Two-photon exchange, single spin asymmetries, and proton form factors

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Radiative Corrections: from medium to high energy experiments

ECT (Trento), July 2022

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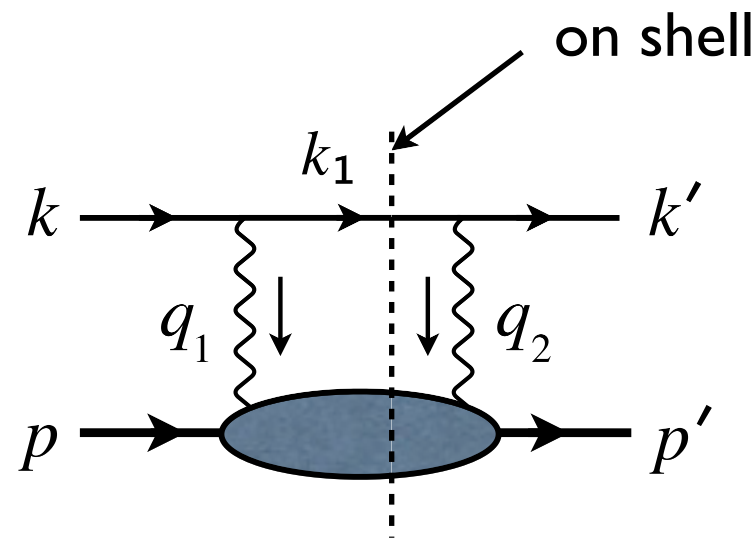
Outline

- Single spin asymmetries (SSA): target normal and beam normal cases
- Current status of two-photon exchange (TPE) calculations for elastic e-p scattering cross sections
- Some issues in extracting proton electric and magnetic form factors using TPE:
  - the G_E/G_M ratio
  - nonlinearity in ε
  - choice of form factor parametrizations for global fits: things to avoid

Dispersive method

Unitarity → $$\mathrm{Im}\langle f|\mathcal{M}|i\rangle \sim \sum_n \int dW_n\, dQ_1^2\, dQ_2^2\ \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle$$

- Imaginary part determined by unitarity
- Uses only on-shell form factors (or helicity amplitudes), directly fit to data
- Real part determined from dispersion relations

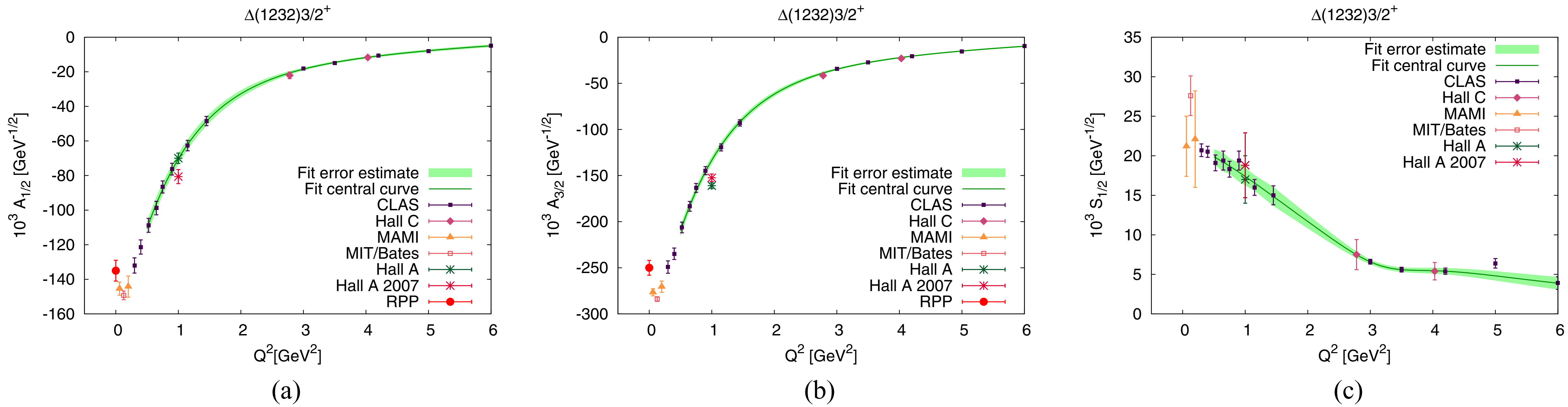


- Resonance intermediate states:

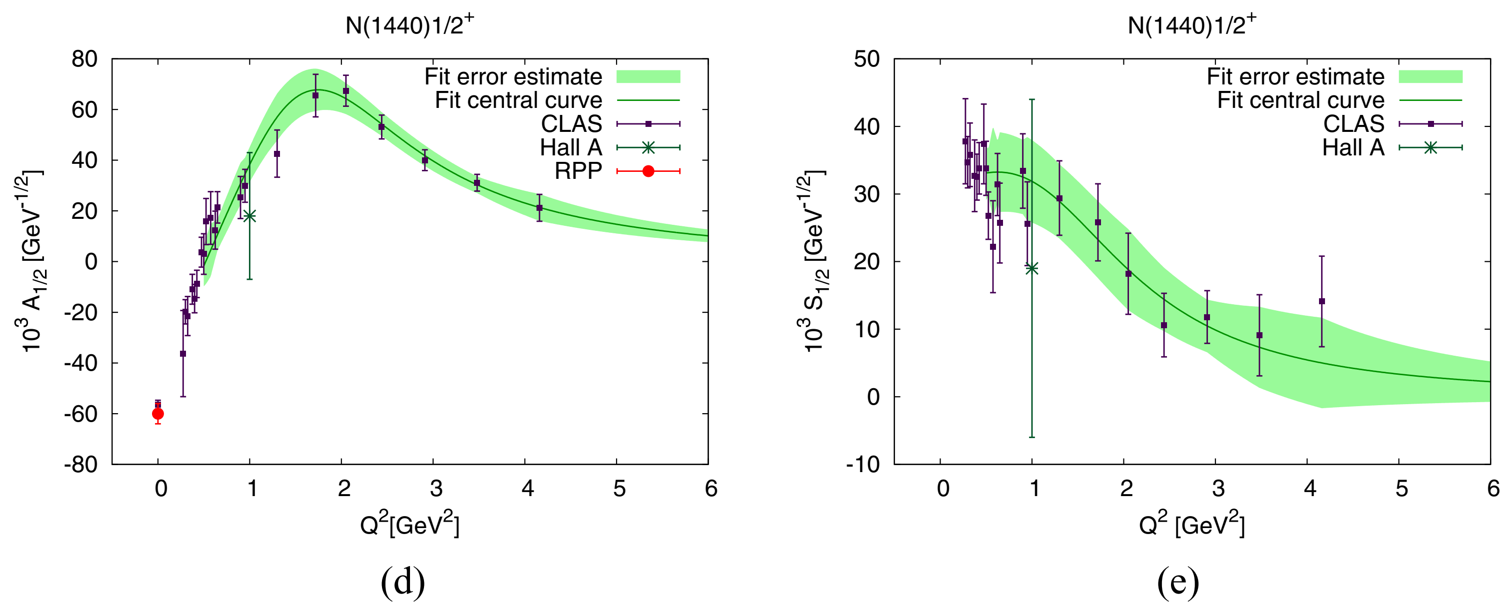
$$\Delta(1232)3/2^+,\ N(1440)1/2^+,\ N(1520)3/2^-,\ N(1535)1/2^-,$$
$$\Delta(1620)1/2^-,\ N(1650)1/2^-,\ \Delta(1700)3/2^-,\ N(1710)1/2^+,\ N(1720)3/2^+$$

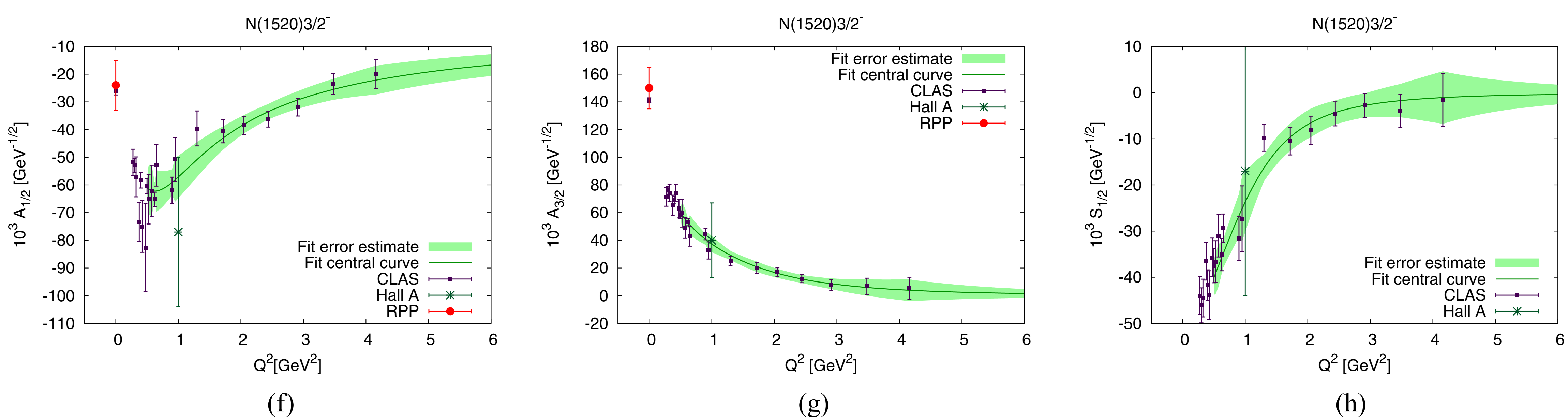
- CLAS exclusive meson electroprodution data for $A_{1/2},\ A_{3/2},\ S_{1/2}$
- Breit-Wigner lineshape

CLAS helicity amplitudes: Δ(1232), N(1440), N(1520)



Hiller Blin *et al.*, PRC**100**, 035201 (2019)





TPE using dispersion relations

Generalized form factors

$$\mathcal{M}_{\gamma\gamma} \to (\gamma_\mu)^{(e)} \otimes \left(F_1'(Q^2,\nu)\gamma^\mu + F_2'(Q^2,\nu)\frac{i\sigma^{\mu\nu}q_\nu}{2M} \right)^{(p)}$$
$$+ (\gamma_\mu\gamma_5)^{(e)} \otimes \left(G_a'(Q^2,\nu)\gamma^\mu\gamma_5 \right)^{(p)}$$

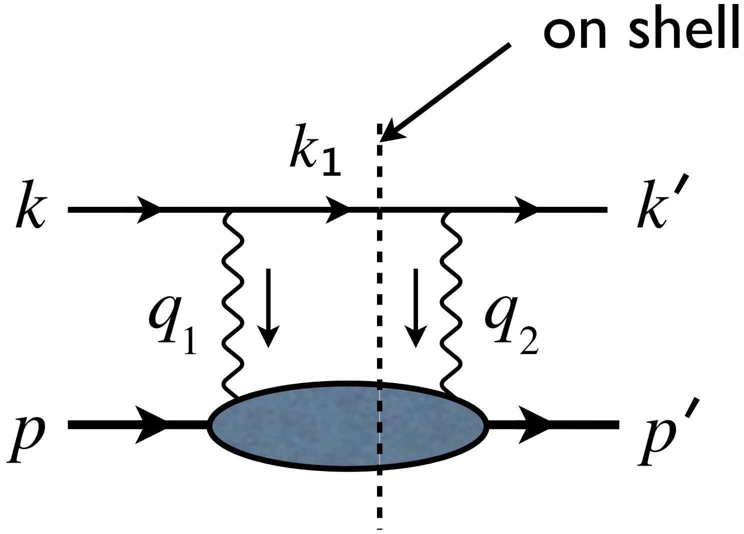
$$\delta_{\gamma\gamma} = 2\mathrm{Re}\, \frac{\varepsilon G_E(F_1' - \tau F_2') + \tau G_M(F_1' + F_2') + \nu(1-\varepsilon)G_M G_a'}{\varepsilon G_E^2 + \tau G_M^2}$$

Dispersion relations

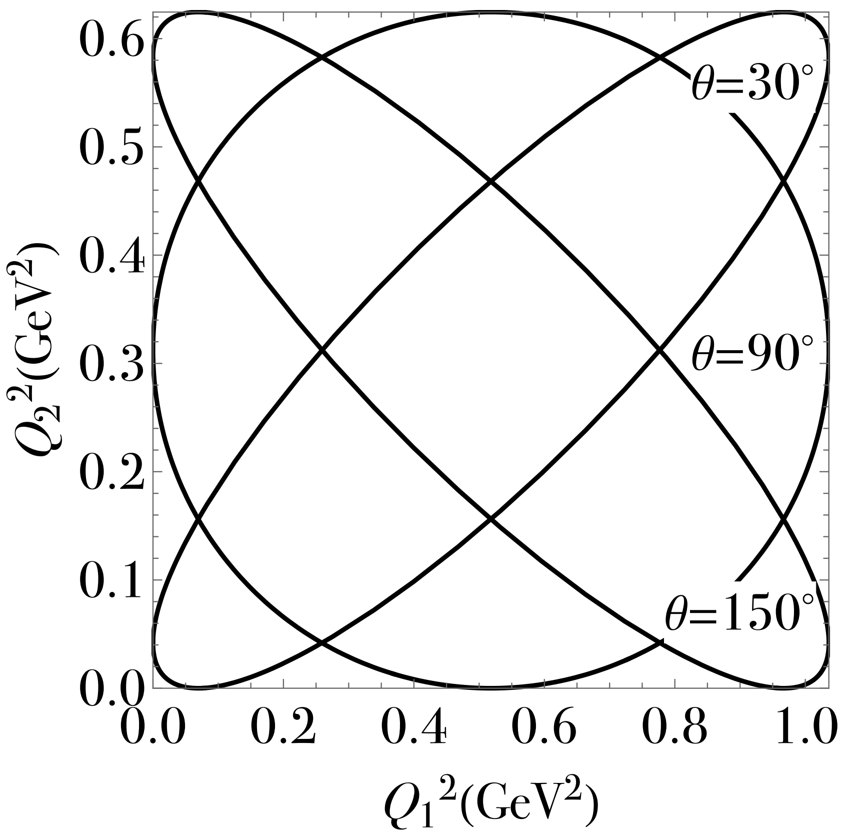
$$\mathrm{Re}\ F_1'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu}{\nu'^2 - \nu^2}\,\mathrm{Im}\ F_1'(Q^2,\nu')\,,$$
$$\mathrm{Re}\ F_2'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu}{\nu'^2 - \nu^2}\,\mathrm{Im}\ F_2'(Q^2,\nu')\,,$$
$$\mathrm{Re}\ G_a'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{-\tau}^{\infty} d\nu'\ \frac{\nu'}{\nu'^2 - \nu^2}\,\mathrm{Im}\ G_a'(Q^2,\nu')\,.$$

Integral extends into ``unphysical region'' down to zero energy ($\cos\theta < -1$)

Direct measurements of Im part



This is all in the physical region.



