

# Proton Charge Radius from PRad Experiment at Jefferson Lab

A. Gasparian

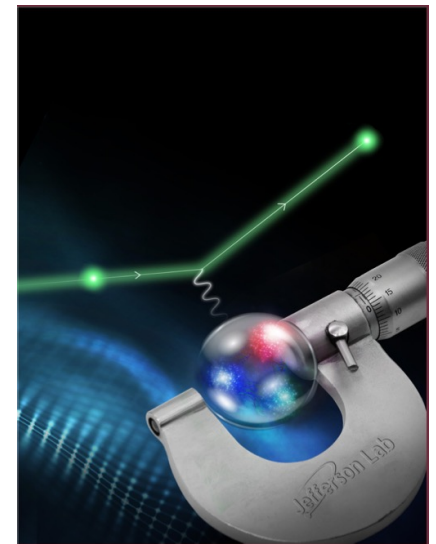
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for the PRad collaboration

## Outline

- the PRad approach for a new ep-experiment
- PRad experiment and the results
- new planned experiment, PRad-II
- summary

**PR**oton  
Radius



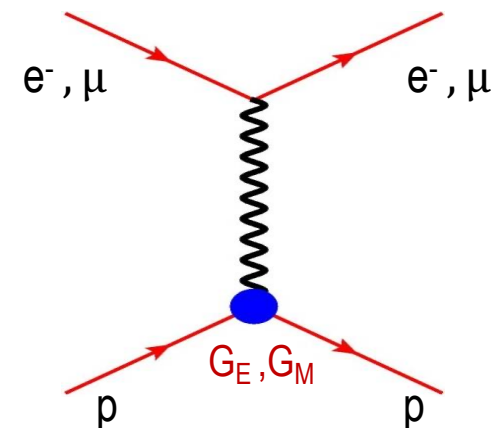
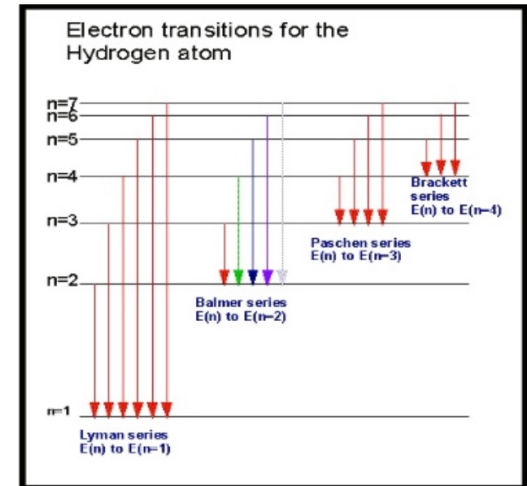
# Proton Charge Radius

One of the most fundamental quantities in physics:

- atomic physics:
  - ✓ precision atomic spectroscopy (QED, Lamb shifts, **Rydberg constant  $R_\infty$** );
  - ✓  $r_p$  is strongly correlated to  $R_\infty$
- nuclear physics:
  - ✓ QCD, test of nuclear/particle models
- connects atomic and subatomic physics.

Methods to measure the Proton rms charge radius ( $r_p$ ):

- Hydrogen spectroscopy (lepton-proton bound state, **Atomic Physics**):
  - ❖ regular hydrogen
  - ❖ muonic hydrogen
- Lepton-proton elastic scattering (**Nuclear Physics**):
  - ❖ ep- scattering (like PRad)
  - ❖  $\mu p$ - scattering (like MUSE)

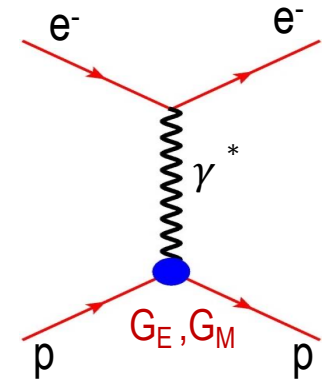


# Proton Radius from $ep \rightarrow ep$ Scattering Experiments

- In the limit of first Born approximation the elastic  $ep$  scattering (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left( \frac{E'}{E} \right) \frac{1}{1+\tau} \left( G_E^p{}^2(Q^2) + \frac{\tau}{\varepsilon} G_M^p{}^2(Q^2) \right)$$

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \tau = \frac{Q^2}{4M_p^2} \quad \varepsilon = \left[ 1 + 2(1+\tau) \tan^2 \frac{\theta}{2} \right]^{-1}$$



- Structureless proton:

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{\alpha^2 [1 - \beta^2 \sin^2 \frac{\theta}{2}]}{4k^2 \sin^4 \frac{\theta}{2}}$$

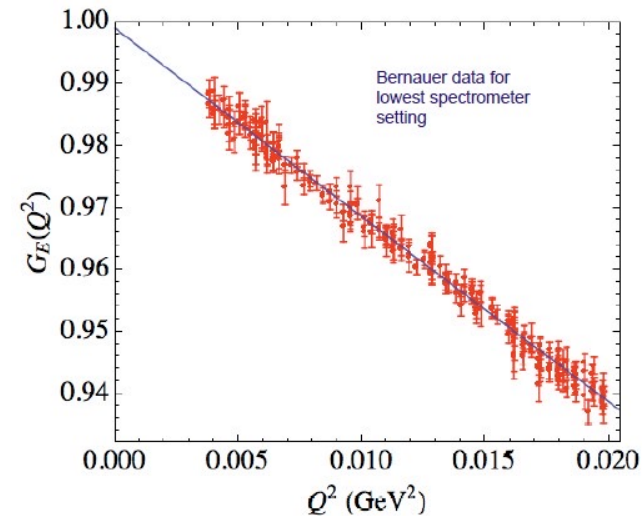
- $G_E$  and  $G_M$  can be extracted using Rosenbluth separation
- for extremely low  $Q^2$ , the cross section is dominated by  $G_E$
- Taylor expansion of  $G_E$  at low  $Q^2$

$$G_E^p(Q^2) = 1 - \frac{Q^2}{6} \langle r^2 \rangle + \frac{Q^4}{120} \langle r^4 \rangle + \dots$$

definition of the proton rms charge radius  $\rightarrow$

derivative at  $Q^2 = 0$ :

$$\langle r^2 \rangle = -6 \left. \frac{dG_E^p(Q^2)}{dQ^2} \right|_{Q^2=0}$$



Mainz low  $Q^2$  data set  
Phys. Rev. C 93, 065207, 2016

# First Measurement of the Proton Radius

- Robert Hofstadter, experiments in 1955-1956
  - ✓ ep-elastic scattering
  - ✓  $E_e = 188 \text{ MeV}$  electron beam
  - ✓ at Stanford University
- Nobel prize in 1961:
 

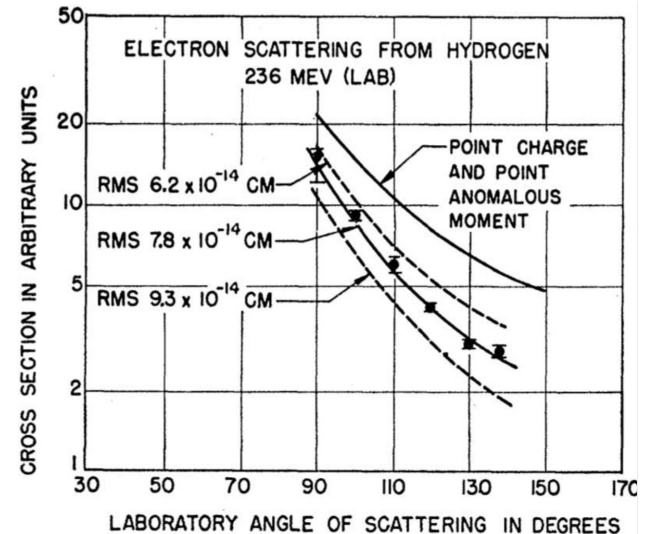
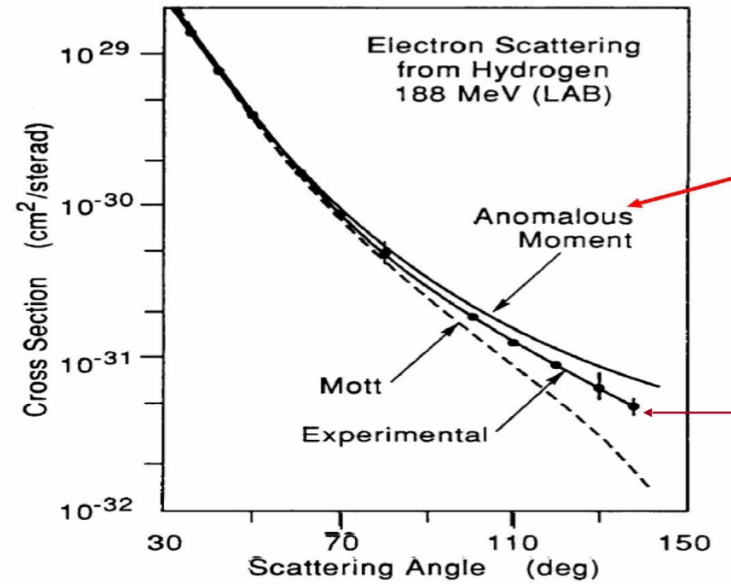
*“for his pioneering studies of electron scattering in atomic nuclei and for his consequent discoveries concerning the structure of nucleons”*

*“proton has a diameter of  $0.74 \mp 0.24 \times 10^{-13} \text{ cm}$ ”*

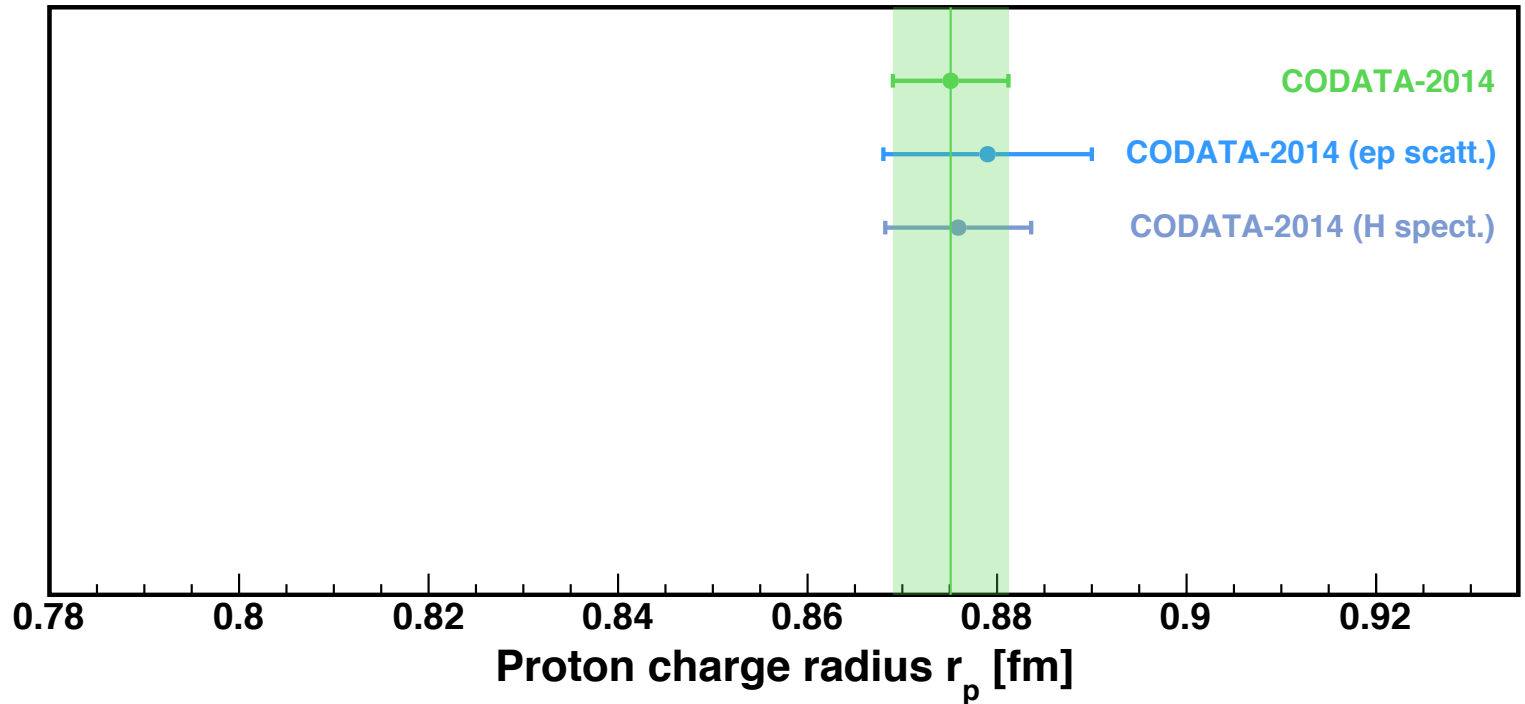
$r_p = 0.74 \text{ fm}$  with a 32% uncertainty

Hofstadter, McAllister, Phys. Rev. 98, 217 (1955).  
 Hofstadter, McAllister, Phys. Rev. 102, 851 (1956)

- Over 50 years of experimentation!
  - ✓ started from  $0.74 \text{ fm}$
  - ✓ ended to  $0.895 \text{ fm}$  by 2010.
  - ✓ where we are now ???



