Nuclear Equation of State and Neutron Star Mergers

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Collaborators: **Mahmudul Hasan Anik**, **Spencer Beloin**, **Xingfu Du**, **Jesse Farr**, Stefano Gandolfi, Sophia Han, Craig Heinke, Jeremy Holt, Jacob Lange, **Zidu Lin**, Jérôme Margueron, Zach Meisel, Richard O'Shaughnessy, **Satyajit Roy**, Ingo Tews, **Gema Villegas**,

Outline

- Diversion: Decision Theory
- Multimessenger inference: $P(\varepsilon)$ and pre-merger
- New (n_B, Y_e, T) EOS tables: input for post-merger
- NP3M
- Summary

Diversion: Decision Theory

Taking a Step Back

- Experiment ⇔ data analysis ⇔ theory

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- Decision theory can help us be more efficient

Decision Theory

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- Measure the improvement across many theoretical models by measuring the decrease in the uncertainty of an experiment
- Maximize the information obtained in one nuclear mass measurement at FRIB. Information gain is the KL divergence:

$$D_{\mathrm{KL}}(P_{\mathrm{post}}||P_{\mathrm{prior}}) = \sum_{x \in X} P_{\mathrm{post}}(x) \ln\left(\frac{P_{\mathrm{post}}(x)}{P_{\mathrm{prior}}(x)}\right)$$

Farr, Meisel, and Steiner (2022), with future contributions from Jain and Nazarewicz

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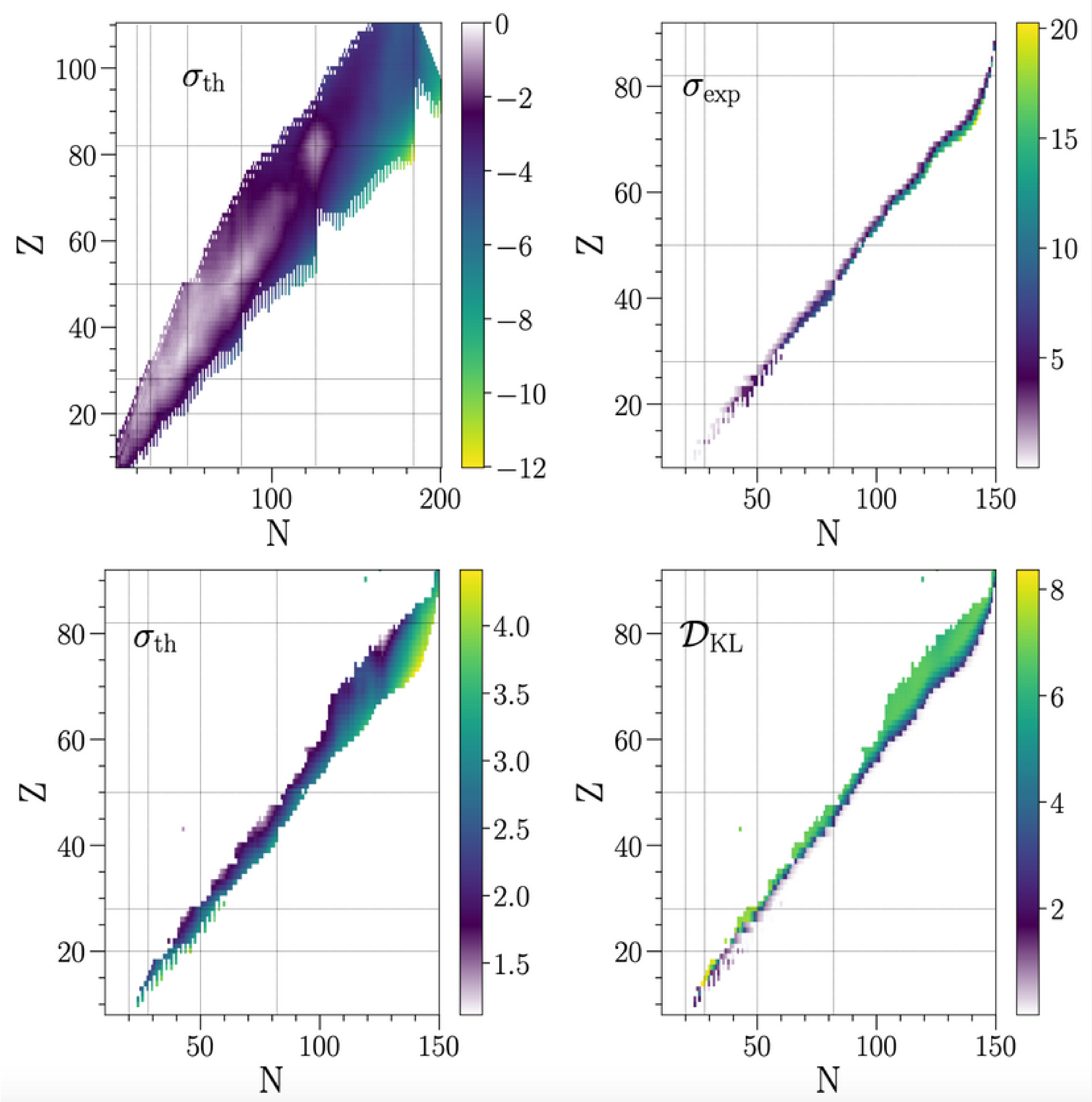
- Presume one-dimensional uncorrelated Gaussians for now

Results for FRIB Masses w.r.t. to Theory Masses

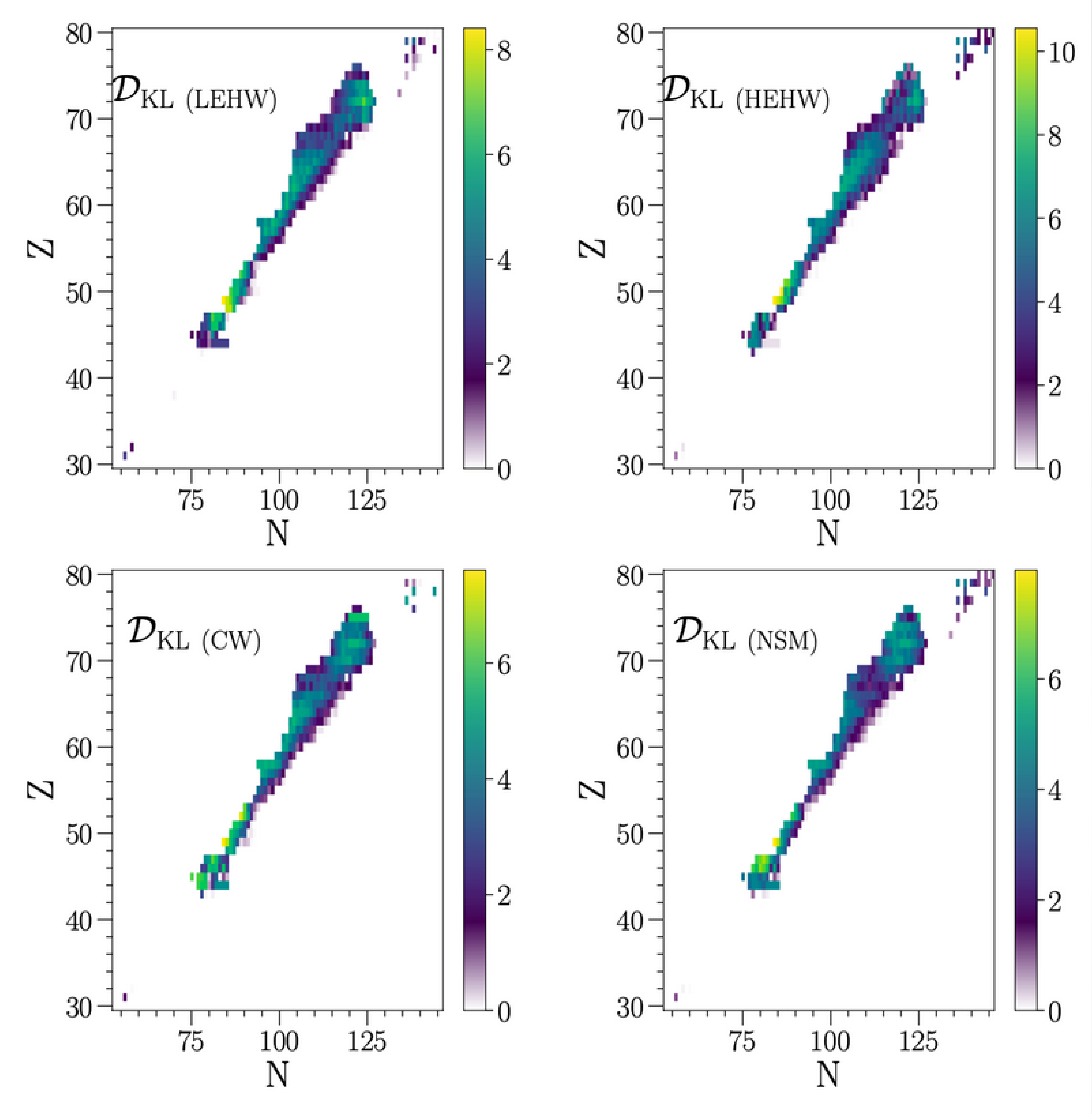


Isotope	Z	N	$\sigma_{\rm ex}$	$\sigma_{\rm th}$	$\sigma_{\rm post}$	$D_{\rm KL}$
^{42}Si	14	28	3.94×10^{-4}	2.78	3.94×10^{-4}	8.36
^{41}Si	14	27	3.82×10^{-4}	2.63	3.82×10^{-4}	8.34
^{43}P	15	28	4.01×10^{-4}	2.39	4.01×10^{-4}	8.19
^{45}S	16	29	4.19×10^{-4}	2.30	4.19×10^{-4}	8.11
^{50}Ar	18	32	4.92×10^{-4}	2.66	4.92×10^{-4}	8.10
^{47}Cl	17	30	4.41×10^{-4}	2.33	4.41×10^{-4}	8.07
^{49}Ar	18	31	4.58×10^{-4}	2.29	4.58×10^{-4}	8.02
^{44}P	15	29	5.94×10^{-4}	2.51	5.94×10^{-4}	7.85
^{46}S	16	30	6.42×10^{-4}	2.63	6.42×10^{-4}	7.82

