Current status of the ULQ^2 experiment and required corrections

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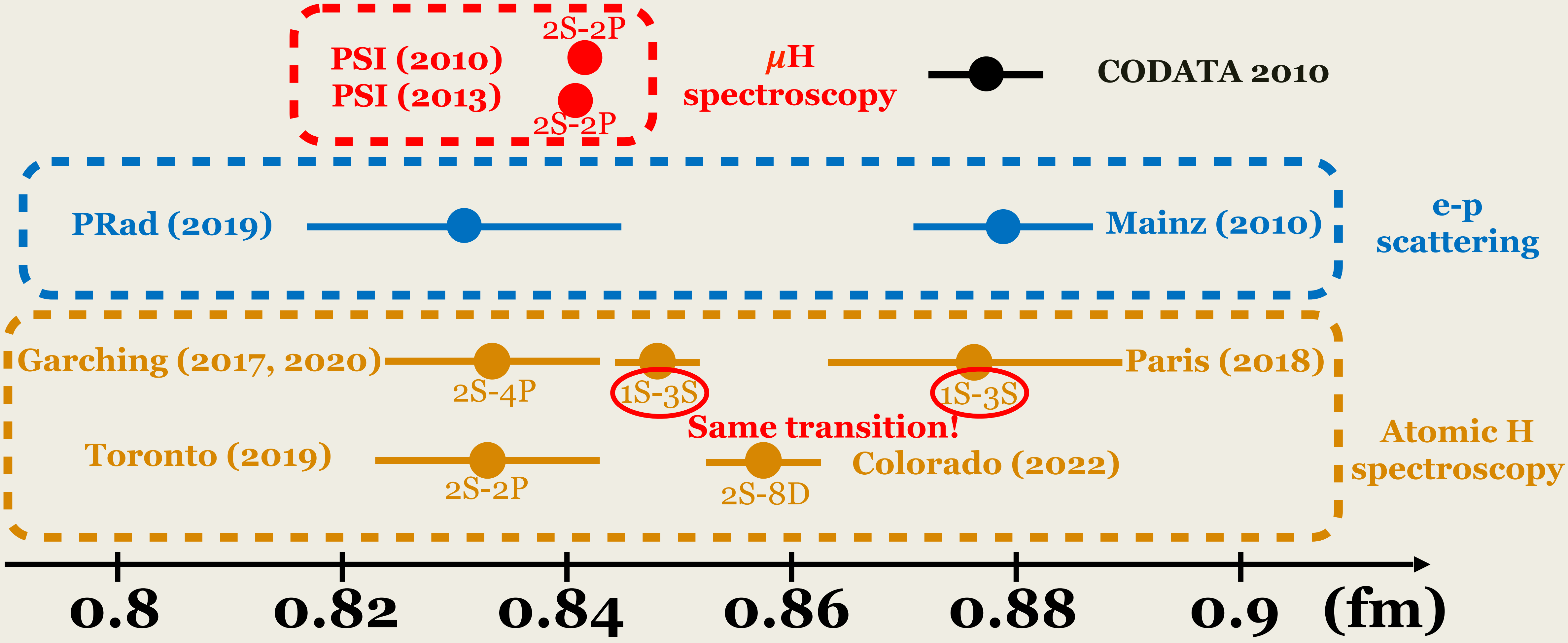
I. Proton charge radius puzzle

Proton charge radius puzzle



Proton charge radius puzzle

- The proton charge radius is one of the most primary characteristics of the nucleon.

- The proton charge radius is greatly related to the Rydberg constant (R_∞), one of the constant determined with the highest accuracy:

$$E_{n,l} = \alpha R_\infty + \beta < r_p^2 > \delta_{l,0}$$

Interaction between the S-orbitals and the nucleus.

- The cause of the discrepancy is not yet fully understood.

  ➡ Lack of precision or errors during some previous measurements?

II. Determination of the proton charge radius using electron scattering

Electric form factor and proton charge radius

Momentum transfer:

$$Q^2 = |\vec{q}|^2 - \omega^2$$

$$Q^2 \sim 4EE' \sin\theta/2^2$$

Ultra-relativistic approximation

$$\tau = \frac{Q^2}{4M_p^2},\ \epsilon^{-1} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

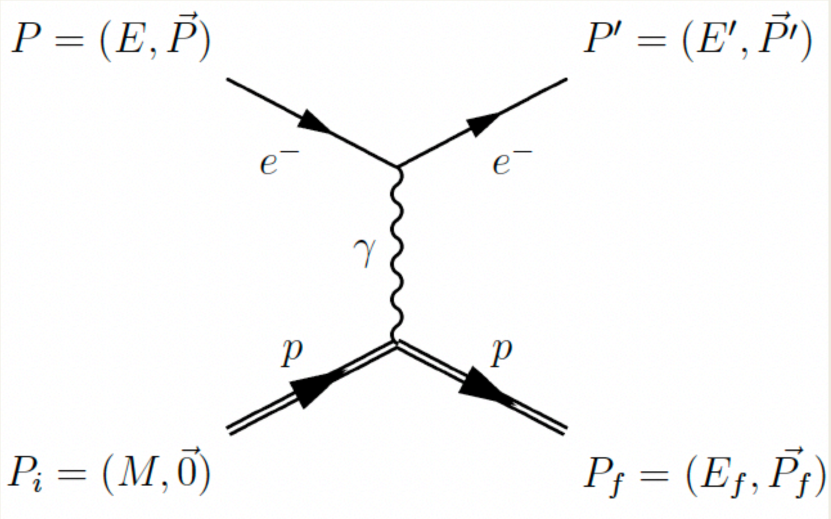
Cross section:

$$\left(\frac{d\sigma}{d\Omega}\right) \approx \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E} \frac{G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2)}{1+\tau}$$

$$G_E(Q^2) = 1 - \frac{Q^2}{3!}\langle r_E^2 \rangle + \frac{Q^4}{5!}\langle r_E^4 \rangle + \ldots$$

$$R_E = \sqrt{\langle r_E^2 \rangle} \equiv \sqrt{-6 \lim_{Q^2\to 0} \frac{dG_E}{dQ^2}}$$

G. Miller, Phys. Rev. C **99** (2019), 035202



Feynman diagram of the leading-order of the electron-proton scattering.

Q^2 as small as possible!

Electric form factor and proton charge radius

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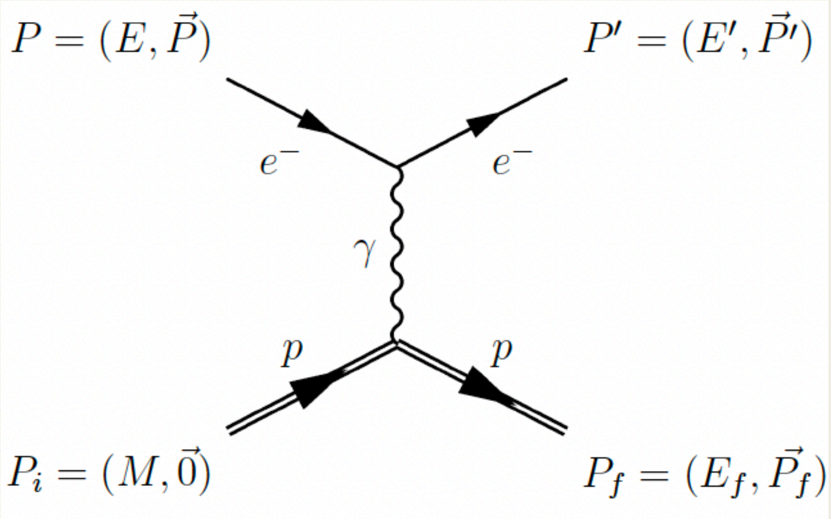
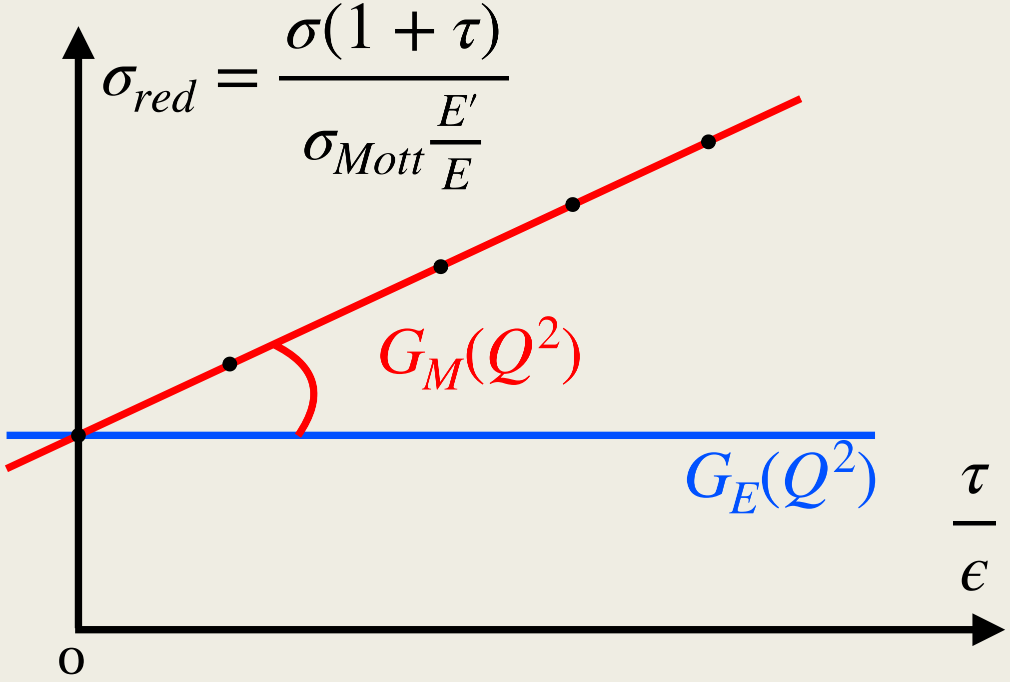


Feynman diagram of the leading-order of the electron-proton scattering.

<u>Rosenbluth separation:</u>

- ❖ Determine separately $G_E(Q^2)$ and $G_M(Q^2)$
- ❖ Measurement of the cross-section with a constant Q^2 and different angles.



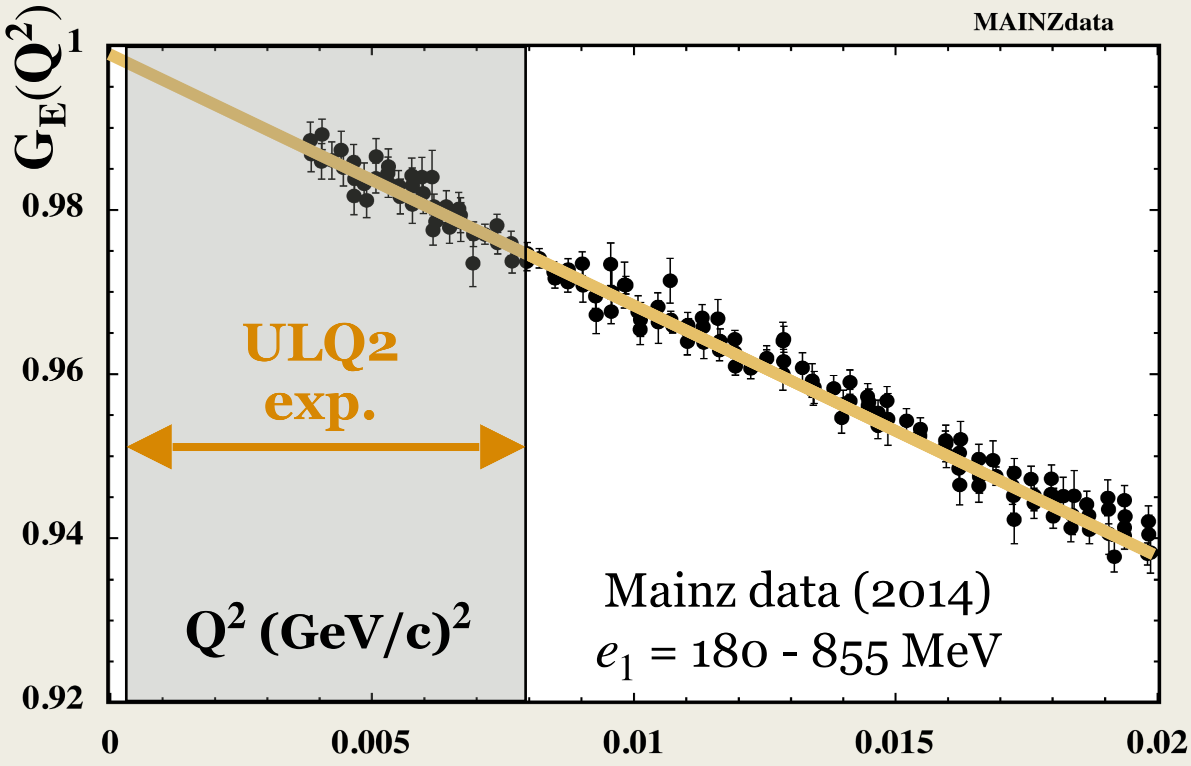
Specifications of the ULQ^2 experiment

Ultra Low Q^2

<u>Characteristics of the experiment:</u>

- Measurement of $G_E(Q^2)$ for extremely small values of Q^2:
  0.0003 (GeV/c)² $\leq Q^2 \leq$ 0.008 (GeV/c)²
- **<u>Lowest beam energy for electron scattering in the world</u>**:
  10 MeV $\leq E \leq$ 60 MeV
- Rosenbluth separation with 30° $\leq \theta \leq$ 150°.
- Polyethylene (CH_2) target.

➡ **Absolute** cross-section measurement with 10^{-3} accuracy.



Momentum transfer range reached during the ULQ^2 experiment.

Absolute cross-section measurement

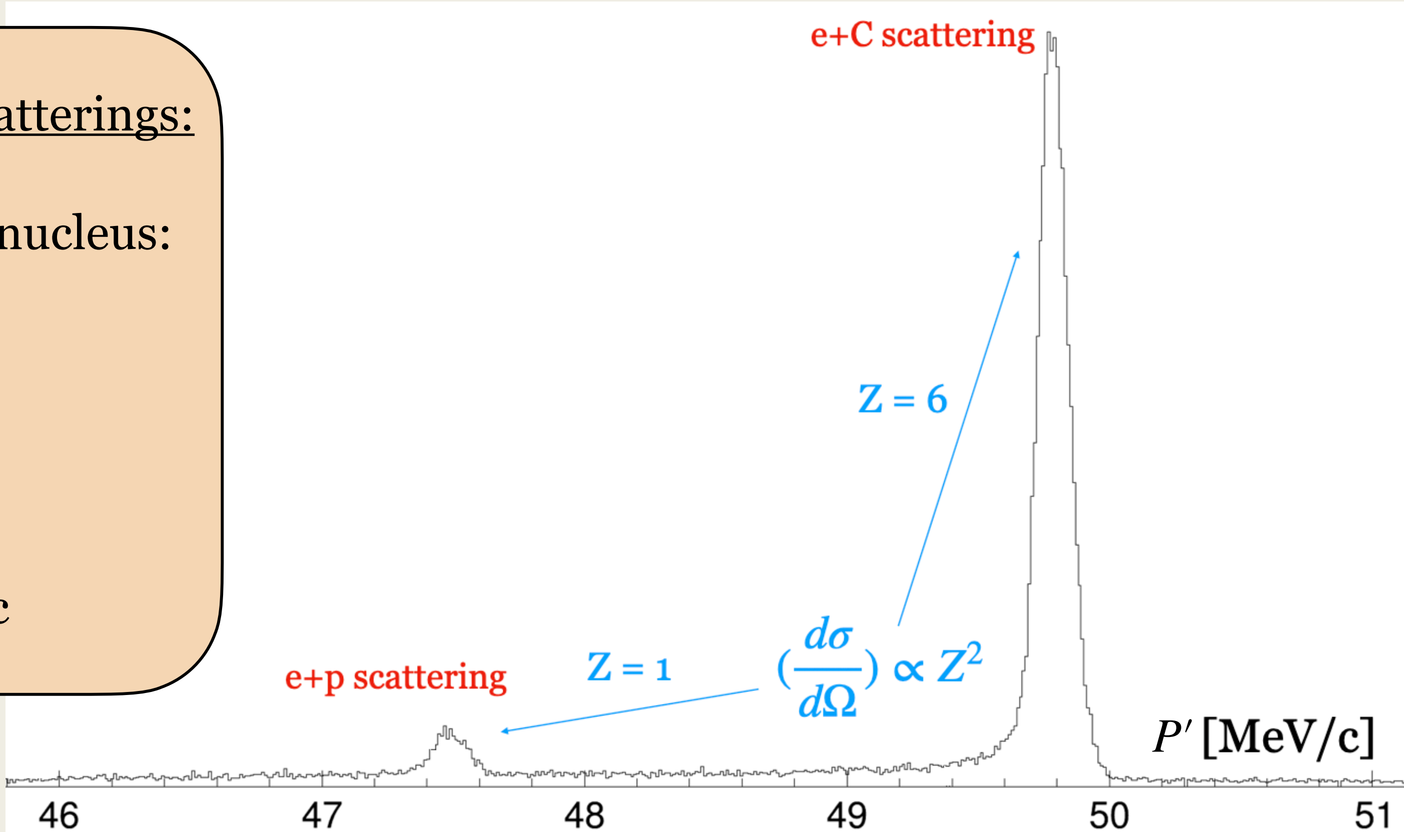
Simultaneous detection of e+p and e+^{12}C scatterings:

- Use of a CH_2 target.
- Momentum of a scattered electron on a X nucleus:

$$P'_X \overset{\text{URL}}{\sim} \frac{E}{1+\frac{2E\sin\theta/2^2}{M_X}}$$

- E = 50 MeV, θ = 90°

➡ $P'_{^{12}C} \sim 49.8$ MeV/c, $P'_H \sim 47.5$ MeV/c



Simultaneous detection of e+p and e+^{12}C scattering with a CH_2 target (Experimental data).

