

---

# Radiative Corrections in SIDIS

Igor Akushevich<sup>1</sup>, Alexander Ilyichev<sup>2</sup>, Stan Srednyak<sup>1</sup>



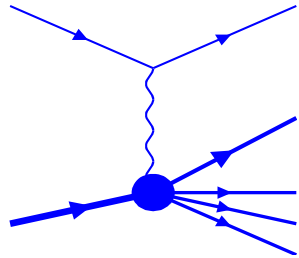
<sup>1</sup> Duke University, Durham, NC USA

<sup>2</sup> Institute for Nuclear Problems,  
Byelorussian State University,  
Minsk Belarus

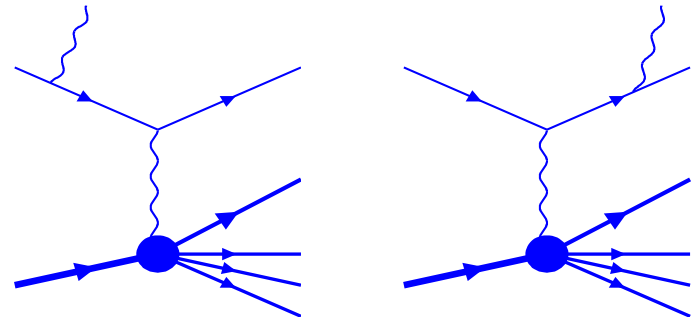
e-mail: [igor.akushevich@duke.edu](mailto:igor.akushevich@duke.edu)

# Contribution to RC in SIDIS

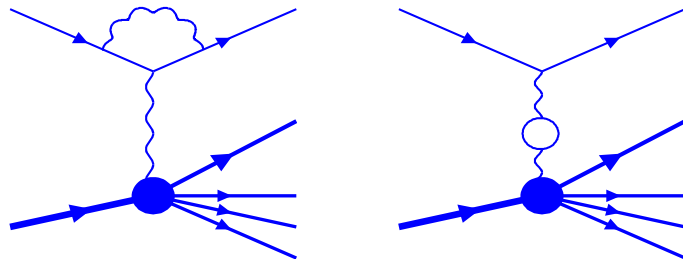
The Born cross section



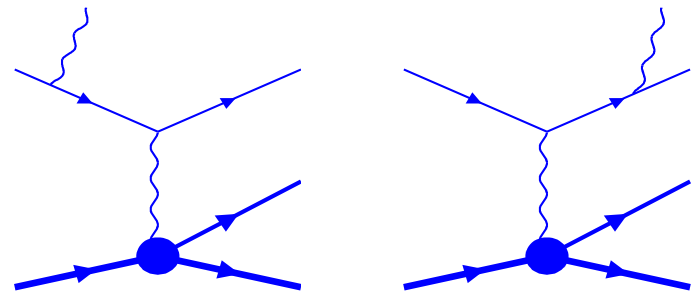
Emission of a radiated photon (semi-inclusive processes)



Loop diagrams



Emission of a radiated photon (exclusive processes)



# Original studies of RC in SIDIS

---

- ➔ The original formalism, simple quark-parton model:
  - ➔ Soroko AV, Shumeiko NM (1989) Radiative effects in deep inelastic scattering of leptons on nucleons in a semi-inclusive experiment. *Soviet Journal of Nuclear Physics*, 49(5); p. 838-844
- ➔ Fortran code POLRAD 2.0, Patch SIRAD
  - ➔ Akushevich I, Ilyichev A, Shumeiko N, Soroko A, and Tolkachev A, (1997) POLRAD 2.0. FORTRAN code for the radiative corrections calculation to deep inelastic scattering of polarized particles. *Computer physics communications*, 104(1-3), pp.201-244.
- ➔ RC to unpolarized SIDIS cross section, angular structure
  - ➔ Akushevich I, Shumeiko N, and Soroko A (1999) Radiative effects in the processes of hadron electroproduction. *European Physical Journal*, C10(4), pp.681-687.
- ➔ Radiative tail from exclusive peak
  - ➔ Akushevich I, Ilyichev A, and Osipenko M (2009) Lowest order QED radiative corrections to five-fold differential cross section of hadron lepton production. *Physics Letters B* 672(1), pp.35-44.
- ➔ General calculation of RC for polarized particles
  - ➔ Akushevich I and Ilyichev A (2019) Lowest order QED radiative effects in polarized SIDIS. *Physical Review D* 100(3), p.033005.

# General task for RC to SIDIS

---

- ➔ Ultimate purpose of this task is to create the code for exact (i.e., leading+next-to-leading) calculation of RC to the SIDIS cross section of electron scattering by the proton target with both particles arbitrary polarized and to develop a Monte Carlo generator based on this code.
- ➔ Stages of the theoretical calculation and practical implementation include the steps:
  - ➔ *Elaborate the covariant hadronic tensor of SIDIS based on the developments of the theoretical groups of Aram Kotzinian (Nuclear Physics B 441 (1995) 234-256), Peter Mulders (Phys.Rev. D49 (1994) 96-113) and Alessandro Bacchetta (JHEP 0702:093,2007).*
  - ➔ *Calculate the SIDIS cross sections using the elaborated hadronic tensor, compare analytic expressions for the Born cross section between the results obtained by researchers of these groups and understand possible discrepancies.*
  - ➔ *Calculate the coefficients from contraction of leptonic tensor involving RC with tensor structures from the hadronic tensor and extract the infrared divergence.*
  - ➔ *Calculate loop diagrams and obtain the infrared-free cross section of RC.*
  - ➔ *Code these coefficients and complete the code for RC due to DIS radiative tail.*
  - ➔ *Obtain the hadronic tensor of exclusive process, calculate the contractions for the kinematics of exclusive process and implement this part of RC to the code.*
  - ➔ *Implement set of available four-dimensional DIS and three-dimensional exclusive SFs, investigate the model dependence of the RC, and identify the kinematical regions where uncertainty in SFs could result in significant effect on RC.*
  - ➔ *Develop a Monte Carlo generator that will generate the channel of scattering (non-radiative, radiative DIS, and radiative exclusive scattering) and the kinematical variables of the additionally radiated photon.*

# Current Status of Our Calculations

---

- ➔ All analytic calculations were performed without any approximations and presented in our paper:
  - ➔ Akushevich, Igor, and Alexander Ilyichev. “Lowest order QED radiative effects in polarized SIDIS” *Physical Review D* 100, no. 3 (2019): 033005.
- ➔ Fortran code was created by Alexander Ilyichev
  - ➔ SIDIS SF were implemented as in (Wandzura-Wilczek-type approximation):
    - ➔ *Bastami, Saman, Harut Avakian, A. V. Efremov, Aram Kotzinian, B. U. Musch, B. Parsamyan, Alexei Prokudin et al. “Semi-inclusive deep-inelastic scattering in Wandzura-Wilczek-type approximation.” *Journal of High Energy Physics* 2019, no. 6 (2019): 7.*
    - ➔ *However, this model is not completely satisfactory: there are kinematical regions with unrealistic estimates of SFs, applicability of the model in entire region necessary for RC has to be reviewed.*
  - ➔ Exclusive SFs were not implemented. Possible approaches are still reviewed.
- ➔ Extraction of the leading log contributions and analysis of shifted kinematics is completed.

# Exact Formulae for RC in SIDIS

The total RC is calculated exactly in our paper (PR D100 (2019) 033005):

$$\sigma_{RC} = \frac{\alpha}{\pi} \exp(\delta_{inf}) \left( \delta_{VR} + \delta_{vac}^l + \delta_{vac}^h \right) \sigma_B + \sigma_R^F + \sigma_{AMM} + \sigma_R^{ex}$$

Here  $\delta_{VR}$  and  $\delta_{vac}^{l,h}$  come from the radiation of soft photons and the effects of vacuum polarization, the correction  $\delta_{VR}$  is infrared-free sum of factorized parts of real and virtual photon radiation, and  $\sigma_R^F$  and  $\sigma_R^{ex}$  are contributions from the process of emission of an additional real photon:

$$\begin{aligned} \sigma_B &= \frac{\alpha^2 S_x^2}{4Q^4 \sqrt{\lambda_Y} S} \sum_{i=1}^9 \theta_i^B \mathcal{H}_i \\ \sigma_R^F &= -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\max}} dR \sum_{i=1}^9 \left[ \sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1} \mathcal{H}_i}{RQ^4} \right], \\ \sigma_R^{ex} &= -\frac{\alpha^3 S S_x^2}{2^8 \pi^5 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{i=1}^9 \sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex} \theta_{ij} R_{ex}^{j-2}}{(1 + \tau - \mu) \tilde{Q}^4}. \end{aligned}$$

Explicit expressions for the functions  $\theta_{ij}$  are given in Appendix B of our paper.

# Leading, Next-to-Leading, and Exact Contributions to RC

---

By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy. The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where  $A$  and  $B$  do not depend on the electron mass  $m$ .

