ECT* Workshop

Radiative Corrections from medium to high energy experiments

NNLO results with McMule

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methods → Adrian Signer's talk

6+ processes implemented at NNLO

- $\mu \to \nu\bar{\nu}e$ $(*)$ [Engel, Signer, YU 18]
- $e\mu \to e\mu$ $(*)$ and $ee \to \mu\mu$ [Banerjee, Engel, Signer, YU 20], [Kollatzsch, YU 2?], [YU et al. 2?]
- $ep \to ep$ and $\mu p \to \mu p$ $(\#)$ [Banerjee, Engel, Signer, YU 20]
- $e\nu \to e\nu$ $(\#)$
- $e^-e^- \to e^-e^-$ $(*)$ and $e^+e^- \to e^+e^-$ $(\#)$ [Banerjee, Engel, Schalch, Signer, YU 21]
- $ee \to \gamma\gamma$ $(\#)$
- $+$ many more at NLO

$*$: complete

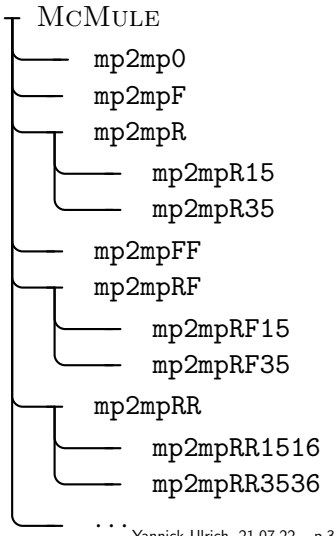
$\#$: complete exepct some hadronic effects

get the code `https://gitlab.com/mule-tools/mcmule`

- compile with `./configure && make`
- define observables in `src/user.f95`
- run all necessary pieces (SLURM script available)
- ℓ-p scattering at NNLO
- plot results with `python`

$\Rightarrow$ examples from all published results online

`gitlab.com/mule-tools/user-library`

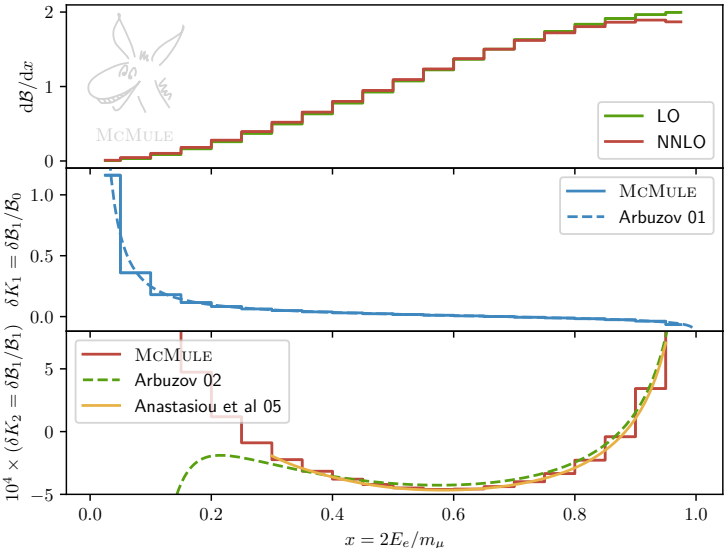


example: $e(q_1)p(q_2) \to e(q_3)p(q_4)$ user-library:l-p-scattering/mesa-legacy/ep2ep_mesa_paper/user.f95