Absolute cross-section measurement

<u>Absolute cross-section of e+p scattering:</u>

Event number

$$\left(\frac{d\sigma}{d\Omega}\right)_{e+p} = \frac{N_{e+p}}{N_e \, N_p \, \Delta\Omega}$$

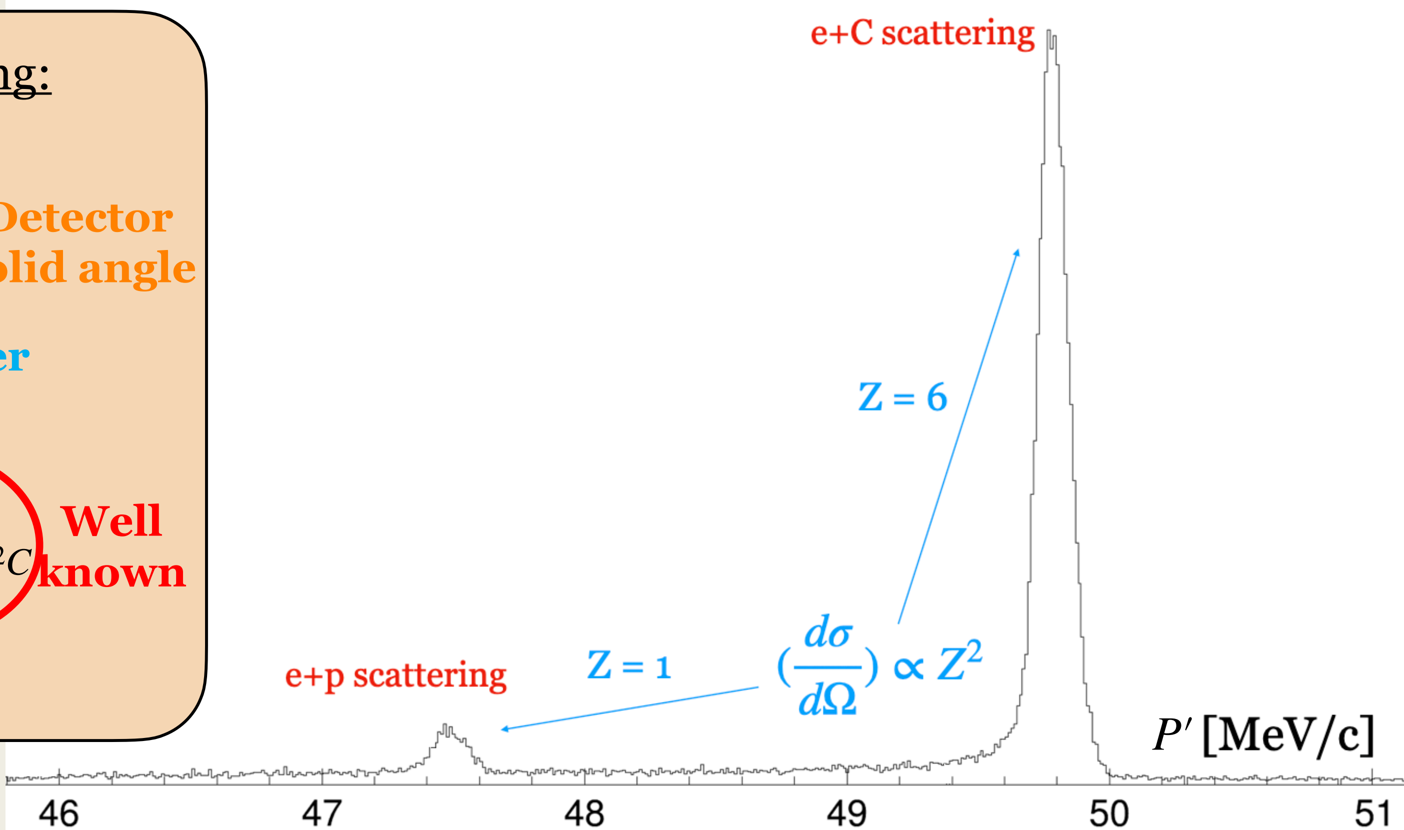
Detector solid angle

Beam dose **Target number**

$$\left(\frac{d\sigma}{d\Omega}\right)_{e+p} = \frac{N_{e+p}/N_{e+{}^{12}C}}{N_p/N_{{}^{12}C}} \left(\frac{d\sigma}{d\Omega}\right)_{e+{}^{12}C}$$

Well known

Ratio of H and C nuclei



Simultaneous detection of e+p and e+^{12}C scattering with a CH_2 target (Experimental data).

Measurement time

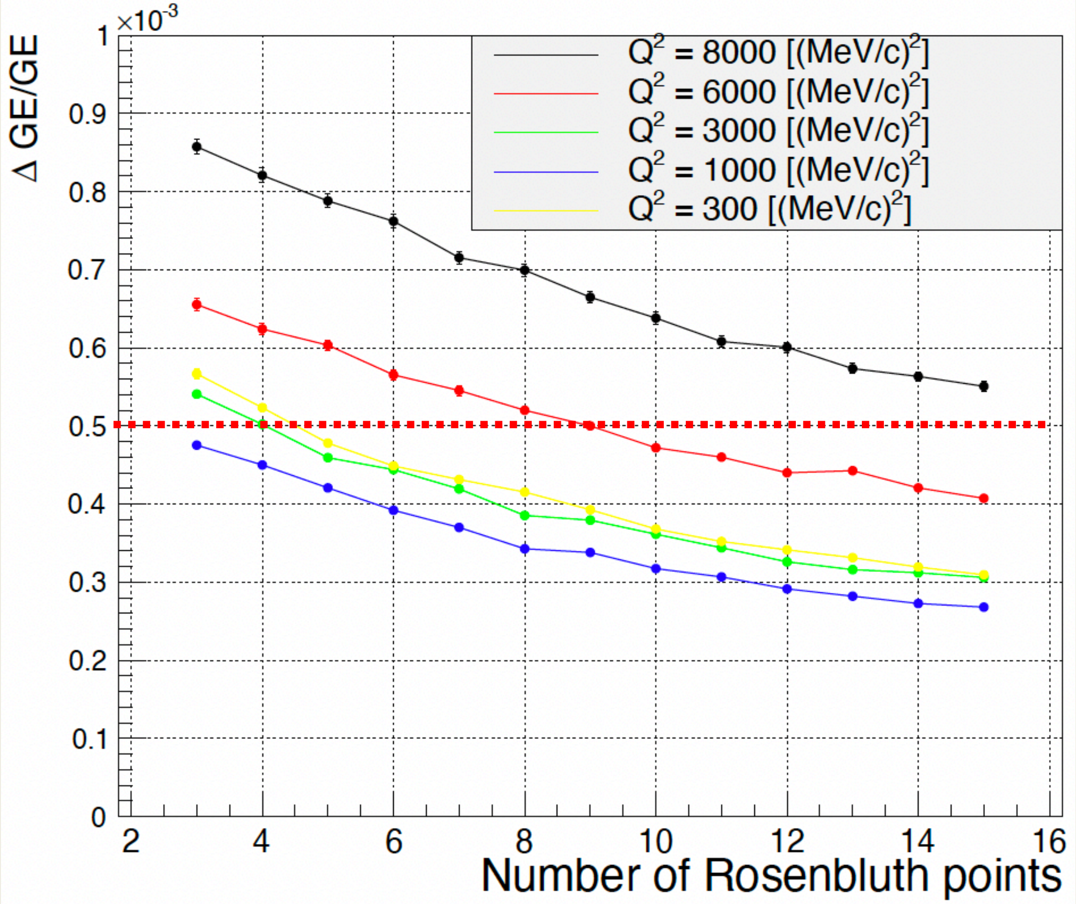
- Goal: Determine R_E with 1 % accuracy

→ Need to have $\frac{\Delta G_E}{G_E} \leq 5 \times 10^{-4}$

→ At least 9 Q^2 data points

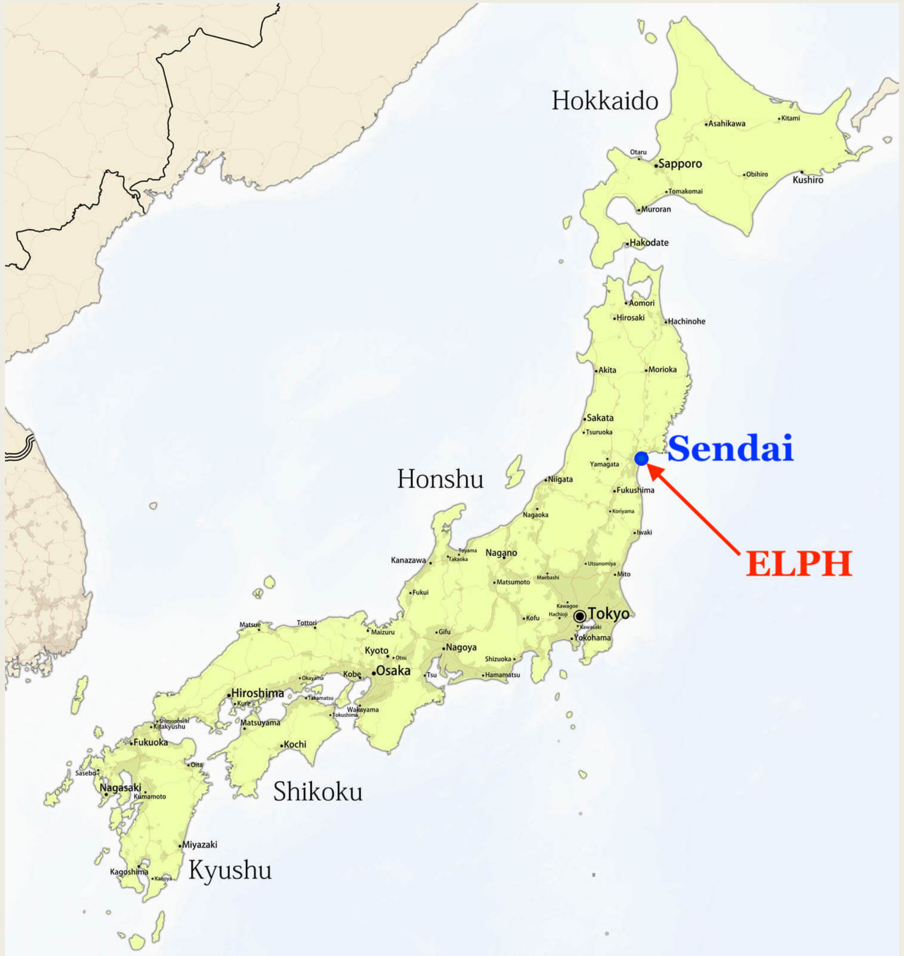
- 1 Q^2 data point = 3-16 Rosenbluth data points
- 1 measurement ~ 12 h

→ Total beam time ~ 1 month



III. Current status of the ULQ^2 experiment

Research Center for Electron-Photon Science (ELPH)



Beam line

<u>ULQ^2 accelerator:</u>

- Energy: E = 10 - 60 MeV
- Energy spread: σ_E/E = 0.06%
- Position spread: σ_x, σ_y = 0.6 mm
- Intensity: I $\leq$ 1 μA
- Pulse duration: $\Delta t \sim 3\ \mu$s
- Pulse frequency: f = 1-300 Hz