Target normal spin asymmetry: same FFs

$$A_n = \sqrt{2\epsilon(1+\epsilon)\tau}\,\frac{1}{\sigma_R}\mathrm{Im}\left\{-G_M(F_1' - \tau F_2' + \nu F_3') + G_E\left(F_1' + F_2' + \sqrt{\frac{2\epsilon}{1+\epsilon}}\nu F_3'\right)\right\}$$

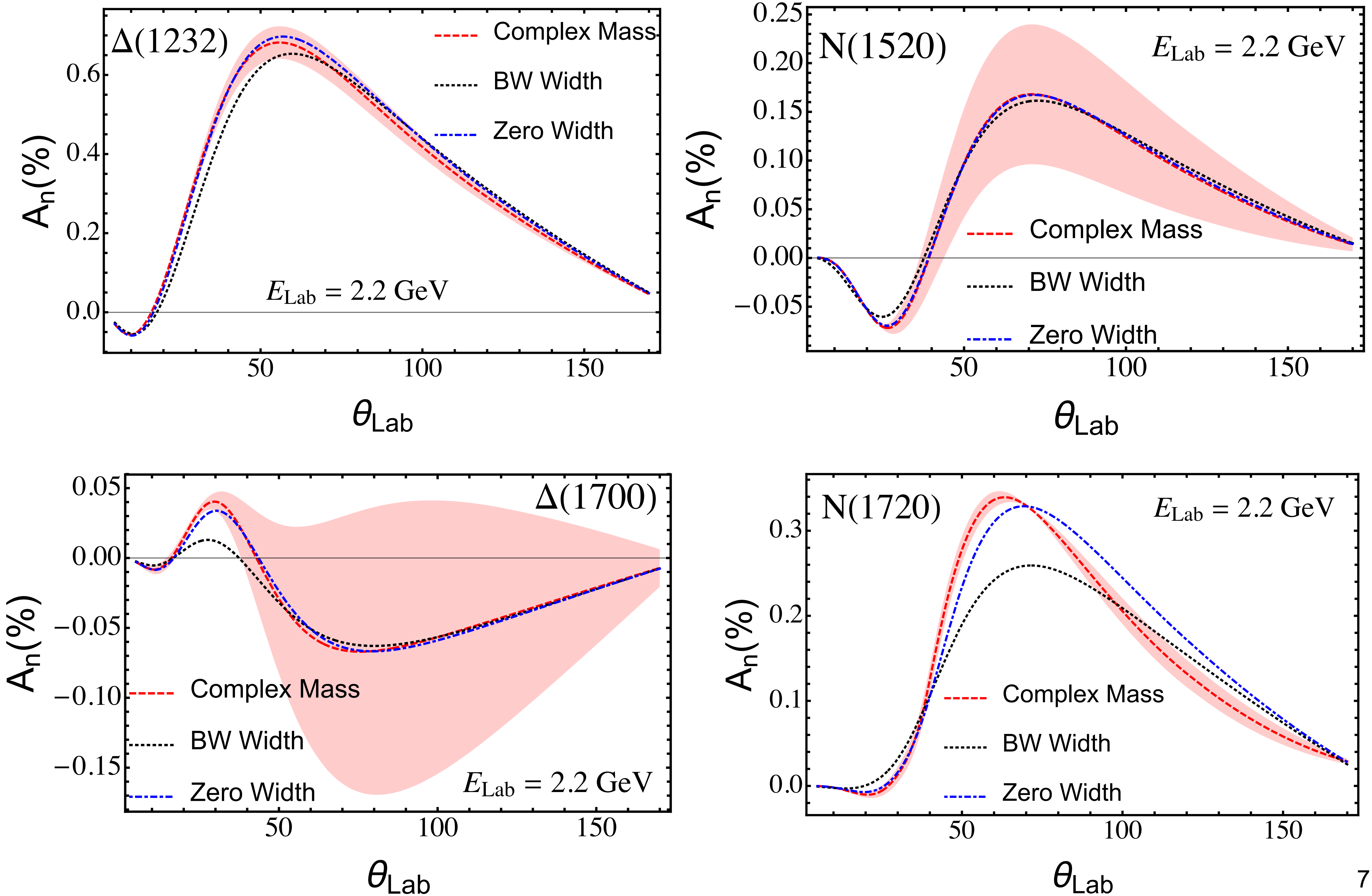
Beam normal spin asymmetry: additional dependence on spin-flip FFs

$$B_n = -\frac{m_e}{M}\sqrt{2\epsilon(1-\epsilon)(1+\tau)}\,\frac{1}{\sigma_R}\mathrm{Im}\left\{\tau G_M F_3' + G_E F_4' + \nu F_1 F_5'\right\}$$

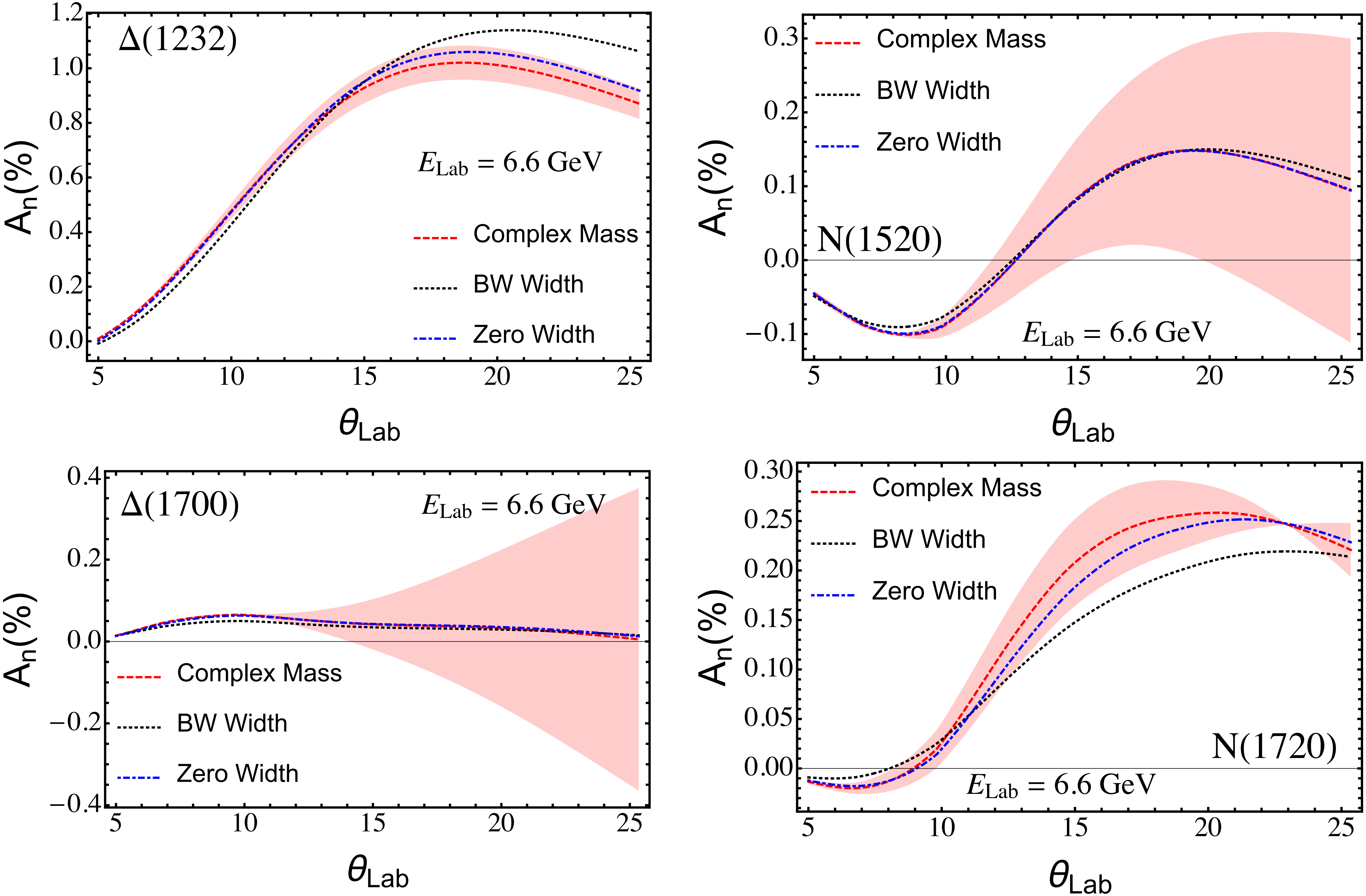
Target normal SSA: 2.2 GeV

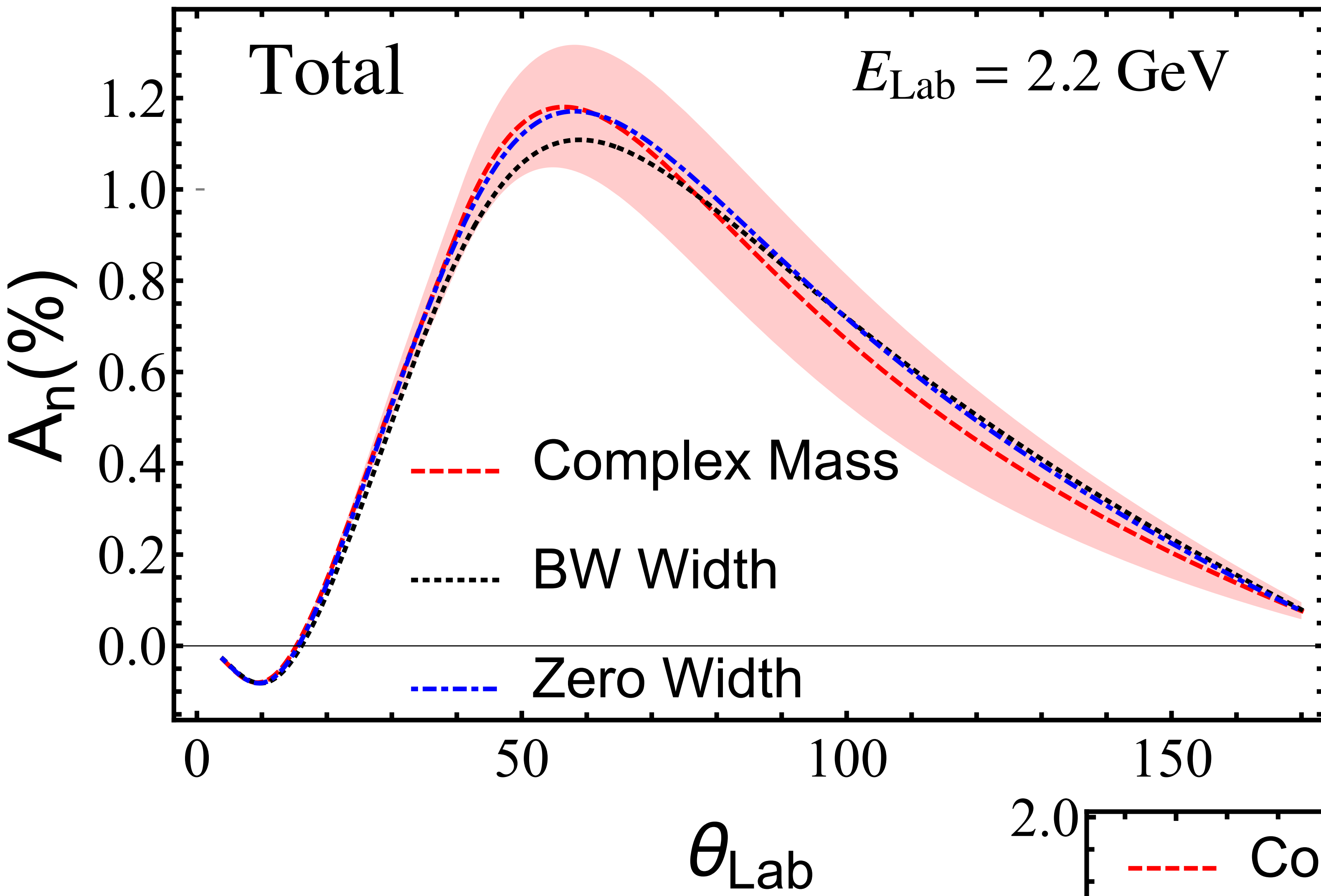
Ahmed, Blunden, Melnitchouk (in preparation)

See earlier talk by Goity

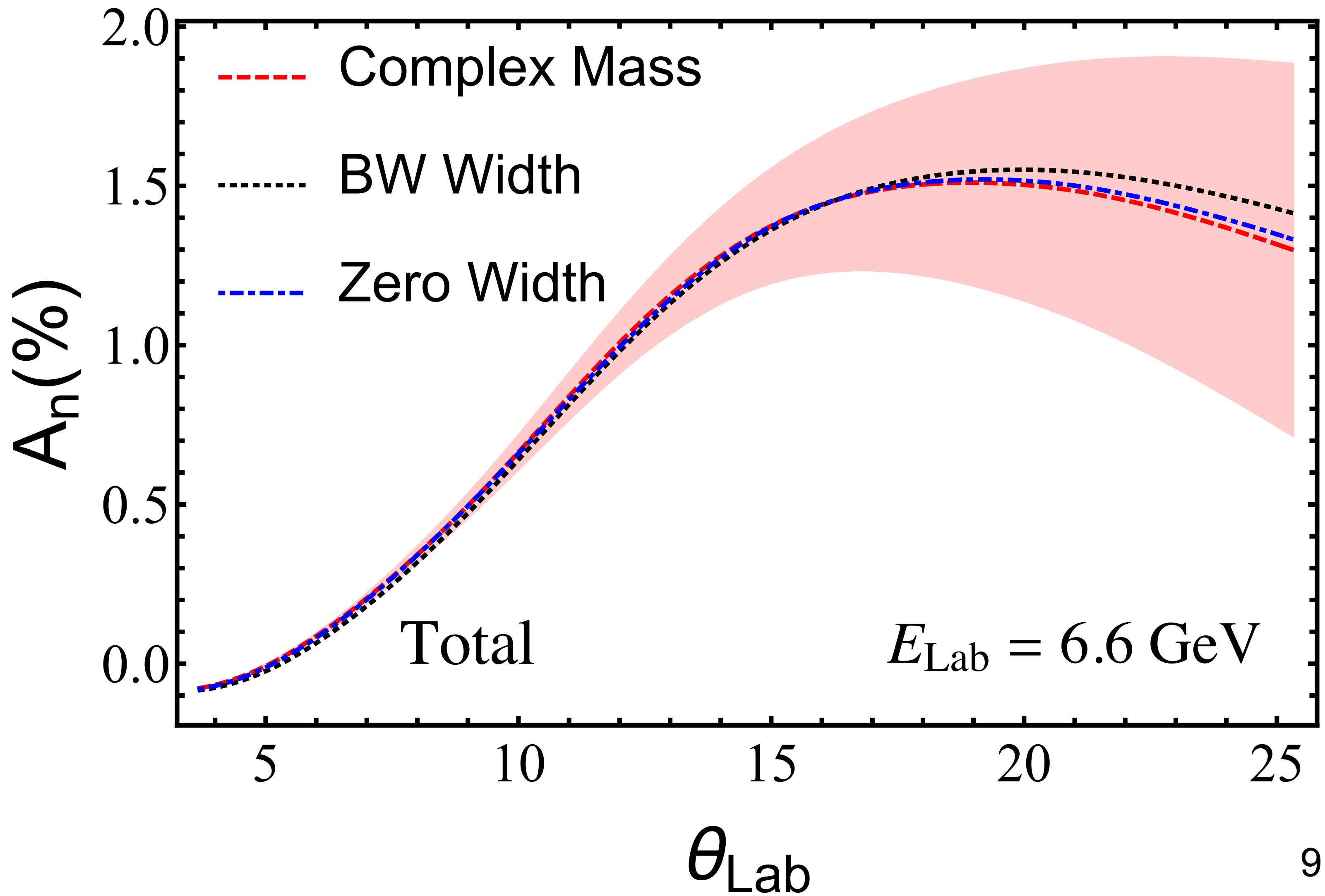


Target normal SSA: 6.6 GeV





- Free of kinematics enhancements at thresholds (e.g. $W^2 \approx s$)
- No significant dependence on width



