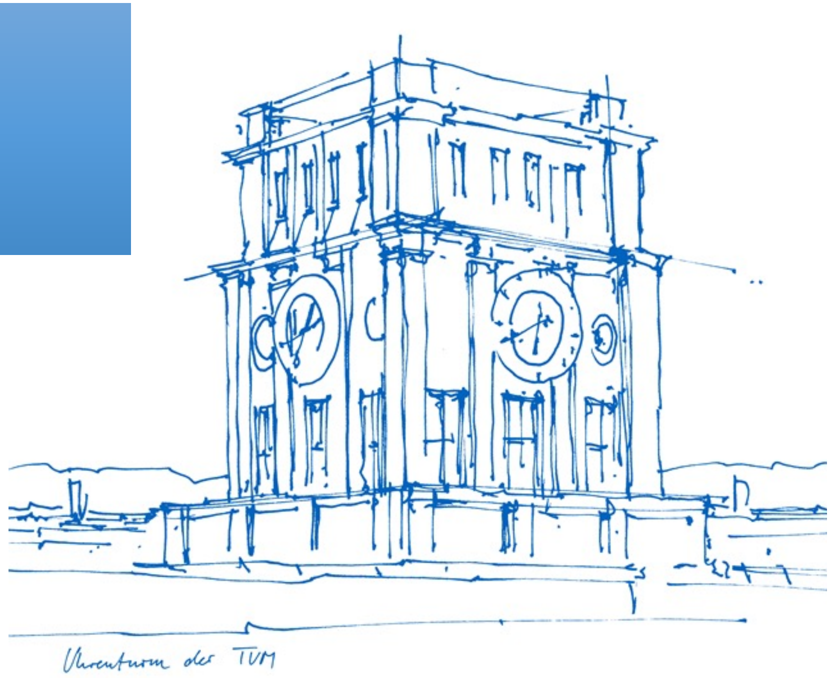
QED Radiative Corrections for the Chiral Anomaly Reaction $\pi^-\gamma \to \pi^-\pi^0$

Jan Friedrich
Institute for Hadronic Structure and Fundamental Symmetries
TU München

FONDAZIONE BRUNO KESSLER
ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

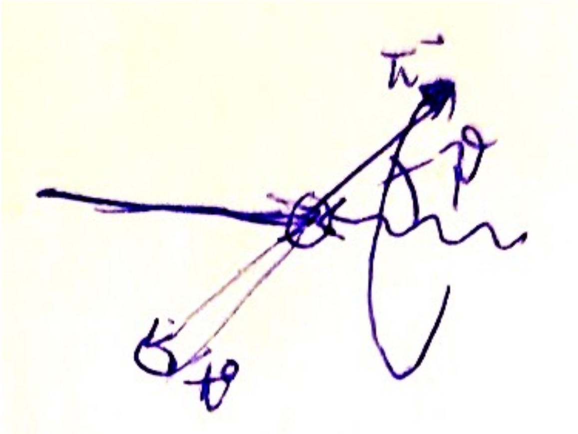
Radiative Corrections from medium to high energy experiments

Slides of the COMPASS analysis part:
Courtesy Dominik Steffen



Plan

1. **Short physics intro**
2. **Process of interest**
3. **One-loop (virtual) photon corrections**
4. **Experiment and data analysis, backgrounds, normalization**
5. **Bremsstrahlung correction**



$\pi^{-}\gamma \to \pi^{-}\pi^{0}(\gamma)$

A comparatively simple case of QED radiative corrections:

- Only one “charged arm”
- Spinless pions

Chiral symmetry of QCD

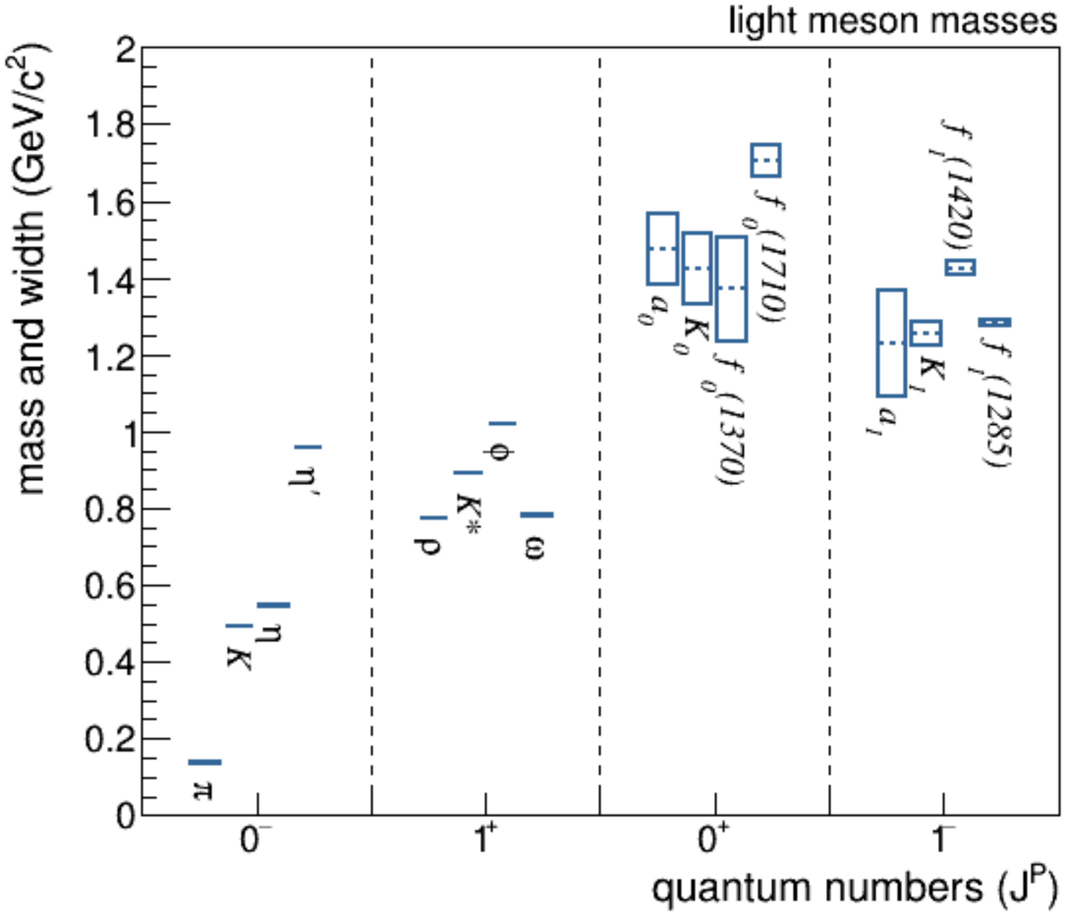
- Lagrange density of QCD:

$$\mathscr{L}_{QCD} = \sum_{\substack{f=\{u,d,\\ c,s,t,b\}}} \sum_{i,j=1}^{N_c} \overline{\psi}_{f,j}(i\gamma^\mu D^j_{i,\mu} - m_f \delta^j_i)\psi^{f,i} - \frac{1}{4}\sum_{a=1}^{N_c^2-1} G^a_{\mu\nu}G^{\mu\nu}_a$$

- Approximate flavor symmetries in chiral limit ($m_u = m_d = m_s = 0$):

$\boldsymbol{SU(3)_R \times SU(3)_L}$

- Left- and right-handed fields decouple for massless particles
- Chirality can directly be translated to parity of particle
  -> mass-degenerate doublets of states with opposite parity
- Why is chiral symmetry not manifested in the spectrum (in contrast to isospin and the eightfold way)?
  -> Nambu-Goldstone mechanism for spontaneous/dynamic breakdown of chiral symmetry



Dynamic breaking of chiral symmetry

Spontaneous symmetry breaking

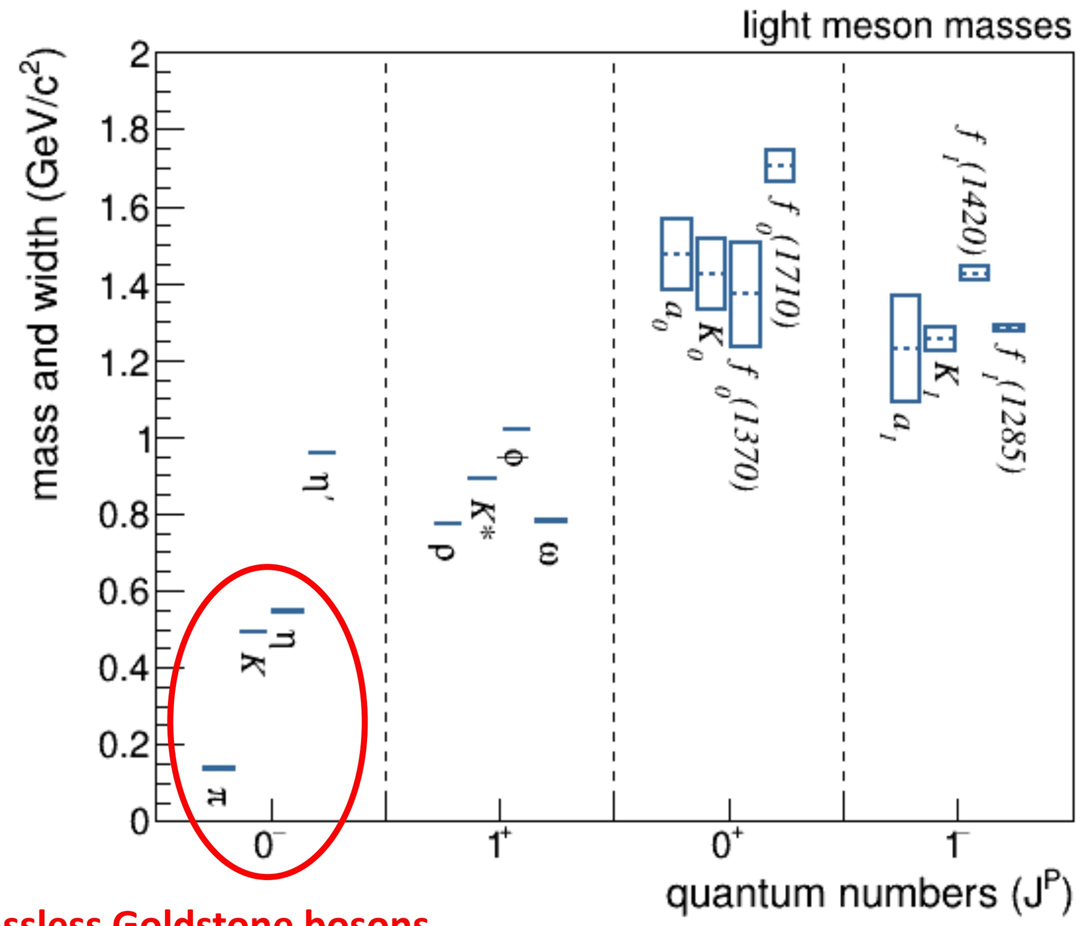
⇒ Eight massless, spinless Goldstone bosons

$$(\pi^{\pm}, \pi^{0}, K^{\pm}, K^{0}, \bar{K}^{0}, \eta)$$

⇒ Explicit breaking of chiral symmetry due to the small quark masses -> Goldstone bosons acquire mass

⇒ $\boldsymbol{SU(3)_R \times SU(3)_L \rightarrow SU(3)_V}$

⇒ Chiral Perturbation Theory: effective Lagrangian with power-counting scheme as low-energy theory for QCD making use of chiral symmetry



(almost) massless Goldstone bosons

The chiral anomaly

- Lagrange density of QCD:

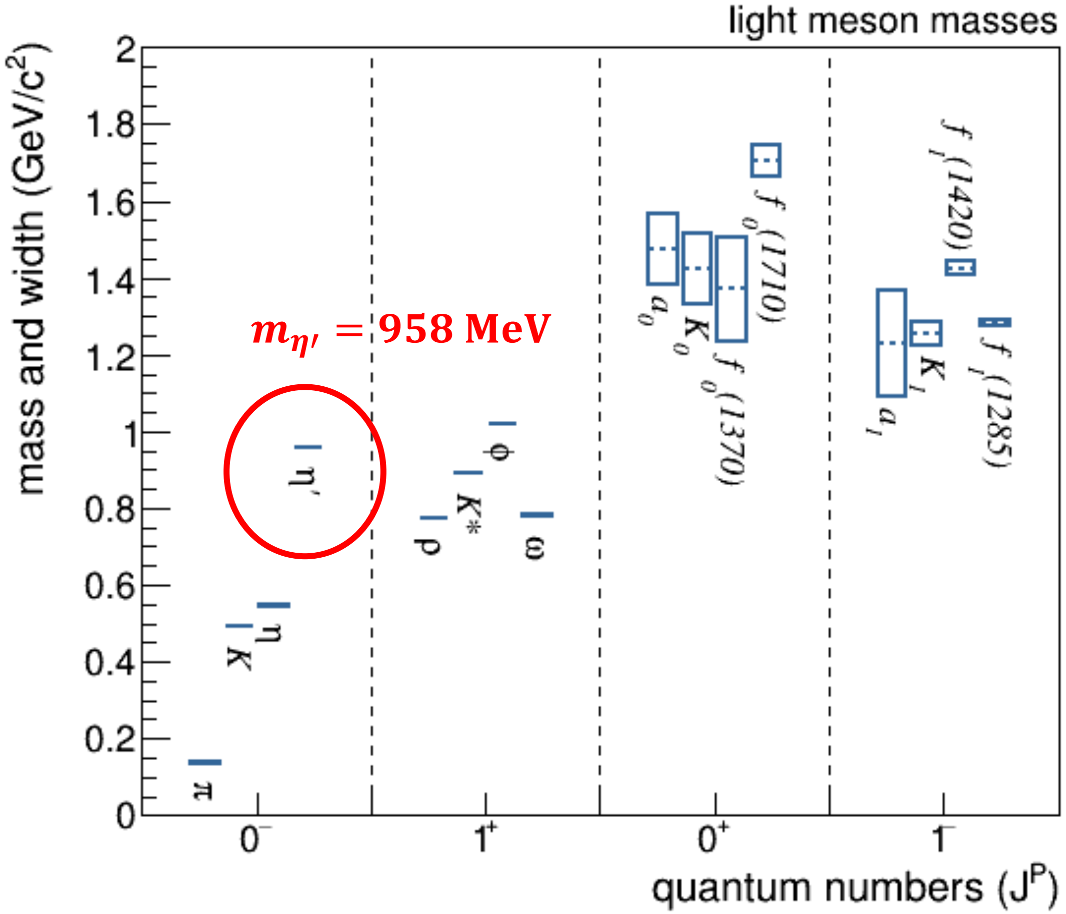
$$\mathcal{L}_{QCD} = \sum_{\substack{f=\{u,d,\\ c,s,t,b\}}} \sum_{i,j=1}^{N_c} \overline{\psi}_{f,j}(i\gamma^\mu D^j_{i,\mu} - m_f \delta^j_i)\psi^{f,i} - \frac{1}{4}\sum_{a=1}^{N_c^2-1} G^a_{\mu\nu}G^{\mu\nu}_a$$

- Features *axial* $U(1)$**-**symmetry in chiral limit**:**

$$\psi(x) \rightarrow e^{i\theta\gamma_5}\psi(x)$$

- No ninth “unnaturally light” meson

- **Anomalous** symmetry breaking: symmetry of the Lagrangian does not lead to conserved Noether currents
- **Anomaly:** Symmetry of classical Lagrangian violated at quantum level

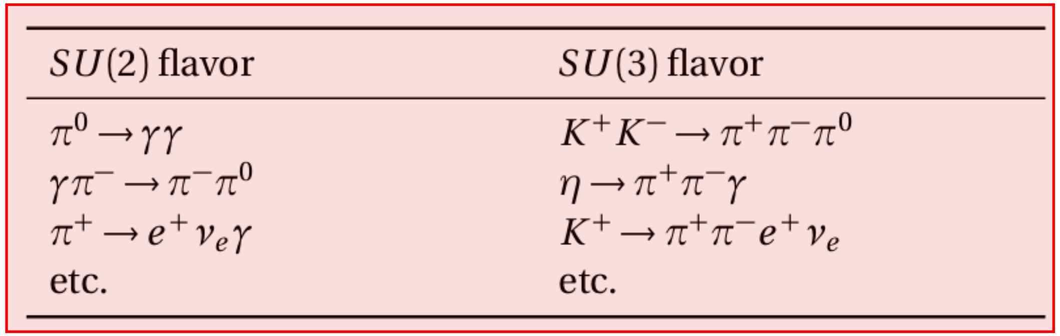


$m_{\eta'} = 958$ **MeV**

Wess-Zumino-Witten term

- Chiral anomaly in ChPT taken into account by Wess-Zumino-Witten (WZW) term

- Describes the coupling of an odd number of Goldstone bosons:

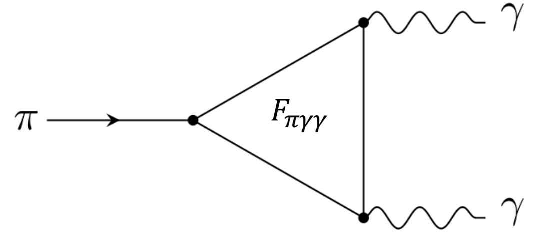


$SU(2)$ flavor	$SU(3)$ flavor
$\pi^0 \to \gamma\gamma$	$K^+K^- \to \pi^+\pi^-\pi^0$
$\gamma\pi^- \to \pi^-\pi^0$	$\eta \to \pi^+\pi^-\gamma$
$\pi^+ \to e^+\nu_e\gamma$	$K^+ \to \pi^+\pi^-e^+\nu_e$
etc.	etc.