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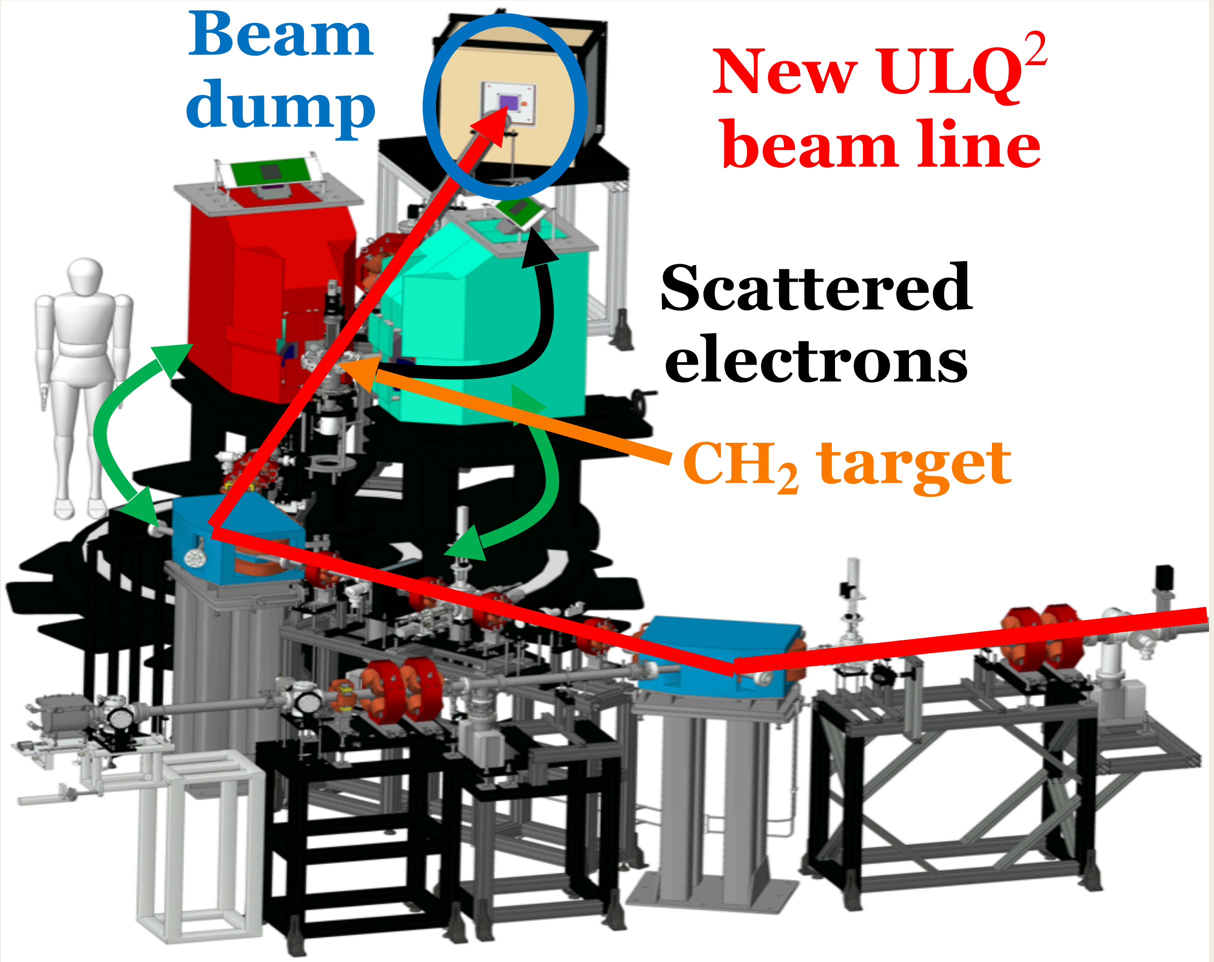
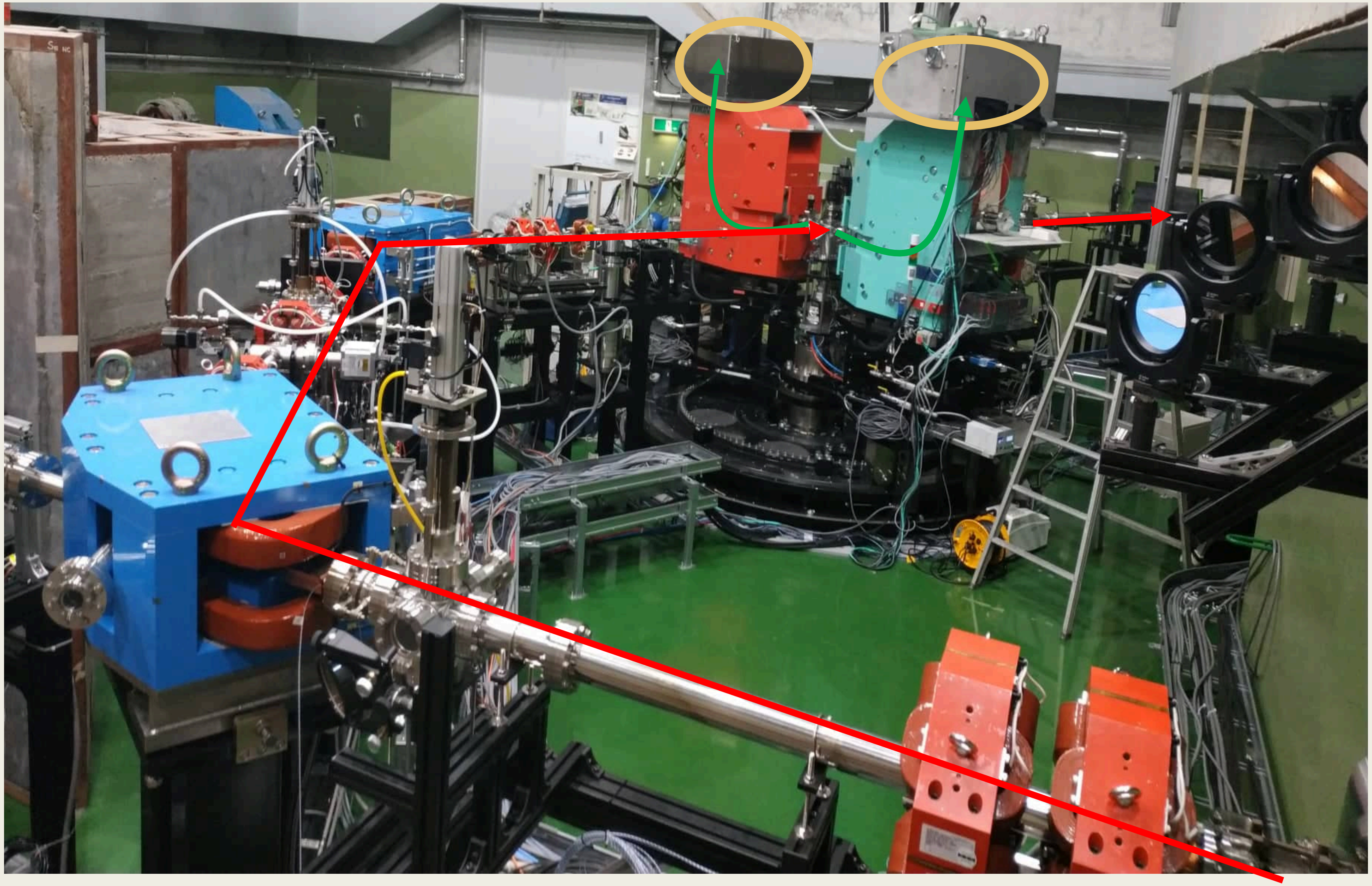
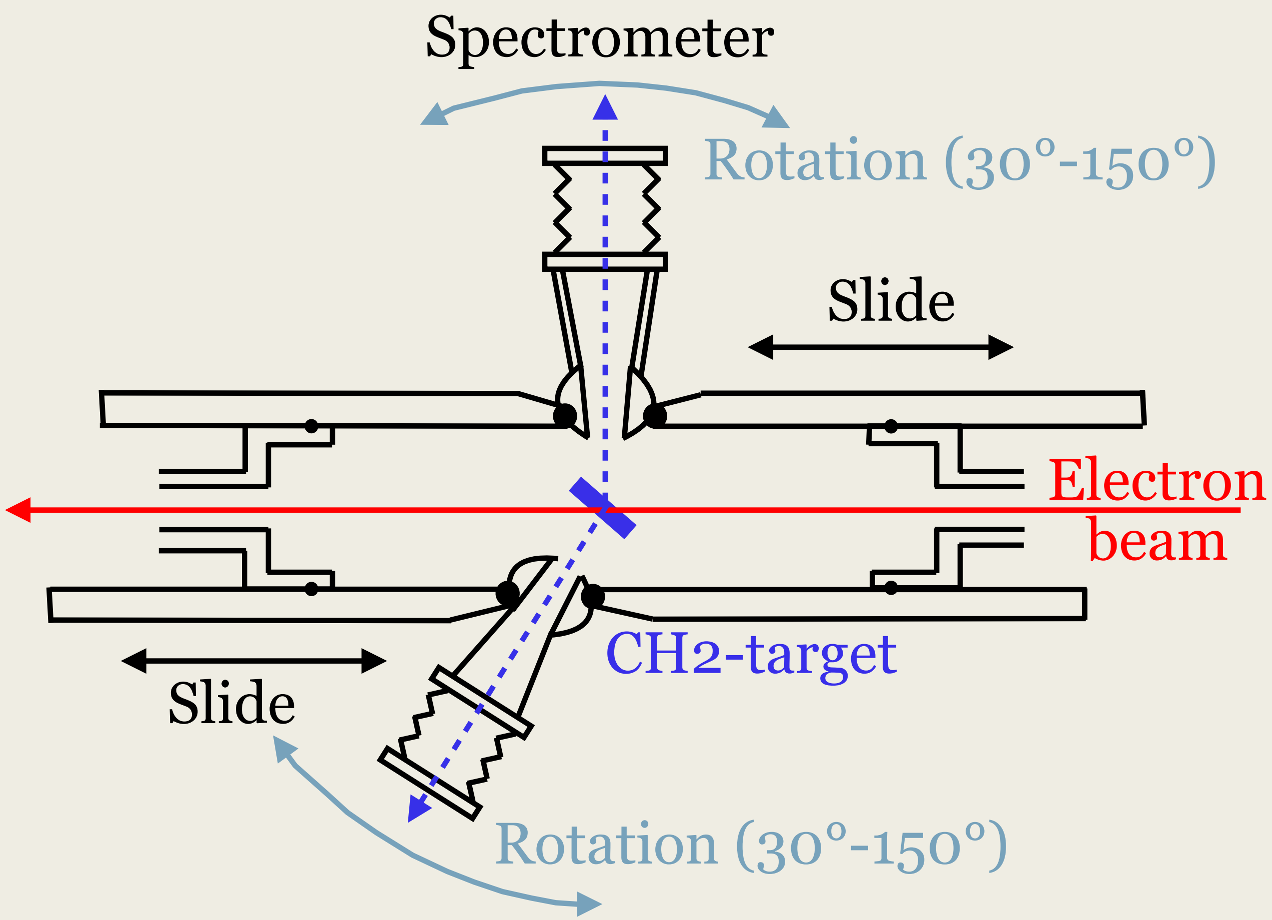
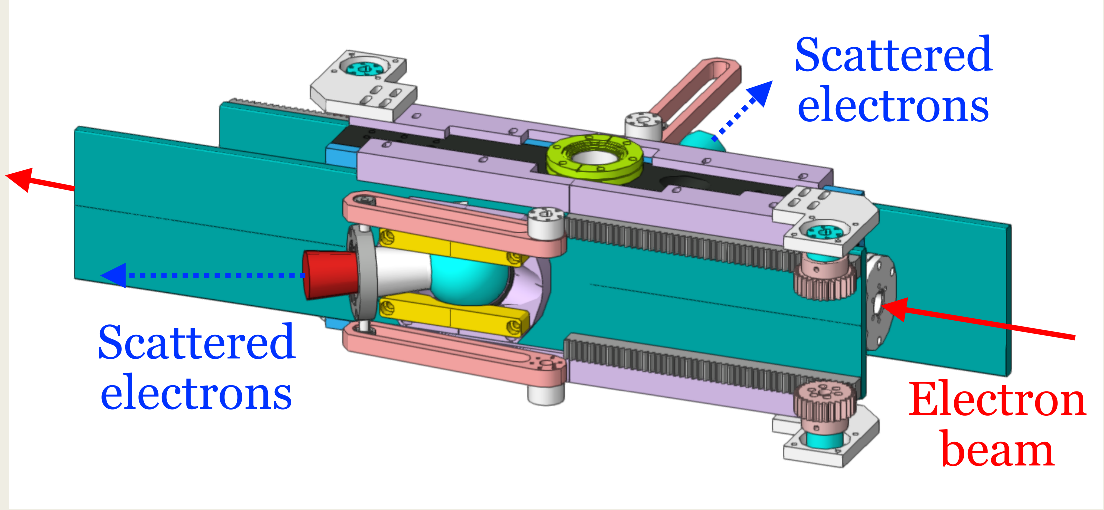


Beam line

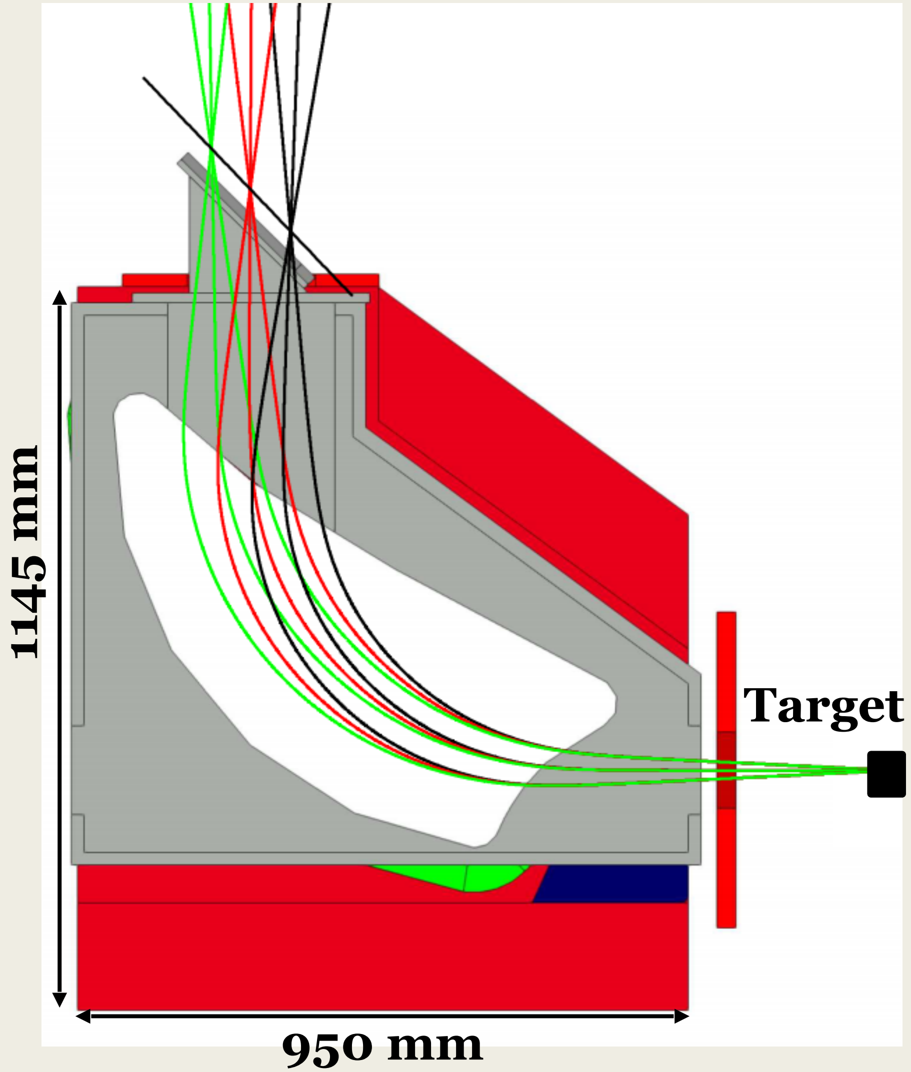


Variable-angle target chamber





Spectrometers



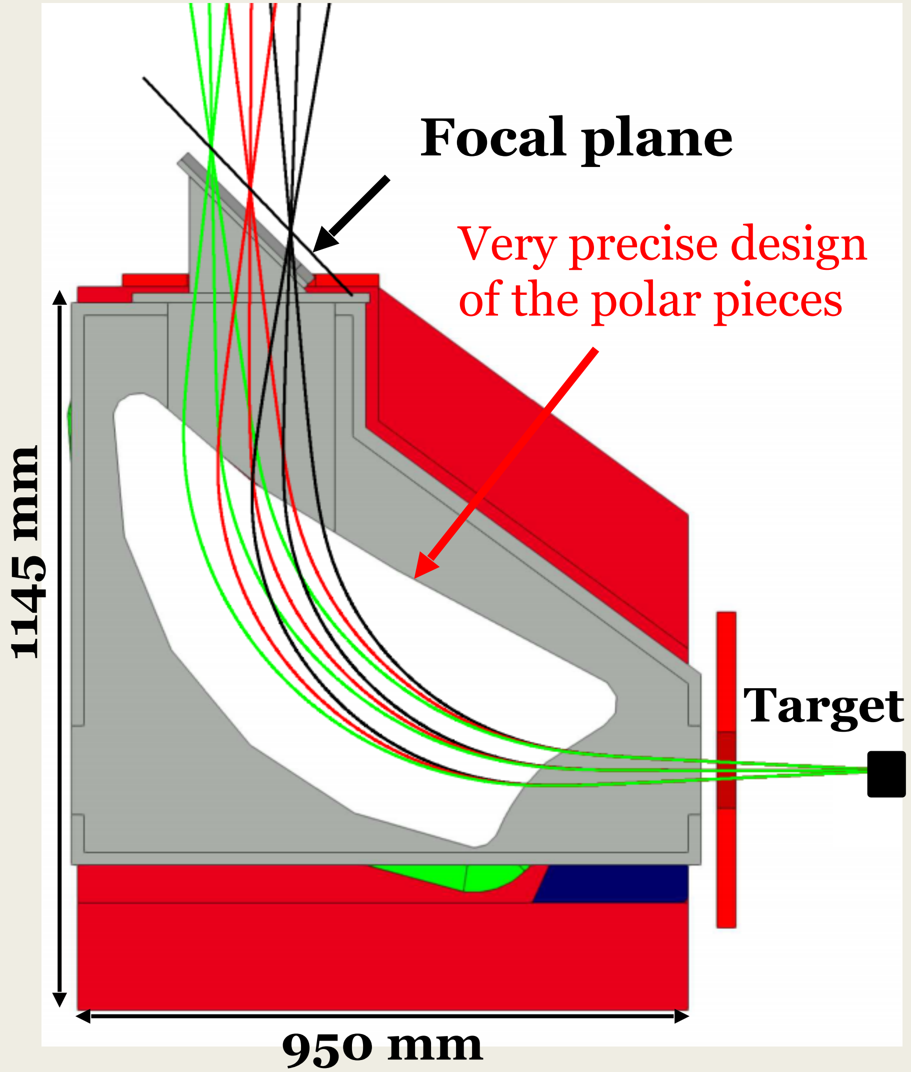
Spectrometer 1 ⟶ Data taking

Spectrometer 2 ⟶ Luminosity monitor

<u>Characteristics:</u>

Height	1145 mm
Length	950 mm
Mass	3.7 t
Curvature radius	0.5 m
Maximum B (I)	0.5 T (300 A)
Angular acceptance	~ 10 mSr
Momentum acceptance	~ 10 %

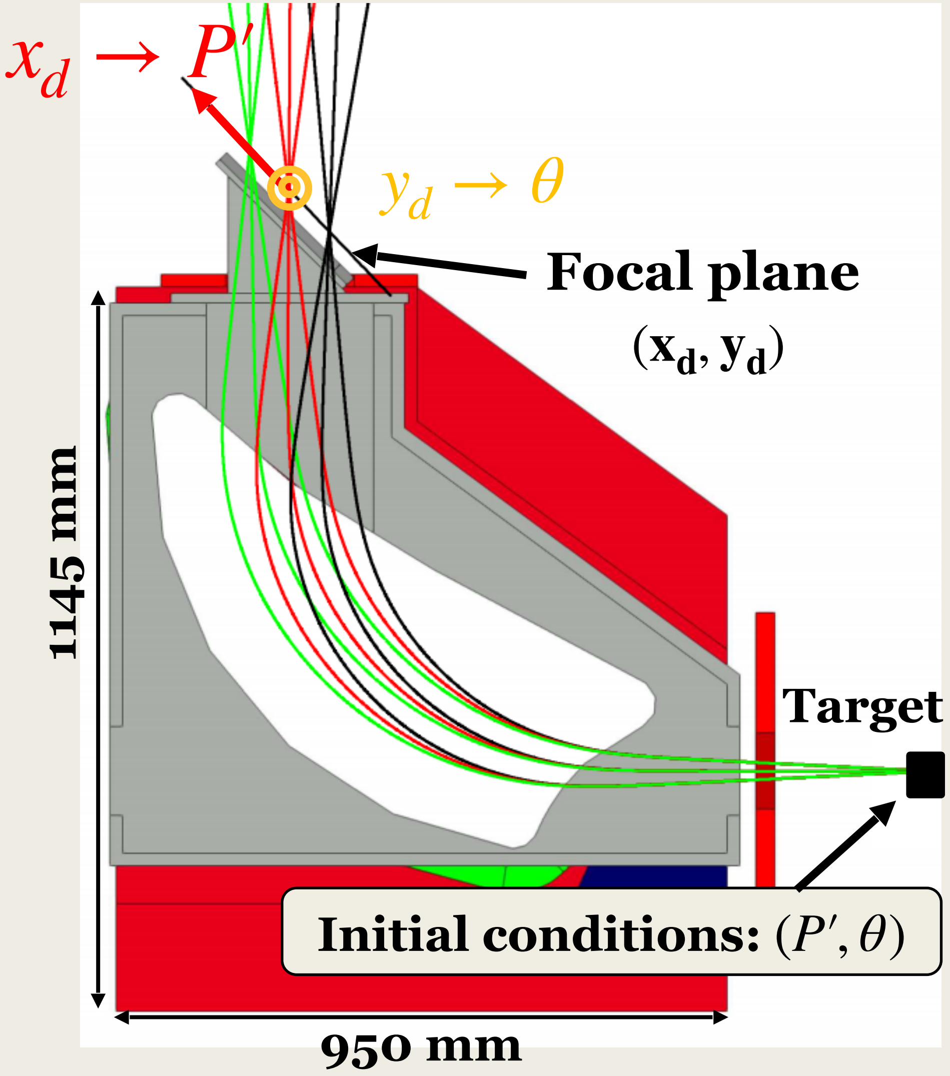
Spectrometers



Measurement in the focal plane:

- ULQ^2 experiment uses very low energy electrons.
  - ➡ Strong multiple scattering: $\langle \theta_{MS} \rangle \propto \dfrac{1}{P'}$
  - ➡ Impossible to determine the path of the electrons.
- Single measurement of the electron position in the focal plane.
- Connected to the target chamber and under vacuum (< 1 mPa).

