Constraining Tidal deformability from terrestrial data

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Neutron stars vis-a-vis Finite Nuclei

Properties	Neutron star	Finite nuclei	Ratio
Force	Gravitational	Nuclear	10^{-40}
Composition	$n, p, e^-, \mu^-, ..$	n, p	
B/A	100 MeV	8 MeV	10^1
Radius	10 -15 km	3-8 fm	10^{18}
Mass	10^{30} kg	1- 300 amu	10^{55}
No. neucleons	10^{57}	1-300	10^{55}
Central no. density	2 - 6 ρ_0	ρ_0	10^1
Asymmetry	$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \sim 0.7$	$\delta = \frac{N-Z}{A} = 0.3$	10^0

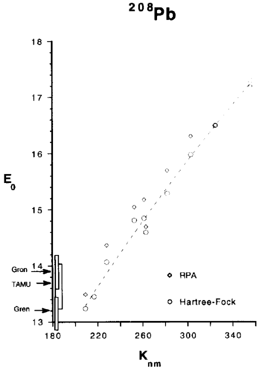
EOS and Nuclear matter parameters

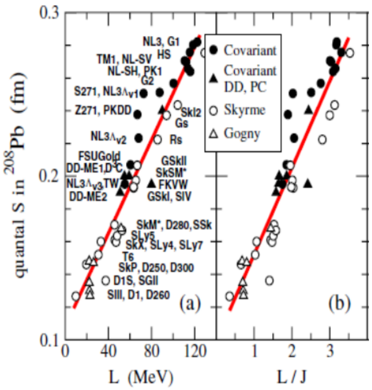
$$
\begin{aligned}
e(\rho,\delta) &= e_{snm}(\rho) + e_{sym}(\rho)\delta^2 \\
e_{snm}(\rho) &= e_0 + \frac{1}{2}K_0\alpha^2 + \frac{1}{3!}Q_0\alpha^3 + \frac{1}{4!}Z_0\alpha^4 + \ldots \\
e_{sym}(\rho) &= J_0 + L_0\alpha + \frac{1}{2}K^0_{sym}\alpha^2 + \frac{1}{3!}Q^0_{sym}\alpha^3 + \ldots \\
\alpha &= \frac{\rho-\rho_0}{3\rho_0} \\
\delta &= \frac{\rho_n-\rho_p}{\rho_n+\rho_p}
\end{aligned}
$$

$e_{\rm snm}(\rho)$ and $e_{\rm sym}(\rho)$ from mean field models
Parameters fitted to basic nuclear properties

Relations of NMPs with EoS

$$
\begin{aligned}
e_0 &= e_{snm}(\rho_0) \\
K_0 &= 9(\rho_0)^2 e''_{snm}(\rho_0) \\
Q_0 &= 27(\rho_0)^3 e'''_{snm}(\rho_0) \\
J_0 &= e_{sym}(\rho_0) \\
L_0 &= 3(\rho_0) e'_{sym}(\rho_0) \\
K_{sym,0} &= 9(\rho_0)^2 e''_{sym}(\rho_0) \\
Q_{sym,0} &= 27(\rho_0)^3 e'''_{sym}(\rho_0)
\end{aligned}
$$





Centelles et.al PRL 102, 122502 (2009)

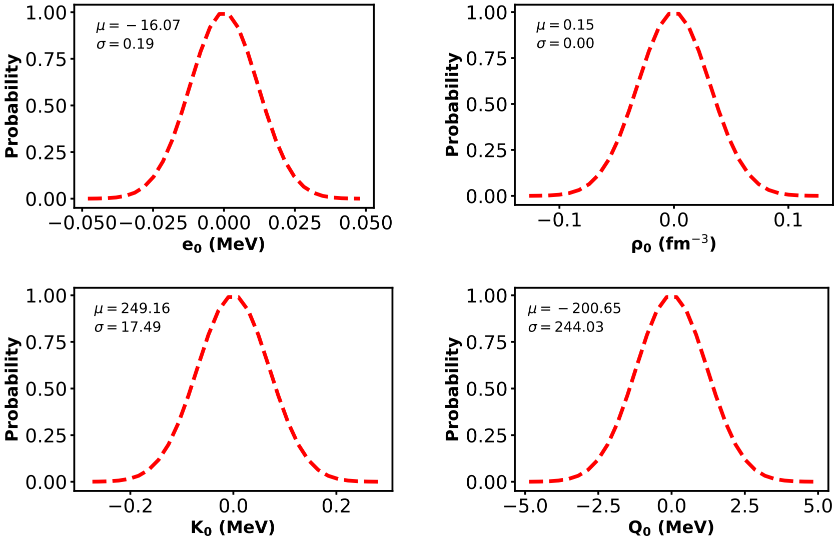
Blaizot et.al NPA 591,(1995)