# Proton Radius before the Puzzle (2010)



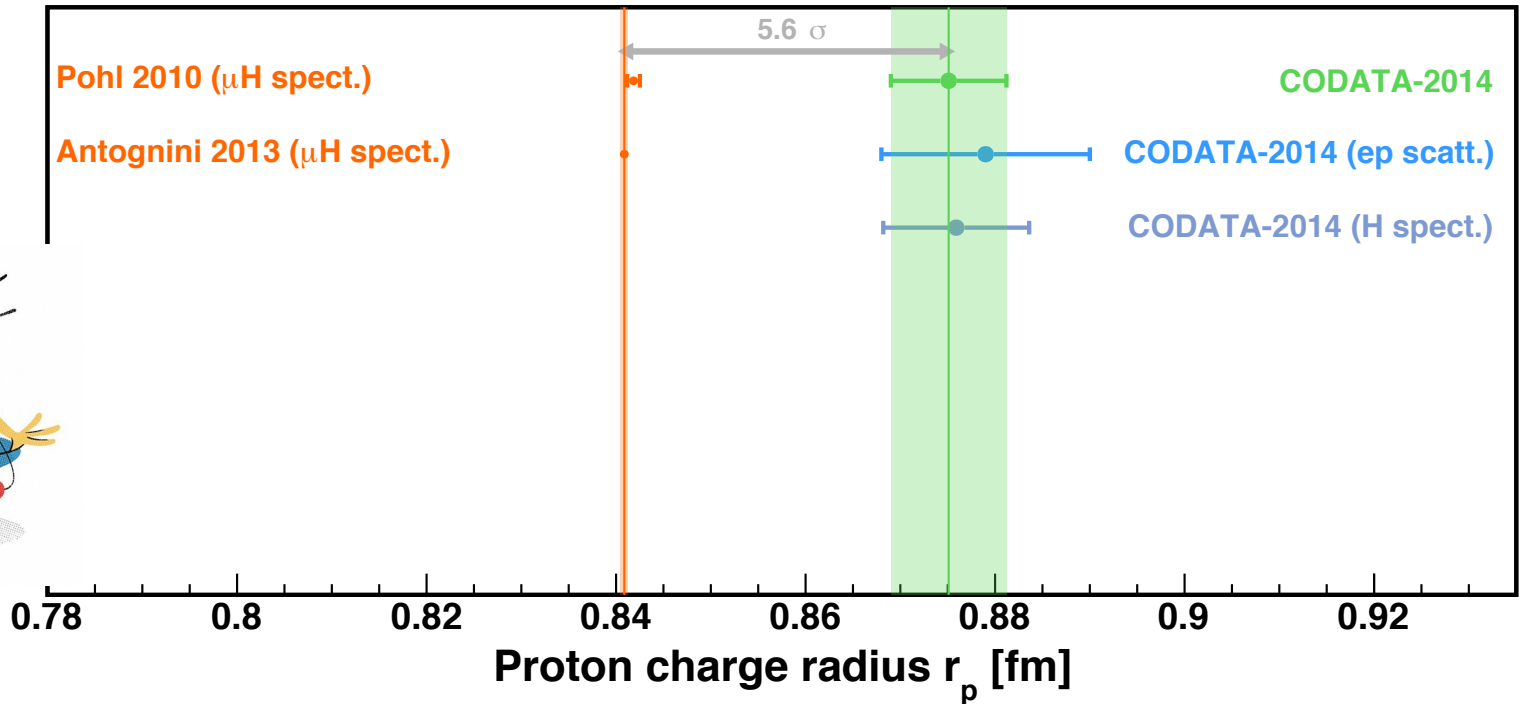
CODATA average:  $0.8751 \pm 0.0061$  fm  
ep-scattering average (CODATA):  $0.879 \pm 0.011$  fm  
Regular H-spectroscopy average (CODATA):  $0.859 \pm 0.0077$  fm

Very good agreement between ep-scattering and H-spectroscopy results !

# The Proton Radius Puzzle before the PRad Experiment (2016)



New York Times



Regular hydrogen average (CODATA):

$0.8751 \pm 0.0061$  fm

Muonic hydrogen (CREMA coll. 2013):

$0.8409 \pm 0.0004$  fm

Muonic hydrogen (CREMA coll. 2010):

$0.84184 \pm 0.00067$  fm

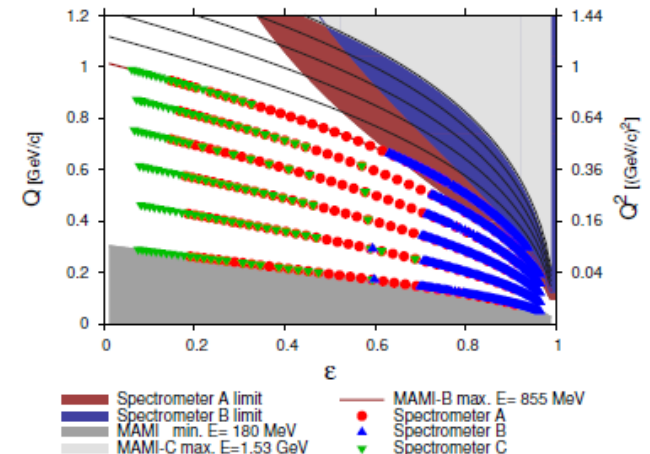
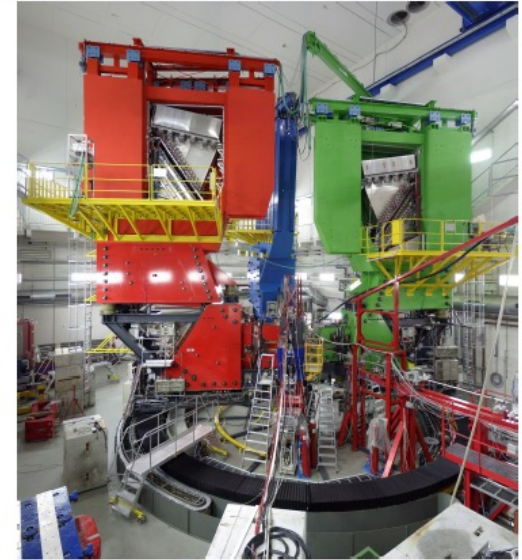
# Possible Resolutions to the *Proton Radius Puzzle*

- Some initial open questions about QED calculations:
  - ❖ additional corrections to muonic-hydrogen. Not found
  - ❖ missing contributions to electronic-hydrogen. Not found
  - ❖ higher moments in electric form factor; Not significant
  - ❖ ...
- Is the ep-interaction the same as  $\mu p$ -interaction (the **lepton universality principle**)?
- New Physics (forces) beyond the Standard Model? Not found yet
  - ✓ many models, discussions, suggestions ...
- Potential solutions:
  - ❖ need new high precision, **high accuracy** experiments:
    - ✓ ep-scattering experiments:
      - reaching extremely low  $Q^2$  range ( $10^{-4}$  GeV/c<sup>2</sup>)
      - possibly with new independent methods PRad at JLab
      - measure absolute cross sections in **ONE experimental setting!**
      - MUSE at PSI, ISR at Mainz, ULQ<sup>2</sup> in Japan, AMBER at CERN ...
    - ✓ ordinary hydrogen spectroscopy experiments:
      - York University in Canada, LKB in Paris, France, CREMA in Germany ...

# Planning a New $ep \rightarrow ep$ Scattering Experiment

- Practically all  $ep$ -scattering experiments were performed with **magnetic spectrometers and  $LH_2$  targets!**
  - ✓ high resolutions but, **very SMALL angular and momentum acceptances:**
    - need many different settings of angle ( $\Theta_e$ ), energies ( $E_e, E'_e$ ) to cover a **reasonable  $Q^2$  fitting interval**
    - normalization of each  $Q^2$  bin
    - their systematic uncertainties
  - ✓ limitation on minimum  $Q^2$ :  **$10^{-3} \text{ GeV}^2/C^2$** 
    - min. scattering angle:  $\theta_e \approx 5^\circ$
    - typical beam energies ( $E_e \sim 1 \text{ GeV}$ )
  - ✓ limits on accuracy of cross sections ( $d\sigma/d\Omega$ ):  **$\sim 2 \div 3\%$** 
    - statistics is not a problem ( $< 0.2\%$ )
    - **control of systematic uncertainties???**
      - ❖ beam flux, target thickness, **windows**,
      - ❖ **acceptances**, detection efficiencies,
      - ❖ ...

Three spectrometer facility of the A1 collaboration:





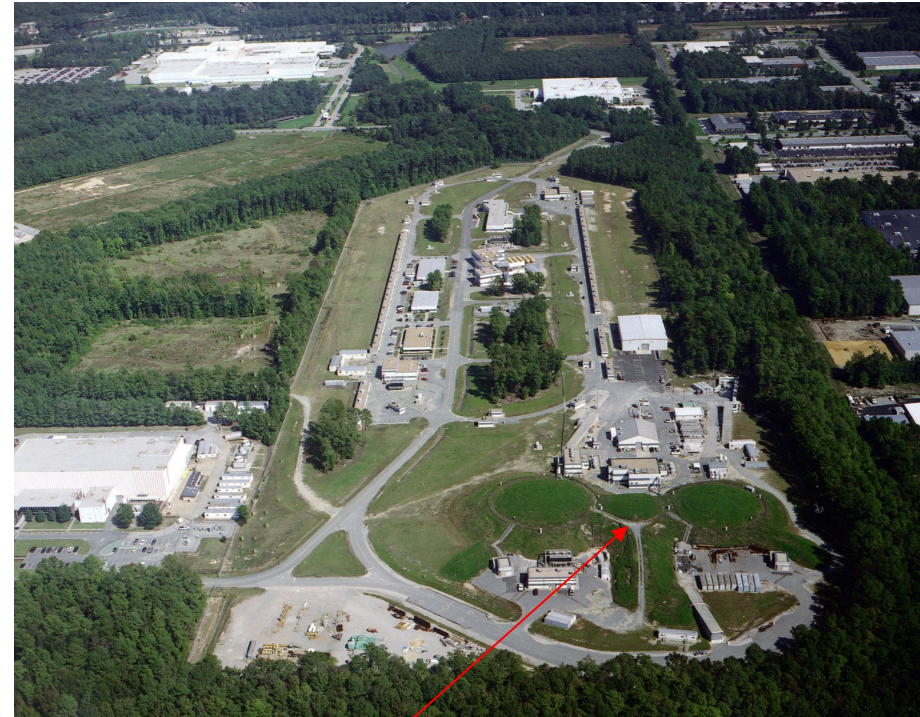
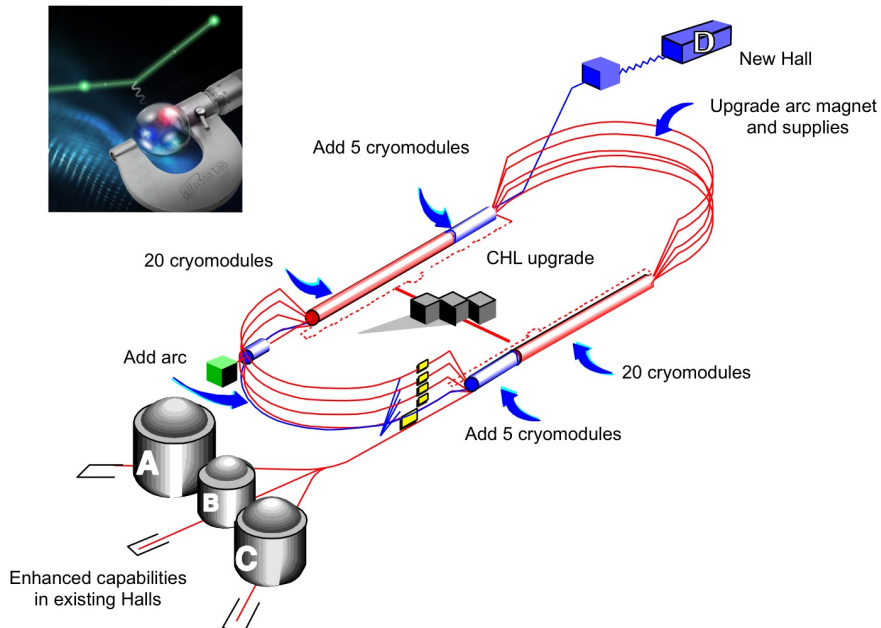
# A Possible Solution: the PRad Experimental Approach

- Use large acceptance, high resolution electromagnetic calorimeter (together with a GEM coordinate detector):
  - ✓ measure all angles in one experimental setting ( $\vartheta_e = 0.6^\circ - 7.0^\circ$ )  
( $Q^2 = 2 \times 10^{-4} \div 6 \times 10^{-2}$ ) GeV/c<sup>2</sup>;
  - ✓ access to smaller angles ( $\vartheta_e \approx 0.6^\circ$ )
  - ✓ calibrate with a well-known QED process: azimuthal symmetry of the calorimeter, simultaneous detection of ee  $\rightarrow$  ee Moller scattering (best known control of systematics).
- Use windowless H<sub>2</sub> gas flow target:
  - ✓ minimize experimental background.
- Use two beam energies only: E<sub>0</sub> = 1.1 GeV and 2.2 GeV to check the consistency of experimental data.

# PRad Experiment Timeline

- ✓ Initial proposal development: 2011-12
- ✓ Approved by JLab PAC39: 2012
- ✓ Funding proposal for windowless H<sub>2</sub> gas flow target (NSF MRI #PHY-1229153) 2012
- ✓ Development, construction of the target: 2012 – 15
- ✓ Funding proposals for the GEM detectors: (DOE awards) 2013
- ✓ Development, construction of the GEM detectors: 2013-15
- ✓ Beam line installation, commissioning, [data taking](#) in Hall B at JLab: January /June 2016
- ✓ [Data analysis](#) 2016 – 2019
- ✓ Publication in [Nature journal](#) November, 2019

# PRad Experiment Performed in Hall B at Jefferson Lab



PRad was performed in Hall B at JLab  
in January – June of 2016

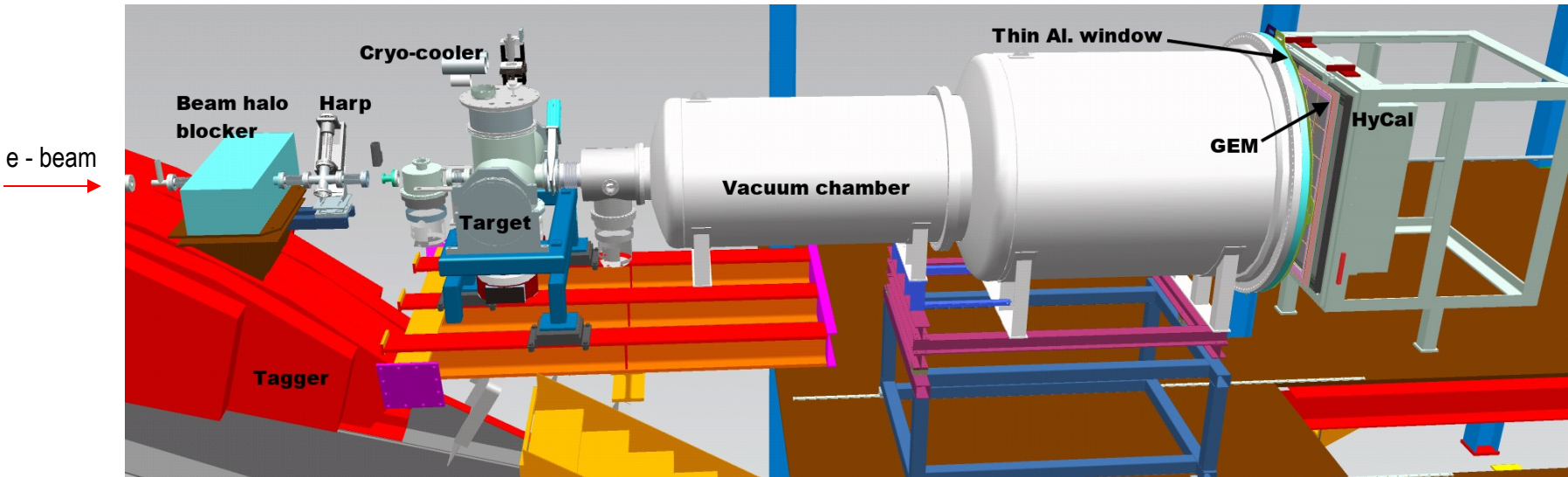
# PRad Experimental Setup in Hall B at JLab (schematics)

