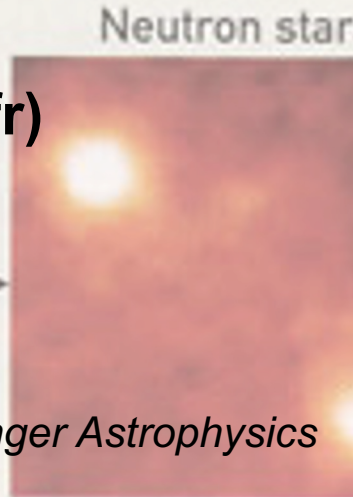
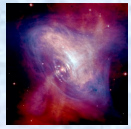


# Nuclear compressibility and equation-of-state impact on compact-star modelling

Anthea F. Fantina (anthea.fantina[AT]ganil.fr)





# Outline

## ❖ Introduction

- Equation of state (EoS) and empirical parameters
- Constraints from theory and experiments

## ❖ Role of the EoS

→ role of the nuclear compressibility ( $K \rightarrow K_{\text{sat}}$  or  $K_{\text{inf}}, K_{\text{sym}}$ )

- in core-collapse supernovae (CCSN)
- in neutron stars (NS)
- in BNS mergers → see [talks by A. Bauswein and A. Steiner](#)

## ❖ Conclusions & open questions



# Micro to macro: jumping across scales

**Microphysics (inputs)**  
(e.g. EoS, weak processes)

pre...

**Astrophysical (macrophysics)  
hydrodynamic/static models**  
(simulations)

...straint

constraint

prediction

**Nuclear theory** (with model parameters)

constraint

prediction

con...

...diction

**Nuclear physics Experiments**  
e.g. nuclear masses, resonances, decay rates, ...

**Astrophysical observations**  
(e.g. GW, NS masses, light curves, v...)



# EoS and empirical parameters

## ▪ Nuclear energy around saturation

→ expansion in density and asymmetry around  $n_{\text{sat}}$  and  $\delta = 0$

$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left( \left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m$$

$$x = (n - n_{\text{sat}})/3n_{\text{sat}}$$

$$\delta = (n_n - n_p)/n$$

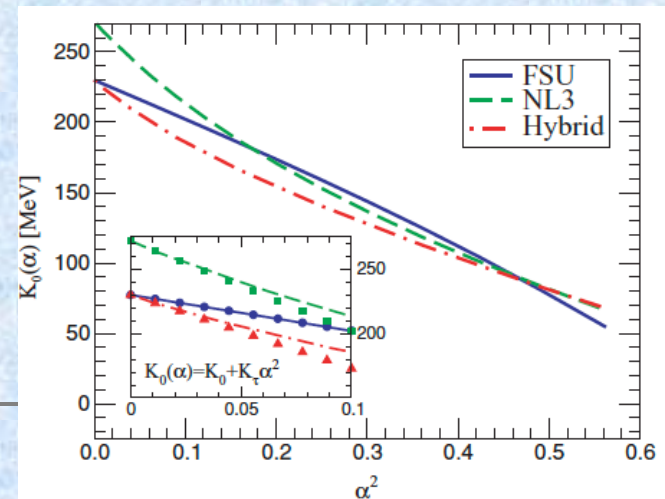
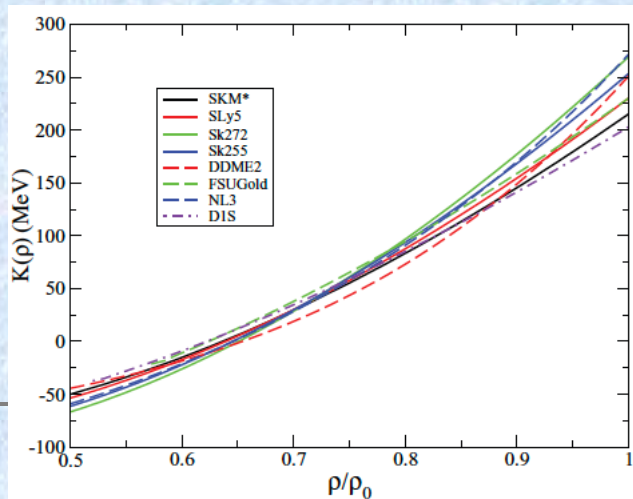
Empirical parameters (bulk)  $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, \mathbf{K}_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, \mathbf{K}_{\text{sym}}, \dots$

but:  $K_{\text{sat}}, K_{\text{sym}}$  : info on compression at  $n_0$  and symmetric matter

→ other parameters contribute if  $n \neq n_0$  and  $\delta \neq 0$ !

$$K(n) = \frac{18}{n} P(n) + 9n^2 \frac{\partial^2 E(n)/A}{\partial n^2}$$

$$K_0(\delta) = K_{\text{sat}} + \left( K_{\text{sym}} - 6L - \frac{Q_{\text{sat}}}{K_{\text{sat}}} L \right) \delta^2 + \mathcal{O}(\delta^4)$$



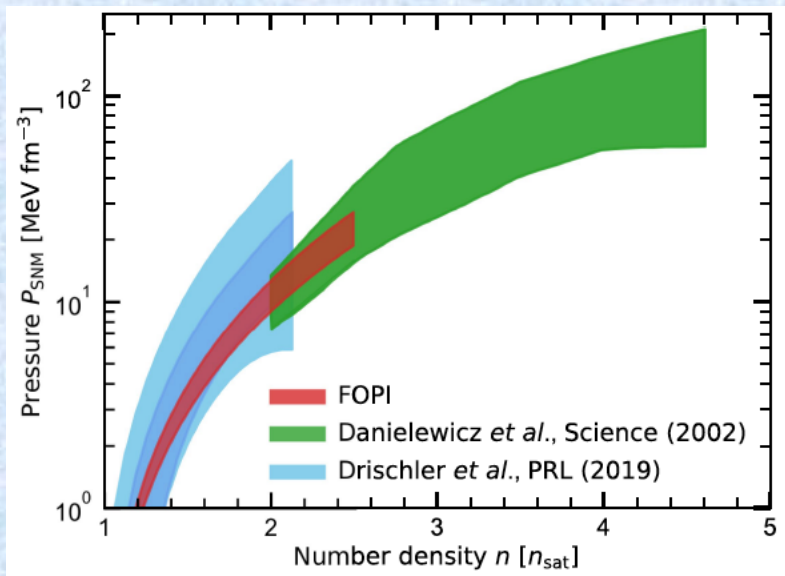


# How can we get constraints?

## Nuclear physics exp./ theory

- Measure of **nuclear properties**:
  - masses and radii of nuclei
  - collective modes, polarizability
  - neutron skins, HIC, flows
  - etc ...
- **ab-initio calculations**

→ “low” density (better in nucleonic sector)

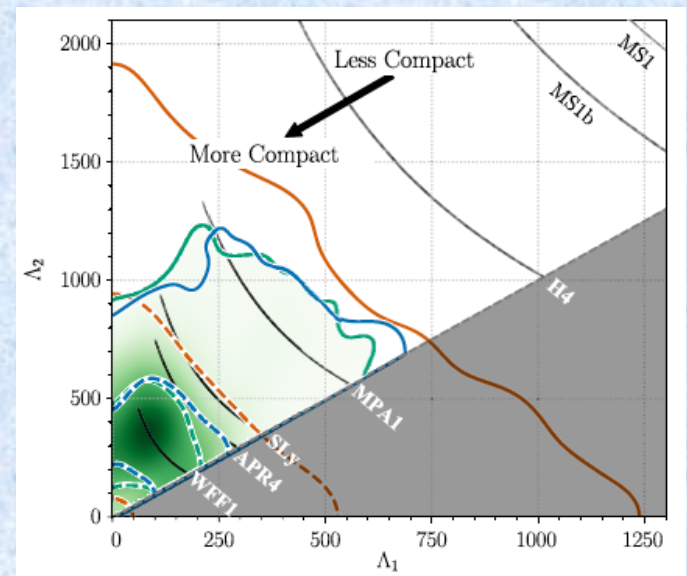


Huth et al., Nature 606, 276 (2022)

## Astrophysical observations

- Measure of **NS properties**:
  - NS masses and radii (NICER)
  - rotational frequency, oscillation modes
  - cooling, moment of inertia
  - etc ...
- **Gravitational waves**

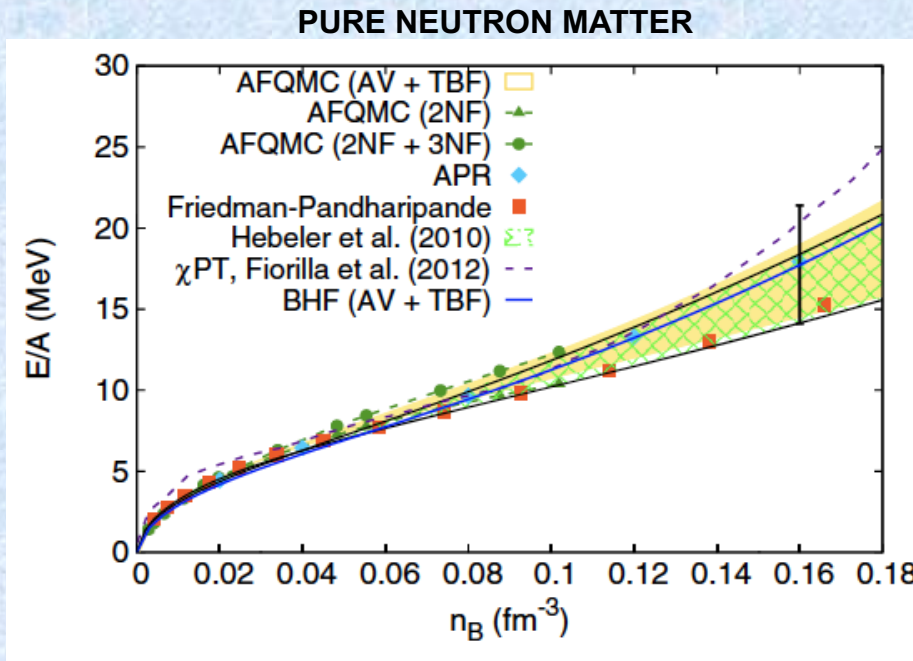
→ “high” density



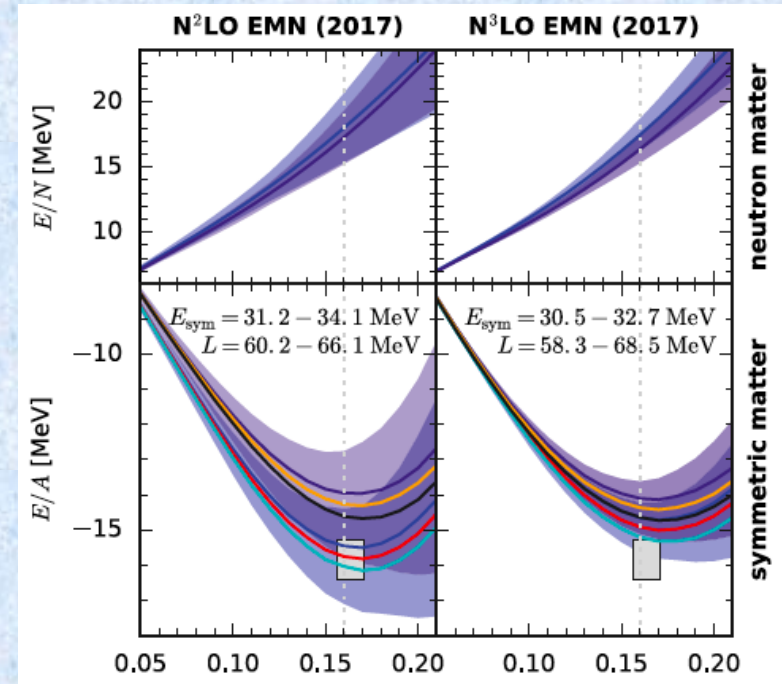
Abbott et al., PRL 121, 161101 (2018)