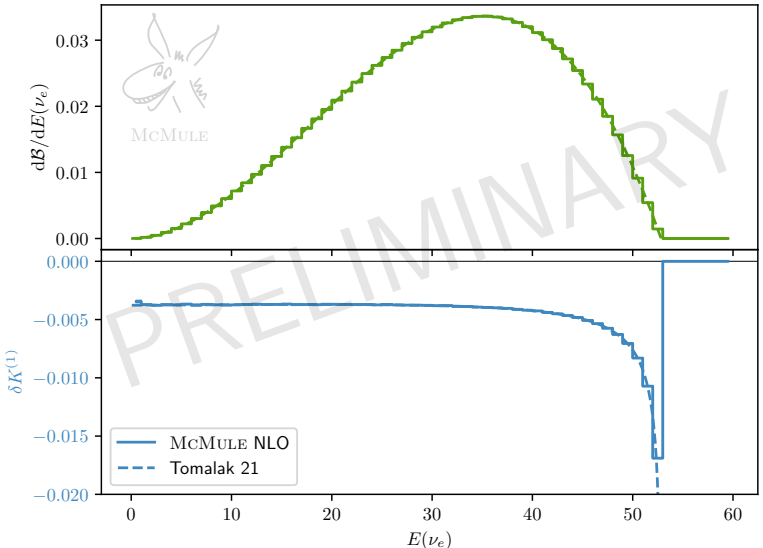
```
integer, parameter :: nr_q = 1
integer, parameter :: nr_bins = 200
real, parameter :: min_val(nr_q) = (/ 25. /)
real, parameter :: max_val(nr_q) = (/ 45. /)

FUNCTION QUANT(q1,q2,q3,q4,q5,q6,q7)

real(kind=prec) :: q1(4),q2(4),q3(4),q4(4),q5(4),q6(4),q7(4)
real(kind=prec), parameter :: ez = (/0.,0.,1.,0./)
real(kind=prec) :: quant(nr_q), thetae

thetae = acos(cos_th(ez,boost_rf(q2, q3)))

pass_cut = .true.
if(thetae < 25*pi/180._prec) pass_cut=.false.
if(thetae > 45*pi/180._prec) pass_cut=.false.

names(1) = "thetae"
quant(1) = 180*thetae/pi
END FUNCTION QUANT
```

- analytic approximations at NNLO $\mathcal{O}(\alpha^2 \log^{\{1,2\}}(m_e/m_\mu))$ [Arbuzov, Melnikov 02]
- numerical loop calculation
  [Anastasiou, Melnikov, Petriello 05]
- first NNLO calculation in McMule to test FKS^2
  [Engel, Signer, YU 18]
- averaged over the neutrinos $\Rightarrow$ currently redoing things to fix that
  [Proust, Rocco, Signer, YU 2?]

mule-tools.gitlab.io/user-library/michel-decay/validation



mule-tools.gitlab.io/user-library/michel-decay/neutrino-spectrum



- largest uncertainty in $(g-2)_\mu$ is HVP
- lattice or $e^+e^- \to$ hadrons data

new-ish proposal: MUonE

[Carloni Calame et al. 15, Abbiendi et al. 17, MUonE Letter of Intent 19]

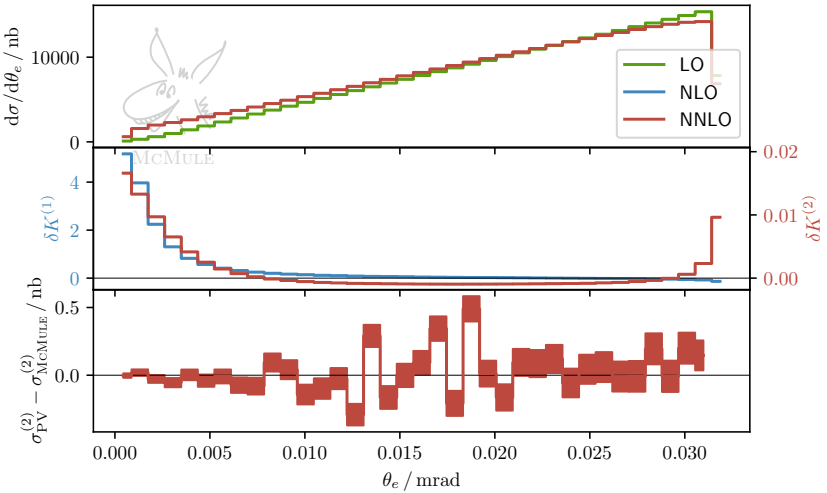
- $150\,\mathrm{GeV}$ muon beam (M1 @ CERN) on Be electrons
- measure $\theta_{e,\mu} \to$ re-construct $t < 0 \to \Delta\alpha_{\mathrm{had}}(t) \to (g-2)_\mu^{\mathrm{HVP}}$
- target precision $10^{-5} \to \mathrm{NNLO} \oplus$ leading $\mathrm{N^3LO} \oplus \mathrm{PS} \oplus \cdots$ [MUonE Theory Initiative 20]