- Effective theory: pion decay constant measured from leptonic decays of the charged pion

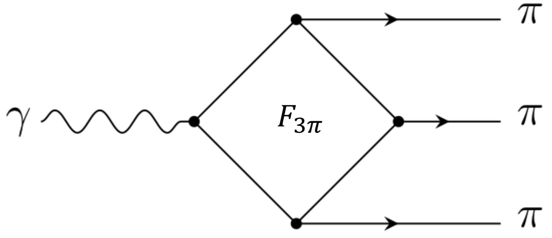
$$(\pi^\pm \to \mu^\pm + \nu)$$

$F_{\pi\gamma\gamma}$



- $F_{\pi\gamma\gamma} = \frac{e^2 N_C}{12\pi^2 F_\pi} = 2.52 \cdot 10^{-2} \mathrm{GeV}^{-1}$

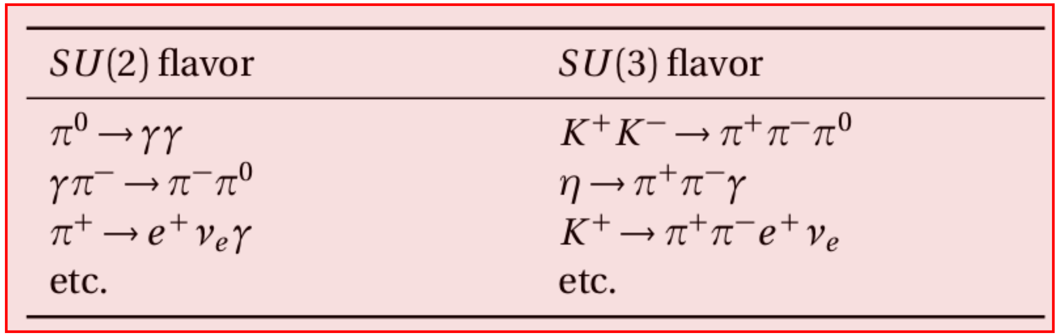
$F_{3\pi}$



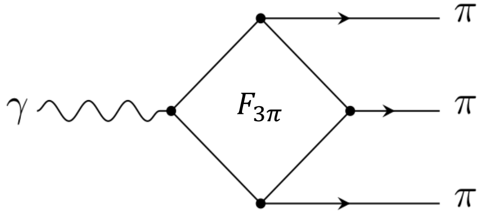
- $F_{3\pi} = \frac{e N_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05) \mathrm{GeV}^{-3}$

Testing the chiral anomaly - $F_{3\pi}$

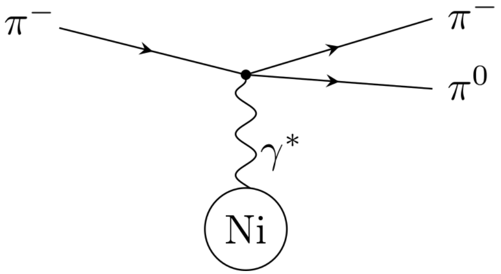
- Processes described by WZW term:



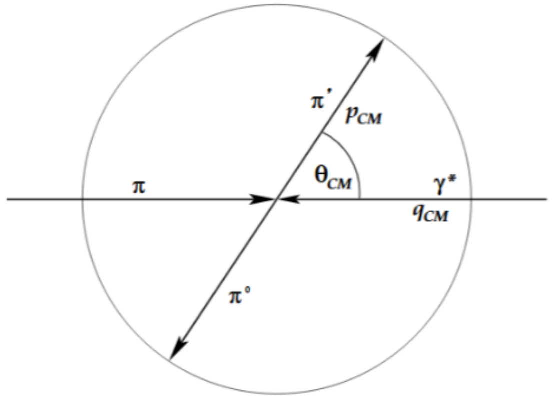
$SU(2)$ flavor	$SU(3)$ flavor
$\pi^0 \to \gamma\gamma$	$K^+K^- \to \pi^+\pi^-\pi^0$
$\gamma\pi^- \to \pi^-\pi^0$	$\eta \to \pi^+\pi^-\gamma$
$\pi^+ \to e^+\nu_e\gamma$	$K^+ \to \pi^+\pi^-e^+\nu_e$
etc.	etc.



- $F_{3\pi}$: Direct coupling of γ to 3π - process proceeds primarily via the chiral anomaly => one of the most definitive tests of low-energy QCD



- Accessible in Primakoff reactions via: $\pi^-\gamma^* \to \pi^-\pi^0$

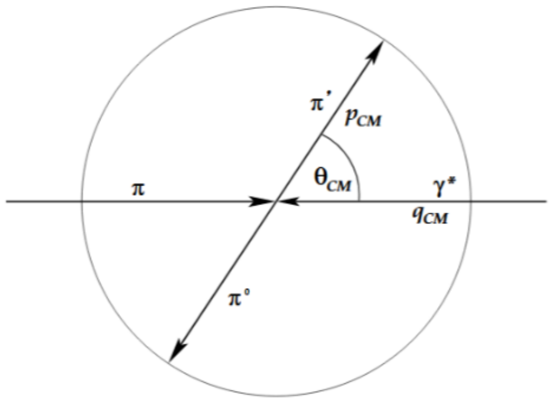


- Problem of explicit chiral symmetry breaking:

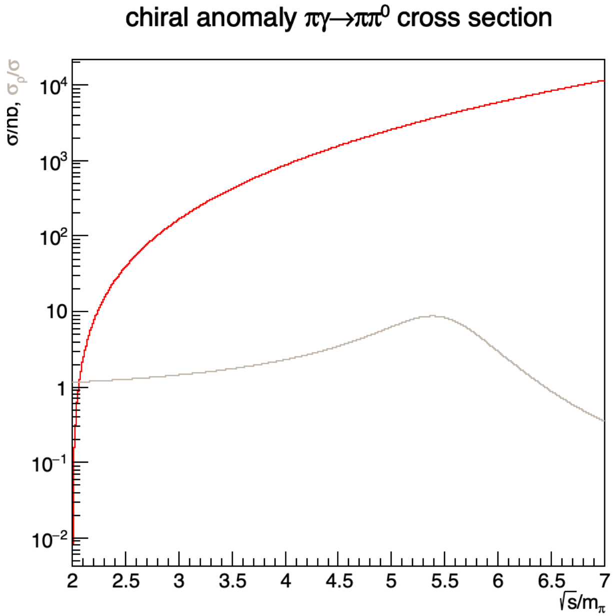
$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3} = F(s = t = u = 0)$$

Reference cross section

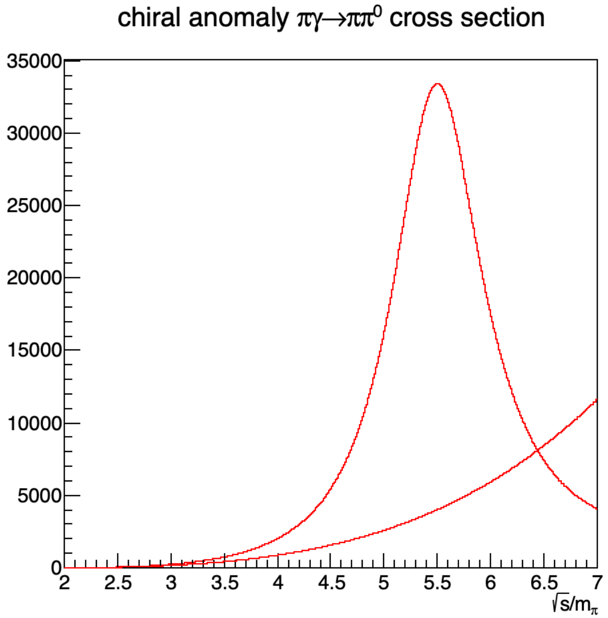
$$\pi^-(p_1) + \gamma(k,\epsilon) \rightarrow \pi^-(p_2) + \pi^0(q)$$



- Embedded in a Primakoff reaction with quasi-real γ^*
- Reference tree-graph cross section
- Main correction at higher s: contribution of the ρ-resonance



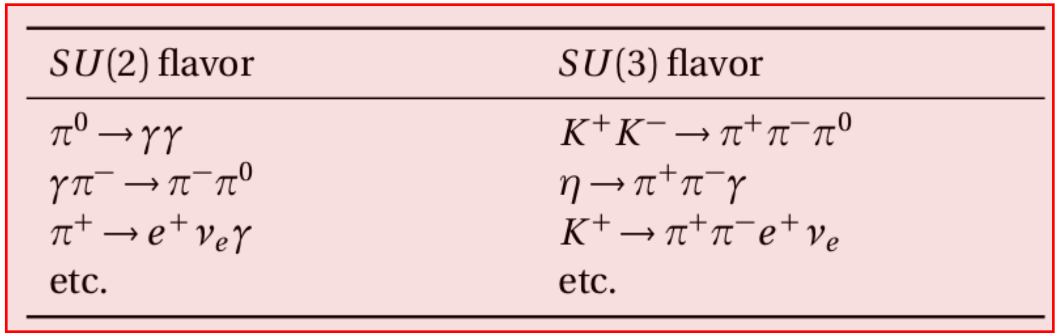
chiral anomaly $\pi\gamma\rightarrow\pi\pi^0$ cross section



chiral anomaly $\pi\gamma\rightarrow\pi\pi^0$ cross section

Testing the chiral anomaly - $F_{3\pi}$

- Processes described by WZW term:



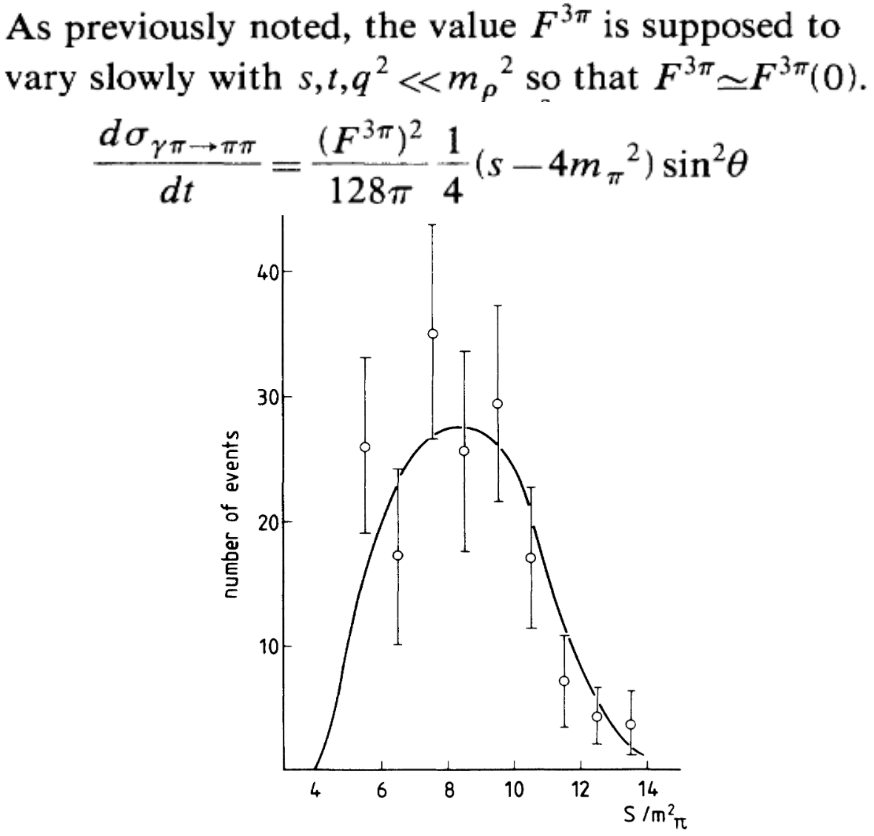
$SU(2)$ flavor	$SU(3)$ flavor
$\pi^0 \rightarrow \gamma\gamma$	$K^+K^- \rightarrow \pi^+\pi^-\pi^0$
$\gamma\pi^- \rightarrow \pi^-\pi^0$	$\eta \rightarrow \pi^+\pi^-\gamma$
$\pi^+ \rightarrow e^+\nu_e\gamma$	$K^+ \rightarrow \pi^+\pi^-e^+\nu_e$
etc.	etc.

- $F_{3\pi}$: Direct coupling of γ to 3π - process proceeds primarily via the chiral anomaly => one of the most definitive tests of low-energy QCD
- Accessible in Primakoff reactions via: $\pi^-\gamma^* \rightarrow \pi^-\pi^0$
- Problem of explicit chiral symmetry breaking:

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3} = F(s = t = u = 0)$$

Previous measurement of $F_{3\pi}$:

[Antipov, Y. *et al.* Phys.Rev. D36 (1987) 101103]
from Serpukhov experiments

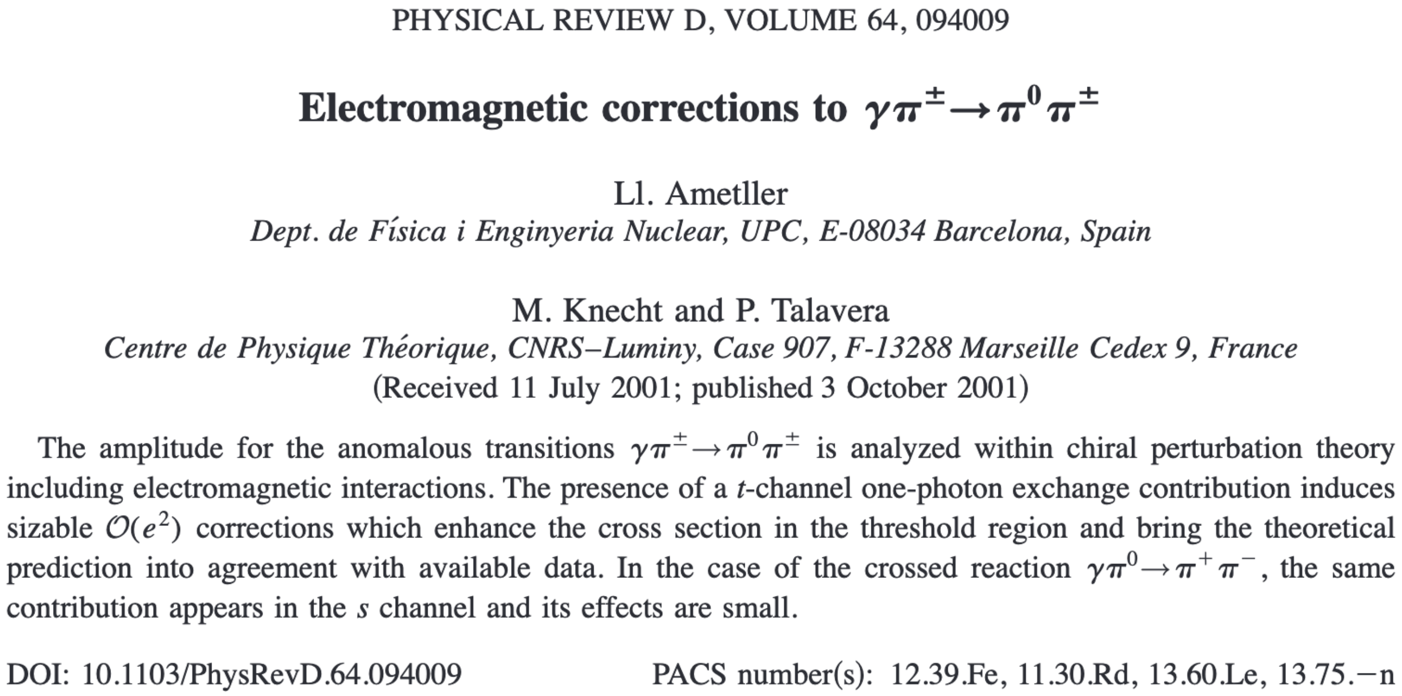


As previously noted, the value $F^{3\pi}$ is supposed to vary slowly with $s,t,q^2 \ll m_\rho{}^2$ so that $F^{3\pi} \simeq F^{3\pi}(0)$.