- ➔ If only  $A$  is kept, this is the leading log approximation.
- ➔ If both contributions are kept (i.e., contained  $A$  and  $B$ ), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Three approaches to extract the leading Log contribution for the SIDIS cross section (i.e., to calculate  $A$ )

- ➔ use our exact formulae, collect all terms that result in leading log after integration over photon angles, combine them into the final expression
  - ➔ extract the poles that correspond to radiation collinear to initial and final electron, integrate over angles, and find the factorizing form traditional for leading log calculations.
  - ➔ use the method of the electron structure functions.
-

# Leading Log: Exatraction from Exact Formulae

---

The exact expression for  $\sigma_R^F$  is:

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\max}} dR \sum_{i=1}^9 \left[ \sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1} \mathcal{H}_i}{RQ^4} \right],$$

Analysis of the integrand and tracing the origin of the leading log allows us to extract the leading log terms from the exact formulae:

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^s R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{S} \frac{1+z_1^2}{z_1(1-z_1)} \theta_i^{Bs}$$

$$\int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{j=1}^{k_i} \theta_{ij}^p R^{j-2} = -4\pi l_m \frac{\sqrt{\lambda_Y}}{X} \frac{1+z_2^2}{1-z_2} \theta_i^{Bp}$$



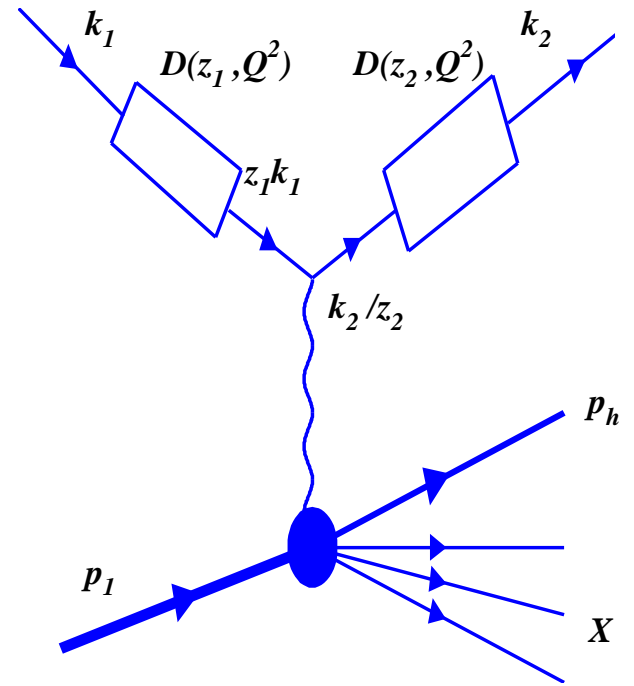
# Leading Log: Extraction the Collinear Poles

---

- ➔ Born cross section:  $\sigma_B(x, Q^2, z, t, \phi_h) = CL_{\mu\nu}(k_1, k_2)W_{\mu\nu}(p, q, p_h)$ .
- ➔ RC cross section  $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int C_p L_{\mu\nu}^{rad}(k_1, k_2, k)W_{\mu\nu}(p, q - k, p_h)d^3k/2k_0$ .
- ➔ the pole for radiation collinear to the initial electron,  $1/k.k_1$ :  
 $L_{\mu\nu}^{rad}(k_1, k_2, k) = A/k.k_1 + Bm^2/k.k_1^2 + \dots$  and keep only the term  $A/k.k_1$ .
- ➔ Substitute  $k = (1 - z_1)k_1$  in  $A$  everywhere including  $W_{\mu\nu}(p, q - k, p_h)$ , neglect terms with  $m^2$  in the numerator.
- ➔ Integrate  $d^3k/k^0/k.k_1$  over angles (resulting in the leading log) and remaining integral over photon energy rewrite as the integral over  $z_1$ .
- ➔ Express the RC cross section as  $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int_{z_{min}^1}^1 C_{pp} dz_1 (1 + z_1^2)/(1 - z_1) f(z_1) L_{\mu\nu}((1 - z_1)k_1, k_2) W_{\mu\nu}(p, q - (1 - z_1)k_1, p_h)$
- ➔ Write the final initial collinear radiation as:  
 $\sigma_{RC}(x, Q^2, z, t, \phi_h) = \int_{z_{min}^1}^1 C_{pp} dz_1 (1 + z_1^2)/(1 - z_1) f(z_1) \sigma_B(\tilde{x}, \tilde{Q}^2, \tilde{z}, \tilde{t}, \tilde{\phi}_h)$
- ➔ Obtain formulae for shifted variables and analyze shifted kinematics.
- ➔ Make similar calculation for the pole for radiation collinear to the final electron.

# Leading Log: Using the Electron Structure Functions

- ➔ The QED radiative corrections to the corresponding cross sections can be written as a contraction of two electron structure functions and the hard part of the cross section.
- ➔ Traditionally, these radiative corrections include effects caused by loop corrections and soft and hard collinear radiation of photons and  $e^+e^-$  pairs.
- ➔ This method can be improved by also including effects due to radiation of one noncollinear photon. The corresponding procedure results in a modification of the hard part of the cross section, which takes the lowest-order correction into account exactly and allows going beyond the leading approximation.



$$\sigma^{in} = \int_{z_1^m}^1 dz_1 D(z_1, Q^2) \int_{\hat{z}_2^m}^1 \frac{dz_2}{z_2^2} D(z_2, Q^2) \sqrt{\frac{\hat{\lambda}_Y}{\lambda_Y} \frac{S_x^2}{\hat{S}_x^2}} \hat{\sigma}_t^B$$

with ESF  $D(z_{1,2}, Q^2) = D^\gamma(z_{1,2}, Q^2) + D_N^{e^+e^-}(z_{1,2}, Q^2) + D_S^{e^+e^-}(z_{1,2}, Q^2)$  (see details in Afanasev, Akushevich, Merenkov (2004) Journal of Experimental and Theoretical Physics, 98(3) 403-416, and references therein).

# The Cross Section in the Leading Log

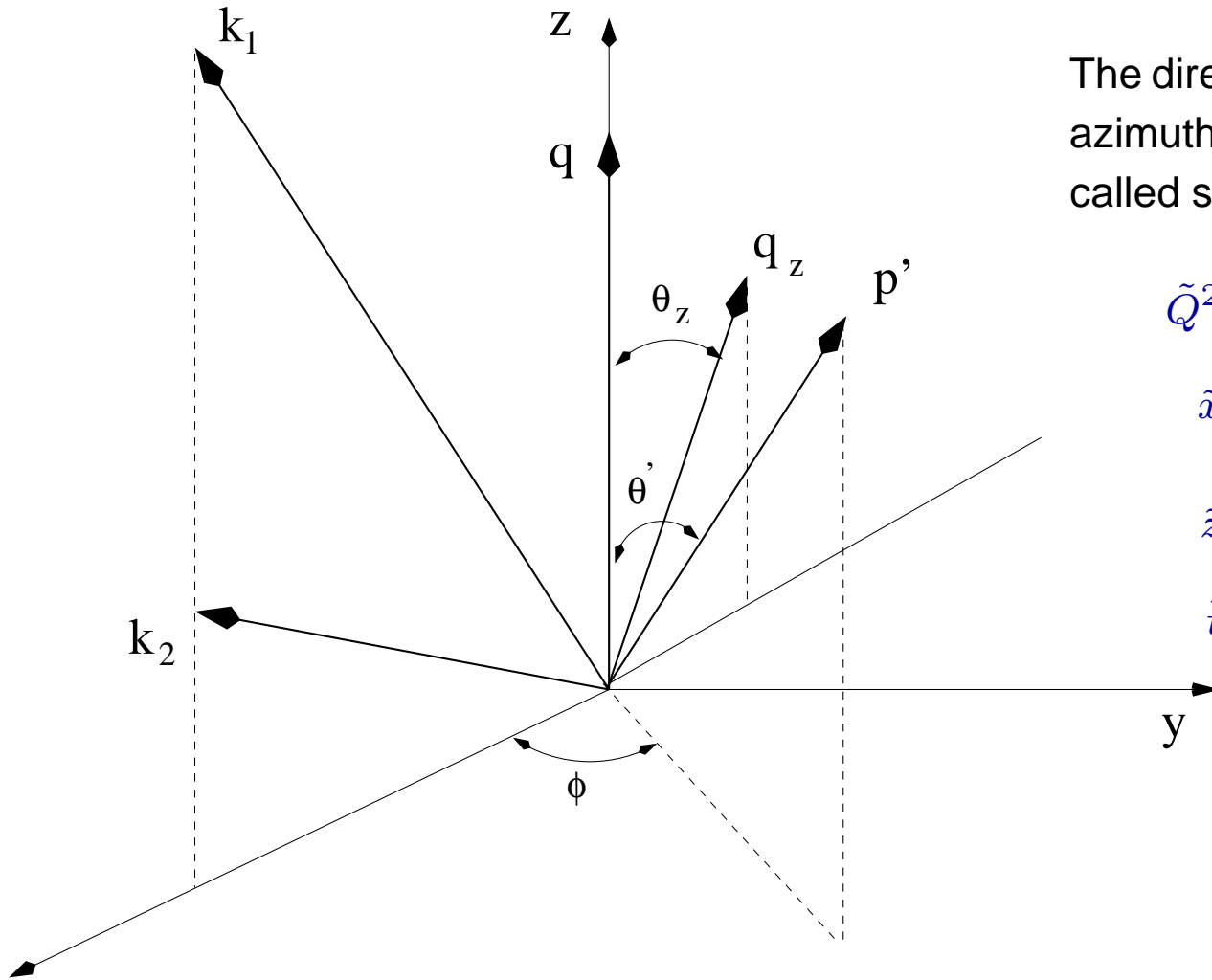
---

The cross section of RC in the leading log approximation is:

$$\begin{aligned}\sigma_{RC}(x, Q^2, z, t, \phi_h) &= \left[ 1 + \frac{\alpha}{\pi} \delta_{\text{vac}} \right] \sigma_B(x, Q^2, z, t, \phi_h) \\ &+ \frac{\alpha}{2\pi} l_m \int_{z_1^m}^1 dz_1 \frac{1+z_1^2}{1-z_1} \left[ \sqrt{\frac{\lambda_Y^s}{\lambda_Y}} \frac{S_x^2}{S_x^s{}^2} \sigma_B(\tilde{x}_1, \tilde{Q}_1^2, \tilde{z}_1, \tilde{t}_1, \tilde{\phi}_{h1}) - \sigma_B(x, Q^2, z, t, \phi_h) \right] \\ &+ \frac{\alpha}{2\pi} l_m \int_{z_2^m}^1 dz_2 \frac{1+z_2^2}{1-z_2} \left[ \frac{1}{z_2^2} \sqrt{\frac{\lambda_Y^p}{\lambda_Y}} \frac{S_x^2}{S_x^p{}^2} \sigma_B(\tilde{x}_2, \tilde{Q}_2^2, \tilde{z}_2, \tilde{t}_2, \tilde{\phi}_{h2}) - \sigma_B(x, Q^2, z, t, \phi_h) \right]\end{aligned}$$

where quantities with tilde are calculated in the shifted kinematics.

# Vectors and angles in the shifted kinematics



The direction of  $q_z$  defines new polar ( $\bar{\theta}$ ) and azimuthal ( $\bar{\phi}$ ) angles of the final hadron (so-called shifted kinematics)

$$\begin{aligned}\tilde{Q}^2 &= z_1 Q^2 \\ \tilde{x} &= \frac{z_1 Q^2}{z_1 S - X} \\ \tilde{z} &= \frac{z S_x}{z_1 S - X} \\ \tilde{t} &= t + (1 - z_1)(Q^2 - V_1)\end{aligned}$$

$$\sin \tilde{\phi}_h = \frac{p_t}{\tilde{p}_t} \sin \phi_h$$

<sup>X</sup>The “shifted” kinematics is completely calculatable, e.g., Akushevich, Ilyichev, Phys. Rev. D85 (2012) 053008

# Vectors and angles in the shifted kinematics

---

Explicit formulae for shifted kinematics:

$$\hat{y} = 1 + \frac{y-1}{z_1 z_2}, \quad \hat{x} = x \frac{z_1 S_x}{z_2 \hat{S}_x}, \quad \hat{S}_x = z_1 S - X/z_2, \quad \hat{Q}^2 = \frac{z_1}{z_2} Q^2, \quad \hat{z} = z \frac{S_x}{\hat{S}_x},$$

$$\hat{t} = t + \left(1 - \frac{z_1}{z_2}\right) Q^2 + (1 - z_1) V_1 + \left(\frac{1}{z_2} - 1\right) V_2$$

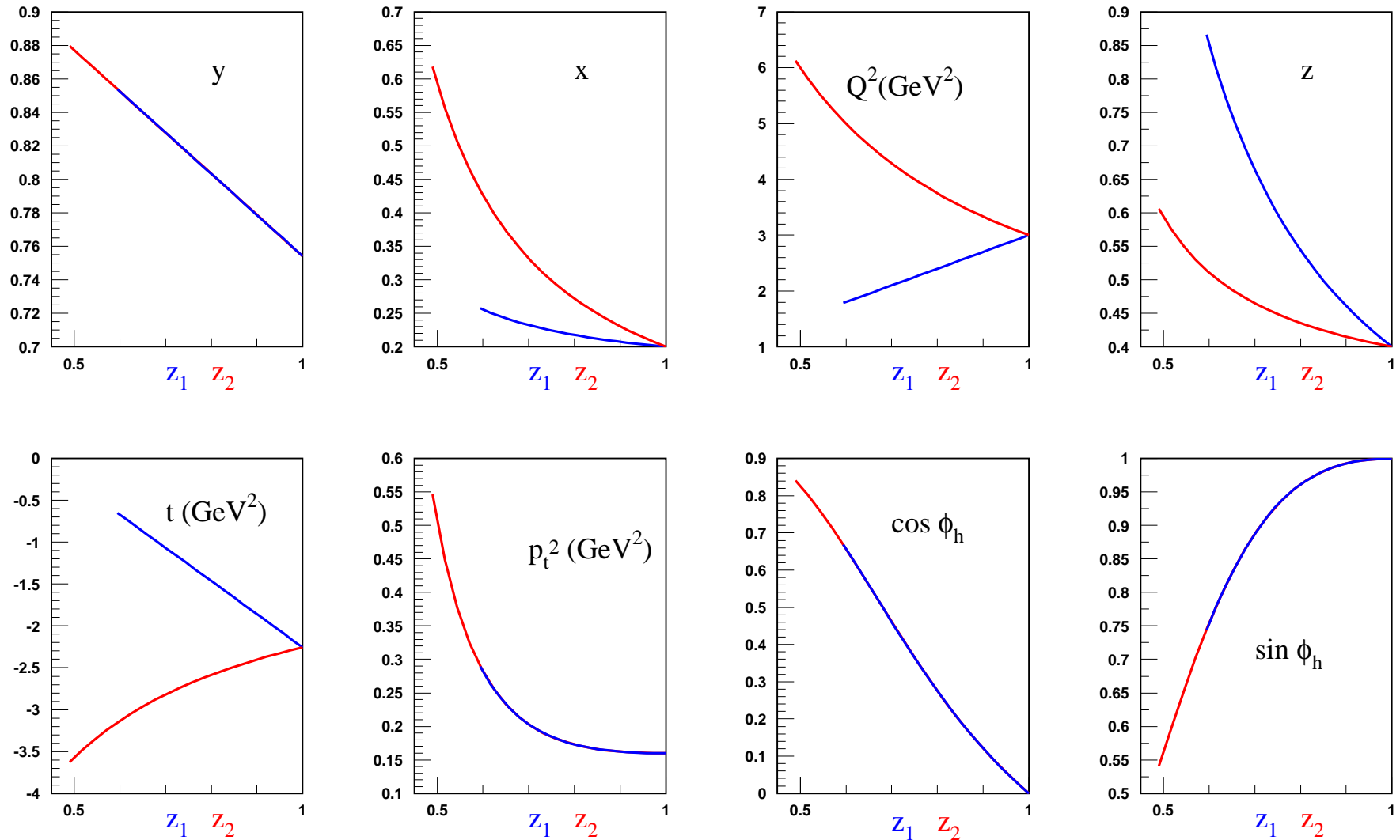
$$\hat{p}_t^2 = \frac{1}{4M^2} \left[ z^2 S_x^2 - \frac{(z S_x (z_1 S - X/z_2) - 2M^2 (z_1 V_1 - V_2/z_2))^2}{4M^2 z_1 Q^2/z_2 + (z_1 S - X/z_2)^2} \right] - m_h^2$$

$$\cos \hat{\phi}_h = \frac{Q^2 (z S_x (z_1 S + X/z_2) - 2M^2 (z_1 V_1 + V_2/z_2)) - (S V_2 - X V_1) (z_1 S - X/z_2)}{2 \hat{p}_t \sqrt{Q^2 (S X - M^2 Q^2) (4M^2 z_1 Q^2/z_2 + (z_1 S - X/z_2)^2)}}$$

$$\sin \hat{\phi}_h = \frac{p_t}{\hat{p}_t} \sin \phi_h.$$

# Variables in the Shifted Kinematics

Variables in shifted kinematics (vs.  $z_{1,2}$ ) for radiation from **initial** (blue) and **final** (red) electron.