μ-e scattering: past work

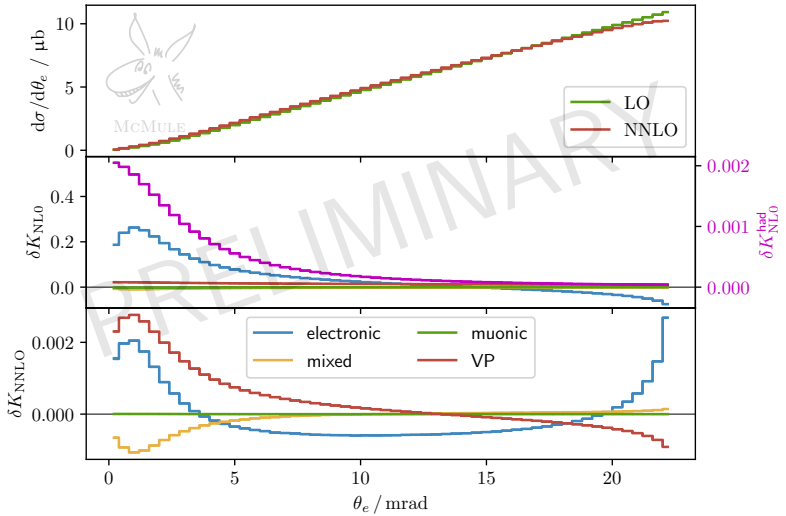
- full EW-NLO
  [Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini 18]
- hadronic corrections
  [Fael 18, Fael, Passera 19, Balzani, Laporta, Passera 22]
- New Physics: contamination & searches
  [Masiero, Paradisi, Passera 20, ...] [di Cortona, Nardi 22, ...]
- dominant NNLO $\Rightarrow$ agreement!
  [Carloni Calame, Chiesa, Hasan, Montagna, Nicrosini, Piccinini 20] & [Banerjee, Engel, Signer, YU 20]
- pair-production and fermion loops
  [Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicrosini, Piccinini 21 & 22]
- two-loop with $m = 0$
  [Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano 22]

compared with [Carloni Calame, Chiesa, Hasan, Montagna, Nicrosini, Piccinini 20]

mule-tools.gitlab.io/user-library/mu-e-scattering/muone



massified $0 \lesssim m$, compared RV+RR [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 2?]



leptonic corrections for e-p and μ-p scattering at NNLO
[Banerjee, Engel, Signer, YU 20]

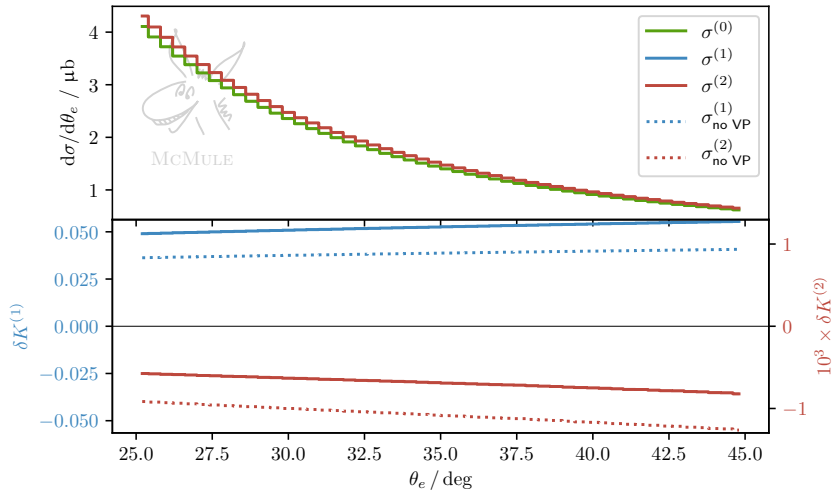
- parametrise proton as form factor $\gamma_\mu \to F_1 \gamma^\mu + F_2 \frac{\mathrm{i}\sigma^{\mu\nu} q_\nu}{2m}$
- part of dominant μ-e scattering $\Rightarrow$ very well checked!
- discrepancy with previous calculation [Bucoveanu, Spiesberger 18]
  - but agree on matrix elements
  - integration the same as μ-e scattering
- trivial to switch e-p to μ-p scattering $F_2 \to 0$
- examples for
  MESA ($p_e = 155\,\mathrm{MeV}$) and MUSE ($p_\mu = 210\,\mathrm{MeV}$)

TPE contribution

- WIP [Engel, Hagelstein, Rocco, Sharkovska, YU ??]