## ■ Main detector elements:

- windowless H<sub>2</sub> gas flow target
- PrimEx HyCal calorimeter
- vacuum box with one thin window at HyCal end
- X,Y – GEM detectors on front of HyCal

## ■ Beam line equipment:

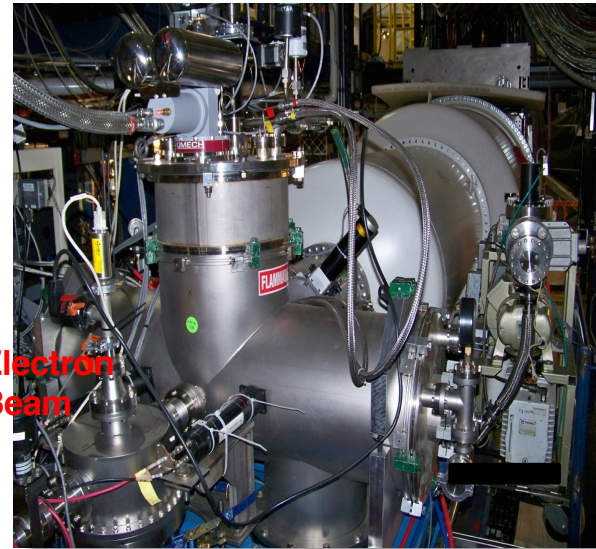
- standard beam line elements (0.1 – 50 nA)
- photon tagger for HyCal calibration
- collimator box (6.4 mm collimator for photon beam, 12.7 mm for e<sup>-</sup> beam halo “cleanup”)
- Harp 2H00 I



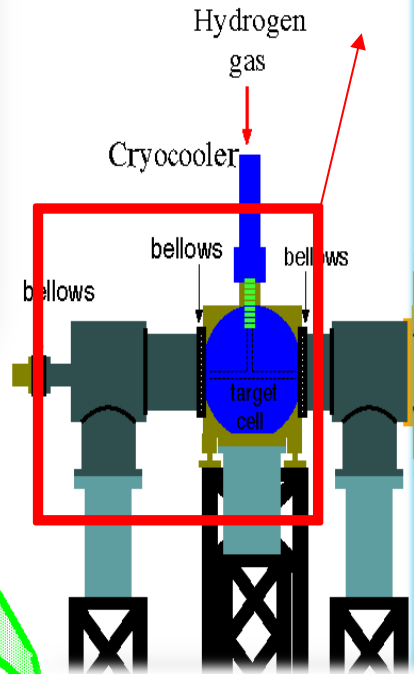
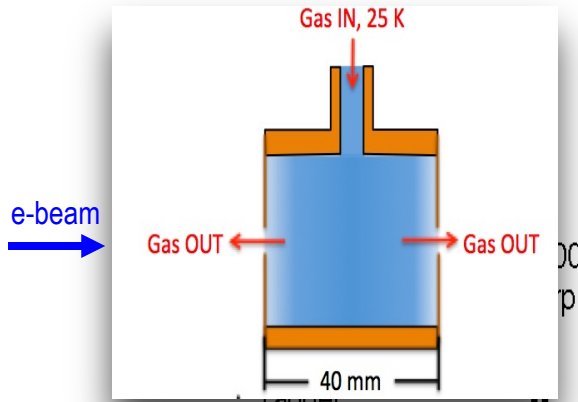


# Windowless Hydrogen Gas Flow Target

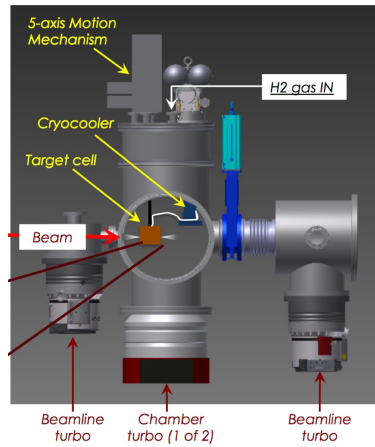
PRad Setup (Side View)



Electron Beam



Electron beam



- 8 cm diam. X 4 cm long target cell
- 2 mm holes open at front and back of kapton foils for the beam passage
- areal density:  $1.8 \times 10^{18}$  H atoms/cm<sup>2</sup>
  - cell pressure: 471 mTorr
  - chamber pressure: 2.34 mTorr: cell vs. chamber pressures: 200:1
  - vacuum tank pressure 0.3 mTorr: cell vs. vacuum tank pressures: 1000:1
  - at temperature: 19.5 K

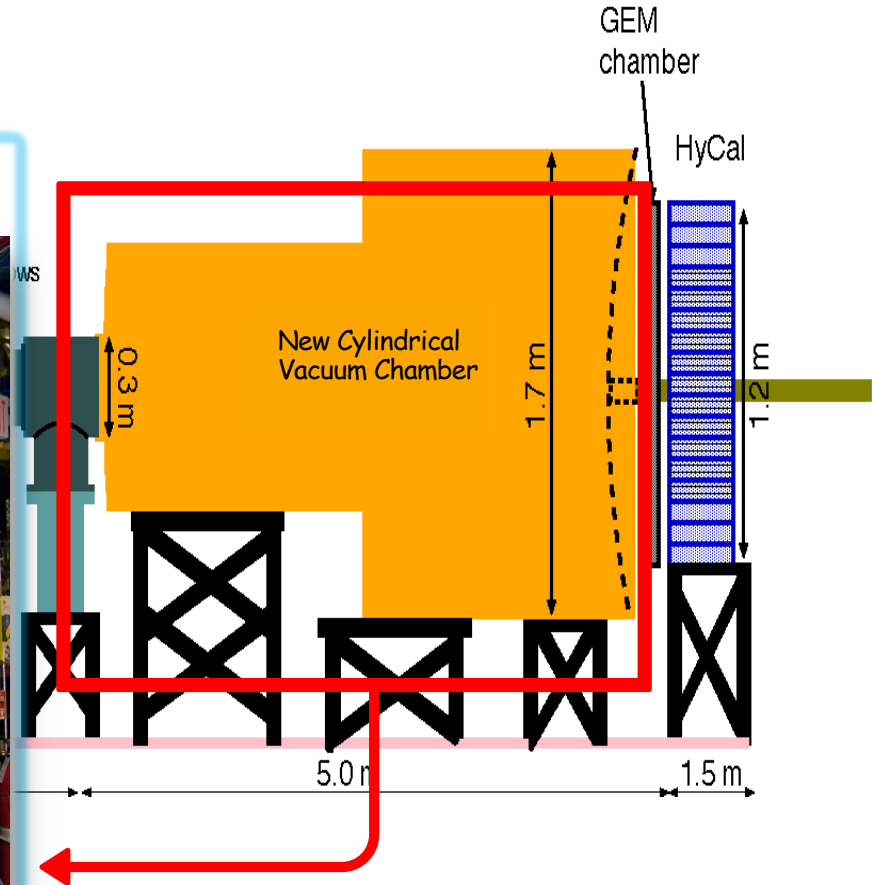
# PRad Experimental Apparatus: Vacuum Chamber

PRad Setup (Side View)

Hydrogen gas

GEM chamber

HyCal



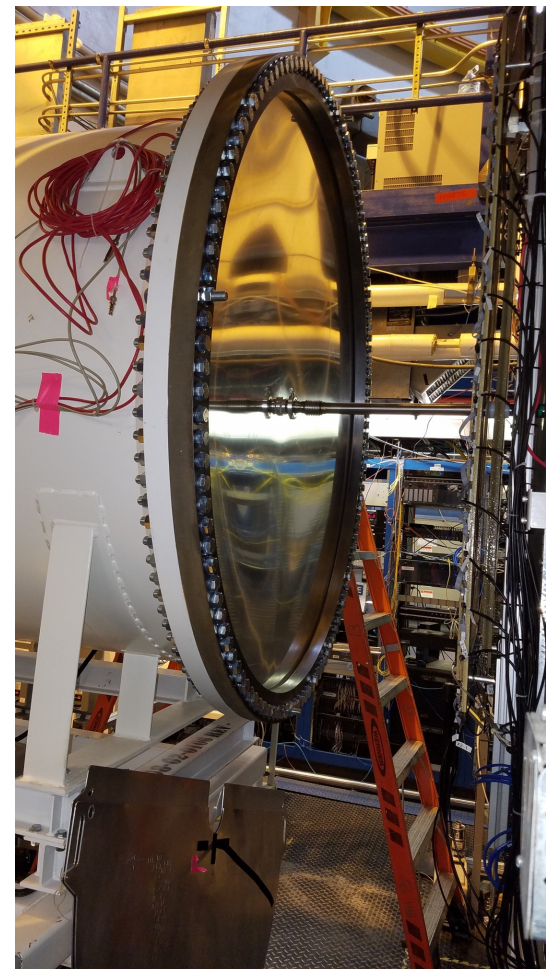
- 5 m long two stages vacuum chamber, 1.7 m diameter, 2 mm Al vacuum window vacuum chamber pressure: 0.3 mTorr



# PRad Experimental Apparatus: Vacuum Chamber and Window



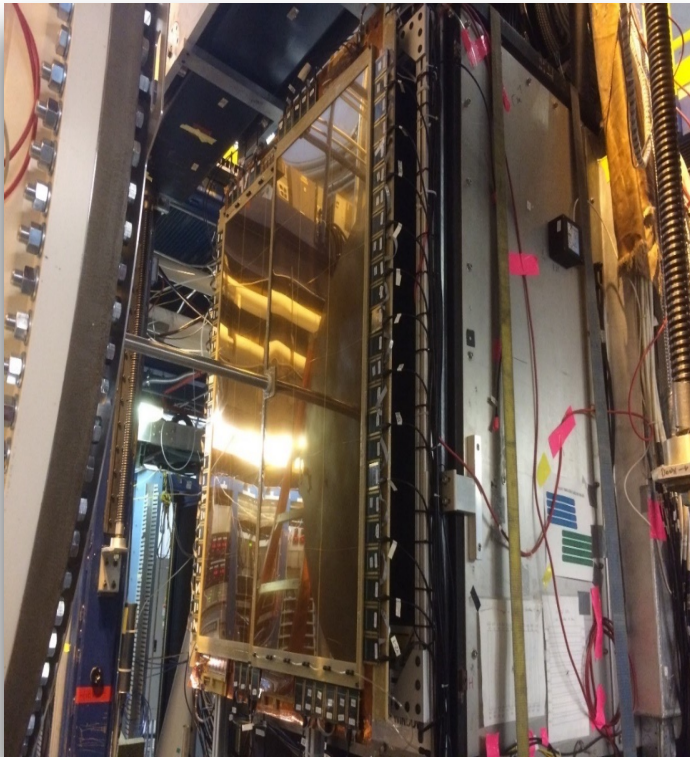
2-stage vacuum box in Hall B beam line



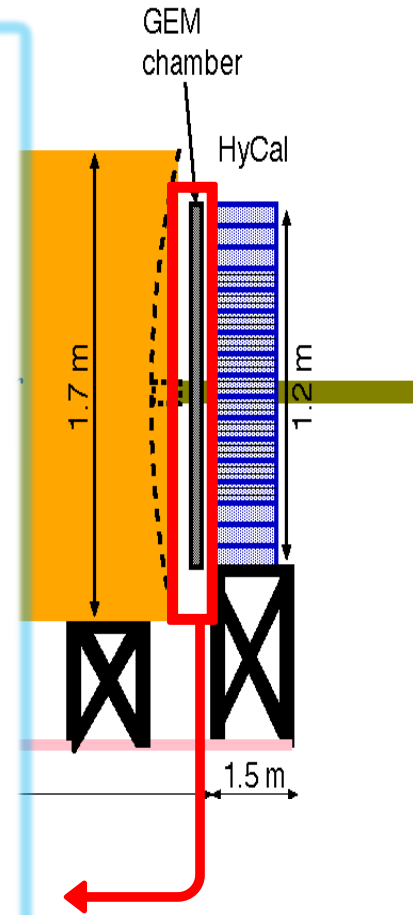
1.7 m diameter, 2 mm Al vacuum window

# PRad Experimental Apparatus: GEM Coordinate Detectors

## PRad Setup (Side View)



- Two large area GEM detectors
- Small overlap region in the middle
- Excellent position resolution ( $72 \mu\text{m}$ )
- Improve position resolution of the setup by  $> 20$  times
- Large improvements in  $Q^2$  determination