# Leading Log: Exclusive Radiative Tail

Exact contribution calculated in our 2019 paper is:

$$\sigma_R^{ex} = -\frac{\alpha^3 S S_x^2}{2^8 \pi^5 \lambda_S \lambda_Y} \int_{\tau_{\min}}^{\tau_{\max}} d\tau \int_0^{2\pi} d\phi_k \sum_{i=1}^9 \sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex} \theta_{ij} R_{ex}^{j-2}}{(1 + \tau - \mu) \tilde{Q}^4}.$$

where  $R_{ex} = \frac{p_x^2 - m_u^2}{1 + \tau - \mu}$ . The Born cross section of exclusive process is:

$$\sigma_{ex}^B = \frac{\alpha^2 S_x}{32\pi^3 Q^4 S \sqrt{\lambda_Y}} \sum_{i=1}^9 \mathcal{H}_i^{ex} \theta_i^B$$

Therefore we have

$$\sigma_{sex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{se}}{\lambda_Y} \frac{S_x^2}{S' S_x^{se}}} \frac{1 + z_{1ex}^2}{1 - z_{1ex}} \sigma_{sex}^B,$$

$$\sigma_{pex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{pe}}{\lambda_Y} \frac{S_x^2}{X' S_x^{pe}}} \frac{1 + z_{2ex}^2}{1 - z_{2ex}} \sigma_{pex}^B,$$

where  $z_{1ex} = 1 - (p_x^2 - m_u^2)/S'$  and  $z_{2ex} = (1 + (p_x^2 - m_u^2)/X')^{-1}$ .

# Contributions to RC in SIDIS

---

- ➔ Radiation by leptons: Leading and Next-to-leading contributions

$$\sigma = \log \frac{Q^2}{m^2} A + B$$

- ➔ Estimates show that for unpolarized RC, NLO can reach 100% of LO contribution: from 15% for large  $z$  to 100% for small  $z$ .
  - ➔ For asymmetries the relative contribution of NLO can be even larger because LO has a tendency to factorize and cancel in numerator and denominator.
- ➔ Radiation by hadrons
    - ➔ Depends on the models we use for RC and specifications of what we plan to extract from the experiment.
  - ➔ Box-type diagrams and interference between radiation by leptons and hadrons
    - ➔ Should be estimates and added to the systematic error.

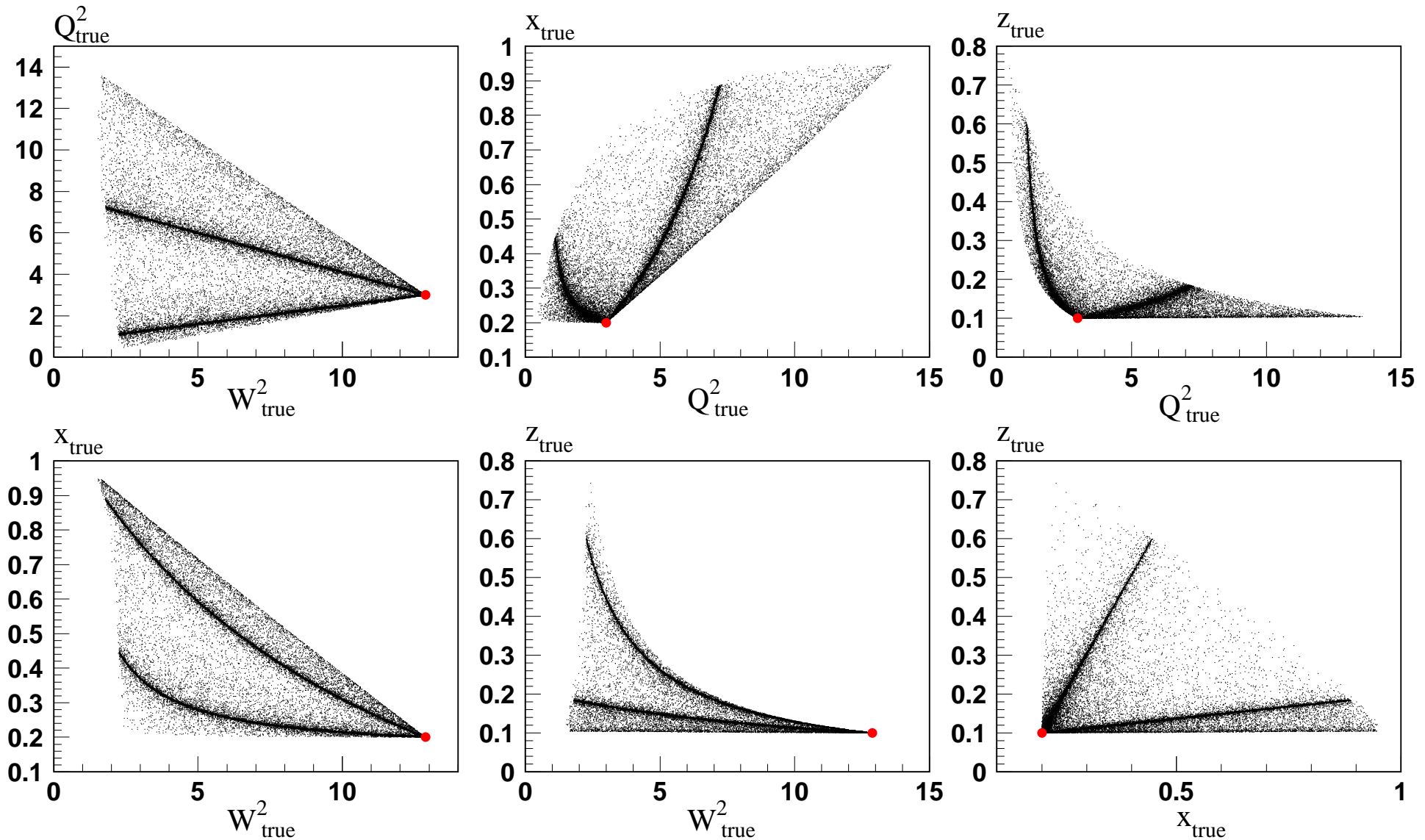


# Why Exact Formulae are Extremely Important

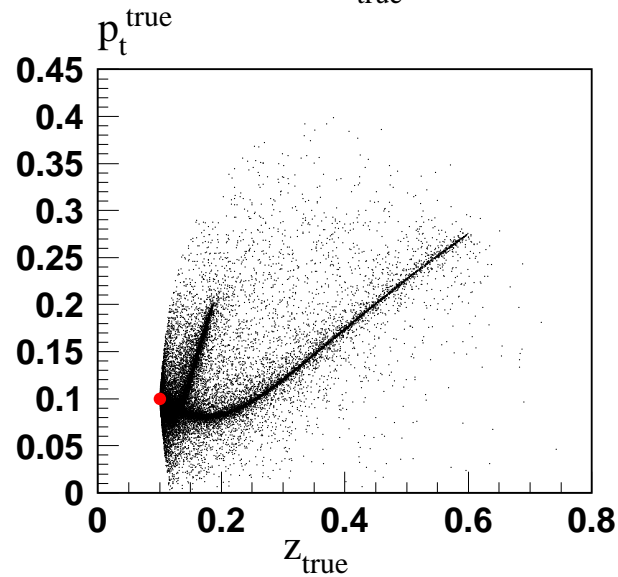
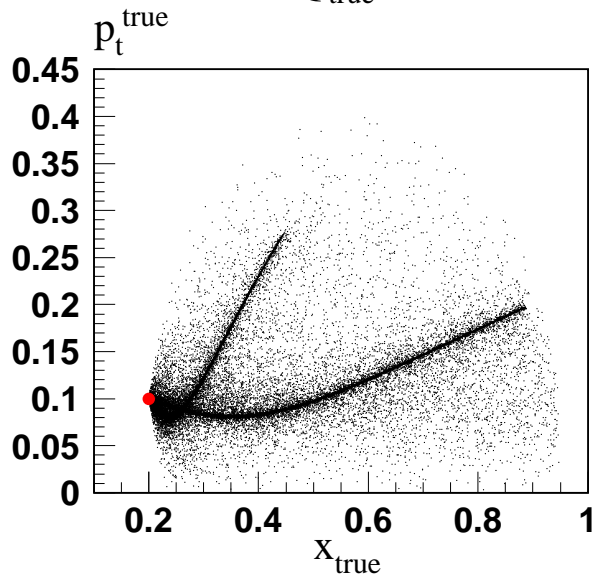
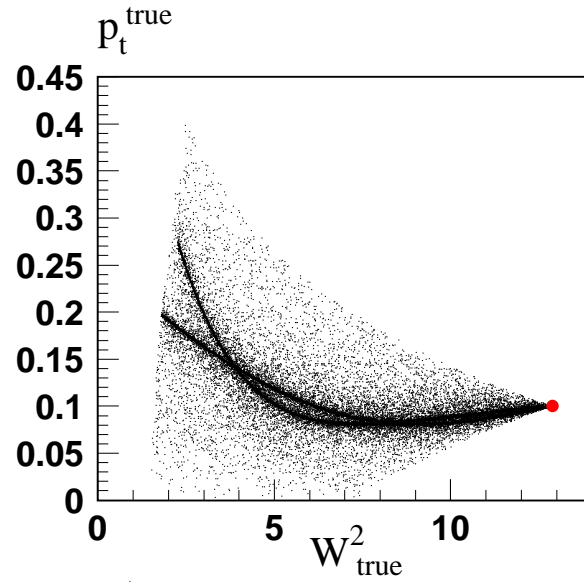
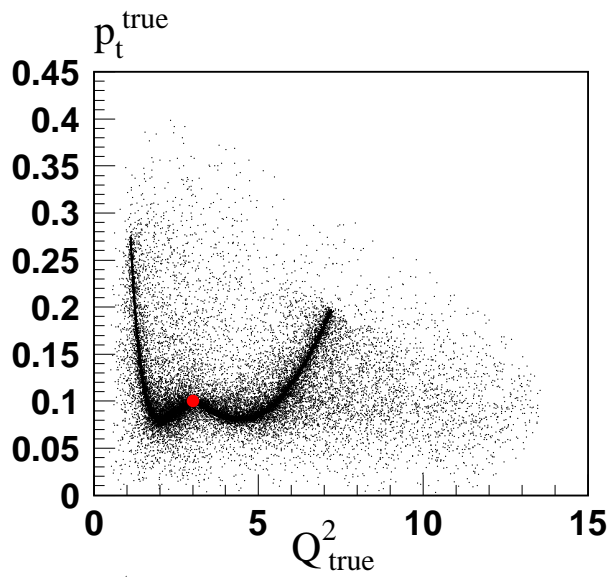
---

