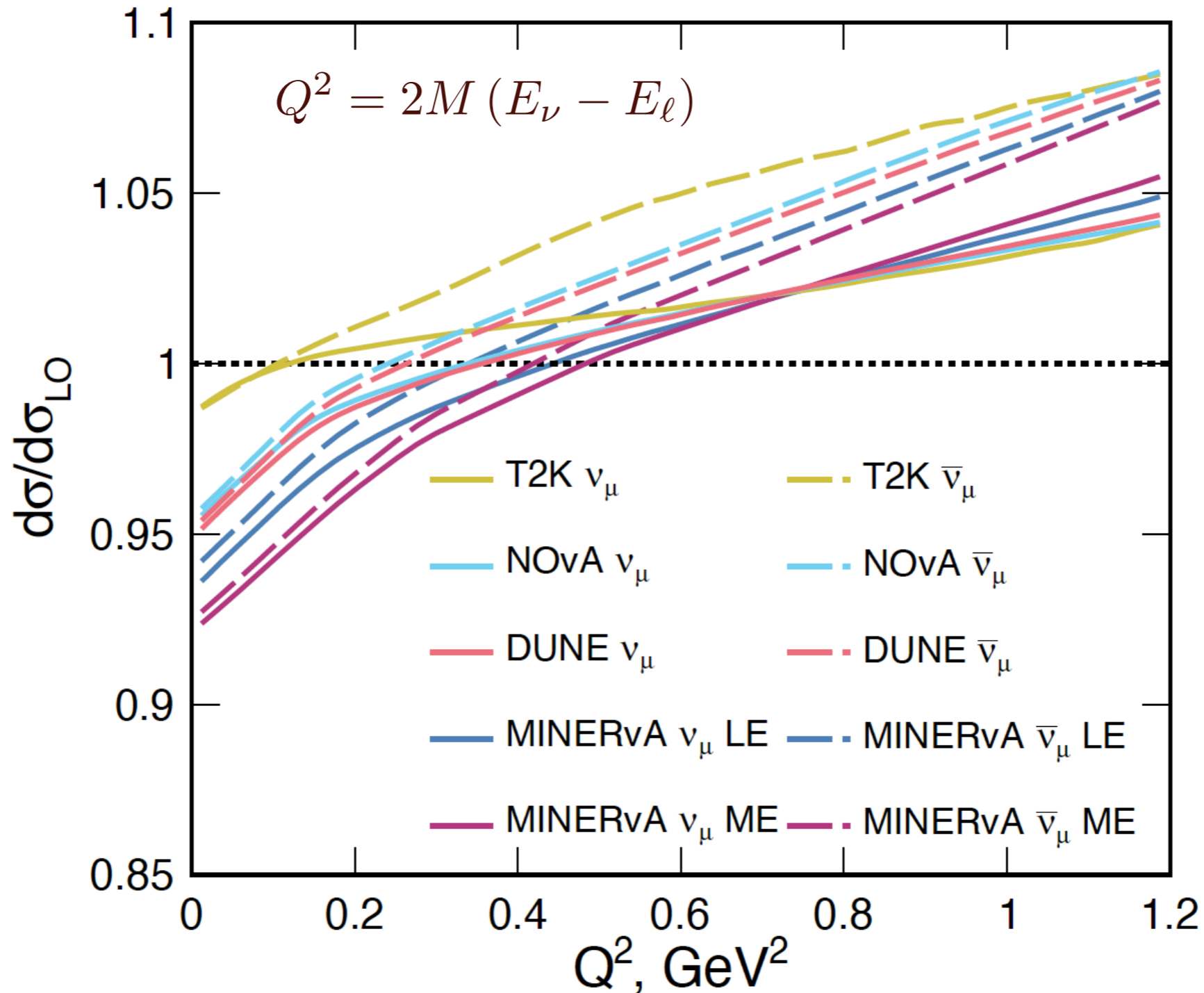
| | E_ν , GeV | | $\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\text{LO}}$, % | $\frac{\sigma_e}{\sigma_\mu} - 1$, % |
|------------|---------------|-------------|--|---------------------------------------|
| T2K/HyperK | 0.6 | ν | 2.47 ± 0.06 | $2.84 \pm 0.06 \pm 0.37$ |
| | | $\bar{\nu}$ | 2.04 ± 0.08 | $1.84 \pm 0.08 \pm 0.20$ |
| NOvA/DUNE | 2.0 | ν | 0.322 ± 0.006 | $0.54 \pm 0.01 \pm 0.22$ |
| | | $\bar{\nu}$ | 0.394 ± 0.003 | $0.20 \pm 0.01 \pm 0.19$ |

TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma(m_\ell \rightarrow 0)}{\sigma(m_\ell = 0)} \approx 1 + Am_\ell^2 + \alpha Bm_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameter squared

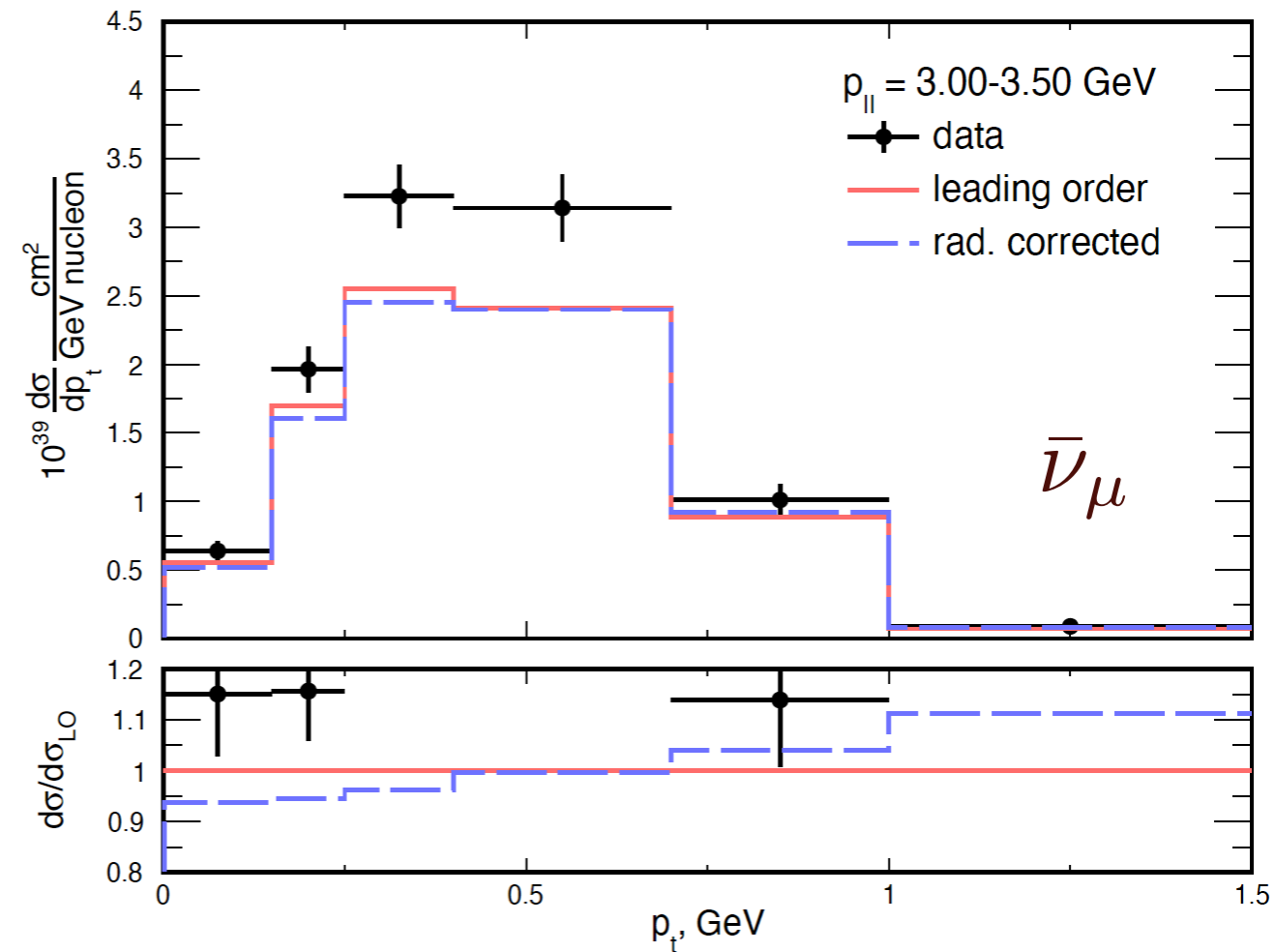
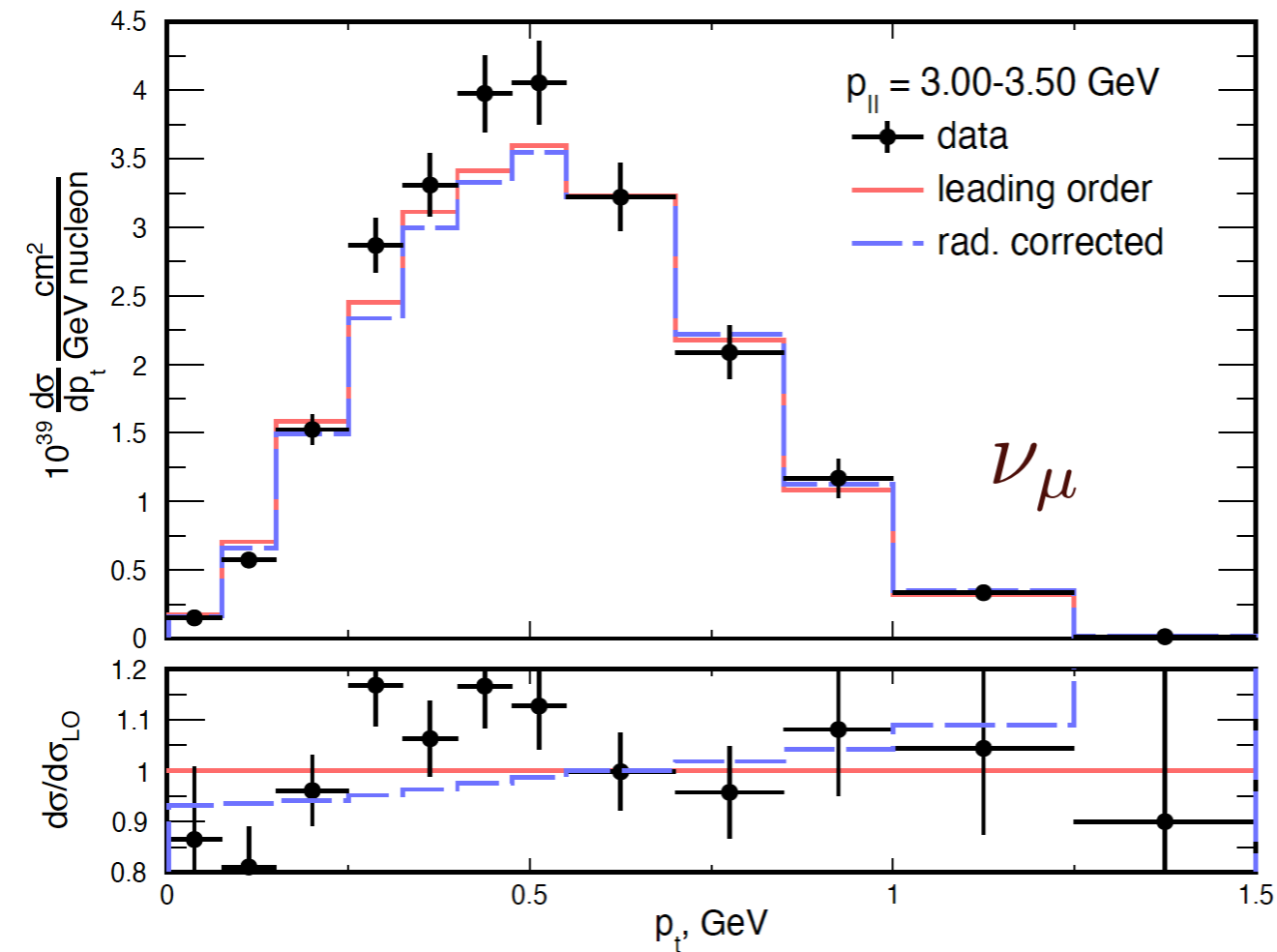
Comparison to data