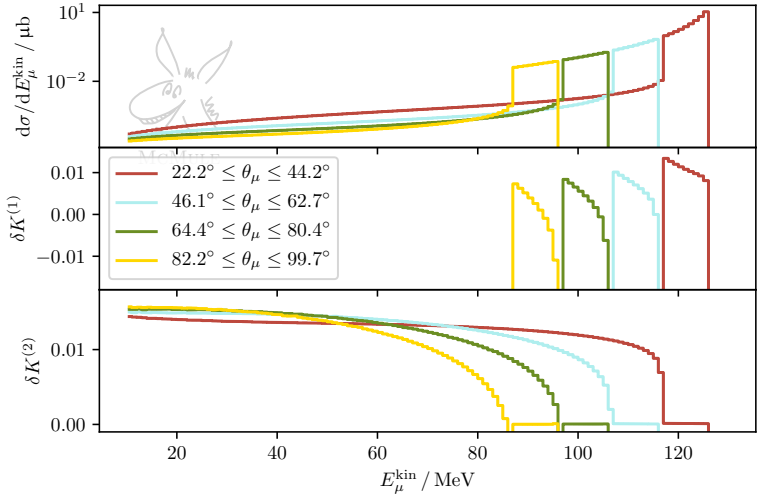
$p_e^{\text{in}} = 155\,\text{MeV},\ E_e^{\text{out}} > 45\,\text{MeV}$

mule-tools.gitlab.io/user-library/l-p-scattering/mesa-legacy



$p_\mu^{\text{in}} = 210\,\text{MeV},\ 20° < \theta_\mu < 100°$

mall mule-tools.gitlab.io/user-library/l-p-scattering/muse-legacy



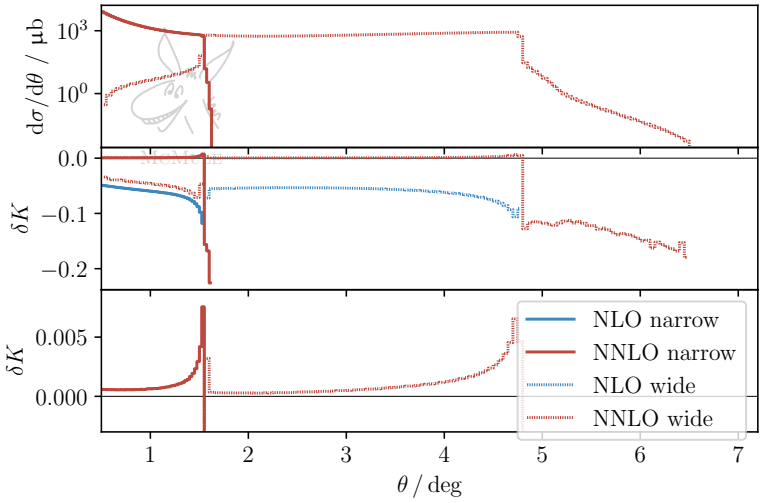
- used for normalisation by PRad ($E = 1.4\,\mathrm{GeV}$)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}\right)_{ep} = \left[\frac{N_{\mathrm{exp}}(ep \to ep, \theta_i)}{N_{\mathrm{exp}}(ee \to ee, \theta_i)}\right]\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}\right)_{ee}$$

- massless two-loop known [Bern, Dixon, Ghinculov 00]
- NLO calculations [Kaiser 10, Akushevich, Gao, Ilyichev, Meziane 15], and generators [Afanasev, Chudakov, Ilyichev, Zykunov 06, Epstein, Milner 16]
- NNLO with McMule using NTS stabilisation & massification [Banerjee, Engel, Schalch, Signer, YU 21]
- validated by comparing $e^+e^- \to e^+e^-$ to Babayaga ($\mathrm{NLO} \oplus \mathrm{PS}$) [Carloni Calame, Montagna, Nicrosini, Piccinini 04]
- two indistinguishable particles $\Rightarrow$ 'narrow' and 'wide' electron

$0.5° < \theta_i < 6.5°$, $\Delta E < 130\,\text{MeV}$, $\Delta\phi < 7.35°$

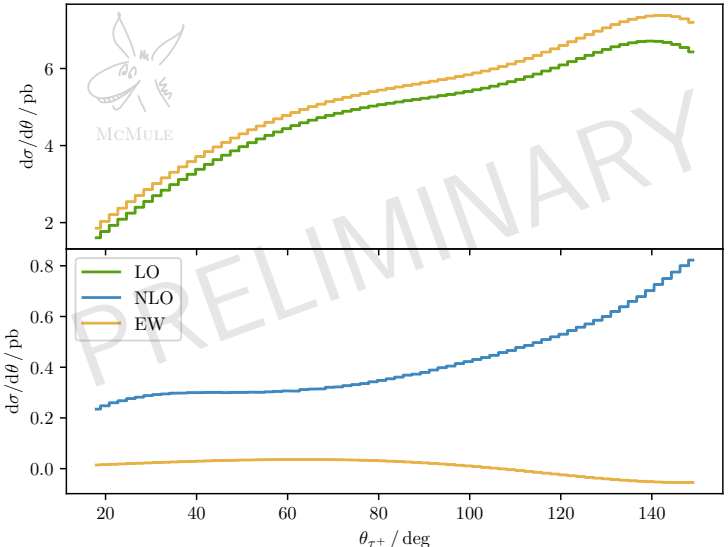
mule-tools.gitlab.io/user-library/moller-scattering/prad