- ➔ We believe that RC correction in data analyses have to be calculated using the exact formulae.
- ➔ We never know that the leading log formulae are sufficient in a certain bin before we prove this by comparison to the exact formulae.
- ➔ We can use the codes with the exact formulae to extract leading log formulae numerically (based on  $\sigma = \log \frac{Q^2}{m^2} A + B + O(m^2/Q^2)$ ), and therefore to test codes that provide leading log estimates of RC.
- ➔ Our numeric experiments show that the difference in RC calculated approximately and exactly can be more than 10% for RC >4%.
- ➔ Monte Carlo generators with generation of the angles of the radiated photons cannot be created using leading log formulae.

# Kinematic Regions for SIDIS SFs ( $x, Q^2, W^2, z$ )



# Kinematic Region for SIDIS SFs (pt)

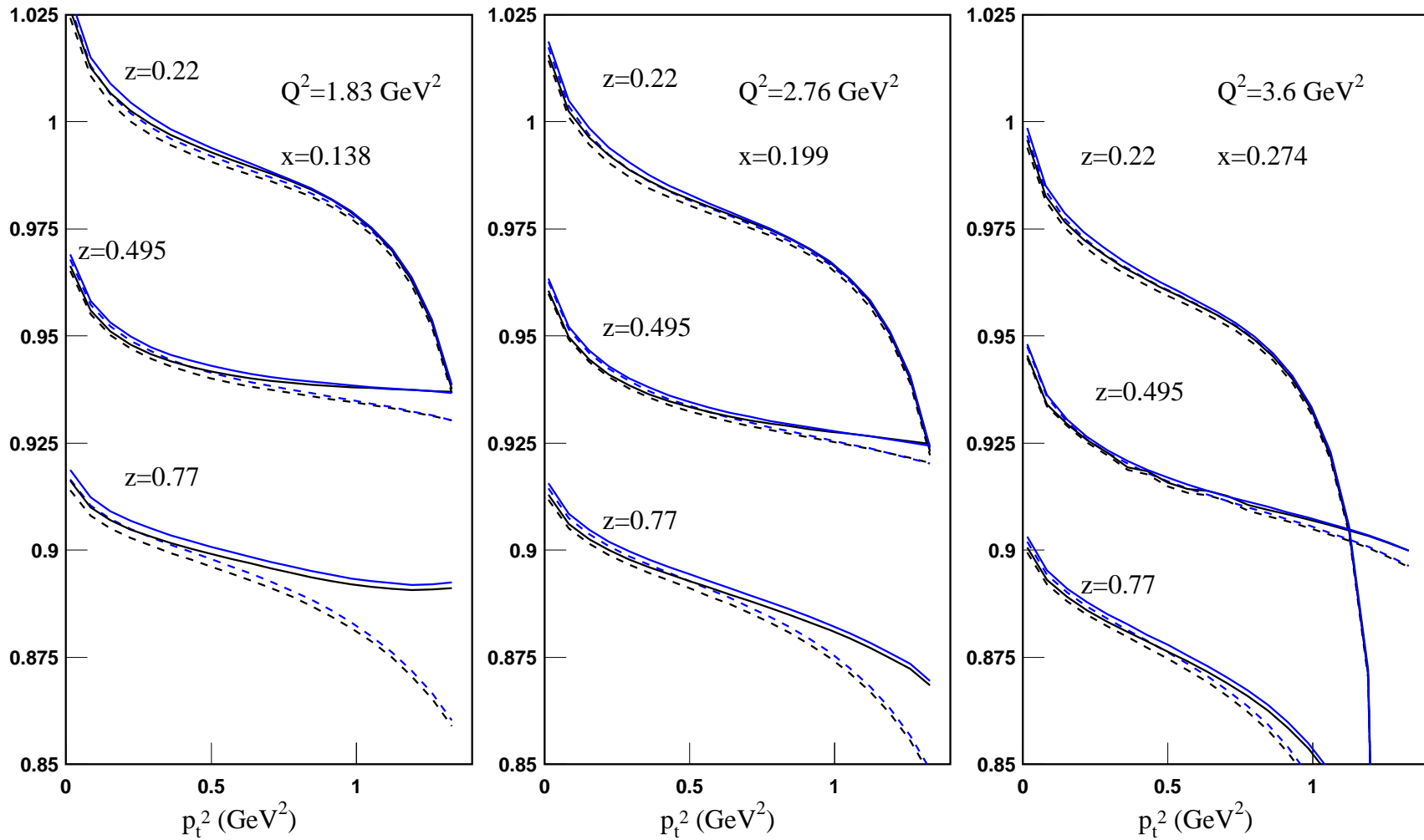


# Conclusions from prior analyses

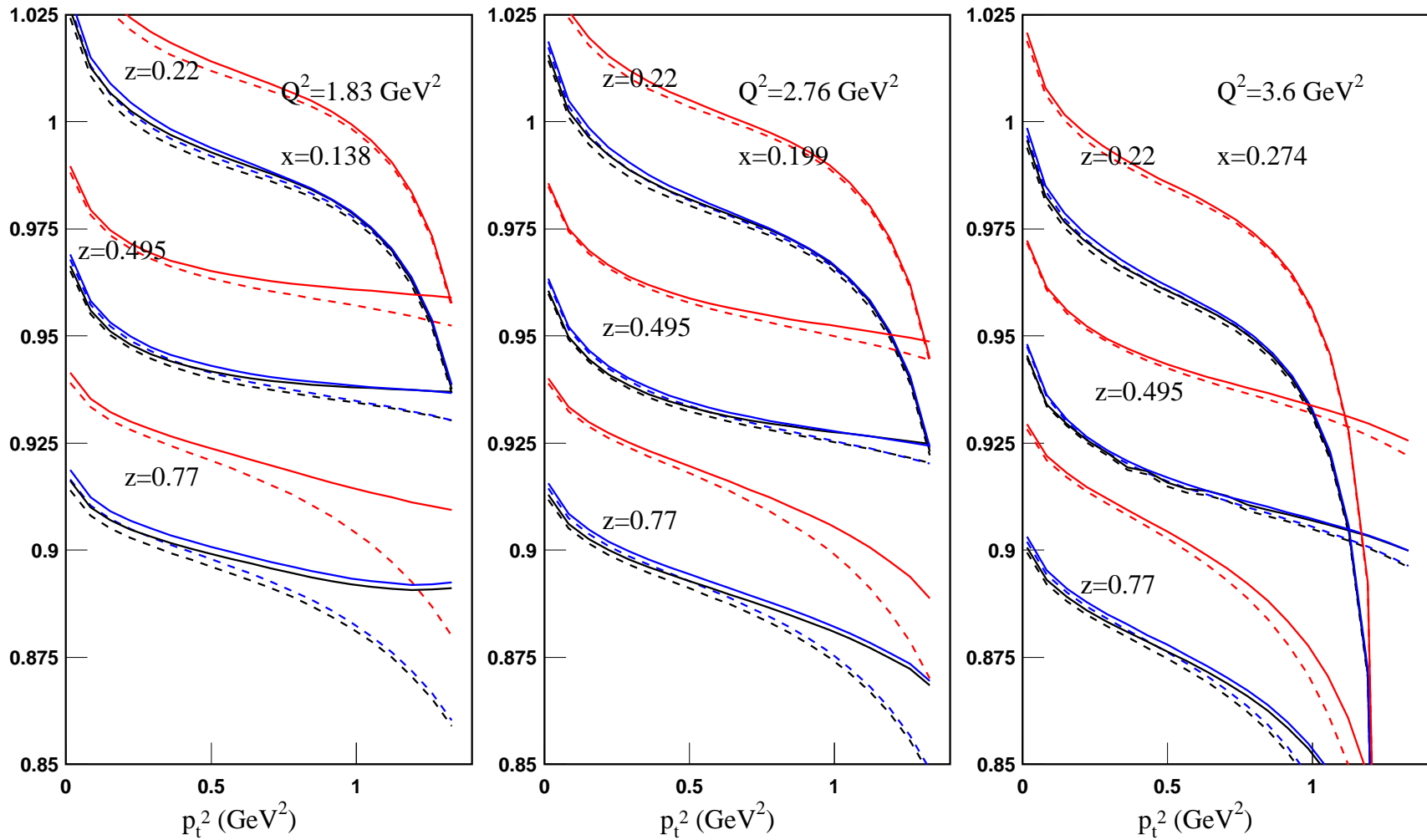
---

- ➔ RC to azimuthal asymmetry can reach 10-20%. We note that different input of SFs produce different corrections (even by sign).
- ➔ Exclusive radiative tail is important contribution to RC. there are kinematical regions (e.g., small  $M_X^2$  where it is a dominant contribution.
- ➔ The RC may be very significant at large PT.
- ➔ There exist effects not observed at the level of the Born cross section (i.e.,  $\langle \cos(3\phi) \rangle$ ).
- ➔ RC to Sivers and Collins asymmetries can reach several dozens of percents and is expected to be sensitive to SF input.

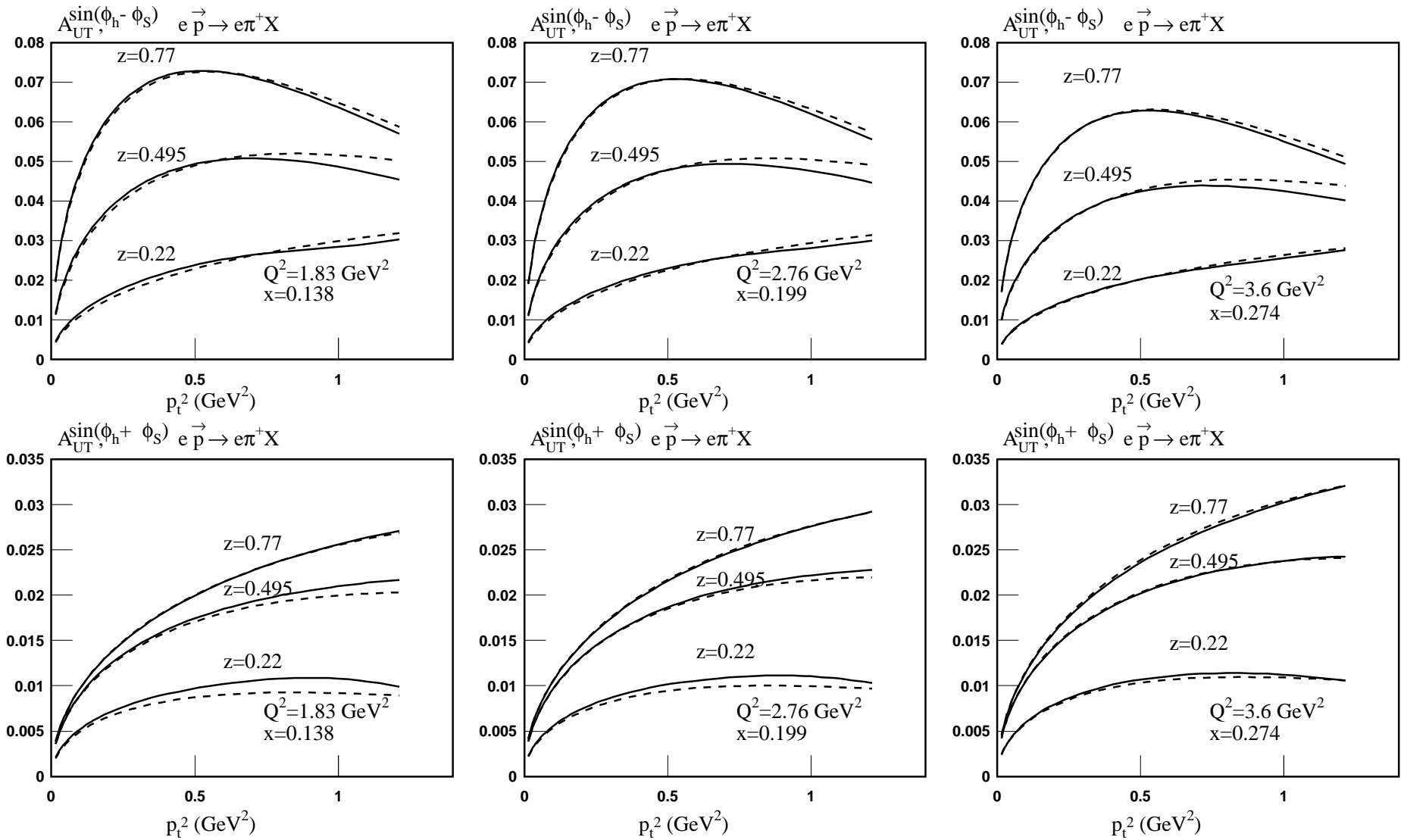
# RC to Unpolarized Cross Section



# RC to Unpolarized Cross Section



# RC to Collins and Sivers Asymmetries



# Open Points

---