- lepton energy spectra with kinematics from the lepton leg
- generator NEUT + flux average over typical experiments

Comparison to data

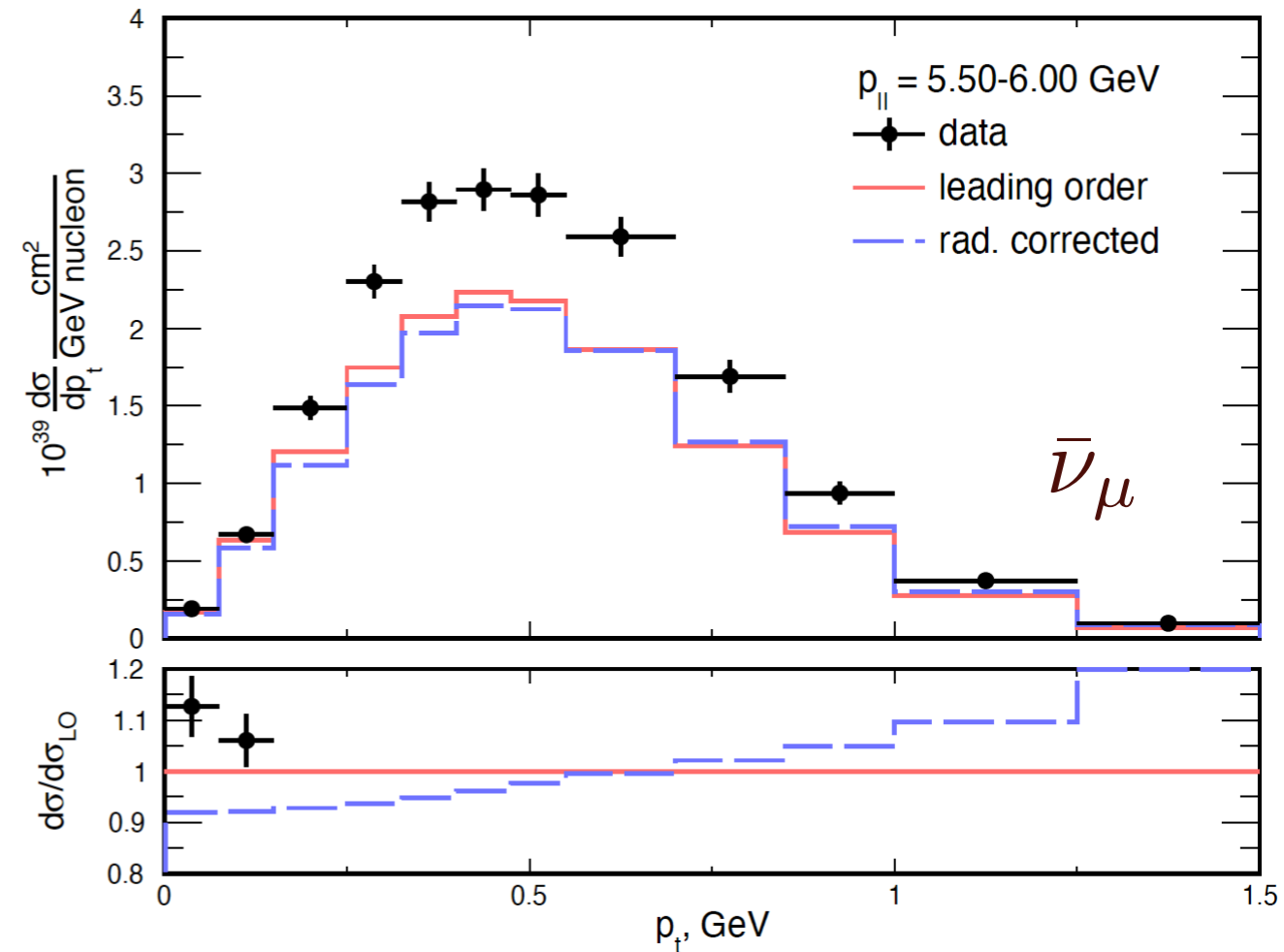
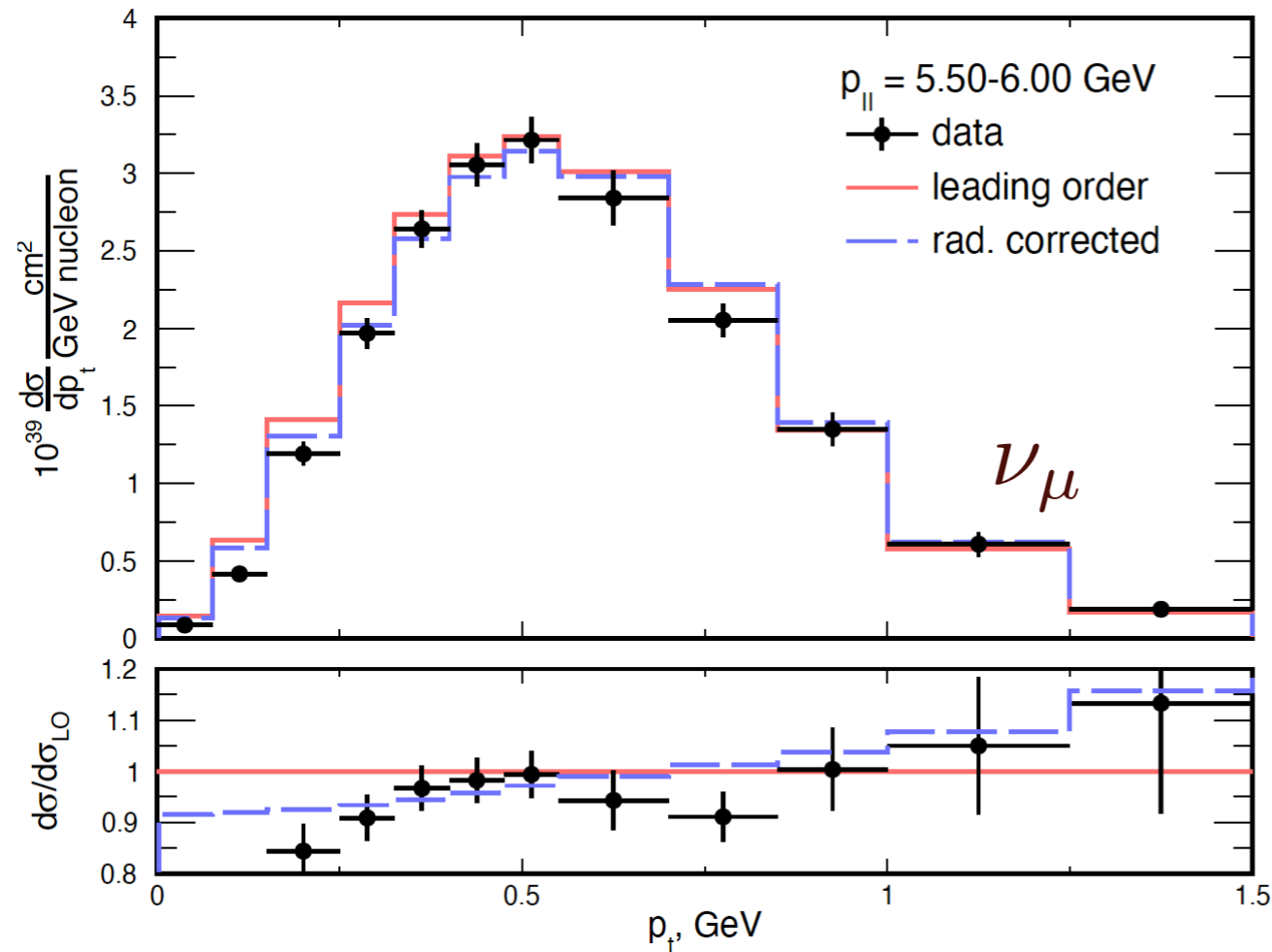
- low-energy flux data from MINERvA@FERMILAB



- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Comparison to data

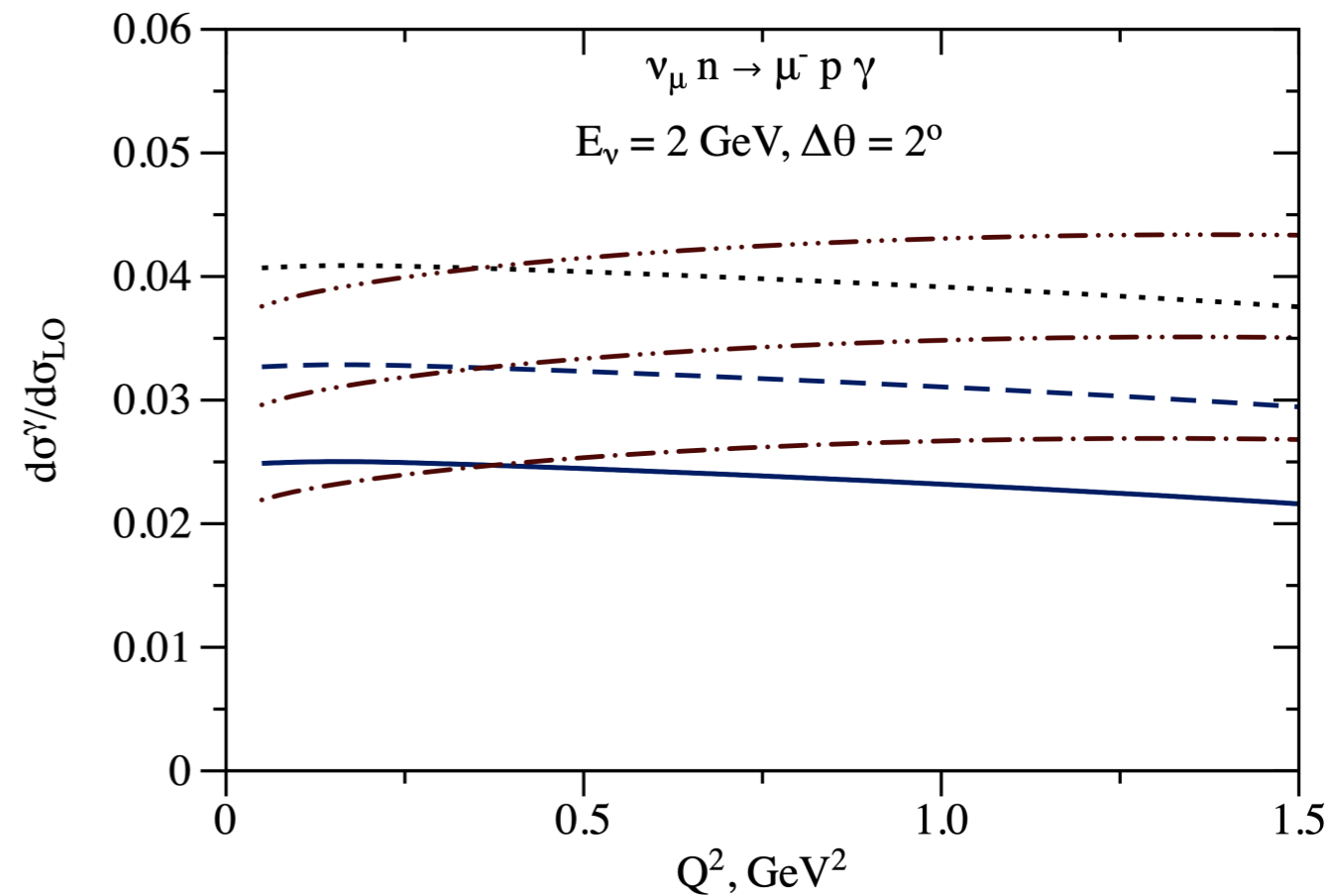
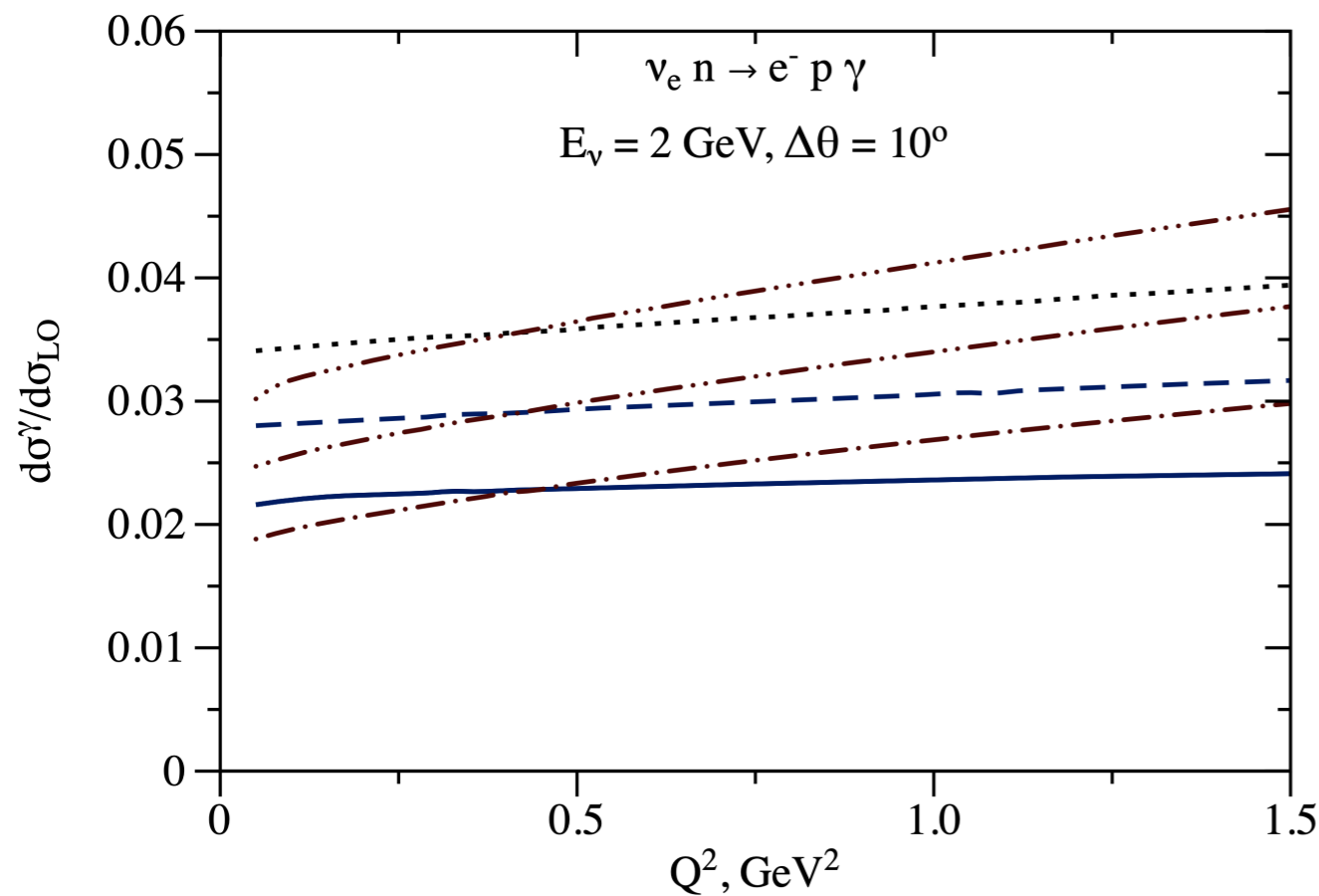
- medium-energy flux data from MINERvA@FERMILAB



- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Radiation of hard photons

- model-dependent description for radiation of hard photons

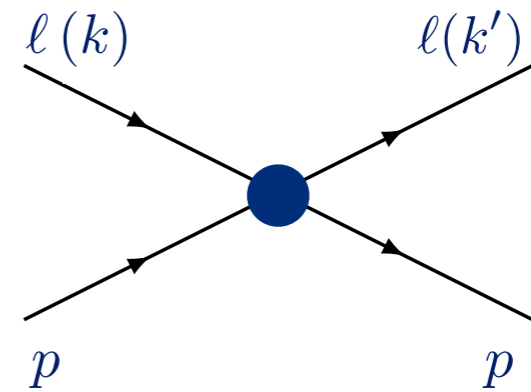
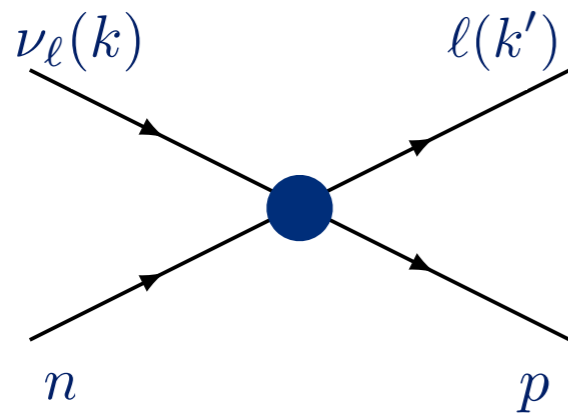


“Blunden calculation”

- photon energies are above 20, 40, and 80 MeV: default vs “SIFF”

“hadronic model”

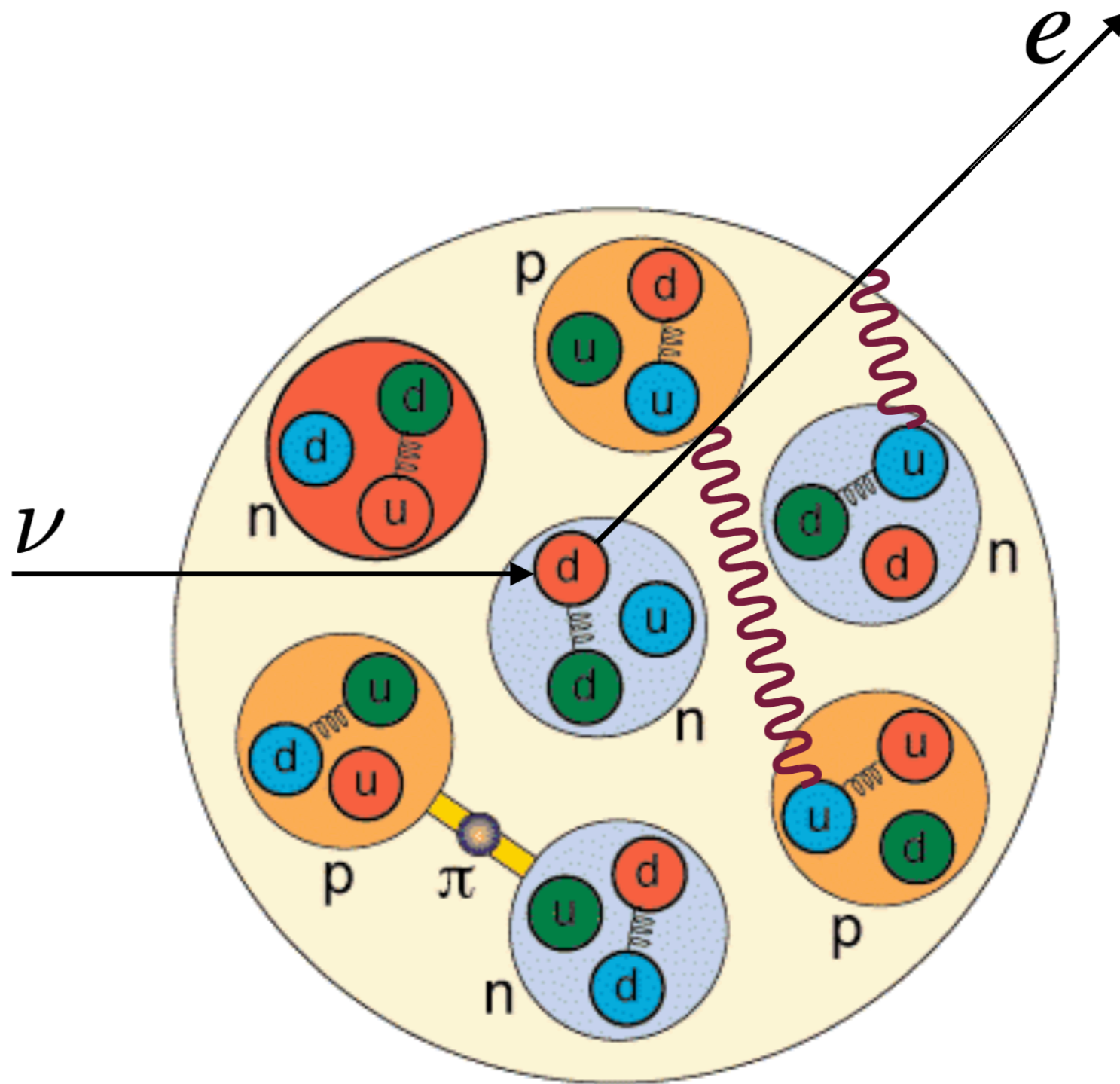
- %-level radiation of non-collinear hard photons
- 10^{-4} flavor misidentification rate for NOvA&T2K kinematics