$$\frac{d\sigma_{\gamma\pi\rightarrow\pi\pi}}{dt} = \frac{(F^{3\pi})^2}{128\pi}\frac{1}{4}(s-4m_\pi{}^2)\sin^2\theta$$

$$\Rightarrow F_{3\pi} = (12.9 \pm 0.9 \pm 0.5)\ \mathrm{GeV}^{-3}$$

Calculation of Radiative Corrections



PHYSICAL REVIEW D, VOLUME 64, 094009

Electromagnetic corrections to $\gamma\pi^{\pm}\rightarrow\pi^{0}\pi^{\pm}$

Ll. Ametller
Dept. de Física i Enginyeria Nuclear, UPC, E-08034 Barcelona, Spain

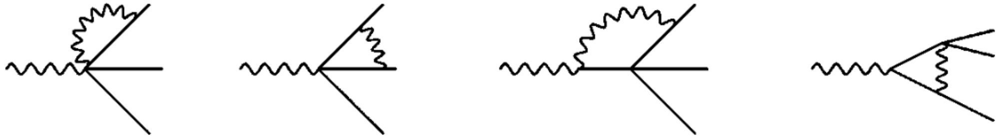
M. Knecht and P. Talavera
Centre de Physique Théorique, CNRS–Luminy, Case 907, F-13288 Marseille Cedex 9, France

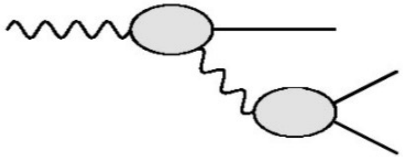
(Received 11 July 2001; published 3 October 2001)

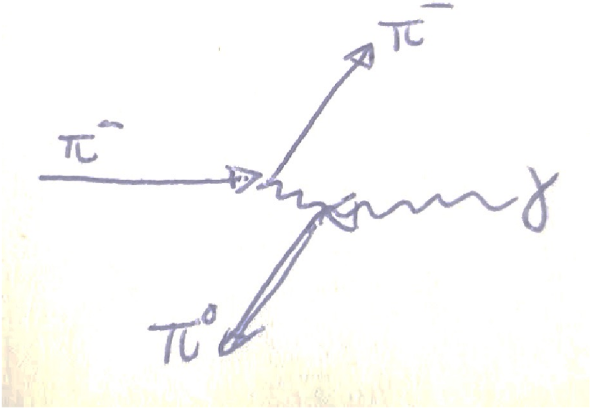
The amplitude for the anomalous transitions $\gamma\pi^{\pm}\rightarrow\pi^{0}\pi^{\pm}$ is analyzed within chiral perturbation theory including electromagnetic interactions. The presence of a t-channel one-photon exchange contribution induces sizable $\mathcal{O}(e^{2})$ corrections which enhance the cross section in the threshold region and bring the theoretical prediction into agreement with available data. In the case of the crossed reaction $\gamma\pi^{0}\rightarrow\pi^{+}\pi^{-}$, the same contribution appears in the s channel and its effects are small.

DOI: 10.1103/PhysRevD.64.094009 PACS number(s): 12.39.Fe, 11.30.Rd, 13.60.Le, 13.75.−n

Ametller, L. *et al.* Phys.Rev. D64 (2001) 094009



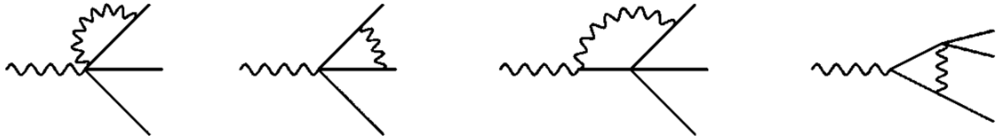




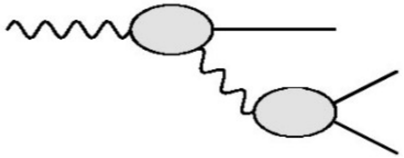
Calculation of Radiative Corrections

- Recap of the paper by N. Kaiser
- Compactification, simplifications, corrections
- Application to COMPASS data: work in progress

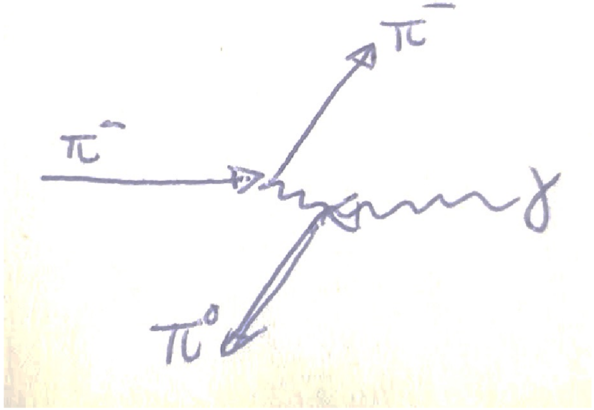
Ametller, L. *et al.* Phys.Rev. D64 (2001) 094009



$$\frac{1}{2}\sum_{\mathrm{pol}}|T|^2 = \frac{\alpha m_\pi^6}{32\pi^3 f_\pi^6}(stu-1)\left\{1-\frac{4e^2 f_\pi^2}{m_\pi^2 t}+2\mathrm{Re}\,G_\gamma(s,t,u)\right\}$$

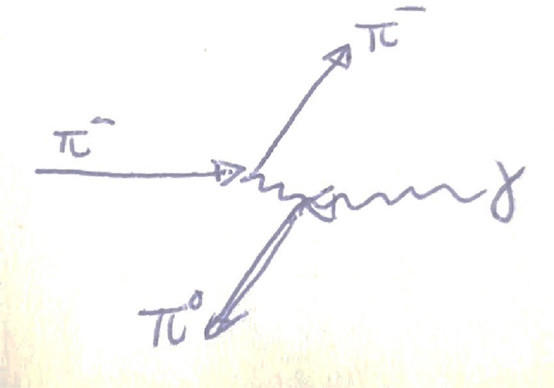


- s, t, u here: unit-free Mandelstam variables $s = (p_1 + k)^2/m_\pi^2$ etc.
- Photon exchange graph is the dominant correction

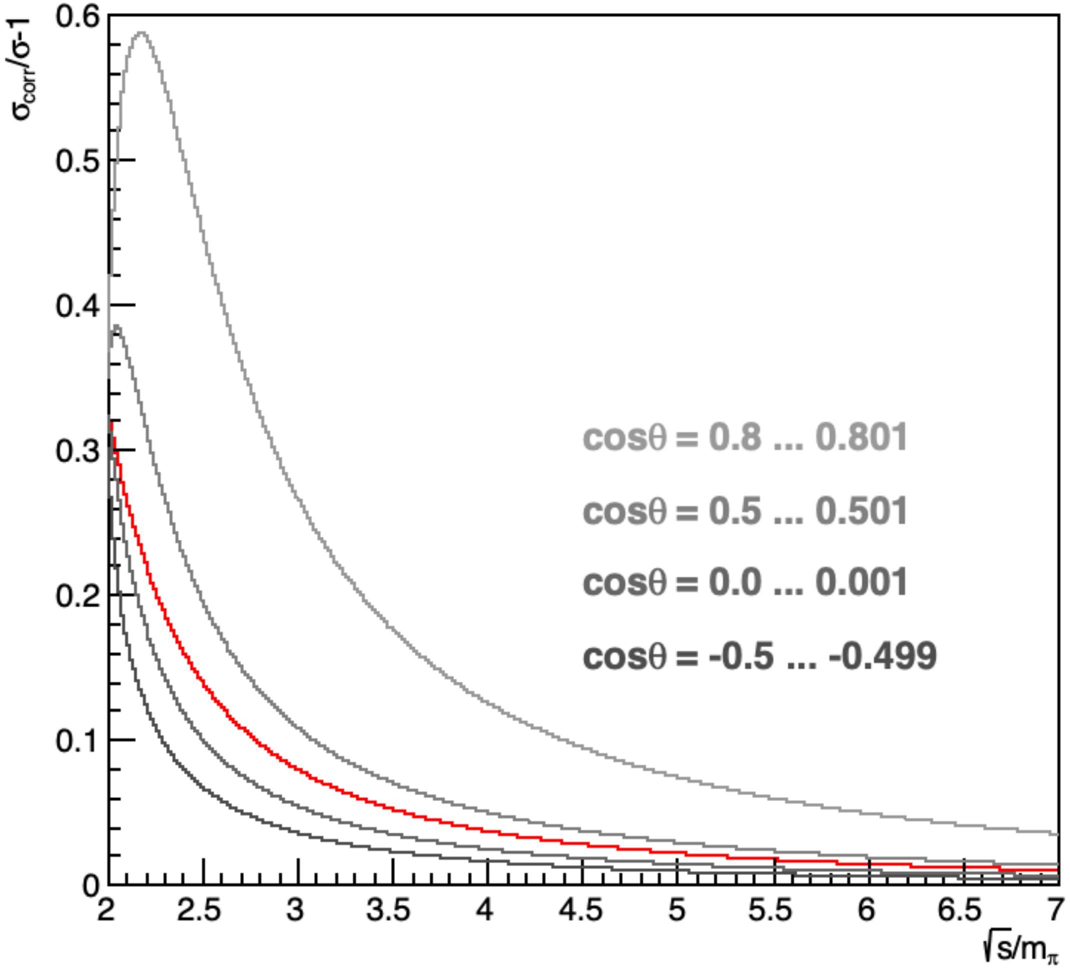


Photon exchange graph

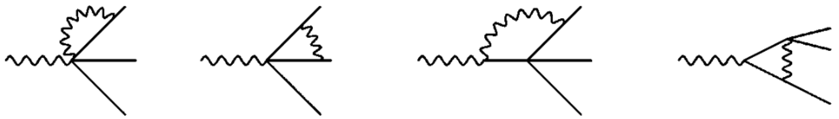
$$\frac{1}{2}\sum_{\text{pol}}|T|^2 = \frac{\alpha m_\pi^6}{32\pi^3 f_\pi^6}(stu-1)\left\{1-\frac{4e^2 f_\pi^2}{m_\pi^2 t}+2\mathrm{Re}\,G_\gamma(s,t,u)\right\}$$



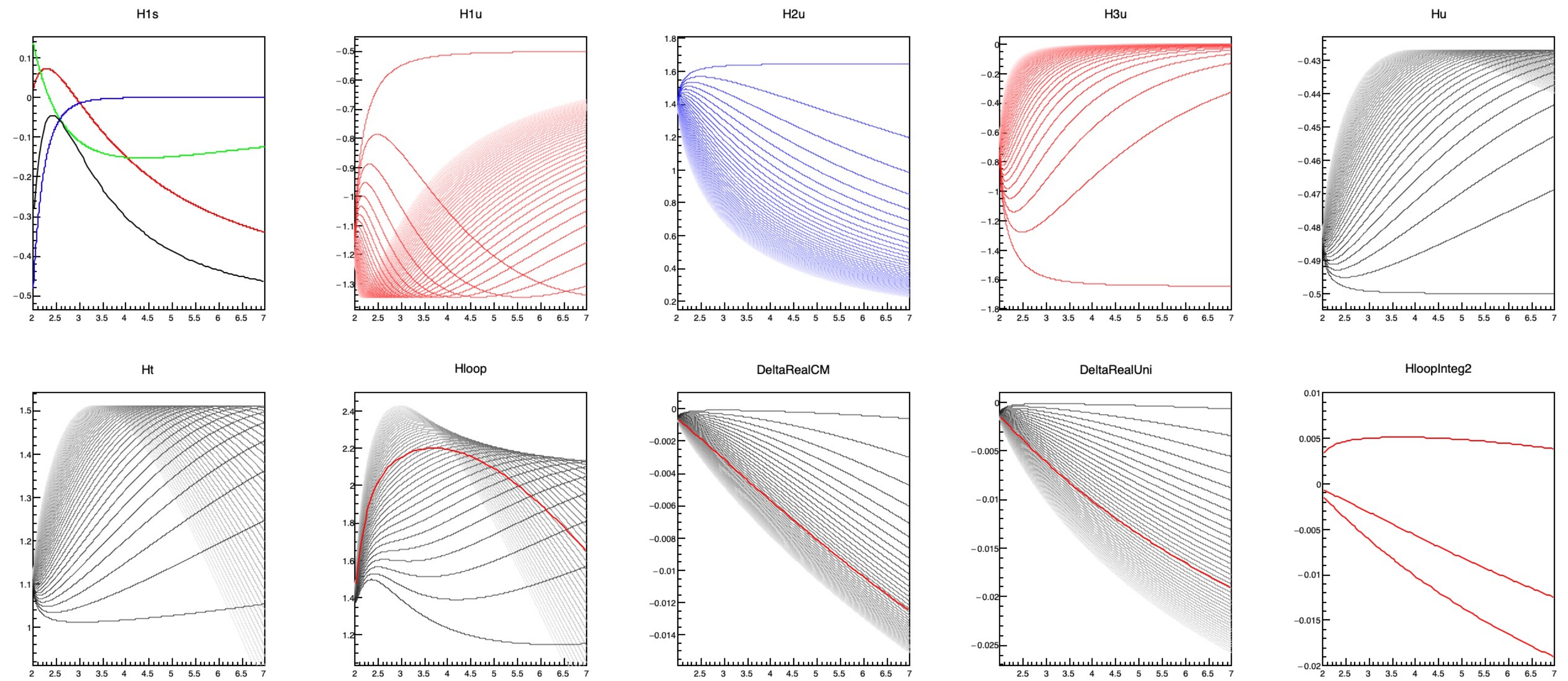
- Pole for $t \to 0$
- Integrated correction amounts to about 32% at threshold
- For kinematics with incomplete θ coverage, the correction has to be implemented differential in θ (integrate covered region or in Monte Carlo)



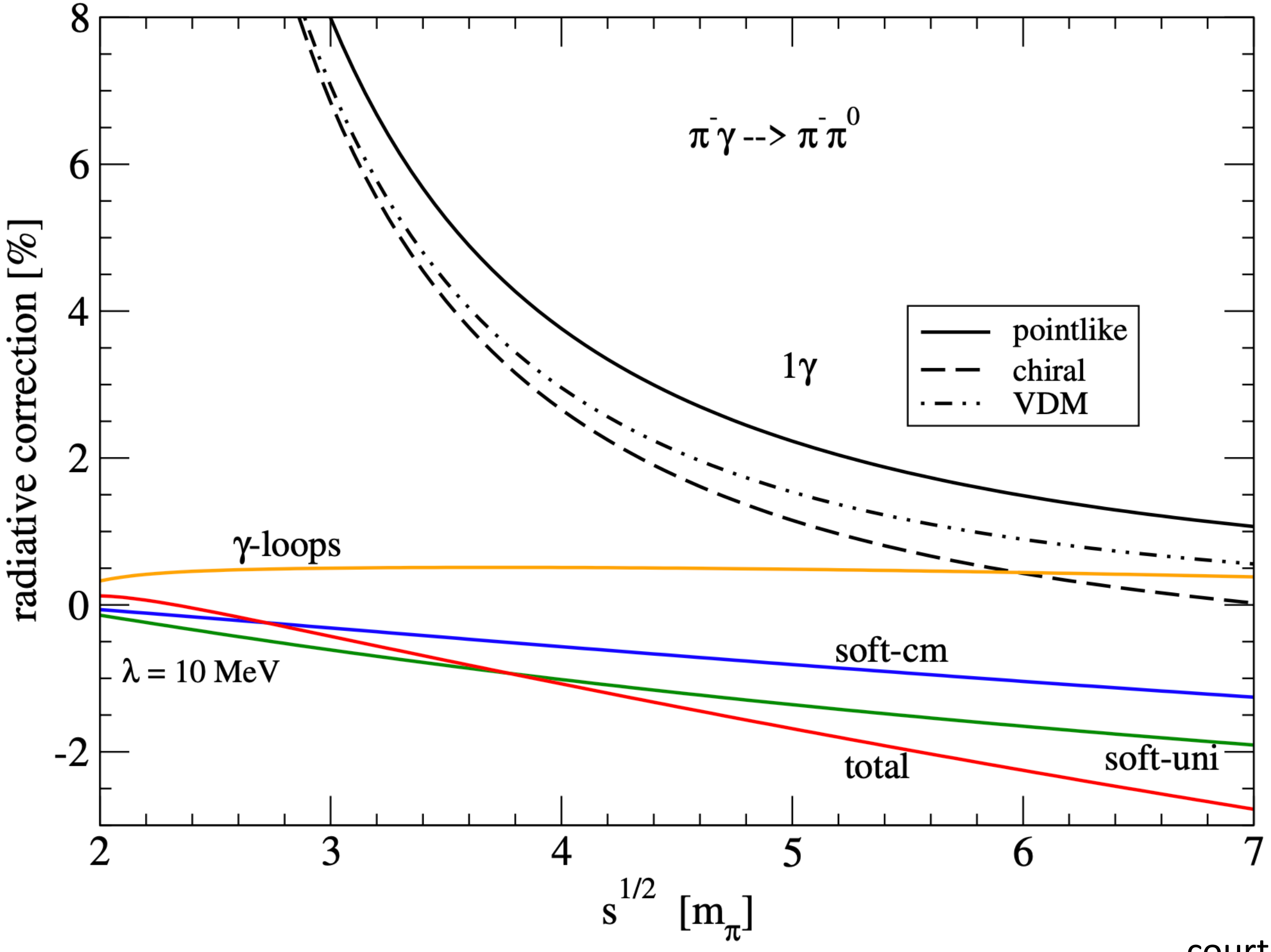
The other virtual corrections on one-photon level