Beam normal

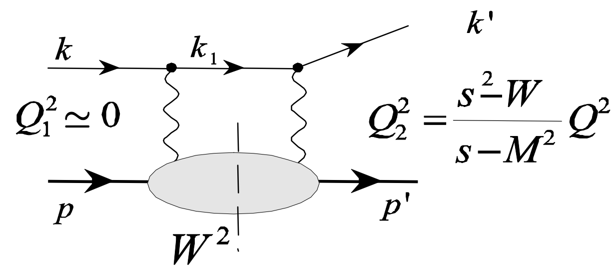
Kinematical limits

- Pasquini & Vanderhaeghen (2004)
- Afanasev & Merenkov (2004)

$Q^2_1 \simeq 0,\ Q^2_2 \neq 0$

↓

$k \,//\, k_1$

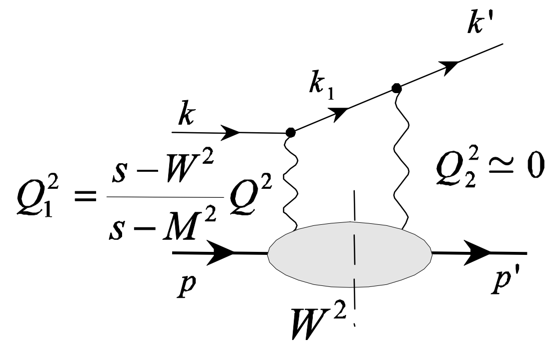


Quasi - VCS

$Q^2_1 \neq 0,\ Q^2_2 \simeq 0$

↓

$k_1 \,//\, k'$

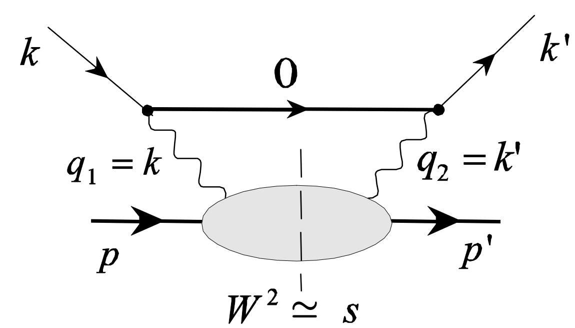


Quasi - VCS

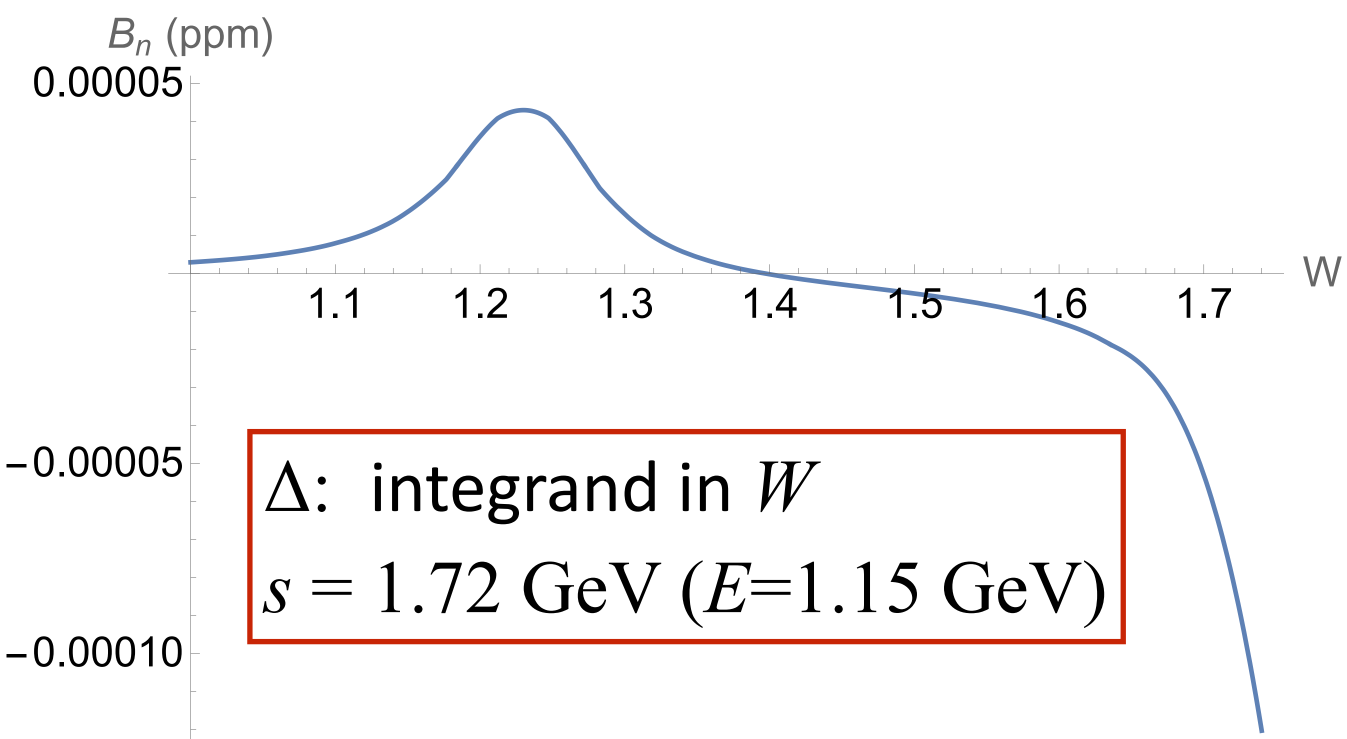
$Q^2_1 \simeq 0,\ Q^2_2 \simeq 0$

↓

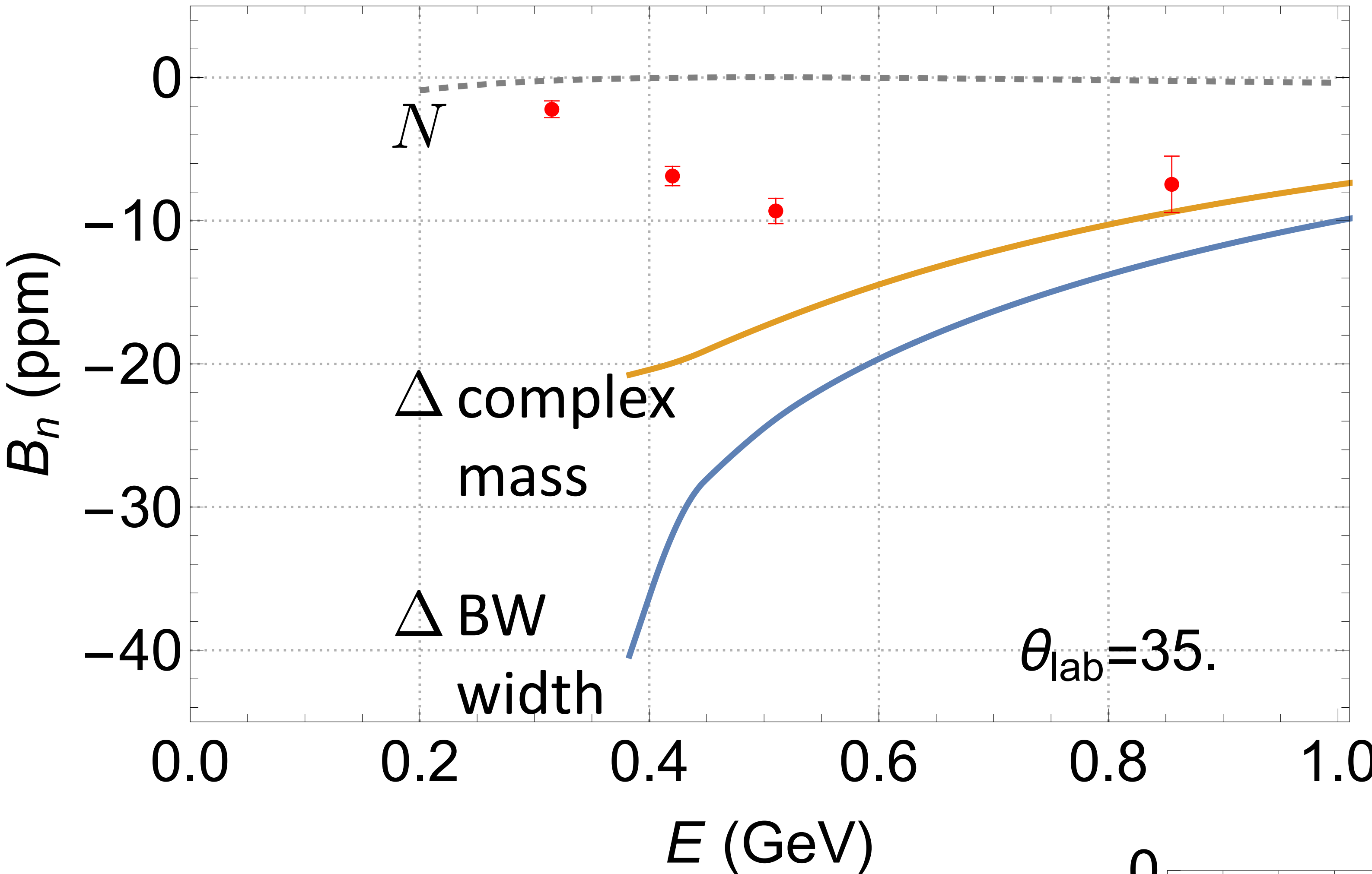
$k_1 = 0,\ W = \sqrt{s} - m_e$



Quasi - RCS

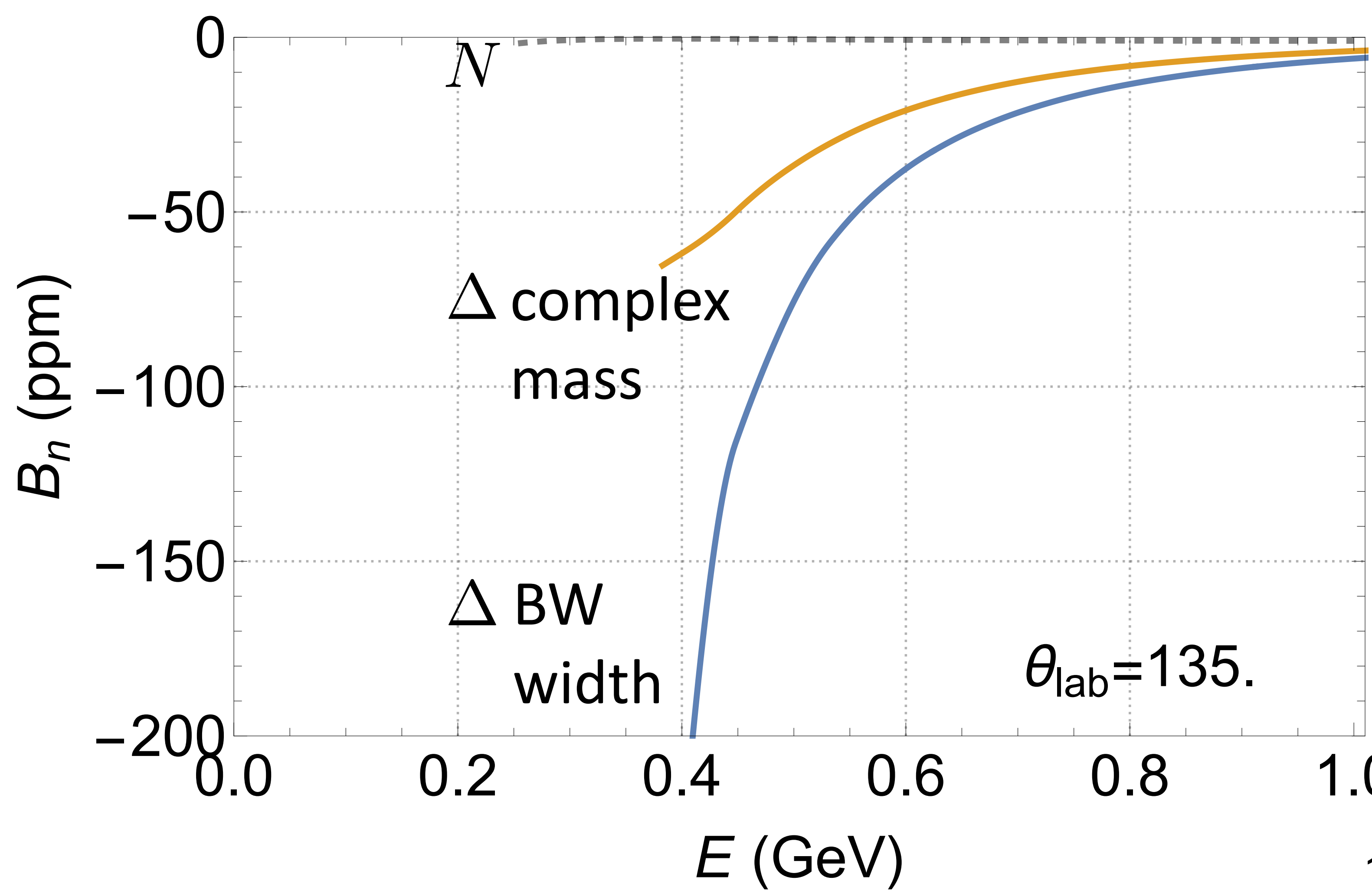


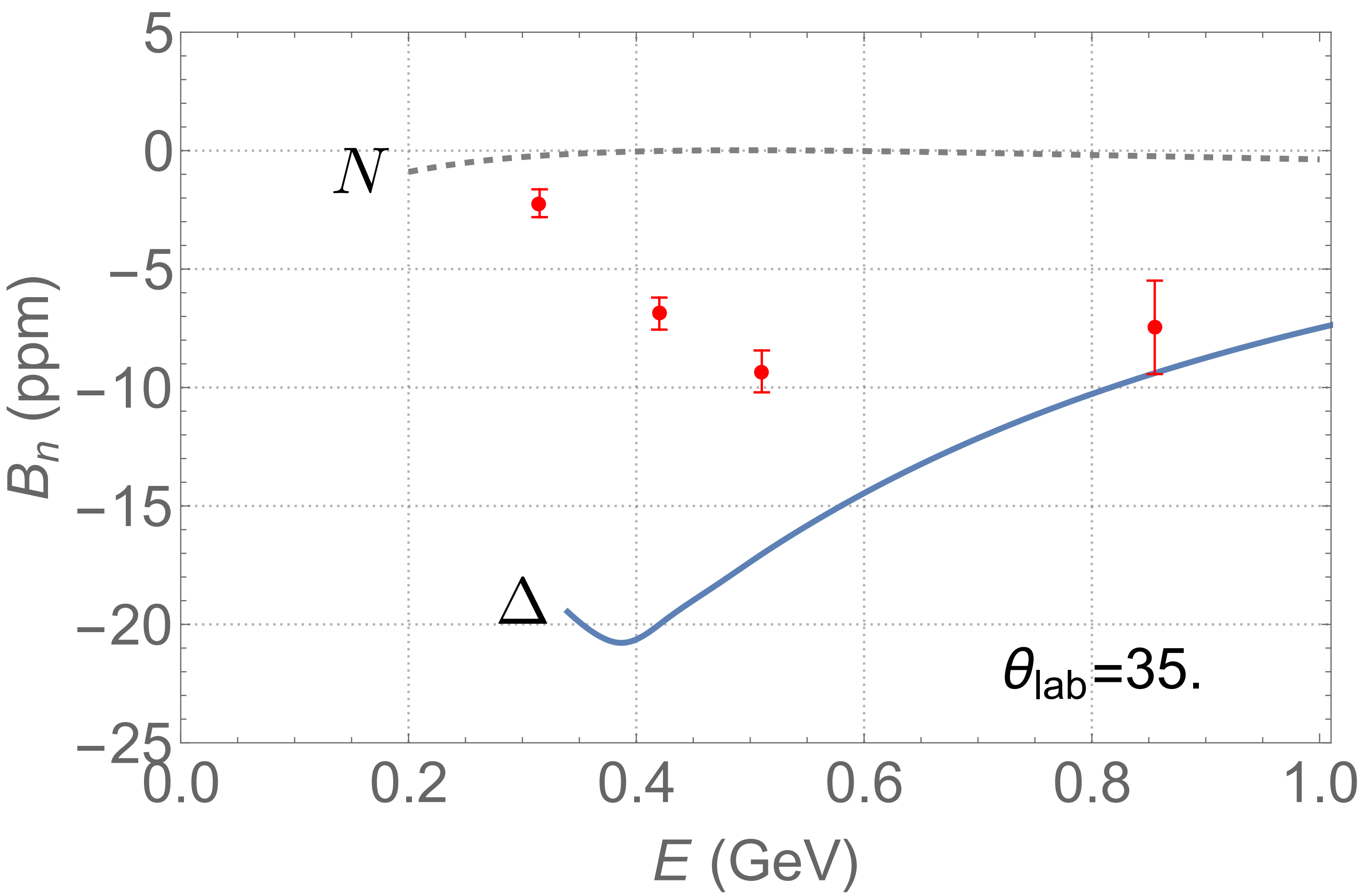
- Quasi-singular behaviour at $W^2 \approx s$
- Analogous to real Compton scattering
- BW lineshape not enough to kill it
- Explore alternatives (e.g. complex pole mass)