QED nuclear medium effects in neutrino-nucleus and electron-nucleus scattering

O. T. and Ivan Vitev (arXiv: 2206.10637)

QED medium effects

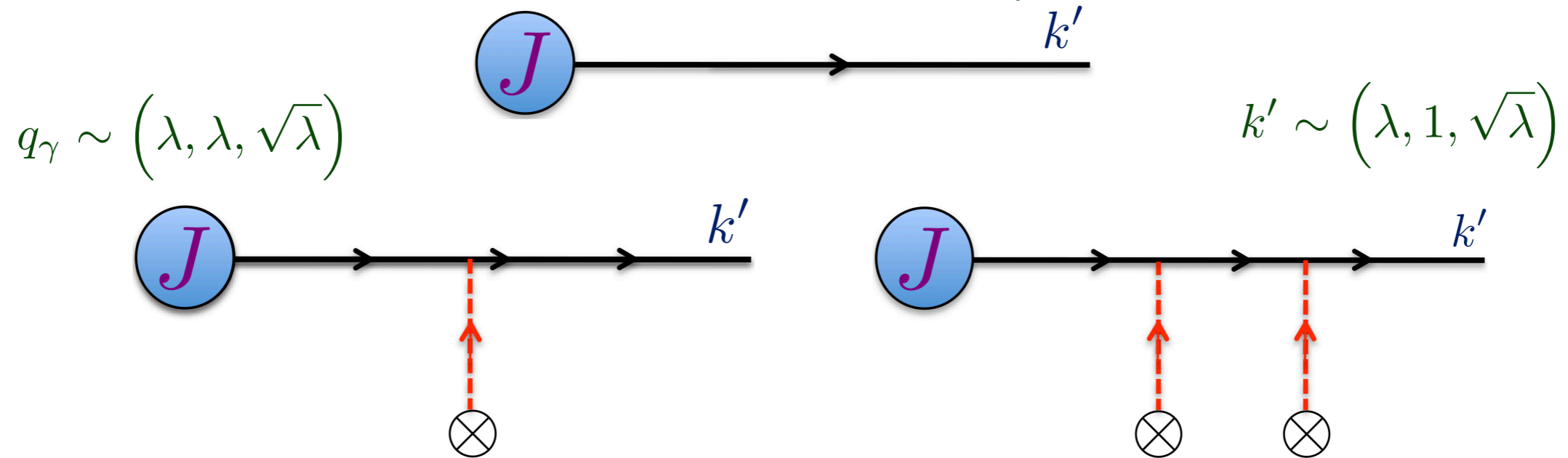


- charged lepton exchanges photons with nuclear medium

SCET_G formulation

- forward scattering is dominant process
- Glauber photons exchanged with a **nuclear charge distribution**

QCD: G. Ovanesyan and I. Vitev, JHEP 06 080 (2011)



- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k}, \vec{k}' - \vec{q}_\perp) - \sigma_0(\vec{k}, \vec{k}') \right)$$

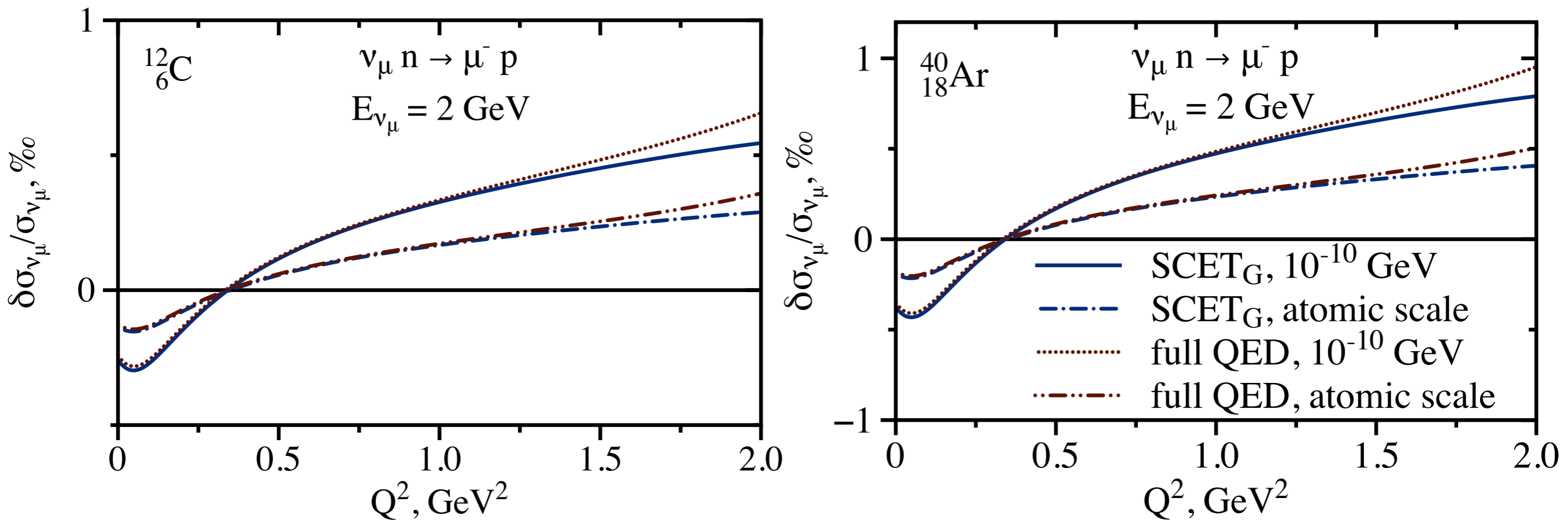
- leading-order cross sections are distorted
- EFT and full QED calculations are performed

Neutrino scattering

IR regularization

$$v(q_{\perp}^2) = \frac{e^2}{q_{\perp}^2 + \lambda^2}$$

- relative correction per nucleon



flavor-independent at GeV energies

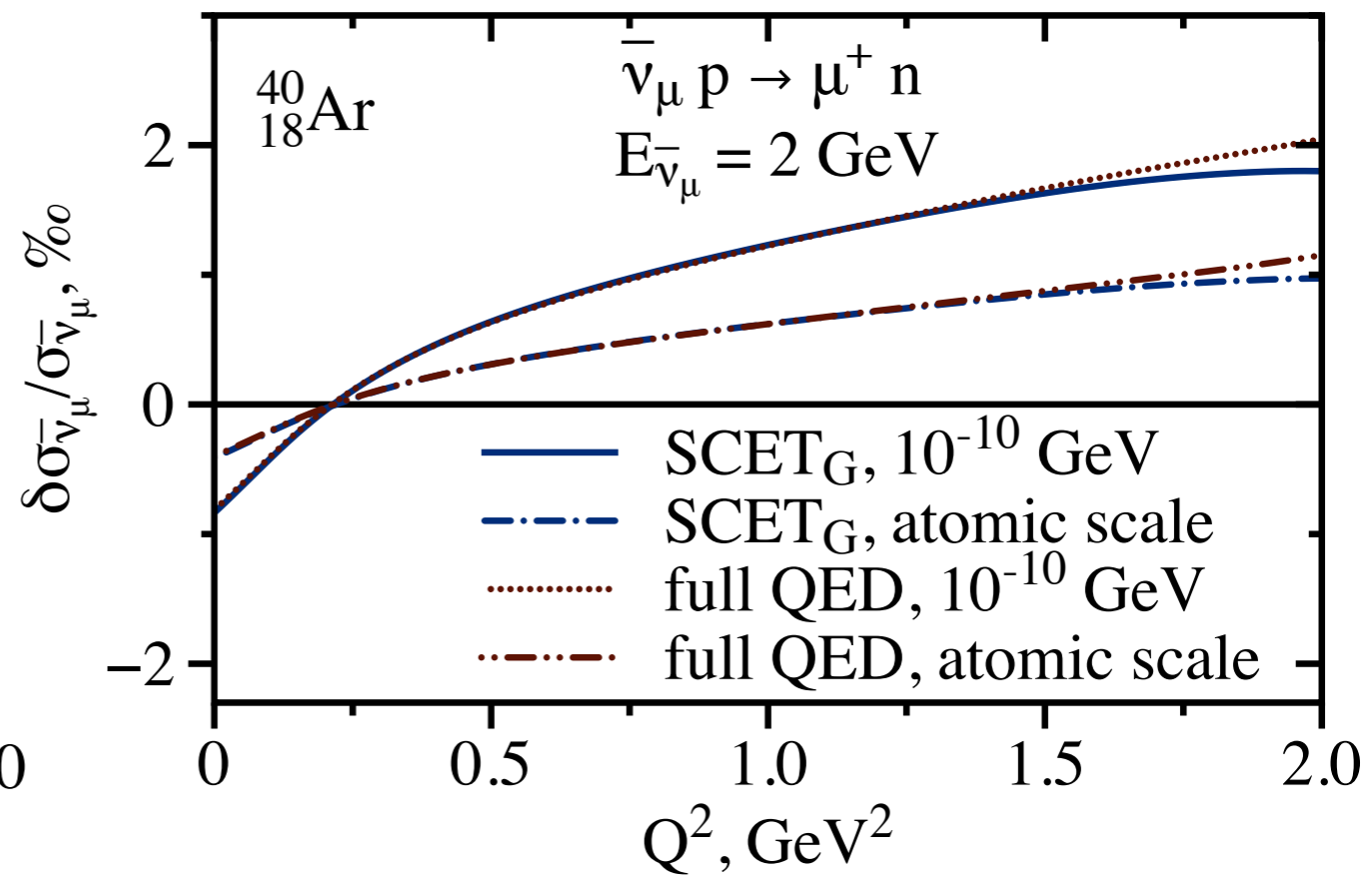
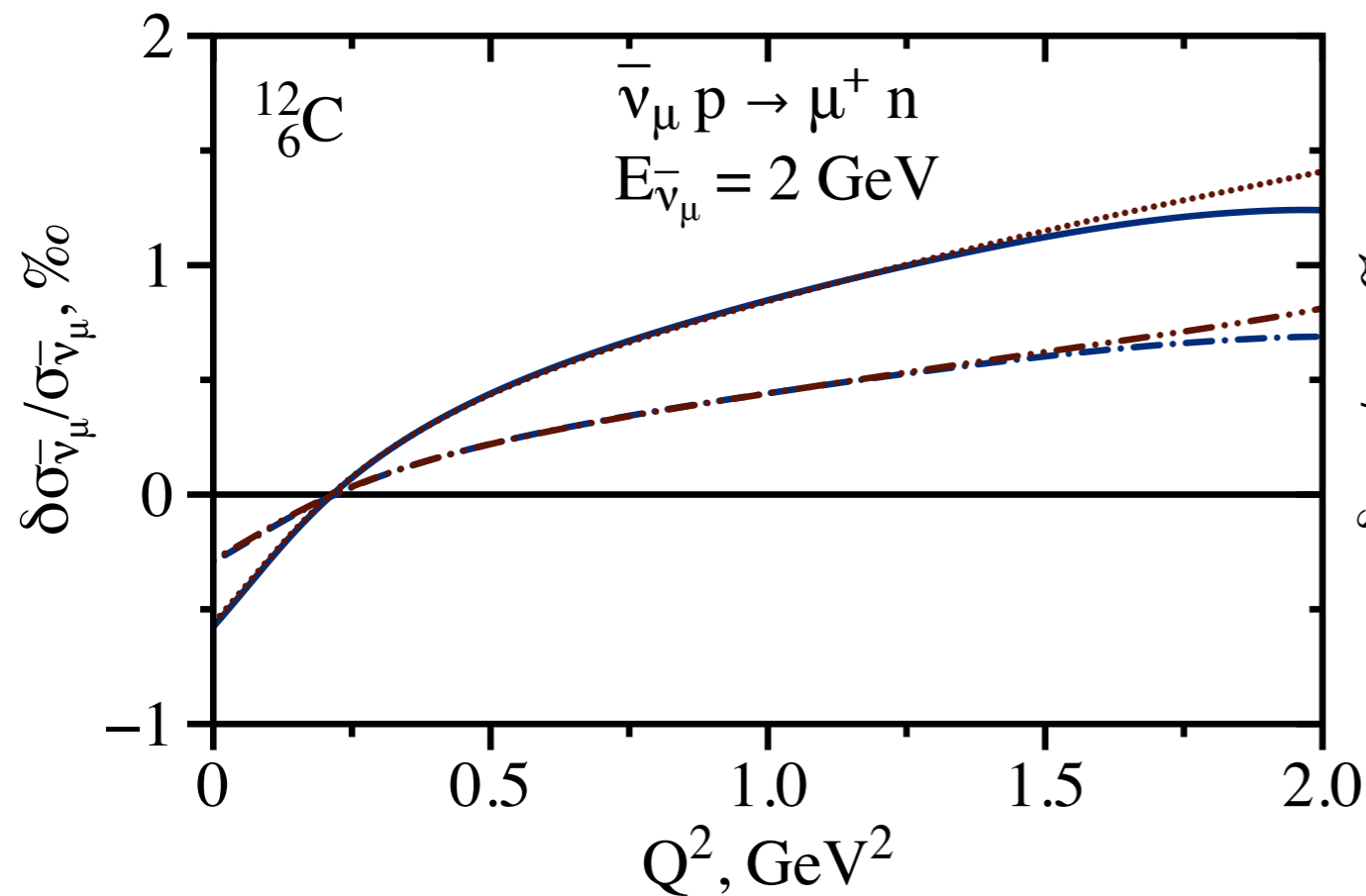
- permille-level distortion of cross sections: $\mathcal{O}(\alpha^2)$ correction
- smaller correction to inclusive cross section