**Exclusive SFs for polarized case** *We are working on it, but we will need several models to test model-dependence. Any help will be appreciated.*

**SIDIS SFs** *We currently implemented WW approach, however, additional cross checks are required for specific regions. Bounds of applicability of WW are needed to be defined and discussed. Additional models would be helpful as well.*

**Iteration procedure of RC** *We need to decide whether iteration procedure of RC will be used in data analyses. We strongly recommend this (especially for asymmetry measurements) to avoid an additional bias due to RC procedure. The bias is proportional to the difference between the values of Born asymmetries in the given bin: i) finally extracted and ii) used in RC codes. These values coincide by definition when the iteration procedure is used.*

**Comparison and agreement between teoretical calculations** *We need to complete analytic and numeric comparison to leptonic RC calculated by Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, and N. Sato (JLAB-THY-21-3489).*

**Hadronic corrections** *Discuss and decide whether and how non-leptonic corrections (including box diagrams and emission by hadrons) will be calculated.*

**Higher order corrections** *We need to discuss approaches for higher order corrections, such as exponentiation, electron SFs, etc. Pay specific attention to the region with small  $M_x^2$ .*

**Monte Carlo** *Approaches to MC generators has to be discussed and implemented.*

---



# Monte Carlo Generator RADGEN

---

We constructed the Monte Carlo generator RADGEN using POLRAD 2.0 (IA, Boettcher, Ryckbosch, hep-ph/9906408)

The cross section is represented in the sum of two positively definite contributions