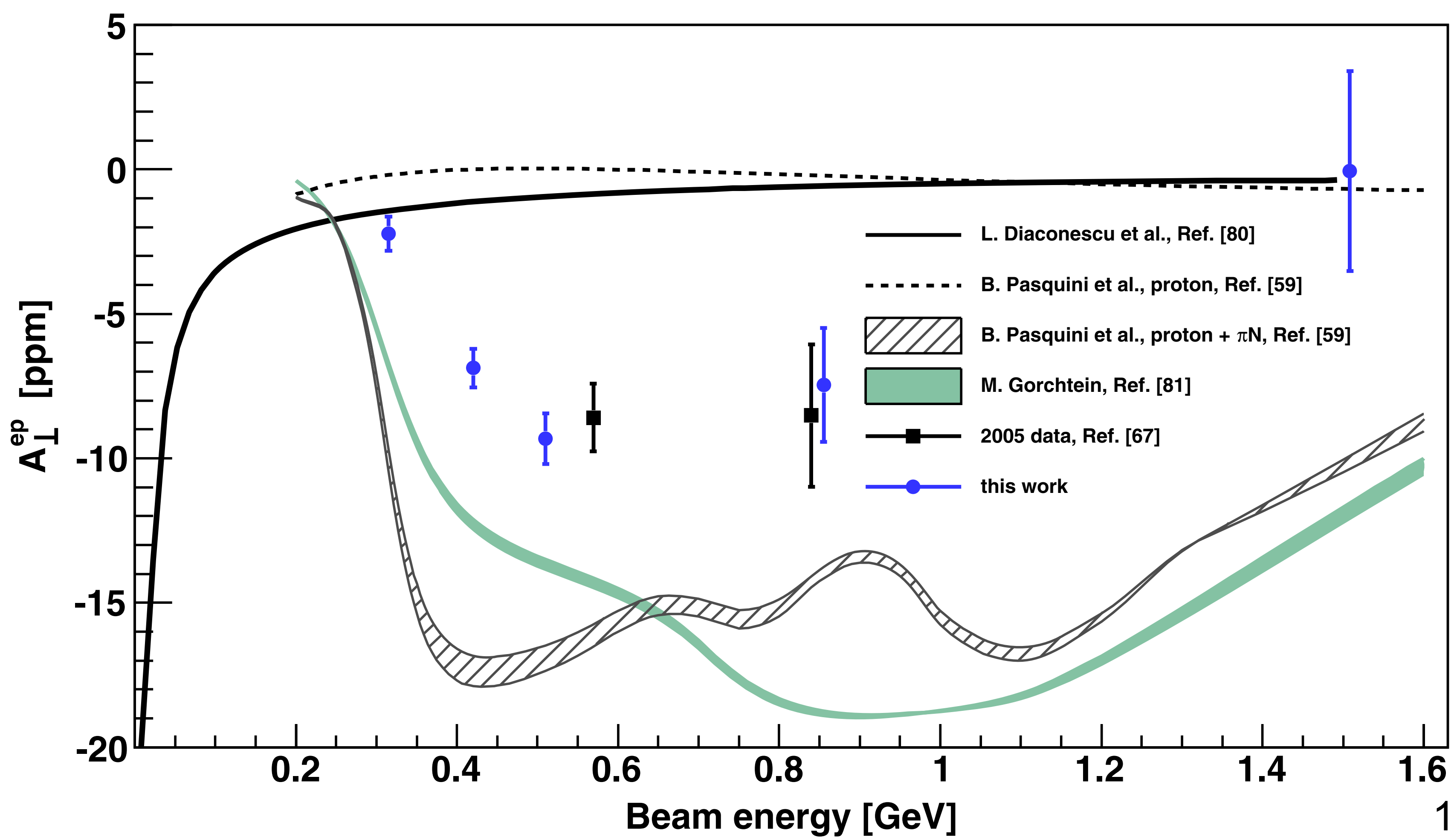
- Breit-Wigner form shows quasi-singular behaviour at Δ threshold (0.34 GeV)

- Quasi-singular behaviour even more pronounced at larger angles (higher Q^2)



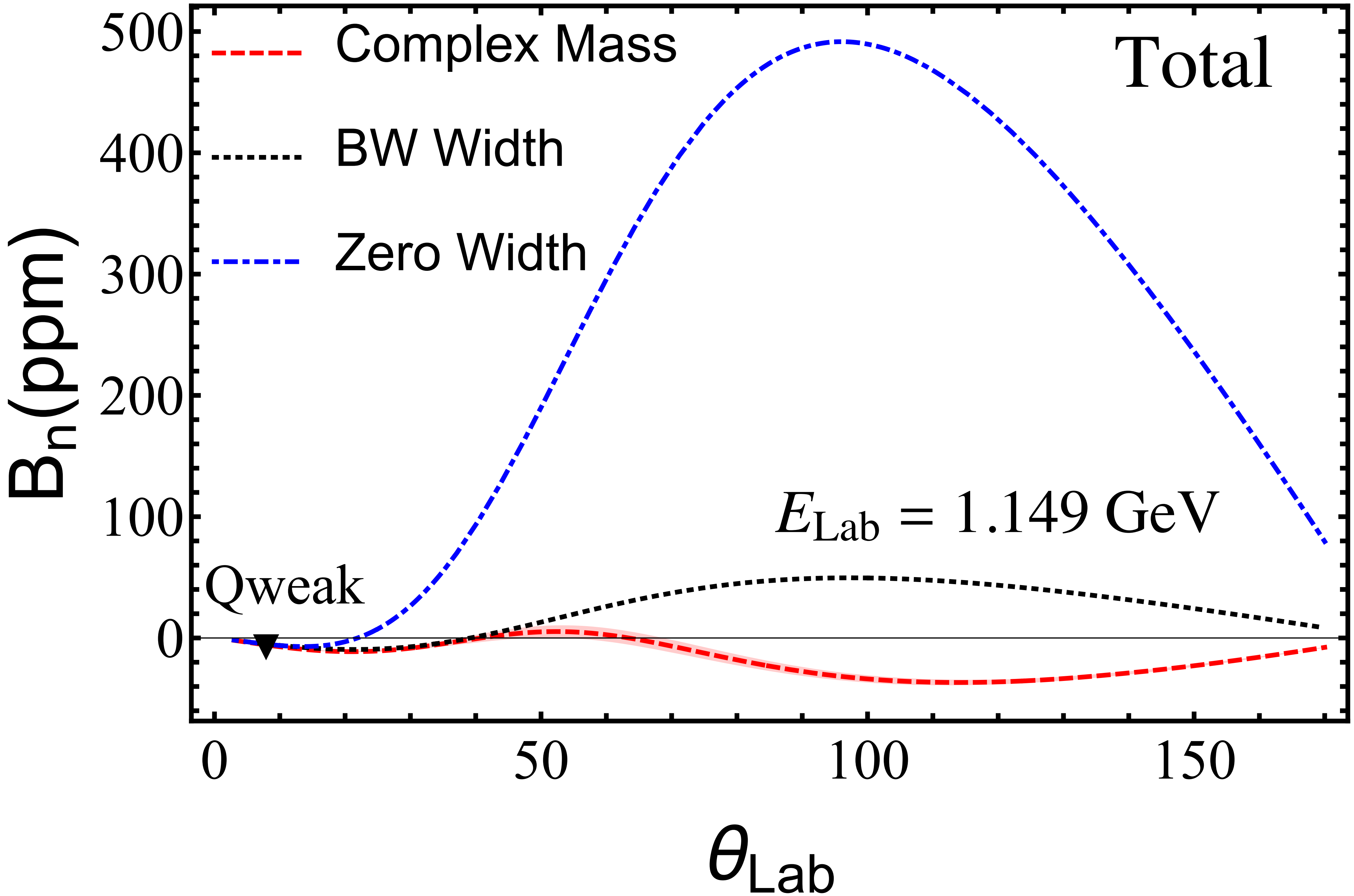


- Overshoots data at low energies and forward angles



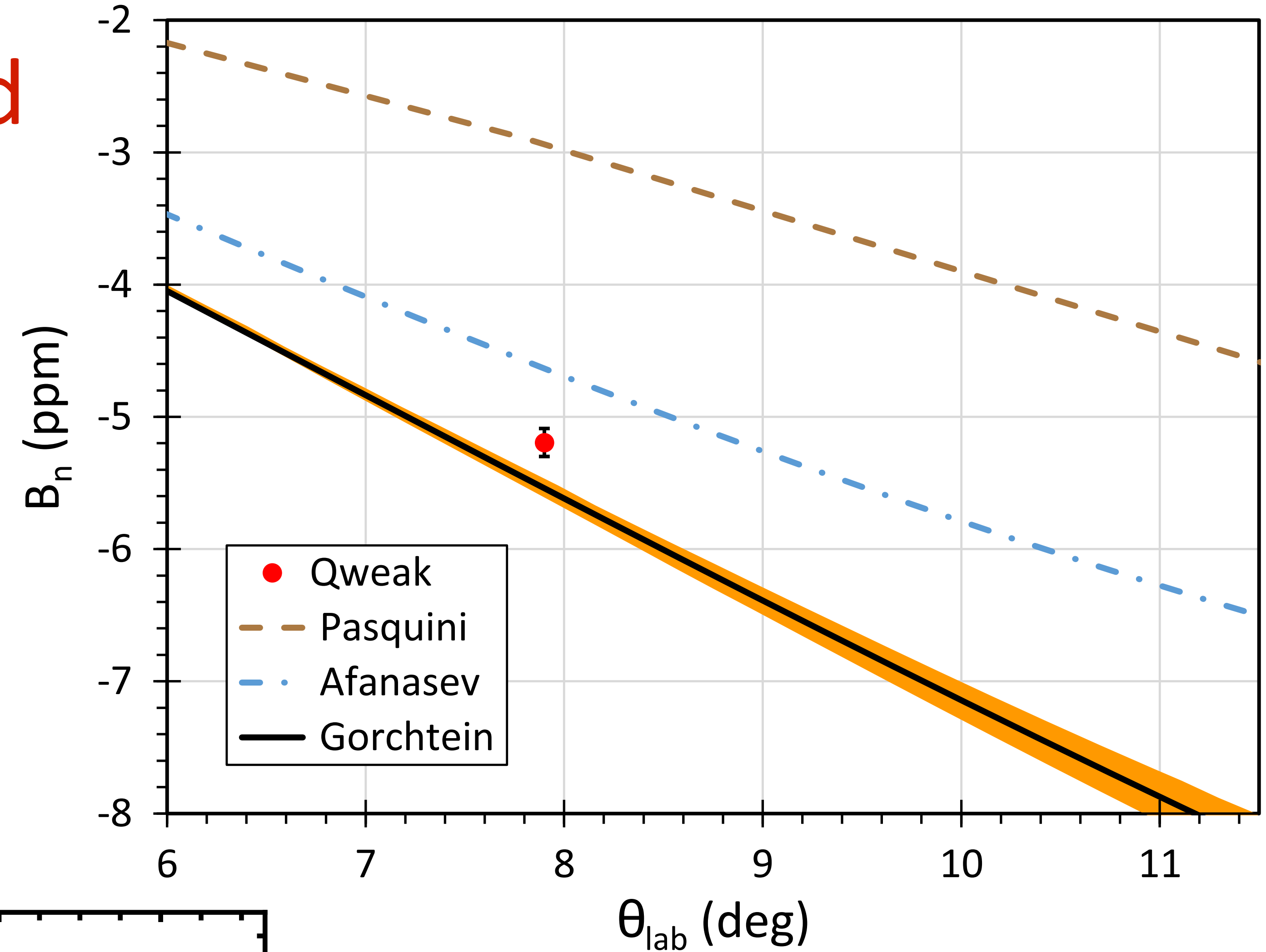
Guo *et al.*, PRL**124**, 122003 (2020)