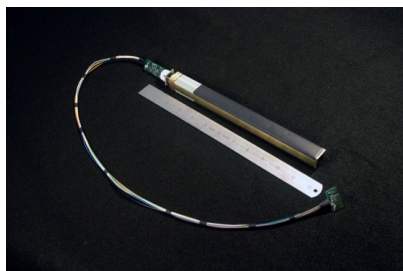
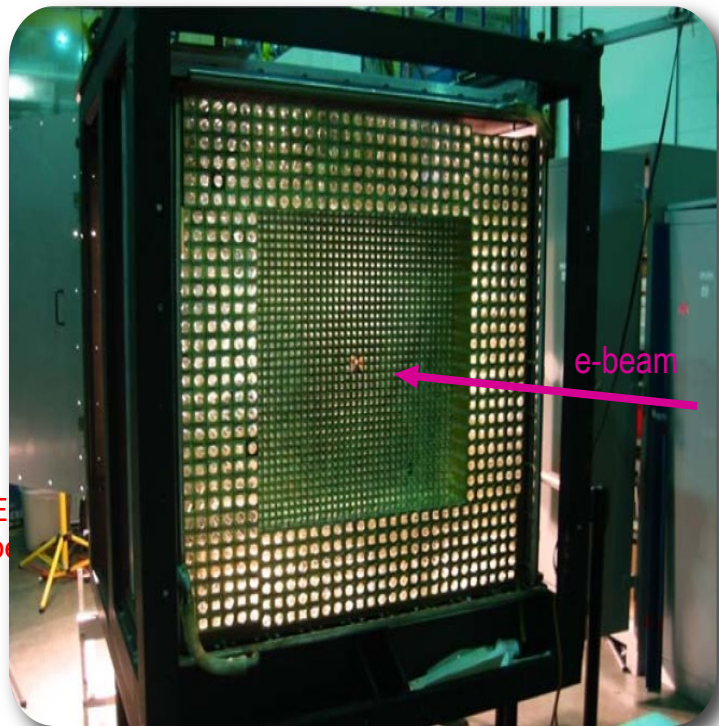
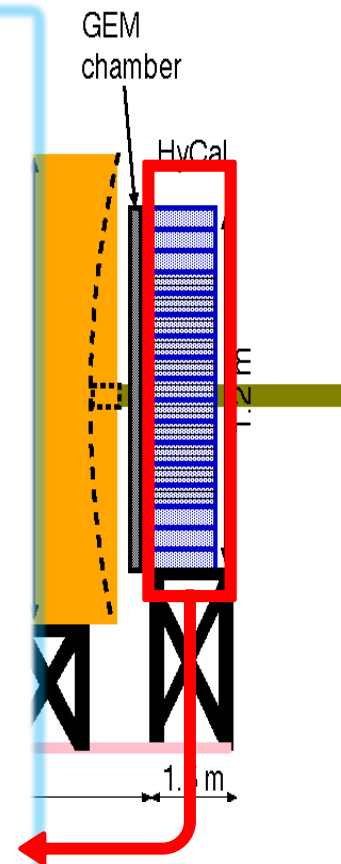




# PRad Experimental Apparatus: HyCal El. Mag. Calorimeter

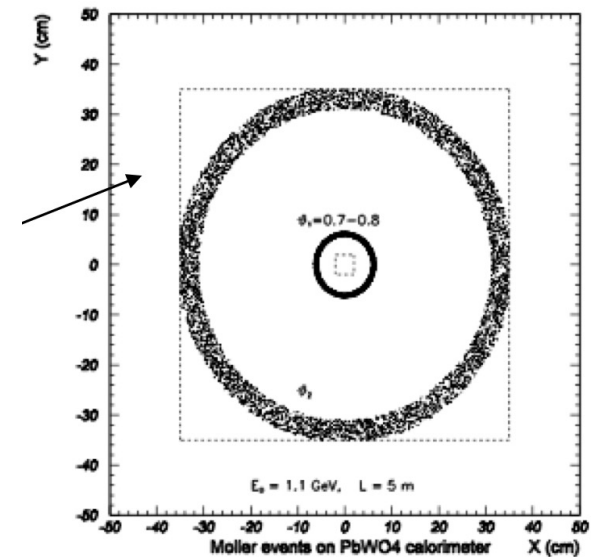
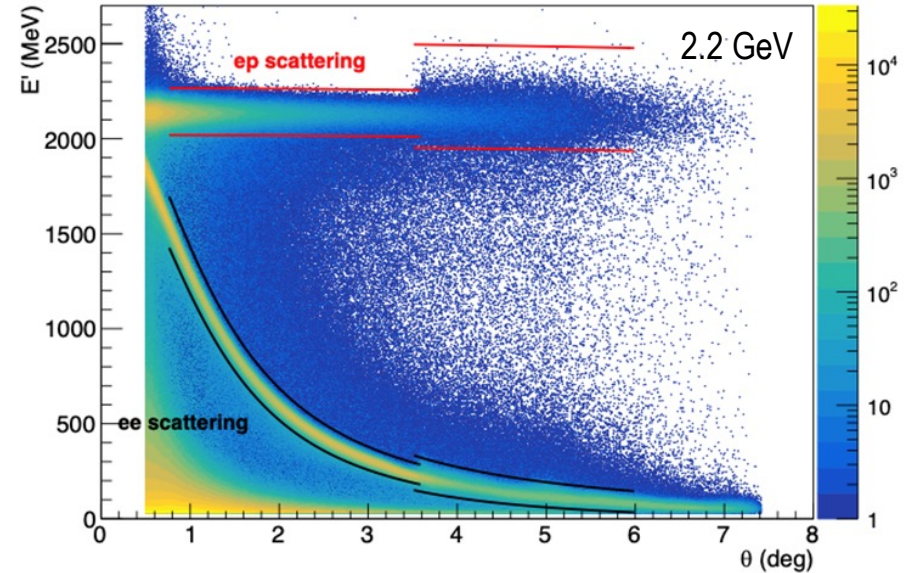
## PRad Setup (Side View)

- hybrid EM calorimeter (HyCal)
  - ✓ inner 1156 PbWO<sub>4</sub> modules.
  - ✓ outer 576 lead glass modules.
- 5.8 m from the target.
- scattering angle coverage: ~ 0.6° to 7.5°
- full azimuthal angle coverage
- high resolution and efficiency
  - ✓ 2.5% at 1 GeV for crystal part
  - ✓ 6.1% at 1 GeV for lead glass part
- energy calibration done with tagged photons



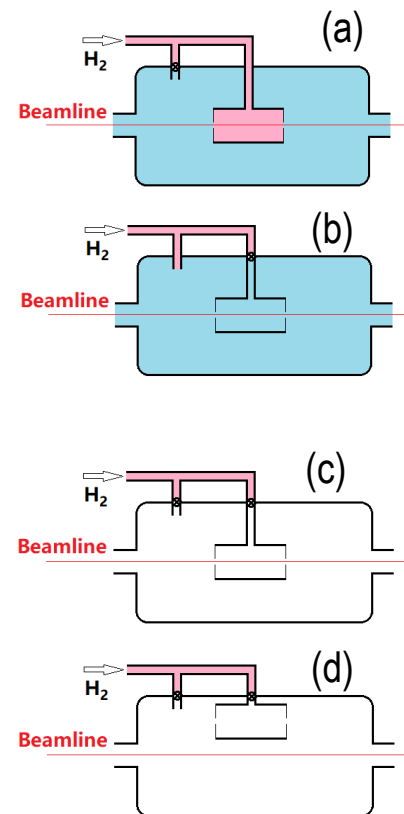
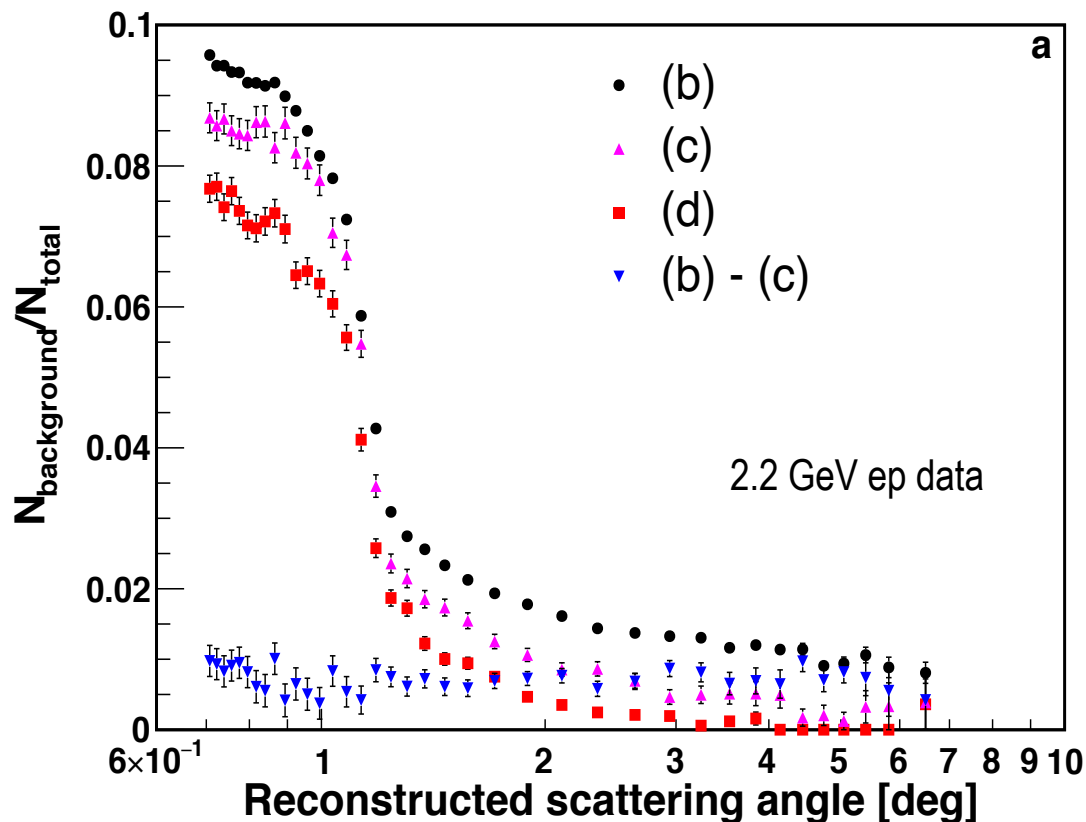
# Data Analysis: Event Selection

- Experimental data was taken with two beam energies:
  - ✓ 1.1 GeV (604 M events)
  - ✓ 2.2 GeV (756 M events)
- For all events, require hit matching between GEMs and HyCal
- For *ep* and *ee* events, apply angle dependent energy cut based on kinematics:
  - cut size depend on local detector resolution
- For *ee*, if requiring double-arm events, apply additional cuts:
  - ✓ elasticity
  - ✓ co-planarity
  - ✓ vertex z (kinematics)



# Data Analysis: Empty Target Runs for Background Subtraction

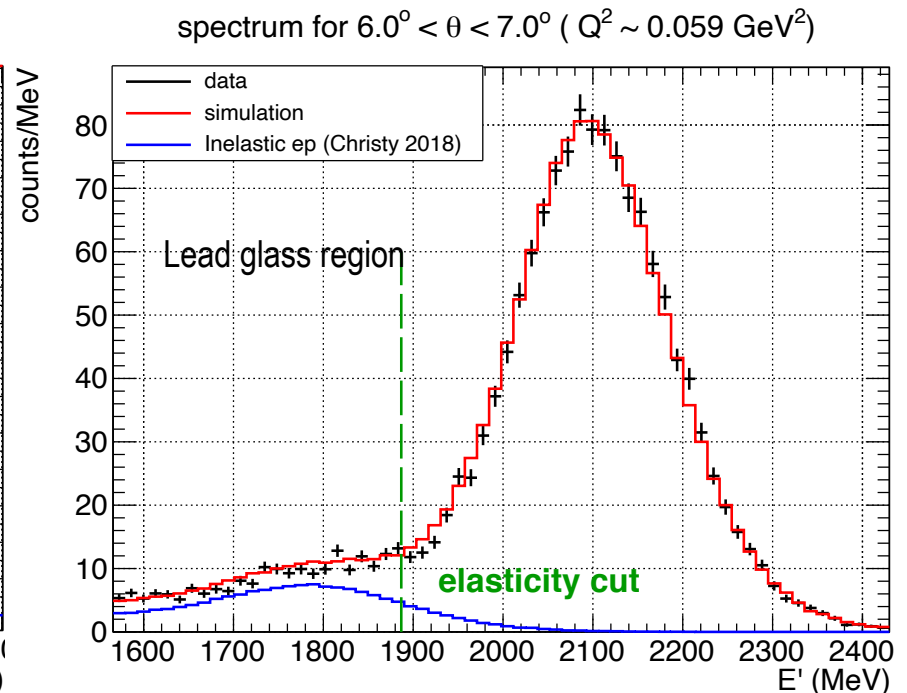
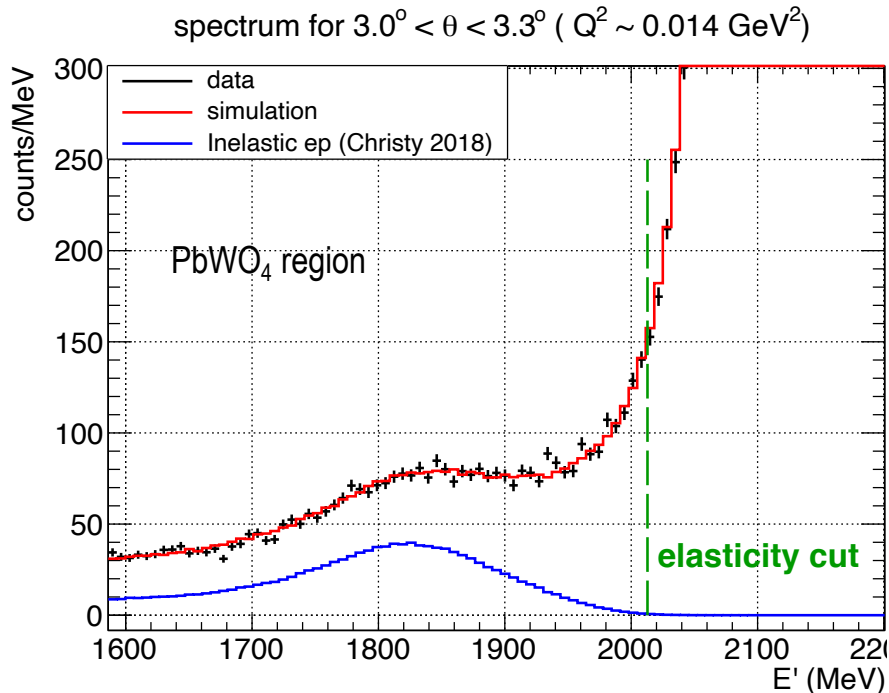
- ep background rate  $\sim 10\%$  at forward angle ( $< 1.1$  deg, dominated by upstream beam halo blocker), less than 2% otherwise
- ee background rate  $\sim 0.8\%$  at all angles



- ✓ Residual hydrogen gas: hydrogen gas filled during background runs

# Data Analysis: *ep*-inelastic Contribution

- Using Christy 2018 empirical fit\* to study inelastic *ep* contribution
- Good agreement between data and simulation
- Negligible for the  $\text{PbWO}_4$  region ( $<3.5^\circ$ )
- Less than 0.2%(2.0%) for 1.1GeV(2.2GeV) in the Lead glass region



\* M. E. Christy and P. E. Bosted, PRC 81, 055213 (2010)