Constraining the NMPs

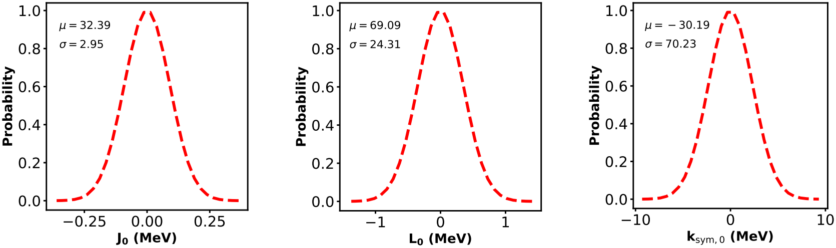
Binding energy, Charge radii $\rightarrow \rho_0, e_0, J_0$
ISGMR $\rightarrow K_0$
Δr_{np}, IVGDR $\rightarrow L_0$
Canonical values for few NMP's,

$$\begin{aligned} \rho_0 &= 0.16\text{fm}^{-3} \\ e_0 &= -16.0 \text{ MeV} \\ K_0 &= 240.0 \text{ MeV} \\ J_0 &= 32\text{MeV} \\ L_0 &= 20 - 80\text{MeV} \end{aligned}$$

Distributions of NMPs from mean-field models

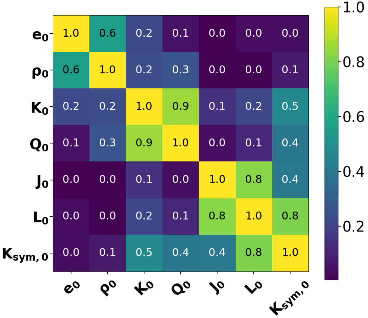


Distributions of NMPs from mean-field models

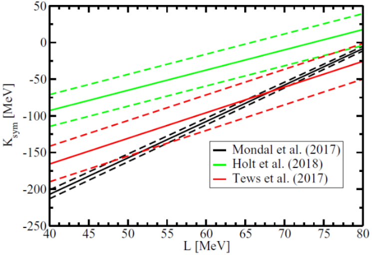


Correlations among Nuclear matter Parameters

B. K. Agrawal, *et.al Phys. Rev. C102, 052801 (2020)*



Bao-An Li, *et.al Phys. Rev. C102, 045807 (2020)*



Tidal deformability for binary system

$$\Lambda = \frac{2}{3}\kappa_2 \left(\frac{R}{m}\right)^5$$

$$EOS \rightarrow sol. TOV\, Eqs. \rightarrow M, R, \kappa_2$$

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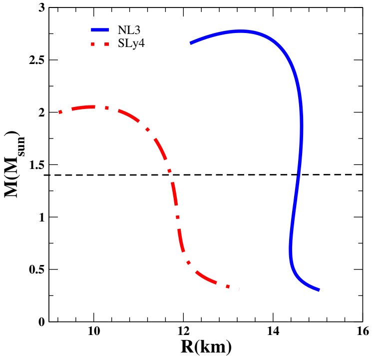