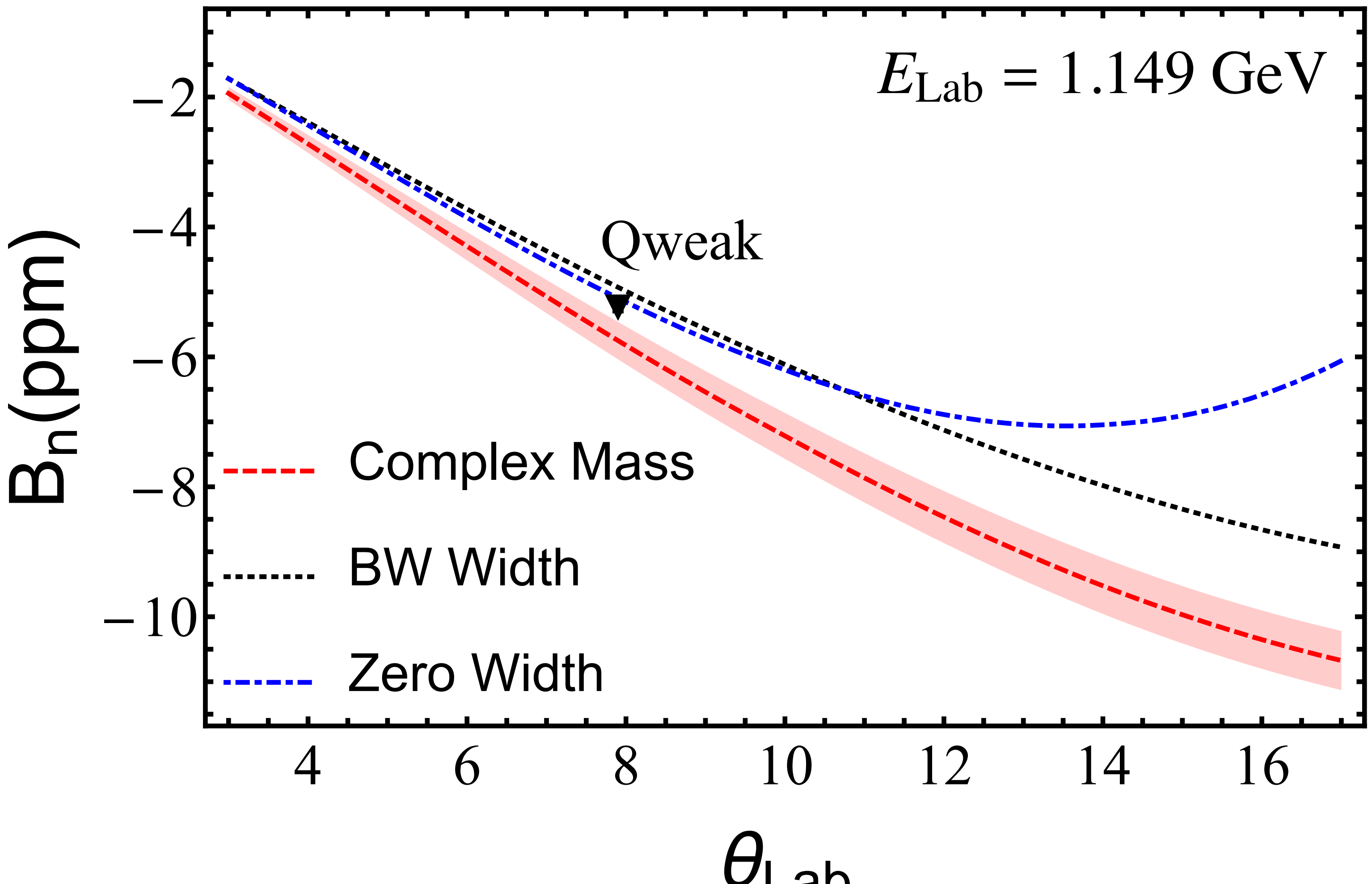
Qweak kinematics



Qweak data point magnified

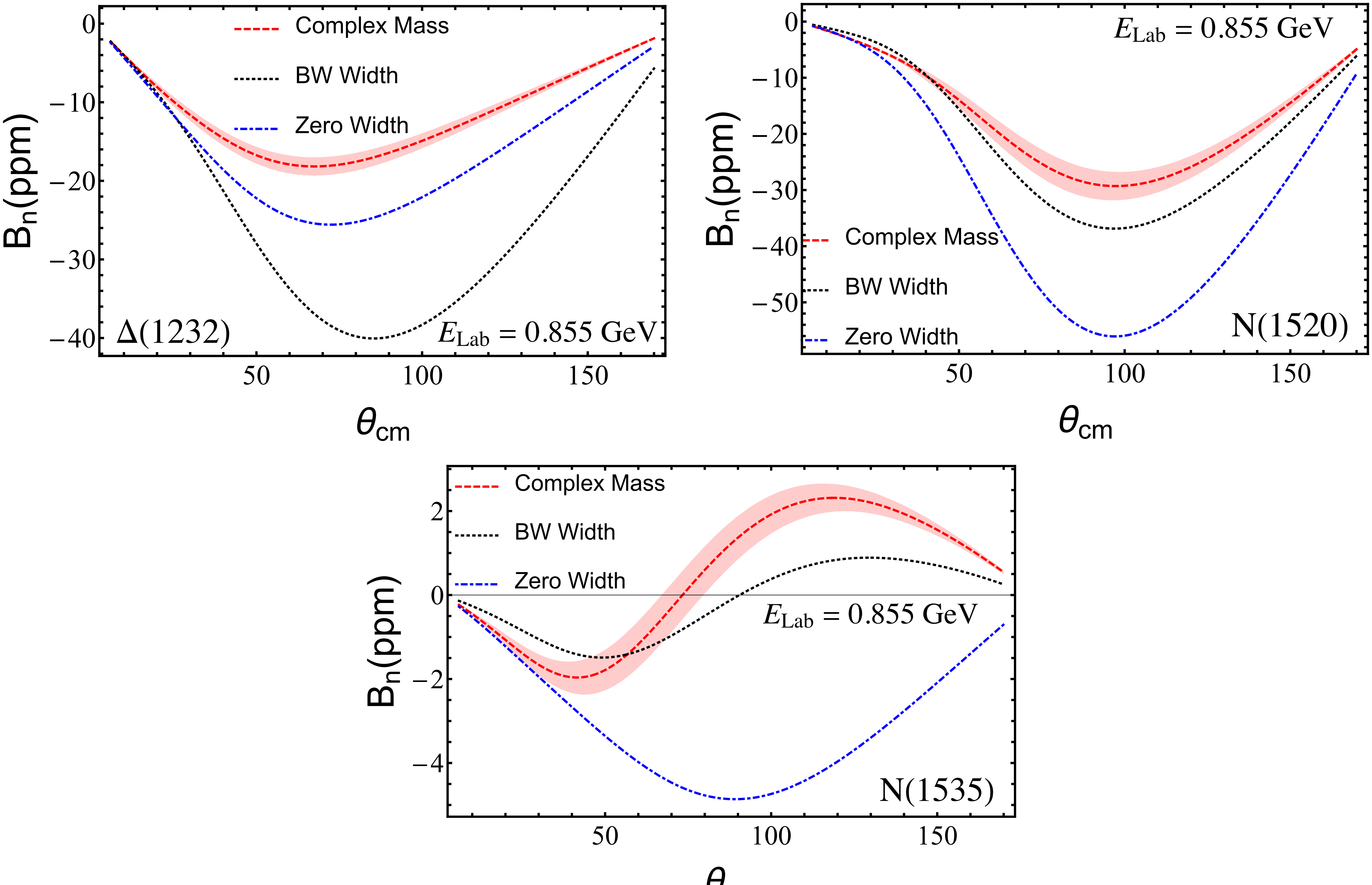
- Afanasev & Merenkov (2004)
- Pasquini & Vanderhaeghen (2004)
- Gorchtein (2006)

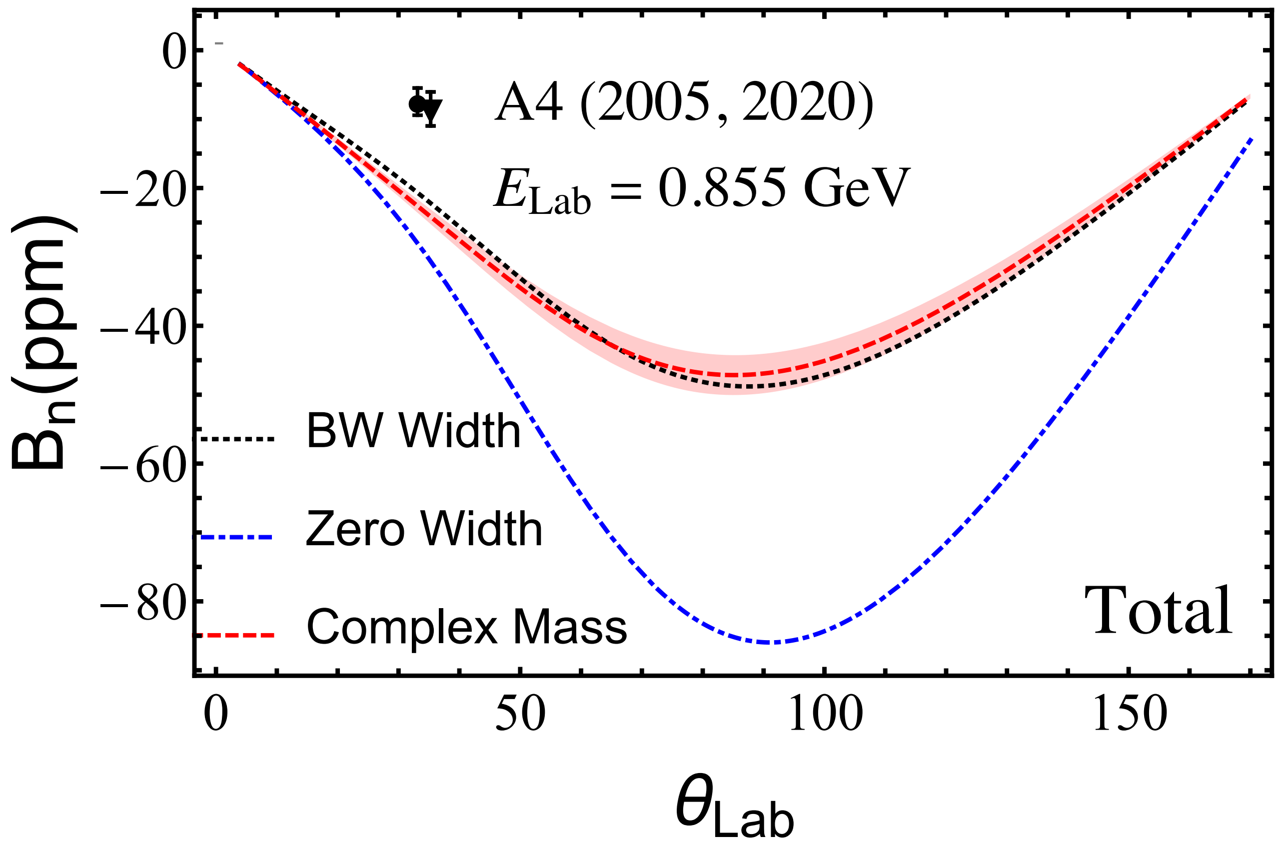


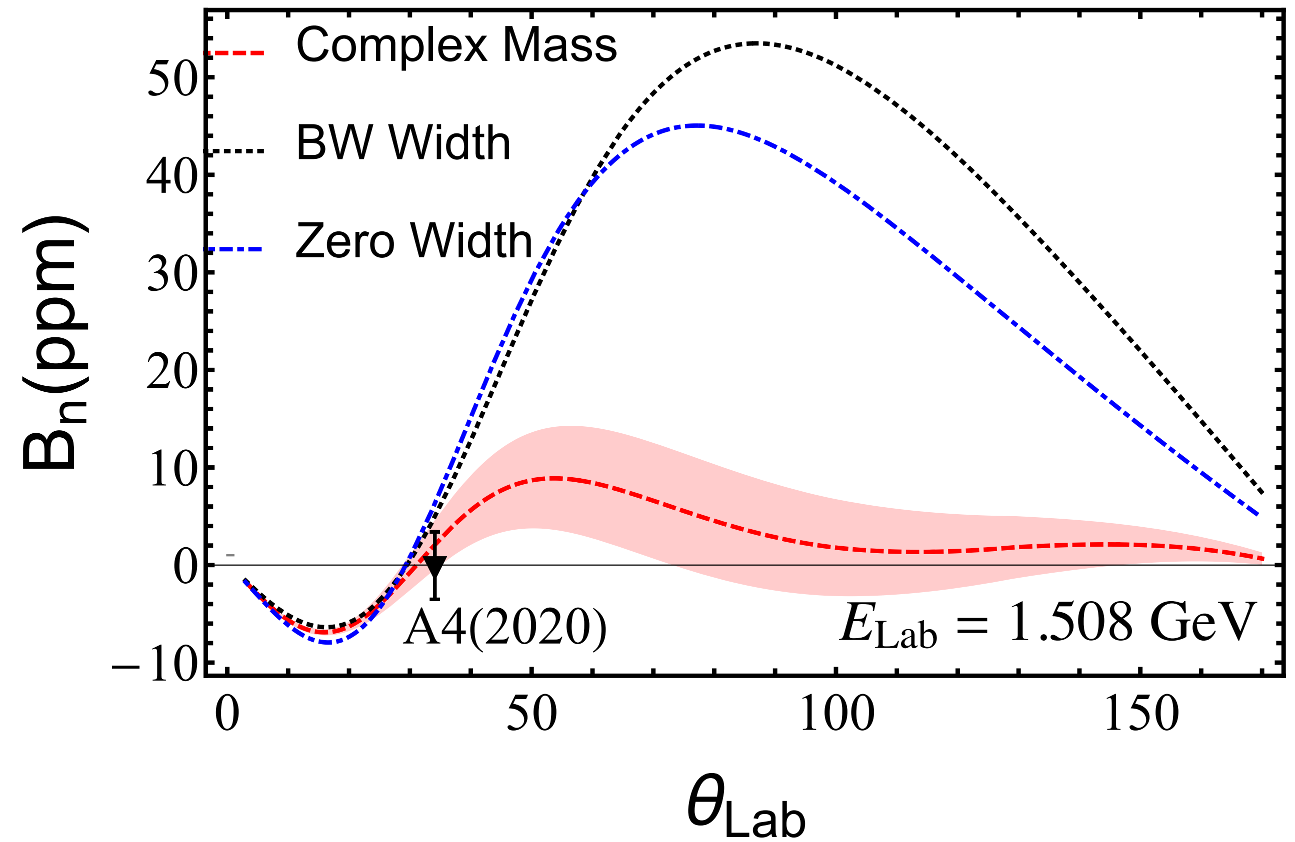


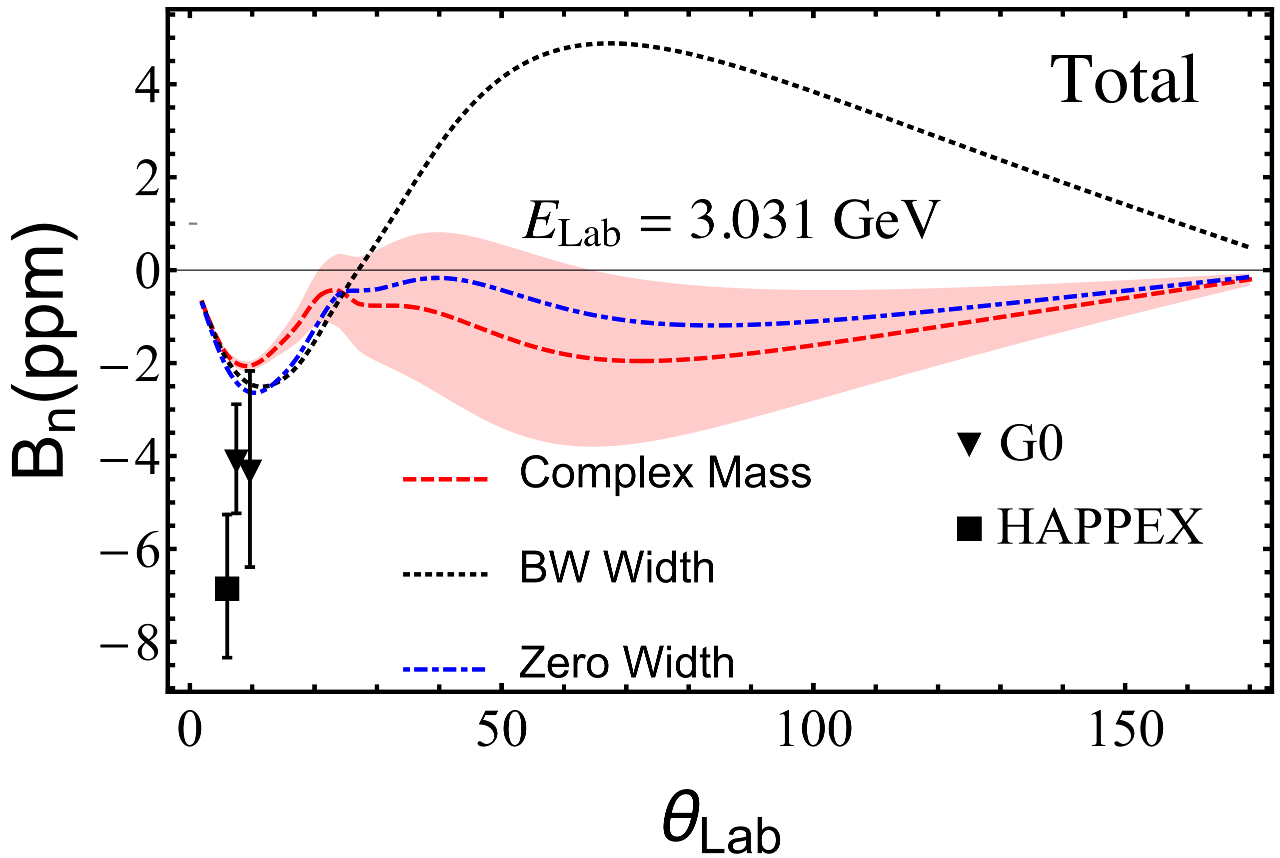
- Comparable with Gorchtein result
- Data point provides strong constraint on theory