- Use estimates for FRIB beam rates to construct plausible mass uncertainties.
- FRIB rates drive information gain because theoretical models are nearly equally bad for all unmeasured nuclei

Farr, Meisel and Steiner (2022)

Results for FRIB Masses w.r.t. to r-Process Masses



Isotope	Z	N	$\sigma_{\rm th}$	$\sigma_{\rm post}$	$D_{\rm LEHW}$
^{133}In	49	84	9.44	1.27×10^{-3}	8.41
^{134}In	49	85	9.35	2.16×10^{-3}	7.87
^{134}Cd	48	86	52.4	0.0126	7.83
^{133}Cd	48	85	23.4	5.72×10^{-3}	7.82
^{135}In	49	86	10.5	4.66×10^{-3}	7.22
^{196}Hf	72	124	8.93	4.50×10^{-3}	7.09
^{136}Sn	50	86	2.37	1.32×10^{-3}	7.00
^{128}Ag	47	81	2.22	1.24×10^{-3}	6.99
^{141}Te	52	89	2.18	1.37×10^{-3}	6.88

- Use r-process sensitivities from Mumpower et al. to determine information gain
- Much larger variation in r-process sensitivities, FRIB rates and theory both contribute to final result

Farr, Meisel, and Steiner (2022)

- Four r-process scenarios, LEHW, HEHW, CW, NSM

Now Back to the Equation of State

In particular, $P(\varepsilon)$ at $T = 0$.

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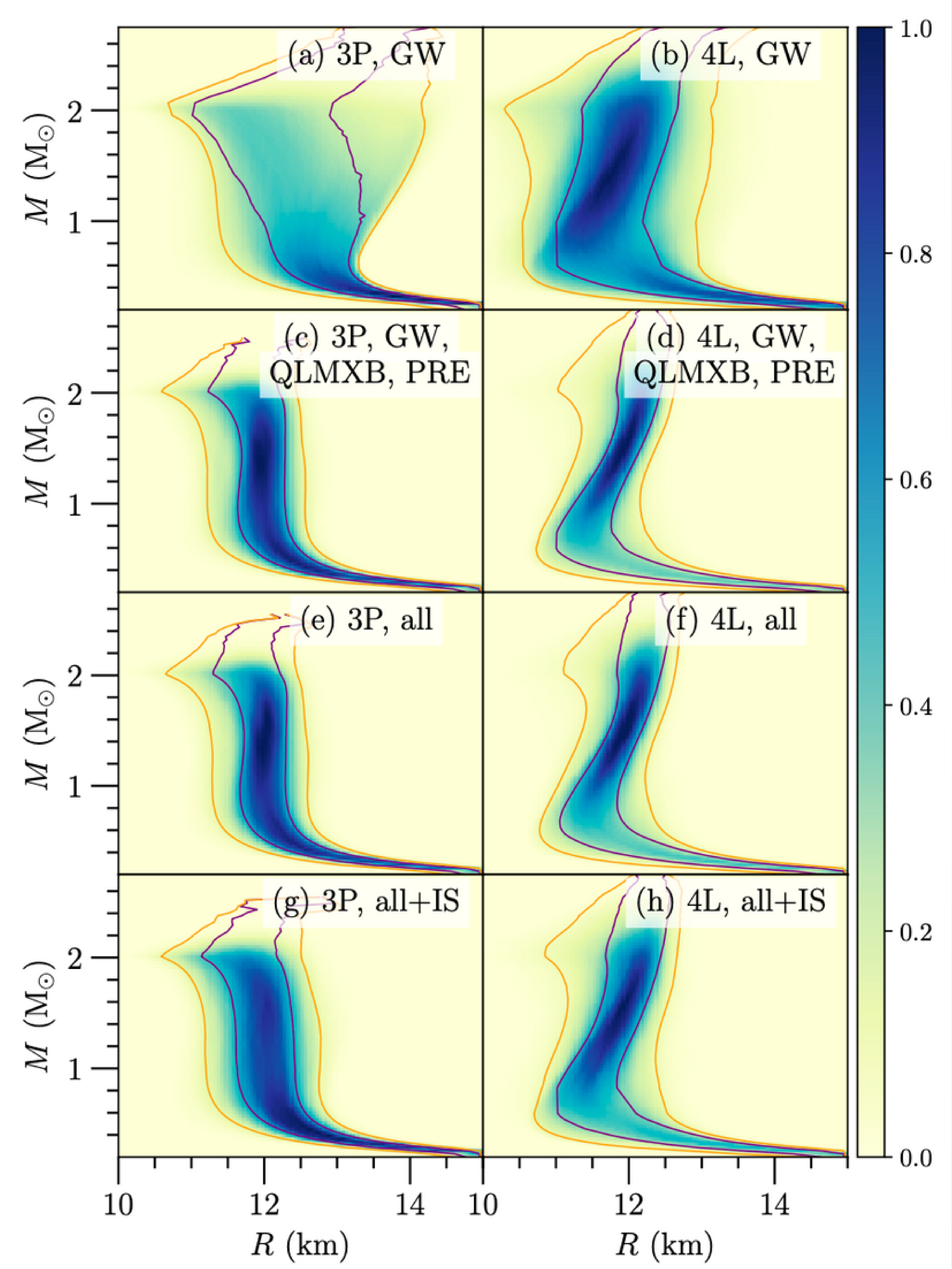
Tightly connected to pre-merger GW signal in a NS-NS merger.

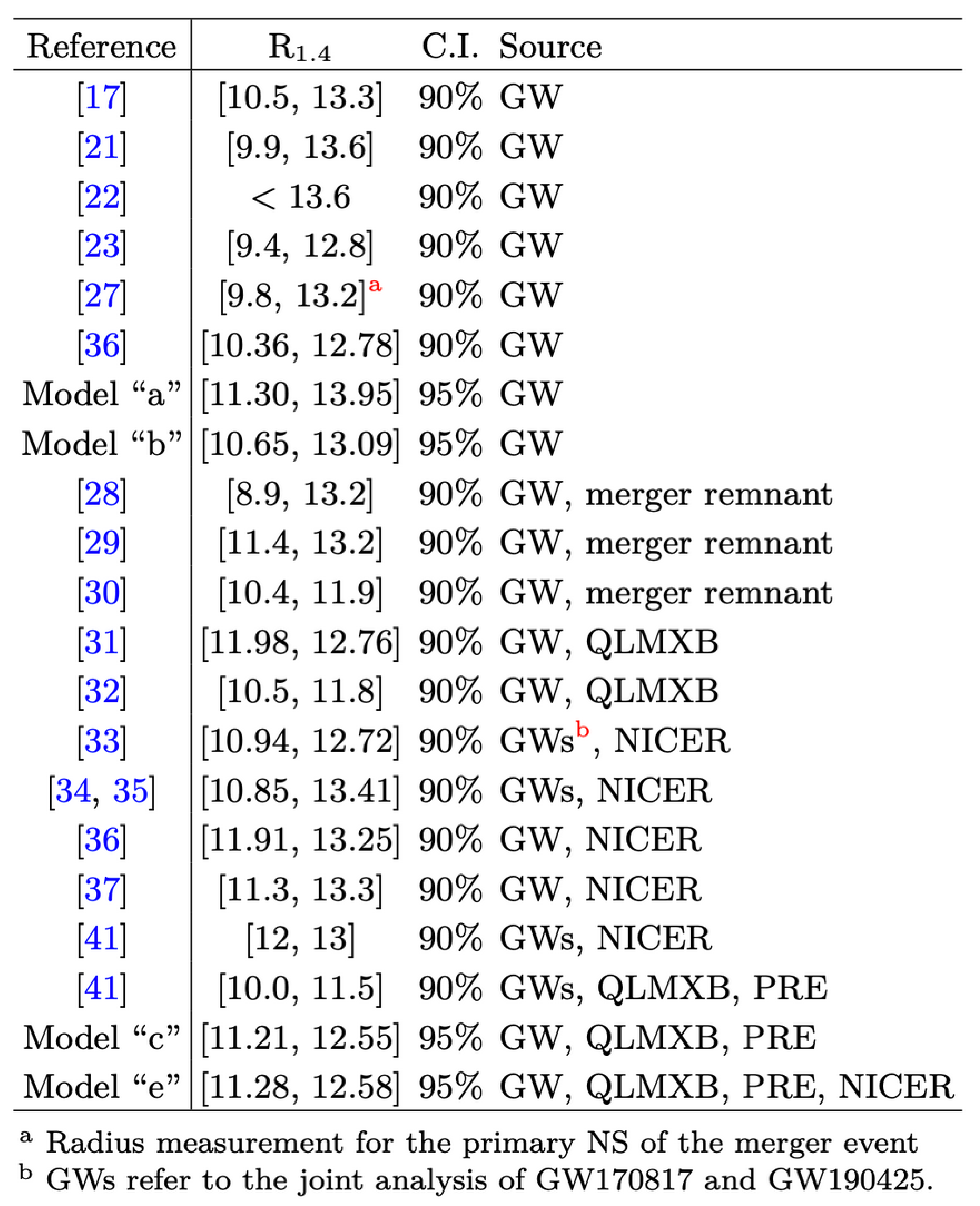
$P(\varepsilon)$ and the pre-merger signal



- Combining GW and electromagnetic constraints
- Bayes + TOV + MCMC
- Few additional assumptions on the EOS (e.g. no differentiability)
- Tested for unknown systematic uncertainties
- Tested variation with maximum mass

Al-Mamun et al., PRL, (2021)

$P(\varepsilon)$ and the pre-merger signal



Reference	$R_{1.4}$	C.I.	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2][a]	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs[b], NICER
[34, 35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

[a] Radius measurement for the primary NS of the merger event

[b] GWs refer to the joint analysis of GW170817 and GW190425.