- Color shading: Different $\cos\theta$ bins, from light (forward, less relevant) to dark (backward, covered by experiment)
- the other virtual corrections also feature $\cos\theta$-dependence, but amount to effects $\lesssim 0.5\%$
- Up to here: Correction could be included as a factor on event-by-event real data, or as a weight in the Monte-Carlo simulation of the experiment, since the kinematics is not distorted



The other virtual corrections on one-photon level



courtesy: Norbert Kaiser

Effect of virtual corrections to the Serpukhov result

- Original Serpukhov result:
  $F_{3\pi} = (12.9 \pm 0.9 \pm 0.5)\ \mathrm{GeV}^{-3}$

Ametller, L. *et al.* Phys.Rev. D64 (2001) 094009

- Applying the correction:

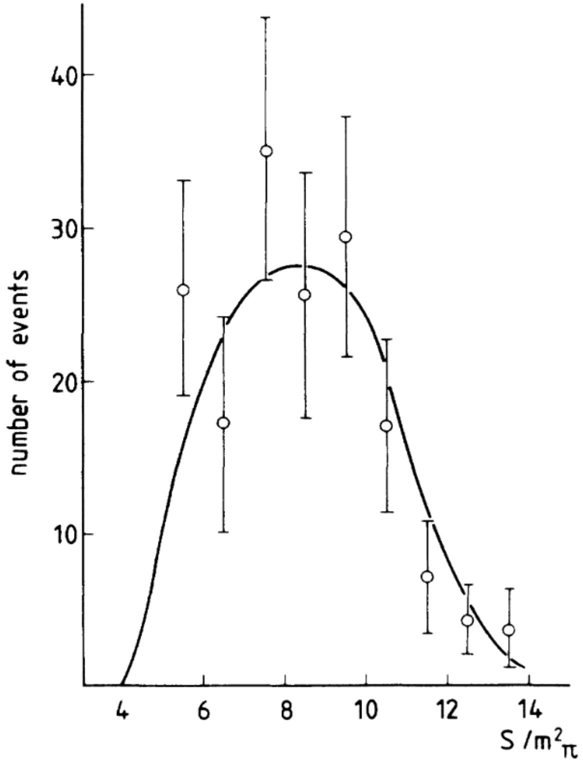
  $F_{3\pi} = (10.7 \pm 1.2)\ \mathrm{GeV}^{-3}$

- Compare to prediction from ChPT:

  $F_{3\pi} = (9.78 \pm 0.05)\ \mathrm{GeV}^{-3}$

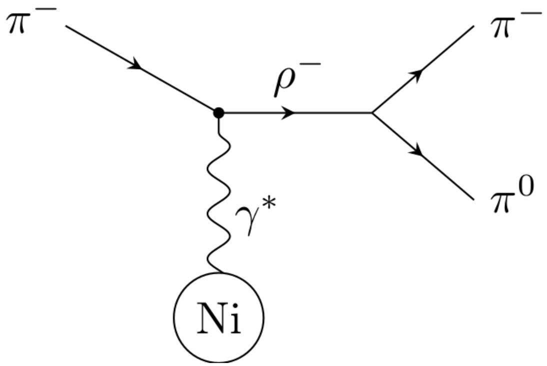
Precision of previous measurements: $\mathcal{O}(10\%)$

$\Rightarrow$ More precise experimental determination desirable

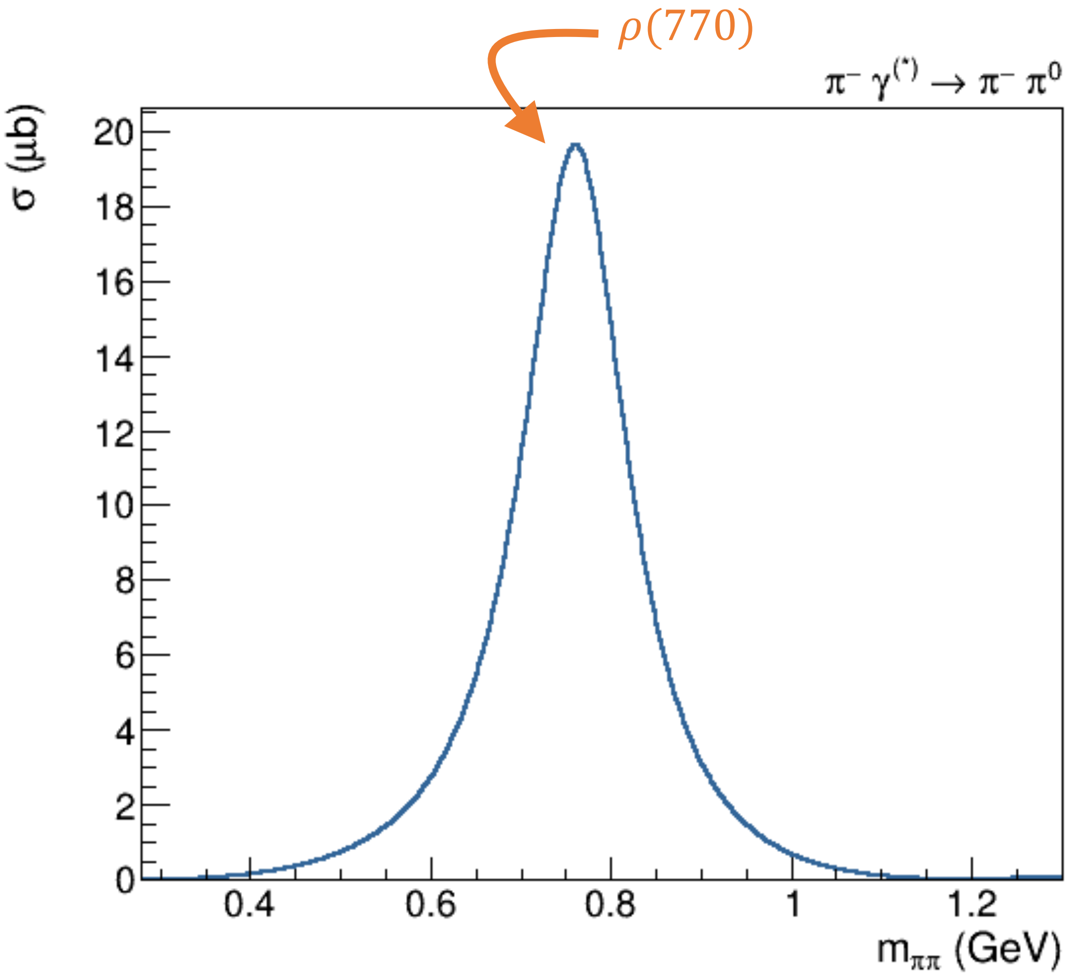


Radiative width of ρ-meson

- Coherent background of $\rho(770)$-production (strong and electro-magnetic)

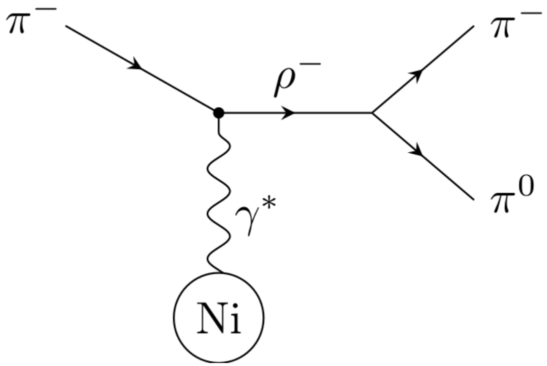


$\Rightarrow$ possibility of extraction of radiative width of ρ-meson: $\Gamma_{(\rho\to\pi\gamma)}/\Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$



Radiative width of $\boldsymbol{\rho}$-meson

- Coherent background of $\rho(770)$-production (strong and electro-magnetic)

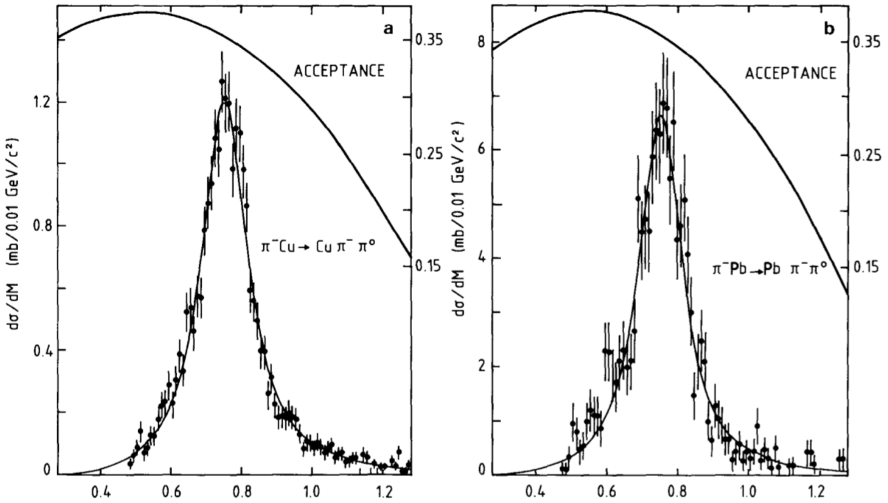


$\Rightarrow$ possibility of extraction of radiative width of ρ-meson: $\Gamma_{(\rho\to\pi\gamma)}/\Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$

Radiative width of $\boldsymbol{\rho}$-meson:

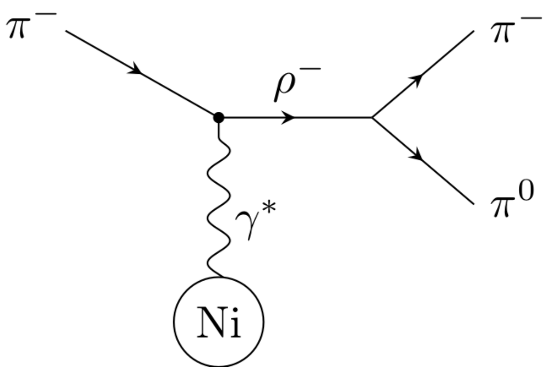
Capraro, L. *et al.* Nucl. Phys. B288 (1987) 659-680 at CERN (SPS):

- From fit of $\mathrm{d}\sigma/\mathrm{d}t$ for ρ production: $\Gamma(\rho \to \pi\gamma) = (81 \pm 4 \pm 4)\ \mathrm{keV}$



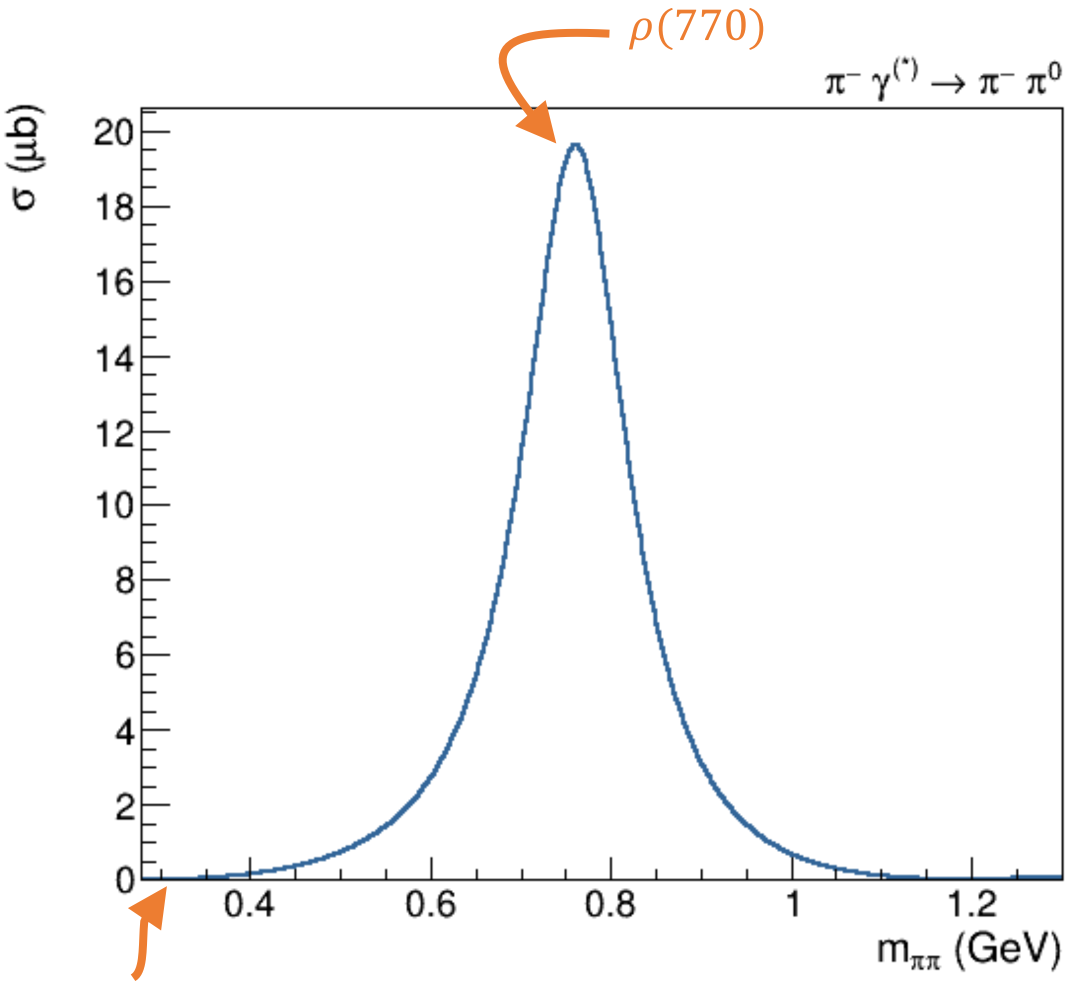
Radiative width of ρ-meson

- Coherent background of $\rho(770)$-production (strong and electro-magnetic)



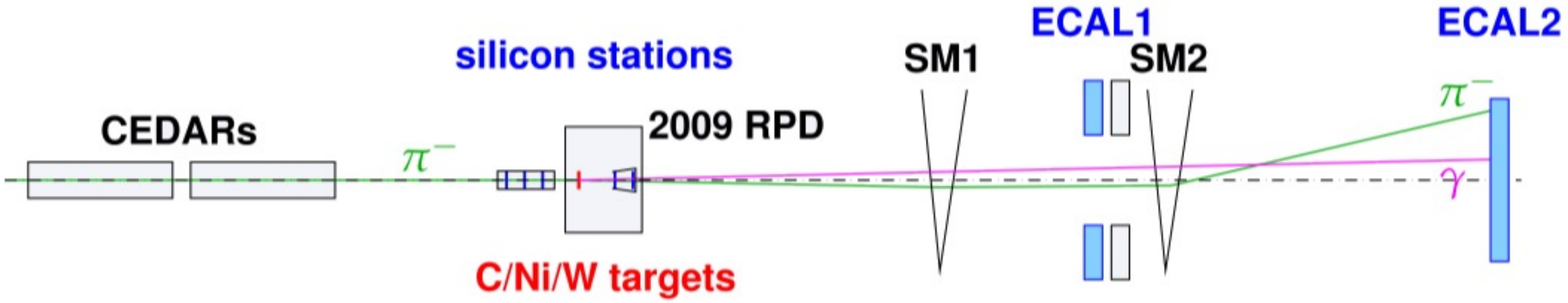
$\Rightarrow$ possibility of extraction of radiative width of ρ-meson: $\Gamma_{(\rho\rightarrow\pi\gamma)}/\Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$

- At kinematic threshold: non-resonant behaviour but chiral anomaly (Serpukhov measurement)
- Interference between Chiral Anomaly and ρ gives additional information