A4 data at 0.855 GeV



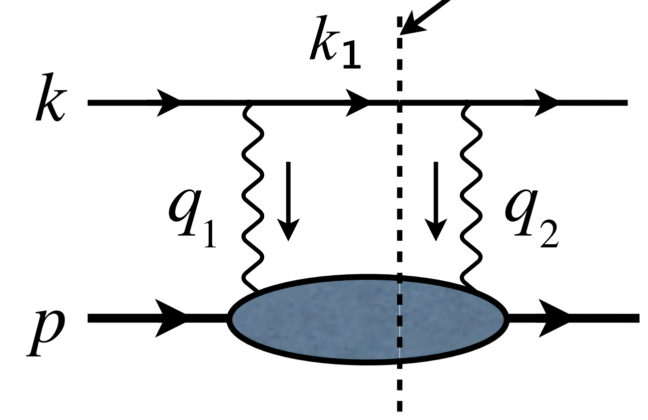






- Overshoots data at low energies and forward angles
- Undershoots data at high energies and forward angles

Re part of TPE: A few technical details



$$\frac{\alpha}{4\pi} Q^2 \frac{1}{i\pi^2} \int d^4 q_1 \; \frac{\mathrm{Im}\left\{L_{\alpha\mu\nu} H^{\alpha\mu\nu}\right\}}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)}$$

$$\xrightarrow{\mathrm{Im}} \quad \frac{s - W^2}{4s} \int d\Omega_{k_1} \; \frac{f\left(Q_1^2, Q_2^2\right) G_1(Q_1^2)\, G_2(Q_2^2)}{\left(Q_1^2 + \lambda^2\right)\left(Q_2^2 + \lambda^2\right)}$$

- L and H are leptonic and hadronic tensors
- f is a polynomial in photon virtual momenta Q_1^2 and Q_2^2
- $G_i(Q_i^2)$ are transition form factors with poles in the complex Q_i^2 plane

Evaluate in unphysical region using contour integration

Tomalak & Vanderhaeghen, EPJA **51**, 24 (2015); Blunden & Melnitchouk, PRC **95**, 065209 (2017)

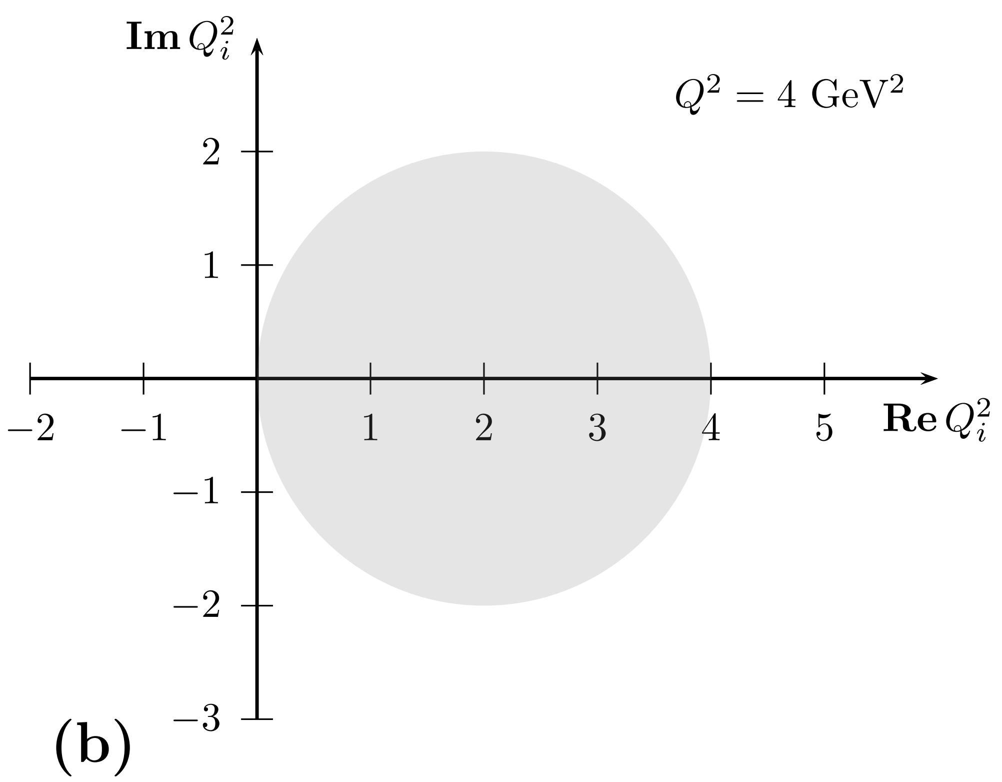
- Numerical contour integration allows for use of general functional forms for transition form factors
- Must be able to analytically continue $G_i(Q_i^2)$ into time-like region

Parametrization of form factors: Good, Bad, and Ugly

1. Good (no poles in time-like region) $Q^{2n};\ e^{-a^2 Q^2};\ \dfrac{1}{(Q^2+\Lambda^2)^n}$

2. Bad (but tolerable): $G(Q_i^2) = \dfrac{\sum_{j=0}^{N} a_j Q_i^{2j}}{\sum_{k=0}^{N+2} b_k Q_i^{2k}}$

   Can have poles in spacelike region of Q_i^2.
   Good if
   $$\left|2\frac{Q_p^2}{Q^2} - 1\right| > 1$$
   but requires special handling beyond that.

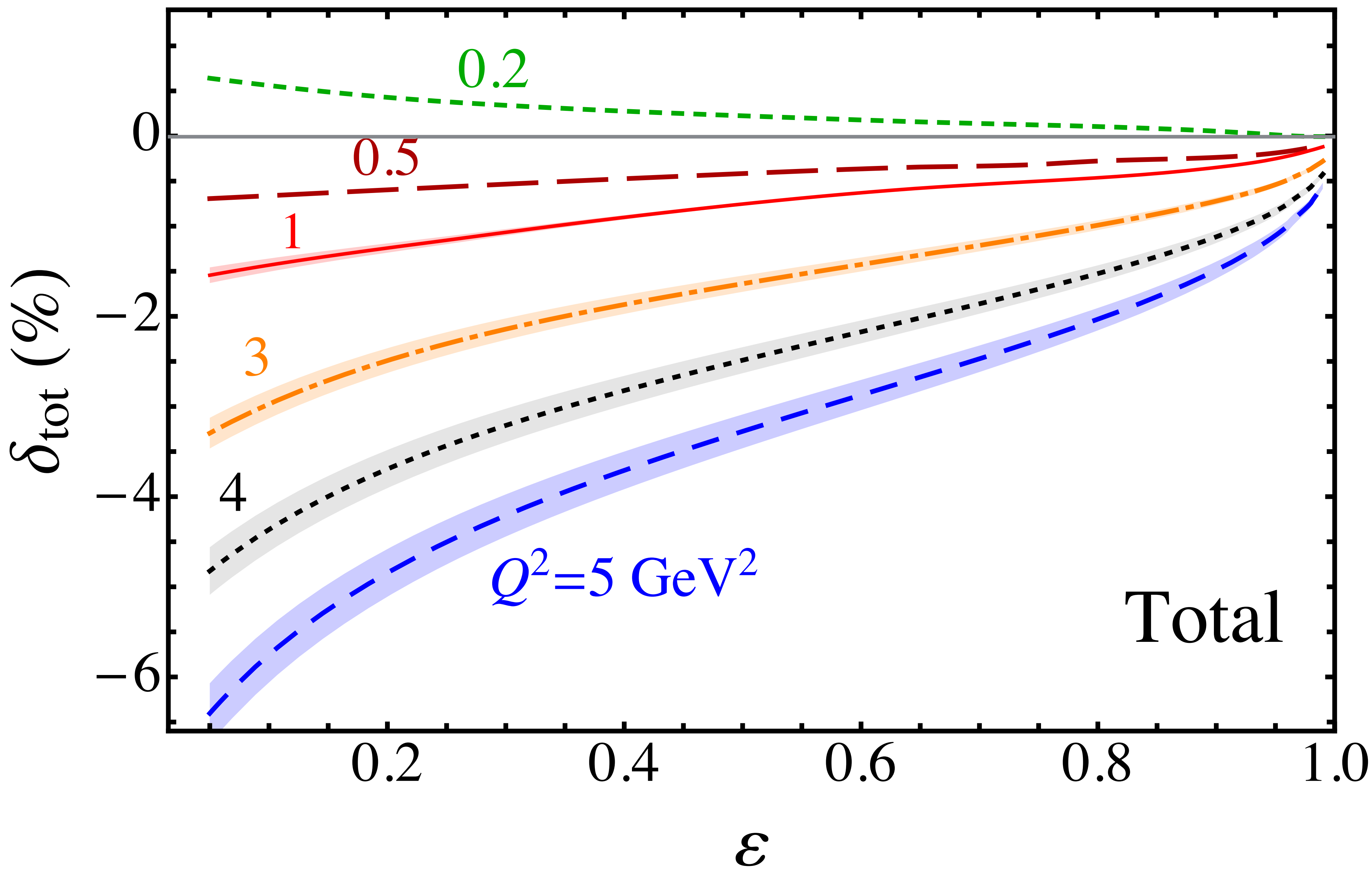


3. Ugly (could try to reparametrize): $\sqrt{Q^2+\Lambda^2}$ has branch cuts; splines

   Gramolin & Nikolenko (2016): $G^2(\tau) \sim 1 - \sum_i a_i \tau^i$ linear fit to σ

   GMp12 (2022): $(\mu_p G_E/G_M)^2 = 1 + c_1\tau + c_2\tau^2$ for convenience of fit

TPE corrections to cross section (CLAS resonances)



- Relative to Maximon-Tjon
- Linear over mid-range of ε values, but curves towards endpoints
- Grid available for interpolation

Ahmed, Blunden & Melnitchouk, PRC**102**, 045205 (2020)

Form factors and the G_E/G_M ratio

SLAC E140/NE11 LT: Walker *et al*, PRD **49**, 5671 (1994); Andivahis *et al*, PRD **50**, 5491 (1994)

Super Rosenbluth LT: Qattan *et al*, PRL **94**, 5671 (1994)

Polarization Transfer (PT): (various)

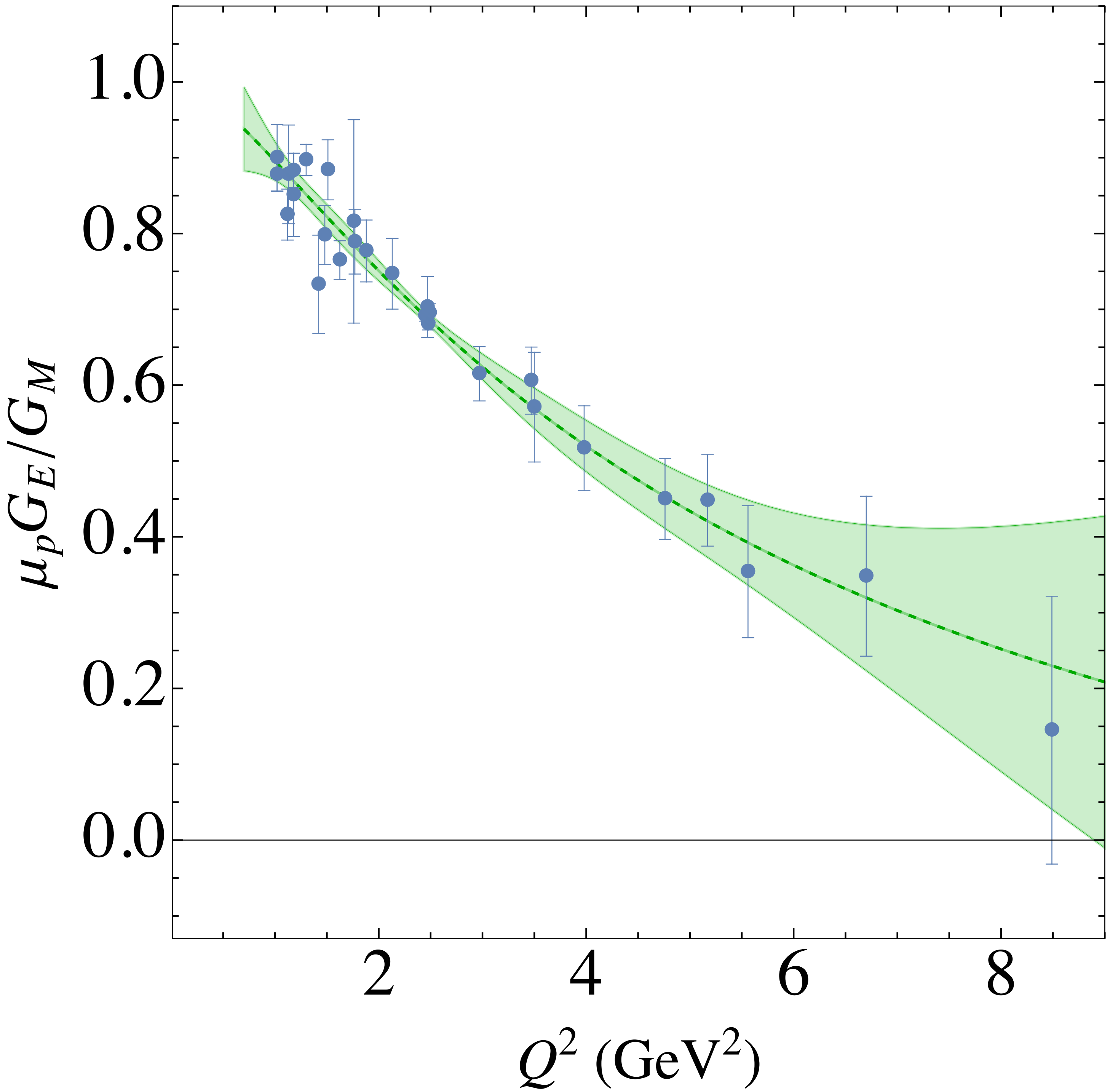
$$1\ \mathrm{GeV}^2 \leq Q^2 \leq 8.83\ \mathrm{GeV}^2$$

- To extract G_E and G_M from LT measurements we should correct the **data** for TPE at the same level as other RCs.

- SLAC: all details of RC are published

- Super Rosenbluth: no RC details are published, not even cross sections!