$$\sigma_{obs} = \sigma_{non-rad} + \sigma_{rad}$$

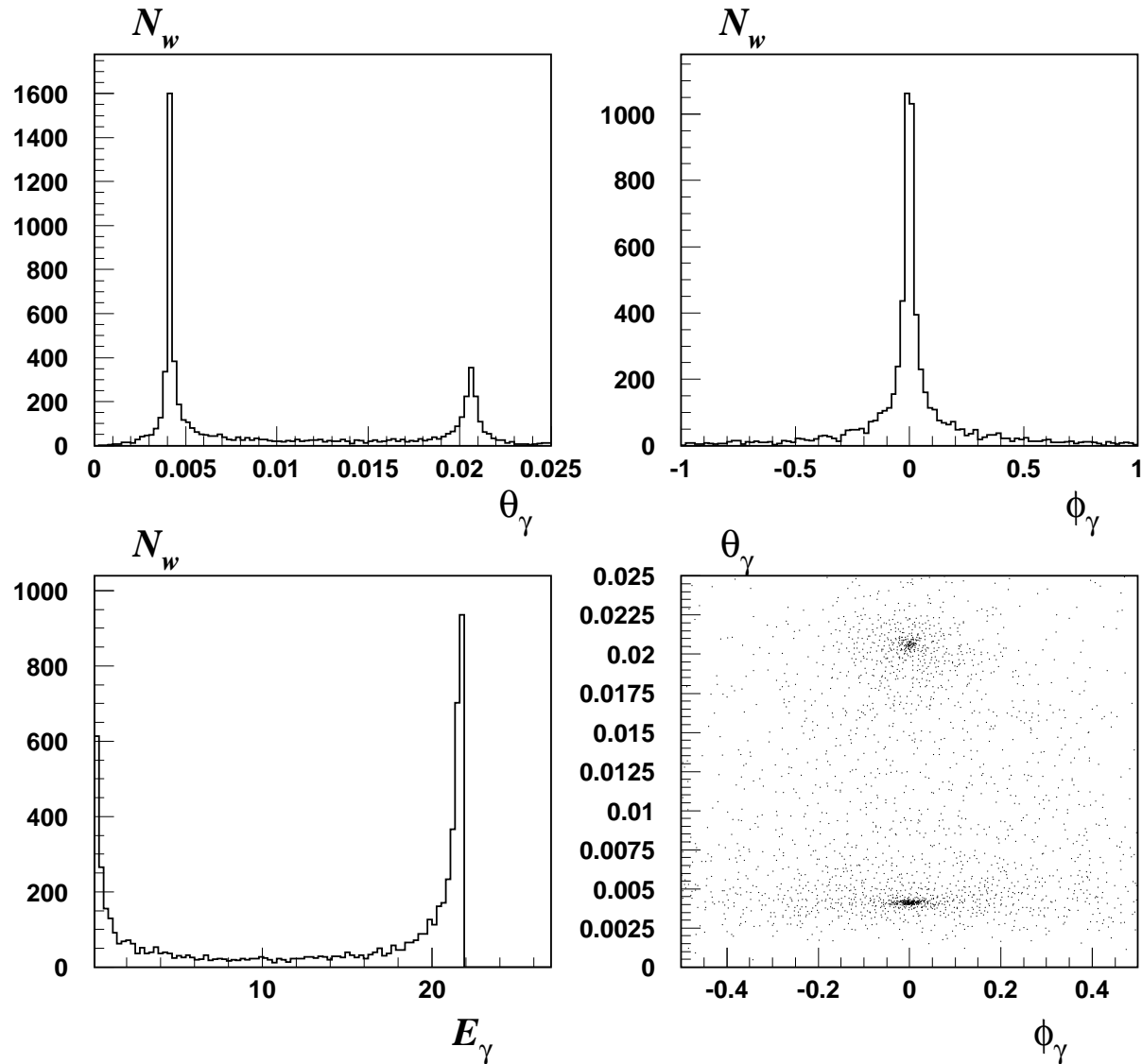
where  $\sigma_{non-rad}$  contains loop diagrams and soft photon emission and  $\sigma_{rad}$  is the contribution of additional hard photon emission with energy larger than a minimal photon energy  $\epsilon_{min}$  associated with resolution in calorimeter.

In spite of introducing the artificial parameter  $\epsilon_{min}$  there is no loosing an accuracy and no acquired dependence of the cross section of this parameter

Two modes for generator operation

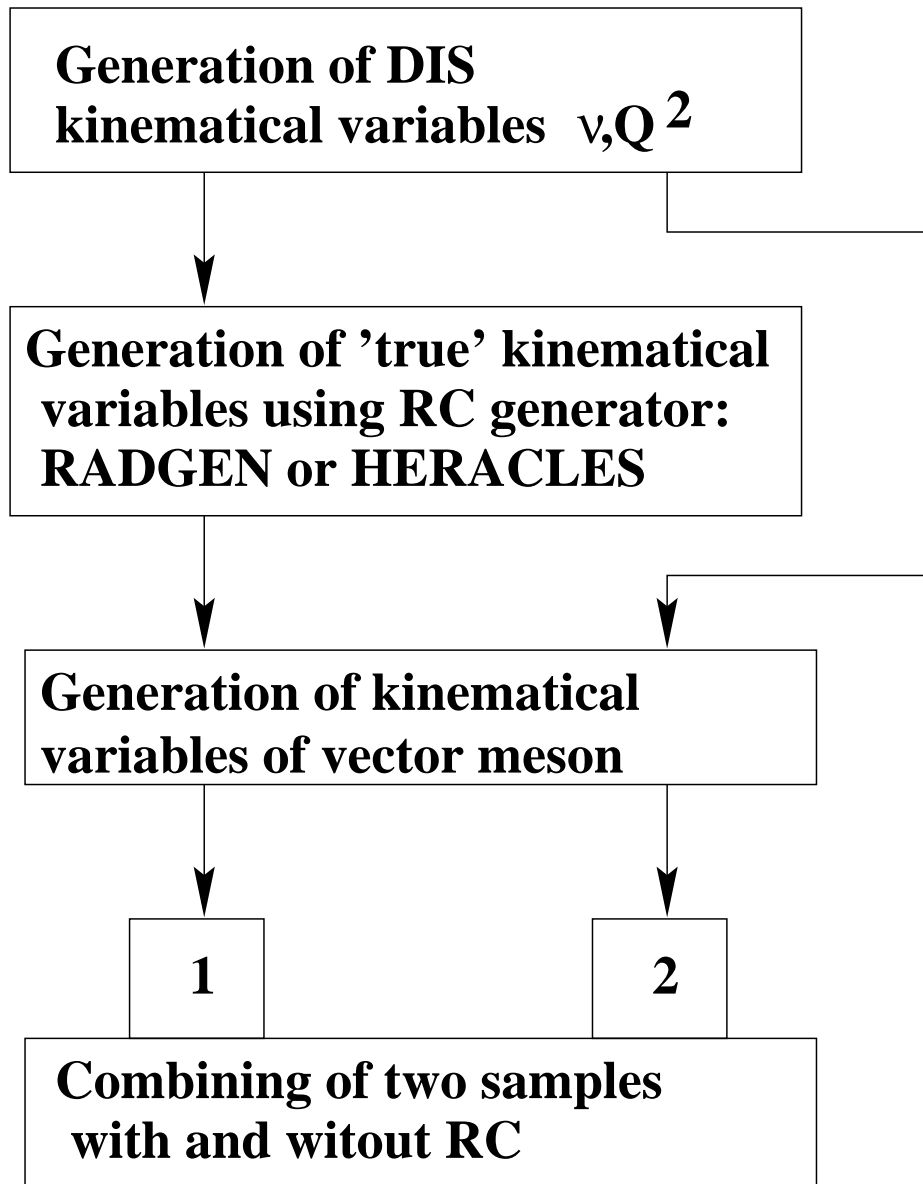
- ➔ integrals are calculated for each event and grid for a simulation is stored
- ➔ look-up table calculated in advance is used for interpolation of the grid

# Monte Carlo Generator RADGEN



# Calculation of RC to SIDIS using Monte Carlo generator

---



Possible scheme of Monte Carlo calculation of the RC factor (Akushevich, hep-ph/9906410)

# Calculation of RC to SIDIS using Monte Carlo generator

---

Can we calculate the RC to SIDIS using generators like RADGEN or DJANGO?

**My answer is NO**

DIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}F_1 + p_\mu p_\nu F_2$$

SIDIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}H_1 + p_\mu p_\nu H_2 + p_{h\mu} p_{h\nu} H_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) H_4$$

The DIS cross section:

$$\sigma = K_1(x, Q^2)F_1(x, Q^2) + K_2F_2(x, Q^2)$$

The SiDIS cross section:

$$\sigma = K_1\tilde{H}_1(x, z, p_T, Q^2) + K_2\tilde{H}_2(x, z, p_T, Q^2) + K_3\tilde{H}_3(x, z, p_T, Q^2) \cos^2 \phi_h + K_4\tilde{H}_4(x, z, p_T, Q^2) \cos \phi_h$$

The contributions involving  $K_3$  and  $K_4$  cannot be reproduced using DIS generators.