# Constraints from nucl. phys.: theo



Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)



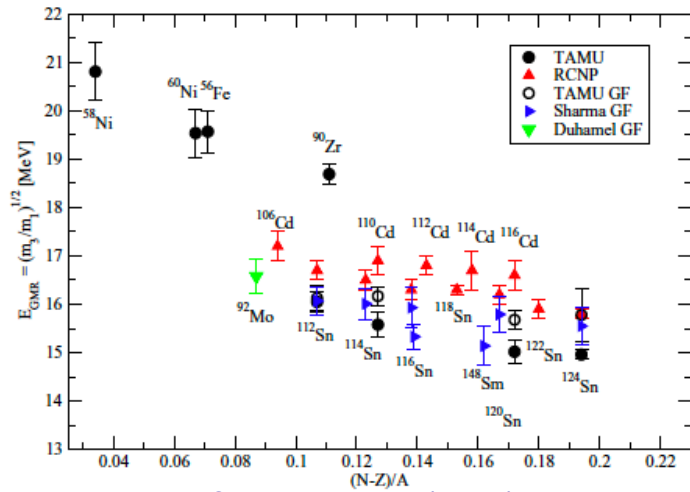
Drischler et al., PPNP 121, 103888 (2021)

- Reasonable agreement of ab-initio (PNM) up to  $\sim$  saturation density
- PNM calculations benchmark for phenomenological models

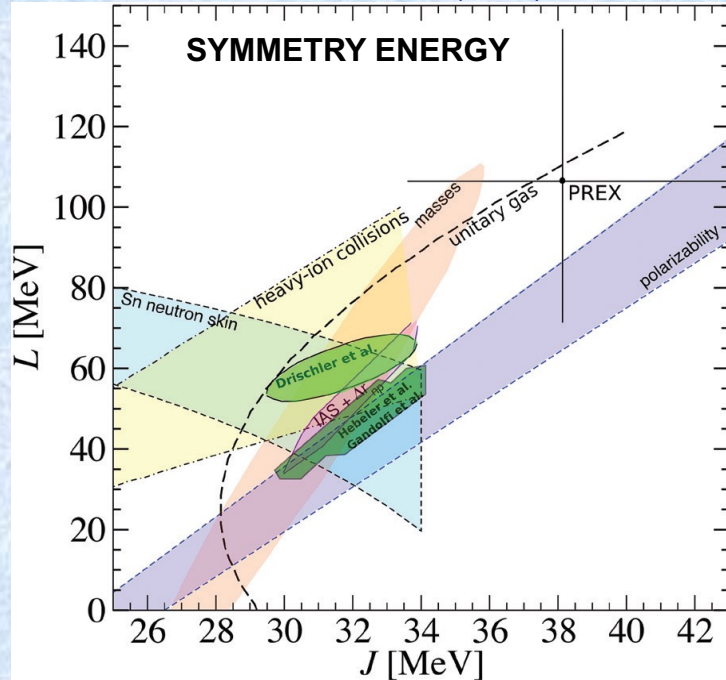
N.B.: for symmetric matter (ab-initio): (i) saturation point difficult to obtain ;  
(ii) larger uncertainties ; (iii) cluster formation at sub-saturation



# Constraints from nucl. phys.: exp (1)

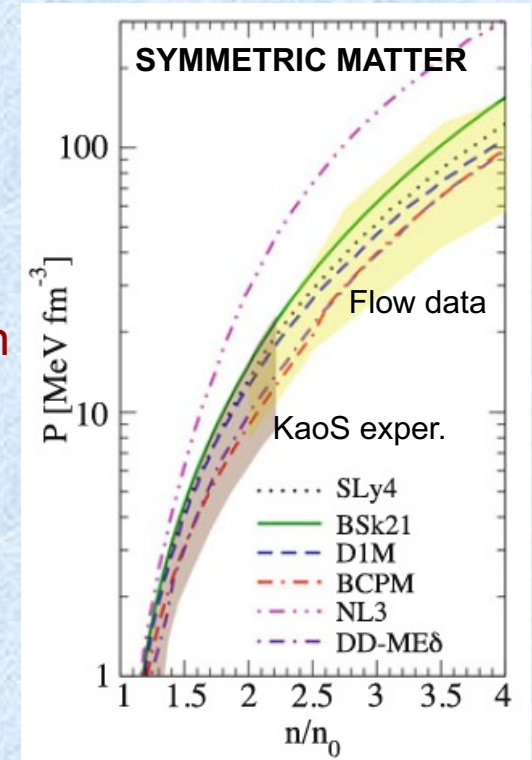


Stone et al., PRC 89, 0044316 (2014)



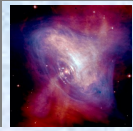
Gulminelli & Fantina, Nucl. Phys. News 31, 9 (2021)

- Constraints at “low” densities
- low-order parameters
- Constraints more on “symmetric” matter
- Not always “clear” constraints
- “tension”



Burgio & Fantina, ASSL 457, 255 (2018)  
 (Flow: Danielewicz et al., Science 2002  
 KaoS: Lynch et al., PPNP 2009)

N.B.: deduced constraints are often *not* raw data, but combined with models  
 → model dependence of constraints !



# Constraints from nucl. phys.: exp (2)

Model	Ref.	$E_{\text{sat}}$ (MeV)	$n_{\text{sat}}$ (fm <sup>-3</sup> )	$K_{\text{sat}}$ (MeV)	$E_{\text{sym}}$ (MeV)
El. scatt.	Wang-99 [55]		0.1607	235 ±15	
LDM	Myers-66 [56]	-15.677	0.136 <sup>a</sup>	295	28.06
LDM	Royer-08 [57]	-15.5704	0.133 <sup>a</sup>		23.45
LSD	Pomorski-03 [58]	-15.492	0.142 <sup>a</sup>		28.82
DM	Myers-77 [59]	-15.96	0.145 <sup>a</sup>	240	36.8
FRDM	Buchinger-01 [60]		0.157 ±0.004		
INM	Satpathy-99 [61]	-16.108	0.1620	288 ±20	
DF-Skyrme	Tondeur-86 [62]		0.158		
DF-Skyrme	Klupfel-09 [63]	-15.91 ±0.06	0.1610 ±0.0013	222 ±8	30.7 ±1.4
DF-BSK2	Goriely-02 [64]	-15.79	0.1575	234	28.0
DF-BSK24, 28,29	Goriely-15 [65]	-16.045 ±0.005	0.1575 ±0.0004	245	30.0
DF-Skyrme	McDonnell-15 [66]	-15.75 ±0.25	0.160 ±0.005	220 ±20	29 ±1
DF-NLRMF	NL3* [67]	-16.3	0.15	258	38.7
DF-NLRMF	PK [68]	-16.27	0.148	283	37.7
DF-DDRMF	DDME1,2 [69,70]	-16.17 ±0.03	0.152 ±0.00	247 ±3	32.7 ±0.4
DF-DDRMF	PK [68]	16.27	0.150	262	36.8
Present		-15.8	0.155	230	32
Estimation		±0.3	±0.005	±20	±2

Margueron et al., PRC 97, 025805 (2018)  
see also Stone et al., PRC 89, 044316 (2014)

**N.B.:** parameter estimation from various analysis  
of experimental data

→ but through different models

→ not straightforward nor unambiguous extraction

Model	Ref.	$Q_{\text{sat}}$ (MeV)	$L_{\text{sym}}$ (MeV)	$K_{\text{sym}}$ (MeV)	$K_{\tau}$ (MeV)
DF-Skyrme	Berdichevsky-88 [71]	30	0		
DF-Skyrme	Farine-97 [72]	-700 ±500			
DF-Skyrme	Alam-14 [31]	-344 ±46	65 ±14	-23 ±73	-322 ±34
DF-Skyrme	McDonnell-15 [66]		40 ±20		
DF-NLRMF	NL3* [67]	124	123	106	-690
DF-NLRMF	PK [68]	-25	116	55	-630
DF-DDRMF	DDME1,2 [69,70]	400 ±80	53 ±3	-94 ±7	-500 ±7
DF-DDRMF	PK [68]	-119	79.5	-50	-491
Correlation	Centelles-09 [73]		70 ±40		-425 ±175
DF-RPA	Carbone-10 [74]		60 ±30		
Correlation	Danielewicz-14 [75]		53 ±20		
Correlation	Newton-14 [76]		70 ±40		
Correlation	Lattimer-14 [77]		53 ±20		
GMR	Sagawa-07 [78]				-500 ±50
GMR	Patel-14 [79]				-550 ±100
Present		300	60	-100	-400
Estimation		±400	±15	±100	±100





# Outline

## ❖ Introduction

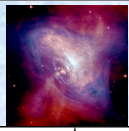
- Equation of state (EoS) and empirical parameters
- Constraints from theory and experiments

## ❖ Role of the EoS

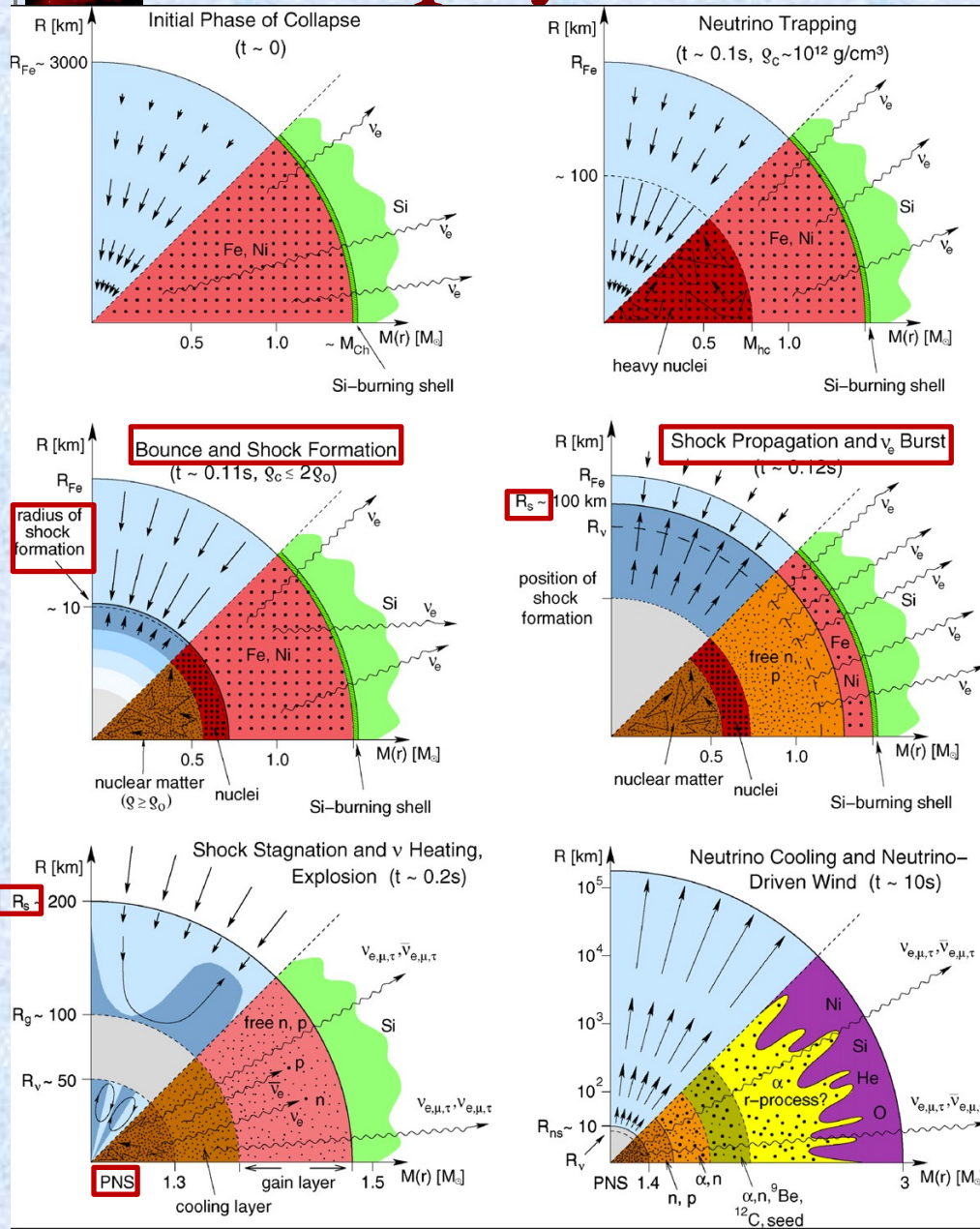
→ role of the nuclear compressibility ( $K \rightarrow K_{\text{sat}}$  or  $K_{\text{inf}}, K_{\text{sym}}$ )