Spectrometers

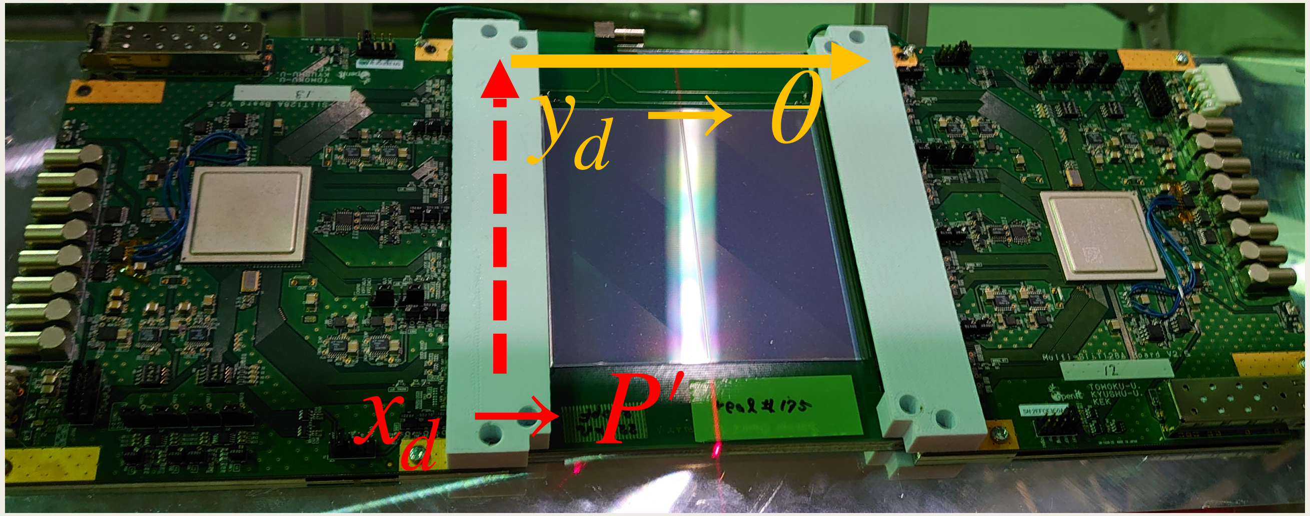


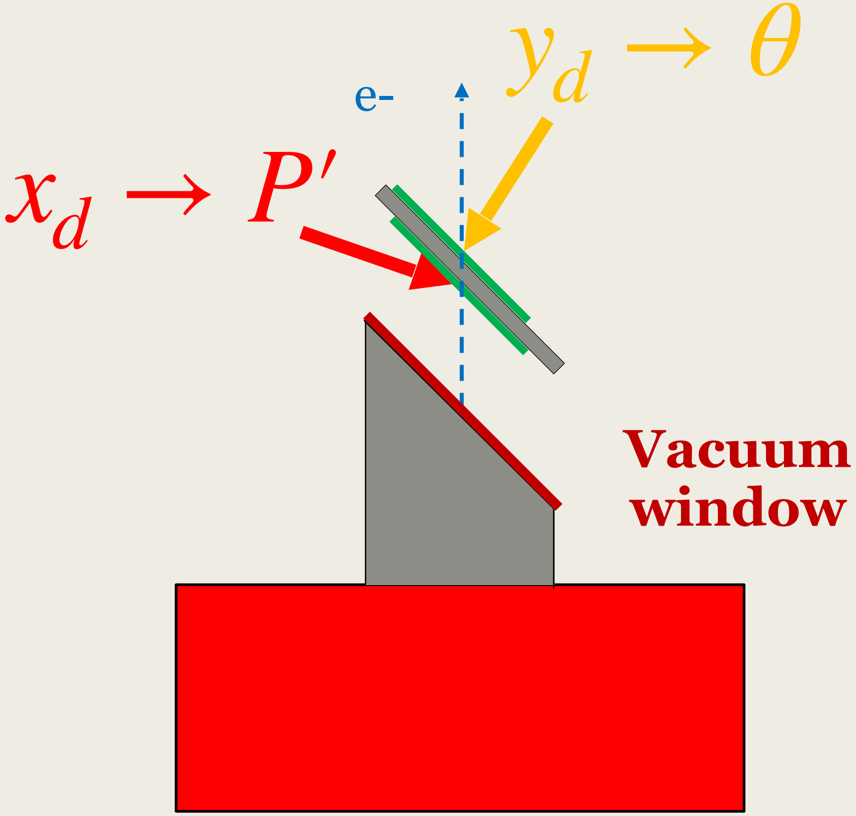
<u>Measurement in the focal plane:</u>

- Electrons focused in the focal plane depending on their momentum p and horizontal scattering angle θ.
- (P', θ) determined from the (x_d, y_d) position of the electrons on the detectors placed in the focal plane.
- To resolve e+p and e+C scattering peaks with Q^2= 0.0003 (GeV/c)^2,

  Momentum resolution: $\sigma_p = \frac{\Delta P}{P} < 10^{-3}$

Detection system

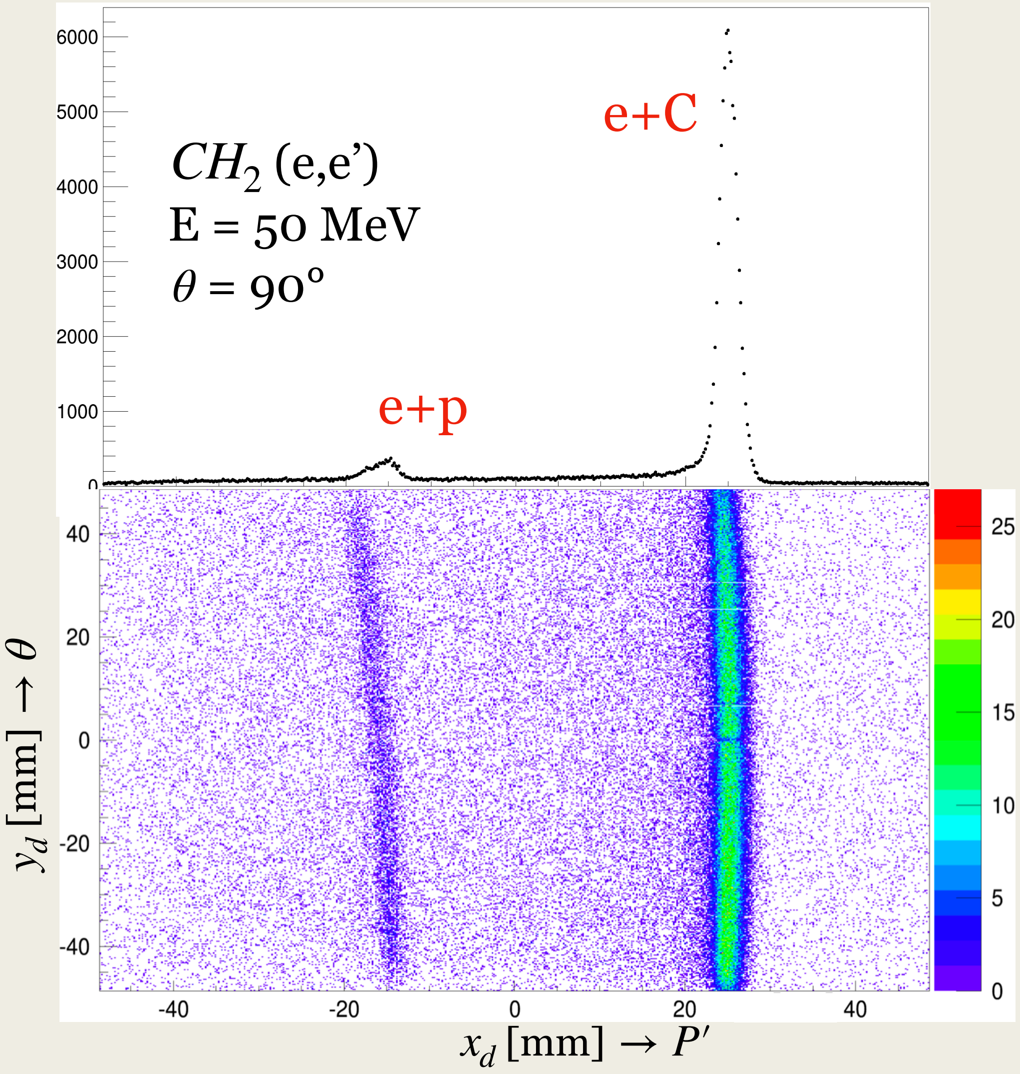


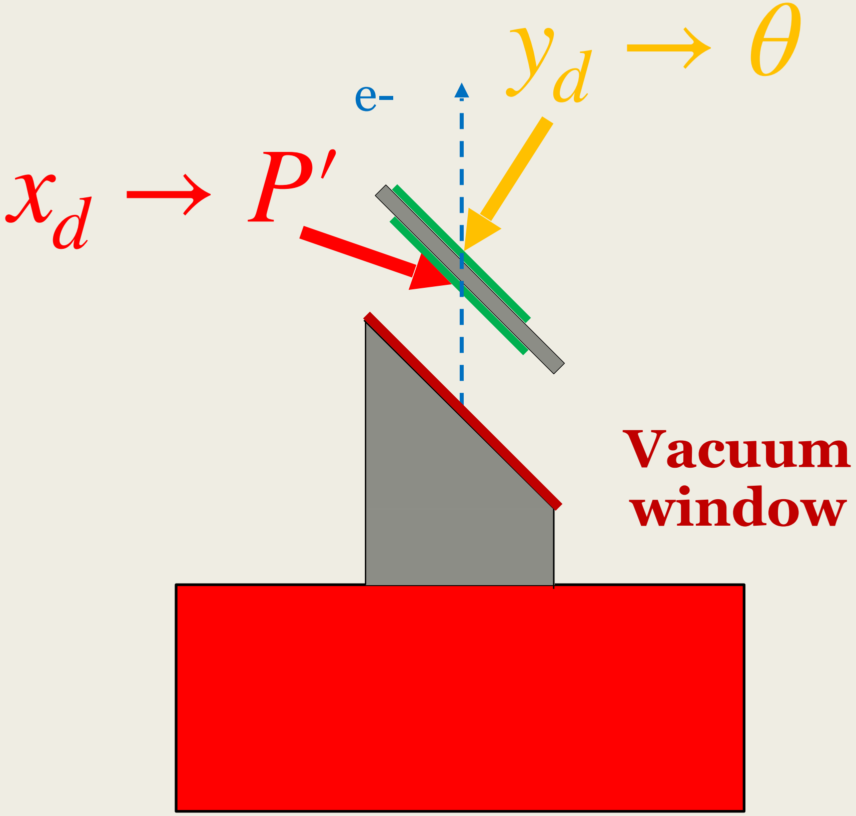


<u>Single Sided Silicon Strip Detectors (SSDs):</u>

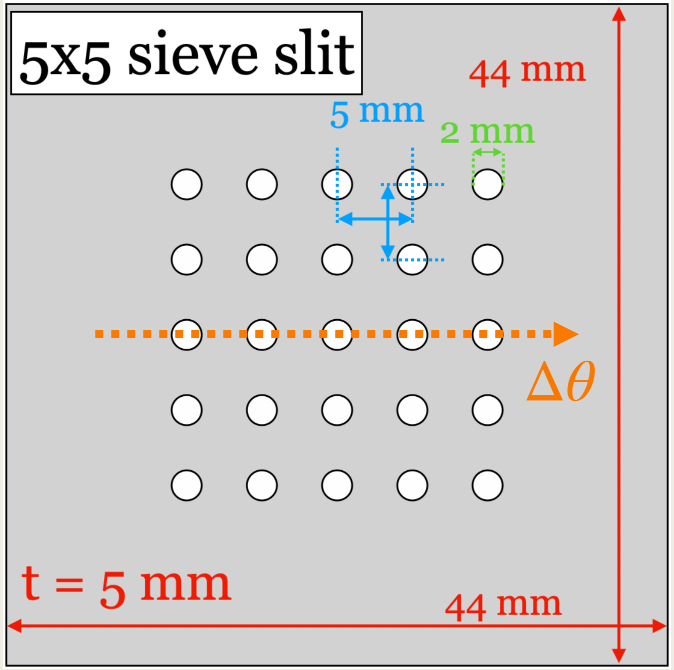
- Developed with the J-PARC muon g-2/EDM collaboration.
- 2 detectors each made of 2 x 512 channels on each spectrometer.
- Located in the focal plane of the spectrometers.
- Channel width: 0.19 mm, thickness: 0.32 mm.

Detection system





Detection system



Momentum dispersion Angular dispersion

Relation between (x_d, y_d) and (P', θ):

$$x_d = (x_d|\delta)\delta + (x_d|\delta^2)\delta^2$$

$$y_d = [(y_d|\Delta\theta) + (y_d|\delta\Delta\theta)\delta]\Delta\theta$$

with $\delta = \dfrac{P' - P_c}{P_c}$ Spectrometer central momentum

$\propto B_C$

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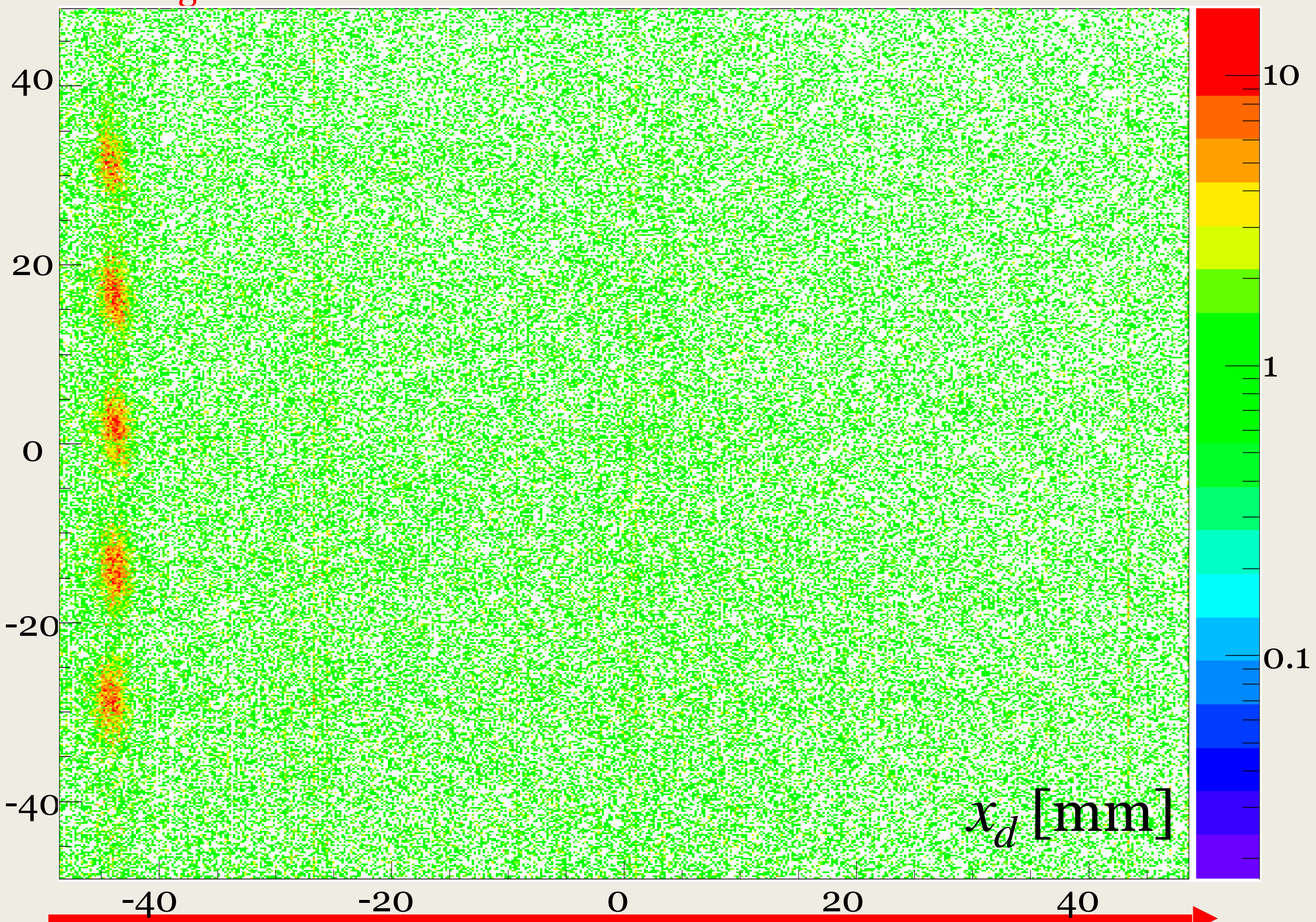
Spectrometer central angle