Antineutrino scattering

- relative correction per nucleon

IR regularization

$$v(q_{\perp}^2) = \frac{e^2}{q_{\perp}^2 + \lambda^2}$$



flavor-independent at GeV energies

- permille-level distortion of cross sections: $\mathcal{O}(\alpha^2)$ correction
- larger correction than for neutrino scattering

SCET_G formulation

- forward scattering is dominant process
- Glauber photons exchanged with a nuclear charge distribution
- add initial-state exchanges, no interference with final-state exchanges
- change: integral along initial lepton direction over charge and potential

$$\delta\sigma_i \sim \int_{\text{lepton line}}^{\text{initial}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k} + \vec{q}_\perp, \vec{k}') - \sigma_0(\vec{k}, \vec{k}') \right)$$

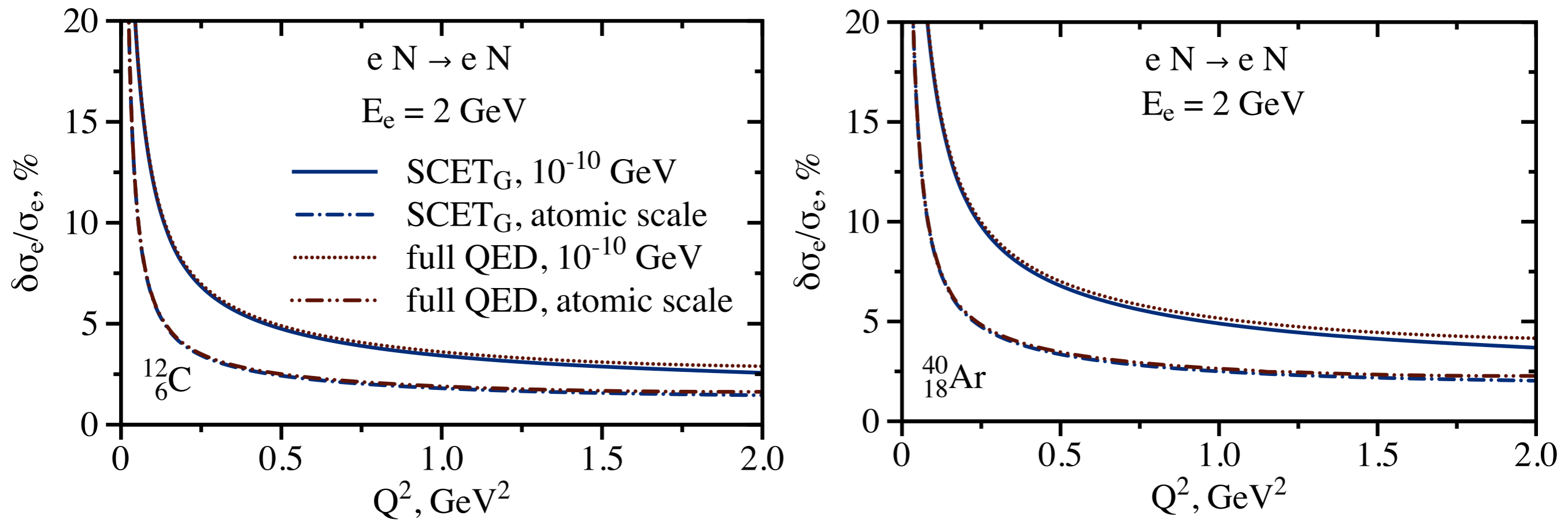
- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z) dz \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0(\vec{k}, \vec{k}' - \vec{q}_\perp) - \sigma_0(\vec{k}, \vec{k}') \right)$$

- leading-order cross sections are modified
- EFT and full QED agree above the lepton mass scale

Electron scattering

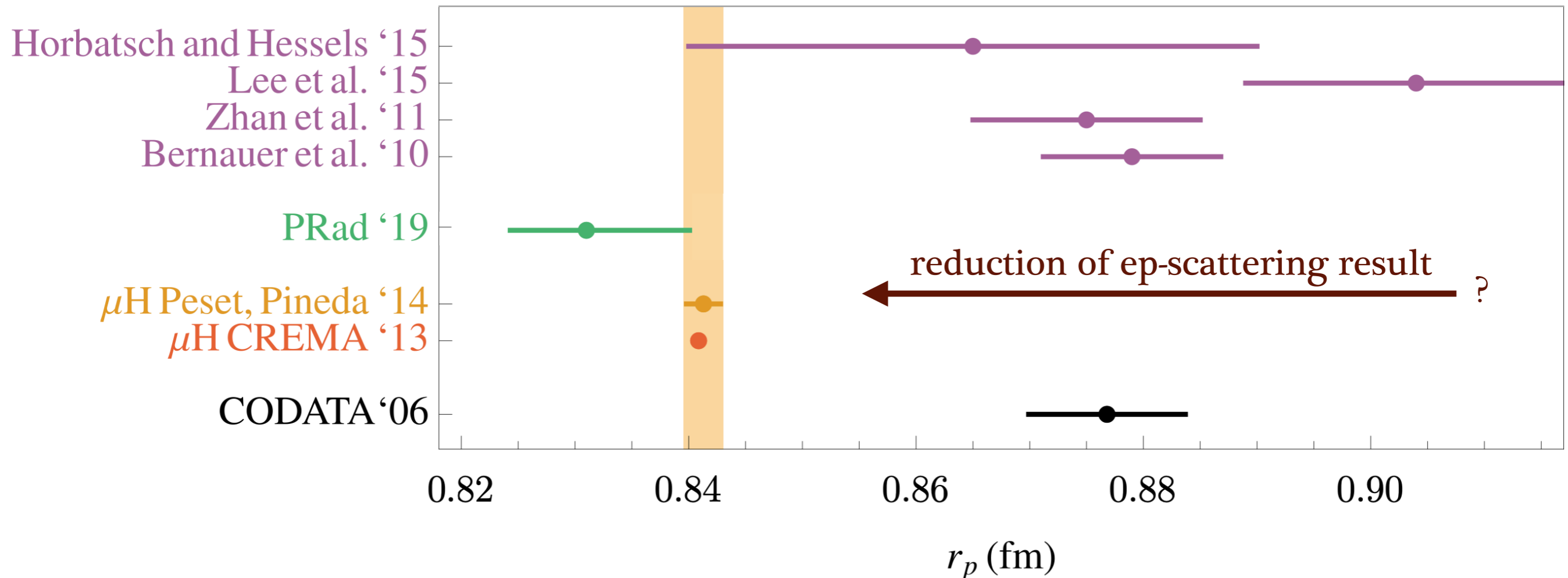
- relative correction per nucleus after incoherent sum over nucleons



- **percent-level** at low momentum transfers: $O(\alpha^2)$ correction
- **critical new effect** for electron scattering experiments

Proton charge radius extraction

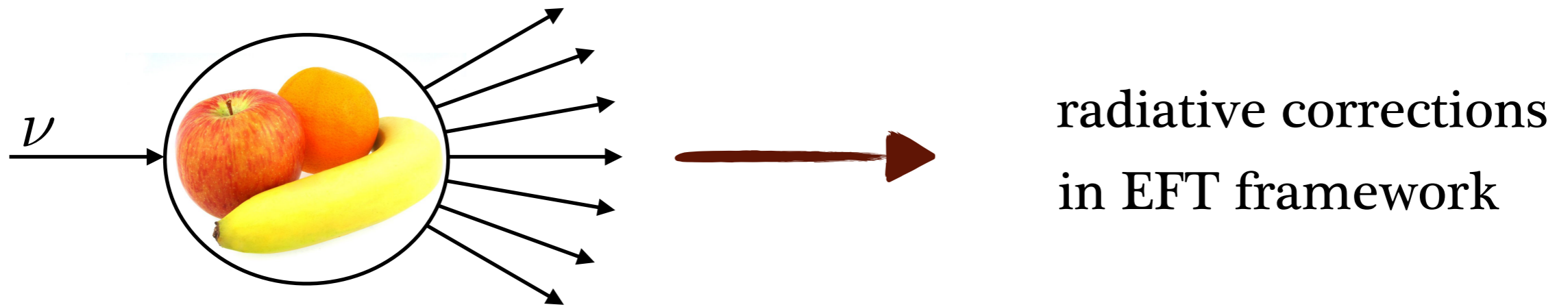
- determinations from elastic **electron-proton scattering data**
- determinations from **the Lamb shift in muonic hydrogen**
- combination of spectroscopic transitions in ordinary hydrogen



Clara Peset, Antonio Pineda and O. T., Prog. Part. Nucl. Phys. (2021)

- **QED medium effects** reduce tensions to **A1@MAMI** data
- calls for a more elaborate analysis of form factors