- in core-collapse supernovae (CCSN)
- in neutron stars (NS)
- in BNS mergers → see talks by A. Bauswein and A. Steiner

## ❖ Conclusions & open questions



# Astrophysical context : CCSN



1. Infall epoch  
→ core collapse

2. Bounce and shock propagation  
→ bounce formation  
→ shock radius

3. Explosion  
→ time of explosion



# EoS in CCSN simulations

Most used EoSs (historically) in CCSN :

- **Hillebrandt & Wolff 1984** : NSE + SNA at higher density  
Skyrme (Ska) interaction for nucleons.  $K_{\text{sat}} = 263 \text{ MeV}$
- **Lattimer & Swesty (LS) 1991** : SNA, nuclei +  $\alpha$  + free n,p + leptons  
CLDM and NR simplified Skyrme-like functional for nucleons,  $\alpha$  Boltzmann gas  
 $K_{\text{sat}} = 180, 220, 375 \text{ MeV}$
- **Shen et al. 1998** : SNA, nuclei +  $\alpha$  + free n,p + leptons  
TF approach, RMF (TM1) for nucleons,  $\alpha$  Boltzmann gas.  $K_{\text{sat}} = 281 \text{ MeV}$
- **SHFo, SHFx 2013**: NSE, nuclei +  $\alpha$  + free n,p + leptons  
RMF for nucleons.  $K_{\text{sat}} = 245, 239 \text{ MeV}$  (but also symmetry energy parameters differ...)

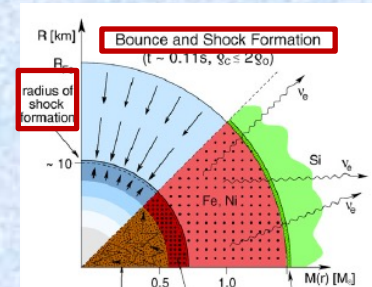
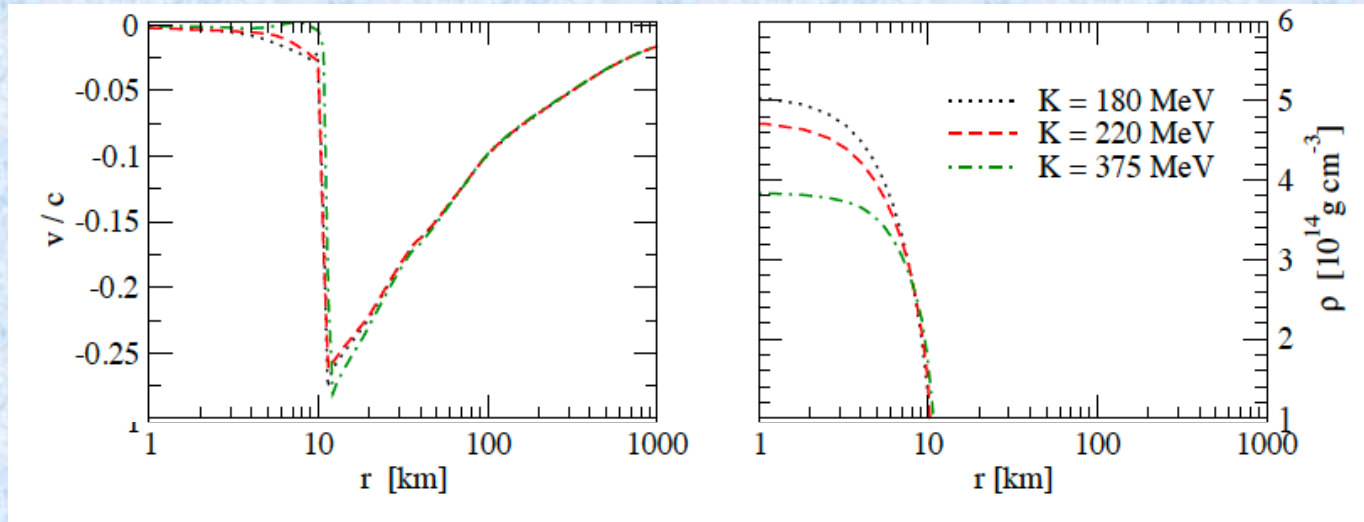


BUT :

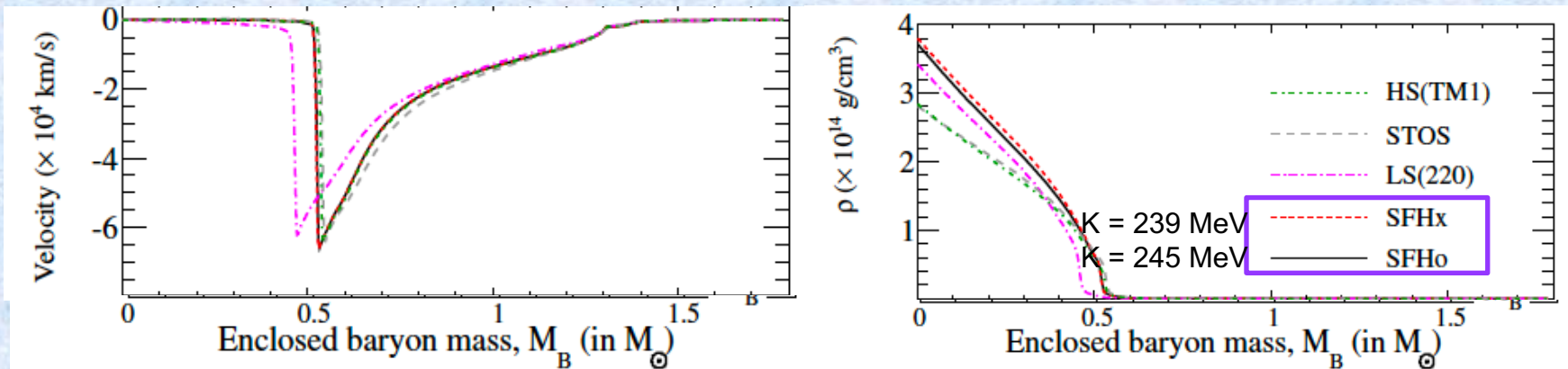
- ❖ when comparing “(in)compressibility”  $\rightarrow$  comparing different models !
- ❖ Mazurek’s law  $\rightarrow$  complex interplay and feedback with hydro/transport



# CCSN simulations: $K$ and bounce



A. F. Fantina, PhD thesis (2010) - 1D GR, Lattimer&Swesty EoS, neutrino leakage-type scheme,  $15 M_{\text{sun}}$  progenitor  
 (see also Suwa et al., ApJ 764, 99 (2013) 1D simulations, Newtonian, LS and Shen EoS,  $15 M_{\text{sun}}$  progenitor,  $v$ : diffusion approx. scheme)

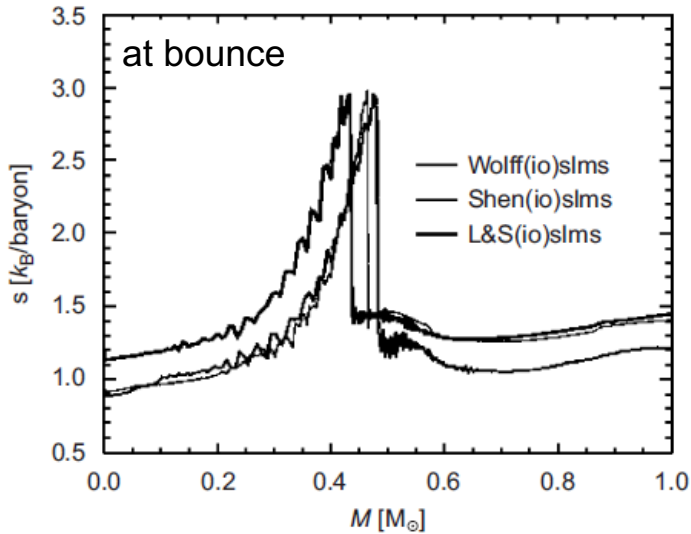


Steiner et al., ApJ 774, 17 (2013) - 1D simulation GR, Boltzmann  $v$  transport,  $11.2 M_{\text{sun}}$  and  $40 M_{\text{sun}}$  progenitors

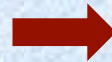
➡ not great impact on dynamics at bounce, impact on matter properties 12



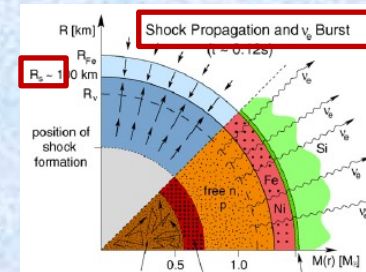
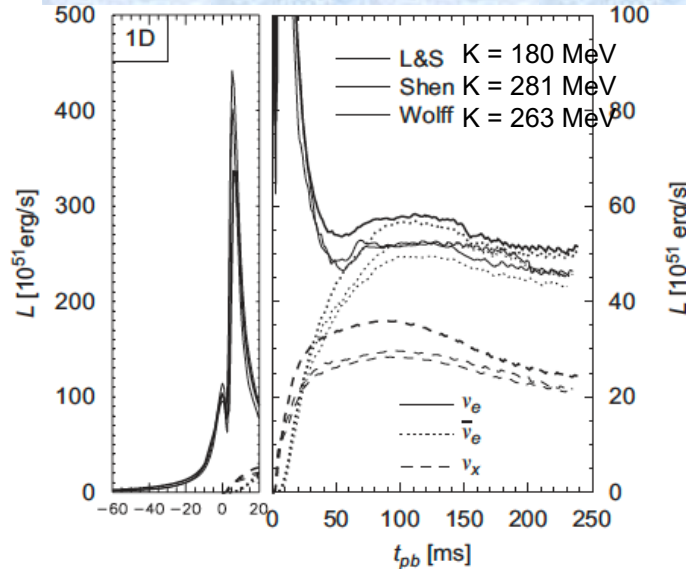
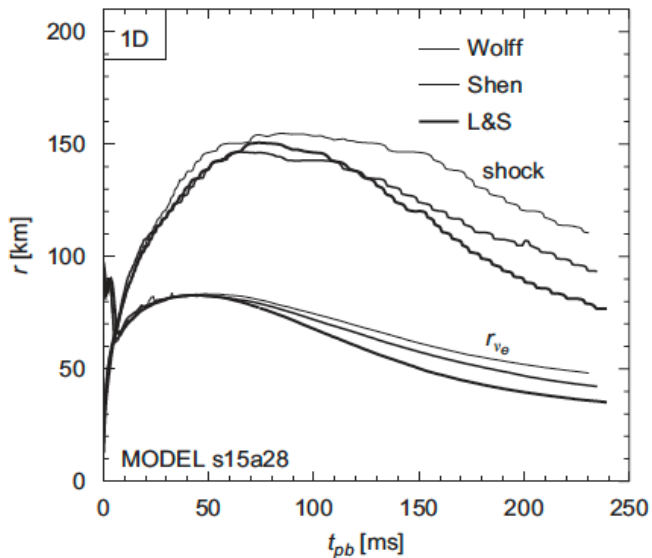
# CCSN simulations: (post-)bounce