Large $B_c \to$ small δ

y_d [mm] $\to \Delta\theta = \theta - \theta_s$



$\delta \sim -0.05$ x_d [mm] $\to \delta$ $\delta \sim 0.05$

Detection system

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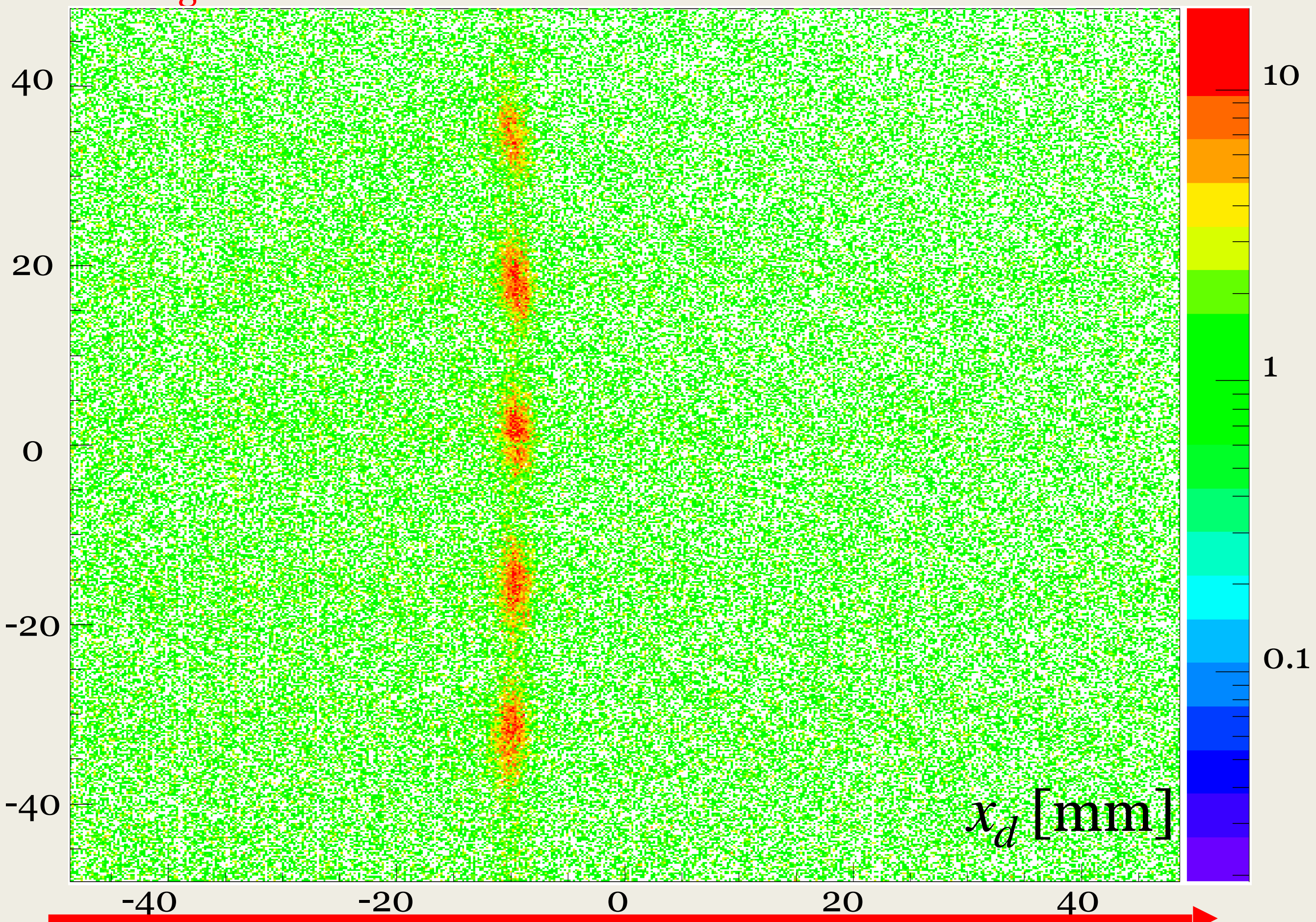
$\propto B_C$ Spectrometer central momentum

Spectrometer central angle

Smaller B_c → larger δ



y_d [mm] → $\Delta\theta = \theta - \theta_s$

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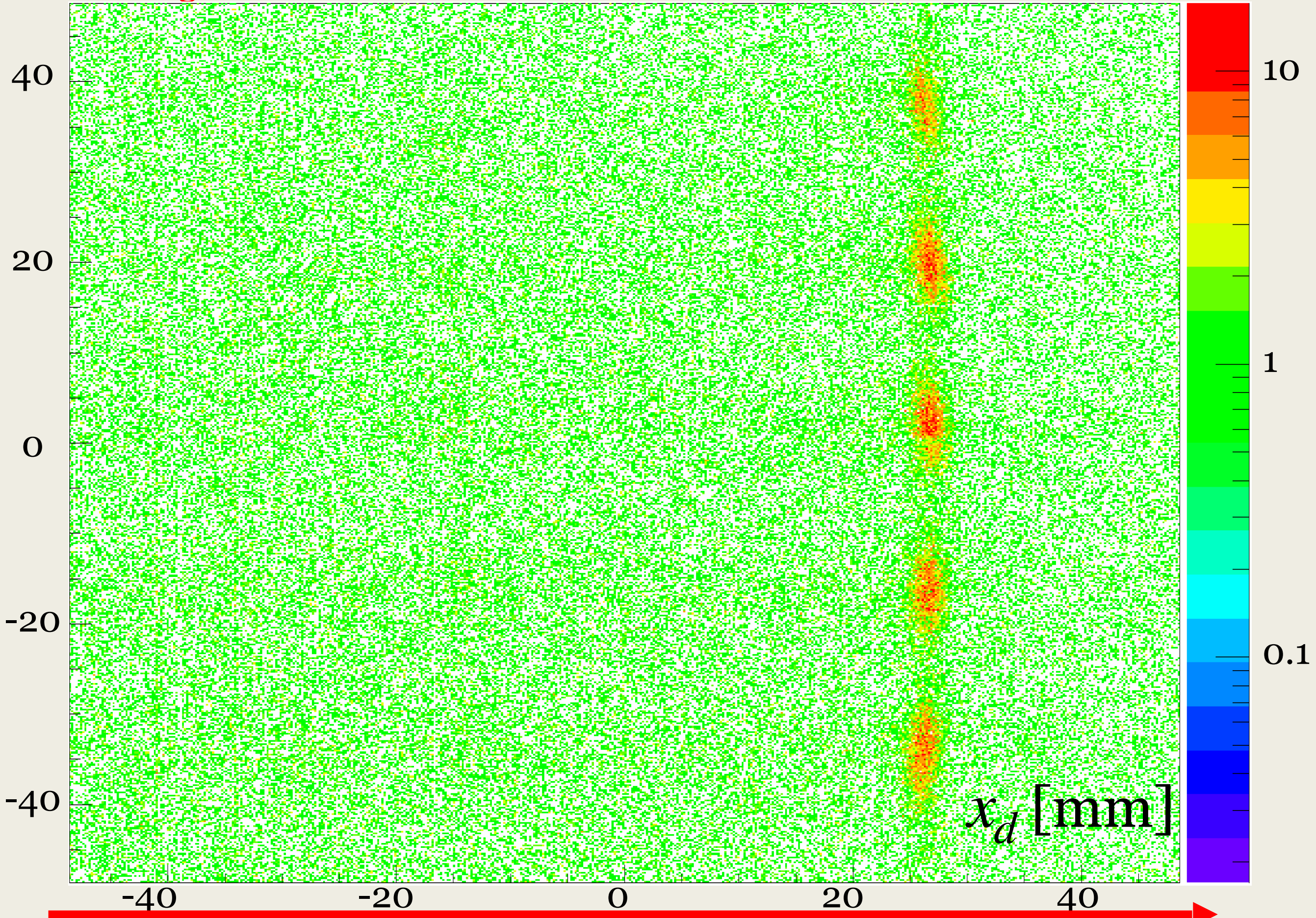
$$\text{with } \delta = \frac{P' - P_c}{\underbrace{P_c}_{\propto B_C}}$$

Spectrometer central momentum

Spectrometer central angle

Small B_c → large δ



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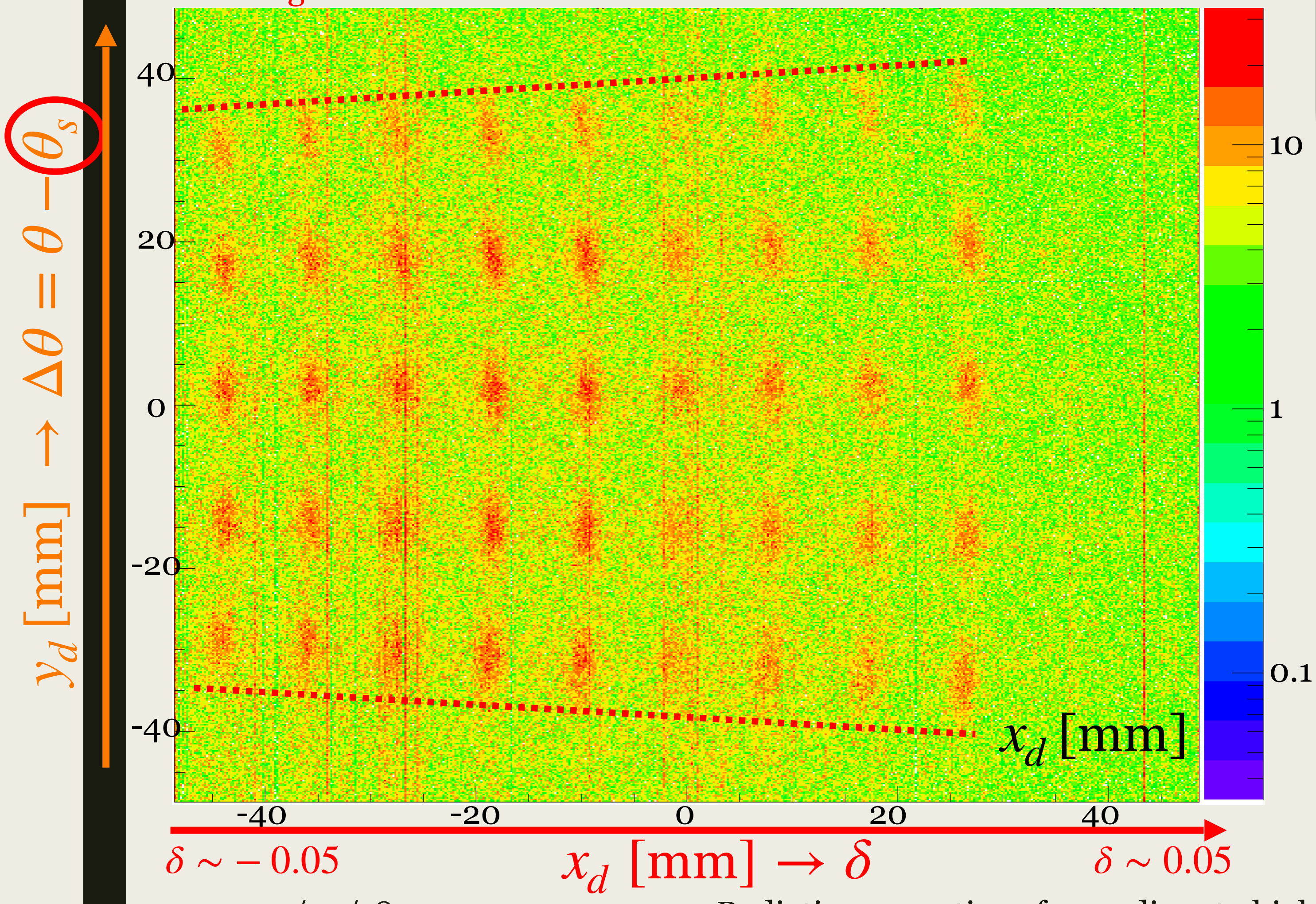
$\propto B_C$

Spectrometer central momentum

Spectrometer central angle

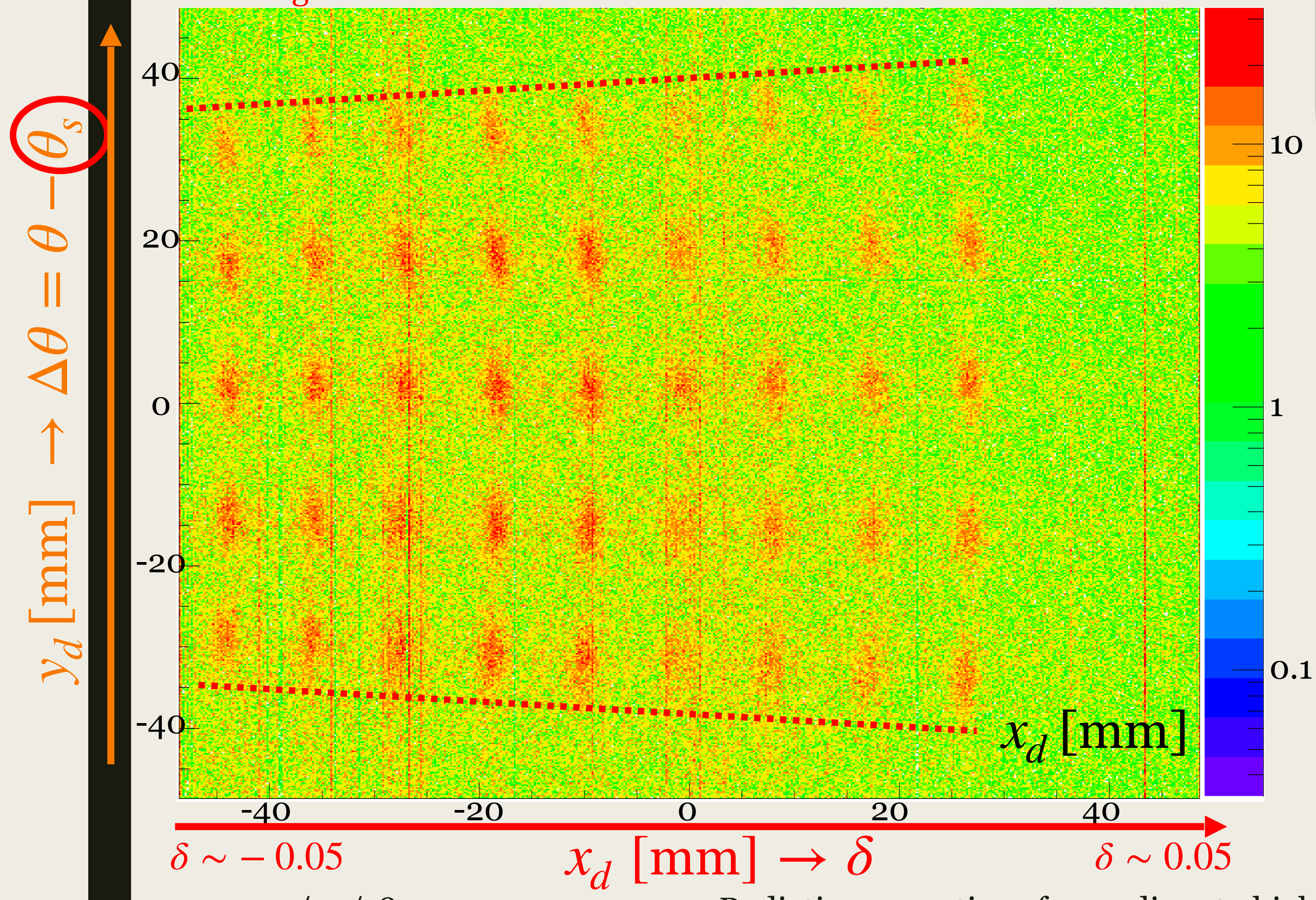
Summary figure



Detection system

Spectrometer central angle

Summary figure



Momentum dispersion Angular dispersion

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$$y_d = [(y_d|\Delta\theta) + (y_d|\delta\Delta\theta)\delta]\Delta\theta$$

with $\delta = \frac{P' - P_c}{P_c}$ Spectrometer central momentum

$\propto B_C$

Commissioning results:

$(x_d|\delta) = 866.1(7)$ mm, $\frac{(x_d|\delta^2)\delta}{(x_d|\delta)} \approx -0.2 \times \delta$ Up to 1% effect

$\sim 10^{-3}$ accuracy

$(y_d|\Delta\theta) = 1.000(4)$ mm/mrad, $\frac{(y_d|\delta\Delta\theta)\delta}{(y_d|\Delta\theta)} \approx 2.0 \times \delta$

Up to 10% effect!

$$\sigma_p = \frac{\Delta p}{p} = 5.6 \times 10^{-4} \leq 10^{-3}$$

→ **The spectrometers fulfill the requirement!**

Detection system

- From x_d, we get δ but not directly P' …

- The beam energy derived from the current of upstream magnets is not precise enough …

Detection system

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- To get E and P', use of C and H peaks:

1st order

$$x_X \sim (x_d|\delta)\frac{P'_X - P_c}{P_c}$$

$$\boxed{R \equiv \frac{P'_C}{P'_H} \sim \frac{1 + \frac{x_C}{(x_d|\delta)}}{1 + \frac{x_H}{(x_d|\delta)}}}$$

Detection system

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URL

$$P'_X \sim \frac{E}{1 + \frac{2E \sin\theta/2^2}{M_X}}$$

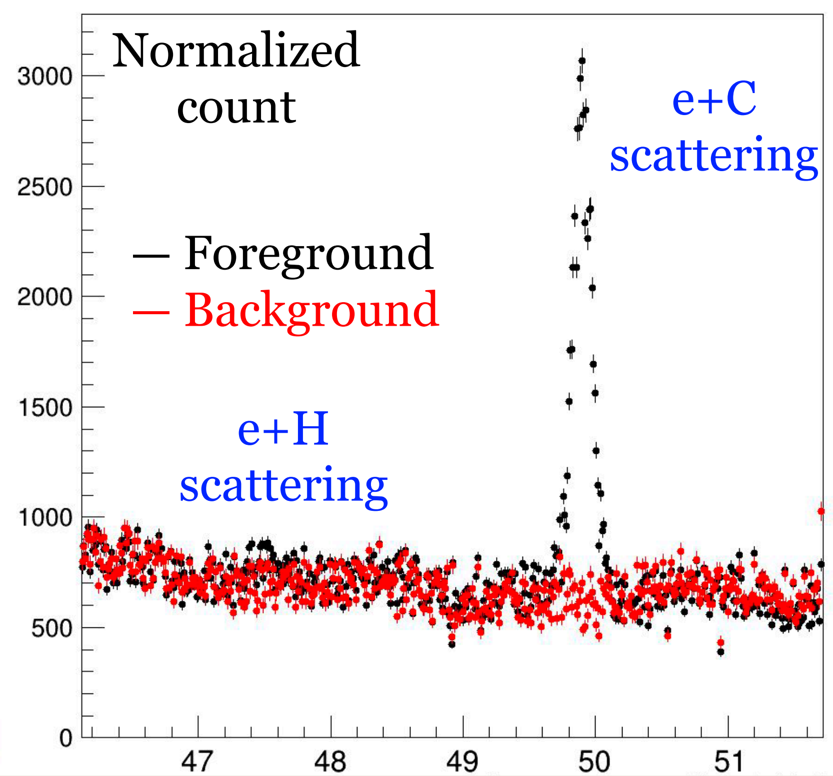
$$R \equiv \frac{P'_C}{P'_H} \sim 1 + 2E \sin\theta/2^2 \left(\frac{1}{M_H} - \frac{1}{M_C}\right)$$

Determination of the beam energy directly from the experimental data!