- Combined electromagnetic and GW-based constraints on NS structure
- What should we make of the range of results?
- Some variation from different data sets

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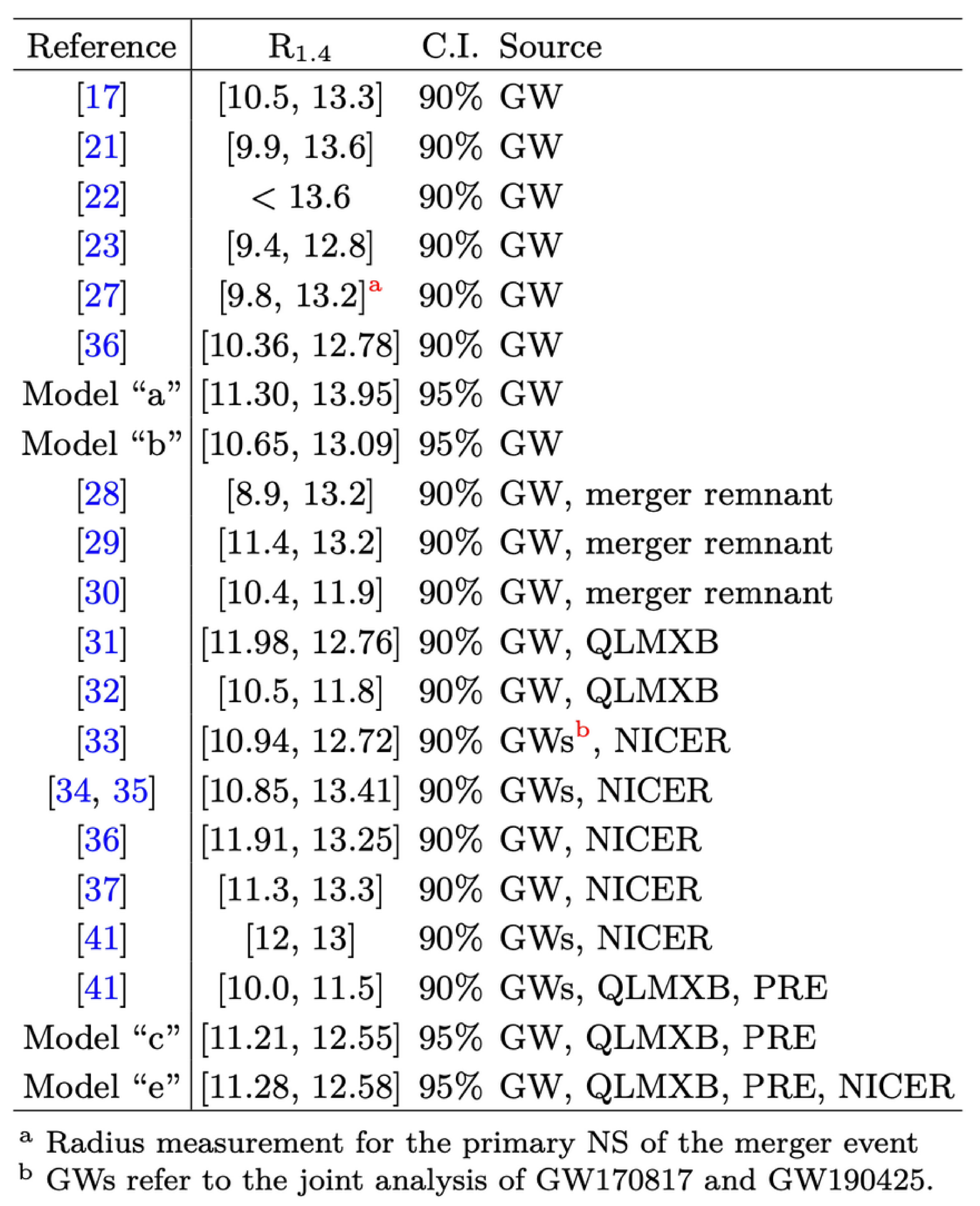
$P(\varepsilon)$ and the pre-merger signal



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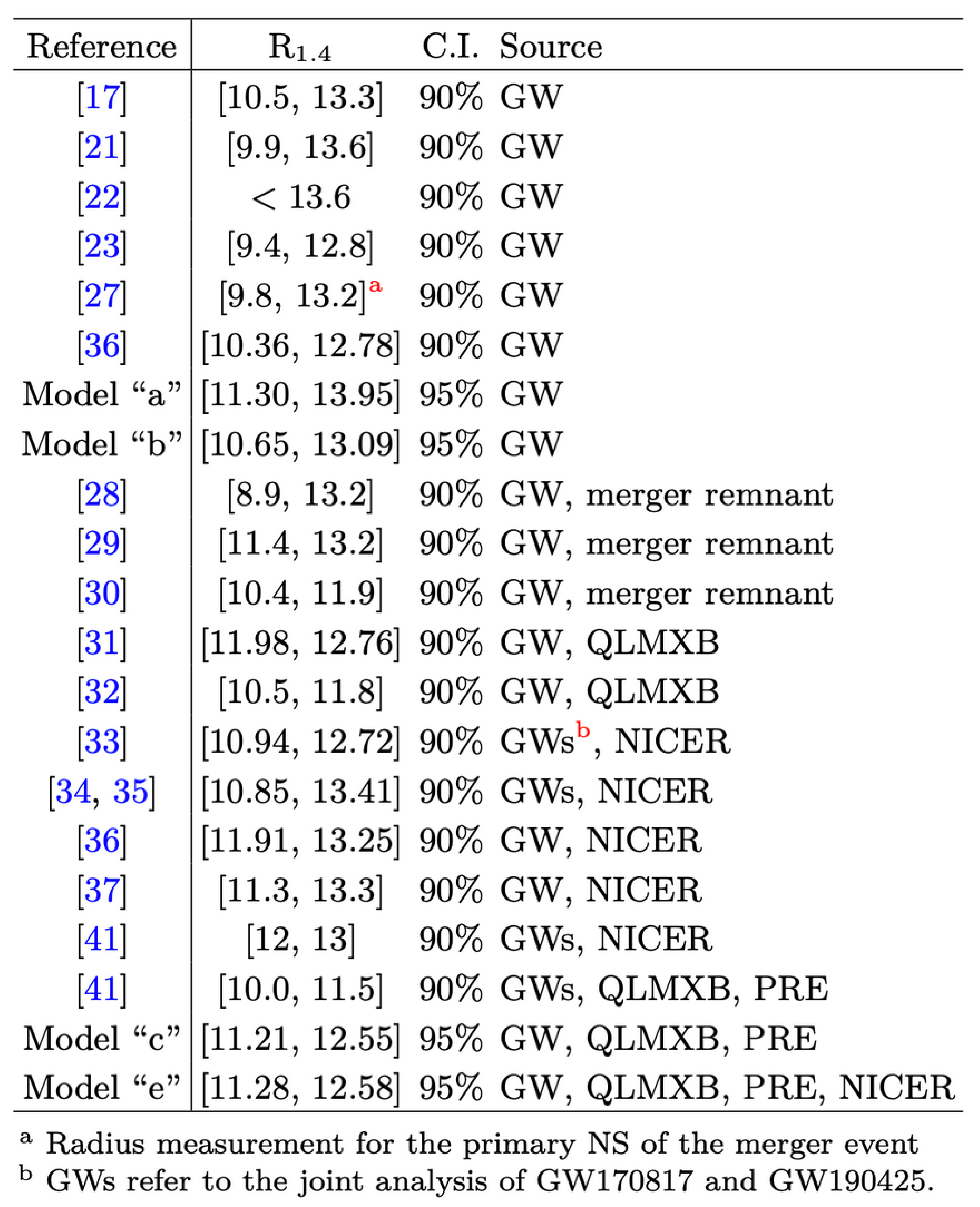