K = 263 MeV  
K = 281 MeV  
K = 180 MeV



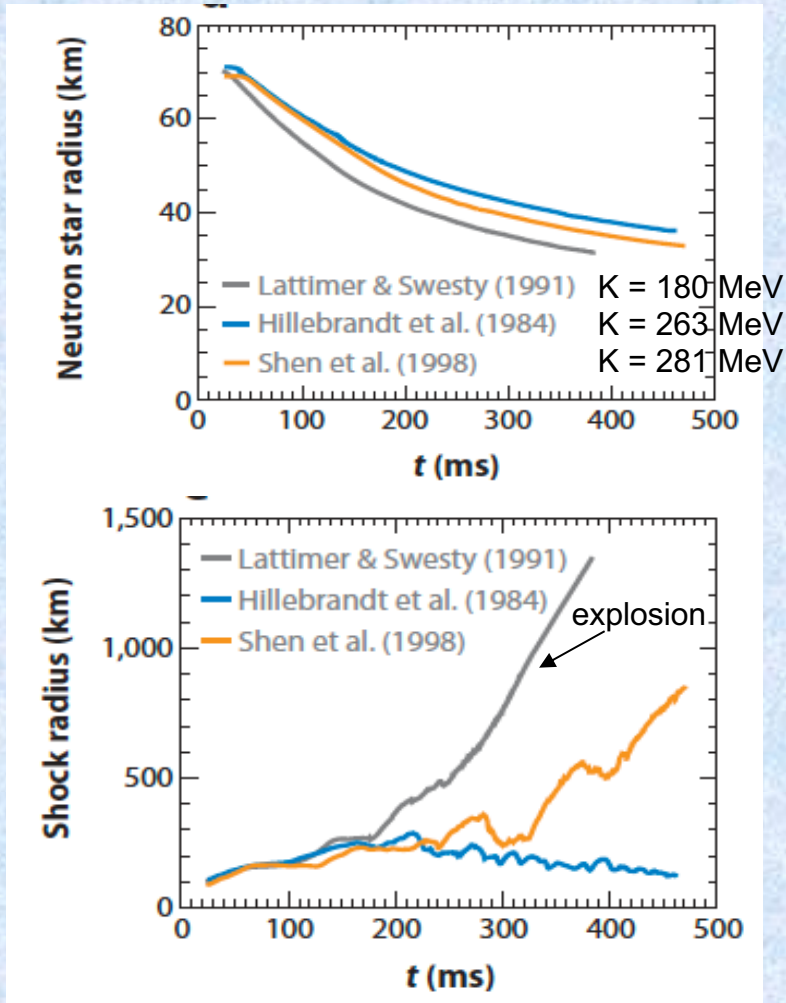
“softer” EoS → smaller inner core  
→ more compact PNS  
→ higher  $\nu$  luminosity



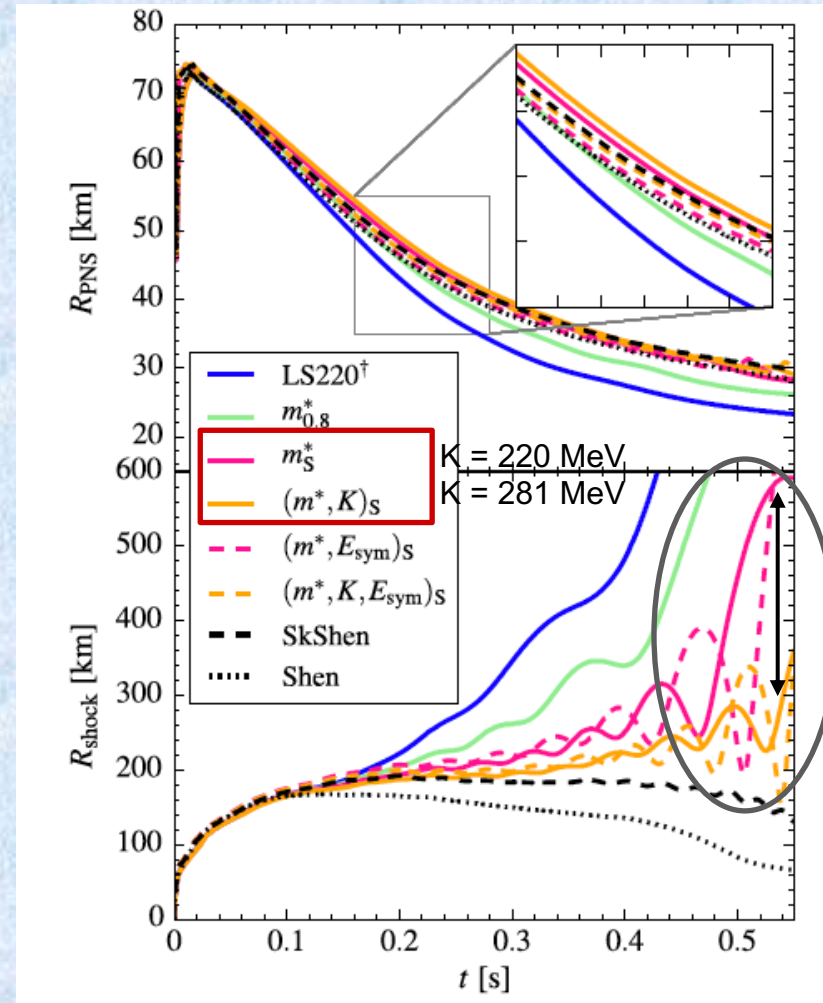
Janka et al., Phys. Rep. 442, 38 (2007) - 1D simulation, 15  $M_{\text{sun}}$  progenitor, “ray-by-ray”  $\nu$  treatment



# CCSN simulations: shock radius, PNS



Janka et al., Annu. Rev. Nucl. Part. Sci. 62, 407 (2012)  
2D simulations, Newtonian,  $11.2 M_{\text{sun}}$  progenitor



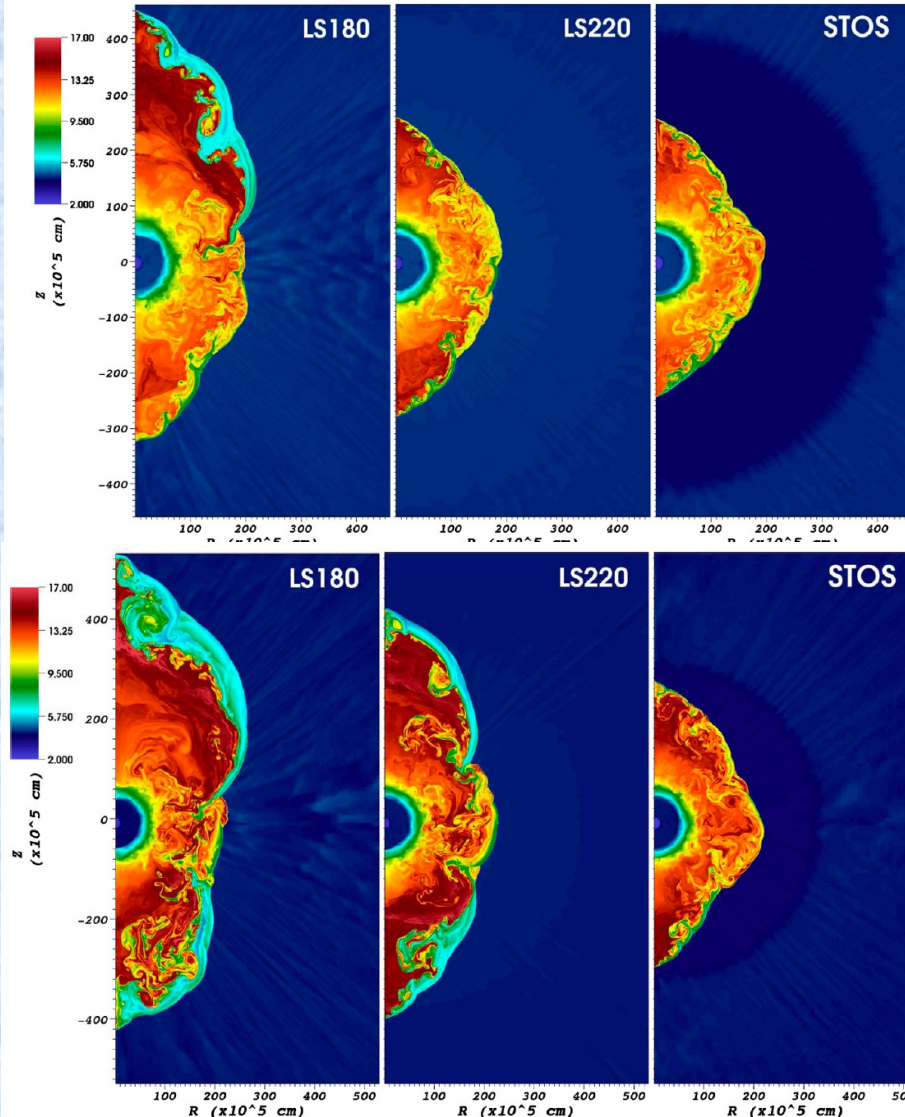
Yasin et al., PRL 124, 097701 (2020)  
1D simulations, multi-species neutrino physics,  $15 M_{\text{sun}}$  progenitor



“softer” EoS  $\rightarrow$  more compact PNS  
 $\rightarrow$  larger shock radii ( $\rightarrow$  explosion?)



# CCSN simulations: instabilities



*Entropy per baryon (colours)*

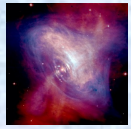
300 ms after bounce

→ larger instabilities for lower  $K_{\text{sat}}$

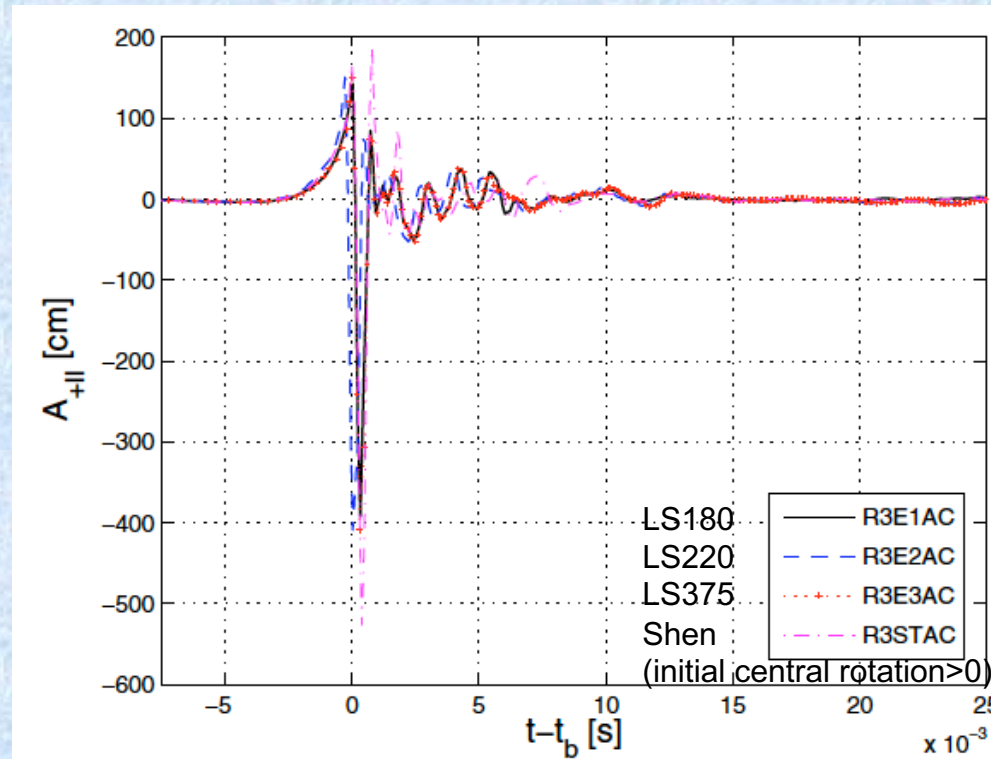
600 ms after bounce

→ shock expansion for LS,  
stationary for Shen et al. EoS

Couch, ApJ 765, 29 (2013). 2D simulations, simplified neutrino physics (fixed  $L_\nu$ ),  $15 M_{\text{sun}}$  progenitor