➡ **Precise determination of Q^2!**

Noise reduction

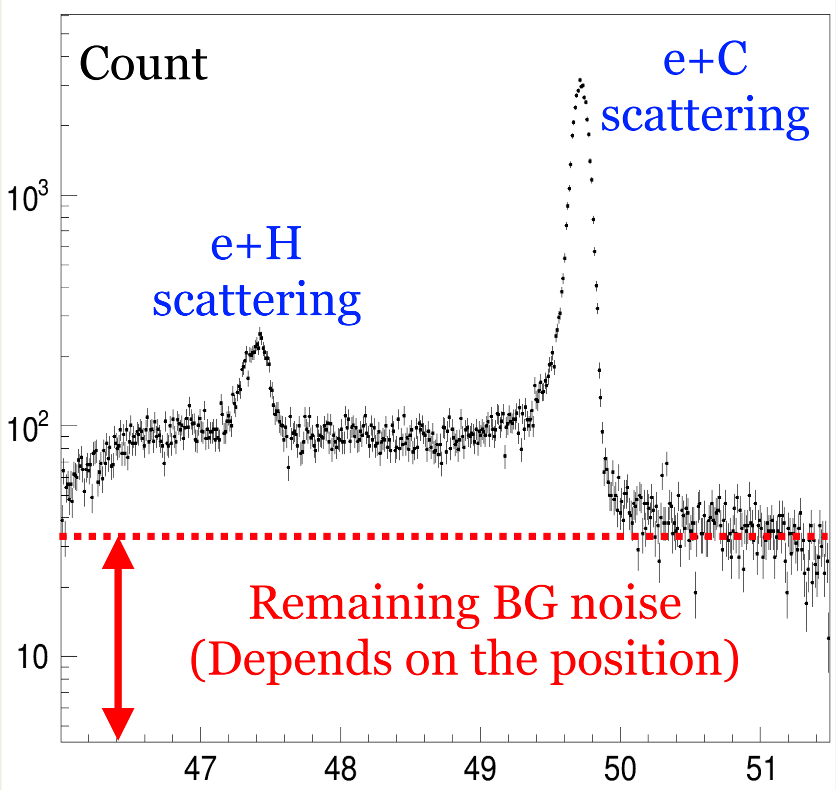
November 2020



Momentum [MeV/c]

Pb-shields around detectors and BG noise sources

November 2021



Momentum [MeV/c]

New noise reduction measures ongoing!

Following experimental tasks

- New more precise measurement of $(x_d | \delta) \rightarrow$ increase E and Q^2 precision
- BG noise study: reduction of the remaining noise
- Detector efficiency study: position dependence with 10^{-3} accuracy

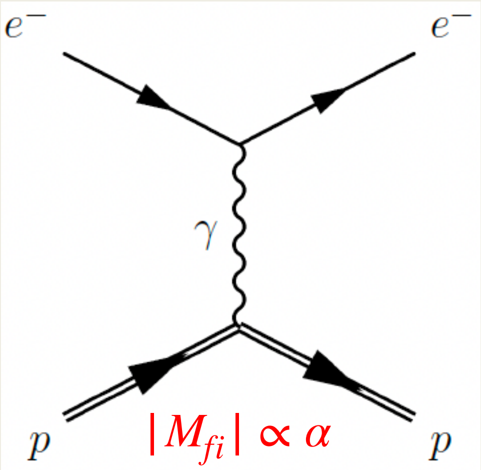
$\Rightarrow$ Physics runs should start at the end of this year!

IV. Different corrections for the ULQ2 experiment

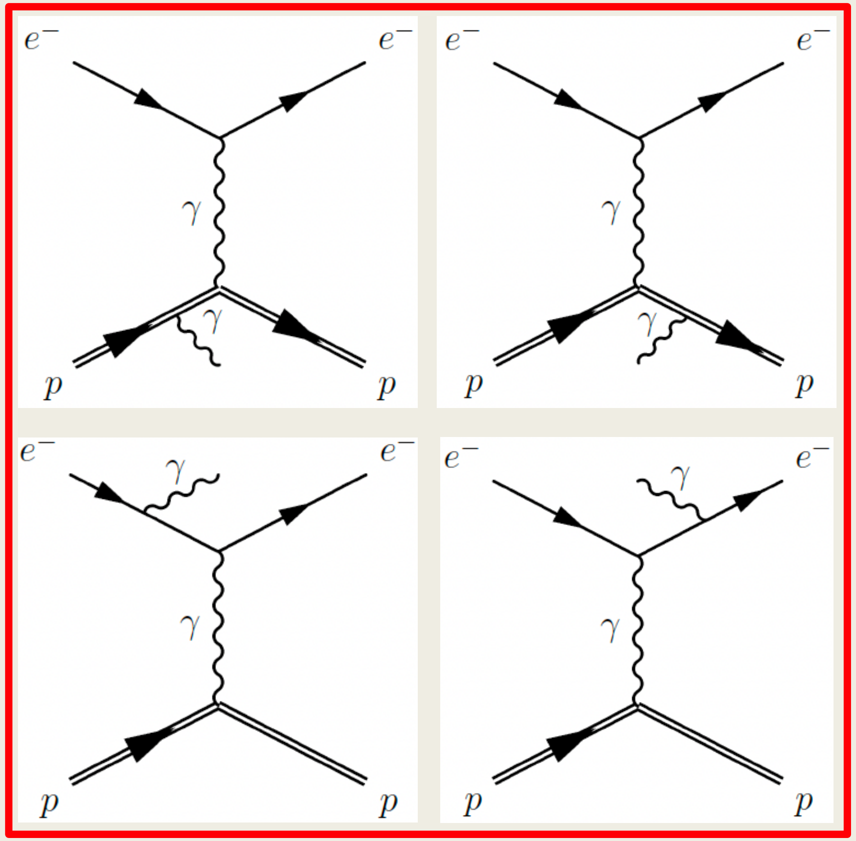
Radiative corrections

- Measure of the cross-section of the leading-order of e+p and e+C with an accuracy of 10^{-3}
- Actually, many other diagrams ($eX \rightarrow eX\gamma$, vertex corrections, …)

Radiative corrections

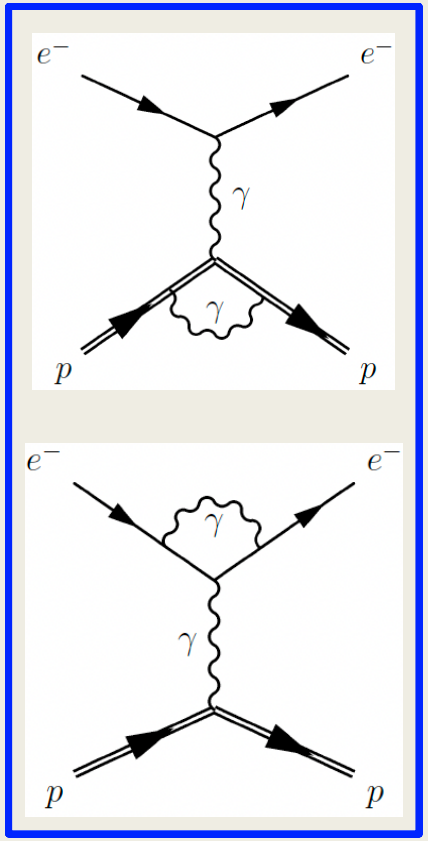


Leading order



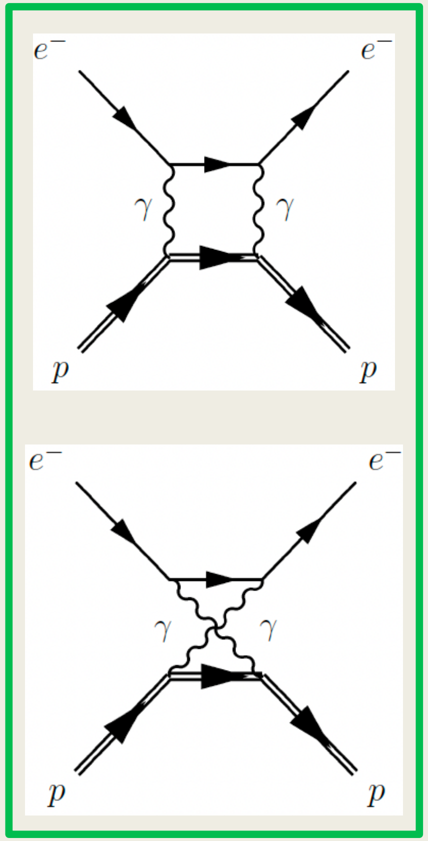
Bremsstrahlung

$|M_{fi}| \propto \alpha^{3/2}$



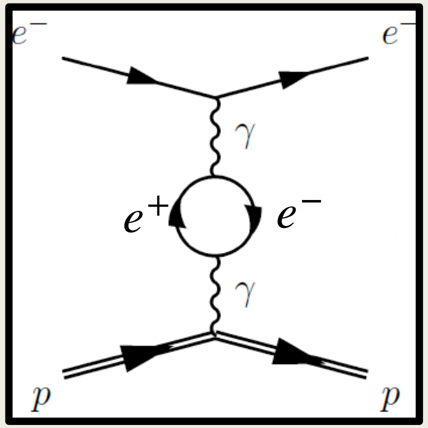
Vertex corrections

$|M_{fi}| \propto \alpha^2$



Two-photon exchange

$|M_{fi}| \propto \alpha^2$



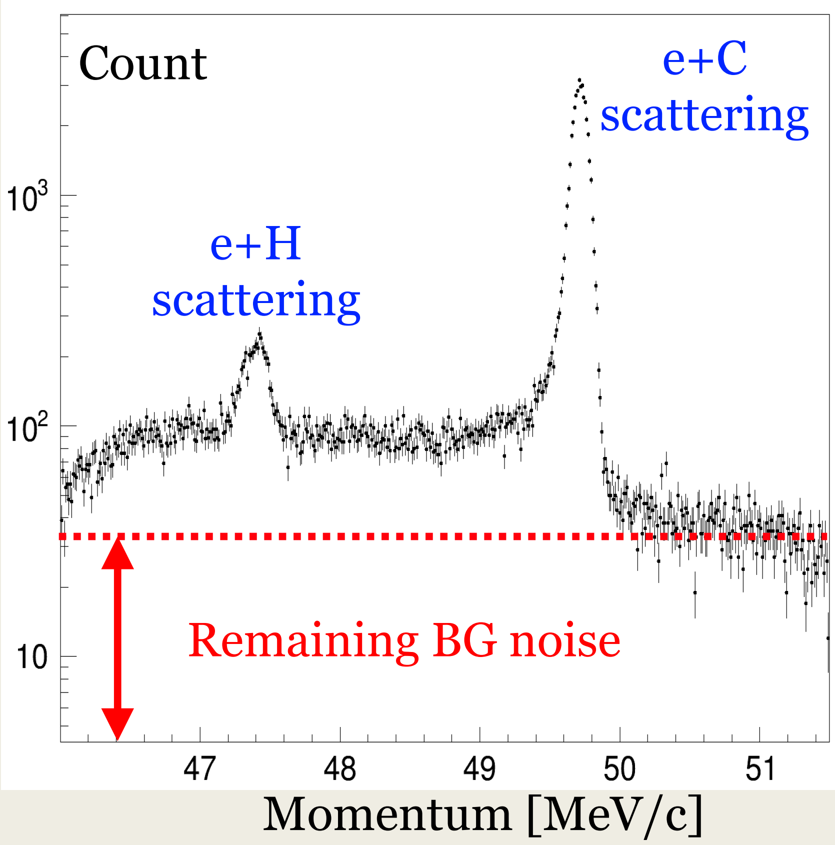
Vacuum polarization

$|M_{fi}| \propto \alpha^2$

+ Higher-order terms

Radiative corrections

- Measure of the cross-section of the leading-order of e+p and e+C with an accuracy of 10^{-3}
- Actually, many other diagrams (eX $\rightarrow$ eXγ, vertex corrections, ...)
- Need to fit the radiative tail of H and especially the radiative tail of C
- The yield of the H peak strongly depends on the parametrization of the C radiative tail ...



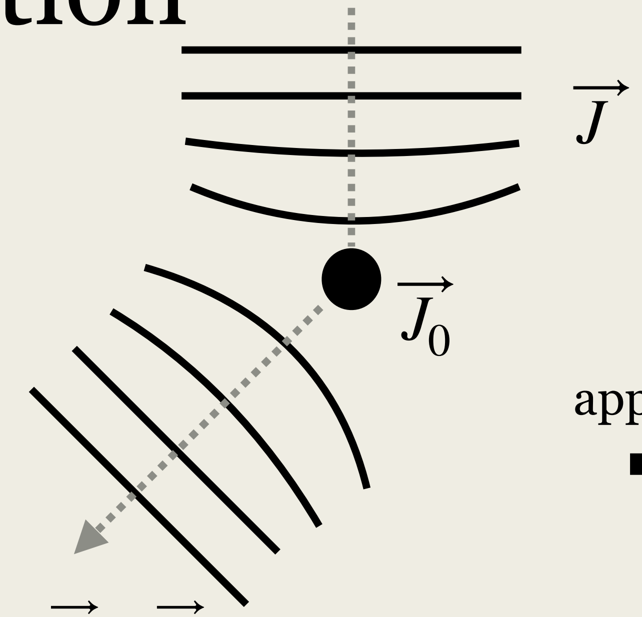
Radiative corrections

- Current parametrization: convolution of three functions: J.Friedrich, Nucl. Inst. Meth. **129** (1975), 505-514
  - $\frac{d^2\sigma}{d\Omega dE_f} = \frac{d\sigma}{d\Omega} g(E_f)$ with $g(E_f) = g^S * g^{rad} * g^{col}(E_f)$
  - $g^S(E_f)$: internal radiative corrections (Schwinger corrections, ...)
  - $g^{rad}(E_f)$: external radiative corrections (Bremsstrahlung, ...)
  - $g^{col}(E_f)$: energy loss in the target (Landau)

$\Rightarrow$ Insufficient parametrization of the radiative tail ...

We welcome your contribution!

Coulomb distortion



Plane-wave distortion
near the nucleus

First order Born
approximation not enough
$\Rightarrow$ Second order

Elastic scattering: $\vec{J_0} + \vec{J} = \vec{J_0}$

Proton nucleus: $J_0 = \frac{1}{2}^+$

$\psi = \begin{pmatrix} F \\ G \end{pmatrix}$ solution of the Dirac equation with a Coulomb potential

Coulomb ($C0$) scattering

Magnetic ($M1$) scattering

$\begin{pmatrix} F \\ G \end{pmatrix} \rightarrow \begin{pmatrix} F \\ G \end{pmatrix}$

$\begin{pmatrix} F \\ G \end{pmatrix} \rightarrow \begin{pmatrix} G \\ F \end{pmatrix}$

The magnetic scattering changes the electron wave spin orientation!

Coulomb distortion

- Classical treatment of the Coulomb distortion: Feshbach correction

$$\left(\frac{d\sigma}{d\Omega}\right)_{Fesh} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E} \frac{(1 + Z\alpha\delta_{Fesh})\epsilon G_E^2(Q^2) + (1 + Z\alpha\delta_{Fesh})\tau G_M^2(Q^2)}{\epsilon(1+\tau)},$$

$$\text{with } \delta_{Fesh} = \frac{\pi \sin\frac{\theta}{2}(1 - \sin\frac{\theta}{2})}{\cos^2\frac{\theta}{2}}, \tau = \frac{Q^2}{4M_p^2} \text{ and } \epsilon^{-1} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

Assumption: Same correction for Coulomb and magnetic scattering.