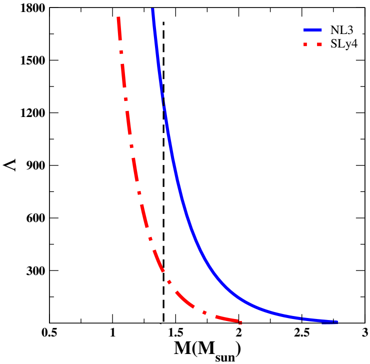

Combined Tidal deformability for binary system

$$\tilde{\Lambda}(q) = \frac{16}{13}\frac{(12q+1)\Lambda_{m_1} + (12+q)q^4\Lambda_{m_2}}{(1+q)^5}$$

$$q = \frac{m_2}{m_1} \leq 1$$

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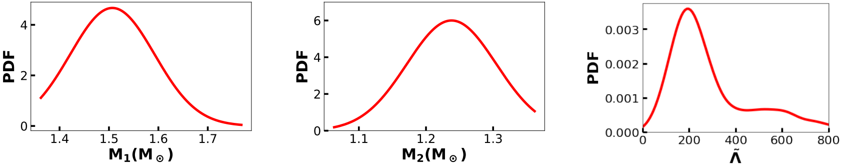

Combined Tidal deformability for binary system

$$\begin{aligned}\frac{\Lambda_{m_2}}{\Lambda_{m_1}} &= q^6 \\ \tilde{\Lambda}(q=0.95) &= 1.17\Lambda_{1.4} \\ m_1 &= 1.4M_\odot, \\ m_2 &= 1.33M_\odot\end{aligned}$$

Current estimation:

$$\Lambda_{1.4} = 190^{+390}_{-120}$$

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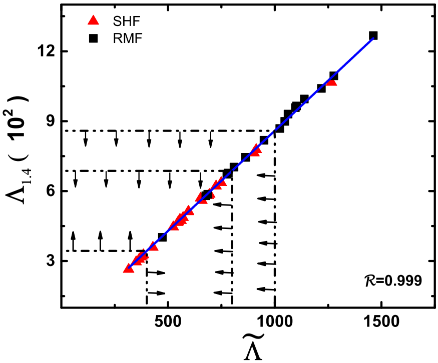
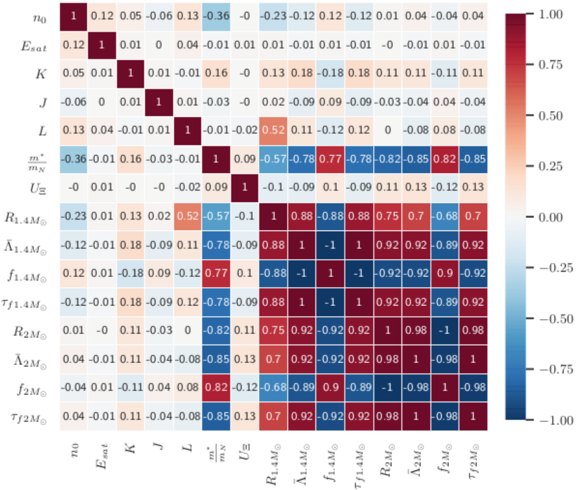
B.K.Agrawal et.al, PRC 98, 035804(2018)