# CCSN simulations: GW signal (1)



Scheidegger et al., A&A 514, A51 (2010)

3D GR simulations with B field, Boltzmann transport,  $15 M_{\text{sun}}$  progenitor

→ impact on GW signal (amplitude in equatorial plane):  
**hard to discriminate**

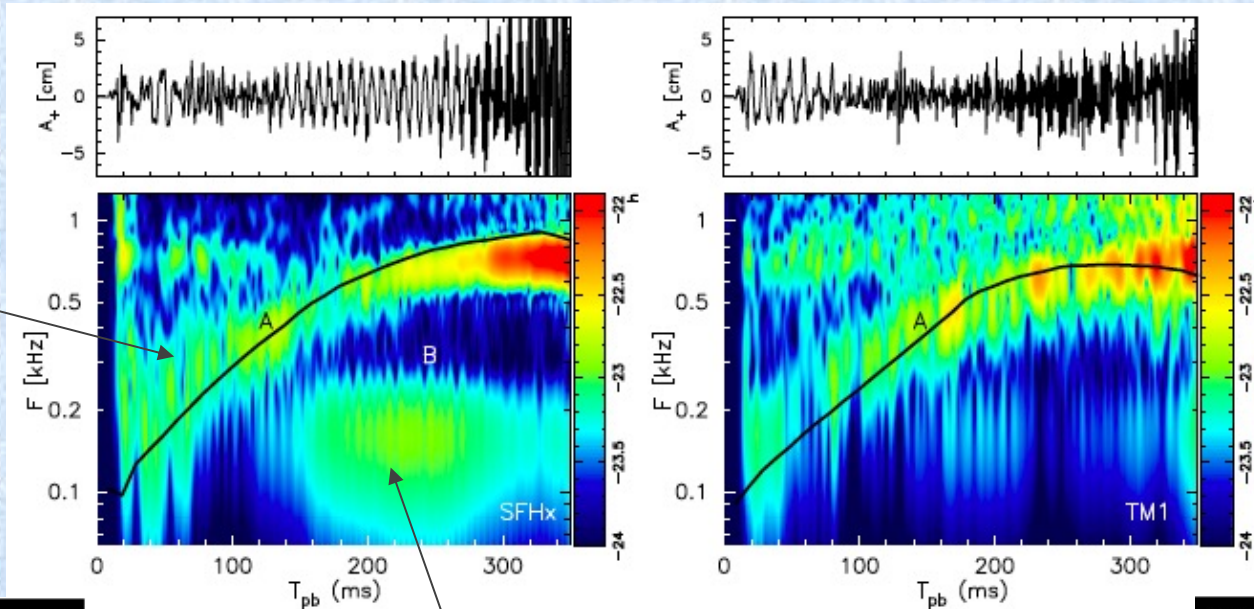




# CCSN simulations: GW signal (2)

SFHx (“softer” EoS)

TM1 (“stiffer” EoS)

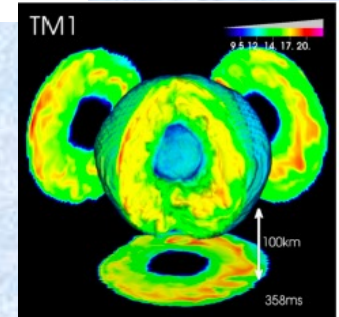
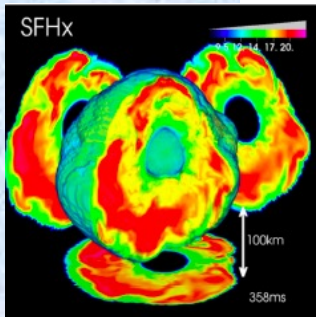


PNS  
(g-mode)  
oscillations

associated to SASI

→ SASI activity higher for “softer” EoS

but: different EoSs !

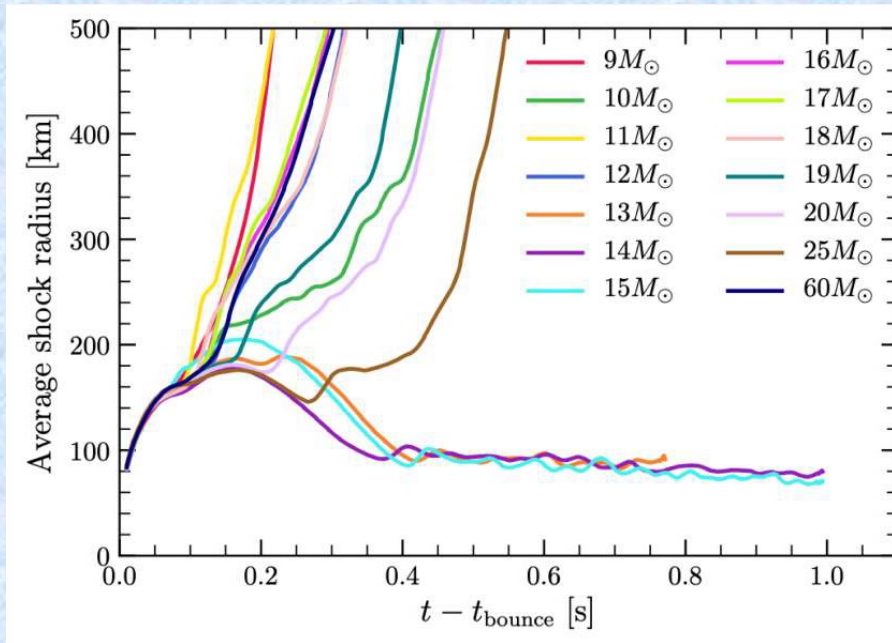


Kuroda et al., ApJL 829, L14 (2016) - 3D GR simulations, 15  $M_{\text{sun}}$  progenitor

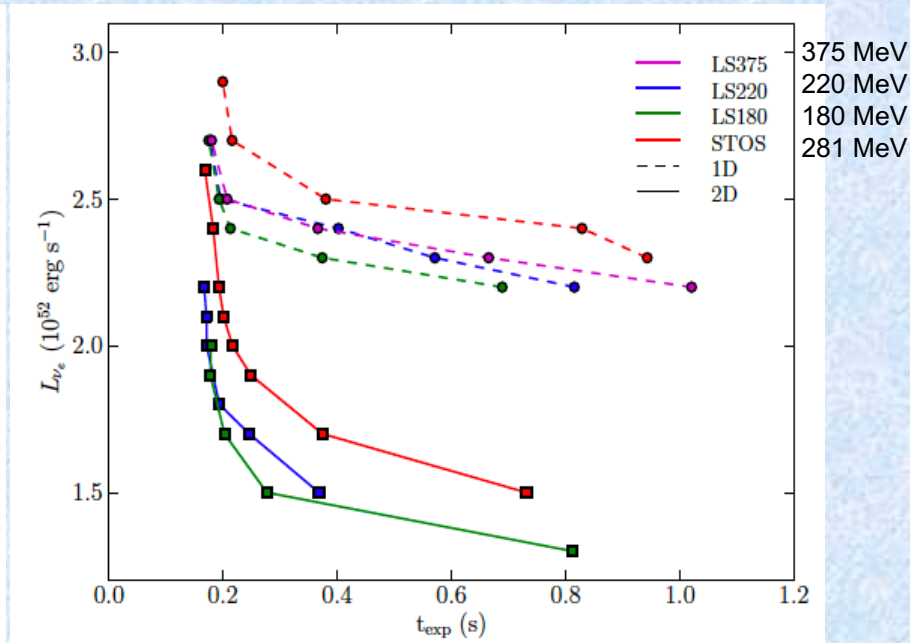


# CCSN simulations: many inputs count!

Other inputs matter ! e.g. progenitor, dimensionality, ...



Burrows et al., MNRAS 491, 2715 (2019), 3D, SFHo EoS



Couch, ApJ 765, 29 (2013). 1D and 2D simulations, simplified neutrino physics (fixed  $L_\nu$ ),  $15 M_{\text{sun}}$  progenitor (see also Pan et al., ApJ 857, 13 (2018))

→ **Progenitor mass dependence:**  
non-monotonic behaviour, dependence on progenitor structure

→ if same EoS, higher  $K_{\text{sat}}$  → later explosion for given  $L_\nu$   
**but : 1D vs 2D dependence !**



# Conclusions and open questions (CCSN)

- ❖ Roughly speaking, "softer" EoS :
  - more compact and faster contracting PNS
  - higher  $\nu$  luminosities
  - larger shock radii → more favourable to explosion

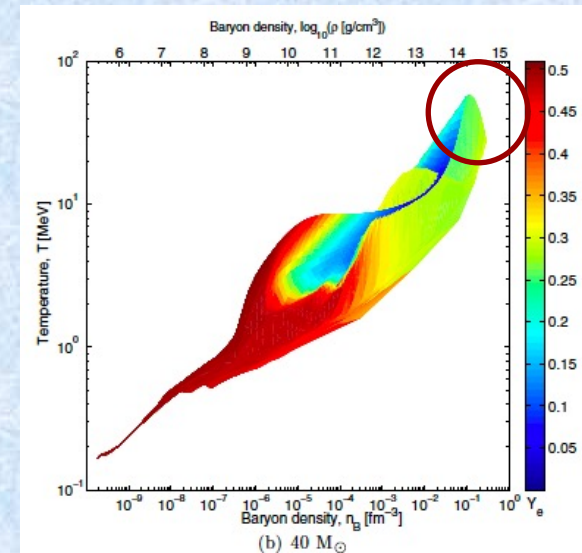
but :

- ✗ Difficult to correlate single nuclear parameters in CC dynamics !
  - EoS models differ from several aspects (nuclear theory, parameters, ...)

- ✗ *Hydro (macro) vs micro effects (also for BNS!)*
  - Consistent treatment of phase transitions challenging
  - Extension of many-body methods and extrapolation (e.g. parameters usually fitted at  $T=0$ )
  - Need of complex multi-D simulations → other effects: hydro instabilities (SASI, ...), progenitors,  $\nu$  treatment, ...