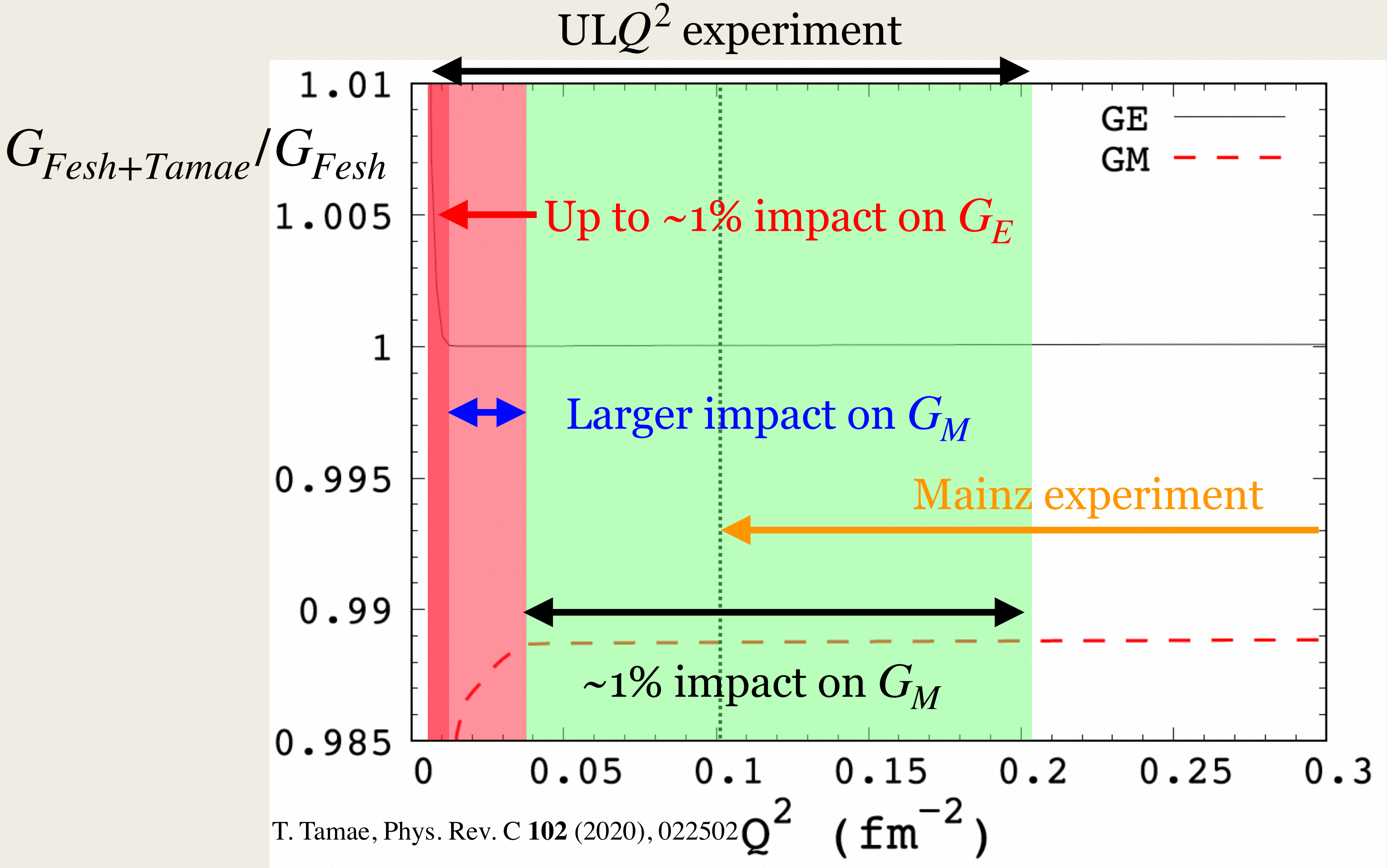
Coulomb distortion

- Tamae's treatment: Different corrections for G_E and G_M T. Tamae, Phys. Rev. C **102** (2020), 022502

$$\left(\frac{d\sigma}{d\Omega}\right)_{Fesh+Tamae} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E} \frac{\left(1 + Z\alpha\delta_{Fesh}\right)\epsilon G_E^2(Q^2) + \left(1 + Z\alpha\delta_{Tamae}\right)\tau G_M^2(Q^2)}{\epsilon(1+\tau)},$$

$$\text{with } \delta_{Fesh} = \frac{\pi \sin\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right)}{\cos^2\frac{\theta}{2}} \text{ and } \delta_{Tamae} = \frac{\pi \sin\frac{\theta}{2}\left(2 + \sin\frac{\theta}{2}\right)}{1 + \sin^2\frac{\theta}{2}}$$

Coulomb distortion



Ratio of form factors where Feshbach and Tamae are applied and only Feshbach correction is applied.

Necessary for the determination of G_M (and G_E at ultra low Q^2)!!

Electron mass

- Ultra-relativistic limit: $m_e \ll E$ ($m_e \to 0$)

$$\left(\frac{d\sigma}{d\Omega}\right)_{URL} = \boxed{\frac{\alpha^2 Z^2 \cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}}\frac{E'}{E}\frac{\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\epsilon(1+\tau)}$$

$\left(\frac{d\sigma}{d\Omega}\right)_{Mott}$

$(1 + Z\alpha\delta_{Fesh})$ $(1 + Z\alpha\delta_{Tamae})$

with $Q^2 = 4EE'\sin^2\frac{\theta}{2}$, $\tau = \frac{Q^2}{4M_p^2}$ and $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$

- At 10 MeV, $\frac{m_e}{E} \sim 0.05$ ➔ URL not justified

Electron mass

- Without the ultra-relativistic limit: E. Borie, arXiv:1207.6651v4

$1 + Z\alpha\delta_{Fesh}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{real} = \frac{\alpha^2 Z^2}{Q^4} \frac{P'/P}{1 + (E - (PE'/P')\cos\theta)/M_p} [(G_E^2(Q^2) \frac{4EE' - Q^2}{1 + Q^2/4M_p^2}$$

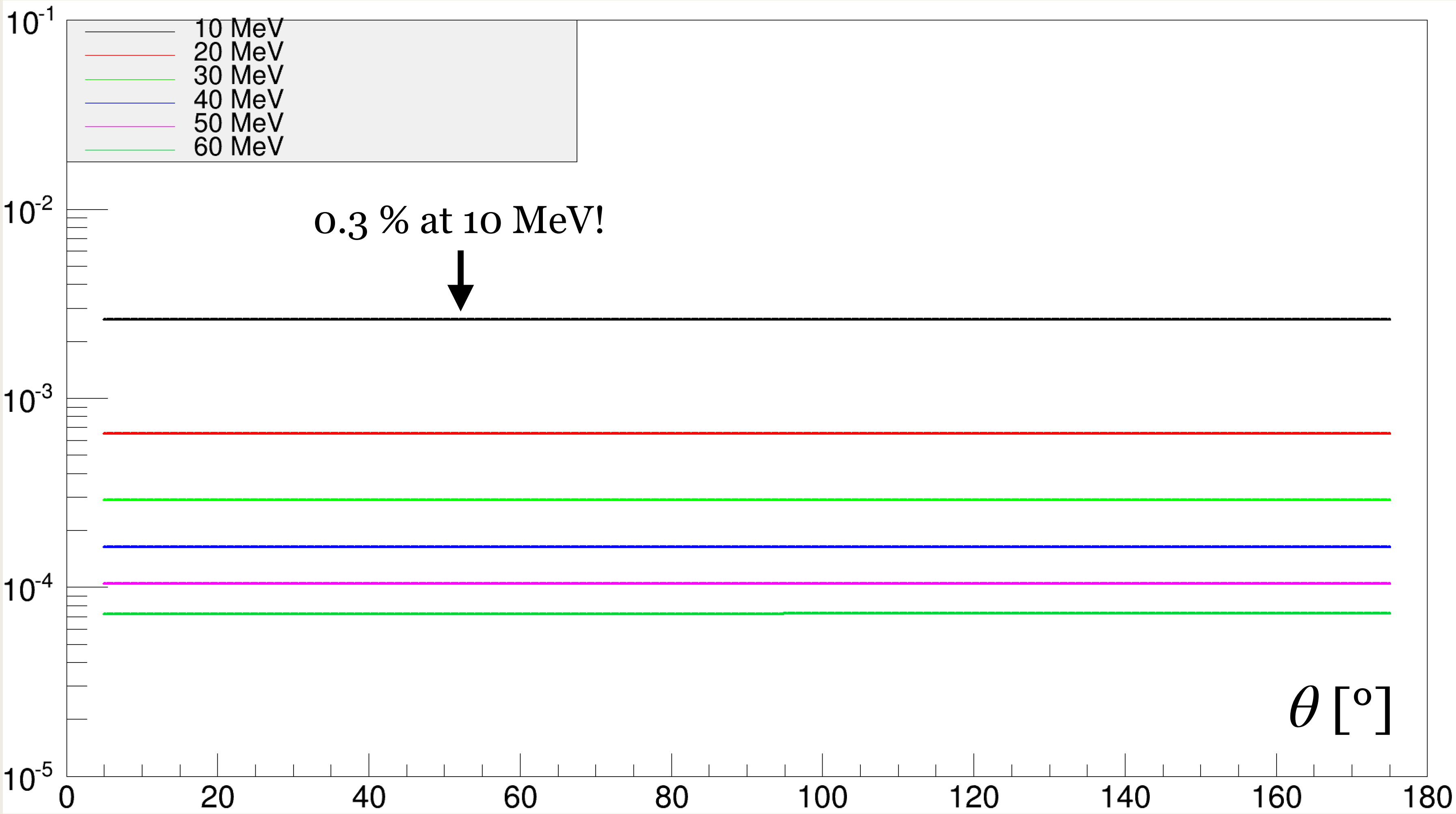
$$+ G_M^2(Q^2)((4EE' - Q^2)(1 - \frac{1}{1 + Q^2/4M_p^2}) + \frac{Q^4}{2M_p^2} - \frac{Q^2 m_e^2}{M_p^2})]$$

$1 + Z\alpha\delta_{Tamae}$

with $Q^2 = (\vec{P} - \vec{P'})^2 - (E - E')^2$

Electron mass

$$\frac{|Q^2_{real} - Q^2_{URL}|}{Q^2_{real}}$$

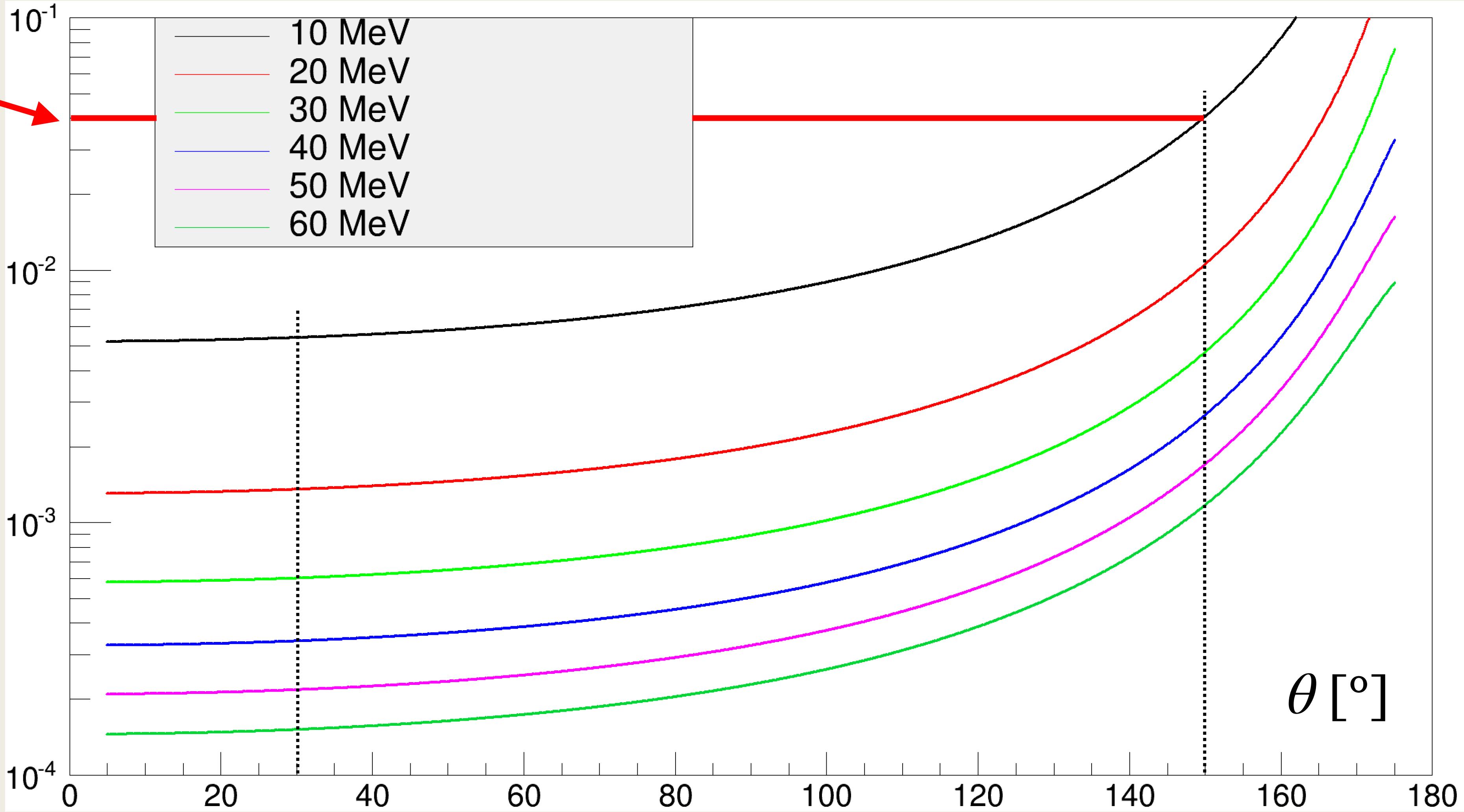


Electron mass

Up to 4 % at 10 MeV!

$$\frac{|(\frac{d\sigma}{d\Omega})_{real} - (\frac{d\sigma}{d\Omega})_{URL}|}{(\frac{d\sigma}{d\Omega})_{real}}$$

(with PWBA and standard-dipole form factors)



Summary

- ❑ Radiative corrections: several % correction
- ❑ Electron mass: up to 4% correction
- ❑ Tamae correction: up to 1% correction on G_E and G_M, especially at low Q^2

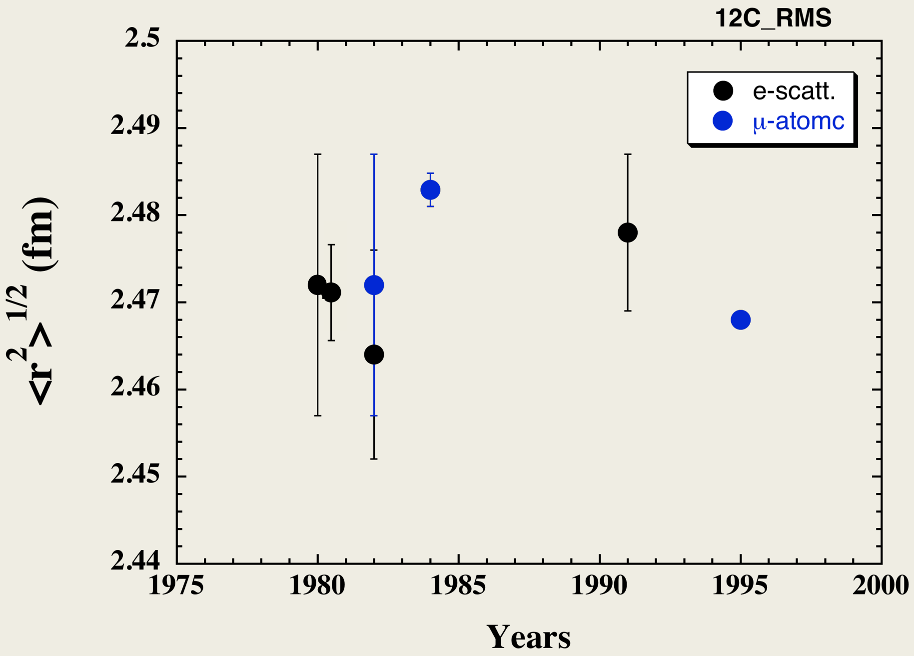
➡ First measurement of the proton form factors starting tomorrow!
We aim at observing the effect of these corrections!