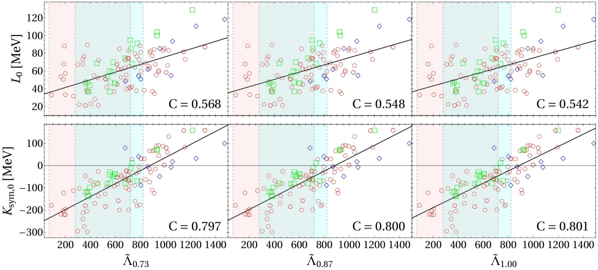
Correlations of NMPs with NS properties

NMP distribution - Uniform
B. K. Pradhan, *et.al arxiv:2203.03141 (2022)*

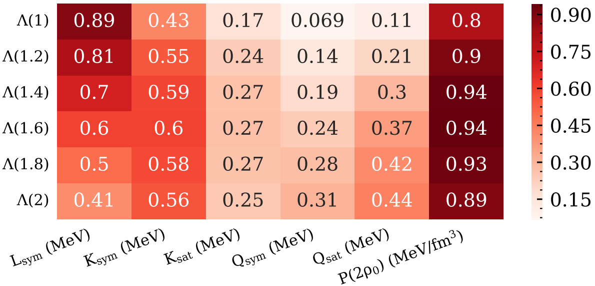


Correlations of NMPs with NS properties

NMP distribution - Uniform
Z. Carson, *et.al Phys. Rev. D99, 043010 (2019)*

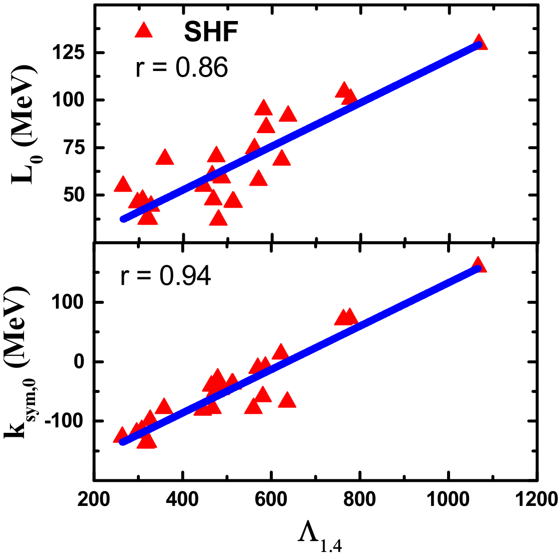


NMP distribution - Gaussian
C. Y. Tsang, *et.al Phys. Rev. C102, 045808 (2020)*



Correlations of NMPs with NS properties

Agrawal et.al CRC press Nuclear Structure Physics, 317-332 2020



Multivariate Gaussian distribution (MVGD) for NMPs

.

$$\begin{aligned} \text{EoS}_n^{\text{Skyrme}} &= \{\text{NMPs}\}_n \\ &\sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \end{aligned}$$

NMPs :$\{e_0, \rho_0, K_0, Q_0, J_0, L_0 K_{\text{sym},0}\}_n$
$N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$: MVGD for NMPs
$\boldsymbol{\mu}$: mean value of the NMPs
$\boldsymbol{\Sigma}$: covariance matrix

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	MVGD	
p_i	μ_i	$\sqrt{\Sigma_{ii}}$
e_0	-16.0	0.25
ρ_0	0.16	0.005
K_0	230.0	20
Q_0	-300	100
J_0	32.0	3
L_0	60.0	20
$K_{\text{sym},0}$	-100.0	100

MVGD-I: $\Sigma_{i,j} = 0, i \neq j$ MVGD-II: $\Sigma_{L_0,K_{\rm sym,0}} \neq 0$

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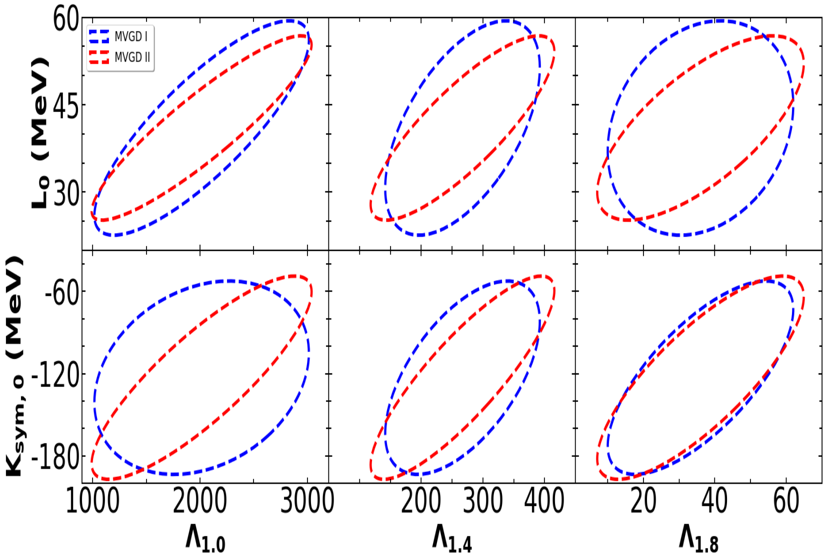


B.K.Agrawal et.al PRC 102 (5), 052801, (2020) Correlation coefficients of Λ_m with L_0 and $K_{sym,0}$ for m = 1.0, 1.4 and 1.8 $M_\odot$.