PT data



- Band is at 90% confidence interval

SLAC formulation: Walker *et al.* PRD **49**, 5671 (1994)

$$\sigma_R^{\text{meas}} = C_{\text{RC}}^{\text{old}} \left(\sigma_R^{\text{Born}}\right)^{\text{old}} = C_{\text{RC}}^{\text{new}} \left(\sigma_R^{\text{Born}}\right)^{\text{new}}$$

$$C_{\text{RC}} = C_L \exp\left(\delta_{\text{RC}} + \delta\right),$$

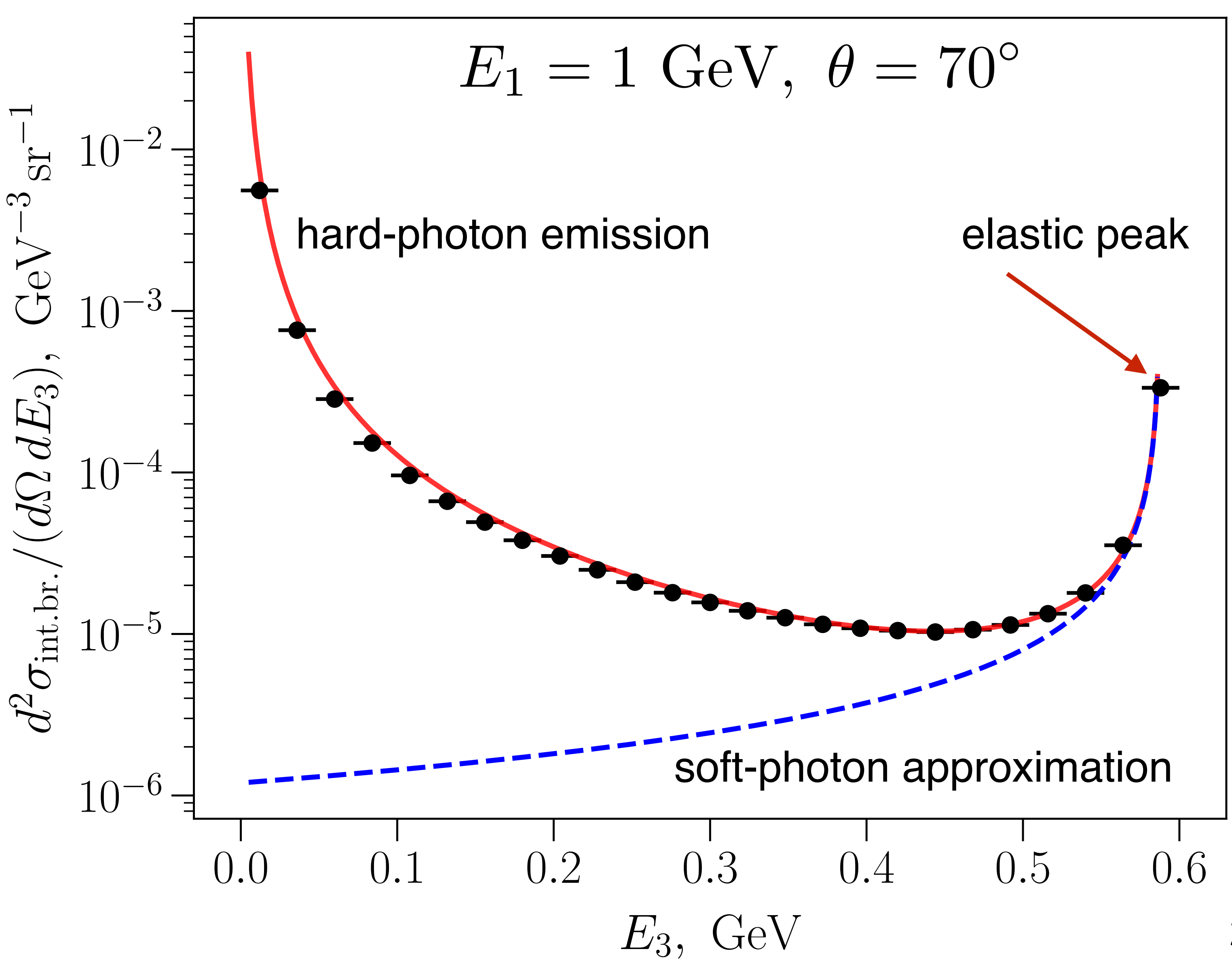
$$\delta_{\text{RC}} = \delta(\text{MTj}) + \delta_{\text{VP}} + \delta_{\text{brem,int}} + \delta_{\text{brem,ext}},$$

$$\delta = \delta_{\text{TPE}} - \delta_{\text{IR}}(\text{MTj})$$

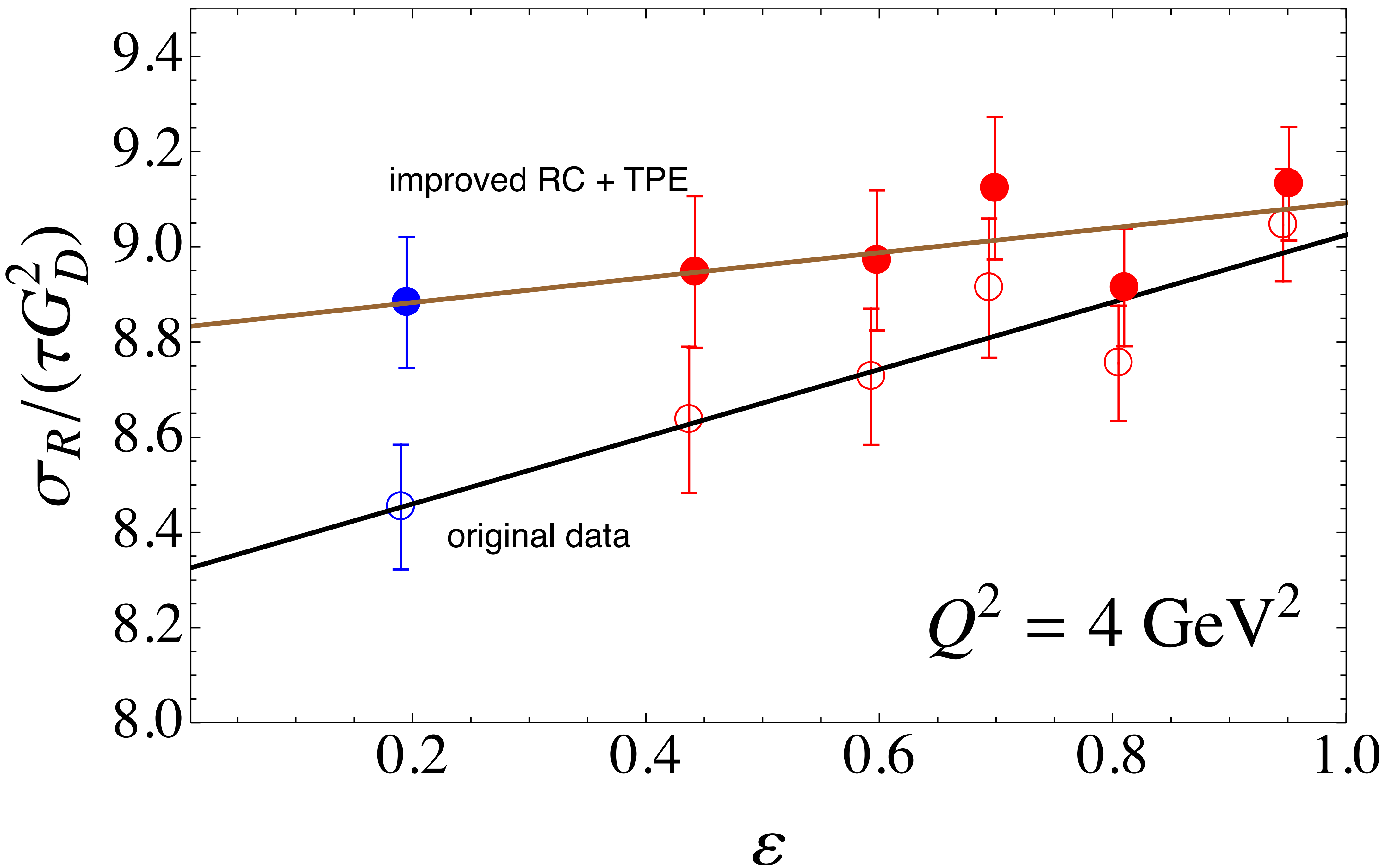
MTs = Mo-Tsai
MTj = Maximon-Tjon

RC improvements: Gramolin & Nikolenko, PRC **93**, 055201 (2016)

- Use exponentiation
- Use Maximon-Tjon instead of Mo-Tsai (no difference at order Z^0)
- Improvements to hard internal and external δ_{brem} bremsstrahlung
- Minor improvements to VP and ionization factor C_L

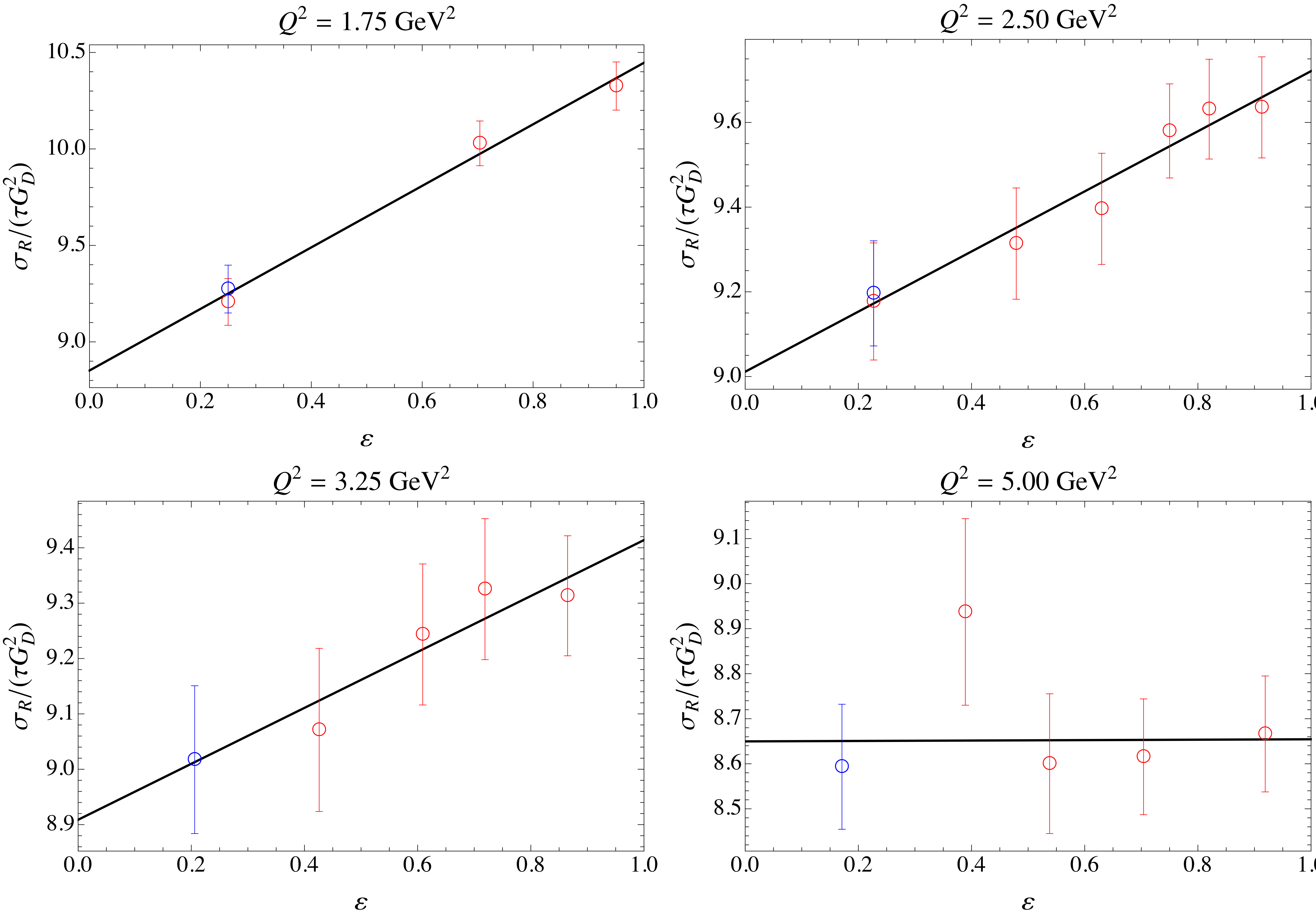


$$\frac{1}{\tau}\sigma_{\rm red} = G_M^2 + \frac{\varepsilon}{\tau}G_E^2$$

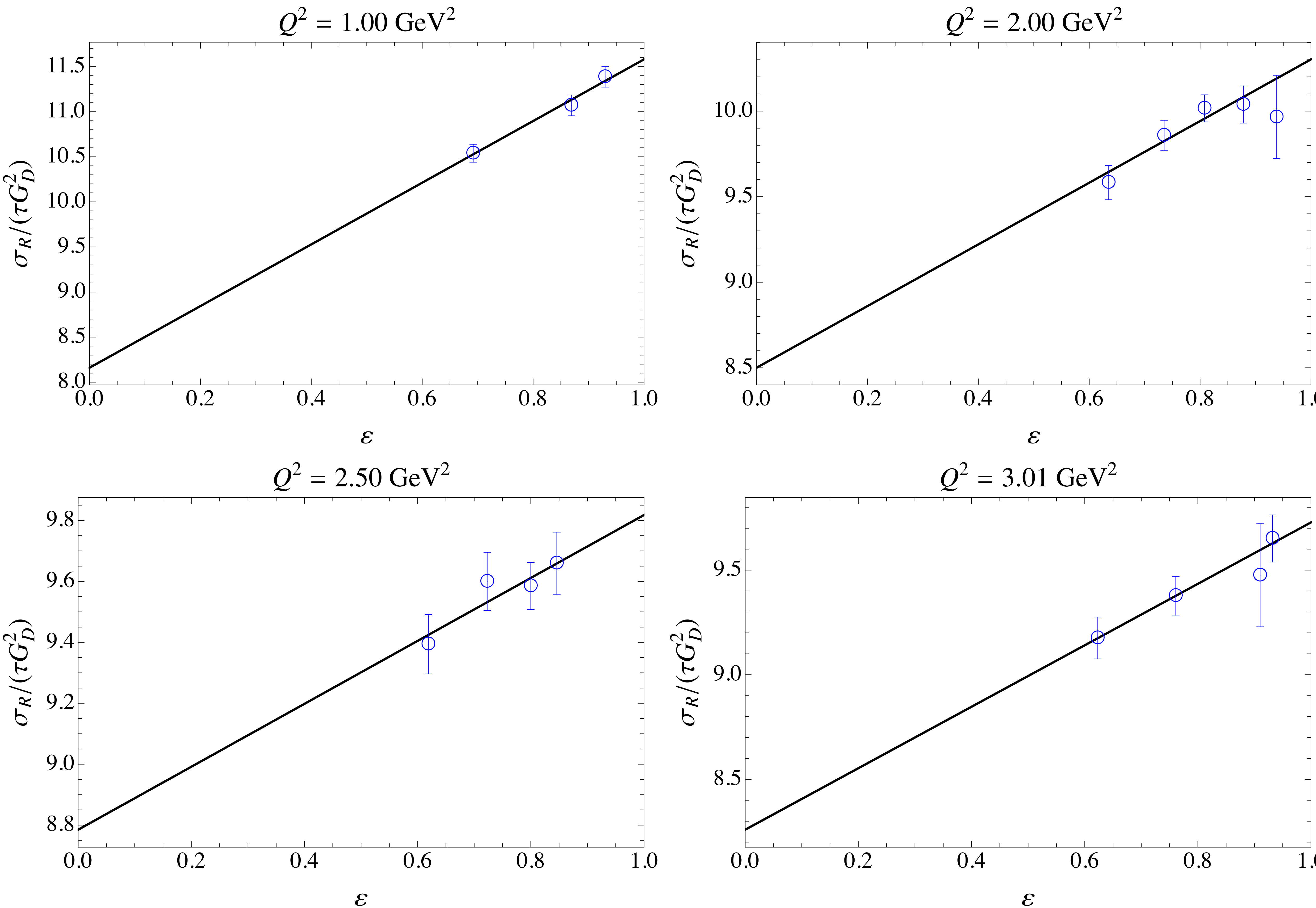


- Example from Andivahis data set at $Q^2 = 4$ GeV^2
- Uses improved RC + our TPE
- No evidence of non-linearity