Low-mass tail:
mainly driven by $F_{3\pi}$

Principle of the COMPASS Measurement

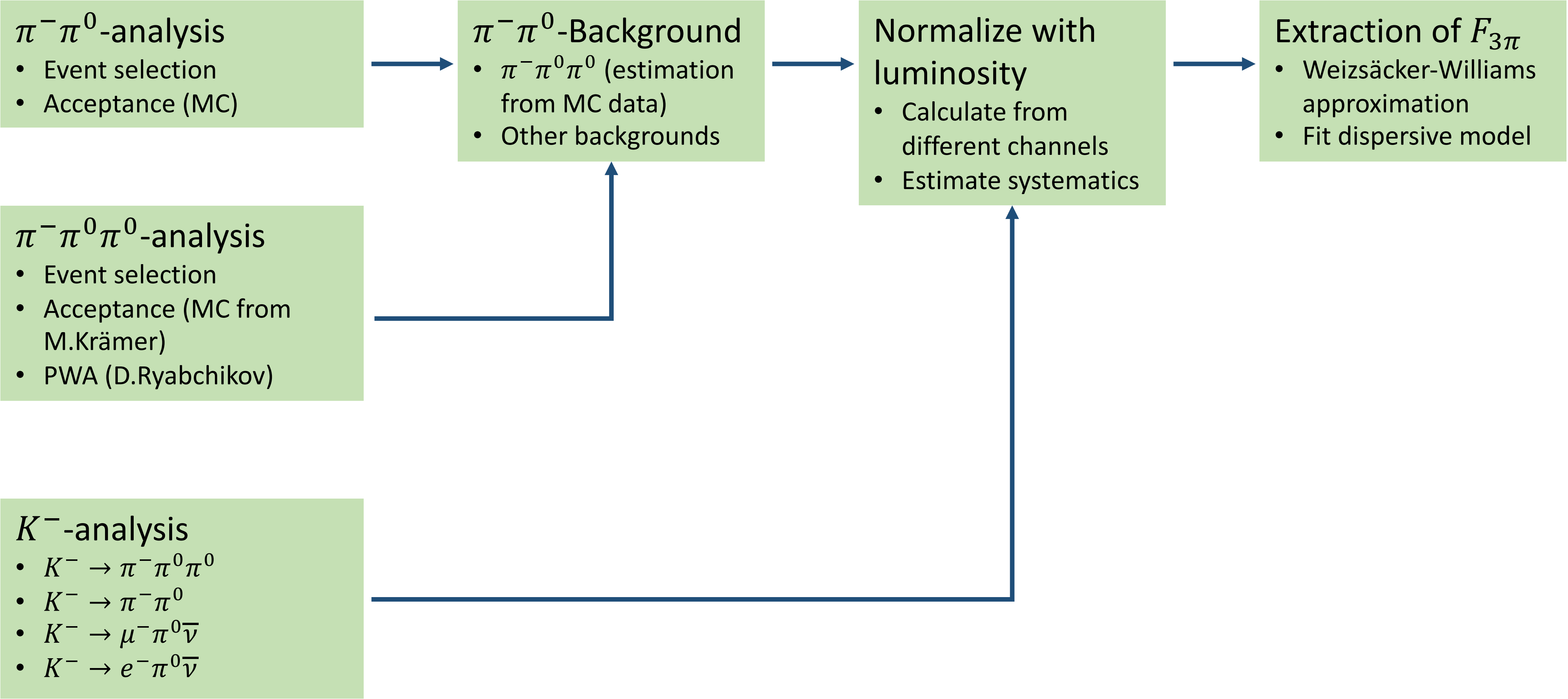


- 190 GeV negative hadron beam: $96.8\%\ \pi^-, 2.4\%\ K^-, 0.8\%\ \bar{p}$
- Beam particle identification by Cherenkov detectors
- 4mm Ni target disk ($\approx 25\%\ \ X/X_0$)
- Measure scattered π^- and produced photons (number of photons depends on final state)
- Select exclusive events at very low Q^2
- For absolute cross-section measurements: Luminosity

  Luminosity determination via free Kaon decays

$$(K^- \longrightarrow \pi^-\pi^0 \quad \text{or} \quad K^- \longrightarrow \pi^-\pi^0\pi^0)$$

Plan of the Analysis



$\pi^-\pi^0$-analysis
- Event selection
- Acceptance (MC)

$\pi^-\pi^0$-Background
- $\pi^-\pi^0\pi^0$ (estimation from MC data)
- Other backgrounds

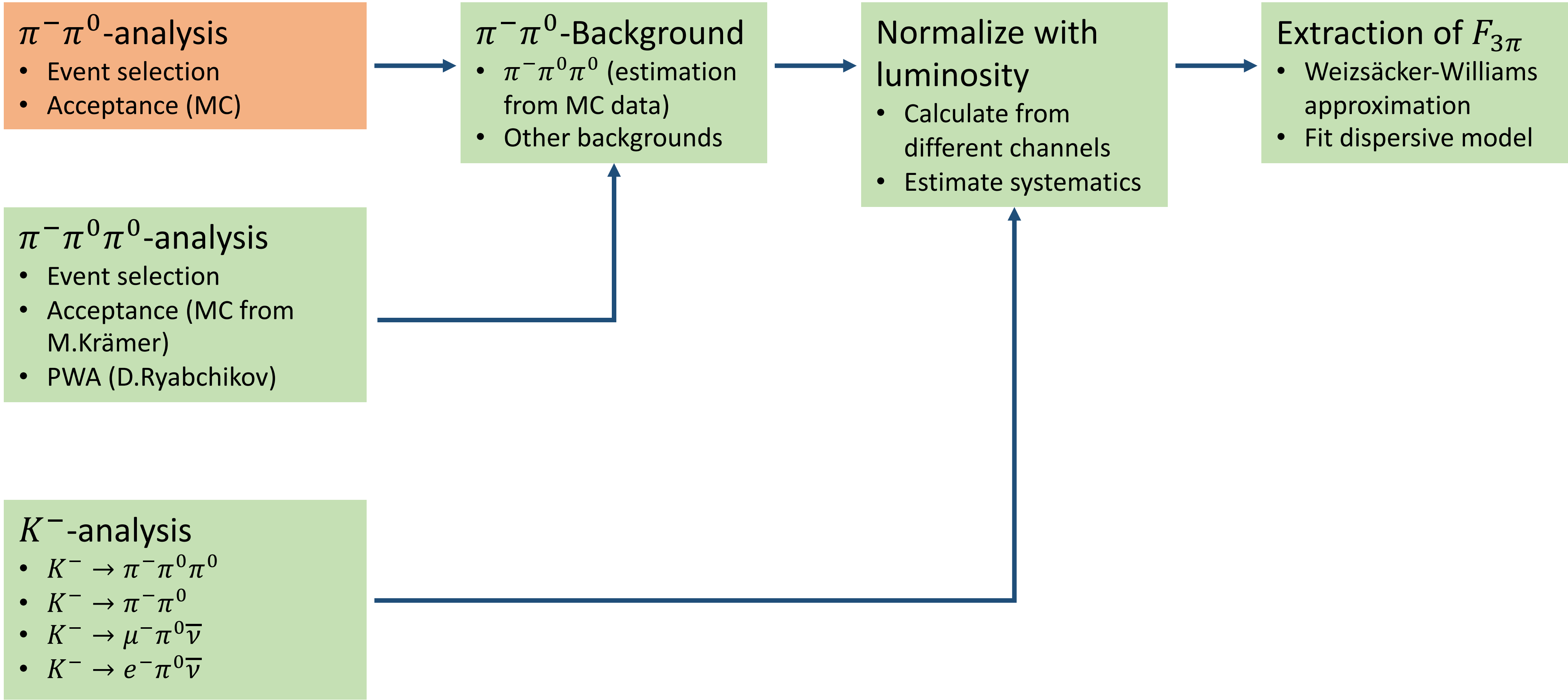
Normalize with luminosity
- Calculate from different channels
- Estimate systematics

Extraction of $F_{3\pi}$
- Weizsäcker-Williams approximation
- Fit dispersive model

$\pi^-\pi^0\pi^0$-analysis
- Event selection
- Acceptance (MC from M.Krämer)
- PWA (D.Ryabchikov)

K^--analysis
- $K^- \to \pi^-\pi^0\pi^0$
- $K^- \to \pi^-\pi^0$
- $K^- \to \mu^-\pi^0\overline{\nu}$
- $K^- \to e^-\pi^0\overline{\nu}$

Plan of the Analysis



$\pi^-\pi^0$-analysis
- Event selection
- Acceptance (MC)

$\pi^-\pi^0$-Background
- $\pi^-\pi^0\pi^0$ (estimation from MC data)
- Other backgrounds

Normalize with luminosity
- Calculate from different channels
- Estimate systematics

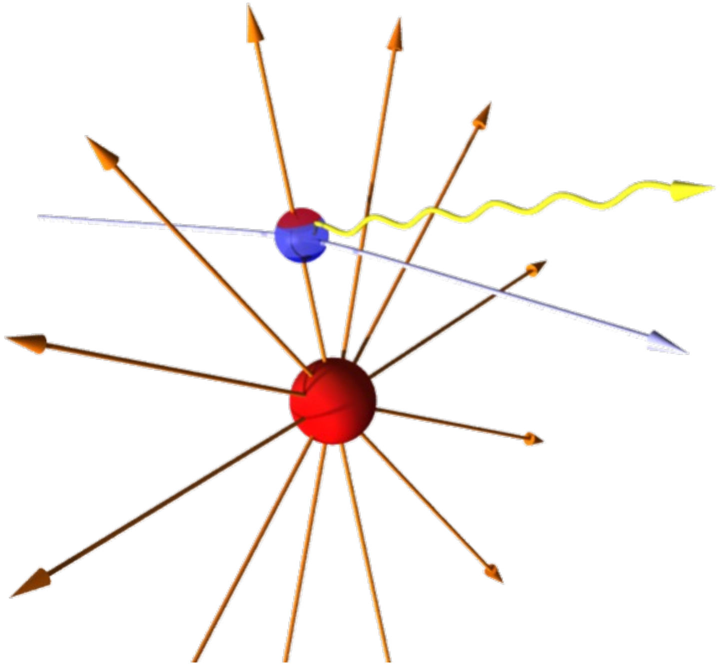
Extraction of $F_{3\pi}$
- Weizsäcker-Williams approximation
- Fit dispersive model

$\pi^-\pi^0\pi^0$-analysis
- Event selection
- Acceptance (MC from M.Krämer)
- PWA (D.Ryabchikov)

K^--analysis
- $K^- \to \pi^-\pi^0\pi^0$
- $K^- \to \pi^-\pi^0$
- $K^- \to \mu^-\pi^0\overline{\nu}$
- $K^- \to e^-\pi^0\overline{\nu}$

Primakoff reactions

- Idea dates back to Henry Primakoff ("photon target")
- Photon is provided by the strong Coulomb field of a nucleus (typical field strength at $d = 5R_{Ni}$: $E \approx 300\ \mathrm{kV/fm}$
- Coulomb field of nucleus as a source of quasi-real ($P_\gamma^2 \ll m_\pi^2$) photons
- Large impact parameters (ultra-peripheral scattering)



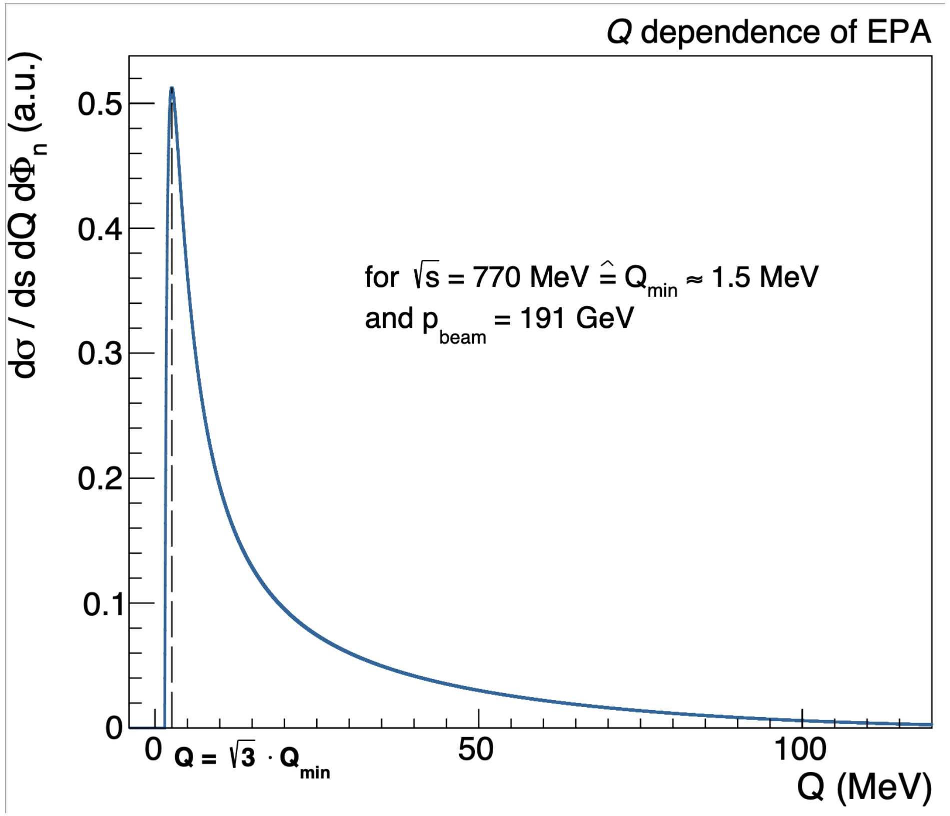
Weizsäcker-Williams approximation

- Coulomb field of relativistic charge ≈ flux of quasi-real photons:

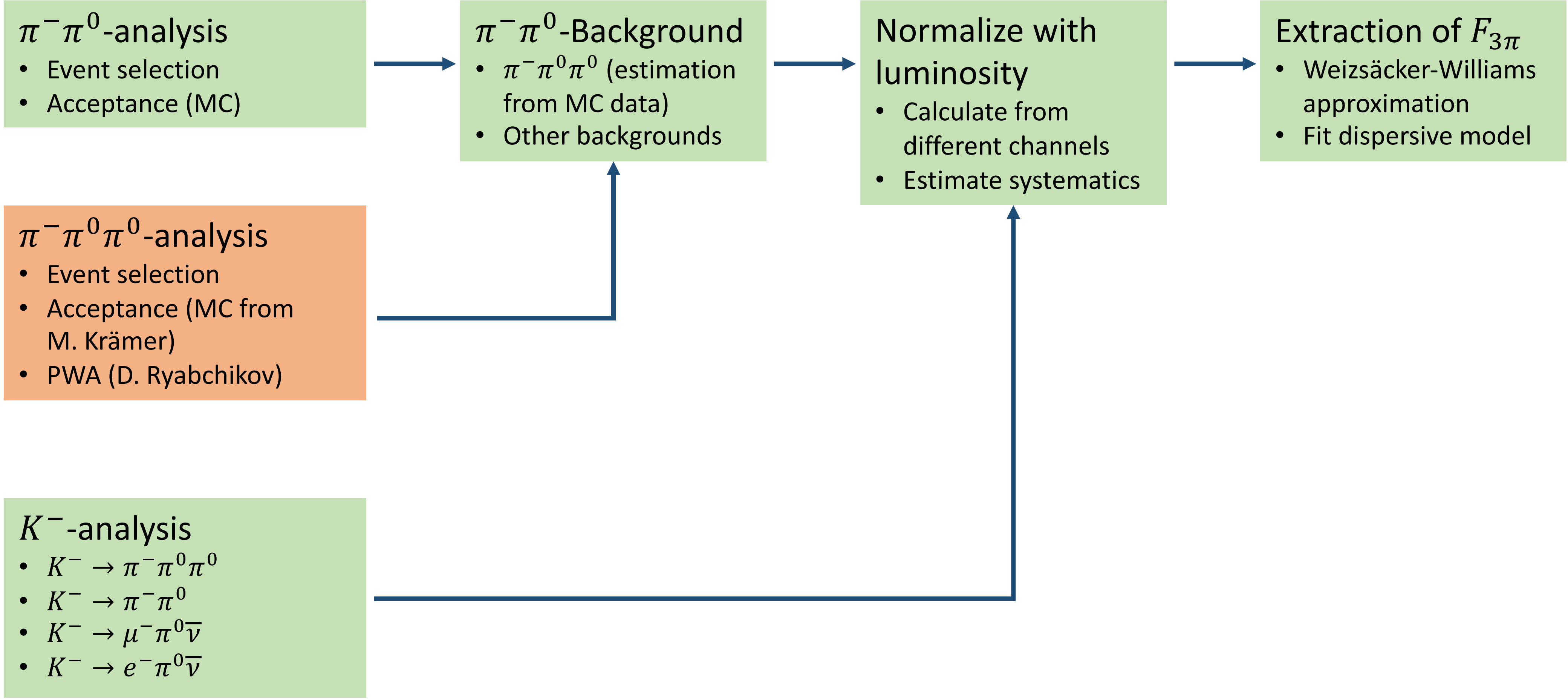
$$\frac{\mathrm{d}\sigma}{\mathrm{d}s\,\mathrm{d}Q^2\,\mathrm{d}\Phi_n} = \underbrace{\frac{Z^2\alpha}{\pi(s-m_\pi^2)}F^2(Q^2)\frac{Q^2-Q^2_{\mathrm{min}}}{Q^4}}_{\text{Flux of quasi-real photons}} \cdot \underbrace{\frac{\mathrm{d}\sigma_{\pi\gamma\to X}}{\mathrm{d}\Phi_n}}_{\text{Cross-section of reaction}}$$