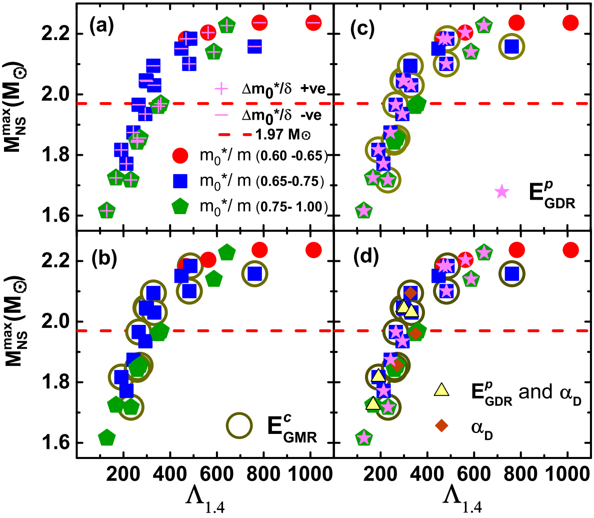
Model		$\Lambda_{1.0}$	$\Lambda_{1.4}$	$\Lambda_{1.8}$
MVGD-I	L_0	0.82	0.56	0.22
	$K_{\mathrm{sym},0}$	0.26	0.58	0.71
MVGD-II	L_0	0.9	0.83	0.7
	$K_{\mathrm{sym},0}$	0.84	0.86	0.86

Correlations of $\Lambda_{1.4}$ with L_0 and $K_{\mathrm{sym},0}$

Model	Dist. NMPs	L_0	$K_{\mathrm{sym},0}$
RMF	Uniform-uncorrelated	0.11	–
Metamodel	Uniform-uncorrelated	0.54	0.80
ELF	Gaussian-uncorrelated	0.7	0.59
Skyrme	Gaussian-uncorrelated	0.56	0.58
Skyrme	Gaussian-correlated	0.84	0.86

Nuclear observables and Tidal deformability

B.K.Agrawal et.al PRC 99, 052801(2019)



Covariance approach

$$\chi^2(\mathbf{p}) = \frac{1}{N_d - N_p}\sum_{i=1}^{N_d}\left(\frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}(\mathbf{p})}{\Delta\mathcal{O}_i}\right)^2,$$

$$\mathcal{C}_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial^2\chi^2(\mathbf{p})}{\partial \mathrm{p}_\alpha \partial \mathrm{p}_\beta}\right)_{\mathbf{p_0}},$$

$$\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}} = \sum_{\alpha\beta}\left(\frac{\partial\mathcal{A}}{\partial \mathrm{p}_\alpha}\right)_{\mathbf{p_0}} \mathcal{C}_{\alpha\beta}^{-1}\left(\frac{\partial\mathcal{B}}{\partial \mathrm{p}_\beta}\right)_{\mathbf{p_0}},$$

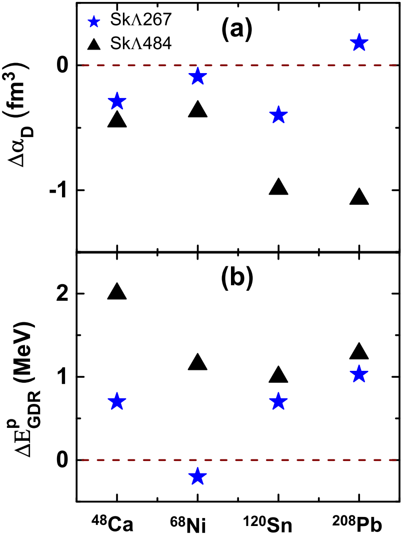
$$c_{\mathcal{AB}} = \frac{\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}}}{\sqrt{\overline{\Delta\mathcal{A}^2}\;\overline{\Delta\mathcal{B}^2}}},$$

Fit data for Skyrme models.