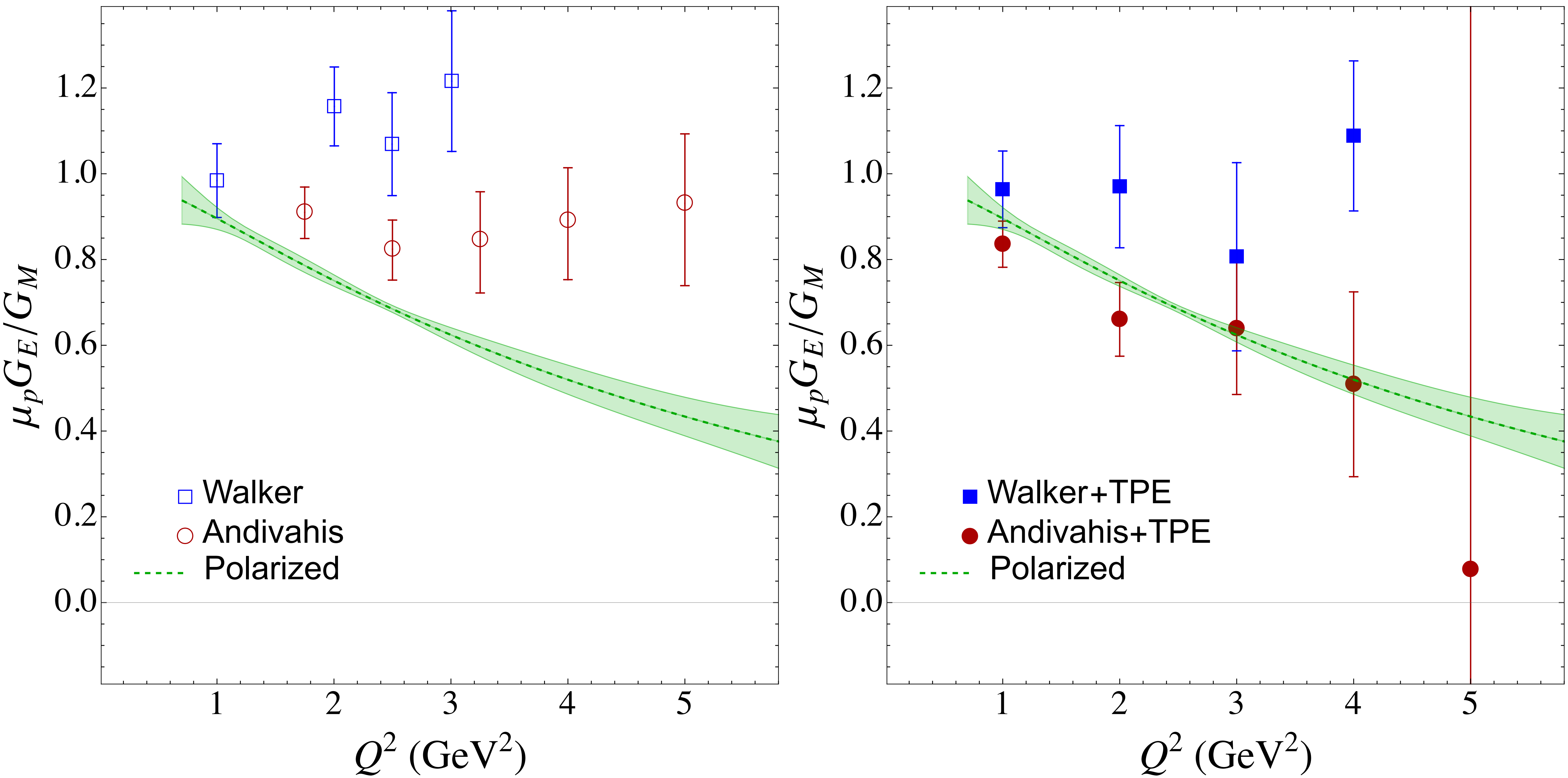
Andivahis: TPE corrected reduced cross sections



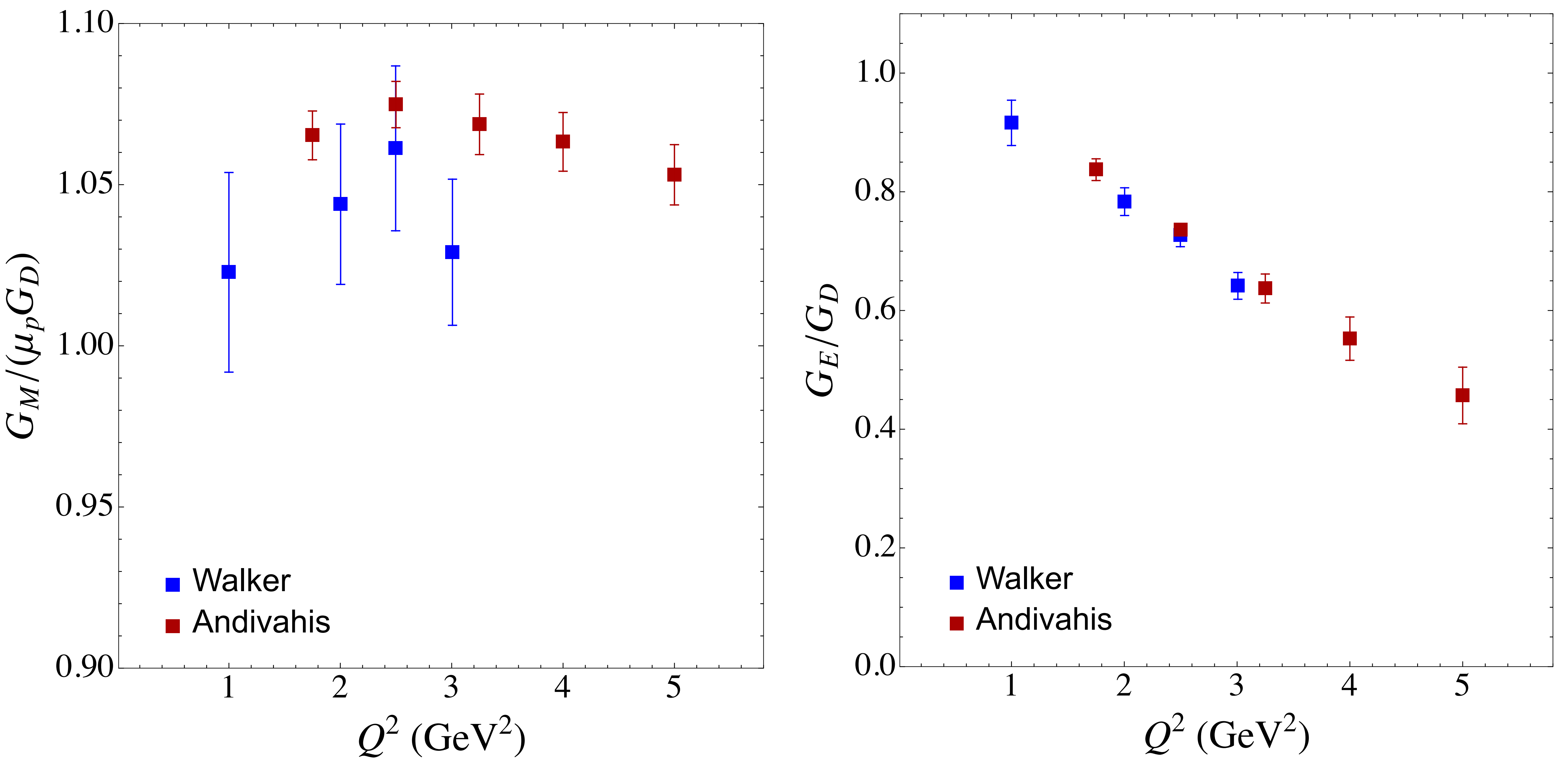
Walker: TPE corrected reduced cross sections



Proton form factor ratio: Rosenbluth

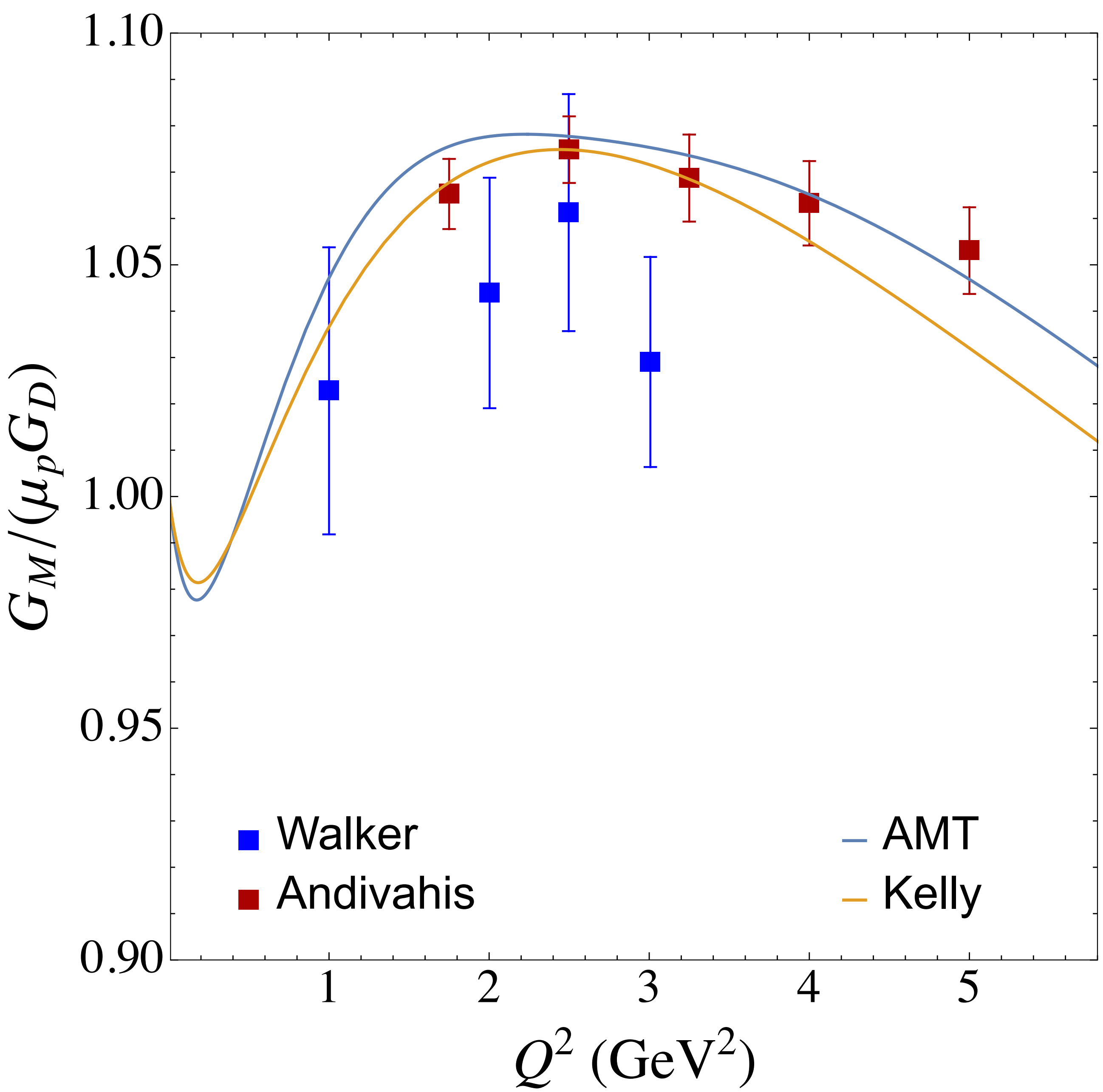


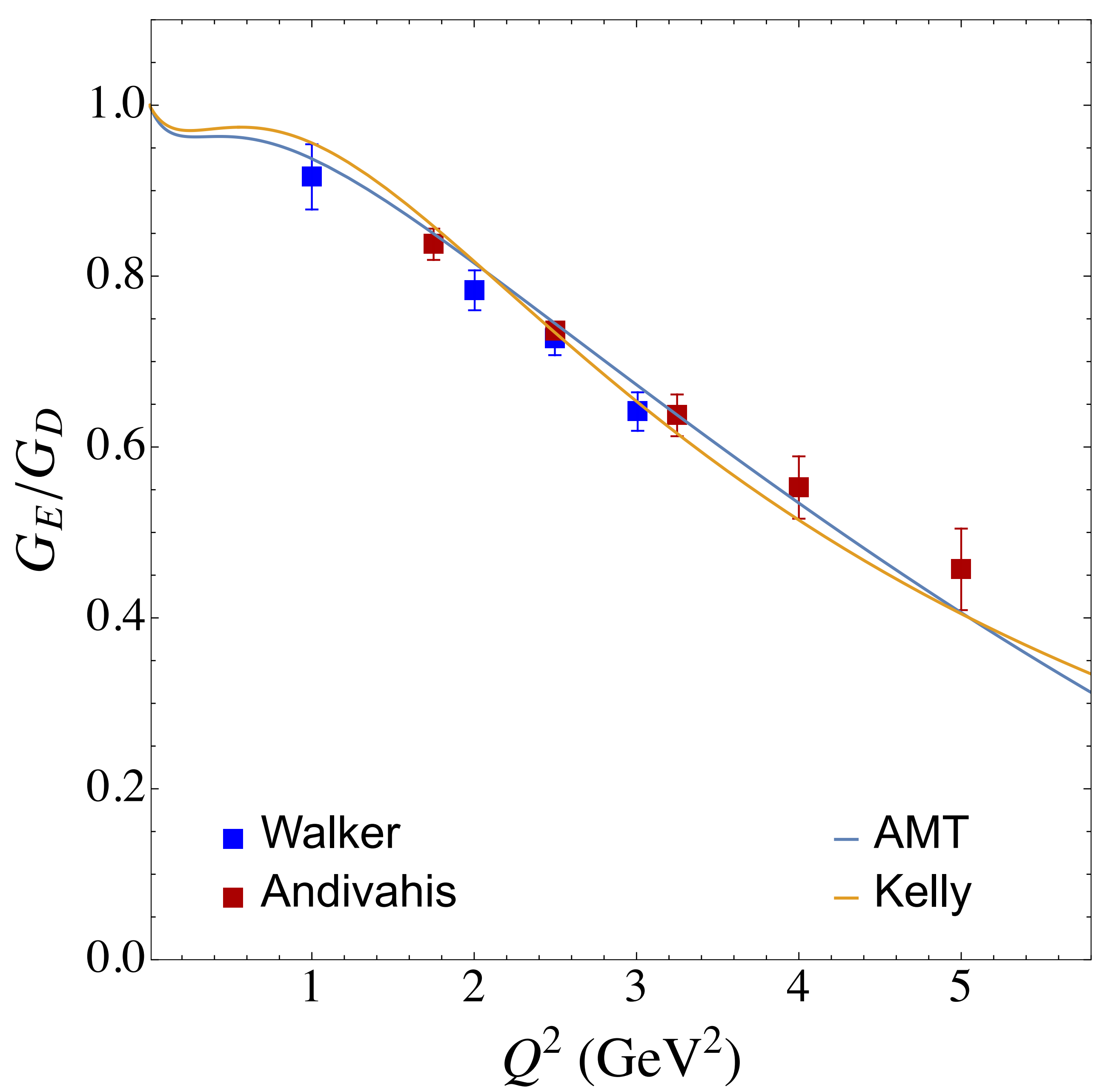
Magnetic and Electric form factors



- For G_E, use G_M and PT ratio (G_E/G_M)

Magnetic and Electric form factors





- For G_E, use G_M and PT ratio (G_E/G_M)
- Kelly (2004) and AMT (2007) parameterizations accounting for TPE (as known at the time)

Summary

- Beam normal SSA interesting and challenging.
  - How to handle quasi-singular behaviour, if at all
- Efforts underway to incorporate electroproduction data throughout the resonance region, including non-resonant background, using CLAS data as input (Tomalak *et al.* have a similar approach using MAID analysis)
- "Results of e^+p/e^-p experiments are by no means definitive."
  - Problems remain at high ε
- Proper inclusion of TPE likely resolve PT and LT differences at high Q^2

Clear need for definitive e^+p measurements at high Q^2, low ε