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- What should we make of the range of results?
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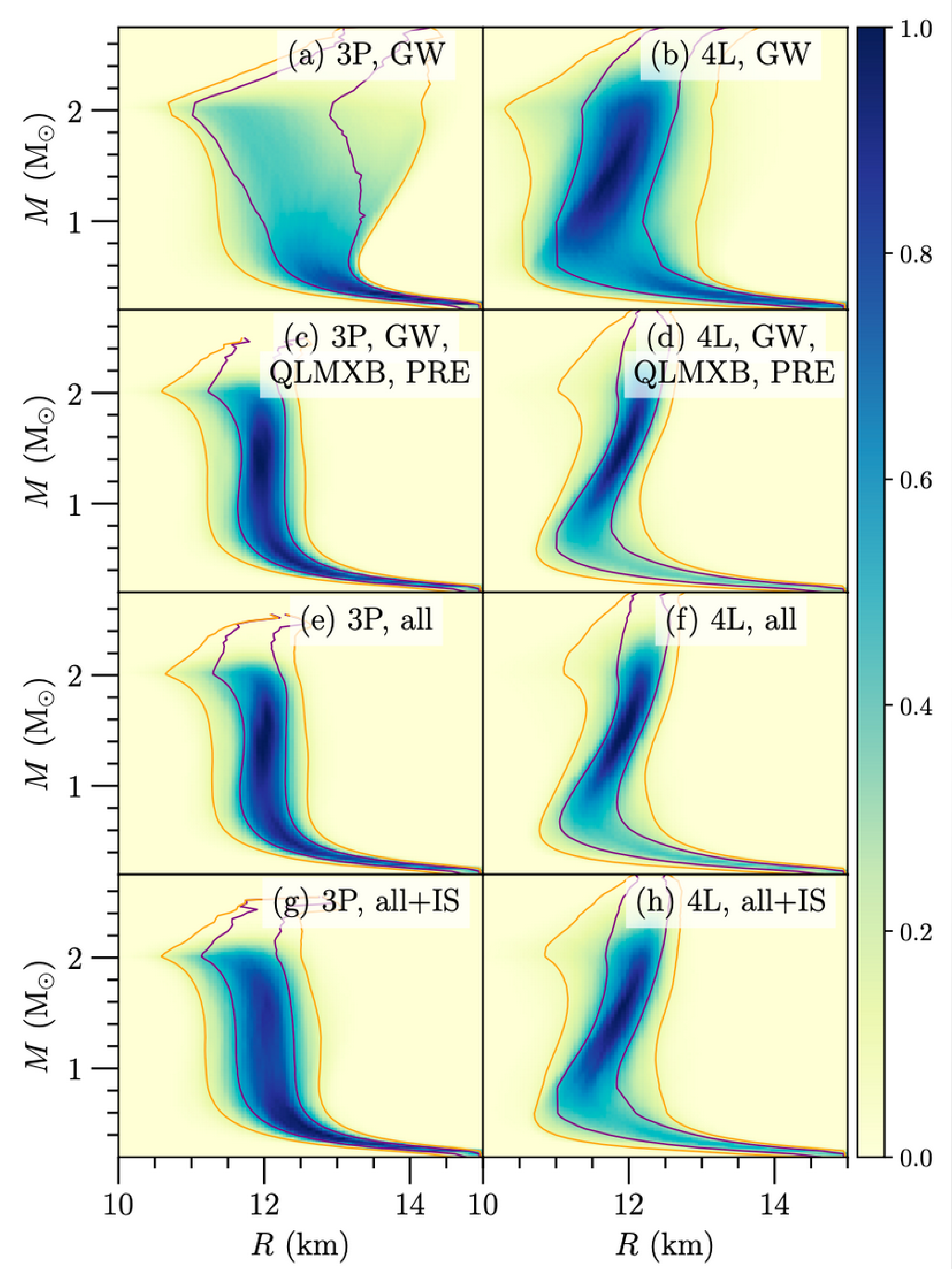
- Similar analysis: Bayes + TOV + MCMC
- Fundamentally, these differences result from **different prior distributions**

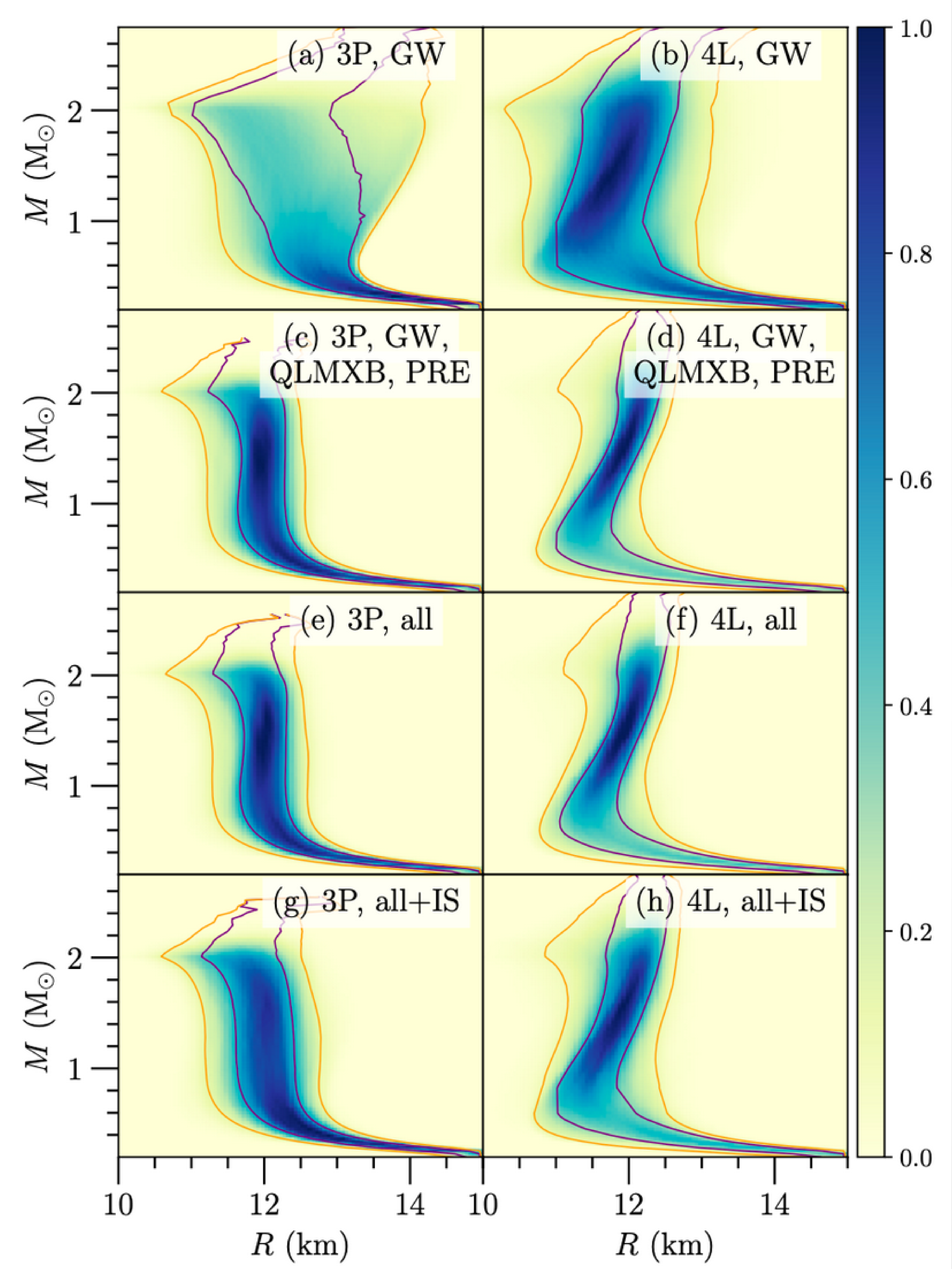
$P(\varepsilon)$ and the pre-merger signal



- Different rows represent different data sets
- Different columns represent different prior choices
- Largest current uncertainty in the EOS is the presence of a phase transition
- One and two-sigma contours vary by 0.5 km or more, depending on the mass
- The posterior radius distribution varies by a factor of a few or more

Al-Mamun et al., PRL, (2021)

$P(\varepsilon)$ and the pre-merger signal



- What is the solution?
- More data!
- GW observations will eventually determine the $T = 0$ pressure-energy density relation with high accuracy
- Composition, finite temperature, transport properties (neutrinos, superfluidity)

Al-Mamun et al., PRL, (2021)

Post-Merger?

See Bauswein's talk earlier today.

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I will talk about the nuclear physics input.

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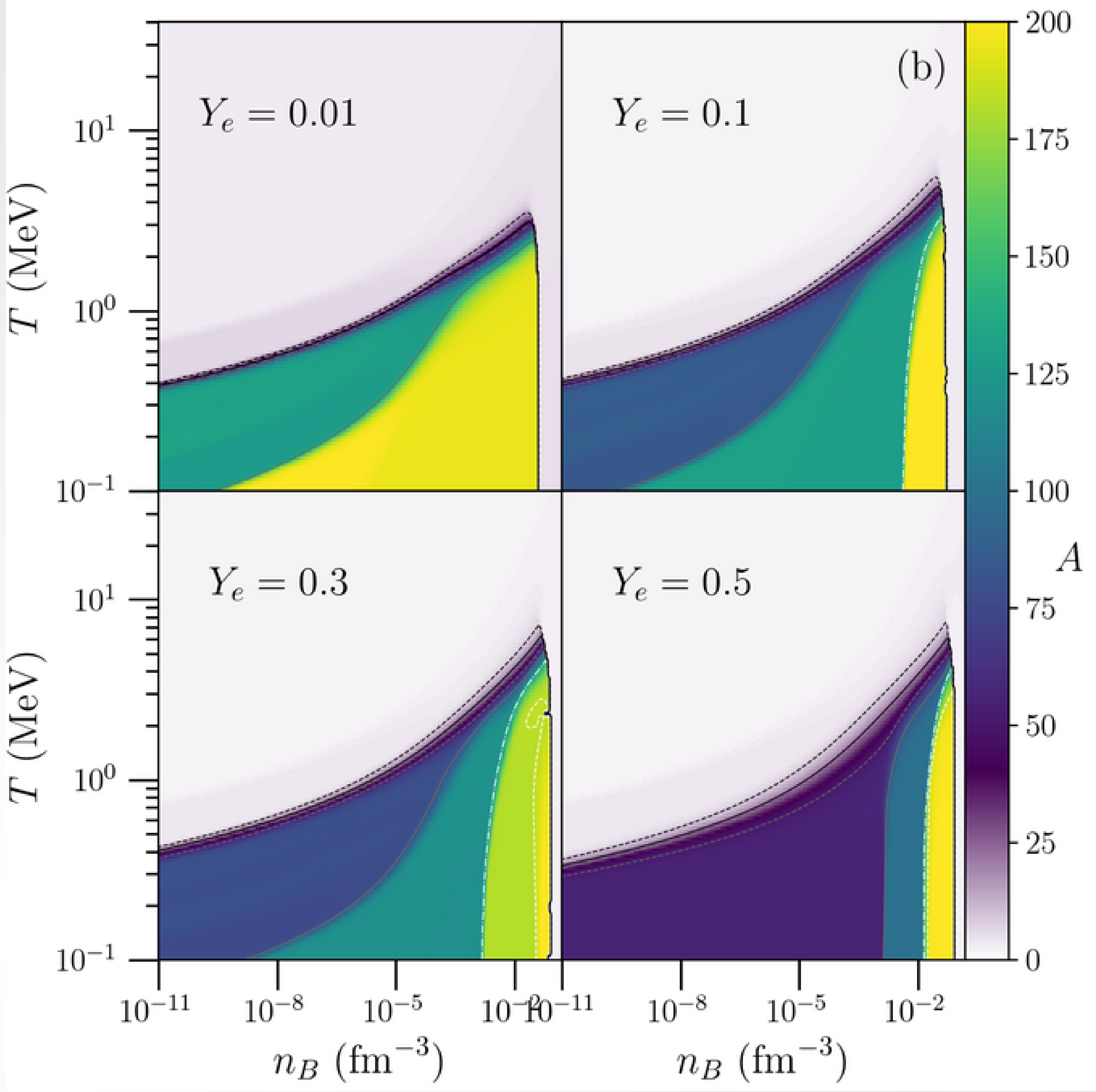
- but we're making progress on both fronts