- Particles scatter off equivalent photons

- Peak at tiny momentum transfers $Q^2 \approx 10^{-5}\mathrm{GeV}^2/c^2$



Plan of the Analysis



$\pi^-\pi^0$-analysis
- Event selection
- Acceptance (MC)

$\pi^-\pi^0$-Background
- $\pi^-\pi^0\pi^0$ (estimation from MC data)
- Other backgrounds

Normalize with luminosity
- Calculate from different channels
- Estimate systematics

Extraction of $F_{3\pi}$
- Weizsäcker-Williams approximation
- Fit dispersive model

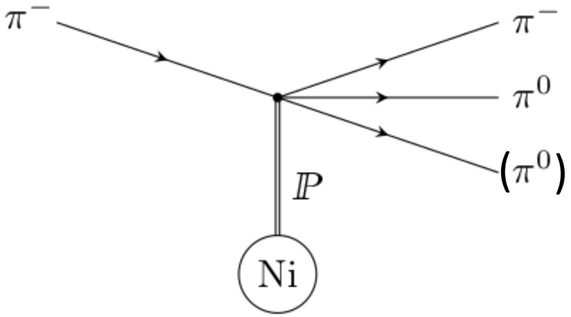
$\pi^-\pi^0\pi^0$-analysis
- Event selection
- Acceptance (MC from M. Krämer)
- PWA (D. Ryabchikov)

K^--analysis
- $K^- \to \pi^-\pi^0\pi^0$
- $K^- \to \pi^-\pi^0$
- $K^- \to \mu^-\pi^0\overline{\nu}$
- $K^- \to e^-\pi^0\overline{\nu}$

Background

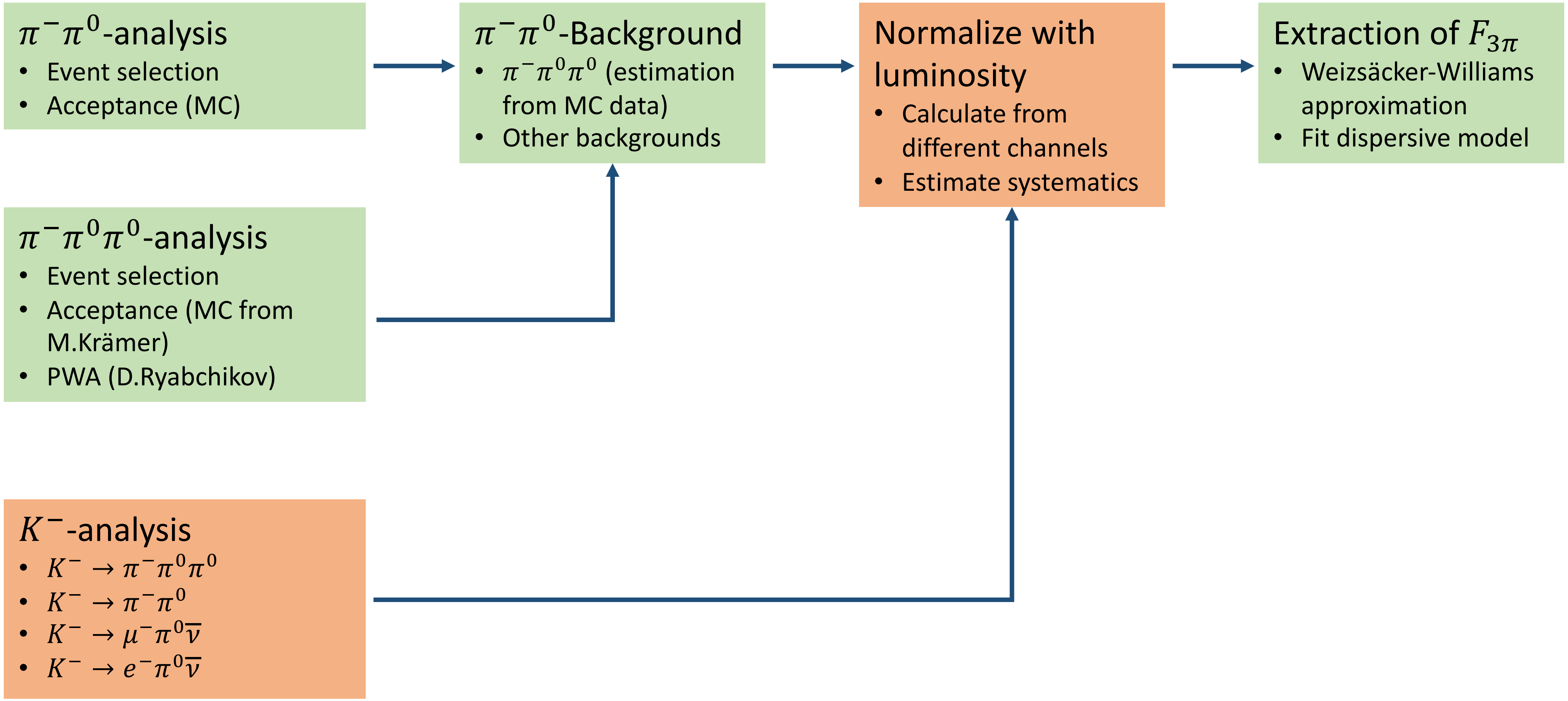
	Primakoff	Strong (ω/π)	Strong (Pomeron)
$\sigma(E_{\text{beam}})$	$\propto \ln(E)$	$\propto 1/E$	$\propto$ const
$\sigma(A_{\text{target}})$	$\propto$ const	$\propto A^{2/3}$	$\propto A^{2/3}$
$\sigma(Z_{\text{target}})$	$\propto Z^2$	$\propto$ const	$\propto$ const
$\sigma(q^2)$	$\propto 1/(q^2 - q^2_{\text{min}})$	depends on qtm nmb	$\propto \exp(-bt')$

Main background:



- $\pi^-\pi^0$-final state forbidden by G-parity
- But: large cross-section for $\pi^-\pi^0\pi^0$-final state => loss of one (soft) π^0
- Approach: determine leakage from 3pi MC data with 2pi event selection

Plan of the Analysis



$\pi^-\pi^0$-analysis

- Event selection
- Acceptance (MC)

$\pi^-\pi^0\pi^0$-analysis

- Event selection
- Acceptance (MC from M.Krämer)
- PWA (D.Ryabchikov)

K^--analysis

- $K^- \rightarrow \pi^-\pi^0\pi^0$
- $K^- \rightarrow \pi^-\pi^0$
- $K^- \rightarrow \mu^-\pi^0\overline{\nu}$
- $K^- \rightarrow e^-\pi^0\overline{\nu}$

$\pi^-\pi^0$-Background

- $\pi^-\pi^0\pi^0$ (estimation from MC data)
- Other backgrounds

Normalize with luminosity

- Calculate from different channels
- Estimate systematics

Extraction of $F_{3\pi}$

- Weizsäcker-Williams approximation
- Fit dispersive model

Luminosity determination

- Needed for absolute cross section measurement: effective integrated luminosity (DAQ dead time taken into account)

Effective luminosity: $L_{eff} = L \cdot (1 - \epsilon_{DAQ})$

- Luminosity can be determined via free kaons in the beam:
  - Use CEDARs to tag kaons
  - Free decays where no material
  - Exclusive events with low (≈ 0) momentum transfer

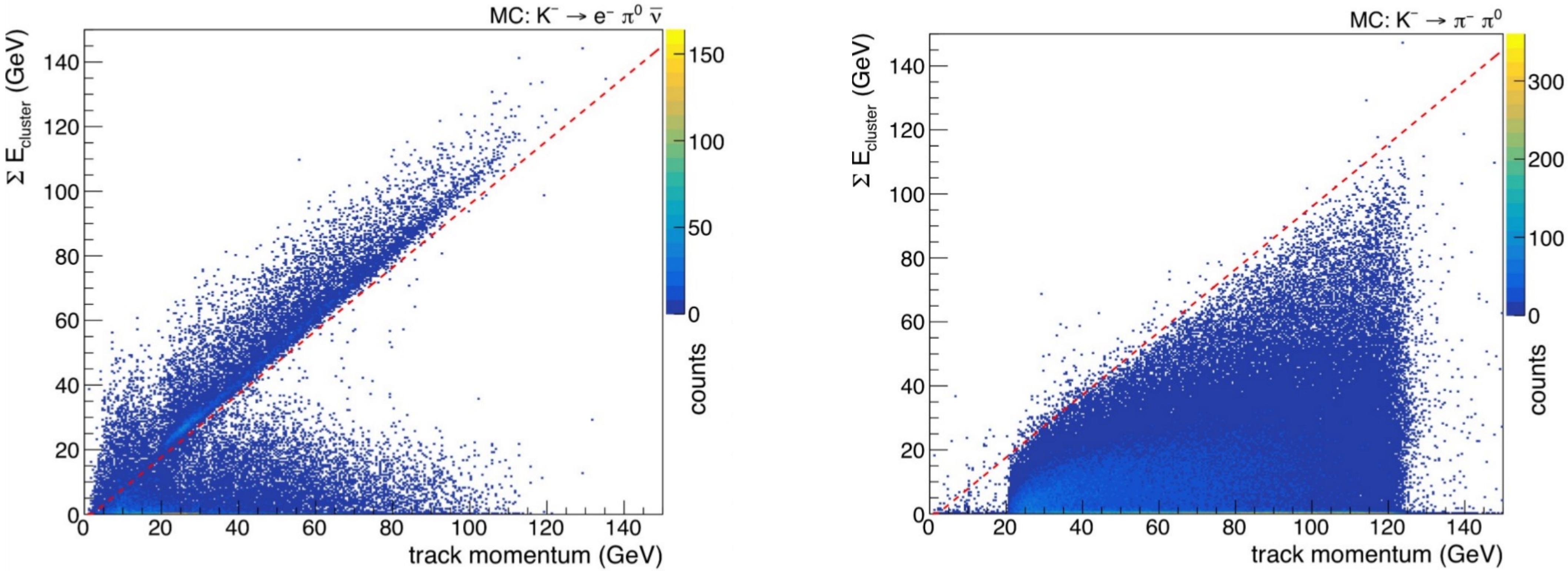
$$\overset{\text{Number of detected kaon decays}}{N_K} = \underset{\text{Pion flux}}{\Phi_\pi} \overset{\text{Kaon/pion flux ratio}}{\frac{n_K}{n_\pi}} \underbrace{BR(K^- \to \pi^0 \pi^-)}_{\text{Branching ratio}} \overbrace{(1 - e^{-\frac{L}{\gamma c \tau}})}^{\text{Probability of kaon decaying in region with length L}} \underset{\text{Acceptance for kaon decays for particular final state}}{\epsilon_K}$$

Kaon decay channels

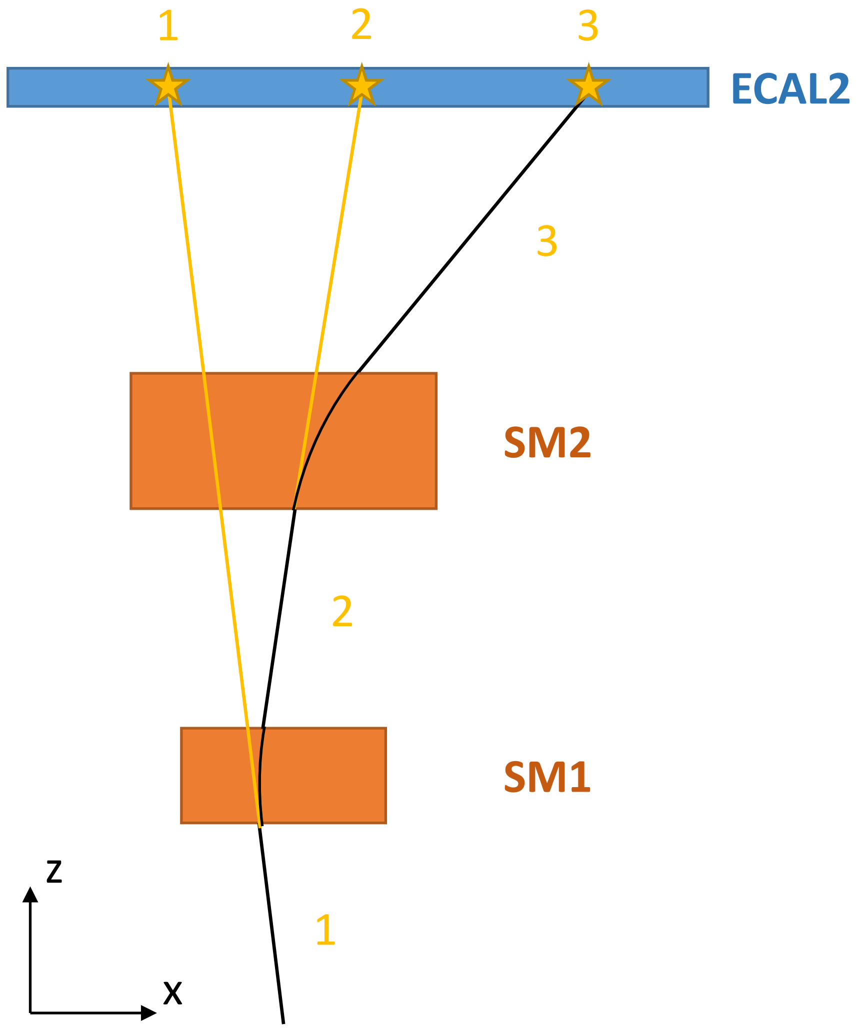
Decay channel	Γ_i/Γ	**Remark**
$K^- \to \mu^- \bar{\nu}_\mu$	(63.56 ± 0.11) %	Does not deposit energy in ECAL2 (Primakoff-trigger)
$\boldsymbol{K^- \to \pi^- \pi^0}$	(20.67 ± 0.08) %	Similar systematics as Primakoff $\pi^- \to \pi^- \pi^0$ channel
$K^- \to \pi^- \pi^- \pi^+$	$(5.583 \ \pm 0.024)$ %	Does not deposit energy in ECAL2 (Primakoff-trigger)
$\boldsymbol{K^- \to e^- \pi^0 \bar{\nu}_e \ (K_{e3})}$	(5.07 ± 0.08) %	Not exclusive, missing energy
$\boldsymbol{K^- \to \mu^- \pi^0 \bar{\nu}_\mu \ (K_{\mu3})}$	(3.352 ± 0.033) %	Not exclusive, missing energy
$\boldsymbol{K^- \to \pi^- \pi^0 \pi^0}$	$(1.760 \ \pm 0.023)$ %	Previous measurement by M. Kraemer, used to determine $^\pi/_K$-ratio in the beam)
others	$< 10^{-4}$	No significant contribution to background expected

π^- / e^- distinction

- Naive idea: E/p in calorimeter
- Possible discrimination line can be identified
- Still: many electrons deposit less energy than expected
- Reason: energy loss due to Bremsstrahlung in the spectrometer