Conclusions



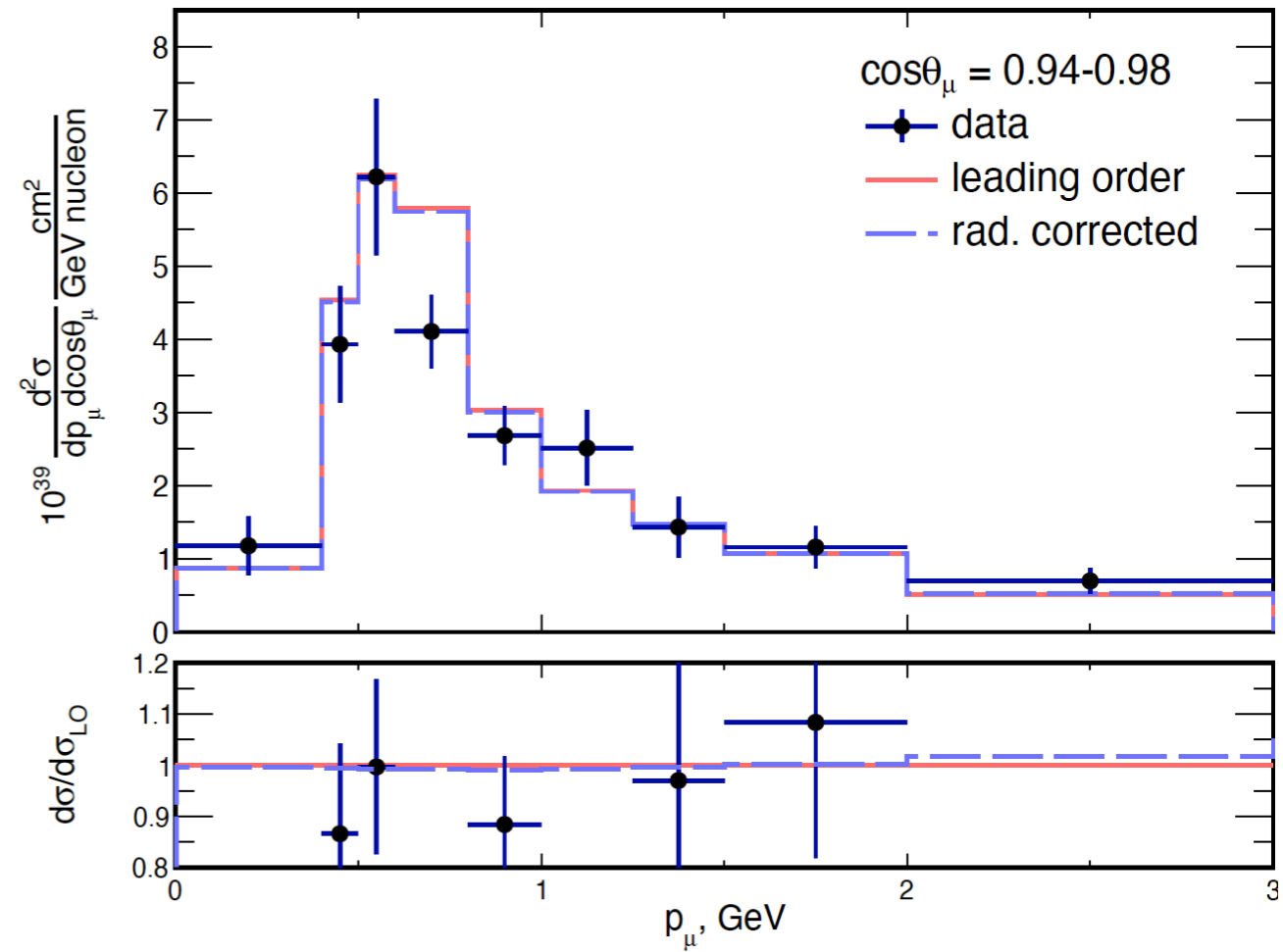
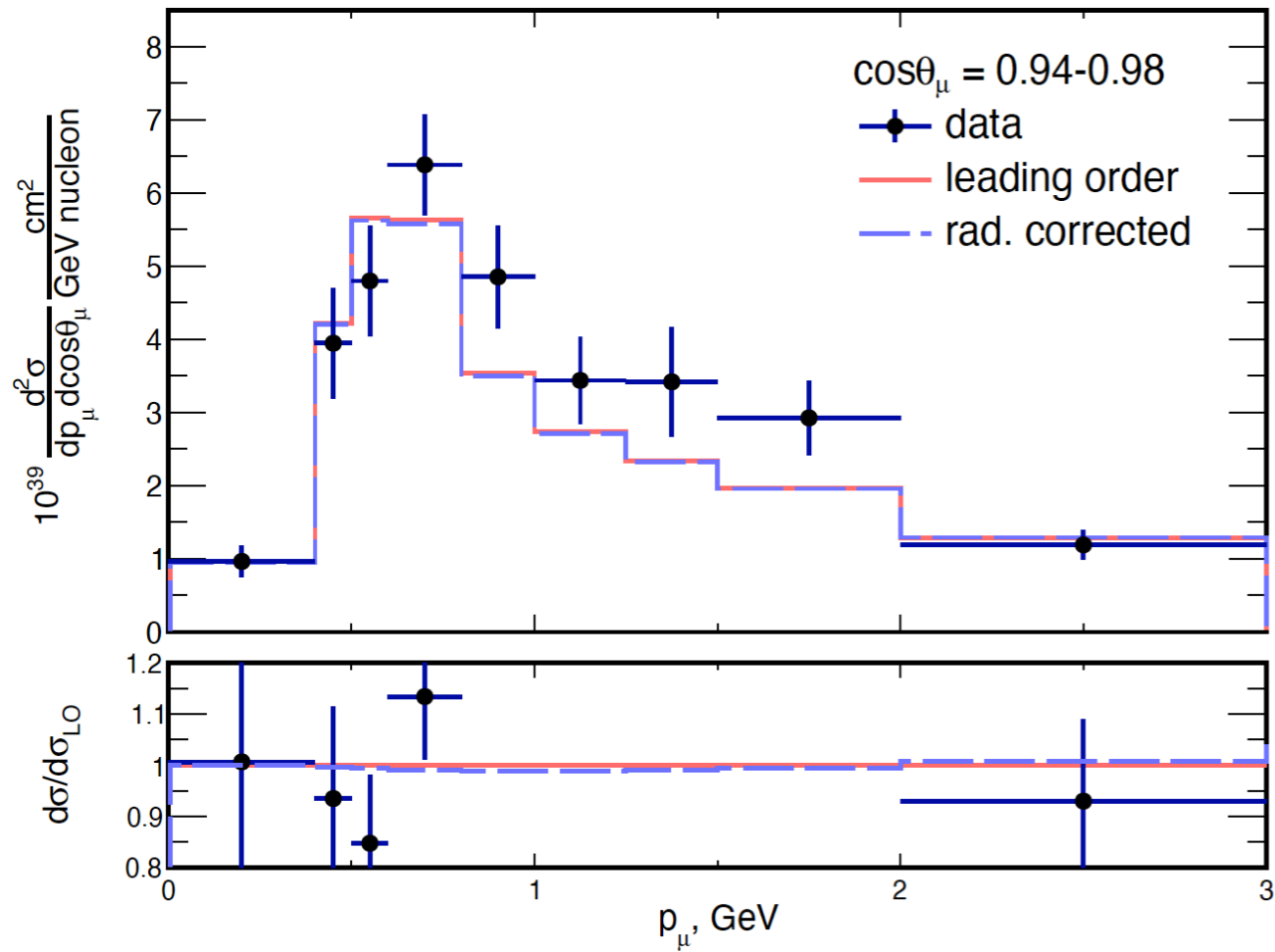
- radiative corrections to neutrino-nucleon cross sections formulated in factorization framework
- charged-current elastic electron vs muon cross-section ratios evaluated from theory with sub-percent uncertainty
- permille-level QED nuclear medium effects in neutrino scattering
- permille- to percent-level QED corrections in electron scattering

Thanks for your attention !!!



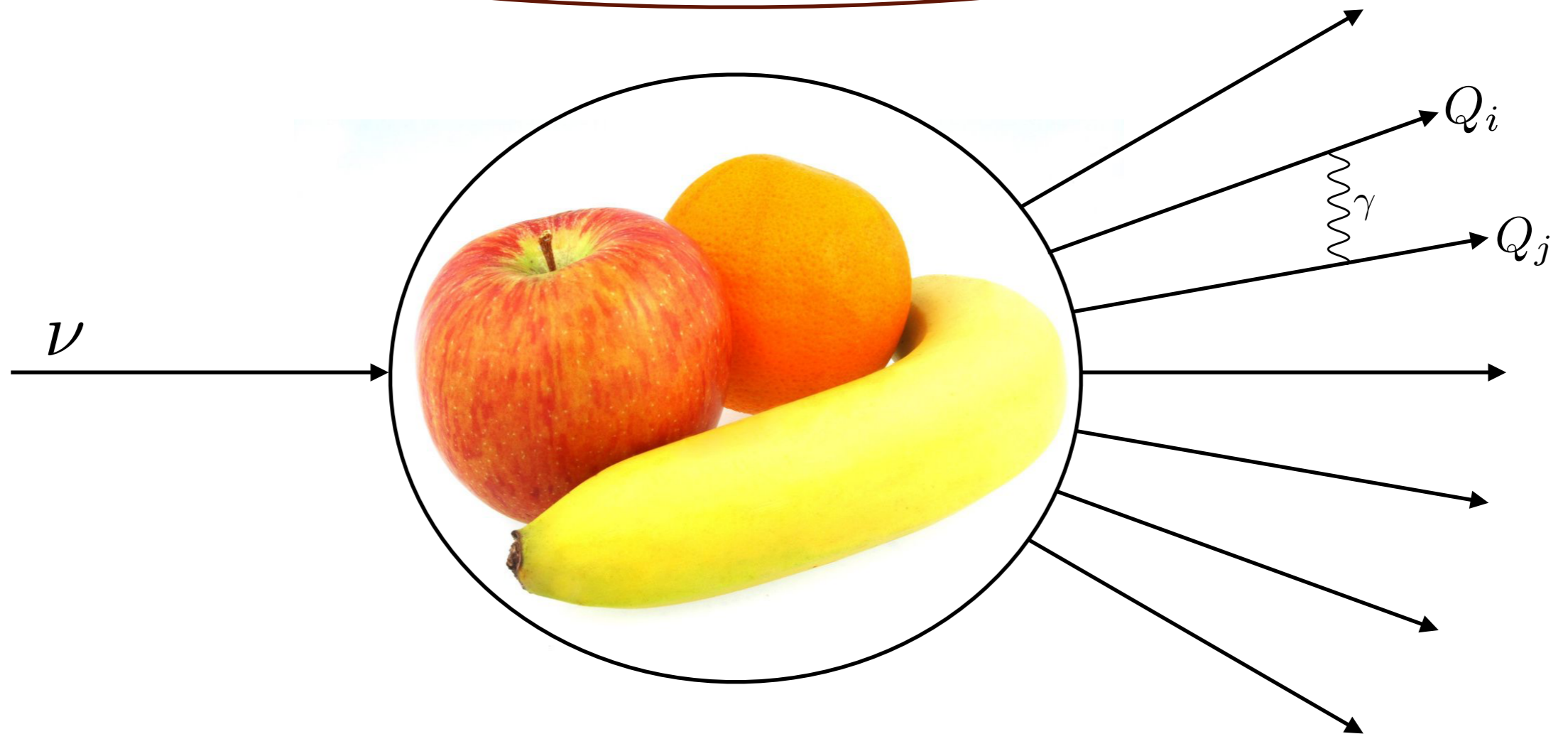
This presentation is based upon work that is supported by the Visiting Scholars Award Program of the Universities Research Association

T2K ND280 data



Electroweak corrections

$$m_e, m_\mu, M, E_\nu \ll M_W, M_Z, m_t, m_H$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \frac{1}{\sin^2 \theta_W}, \ln \frac{M_Z}{M}, \ln \frac{M_t}{M}, \dots$$