# Extraction of the $ep \rightarrow ep$ Elastic Scattering Cross Section

- To reduce the systematic uncertainty, the **ep cross section** was normalized to the **Møller cross section**:

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left[ \frac{N_{\text{exp}}(ep \rightarrow ep \text{ in } \theta_i \pm \Delta\theta)}{N_{\text{exp}}(ee \rightarrow ee)} \cdot \frac{\epsilon_{\text{geom}}^{ee}}{\epsilon_{\text{geom}}^{ep}} \cdot \frac{\epsilon_{\text{det}}^{ee}}{\epsilon_{\text{det}}^{ep}} \right] \left(\frac{d\sigma}{d\Omega}\right)_{ee}$$

- method 1: **bin by bin method** – taking ep/ee counts from the same angle bin
  - method 2: **integrated Moller method** – integrate Møller in a fixed angle range and use it as common normalization for all angle bins
- ✓ **Luminosity cancelled from both methods**
- Radiative effects corrected by Monte Carlo method:

- ✓ GEANT4 based simulation package with full geometry setup
- ✓ event generators with complete calculations of radiative corrections<sup>1),2)</sup>
- ✓ iterative procedure applied for radiative corrections

$$\sigma_{ep}^{\text{Born}(exp)} = \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{\text{exp}} / \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{\text{sim}} \cdot \left(\frac{\sigma_{ep}}{\sigma_{ee}}\right)^{\text{Born}(model)} \cdot \sigma_{ee}^{\text{Born}(model)}$$

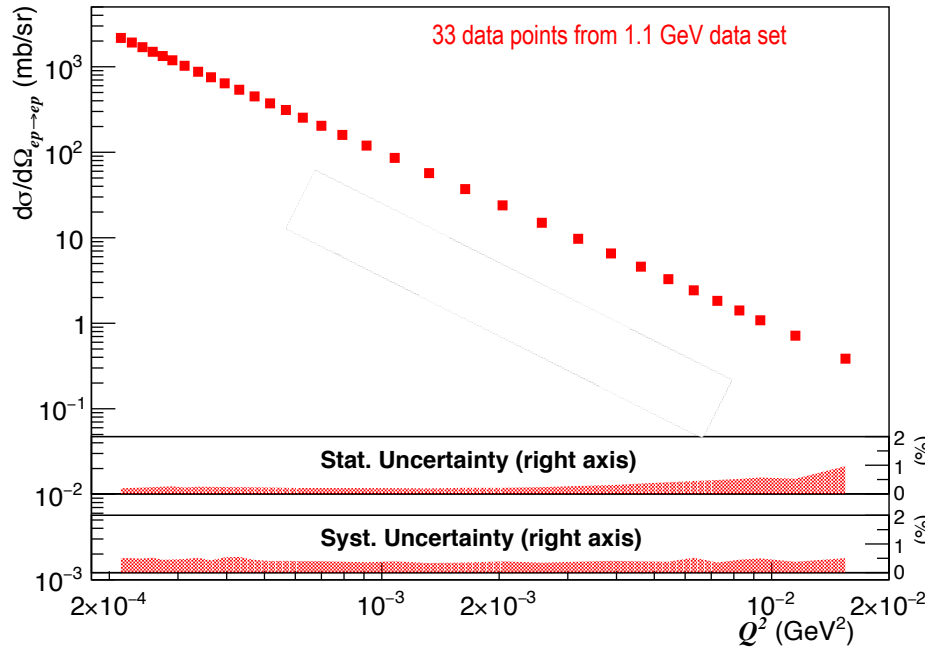
1) A. V. Gramolin et al., *J. Phys. G Nucl. Part. Phys.* 41(2014)115001;

2) I. Akushevich et al., *Eur. Phys. J. A* 51(2015)1 (fully beyond ultra relativistic approximation).

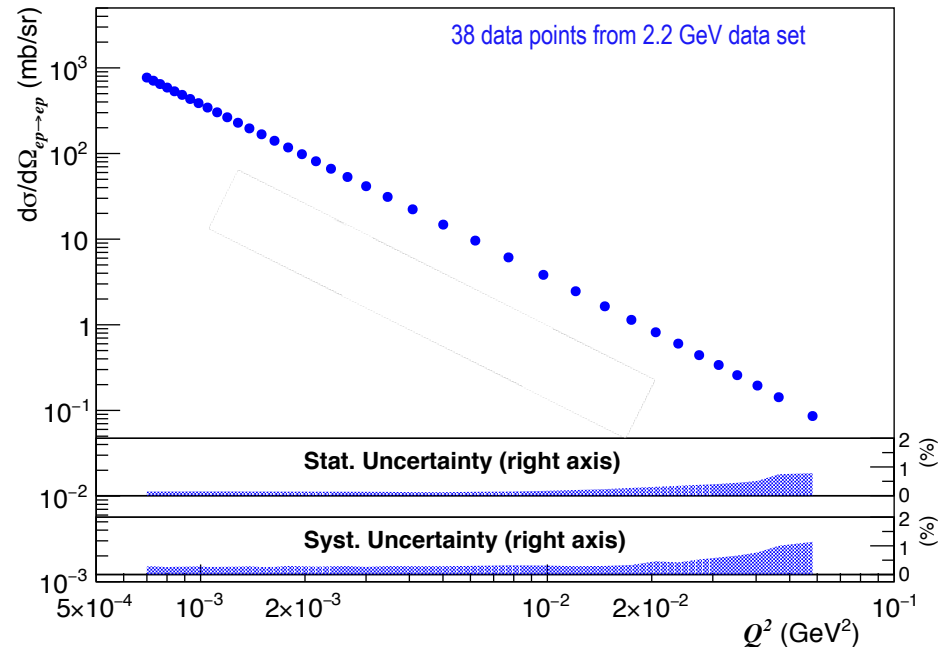
# Extracted $ep \rightarrow ep$ Elastic Differential Cross Sections

- Extracted differential cross sections vs.  $Q^2$ , with 1.1 and 2.2 GeV data.
- Statistical uncertainty:  $\sim 0.2\%$  for 1.1 GeV and  $\sim 0.15\%$  for 2.2 GeV per point.
- Systematic uncertainties:  $0.3\% - 0.5\%$  for 1.1 GeV and  $0.3 - 1.1\%$  for 2.2 GeV per point.

$ep$  elastic scattering cross section (1.1 GeV)



$ep$  elastic scattering cross section (2.2 GeV)



# Fit to Extract the Proton Radius

$n_1$  and  $n_2$  obtained by fitting PRad  $G_E$  to  $\begin{cases} n_1 f(Q^2), & \text{for 1 GeV data} \\ n_2 f(Q^2), & \text{for 2 GeV data} \end{cases}$

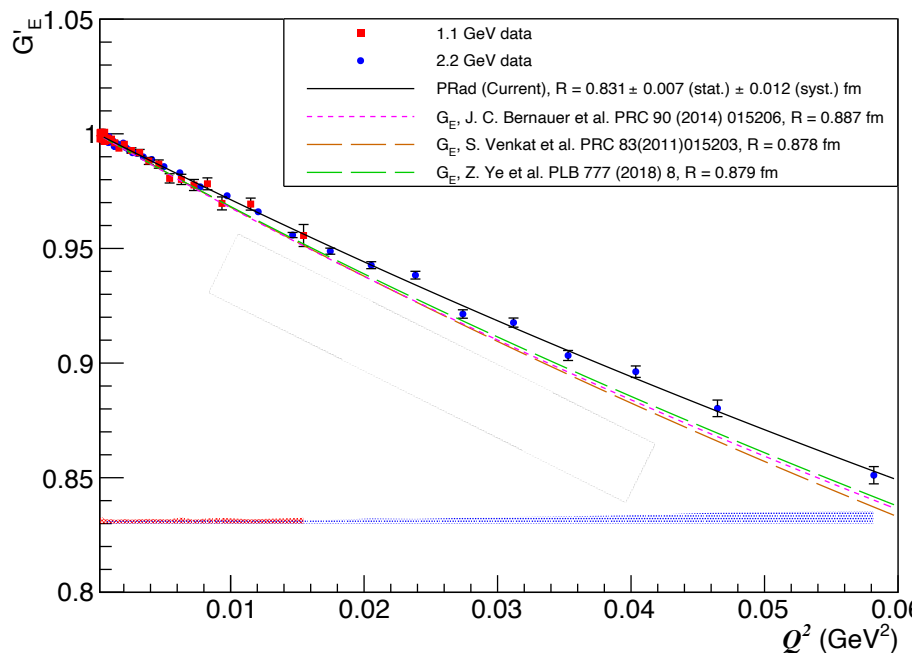
$G'_E$  as normalized electric Form factor:  $\begin{cases} G_E/n_1, & \text{for 1 GeV data} \\ G_E/n_2, & \text{for 2 GeV data} \end{cases}$

Using rational (1,1)

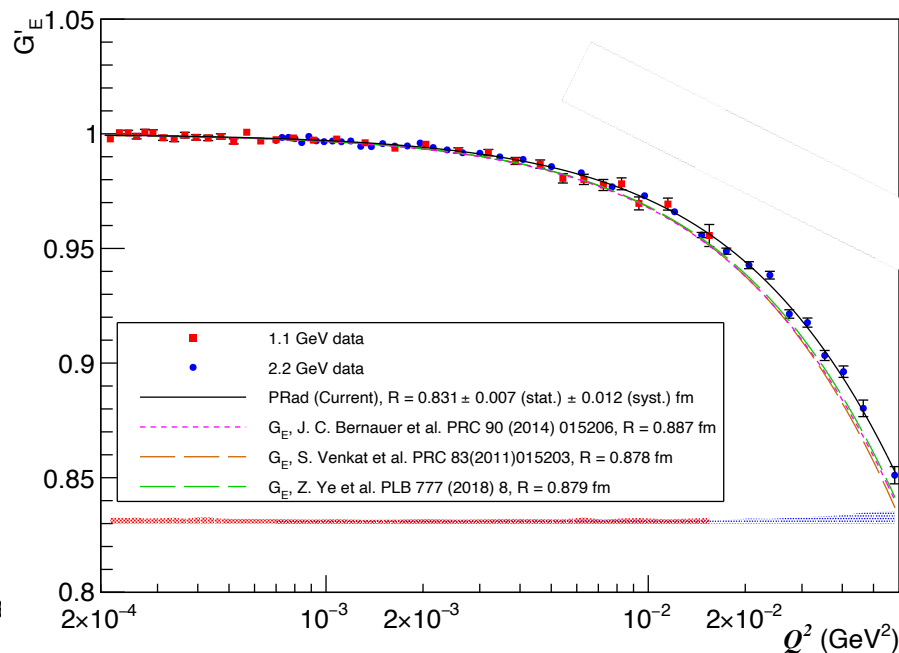
$$f(Q^2) = \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

PRad fit shown as  $f(Q^2)$   $r_p = 0.831 \pm 0.007$  (stat.)  $\pm 0.012$  (syst.) fm