THANK YOU FOR YOUR ATTENTION

^{12}C cross section

- Several measurements of the electric form factor of ^{12}C with electron scattering

- Precise measurement of the carbon charge radius with $\frac{\delta r_C}{r_C} < 10^{-3}$ with $\mu\,^{12}$C



Several re-analysis and compilations since 1995

→ $r_{^{12}C} = 2.4702(22)$ fm

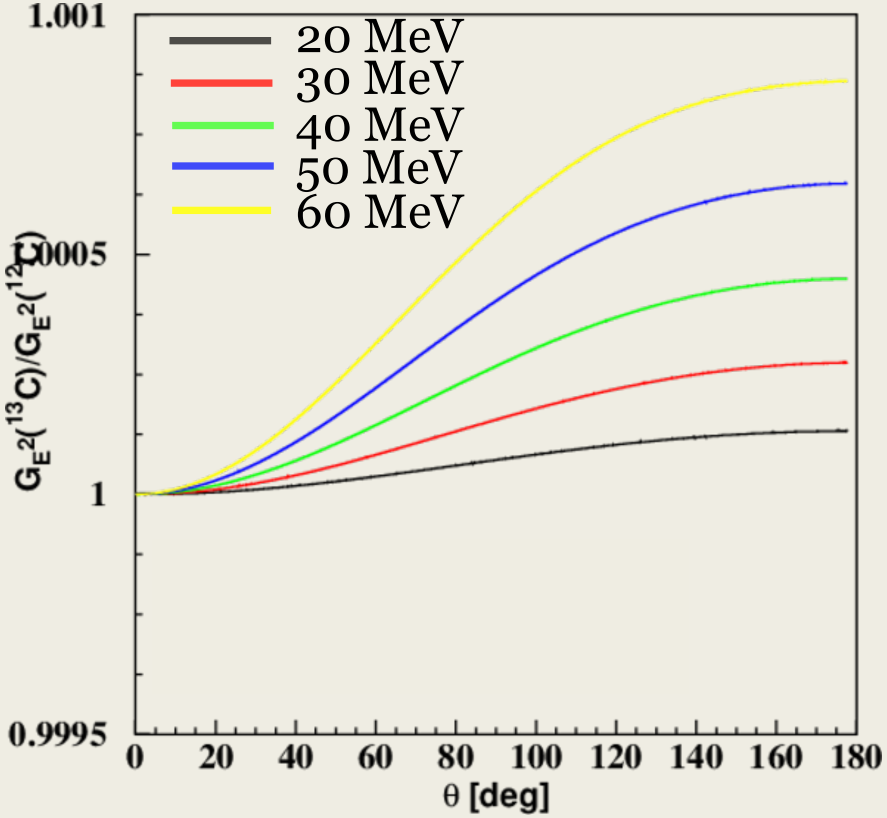
I. Angeli *et al.*, Atom. Data and Nucl. Data Tab. **99** (2013) 69–95

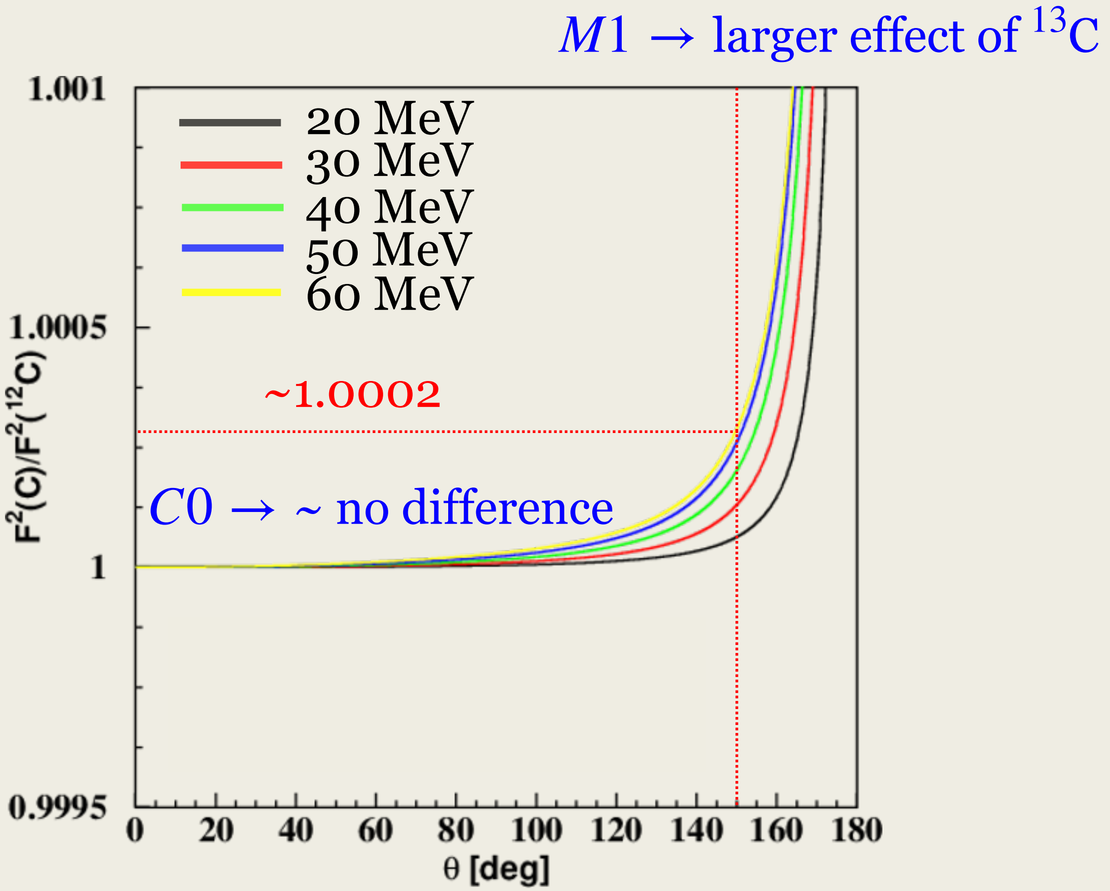
→ Determination of the electric form factor of ^{12}C at low Q^2 with 10^{-3} accuracy

^{12}C vs natural C

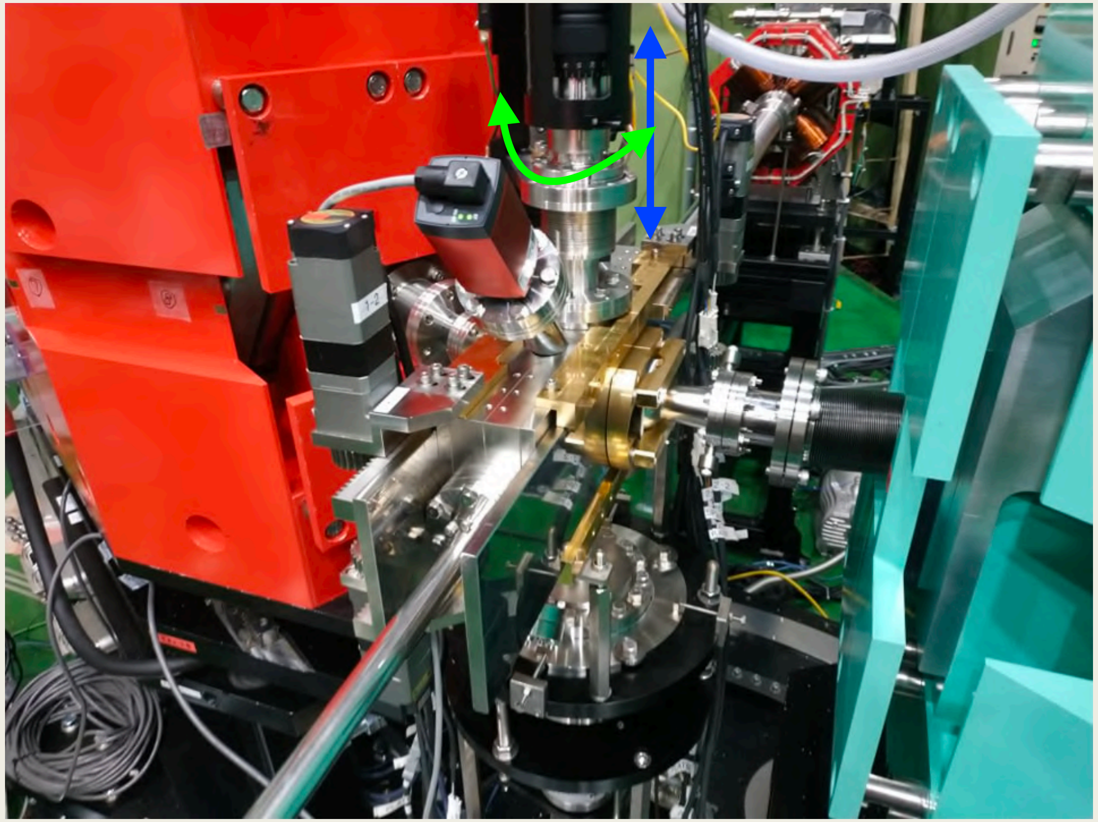
$^{nat}C = 98.9\%^{12}C + 1.1\%^{13}C$

- Very small effect of ^{13}C ~ order of 10^{-4} in the context of the ULQ2 experiment





Variable-angle target chamber



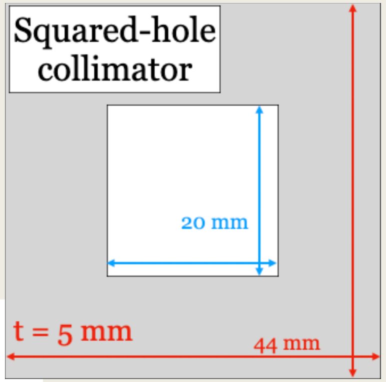
Determination of the momentum dispersion

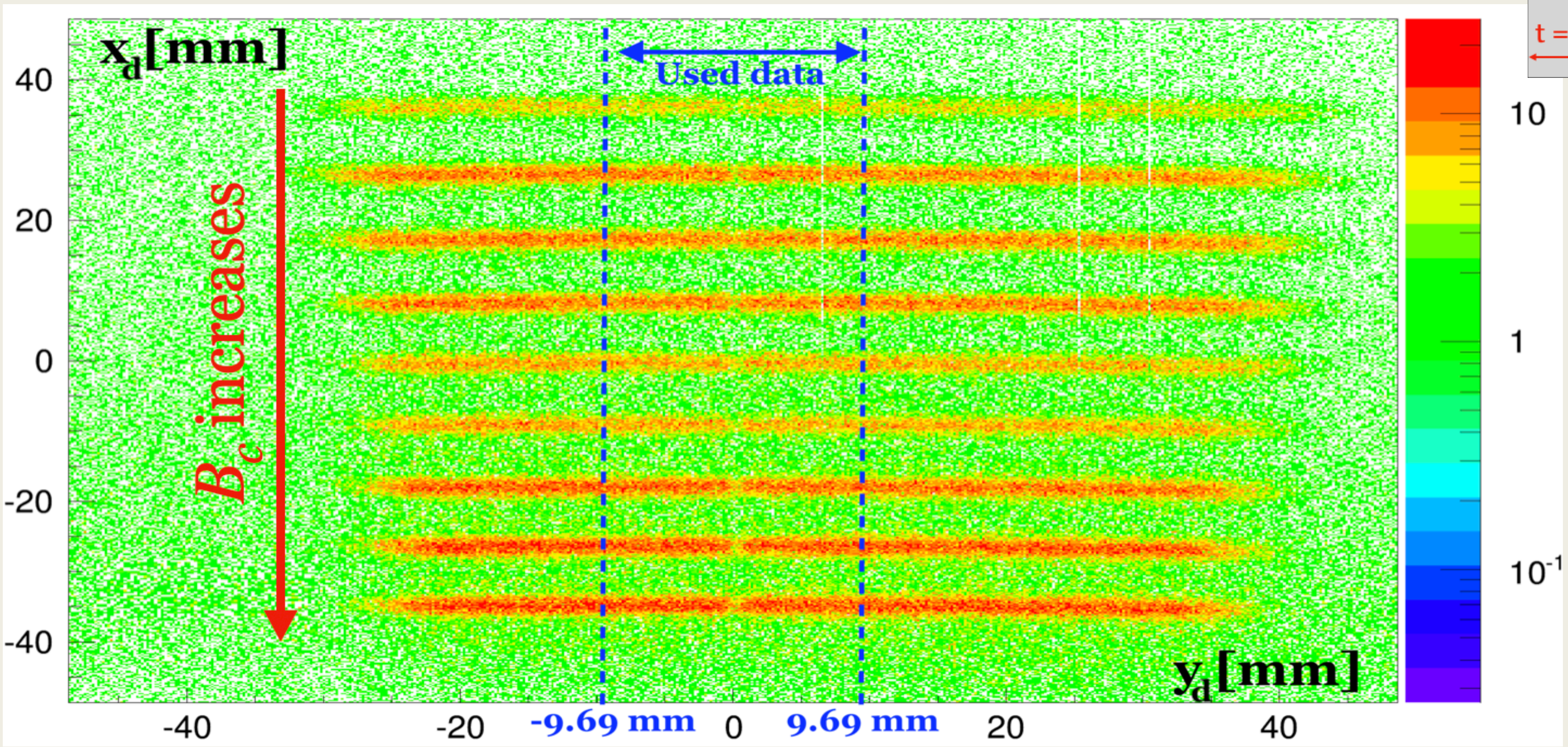
2021/11
CH_2 target (130 um)
θ = 90°
E = 50 MeV
I ~ 10 nA, t = 30 min

SSD position
Fixed term

Momentum dispersion $(x_d|\delta)$:

$$x_d = x_0 + (x_d|\delta)\delta + (x_d|\delta^2)\delta^2,\ \delta = \frac{B_s - B_c}{B_c}$$

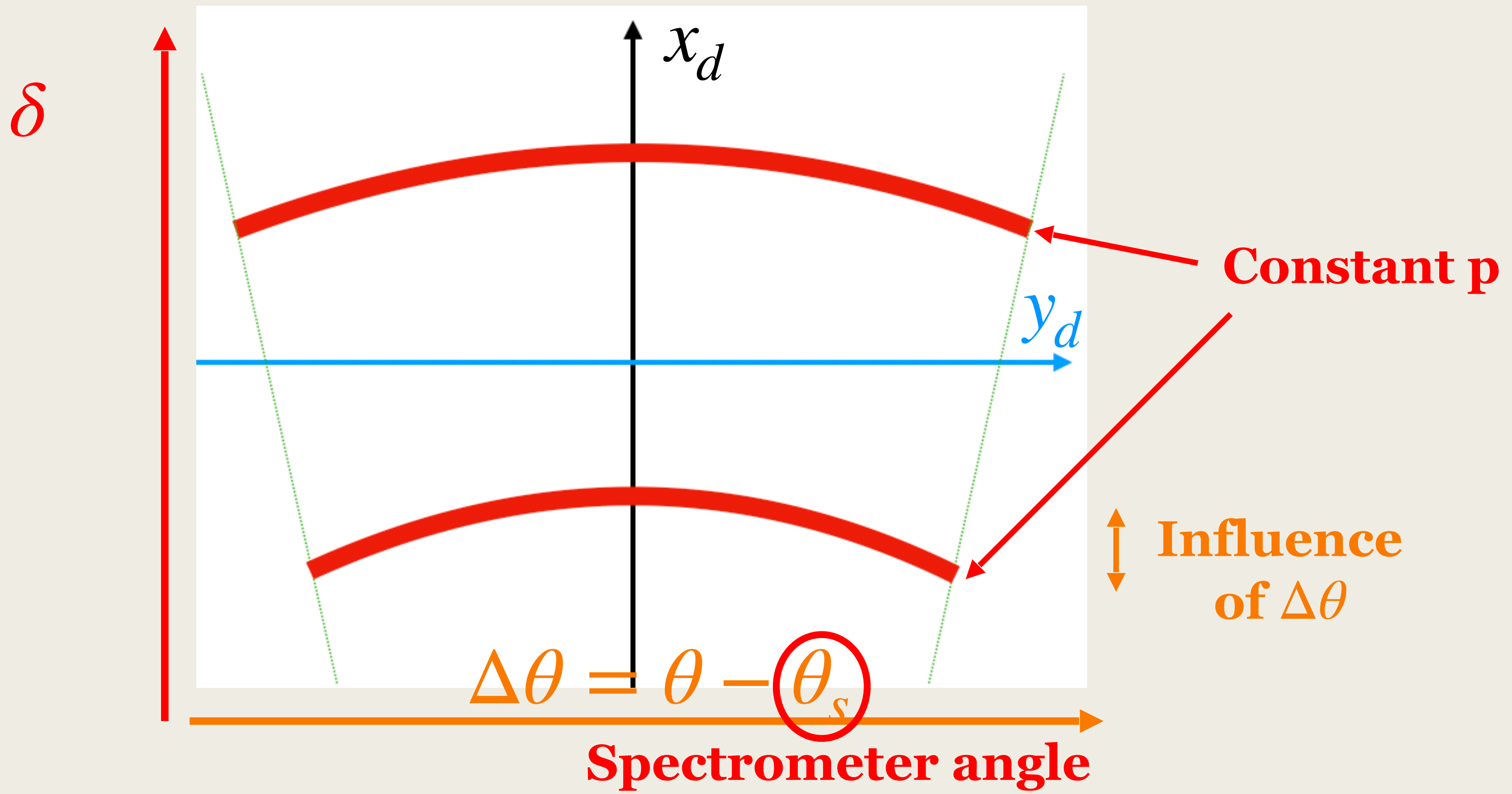




Position of the electrons on the detectors.

Matrix elements:

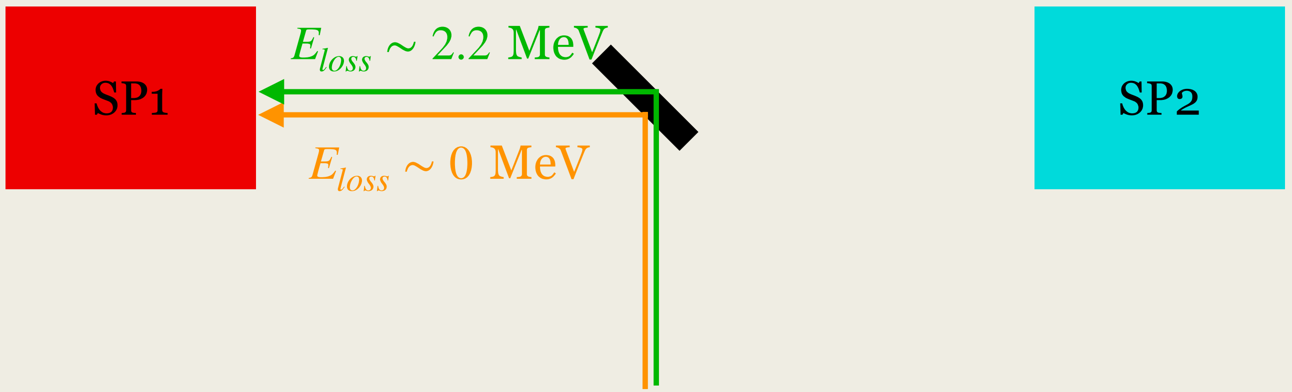
$$x_d = x_0 + (x_d|\delta)\delta + (x_d|\delta^2)\delta^2 + (x|\Delta\theta^2)(\Delta\theta - \theta_0)^2 + (x_d|x_b)x_b,$$
$$y_d = [(y_d|\Delta\theta) + (y_d|\delta\Delta\theta)\delta]\Delta\theta + (y_d|y_b)y_b$$



Detector efficiency

- Use of a 2-mm-thick C target → ΔE_{loss} = 2.2 MeV

- With E=20 MeV, $\dfrac{\Delta E_{loss}}{E} \sim 10\%$ → completely covers the detector surface



Detector efficiency