Quilting an EOS for supernovae and mergers

- Three-dimensional space, $(n_B, Y_e \approx n_p/n_B, T)$
- Canonically, most EOS tables use extrapolation, our method avoids this
- Isospin-symmetric matter near saturation **Laboratory nuclei; NUCLEI collaboration, McDonnell et al. (2015)**
- Neutron-rich matter near saturation **Nuclear theory, e.g. Gandolfi et al. (2012)**
- Nearly non-degenerate matter **Nucleon scattering phase shifts, Horowitz et al. (2006)**

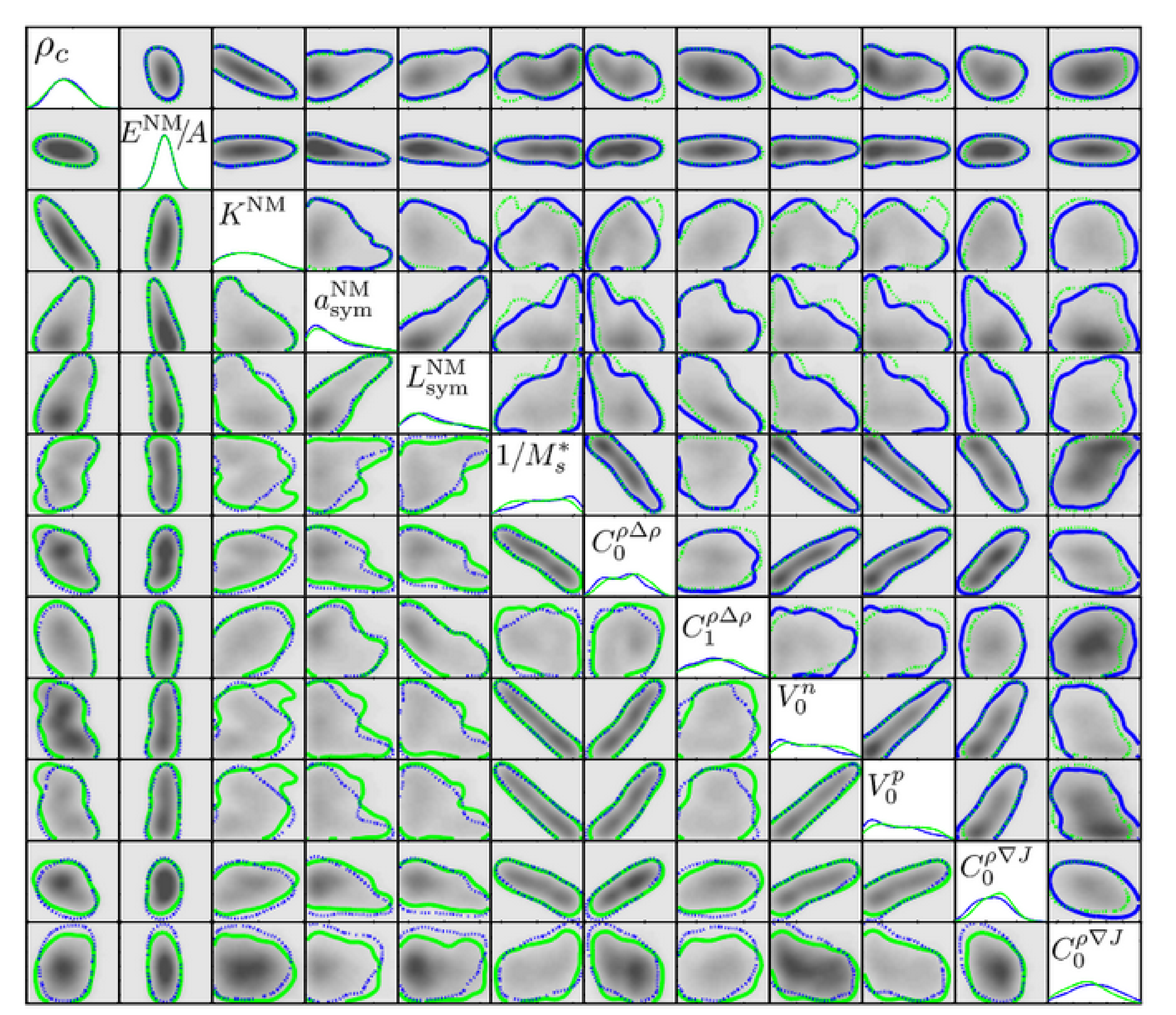


Du et al. (2019, 2022)

- Dense neutron-rich matter **Neutron star observations**
- Hot matter near saturation **Nuclear theory, e.g. Holt et al. (2017)**

Using NUCLEI Posterior Distributions

- Bayesian inference applied to
    - nuclear masses
    - charge radii
    - Odd-even staggering
    - Fission isomer energies
- Generates a probability distribution of Skyrme parameters



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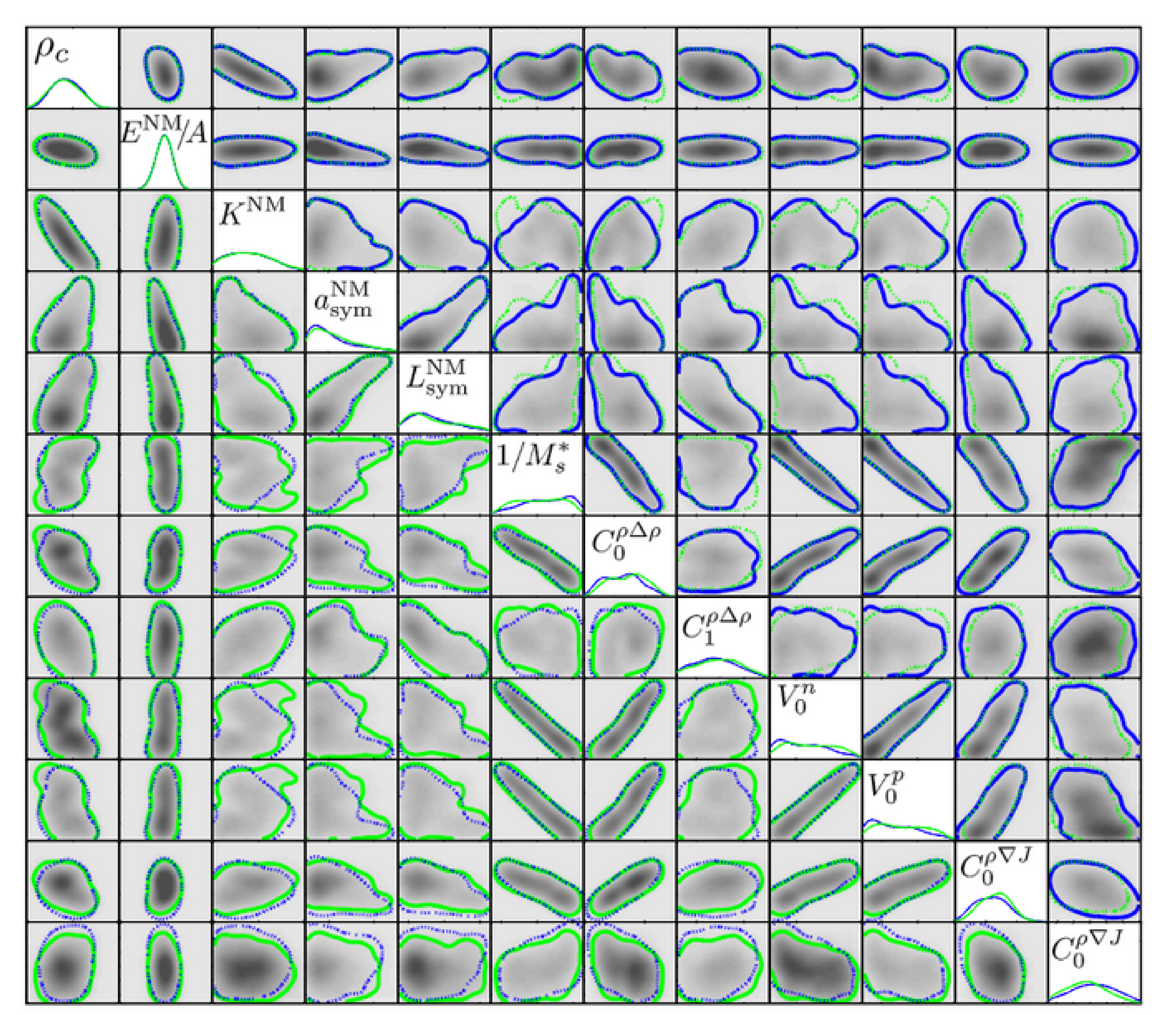