Proton Electric Form Factor  $G'_E$



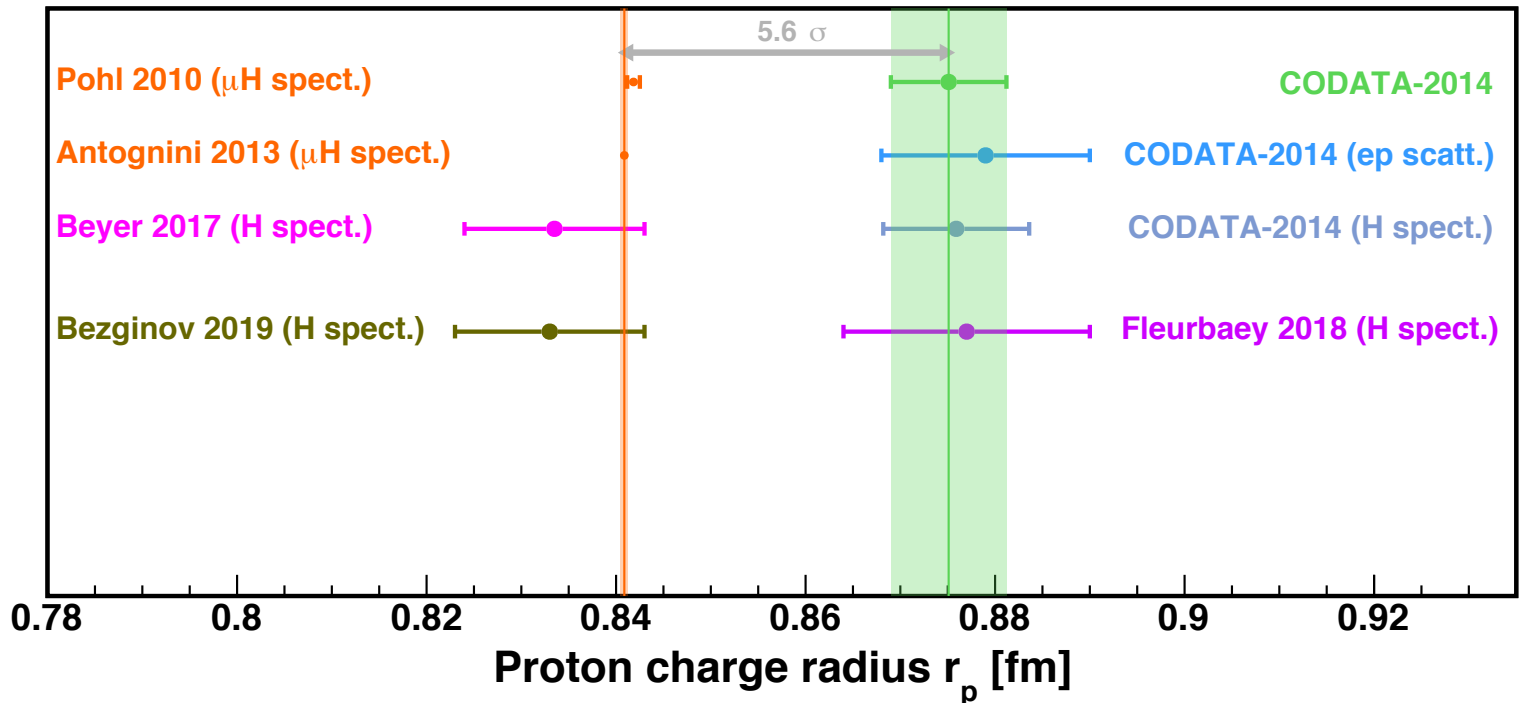
Proton Electric Form Factor  $G'_E$



**PRad final result:  $R_p = 0.831 \pm 0.007$  (stat.)  $\pm 0.012$  (syst.) fm**



# The Proton Radius Puzzle before the PRad Publication



Regular hydrogen average (CODATA):  $0.8751 \pm 0.0061$  fm

Muonic hydrogen (CREMA coll. 2013, PSI):  $0.8409 \pm 0.0004$  fm

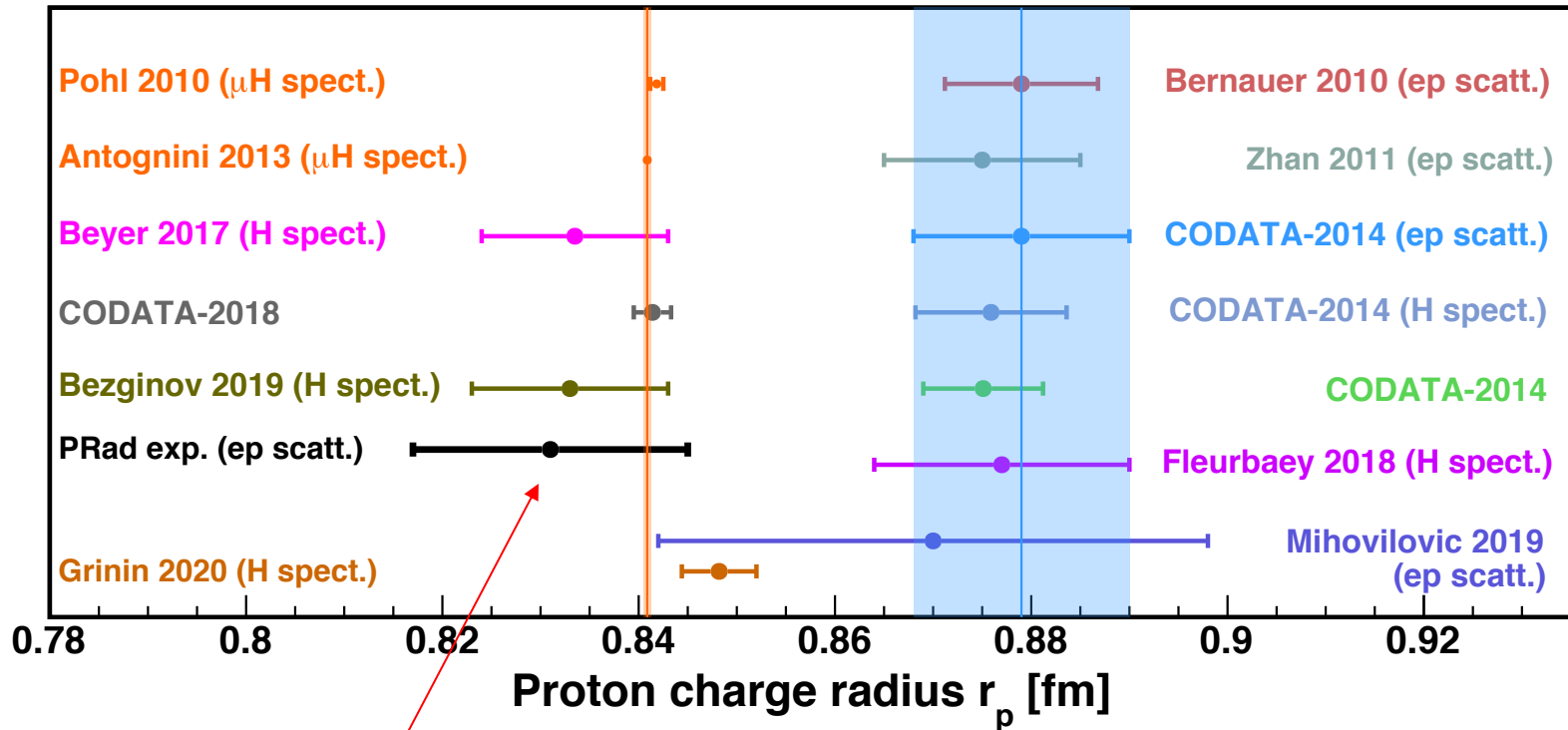
Regular H-spectr. ( $2S \rightarrow 4P$ , Garching, PSI):  $0.8335 \pm 0.0095$  fm

Regular H-spectr. ( $1S \rightarrow 3S$ , LKB, Paris):  $0.877 \pm 0.013$  fm

Regular H-spectr. ( $2S_{1/2} \rightarrow 2P_{1/2}$ , York Un. Canada):  $0.833 \pm 0.010$  fm



# The PRad Final Result on the Radius



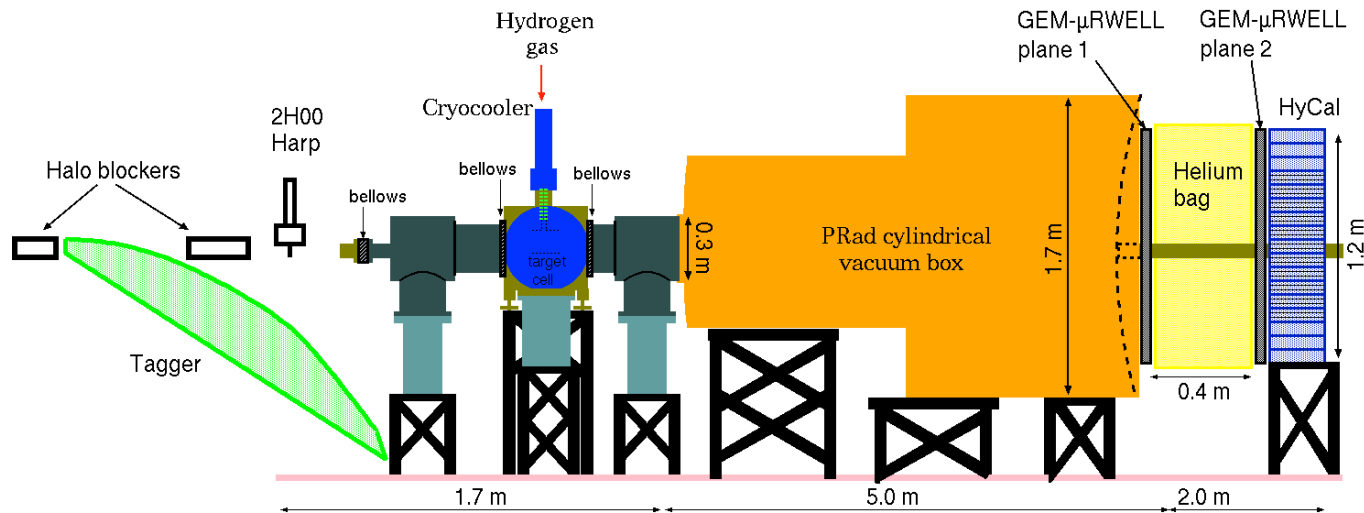
PRad final result:  $R_p = 0.831 \pm 0.007$  (stat.)  $\pm 0.012$  (syst.) fm

published in: Nature 575, 145–150 (2019)

# Planned New Experiment: PRad-II at JLab

- **PRad-II** is planning to improve the PRad accuracy by a factor of **3.8** (to  $\pm 0.43\%$  on rp) by:
  - Significantly improved statistics (4 times less uncertainties);
  - Hardware upgrades:
    - adding full tracking capability (second plane of GEM/ $\mu$ Rwell detectors).
    - small-size scintillator detectors just downstream the target to veto Moller electrons to reach the  **$10^{-5}$  GeV<sup>2</sup> Q<sup>2</sup> range**.
    - adding new “beam halo blocker” just before the Tagger.
    - upgrade DAQ/electronics to fADC based electronics:
    - possible **HyCal upgrade to all PbWO<sub>4</sub> crystals**, essential for the **ep-inelastic background** suppression at relatively higher Q<sup>2</sup> range ( $\approx 10^{-2}$  GeV<sup>2</sup>) and uniformity over full acceptance.

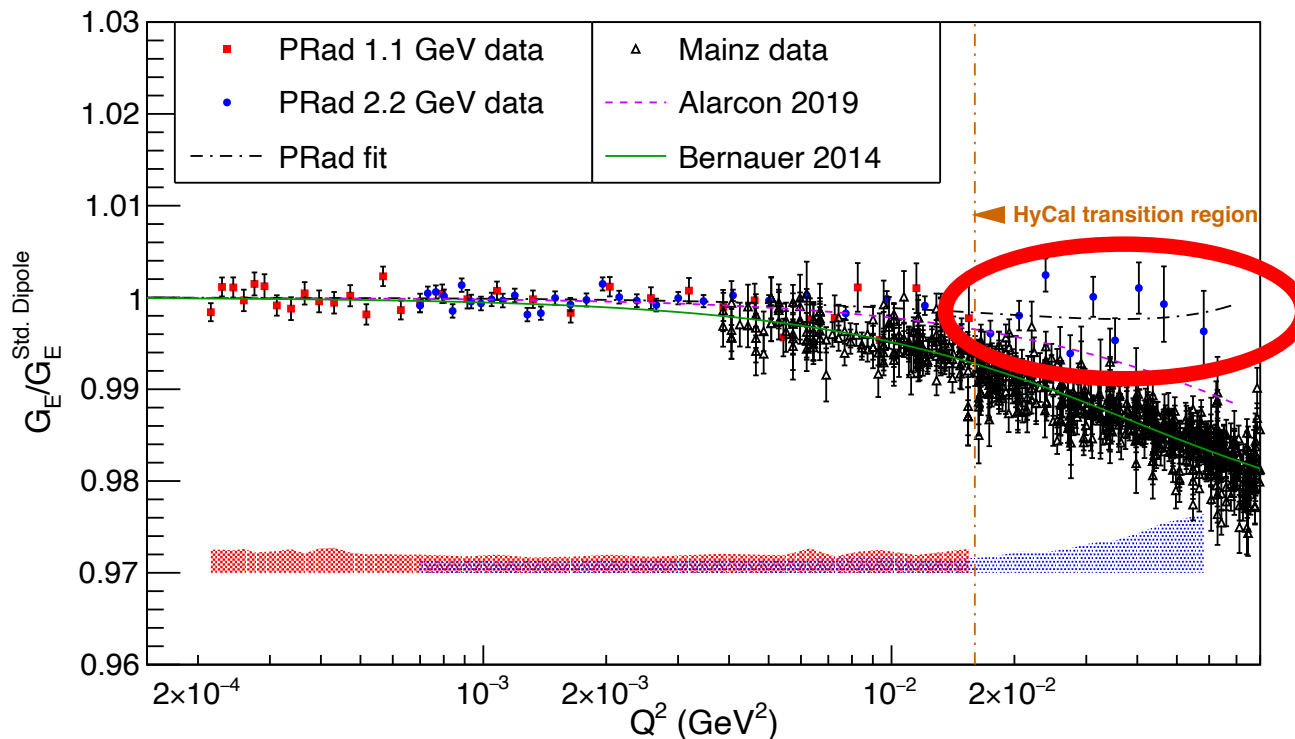
PRad-II Experimental Setup (Side View)



# PRad-II Experiment

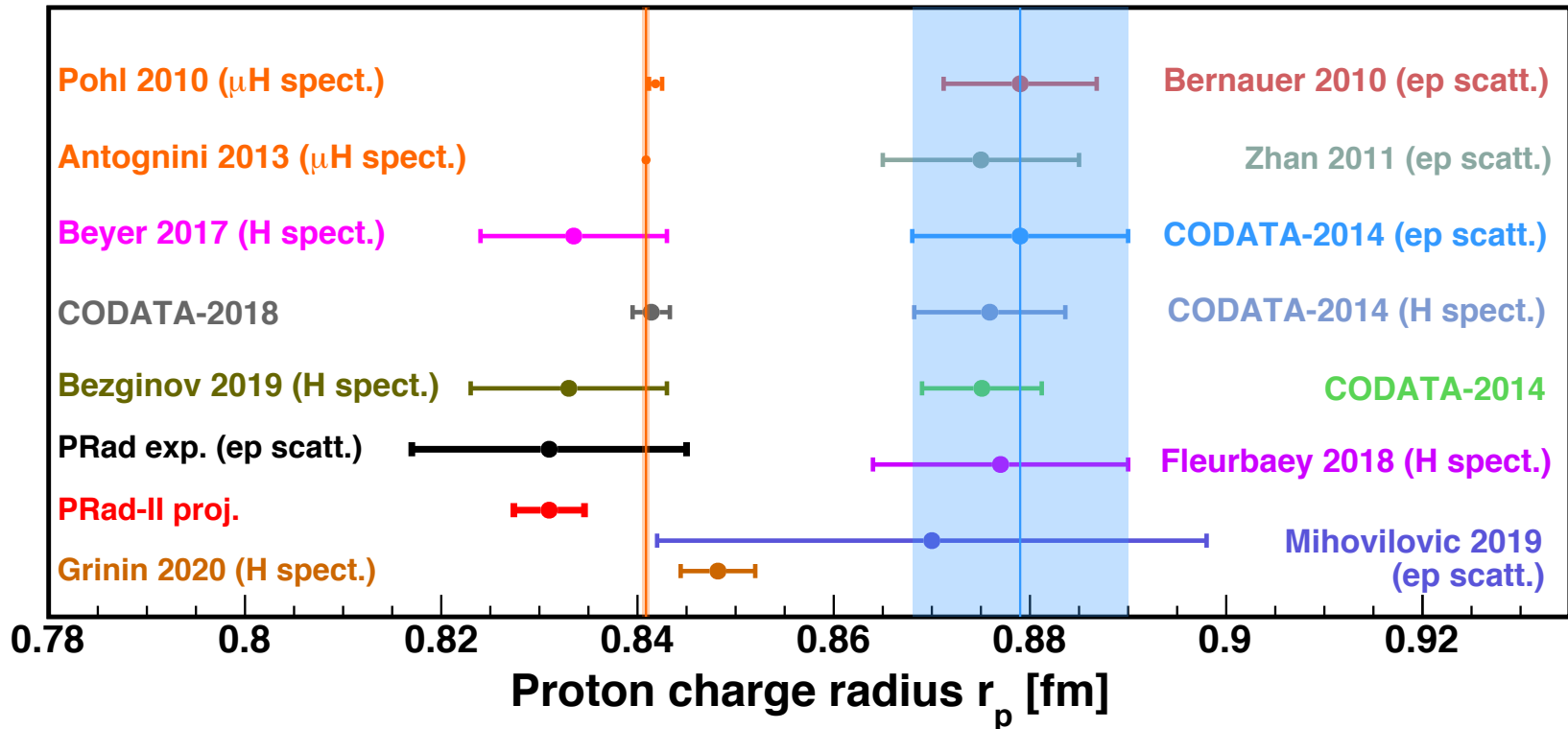
- The new PRad-III includes:
  1. Adding tracking capacity (second GEM plane)
  2. Upgraded HyCal with all high resolution  $\text{PbWO}_4$  modules
  3. Convert to FADC based readout for HyCal
  4. Four times smaller stat. uncertainty
  5. Better RC calculating including NNLO diagrams

Normalized Proton Electric Form Factor  $G_E$



# PRad-II: Projected Result

- Approved by JLab's PAC-48 in August, 2020
- Projected total uncertainty on radius: 0.43%



# PRad-II: Current Status

- Approved by JLab's PAC-48 in August, 2020, (E12-004)
  
- NSF funding proposal was submitted in January 2022:
  - ✓ Major Research Instrumentation (MRI) Track-2 development type proposal, ~\$4M total for:
    - HyCal partial upgrade to PbWO<sub>4</sub> crystals
    - second GEM detector
    - scintillator detectors to veto the Moller electrons at very small Q<sup>2</sup> range
  - ✓ Results are expected before July
  
- Also, for the calorimeter upgrade, we are looking for used crystal detectors from other experiments/institutions matching the PbWO<sub>4</sub> part of the HyCal

# Summary

- PRad was uniquely designed and performed in 2016 to address the “Puzzle”:
  - ✓ data in a large  $Q^2$  range have been recorded with the same experimental setting,  $[2 \times 10^{-4} \div 6 \times 10^{-2}] \text{ GeV}/C^2$ .
  - ✓ lowest  $Q^2$  data set ( $\sim 10^{-4} \text{ GeV}/C^2$ ) has been collected for the first time in ep-scattering experiments;
  - ✓ simultaneous measurement of the Moller and Mott scattering processes has been demonstrated to control systematic uncertainties.
  