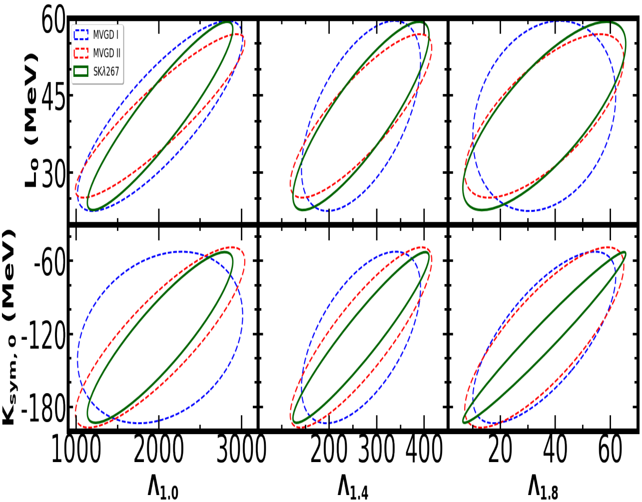
Quantity	Nuclei	Exp. value	Adopted error
Binding energy (MeV)	^{16}O	-127.62	4
	^{40}Ca	-342.05	3
	^{48}Ca	-416.00	1
	^{56}Ni	-483.99	2
	^{100}Sn	-825.30	2
	^{132}Sn	-1102.84	1
	^{208}Pb	-1636.43	1
	$^{24}\mathrm{O} - ^{16}\mathrm{O}$	-41.34	2
charge radii (fm)	^{16}O	2.70	0.04
	^{40}Ca	3.48	0.02
	^{48}Ca	3.48	0.04
	^{56}Ni	3.75	0.18
	^{132}Sn	4.71	0.02
	^{208}Pb	5.50	0.02
α_D (fm^3)	^{48}Ca	2.07	0.22
	^{120}Sn	8.59	0.36
	^{208}Pb	19.60	0.6
E^c_{GMR} (MeV)	^{90}Zr	17.89	0.2
	^{120}Sn	15.70	0.5
	^{208}Pb	14.17	0.28
$M^{\max}_{\mathrm{NS}}$ ($M_\odot$)		2.16	0.15

SkΛ267		
p_i	μ_i	$\sqrt{\Sigma_{ii}}$
e_0	-16.04	0.2
ρ_0	0.161	0.002
K_0	230.2	6.1
Q_0	-366.8	12.0
J_0	31.4	3.1
L_0	41.1	18.2
$K_{\text{sym},0}$	-124.0	70.2

Quantity		SkΛ267	SkΛ484	EXP
Δr_{np} (fm)	^{208}Pb	0.15(05)	0.21 (04)	0.28(0.07)[PREX II]
α_D	^{40}Ca	2.36(16)	2.52(15)	2.07(22)
	^{68} Ni	3.97(12)	4.25(17)	3.88 (31)
	^{120}Sn	8.99(25)	9.58(36)	8.59 (36)
	^{208}Pb	19.42 (50)	20.67(84)	19.6 (0.6)
$E^{cen}_{\rm GMR}$ (MeV)	^{90}Zr	18.36(12)	18.34(13)	17.81(20)
	^{120}Sn	16.46(12)	16.42(13)	15.70(10)
	^{208}Pb	14.04(11)	13.95(12)	14.17 (0.28)
$R_{1.4}$	NS	11.6(1.0)	13.1(1.1)	$12.71^{+1.14}_{-1.19}$ [NICER]
$\Lambda_{1.4}$	NS	267(144)	484(215)	190^{+390}_{-120} [LIGO Virgo]


SkΛ267
SkΛ484
(a)
(b)
$\Delta\alpha_D$ (fm^3)
ΔE^p_{GDR} (MeV)
0
-1
2
1
0
^{48}Ca
^{68}Ni
^{120}Sn
^{208}Pb

$\Lambda - L_0$ and $\Lambda - K_{\mathrm{sym},0}$ correlations from Covariance analysis



Correlation coefficients of Λ_m with L_0 and $K_{sym,0}$ for m $=$ 1.0, 1.4 and 1.8 $M_\odot$.