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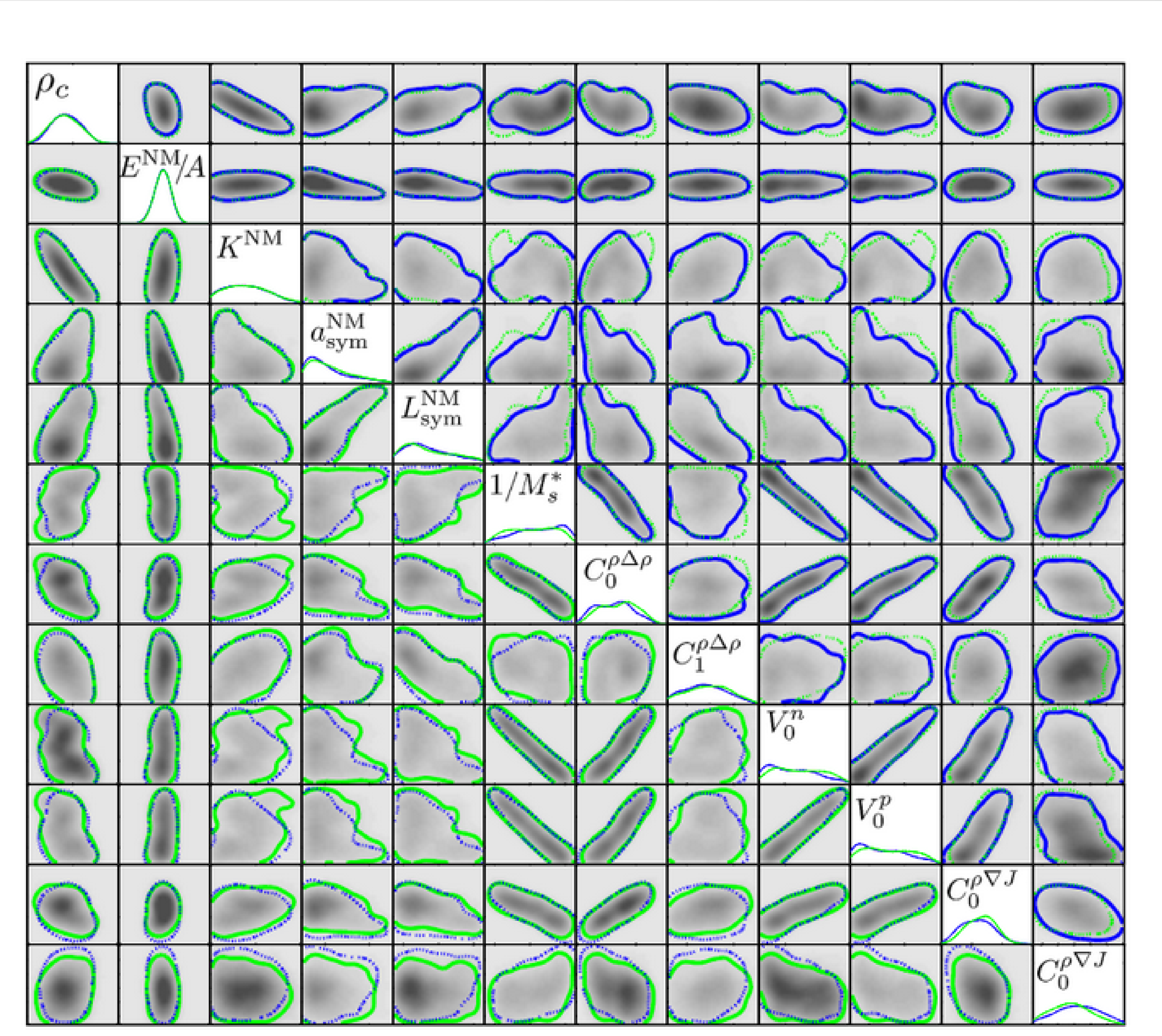

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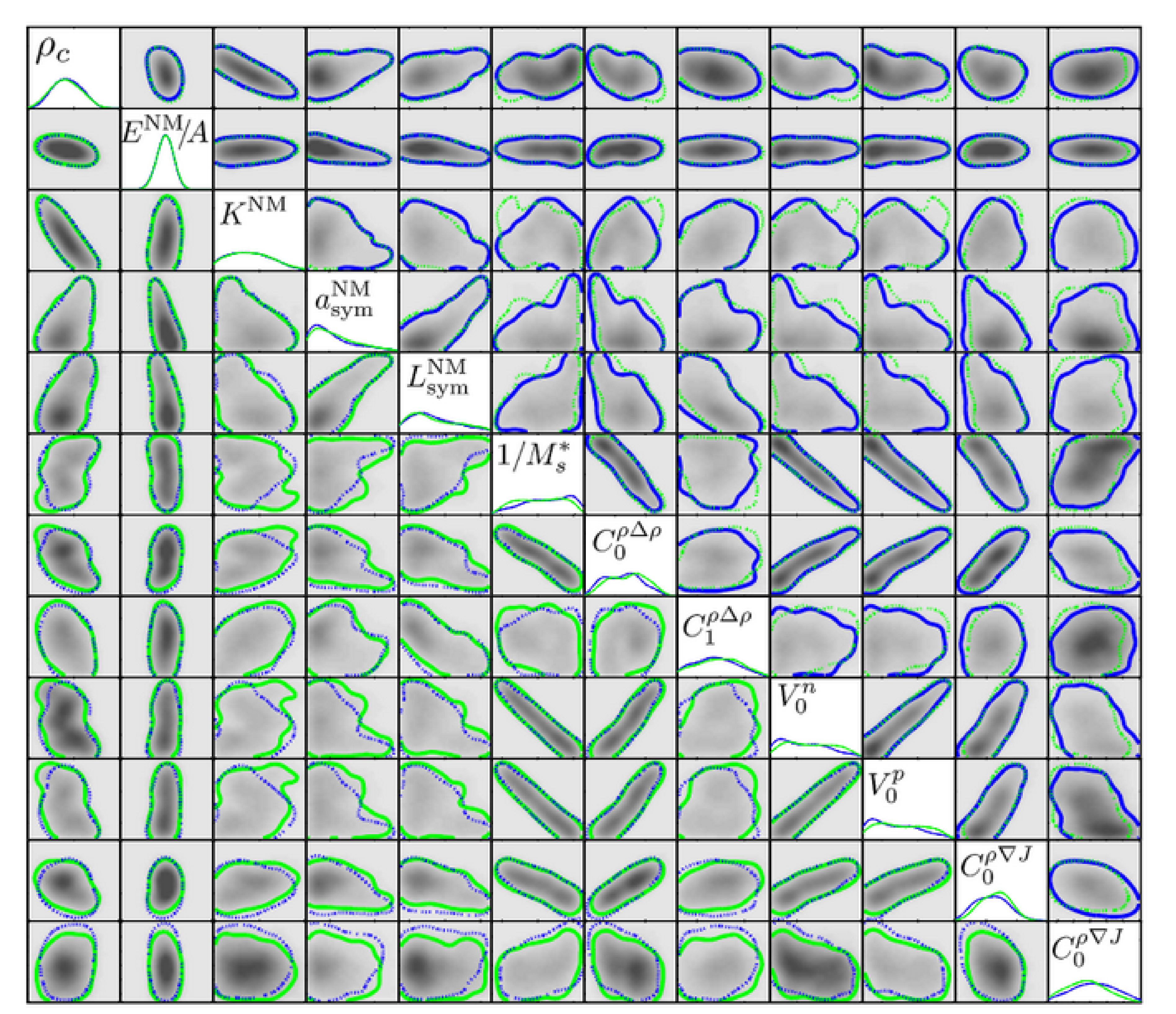

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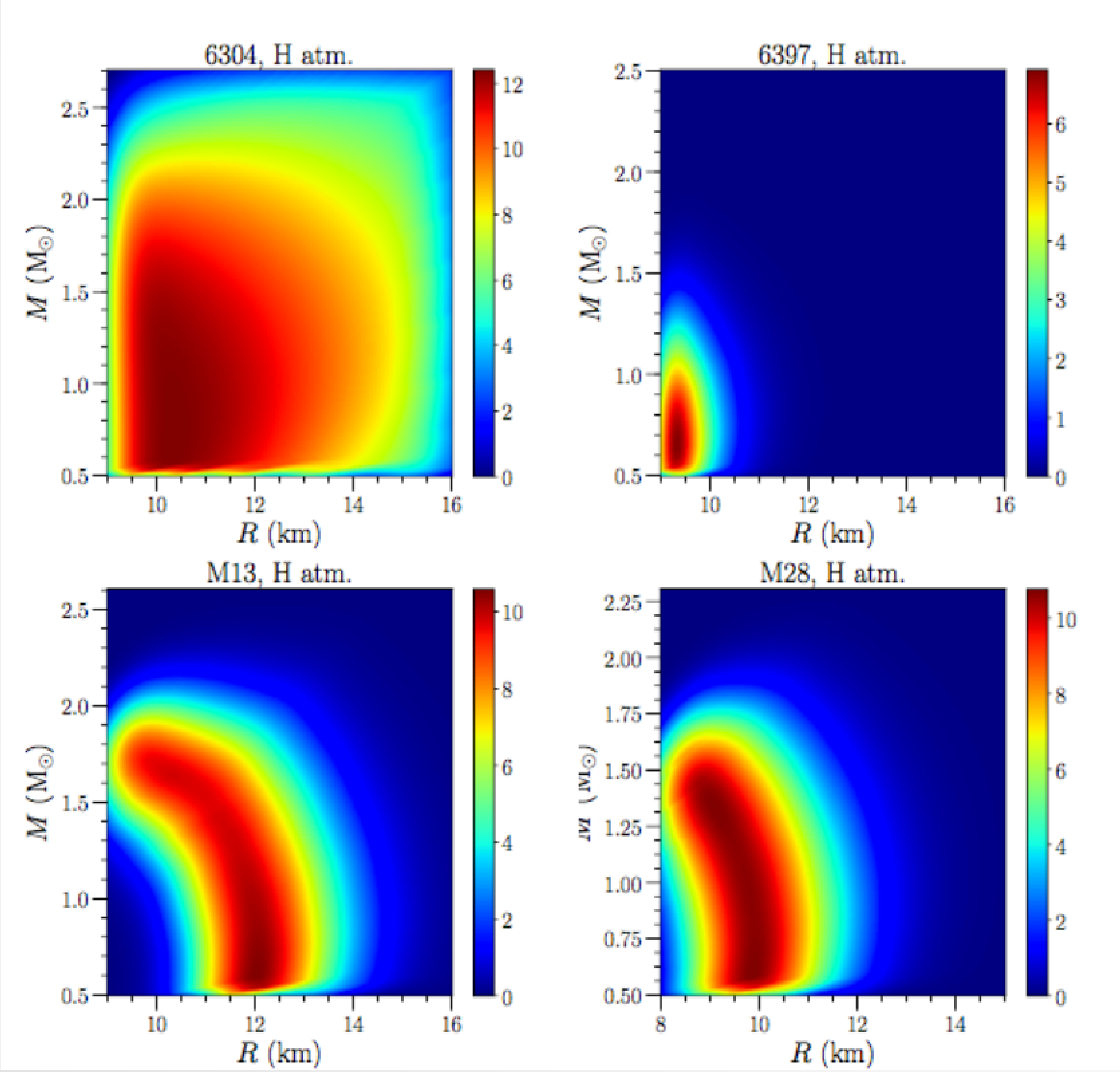
- Go beyond Skyrme/RMF/Gogny?