- PRad final result supports small proton charge radius (Nature 575, 145–150 (2019)):
  - ✓  $R_p = 0.831 \pm 0.007 \text{ (stat.)} \pm 0.012 \text{ (syst.) fm}$  ( $\pm 1.67\%$  total)
  - ✓ significant input in changing the CODATA recommendation on radius.
  
- PRad-II will improve the radius measurement by a factor of 3.8
  - will address the differences between PRad and all modern ep-experiments;
  - will reach the  $Q^2 \sim 10^{-5} \text{ GeV}^2$  range, for the first time in ep-experiments
  - are there any possible systematic uncertainties in  $\mu\text{H}$  results?

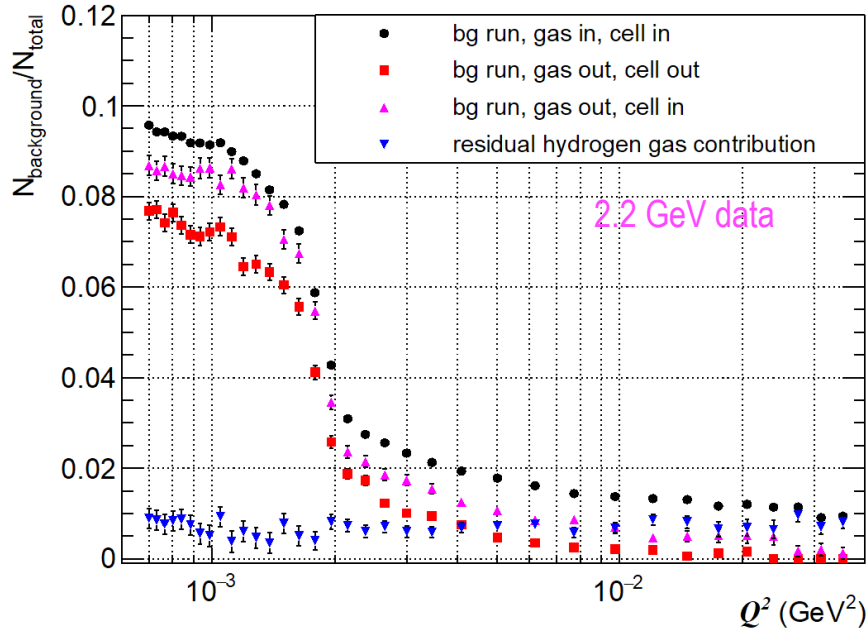
PRad was supported in part by NSF MRI #PHY-1229153 and DOE DE-FG02-03ER41231 awards.  
my research work is supported in part by NSF award: PHY-1812421

Thank you!

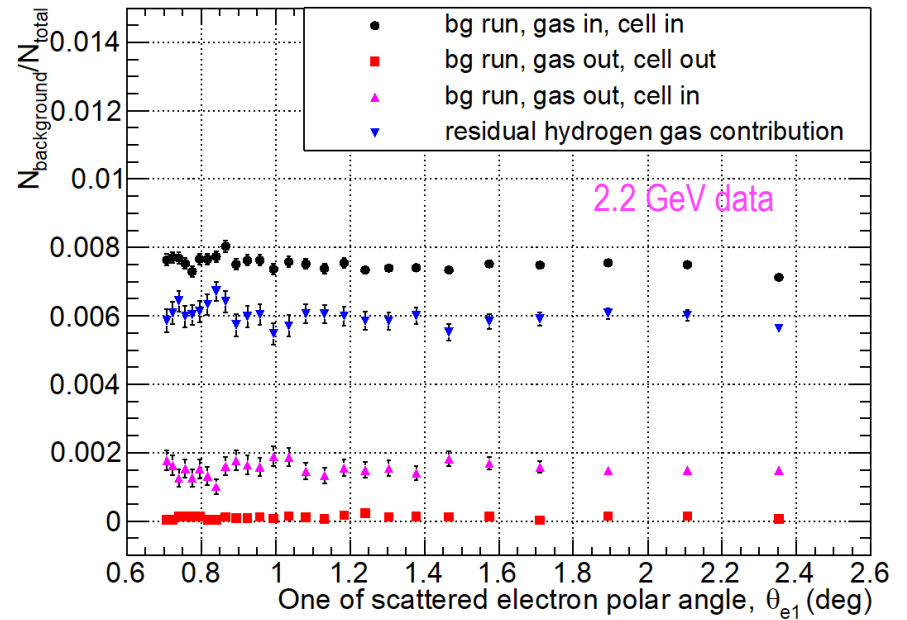
# Data Analysis: Beam Background Subtraction

- $ep$  background rate  $\sim 10\%$  at forward angles ( $<1.3^\circ$ , dominated by upstream “collimator”), less than 2% otherwise.
- $ee$  background rate  $\sim 0.8\%$  at all angles .

$ep$  Background Contribution



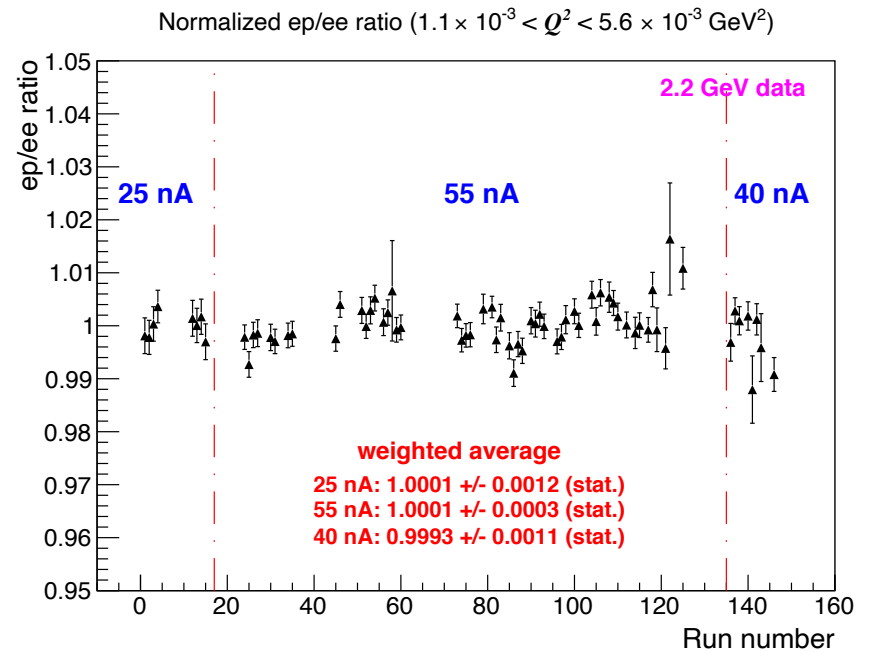
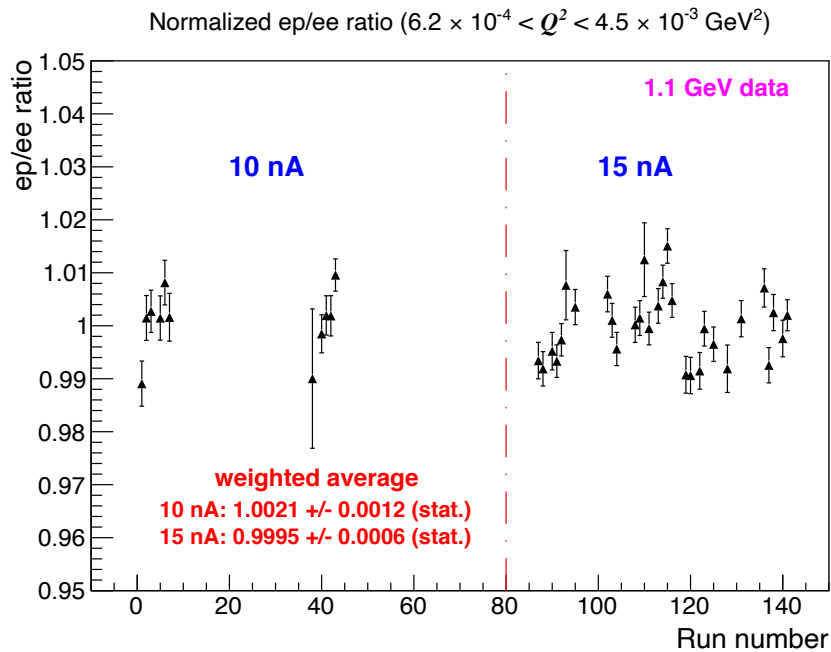
$ee$  Background Contribution





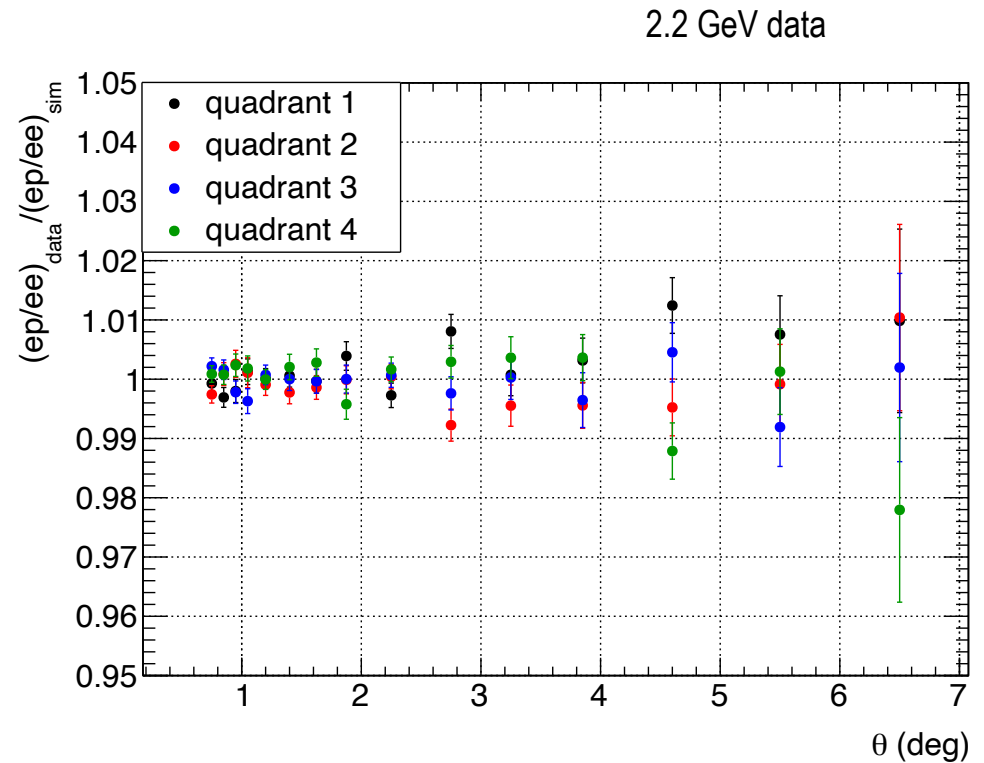
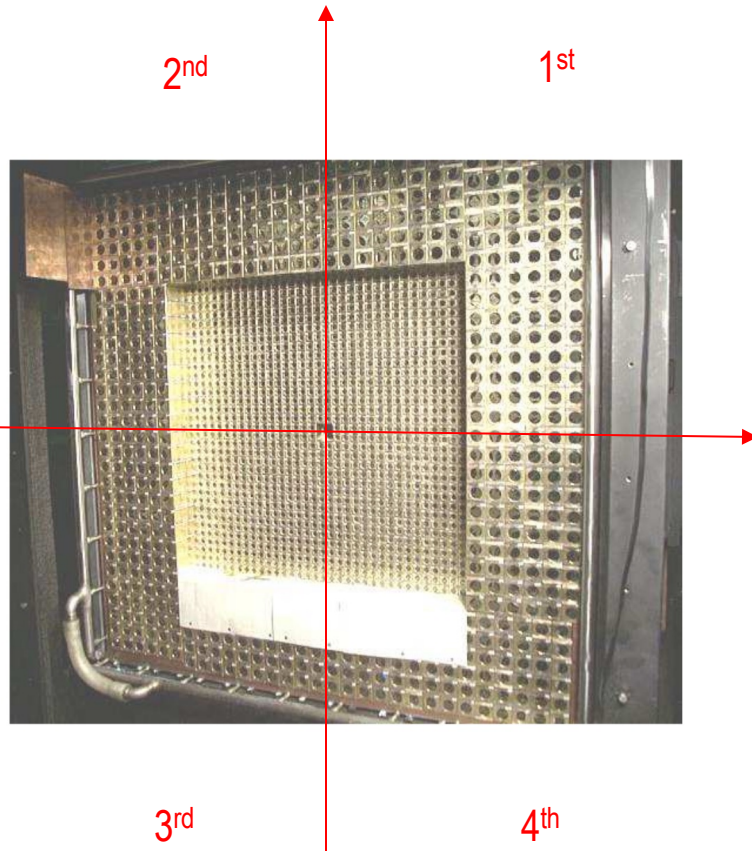
# Data Analysis: Stability vs. Run Number

- Normalized  $ep/ee$  ratio vs. run number, (background subtracted with neighboring empty target runs).
- Sensitive to systematics like time variation of beam line background, ...