---

# Monte Carlo generator of the Radiated Photon for SIDIS

---

- ➔ Generation in DIS using RADGEN
  - ➔ Generate final electron kinematics (i.e.,  $Q^2$  and  $x$ )
  - ➔ Calculate RC in the generated point using RADGEN or POLRAD 2.0
  - ➔ Generate a channel: non-radiated or radiated, i.e., inelastic, elastic, or quasielastic (for nuclear targets)
  - ➔ Generation of the radiated photon kinematics
  
- ➔ Generation in SiDIS using a new Monte Carlo generator
  - ➔ Generate final electron kinematics (i.e.,  $Q^2$  and  $x$ ) and final hadron kinematics (i.e.,  $z$ ,  $p_T$ , and  $\phi_h$ )
  - ➔ Calculate RC in the generated point using HAPRAD 2.0
  - ➔ Generate a channel: non-radiated or radiated, i.e., inelastic, exclusive, or coherent (for nuclear targets)
  - ➔ Generation of the radiated photon kinematics

# SIDIS event generator on radiative corrections

---

A Monte Carlo event generator created by Duane Byer from Duke University, based on the SIDIS RC paper: <https://github.com/duanebyer/sidis>

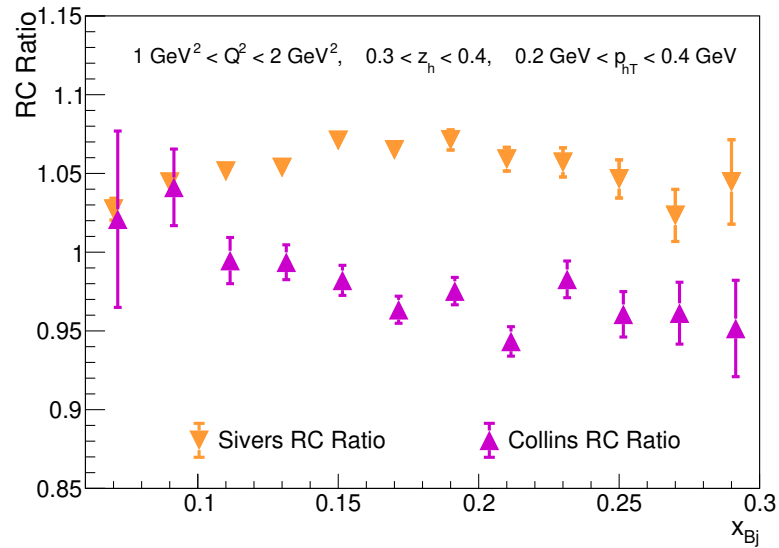
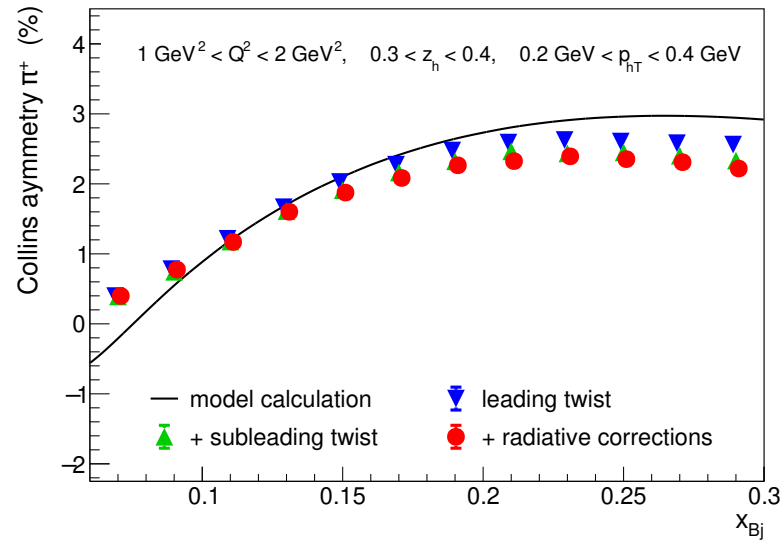
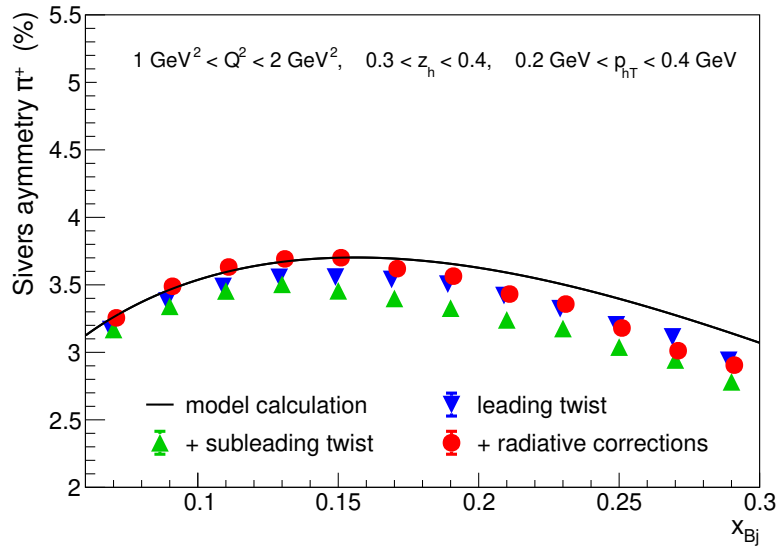
- ➔ Generates events for SIDIS six-fold cross sections computation
- ➔ All eighteen SIDIS structure functions implemented in Gaussian and Wandzura-Wilczek type approximations: S. Bastami et. al., JHEP06, 007 (2019)
- ➔ More fine tuning on the generator should be done for running it in the SoLID experiment's framework, including the neutron target
- ➔ In the meantime, the generator can be used for other experiments from medium to high energies, which also measure the SIDIS processes (CIAS12, COMPASS, etc.)

A respective paper is made by Duane Byer and Vlad Khachatryan, including also Haiyan Gao, Igor Akushevich, Alexander Ilyichev, Chao Peng, Alexei Prokudin, Stan Srednyak, and Zhiwen Zhao

- ➔ Paper soon will be submitted to the archive and Computer Physics Communication
  - ➔ Another paper is in preparation by Akushevich, Ilyichev, and Srednyak on exclusive structure functions in SIDIS
  - ➔ Event generator will be updated in the future for better numerical performance and to include exclusive structure functions
-

# SIDIS event generator on radiative corrections

Examples of extracted Collins and Sivers asymmetries without exclusive SF included



# RC procedure of experimental data in SIDIS

---

The possible (successful) strategy of RC could be developed using our experience in the modeling for DIS. The RC procedure of experimental data should involve an iteration procedure in which the fits of SFs of interest are re-estimated at each step of this iteration procedure.

- ➔ Assume that we have measurements in  $N$  bins: i.e., we measure

$$\beta_i = \{x_i, Q_i^2, z_i, p_{Ti}, \phi_{hi}\}$$

- ➔ We have the integral equation for each bin:

$$\sigma_{extr} = \sigma_{meas} + \alpha C \int dx_{tr} dz_{tr} dQ_{tr}^2 \sum K(\beta, \beta_{tr}) H(x_{tr}, Q_{tr}^2, z_{tr}, p_{T,tr})$$

- ➔ We fit  $\sigma_{meas}$  (or SFs) using measurements in all bins,
  - ➔ We construct the overall fit jointing this fit and a fit from the region beyond the region of the experiment,
  - ➔ We use this overall fit to calculate RC,
  - ➔ We use the equation to obtain  $\sigma_{extr}$  in each bin,
  - ➔ We fit  $\sigma_{extr}$  using measurements in all bins,
  - ➔ ....
  - ➔ Continue till fitting parameters (or  $\sigma_{extr}$ ) do not change.
-



# RC procedure of experimental for SIDIS

---

- ➔ *The fit of SFs are constructed to have the model in the region covered by the experiment*
- ➔ *Use experimental data or theoretical models to construct the models in the regions of softer processes, resonance region, and exclusive scattering*
- ➔ *Check that the constructed models provide correct asymptotic behavior when we go to the kinematical bounds (Regge limit, QCD limit)*
- ➔ *Joint all the models to have continuous function of all four variables in all kinematical regions necessary for RC calculation*
- ➔ *Implement this scheme in a computer code and define the iteration procedure*
- ➔ *If several SFs are measured in an experiment, implement the procedure of their separation in data and model each of them.*
- ➔ *If other SFs are necessary (e.g., unpolarized SFs when spin asymmetries are measured), construct the models for them as well.*
- ➔ *Pay specific attention to exclusive SFs, because the radiative tail from exclusive peak is important (or even dominate) in certain kinematical regions.*
- ➔ *Pay specific attention to  $p_T$  dependence because RC is too sensitive for  $p_T$  model choice.*

# Conclusion

---

- ➔ Newly achieved accuracies in Jlab and new physics studied at Jlab require paying renewed attention to RC calculations and their implementation in data analysis software.
- ➔ For SIDIS RC theoretical efforts are needed both for calculation of SIDIS RC in a bin and for generation of radiated events:
  - ➔ *Hadronic tensor for the SIDIS cross sections in the covariant form is constructed and tested. Further work is required for the hadronic tensor for exclusive processes*
  - ➔ *Exact calculation of RC for the complete SIDIS cross section containing 18 SFs is completed and coding is done, however, implementation of SFs is still in progress.*
  - ➔ *We expect sensitivity of the results for RC to specific assumptions used for constructing SIDIS SFs:*
  - ➔ *Broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SFs as well as SFs in resonance region and exclusive SFs.*
  - ➔ *Iteration procedure with fitting of measured SFs and joining with models beyond SIDIS measurements at each iteration step has to be designed, implemented and involved in data analysis.*
- ➔ Tools for generation of the radiated photon in DIS cannot provide valid generation of the radiated events in SIDIS.
  - ➔ *Such generator can be constructed based on a code for RC in SIDIS in the same way of how RADGEN is constructed based on POLRAD 2.0*