High-Density EOS from Neutron Star Observations



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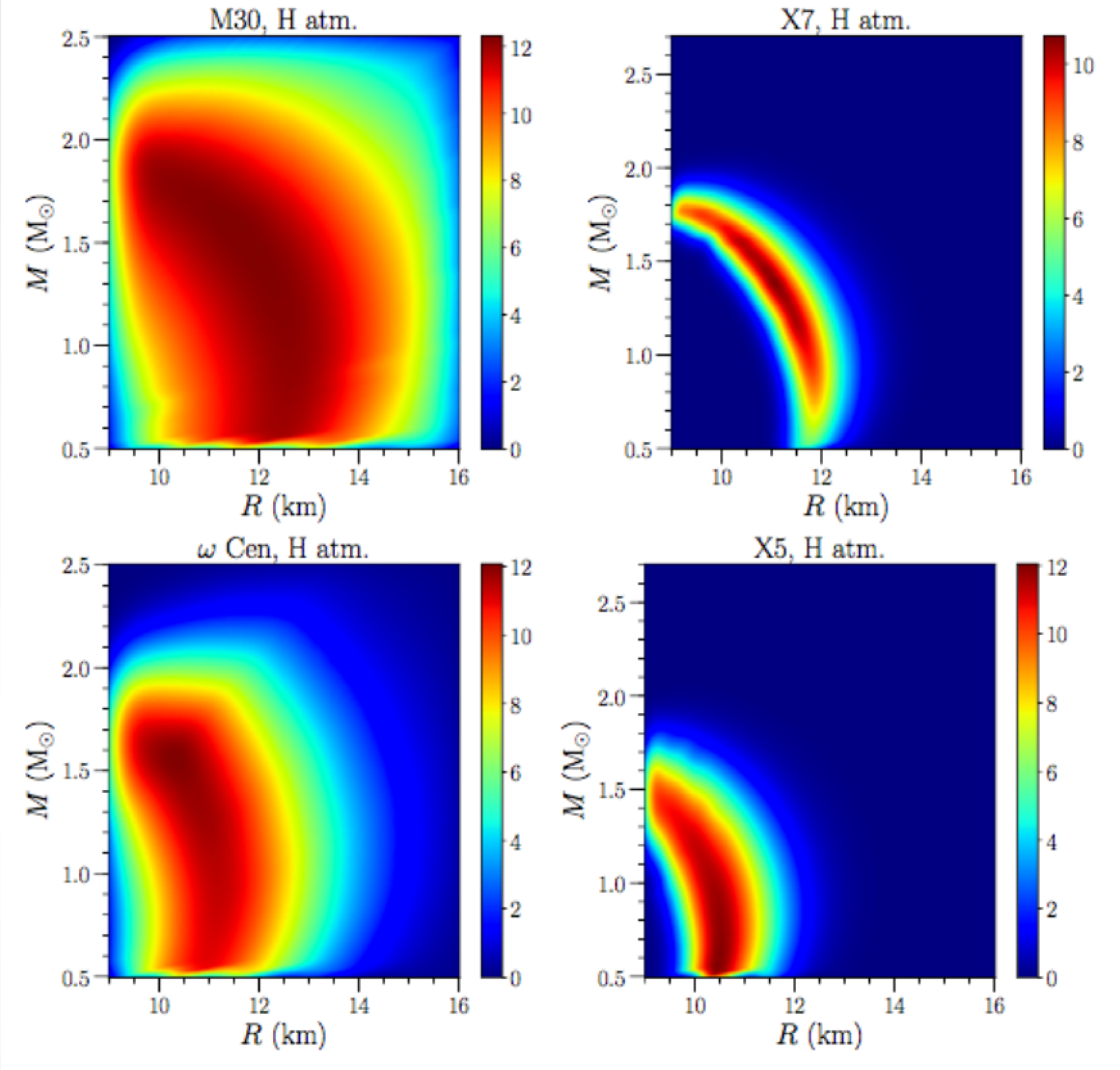


Steiner et al. (2018)

- Neutron star + sun-like main-sequence star
- Not accreting; in globular cluster

- NSs with $M = 2\ \mathrm{M}_\odot$ provide strong constraint
- Some difficult systematic uncertainties

EOS as a Probability Distribution

- Probability distribution for EOSs
- Probability density peaks at lower values because of influence of NS radius observations
- Nine sample EOS tables available now!
- Current work on propagating neutrino opacities so that they are consistent with the underlying EOS
- Variation by larger than an order of magnitude already at the nuclear saturation density
- Currently working on core-collapse simulations with these new EOSs