		$\Lambda_{1.0}$	$\Lambda_{1.4}$	$\Lambda_{1.8}$
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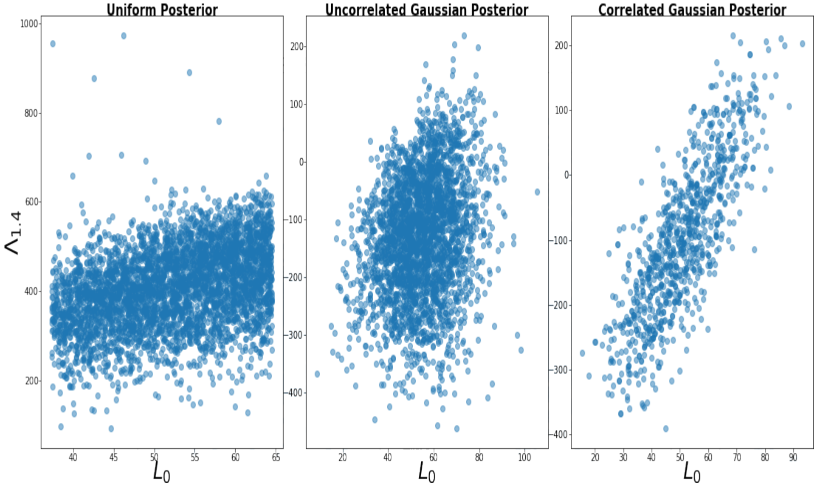
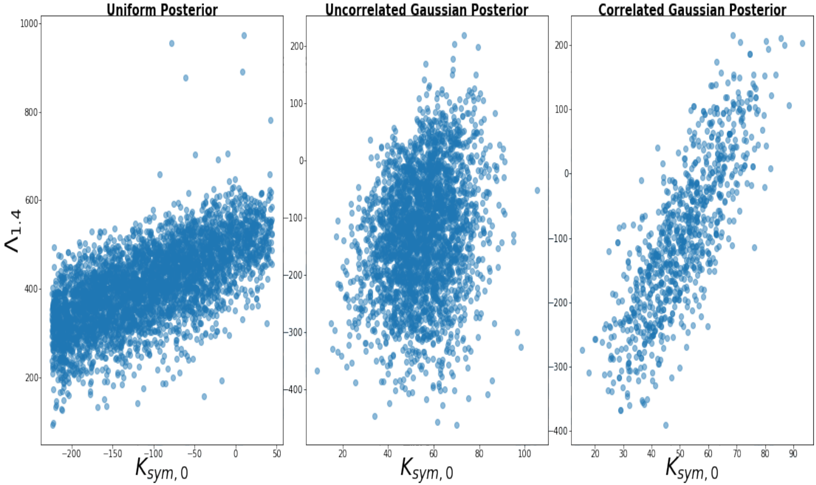
	$K_{\rm sym,0}$	0.84	0.86	0.86
SkΛ267	L_0	0.92	0.85	0.76
	$K_{\rm sym,0}$	0.89	0.94	0.98



Uniform Posterior
Uncorrelated Gaussian Posterior
Correlated Gaussian Posterior
$\Lambda_{1.4}$
L_0
L_0
L_0


Uniform Posterior
Uncorrelated Gaussian Posterior
Correlated Gaussian Posterior
$\Lambda_{1.4}$
$K_{sym,0}$
$K_{sym,0}$
$K_{sym,0}$

Conclusions

- Tidal deformability display strong correlations with the slope and curvature of symmetry energy, provided, finite nuclei constraints are appropriately accounted.
- Correlations of tidal deformability with slope of symmetry energy decreases with the mass of the neutron stars.
- Fit to an extended set of data for finite nuclei complemented with the maximum mass of neutron star yields tidal deformability for the neutron star with $1.4M_{\odot}$ about 100 - 400 (68%) confidance interval.

Collaborators

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