- electroweak corrections can be included in low-energy interactions

couplings of **effective Lagrangian** are precisely determined

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = -\bar{\nu}_l \gamma_\mu P_L \nu_l \cdot \bar{f} \gamma^\mu (c_L^{\nu_l f} P_L + c_R^{\nu_l f} P_R) f$$

$$\mathcal{L}_{\text{eff}}^{\text{CC}} = -2\sqrt{2}G_F \sum_{l \neq l'} \bar{\nu}_{l'} \gamma^\mu P_L \nu_l \bar{l} \gamma_\mu P_L l' - c^{qq'} \sum_{q \neq q'} \bar{l} \gamma^\mu P_L \nu_l \bar{q} \gamma_\mu P_L q'$$

Neutrino-lepton, neutrino-quark scattering

O.T. and Richard J Hill, Phys. Lett. B 805, 3, 135466 (2020)

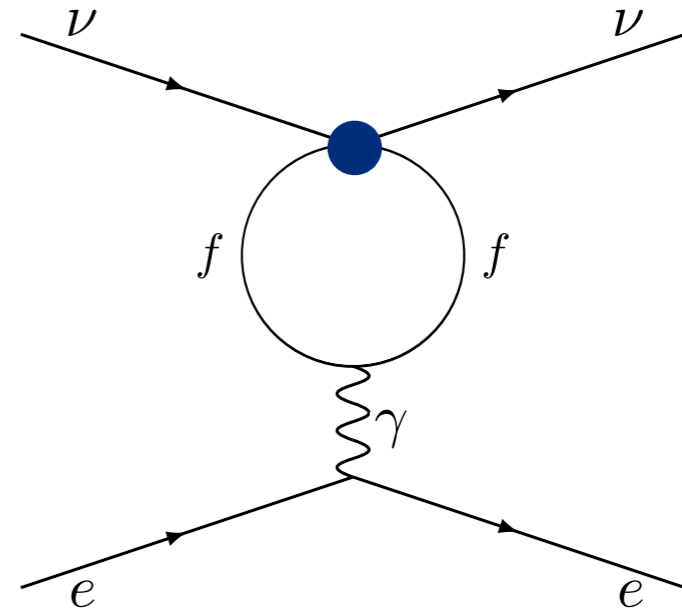
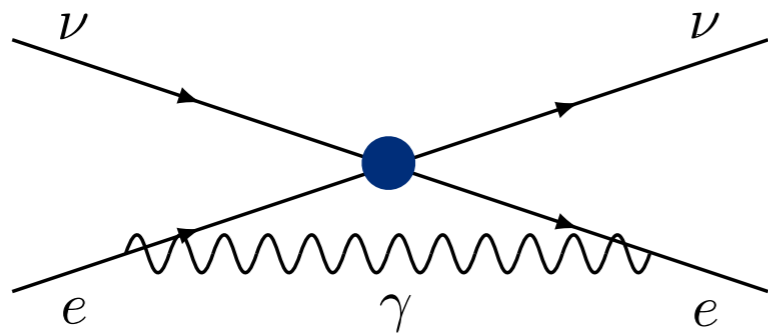
known at permille level



leading in G_F terms with loop expansion in α, α_s within Standard Model

poster at Neutrino 2020:

<https://youtu.be/mrW4aYjP57w>

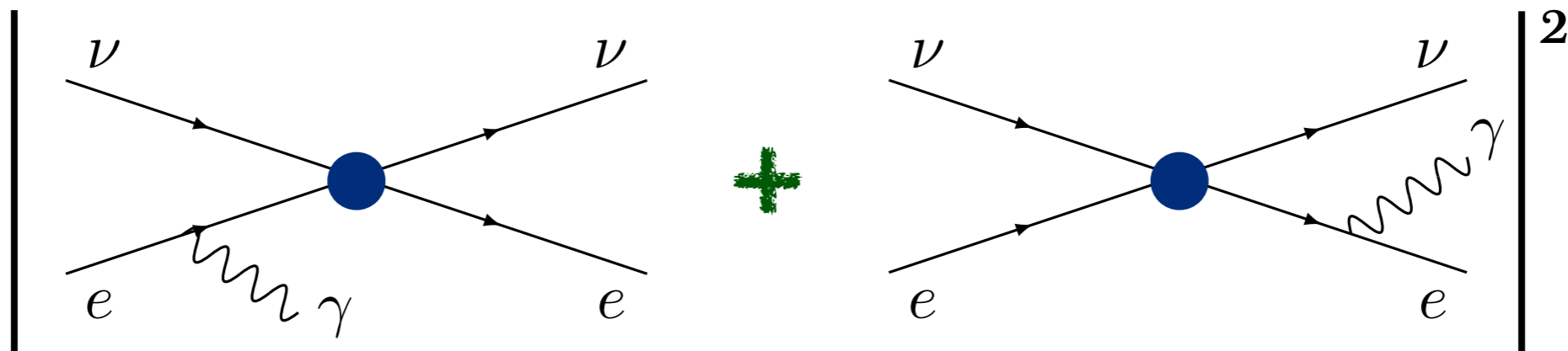


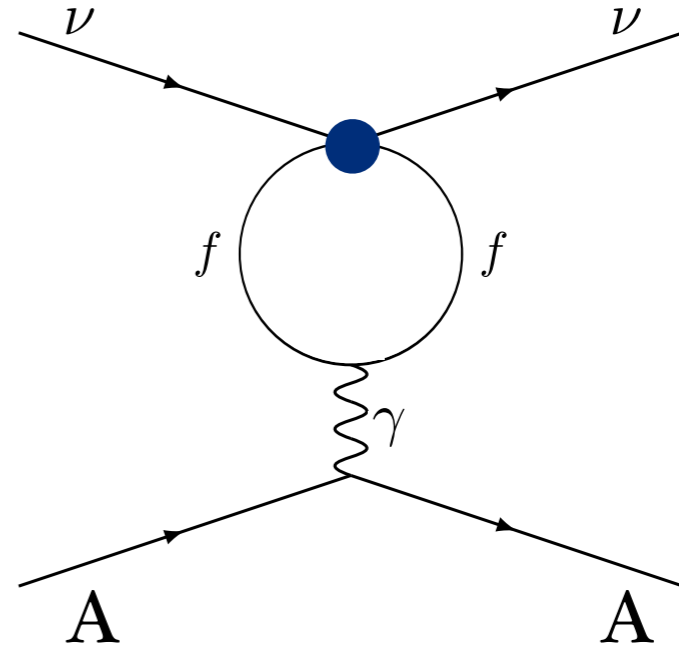
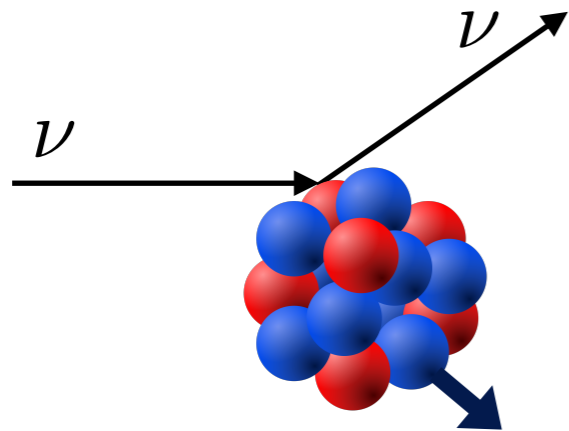
Neutrino-electron scattering

O.T. and Richard J Hill, Phys. Rev. D 101 3, 033006 (2020)

percent-level predictions for MINERvA

known analytically at permille level for NOvA and DUNE





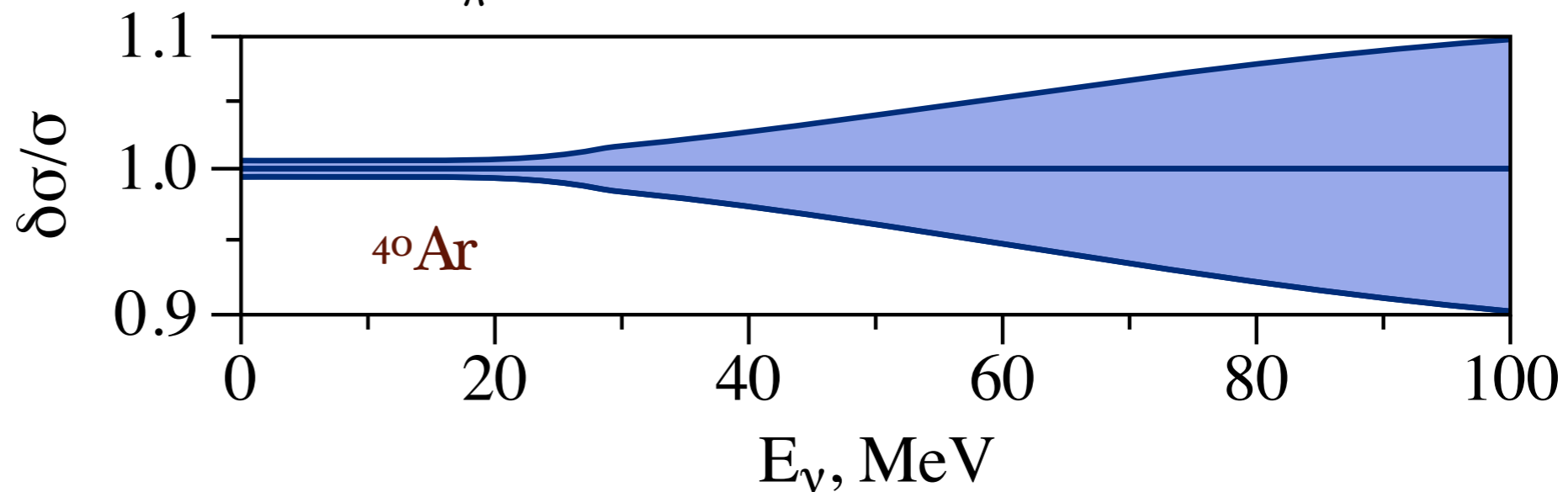
Coherent elastic neutrino-nucleus scattering

O.T., Pedro Machado, Vishvas Pandey and Ryan Plestid, JHEP 2102, 097 (2021)

$$F_W(Q^2) \rightarrow F_W(Q^2) + \frac{\alpha}{\pi} [\delta^{\nu e} + \delta^{\text{QCD}}] F_{\text{ch}}(Q^2)$$

flavor-dependent
at percent level

for Coherent and CCM

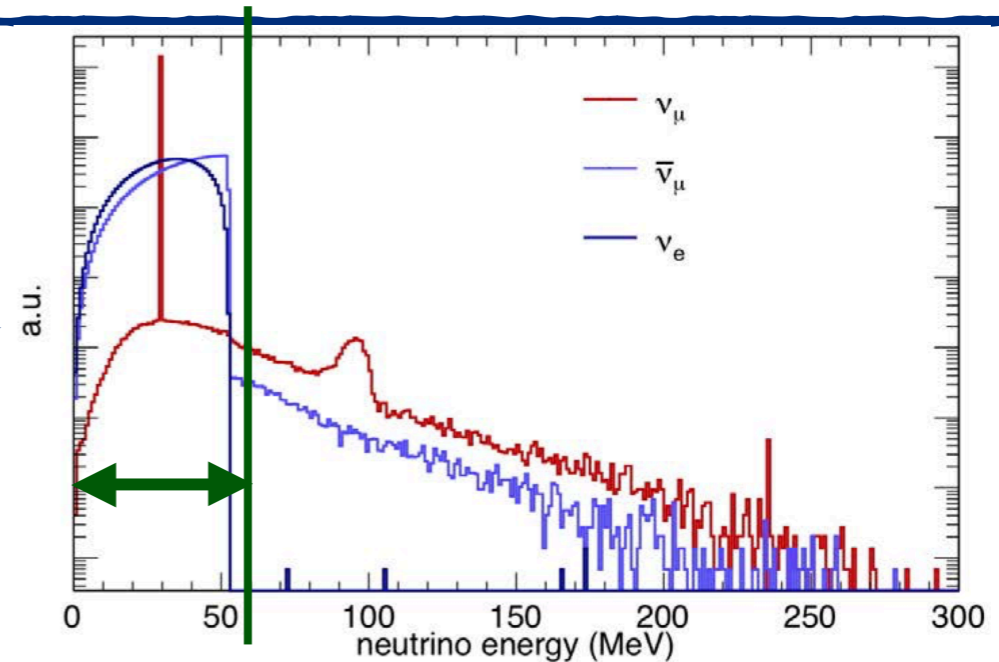


flavor-dependence at tree-level

energy spectra from π DAR \rightarrow

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



Akimov et al., Science 357 6356, 1123-1126 (2017)