- ➔ ✓ no strong conclusive statements can be drawn
- ✓  $K_{\text{sat}}$  not the (only) key parameter

- ➔ ✓ need of systematic studies / simulations



Fischer et al., ApJSS 194, 39 (2011)



# Astrophysical context : NS

Mature (cold) NS  $\rightarrow$  cold catalysed matter (full equilibrium  $\rightarrow$  ground state)

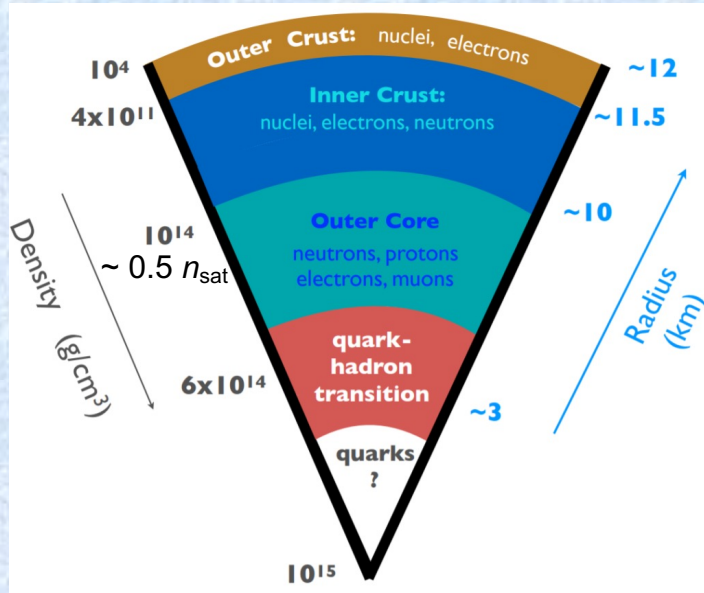
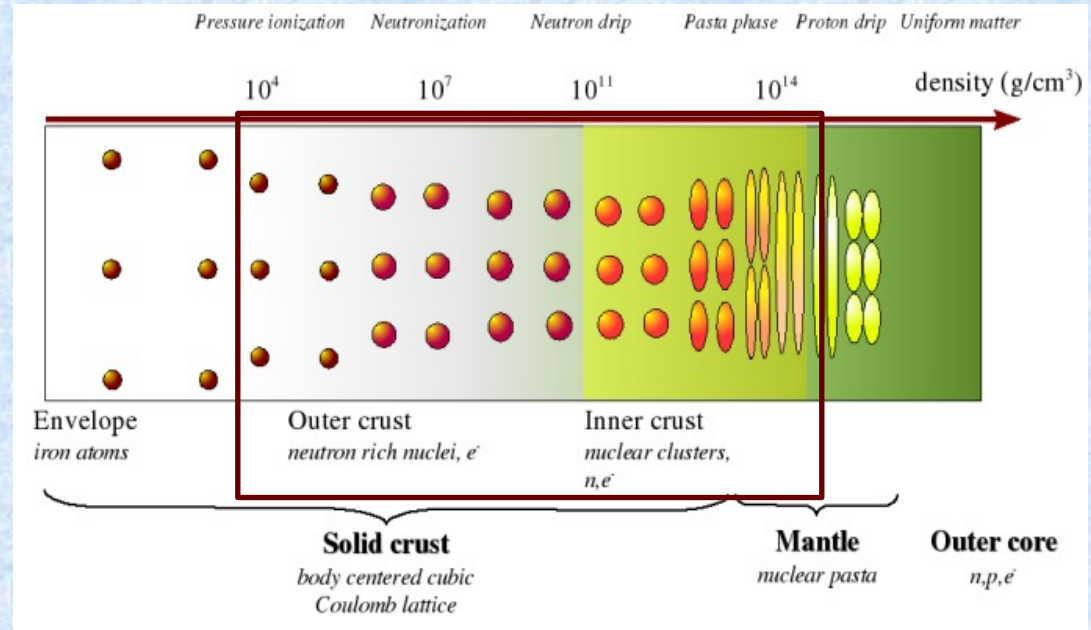


Image Credit: 3G Science White Paper

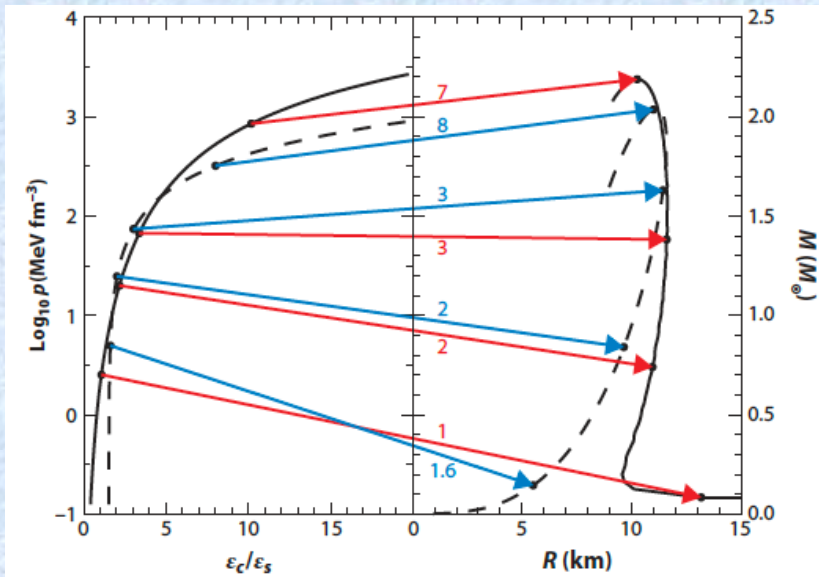


Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008)  
 see also : Chamel & Blaschke, ASSL 457, 337 (Springer, 2018)

If “mature” (cold) NSs  $\rightarrow T = 0$  and  $\beta$  equilibrium  $\rightarrow$  “easier” (ground-state energy)  
but: still challenging because of different states of matter and range of density



# EoS and NS properties

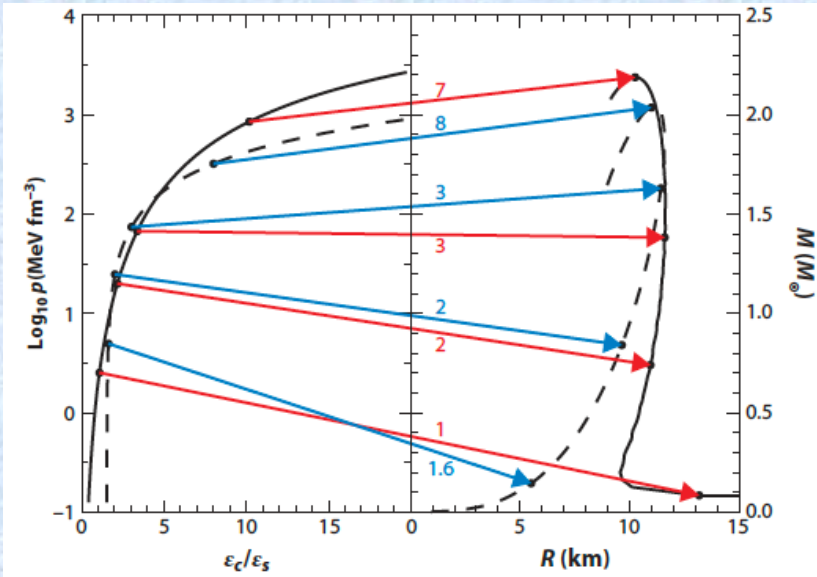


Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

- ✓ GR  $\rightarrow$  one-to-one correspondence  
EoS  $\leftrightarrow$  NS static properties  $M(R)$ ,  $\Lambda(M)$ ...  
(non-rotating mature NS)
- ✓ Different EoSs  $\leftrightarrow$  different NS properties  
 $\leftrightarrow$  different observational signals (GW,...)  
 $\rightarrow$  trace back to EoS and composition ?



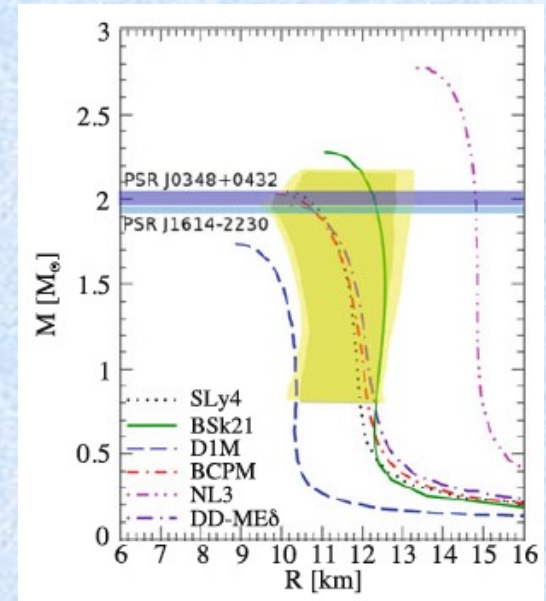
# EoS and NS properties



Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

- ✓ GR  $\rightarrow$  one-to-one correspondence  
EoS  $\leftrightarrow$  NS static properties  $M(R)$ ,  $\Lambda(M)$ ...  
(non-rotating mature NS)
- ✓ Different EoSs  $\leftrightarrow$  different NS properties  
 $\leftrightarrow$  different observational signals (GW,...)  
**?**  
 $\rightarrow$  trace back to EoS and composition ?

- but:
- ✗ EoS model dependent !
  - ✗ no ab-initio dense-matter calculations in all regimes  
 $\rightarrow$  phenomenological models  
(many-body approach + functional)
  - ✗ composition  $\leftrightarrow$  EoS  $\rightarrow$   $M(R)$  ?  
(e.g. masquerade effect!)



Burgio & Fantina, ASSL 457, 255 (2018)

for a review see e.g. Oertel et al., Rev. Mod. Phys. 89, 015007 (2017), Burgio & Fantina, ASSL 457, 255 (2018), Blaschke & Chamel, ASSL 457, 337 (2018)

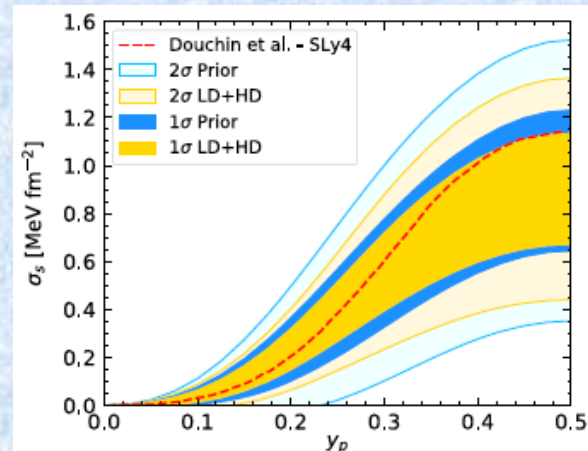


# EoS: meta-model (nucleons only)

- **Meta-model** approach for nucleons : flexible functional (“quasi” agnostic)  
 → expansion in density and asymmetry around  $n_{\text{sat}}$  and  $\delta = 0$  (with  $m_q^*$  included)

$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left( \left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m \quad \begin{aligned} x &= (n - n_{\text{sat}})/3n_{\text{sat}} \\ \delta &= (n_n - n_p)/n \end{aligned}$$

- Empirical parameters (bulk)  $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$  } ~ 15 – 20 parameters
- If one wants to model the crust → + surface and Coulomb term (CLDM)  
 → surface parameters ( $\sigma_0, \sigma_{0,c}, \beta, b_s, \rho$ )



Dinh Thi et al., EPJA 57, 296 (2021) – surface tension



# NS: model dependence of observables

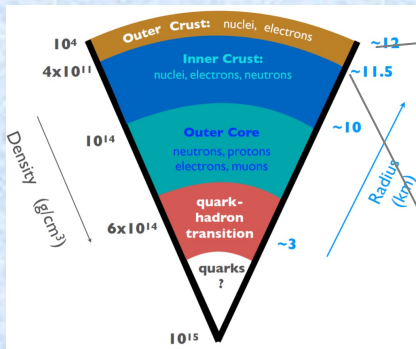
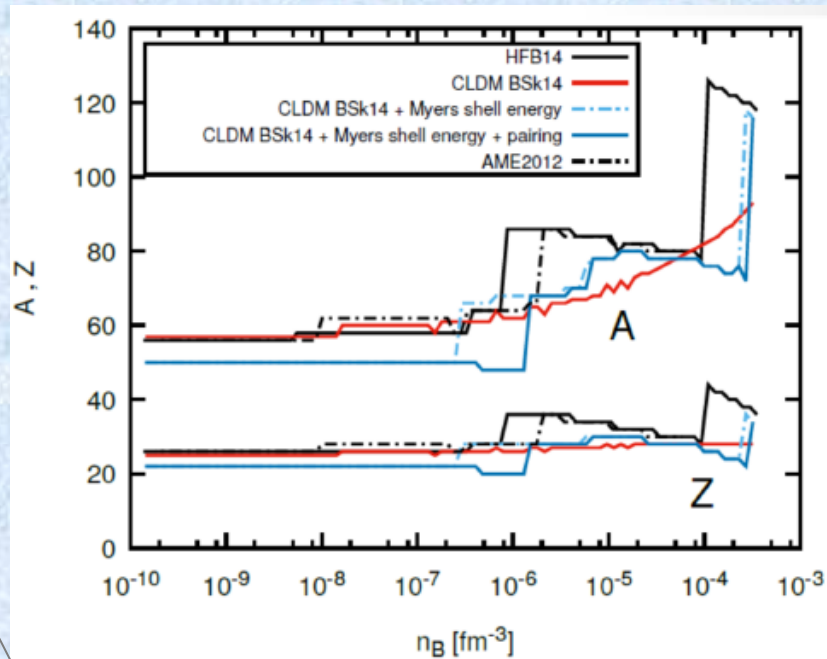