Miscellaneous

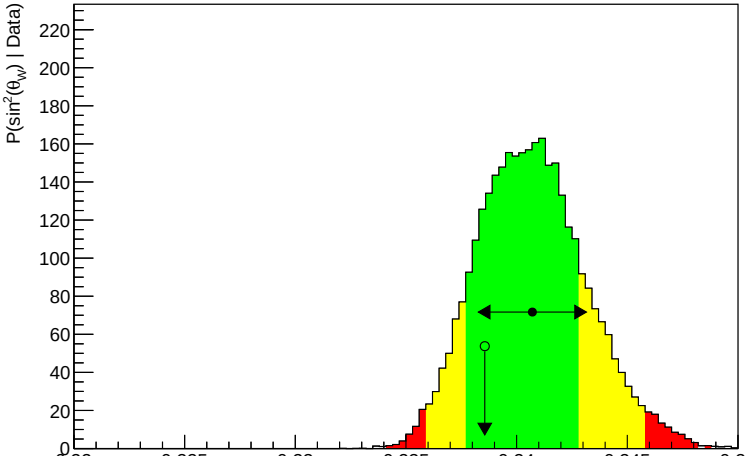
- $ee \to \tau\tau$ EW-NLO [Kollatzsch, YU 2?]
- $e\nu \to e\nu$ at NLO used for event generation
- best possible theory spectrum for $\mu \to \nu\bar{\nu}e$

$\sqrt{s} = m_{\Upsilon(4s)} = 10.5\,\text{GeV} \Rightarrow$ EW effects



$\sin^2\theta_W$ @ DUNE [de Gouvêa, Machado, Perez-Gonzalez, Tabrizi 19]

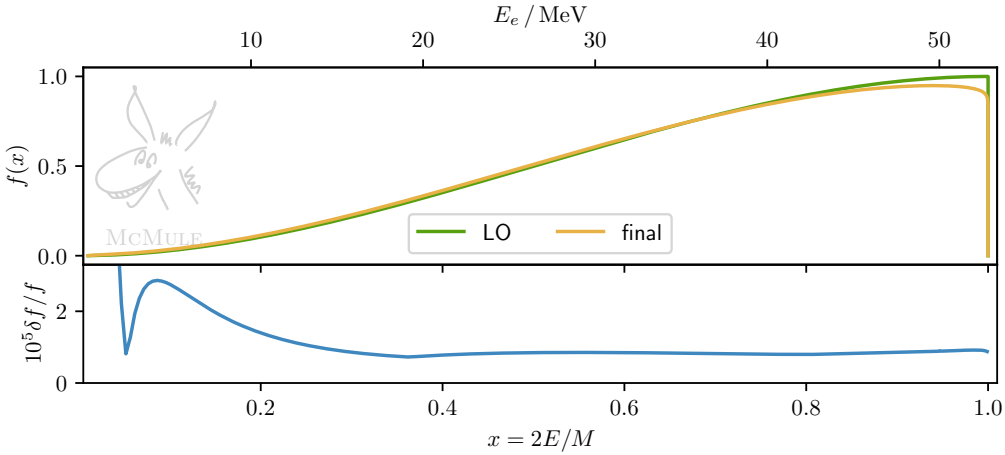
- what is the best observable?
- what influence do flux uncertainties and radiative corrections have? $\Rightarrow$ full (N)NLO

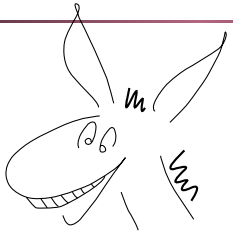


- ALPs in the muon decay $\mu \to eX$

$\Rightarrow$ need $\mathcal{O}(10^{-5})$ at end point of E_e if $m_X \ll m_\mu$

- large logarithms due to soft-photons $\Rightarrow$ resummation





MCMULE

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