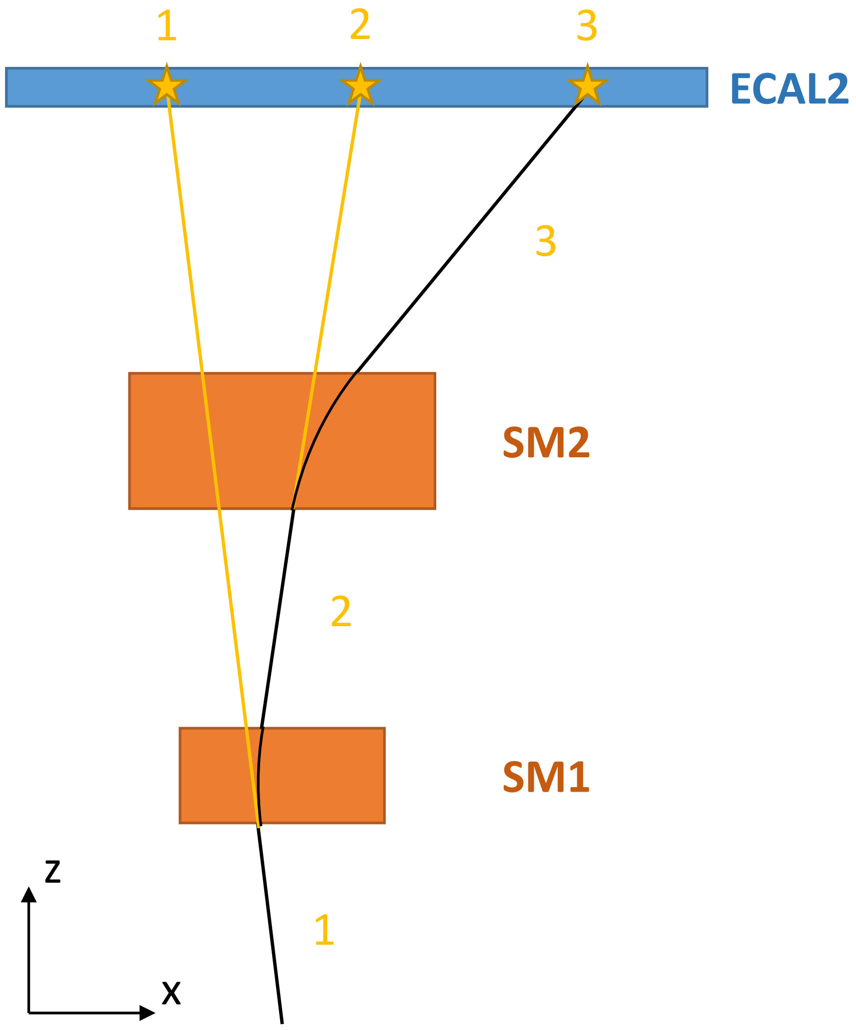


External Bremsstrahlung

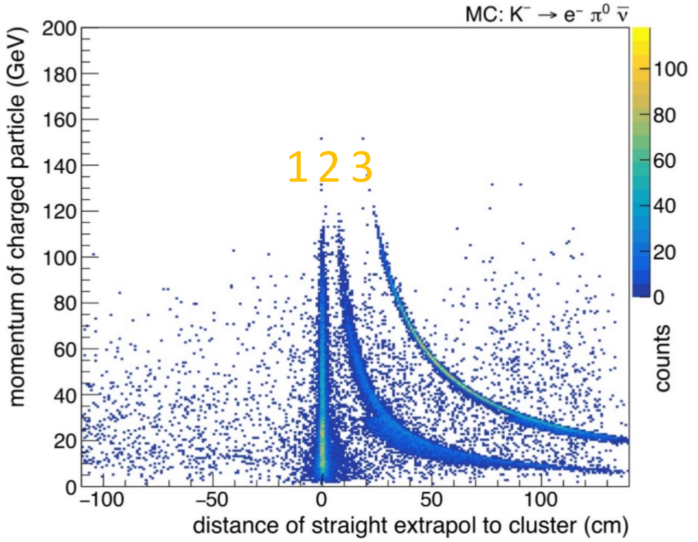


- Charged particle radiates photons while propagating through matter
- Deflection in dipole magnets dependent on momentum of charged track
- 3 distinct regions with increased probability for Bremsstrahlung
- Electrons have higher probability for Bremsstrahlung than pions

External Bremsstrahlung

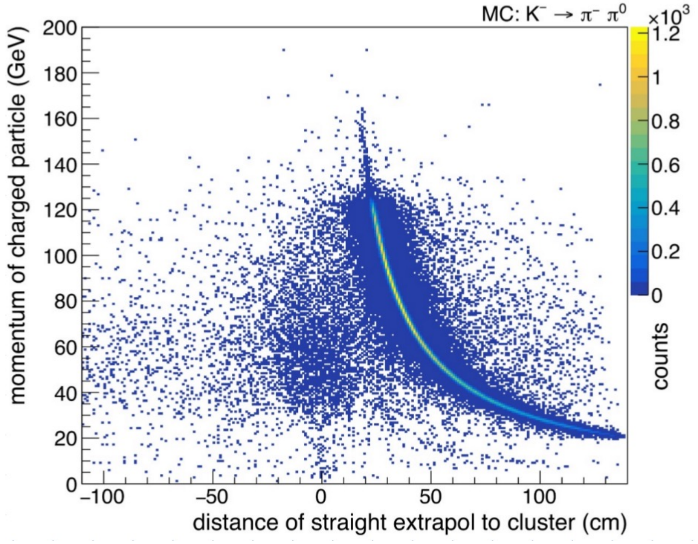


Electrons

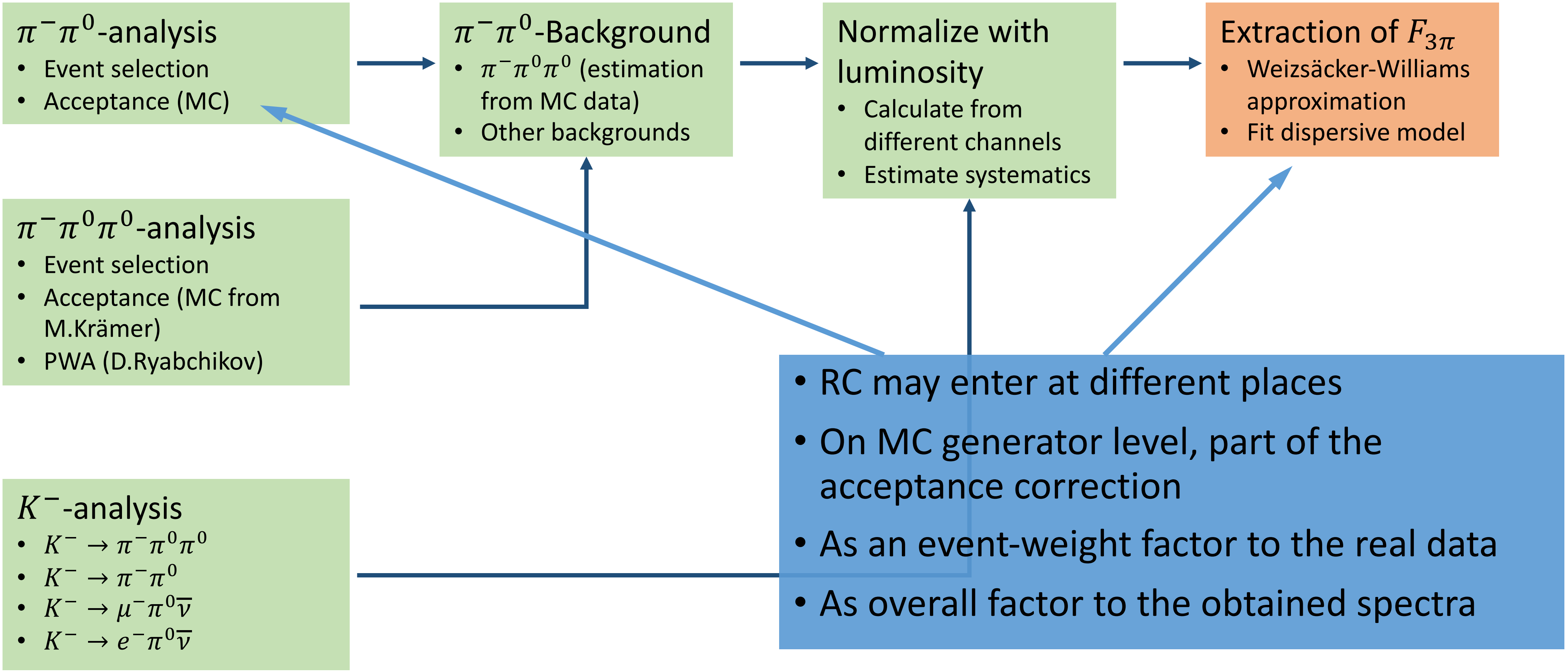


vs

pions



Internal Radiative Corrections: how to implement



$\pi^-\pi^0$-analysis
- Event selection
- Acceptance (MC)

$\pi^-\pi^0$-Background
- $\pi^-\pi^0\pi^0$ (estimation from MC data)
- Other backgrounds

Normalize with luminosity
- Calculate from different channels
- Estimate systematics

Extraction of $F_{3\pi}$
- Weizsäcker-Williams approximation
- Fit dispersive model

$\pi^-\pi^0\pi^0$-analysis
- Event selection
- Acceptance (MC from M.Krämer)
- PWA (D.Ryabchikov)

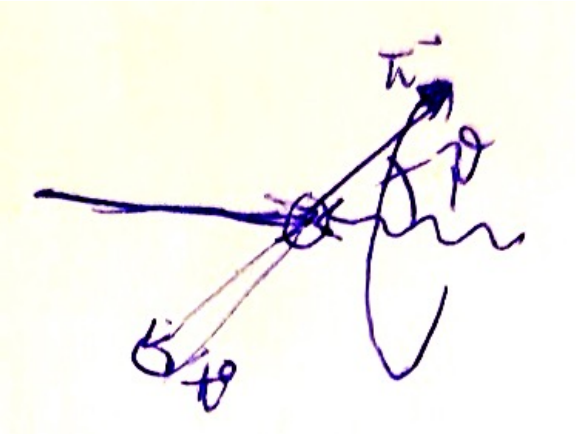
K^--analysis
- $K^- \to \pi^-\pi^0\pi^0$
- $K^- \to \pi^-\pi^0$
- $K^- \to \mu^-\pi^0\overline{\nu}$
- $K^- \to e^-\pi^0\overline{\nu}$

- RC may enter at different places
- On MC generator level, part of the acceptance correction
- As an event-weight factor to the real data
- As overall factor to the obtained spectra

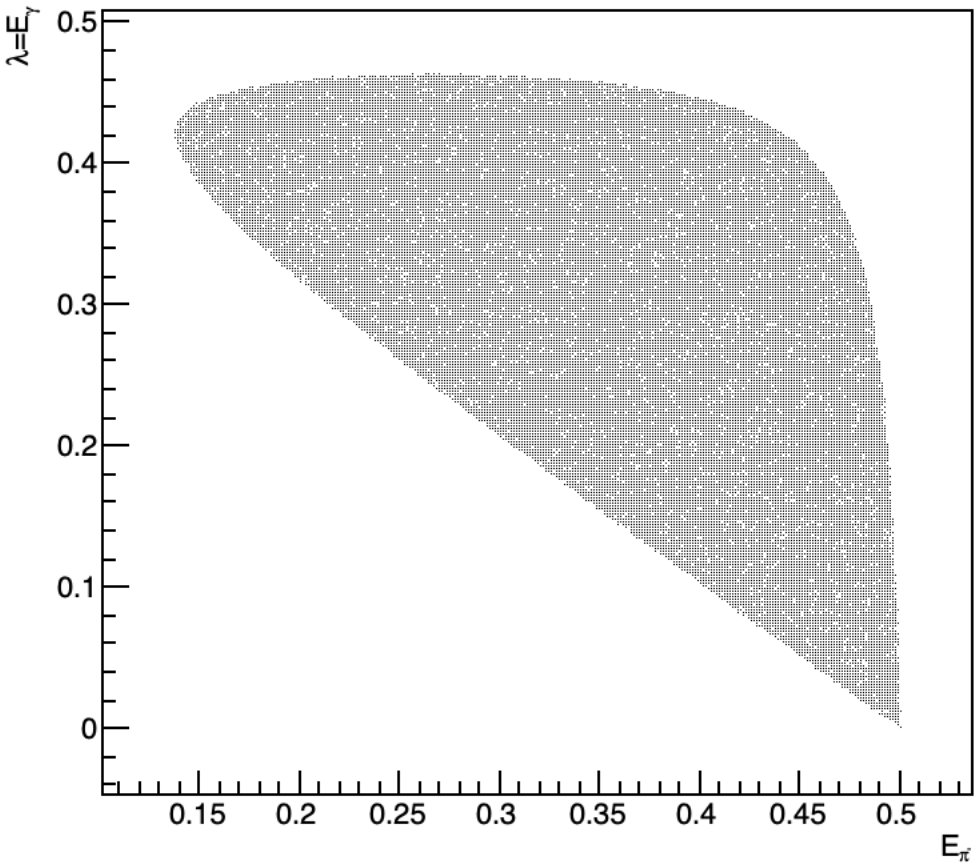
Internal Bremsstrahlung

$$\pi^- \gamma \rightarrow \pi^- \pi^0 \, \gamma$$

- Full phase space for this final state to be simulated
- Take out divergent corner $\lambda = E_\gamma \rightarrow 0$
- Control the transition from “ultra-soft” to soft photons: hand-over point when soft-photon approximation is still sufficiently well describing cross section (here: on the level of 1 MeV)



$\sqrt{s} = 1$ GeV



Internal Bremsstrahlung

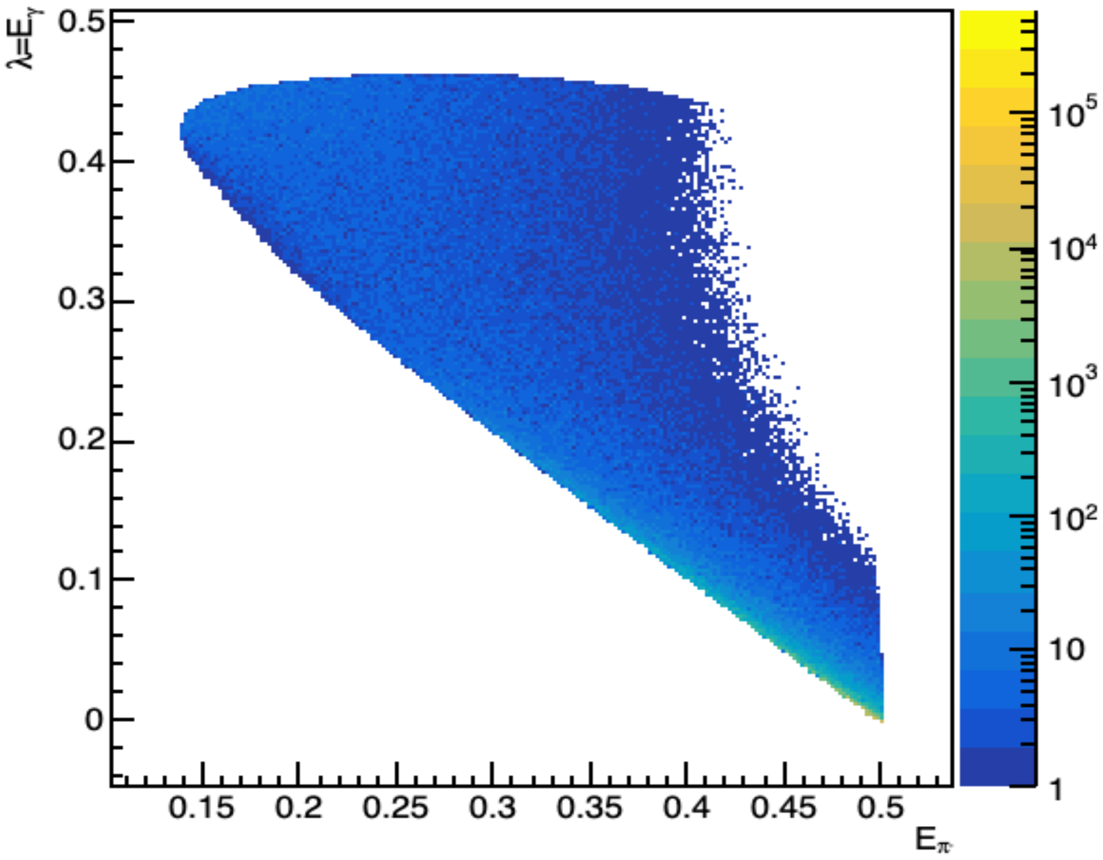
$$\pi^- \gamma \to \pi^- \pi^0 \gamma$$

- In the Monte Carlo, those events enter at the same level as the non-radiative events
- This allows to investigate bin migration and other non-factorizable effects
- The $1/\lambda$ divergence problem has nothing to do with detector resolution (despite bad resolution may come as a problem on top) – it simply means that photon emission becomes increasingly probable

$$\pi^- \gamma \to \pi^- \pi^0 \gamma\gamma$$

- How to treat double-internal radiation and two-photon loops?