Image Credit: 3G Science White Paper



F. Gulminelli's talk @GMR workshop (2020)

➤ composition → dependence on many-body method





# NS: model dependence of observables

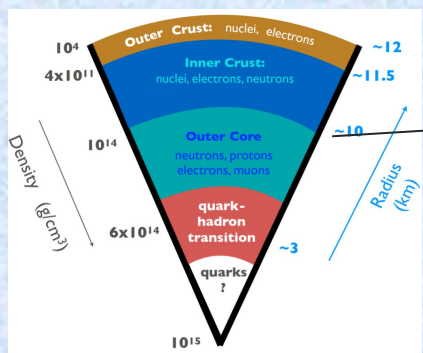
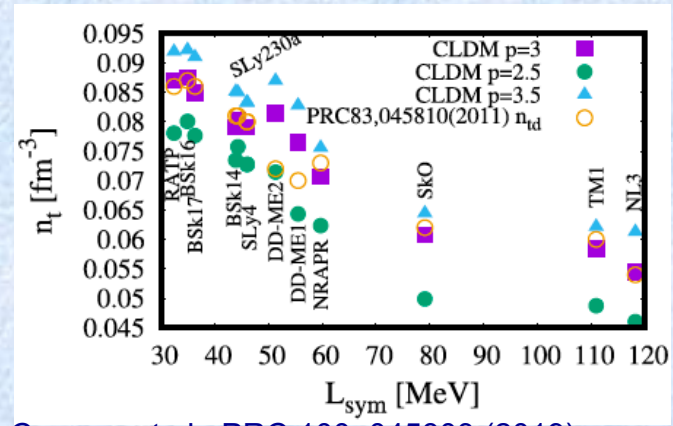
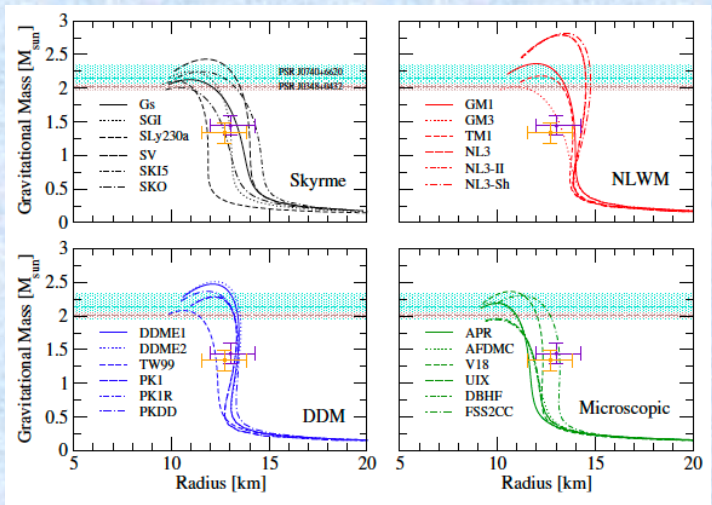


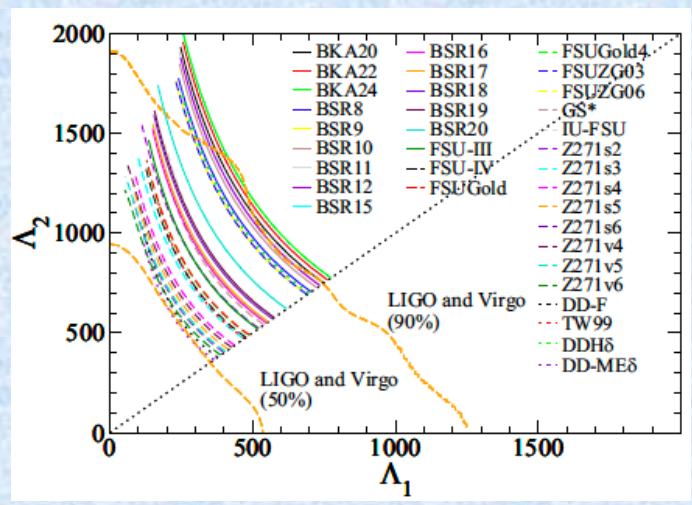
Image Credit: 3G Science White Paper



Carreau et al., PRC 100, 045803 (2019)



Burgio & Vidana, Universe 6, 119 (2020)



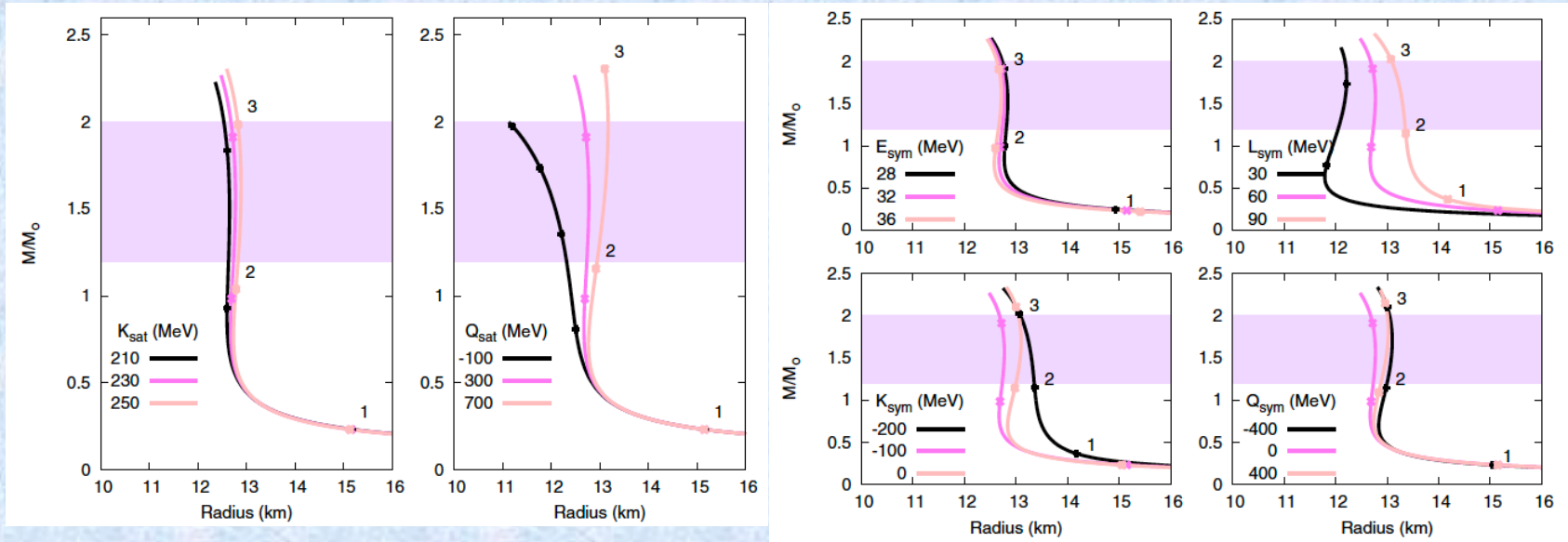
Lourenço et al., PRC 99, 045202 (2019)

- composition → dependence on many-body method
- global observables → dependence on the functional

but: comparison of very different models ( $\neq$  parameters,  $\neq$  many-body method) !  
 → which parameter(s) matter?

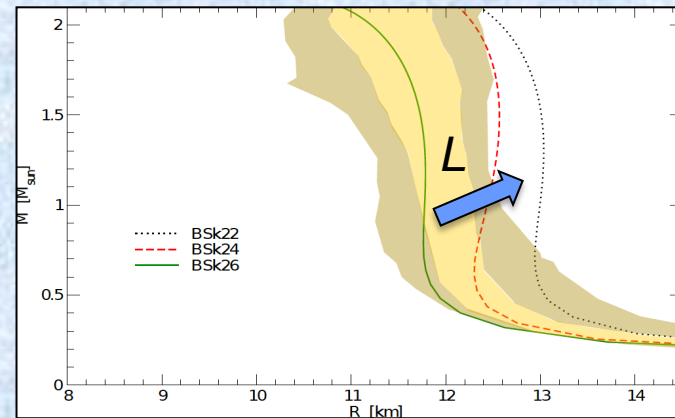


# NS: impact of IS/IV parameters



Margueron et al., PRC 97, 025806 (2018) – meta-model

- impact of isovector parameters
- impact of high-order parameters



Pearson et al., Eur. Phys. J. A 50, 43 (2014)  
(Skyrme-type models)



# EoS: meta-model + Bayesian

- **Meta-model** approach for nucleons : flexible functional (“quasi” agnostic)  
 → expansion in density and asymmetry around  $n_{\text{sat}}$  and  $\delta = 0$  (with  $m_q^*$  included)

$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left( \left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m \quad \begin{array}{l} x = (n - n_{\text{sat}})/3n_{\text{sat}} \\ \delta = (n_n - n_p)/n \end{array}$$

- Empirical parameters (bulk)  $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$  } ~ 15 – 20 parameters
- If one wants to model the crust → + surface and Coulomb term (CLDM)  
 → surface parameters ( $\sigma_0, \sigma_{0,c}, \beta, b_s, \rho$ )

- Apply filters in Bayesian analysis

$$p_{\text{post}}(\vec{X}) = \mathcal{N} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X}) e^{-\chi^2(\vec{X})/2} p_{\text{prior}}(\vec{X})$$

Low-Density filters  
 → ab-initio (EFT)  
 (e.g. Drischler et al,  
 PRC 93, 054316 (2016))

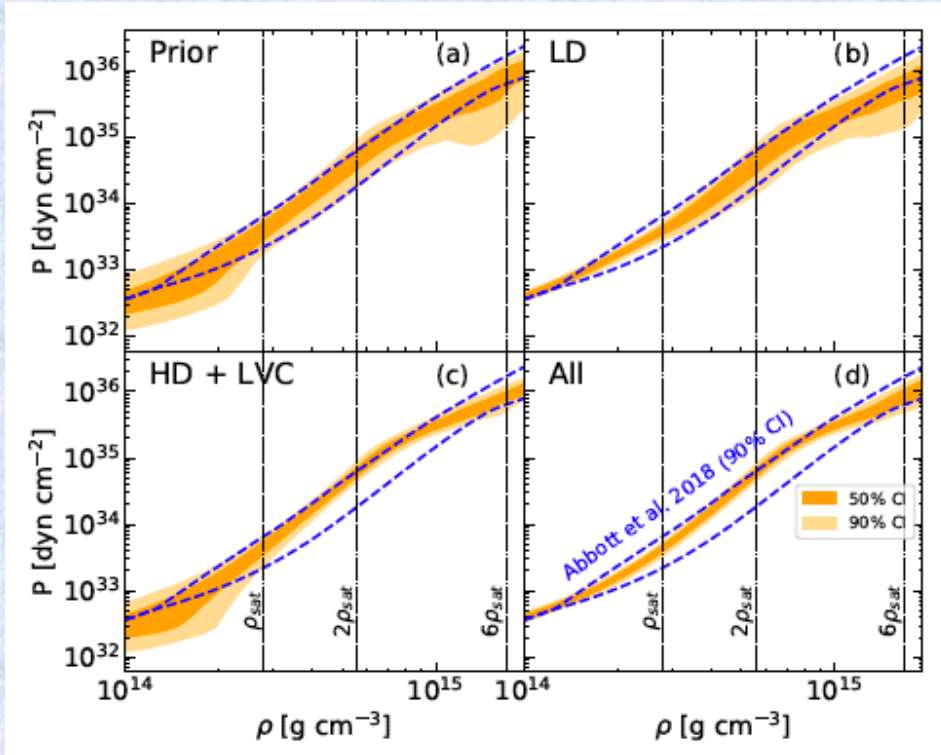
High-Density filters  
 → causality, stability,  
 $M_{\text{NS,max}}, e_{\text{sym}} > 0$   
 (NICER, tidal from GW)