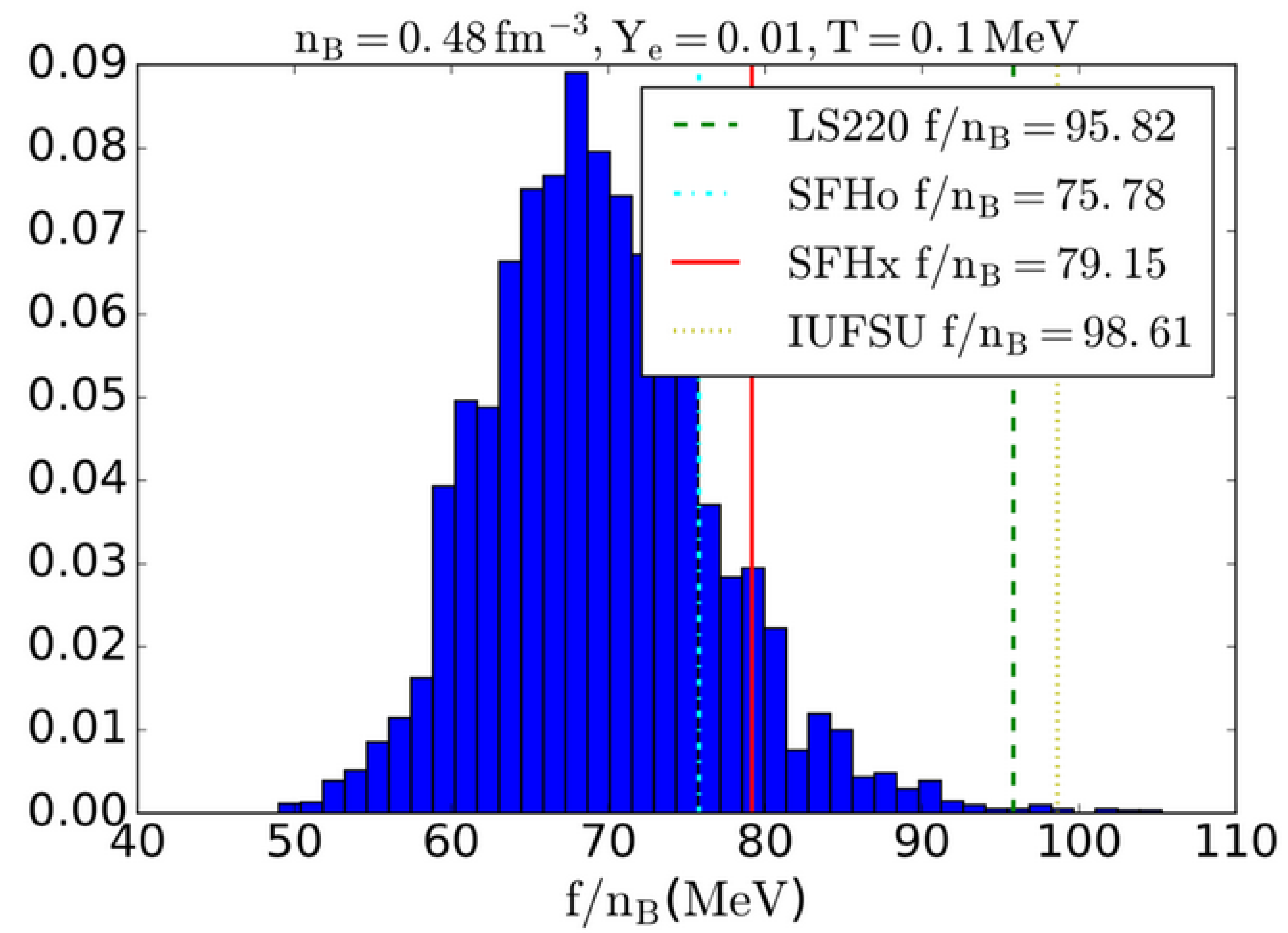
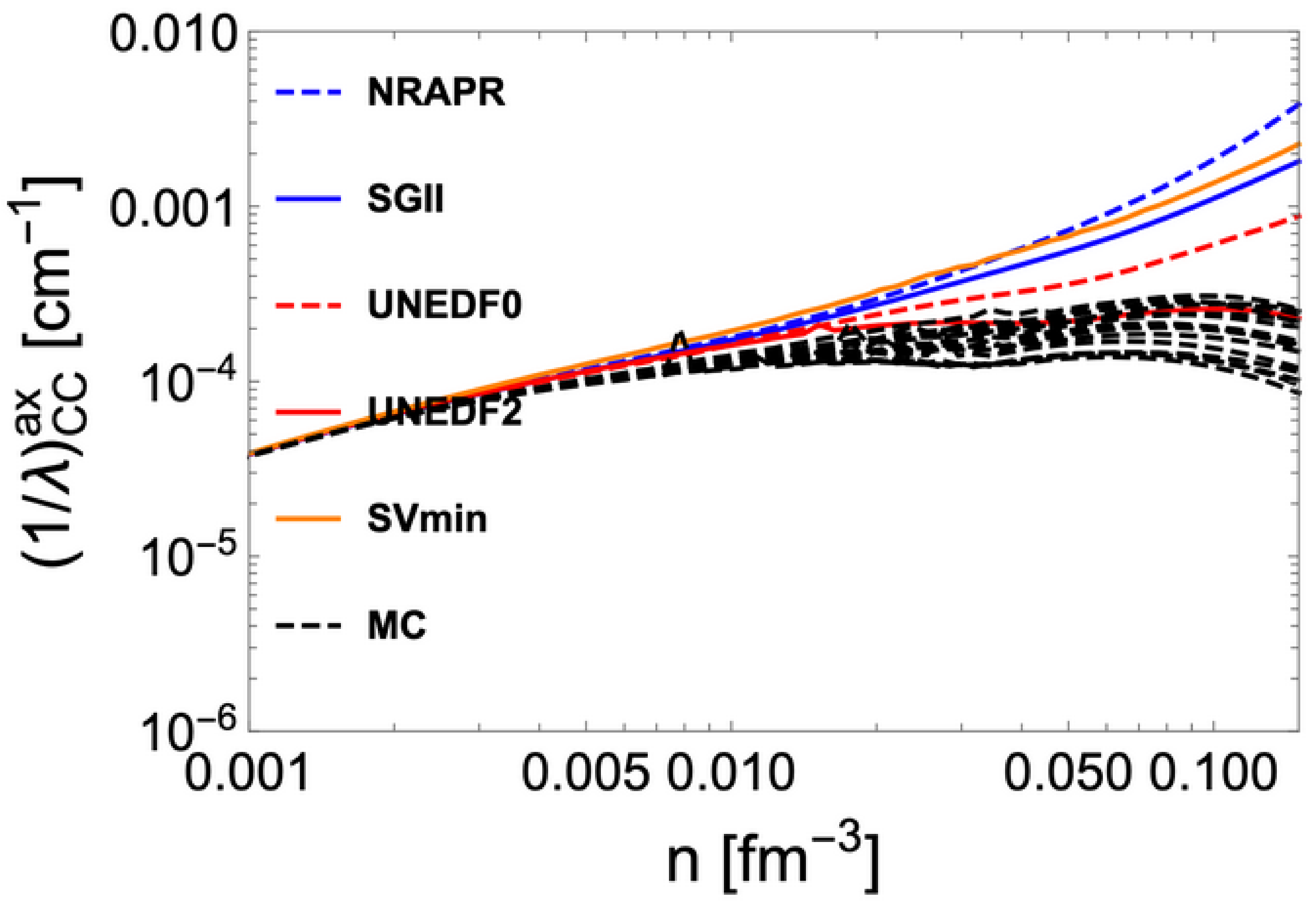


FIG. 6. The probability distribution for the free energy per baryon at $n_B = 0.48\ \mathrm{fm}^{-3}$, $Y_e = 0.10$, and $T = 0.1$ MeV.

Du et al. (2019, 2022)



Lin, Steiner, Margueron (in prep.)

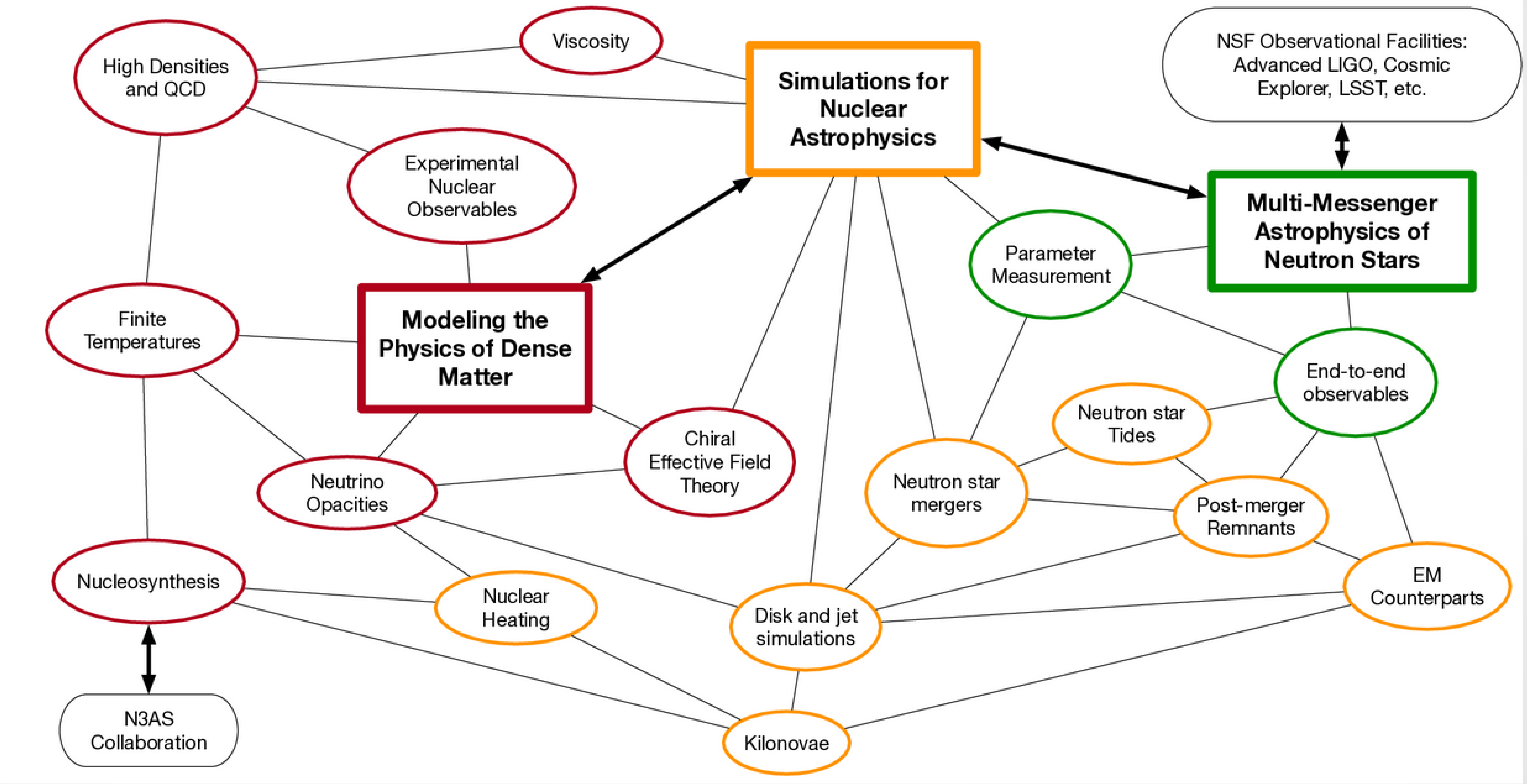
NP3M

What can we do with $3.25M?

Nuclear Physics of Multi-Messenger Mergers

- Understanding neutron star mergers will require a coordinated effort between many communities

NP3M



High Densities and QCD

Viscosity

Simulations for Nuclear Astrophysics

NSF Observational Facilities: Advanced LIGO, Cosmic Explorer, LSST, etc.

Experimental Nuclear Observables

Multi-Messenger Astrophysics of Neutron Stars

Parameter Measurement

Finite Temperatures

Modeling the Physics of Dense Matter

End-to-end observables

Neutron star Tides

Chiral Effective Field Theory

Neutrino Opacities

Neutron star mergers

Post-merger Remnants

Nucleosynthesis

Nuclear Heating

Disk and jet simulations

EM Counterparts

N3AS Collaboration

Kilonovae

- Nuclear structure theory, low-energy nuclear theory, high-energy nuclear theory, nuclear experiment, astrophysics theory, astronomical observations, gravitational wave observations
- Join us at np3m.org!

Summary

- Decision theory: how to optimize a FRIB mass measurement
- Leveraging neutron star observations to learn about QCD and the nucleon-nucleon interaction
- Constraints on mass-radius curve and EOS
- Understand the approximate size and the physical origin of the uncertainties in our posteriors
- Generating new nuclear physics input for merger simulations — new tables!
- Propagating uncertainties through EOS and neutrino uncertainties
- NP3M - A new collaboration to address this science at the interface of nuclear physics and neutron star mergers

Very exciting future! FRIB, P/CREX, GWs, NICER, Strobe-X, IXPE, and more!