# Data Analysis: Azimuthal Uniformity

- Ratio  $(ep/ee)_{dat} / (ep/ee)_{sim}$  vs. azimuthal quadrants
- Sensitive to detector efficiency, beam position, tilting angles, ...



# Extracted Proton Electric Form Factor, $G_E$ vs. $Q^2$

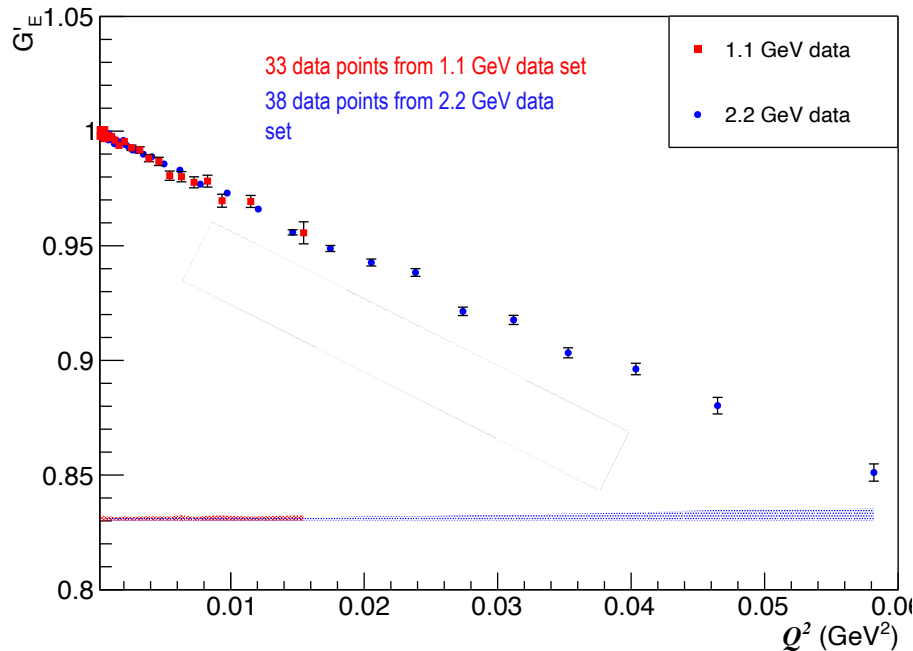
$n_1$  and  $n_2$  obtained by fitting PRad  $G_E$  to  $\begin{cases} n_1 f(Q^2), & \text{for 1 GeV data} \\ n_2 f(Q^2), & \text{for 2 GeV data} \end{cases}$

$G'_E$  as normalized electric Form factor:  $\begin{cases} G_E/n_1, & \text{for 1 GeV data} \\ G_E/n_2, & \text{for 2 GeV data} \end{cases}$

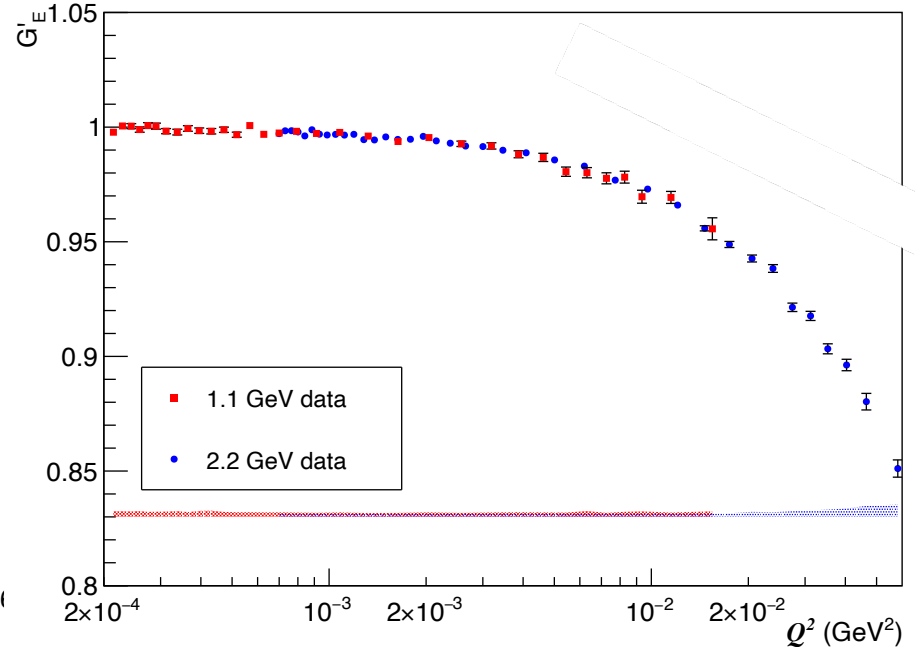
Using rational (1,1)

$$f(Q^2) = \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

Proton Electric Form Factor  $G'_E$



Proton Electric Form Factor  $G'_E$



$$n_1 = 1.0002 \pm 0.0002(\text{stat.}) \pm 0.0020(\text{syst.}),$$

$$n_2 = 0.9983 \pm 0.0002(\text{stat.}) \pm 0.0013(\text{syst.})$$

# PRad Systematic Uncertainties

Item	$r_p$ uncertainty [fm]	$n_1$ uncertainty	$n_2$ uncertainty
Event selection	0.0070	0.0002	0.0006
Radiative correction	0.0069	0.0010	0.0011
Detector efficiency	0.0042	0.0000	0.0001
Beam background	0.0039	0.0017	0.0003
HyCal response	0.0029	0.0001	0.0001
Acceptance	0.0026	0.0001	0.0001
Beam energy	0.0022	0.0001	0.0002
Inelastic $ep$	0.0009	0.0000	0.0000
$G_M^p$ parameterization	0.0006	0.0000	0.0000
Total	0.0115	0.0020	0.0013

# Recent Developments in Fitting Procedures

- The input form factors (with known  $r_p$ ) are used to generate pseudo data using PRad kinematic range and uncertainties.
- All combinations of input functions and fit functions can then be tested repeatedly against regenerated pseudo data.
- Since the input radius is known, this allowed to find fitting functions that are robust for proton radius extractions in an objective fashion.

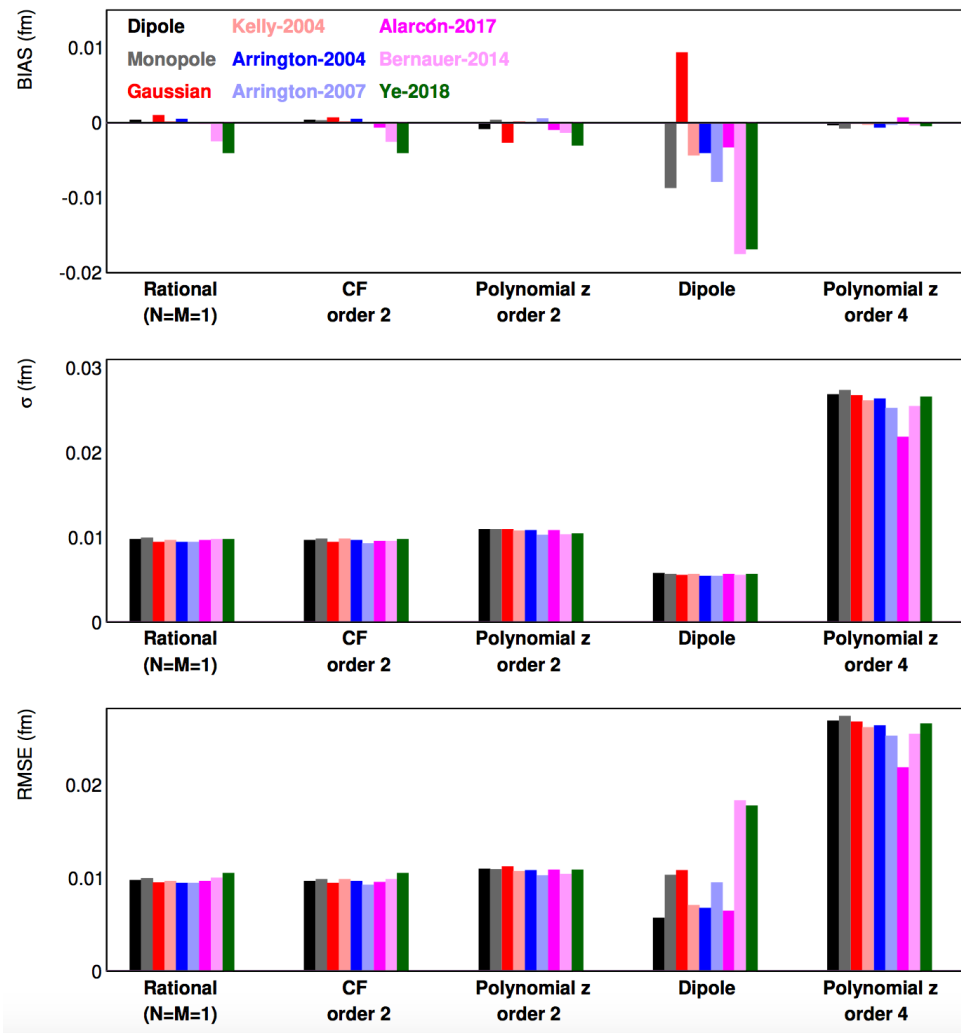
➤ The following fitters:

- ✓ two-parameter rational function
- ✓ two-parameter continued fraction
- ✓ second-order polynomial expansion of  $z$

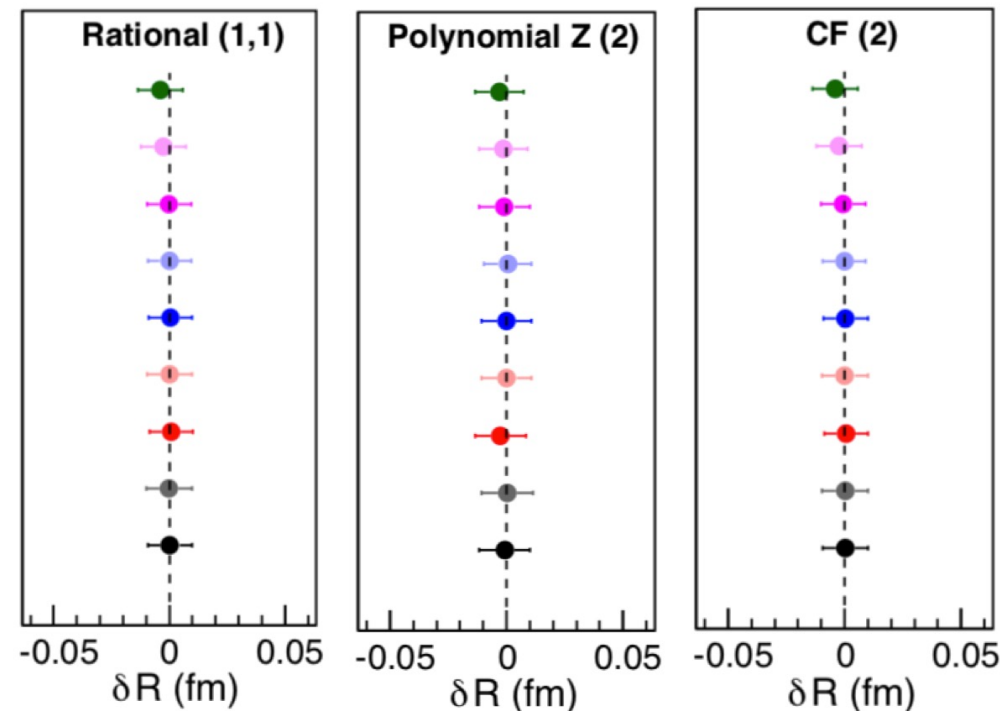
are identified as **robust fitters** with small uncertainties

$$\text{RMSE} = \sqrt{\text{bias}^2 + \sigma^2}$$

- X. Yan, et al.  
*"Robust extraction of the proton charge radius from electron-proton scattering data"*, *PRC* 98, 2, 025204, 2018



# Recent Developments in Fitting Procedures



- Ye-2018
- Bernauer-2014
- Alarcón-2017
- Arrington-2007
- Arrington-2004
- Kelly-2004
- Gaussian
- Monopole
- Dipole

Rational (1,1)

$$p_0 \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$


---

2<sup>nd</sup> order z transformation

$$p_0 (1 + p_1 z + p_2 z^2)$$

$$z = \frac{\sqrt{T_c + Q^2} - \sqrt{T_c - T_0}}{\sqrt{T_c + Q^2} + \sqrt{T_c - T_0}}$$


---

2<sup>nd</sup> order continuous faction

$$p_0 \frac{1}{1 + \frac{p_1 Q^2}{1 + p_2 Q^2}}$$

The robustness = root mean square error (RMSE)

$$\text{RMSE} = \sqrt{(\delta R)^2 + \sigma^2},$$



# PRad Collaboration



A part of the PRad collaboration  
in December, 2019 at JLab

- Currently 14 collaborating universities and institutions:

Jefferson Laboratory, NC A&T State University,  
Duke University, Idaho State University,  
Mississippi State University, Norfolk State University,  
University of Virginia, Argonne National Laboratory,  
University of North Carolina at Wilmington, Hampton University,  
College of William & Mary, Tsinghua University, China,  
Old Dominion University, ITEP Moscow, Russia.

- **Graduate students:**

Chao Peng (Duke), Weizhi Xiong (Duke),  
Xinzhan Bai (UVa), Li Ye (MSU)

- **Postdocs:**

Chao Gu (Duke), Xuefei Yan (Duke), Mehdi Meziane  
(Duke), Zhihong Ye (Duke), Tyler Hague (NC A&T SU),  
Maxime Lavilain (NC A&T), Krishna Adhikari (MSU),  
Latif-ul Kabir (MSU), Chandra Akondi (NC A&T)