$\sqrt{s} = 1\ \mathrm{GeV}$



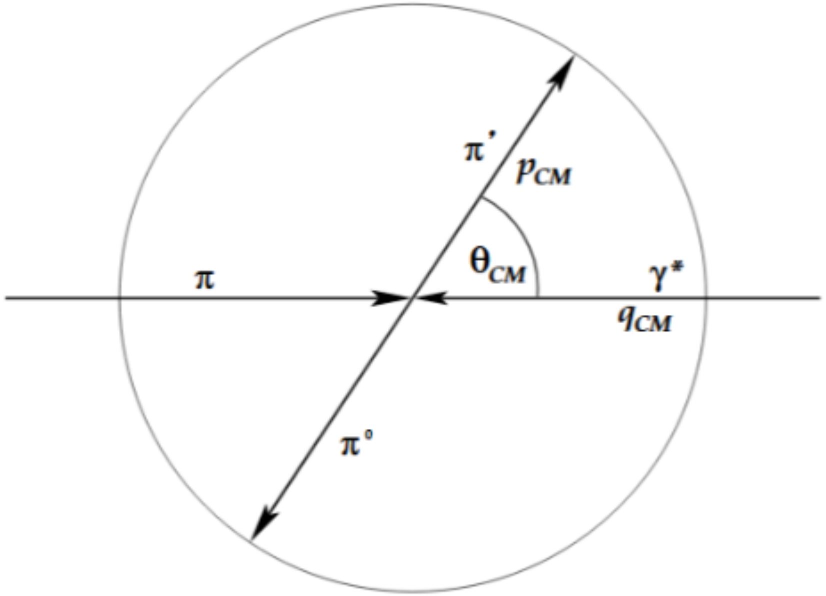
Conclusions

- Every precision experiment demands its own specific QED radiative corrections
- The most complete way of correction comprises
  - Monte-Carlo simulation of radiative events
  - Fitting of the free parameters of interest
  - Iteration of the procedure, demonstration of convergence
- Full documentation of the effects, e.g. comparison off corrections on/off

Thank you for your attention

Reconstruction of $\boldsymbol{Q^2}$

- Reconstruction of momentum transfer: $q^\mu = P^\mu_{\pi'} + P^\mu_{\pi^0} - P^\mu_\pi$
- Mainly limited by (calorimetric) measurement of π^0-energy
- Imposing exclusivity: $\Delta E = 0$ by rescaling cluster energies accordingly $\Rightarrow q_\parallel \approx 0$
- Imposing correct π^0-mass: $m_{\gamma\gamma} = 2\sqrt{E_1 E_2}\sin\frac{\vartheta}{2} = m_{\pi^0_{\mathrm{PDG}}}$



Dispersive framework

- Dispersive framework to deduce $F_{3\pi}$ from a fit to the full data set up to 1.2 GeV including the $\rho(770)$-resonance:

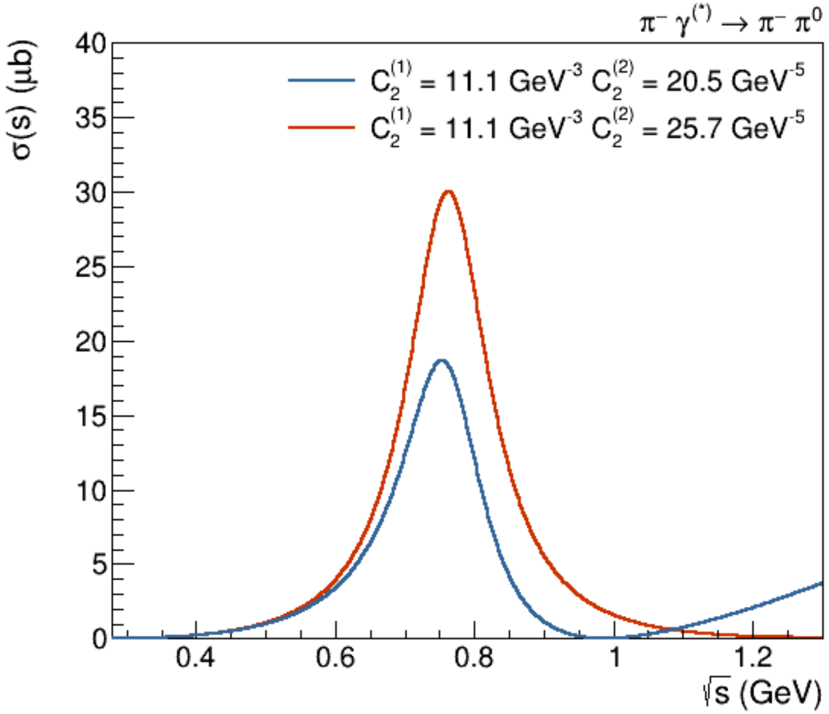
$$\sigma(s) = \frac{(s-4m_\pi^2)^{3/2}(s-m_\pi^2)}{1024\pi\sqrt{s}} \int_{-1}^{1} \mathrm{d}z(1-z^2)|\mathcal{F}(s,t,u)|^2$$

With

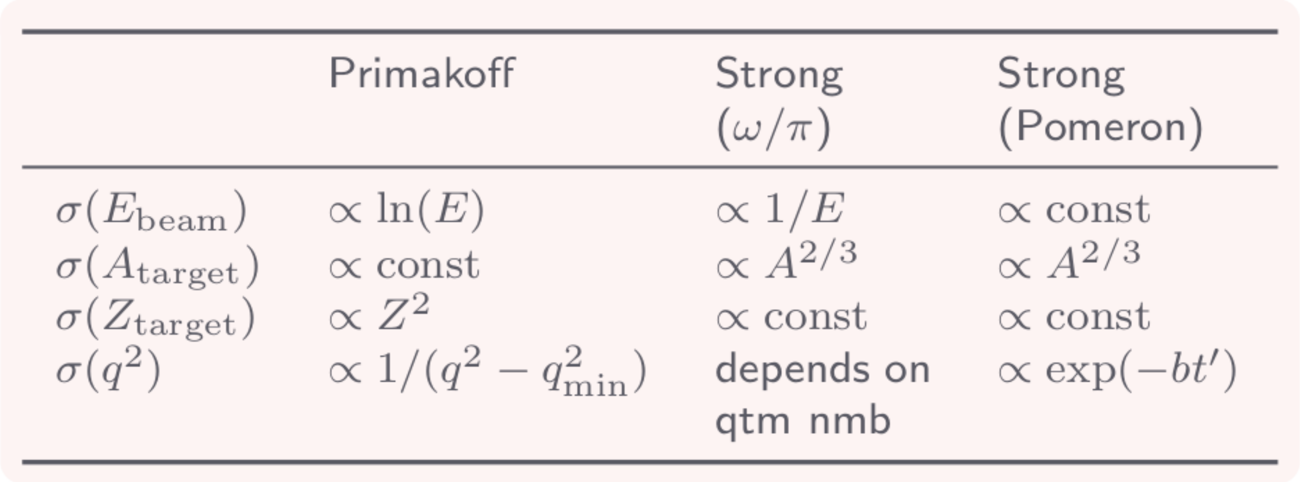
$$\mathcal{F}(s,t,u) = C_2^{(1)}\mathcal{F}_2^{(1)}(s,t,u) + C_2^{(2)}\mathcal{F}_2^{(2)}(s,t,u) - \frac{2e^2F_\pi^2F_{3\pi}}{t}$$

$C_2^{(1)}, C_2^{(2)}$: fit parameters

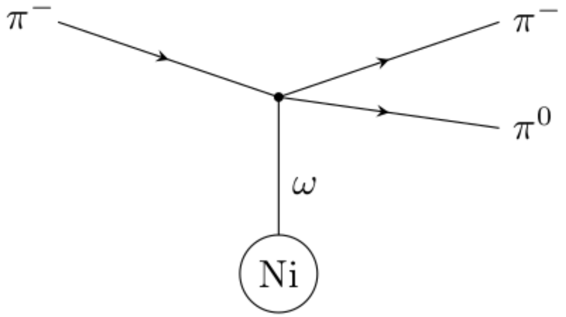
$\mathcal{F}_2^{(1)}(s,t,u), \mathcal{F}_2^{(2)}(s,t,u)$: provided by theory colleagues (Kubis, Hoferichter)



Different sources of background



	Primakoff	Strong (ω/π)	Strong (Pomeron)
$\sigma(E_{\text{beam}})$	$\propto \ln(E)$	$\propto 1/E$	$\propto \text{const}$
$\sigma(A_{\text{target}})$	$\propto \text{const}$	$\propto A^{2/3}$	$\propto A^{2/3}$
$\sigma(Z_{\text{target}})$	$\propto Z^2$	$\propto \text{const}$	$\propto \text{const}$
$\sigma(q^2)$	$\propto 1/(q^2 - q^2_{\text{min}})$	depends on qtm nmb	$\propto \exp(-bt')$

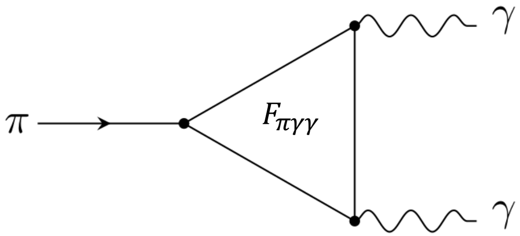


- Different intermediate resonances possible, e.g. via ρ_3 or b_1
- Investigations for $\pi^-\omega \rightarrow \pi^-\pi^0\gamma$ ongoing

First test of the chiral anomaly: π^0 lifetime

- First definitive measurement of π^0-lifetime in 1963:

$$\tau_{\text{exp}}(\pi^0) = (9.5 \pm 1.5) \cdot 10^{-17}\text{s} \neq \tau_{\text{PCAC}}(\pi^0) \approx 10^{-13}\text{s}$$



- Adler, Bell, Jackiw, Bardeen 1969: calculation of triangle diagram

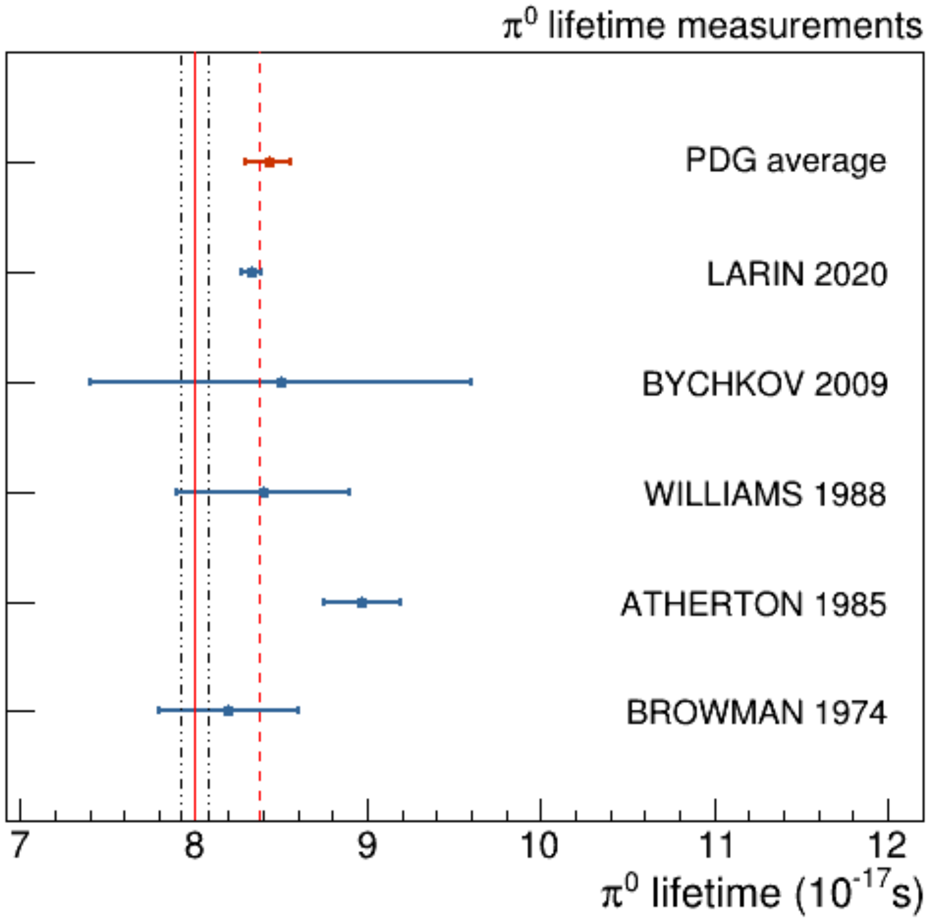
$$\Gamma^{\text{anom}}(\pi^0 \to \gamma\gamma) = F^2_{\pi\gamma\gamma} \cdot \frac{m^3_{\pi^0}}{64\pi} = \left(\frac{e^2 N_c}{12\pi^2 F_\pi}\right)^2 \frac{m^3_{\pi^0}}{64\pi} = 7.75\,\text{eV}$$

$$\tau(\pi^0) = \text{BR}(\pi^0 \to \gamma\gamma) \cdot \frac{\hbar}{\Gamma^{\text{anom}}(\pi^0 \to \gamma\gamma)}$$

$$= 8.38 \cdot 10^{-17}\,\text{s}$$

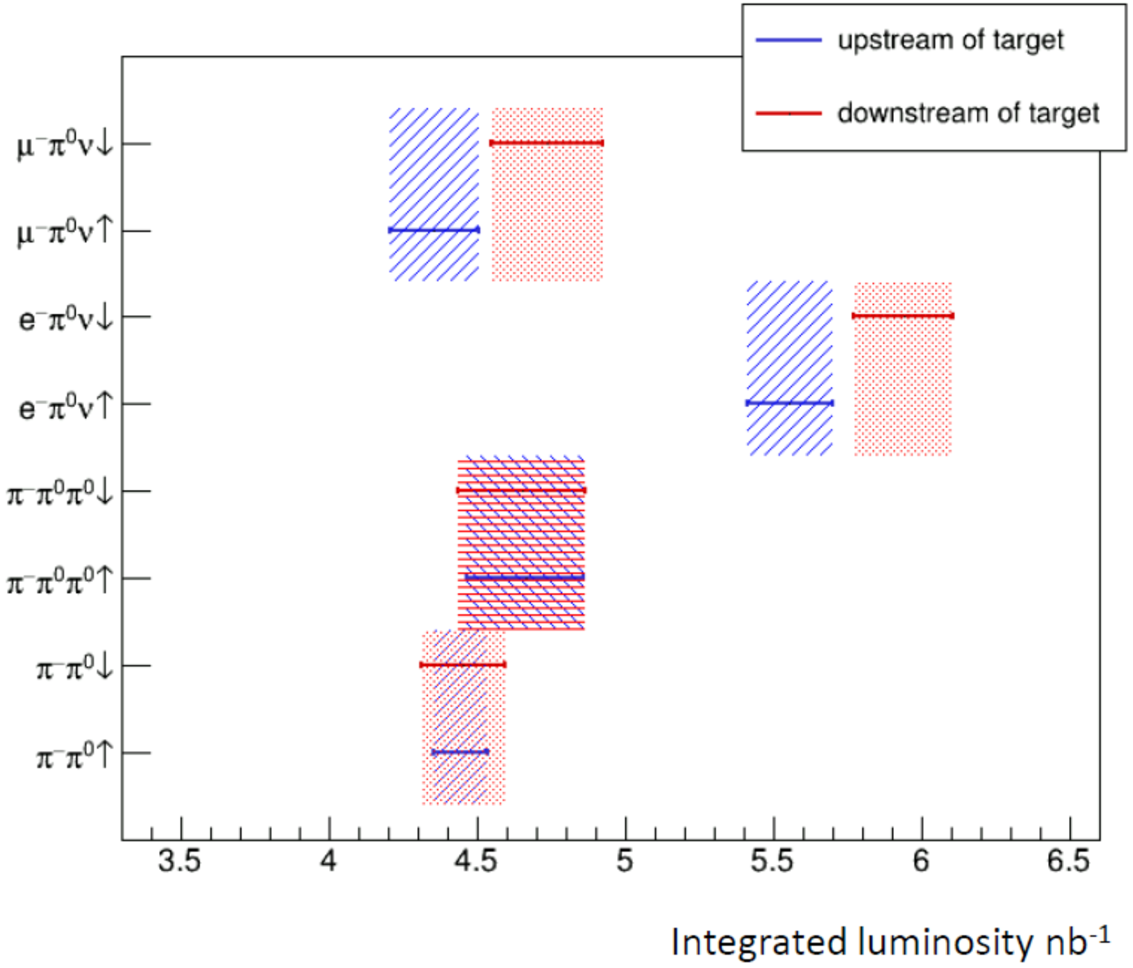
- Moussalam and Kampf 2009: NLO-calculation in chiral perturbation theory

$$\tau_{\text{NLO}}(\pi^0) = (8.04 \pm 0{,}11) \cdot 10^{-17}\text{s}$$



Luminosity

- Values for luminosity from 2 and 3-pion decay channels in better agreement.
- Difference in up-/downstream values for semi-leptonic decays -> hint to problem
- Values for semi-leptonic decays (K_{e3} and $K_{\mu3}$) depend on cuts on phase space -> not entirely understood



Values need to be updated
(event selection, ECAL corrections, etc.)