Neutrinos from muon, pion and kaon decays

O. T., Phys. Lett. B 829, 137108 (2022)

$$\pi^+ \rightarrow \mu^+ \nu_\mu \gamma$$

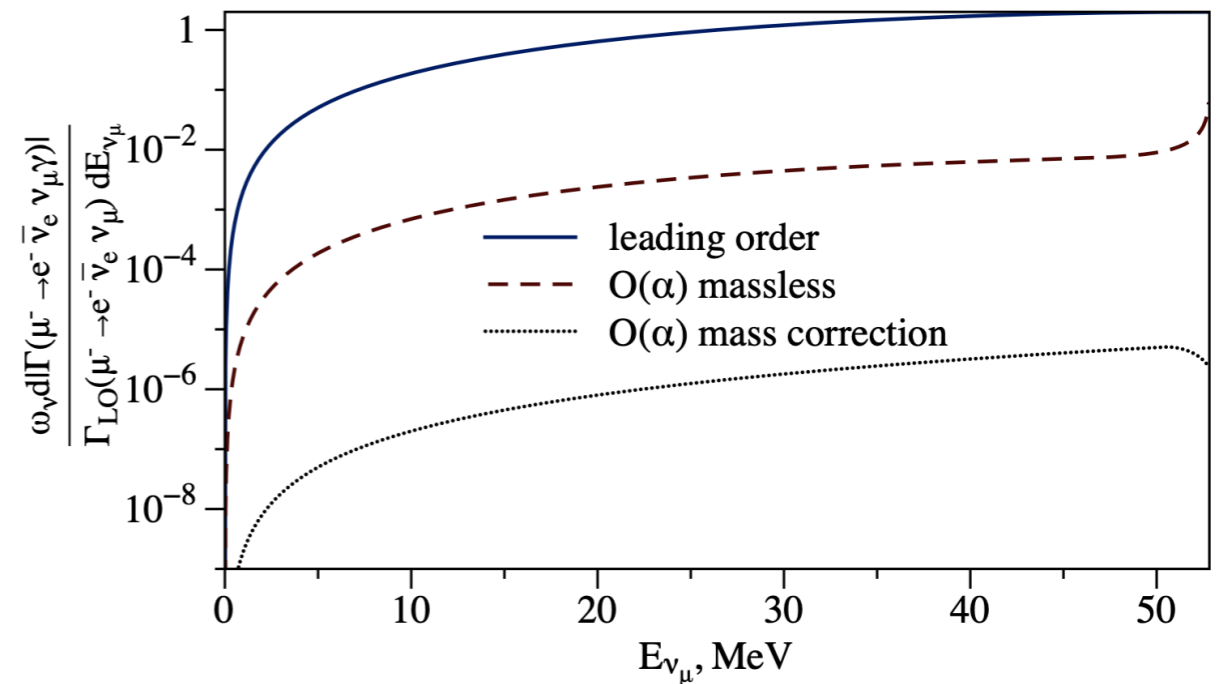
< 0.1 ‰

$$K^+ \rightarrow \mu^+ \nu_\mu \gamma$$

flavor-dependence is clarified to permille level analytically



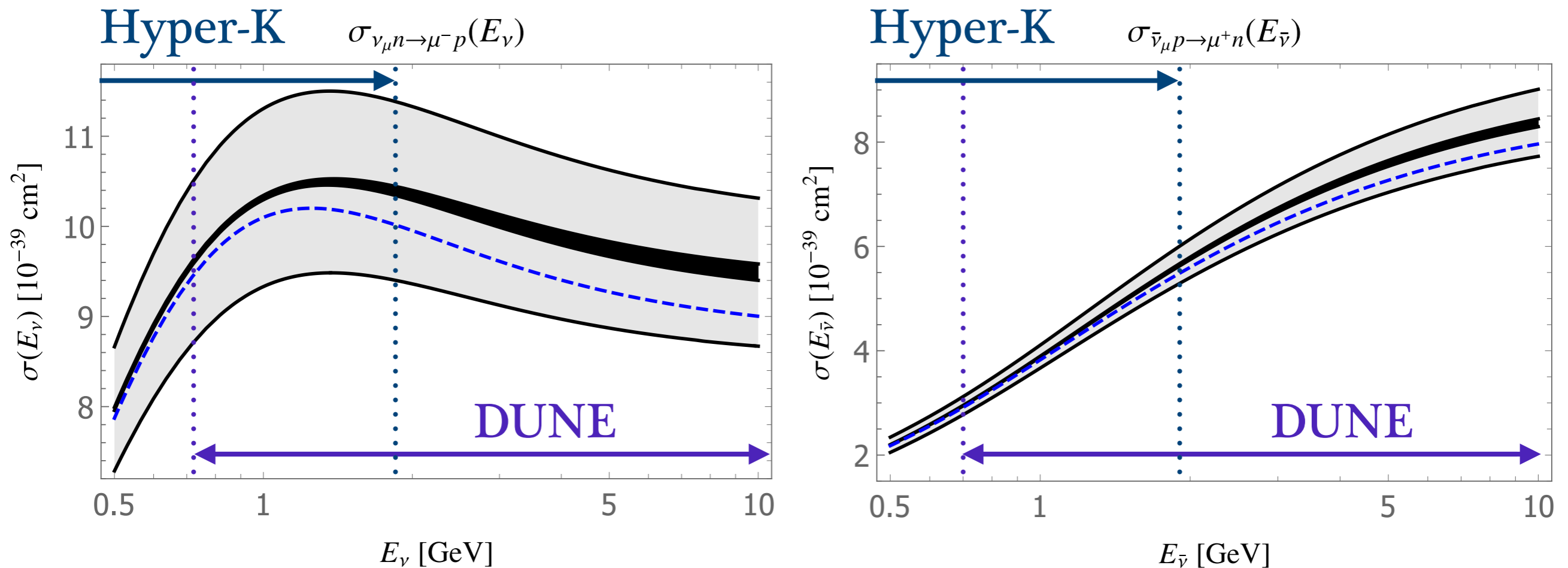
$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma \quad 3-4 \text{ ‰}$$



first QED/EW form factors with different mass

CCQE scattering cross section

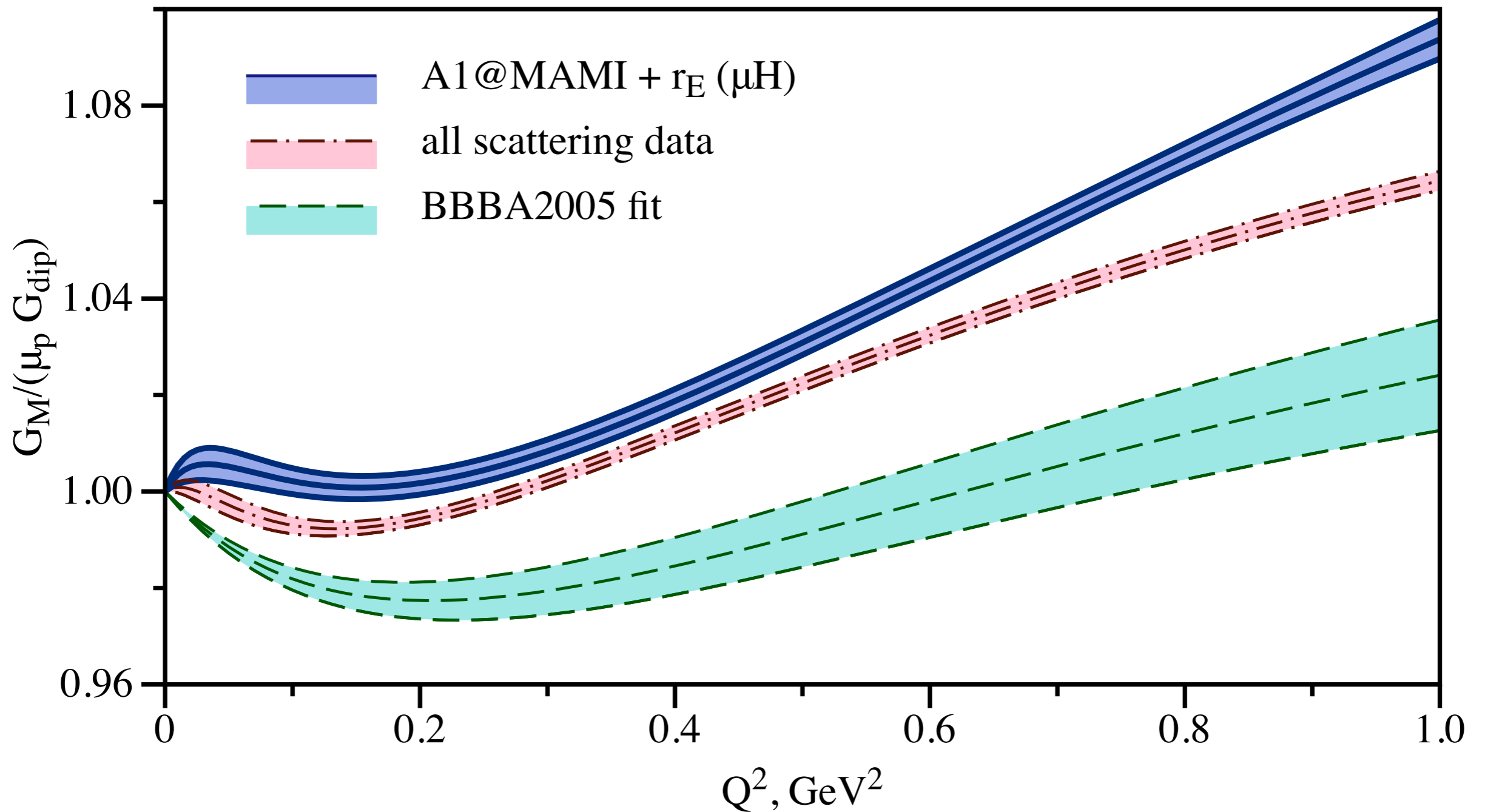
- dark band: uncertainty of fit with A_1 @MAMI data and charge radius
- light band: uncertainty of axial form factor
- blue line: BBBA2005 fit of electromagnetic form factors



- knowledge of vector structure stops a progress in studies of axial

Origin of difference

- fits of proton magnetic form factor:



- proton magnetic form factor has to be precisely measured