nuclear masses  
 (AME2016)  
 → surf. param. ( $\sigma_0, \sigma_{0,c}, \beta, b_s, \rho$ )

flat non-informative prior  
 → span large parameter space

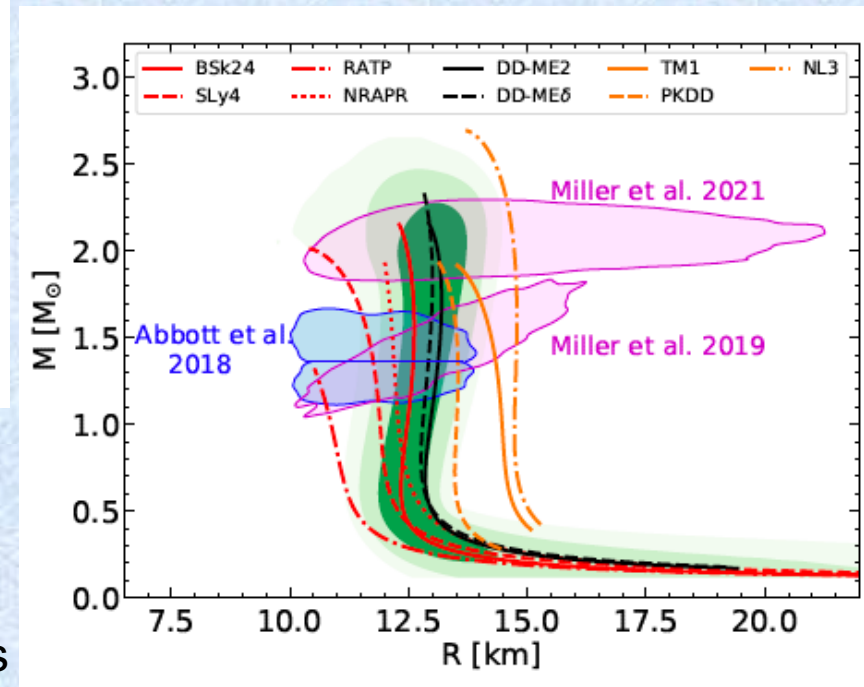


# EoS : effect of LD/HD constraints



Dinh Thi et al., Universe 7, 373 (2021)

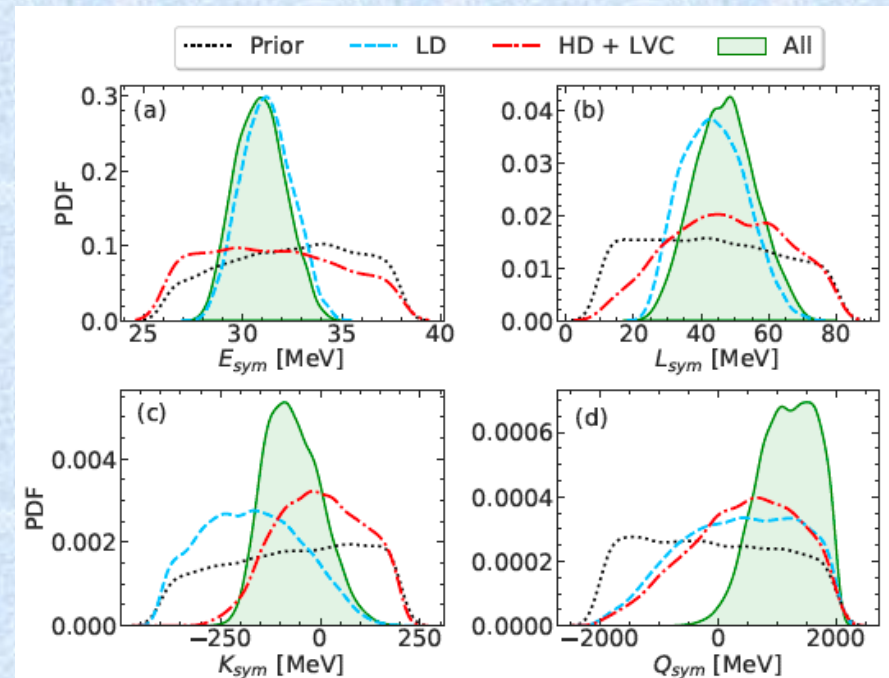
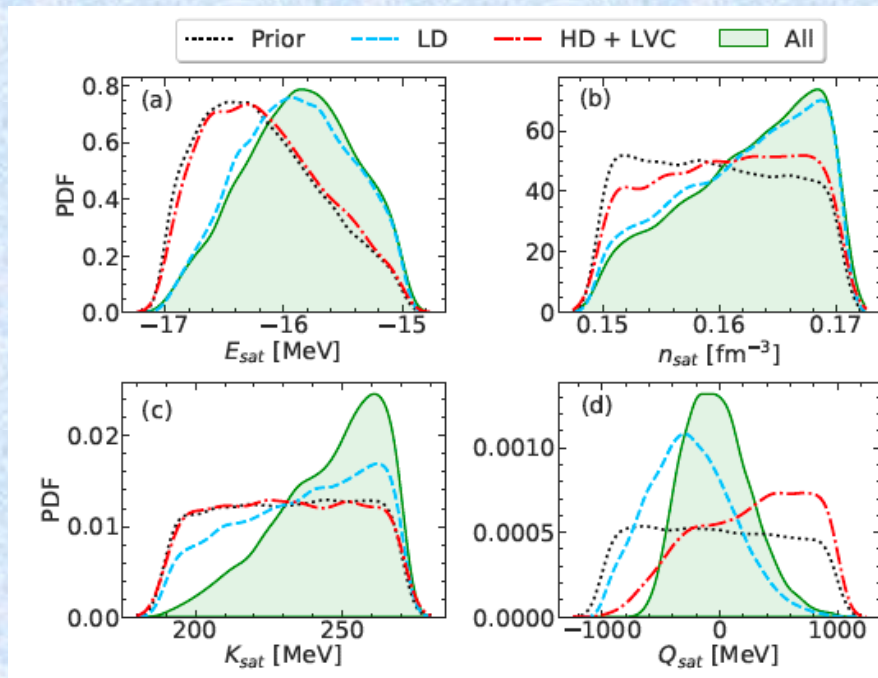
- posterior compatible with observations
- but: some popular models are not !
- nucleonic hp compatible with observations



Dinh Thi et al., A&A 654, A114 (2021)



# NS: empirical parameters



Dinh Thi et al., Universe 7, 373 (2021)

- HD constraints have almost no impact on low-order parameters, but impact on high-order parameters (poorly constrained by experiments)
- LD constraints impact isovector and high-order parameters (also effective at low density)

see also B.A. Li's talk



# NS: correlations and empirical param.

Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021)

## CRUST-CORE TRANSITION

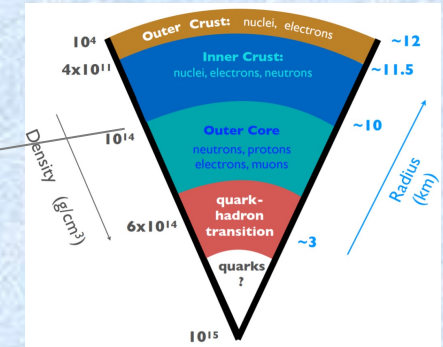
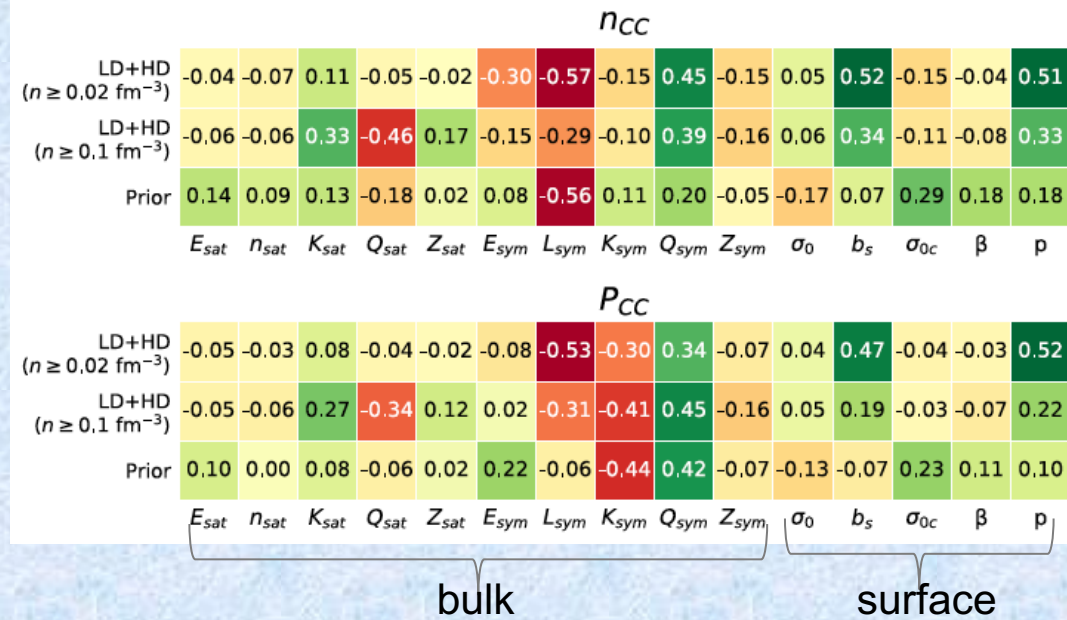
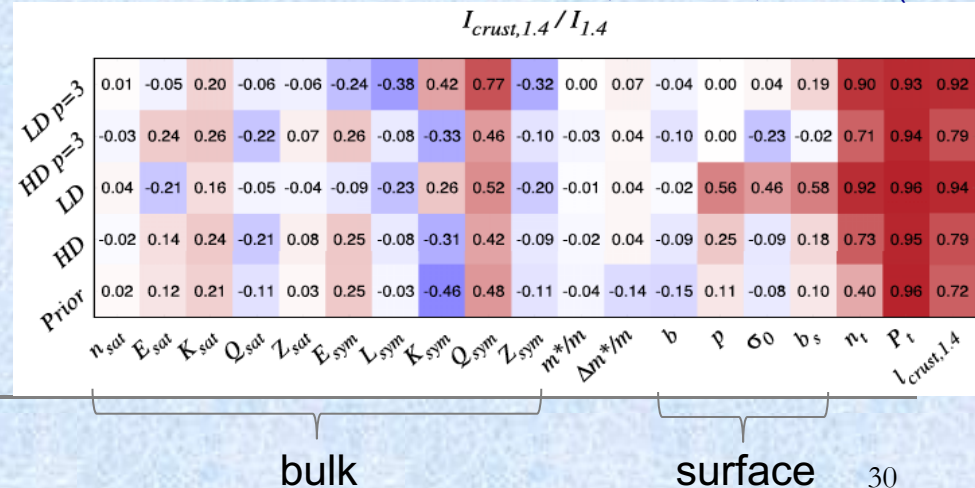


Image Credit: 3G Science White Paper

## CRUSTAL MOMENT OF INERTIA

Carreau et al., PRC 100, 055803 (2019)

- importance of parameters (both *bulk* and *surface*) on observables
- importance of higher-order parameters



see also Balliet et al., ApJ 918, 79 (2021)



# NS: beyond empirical parameters

## LOW-DENSITY EOS

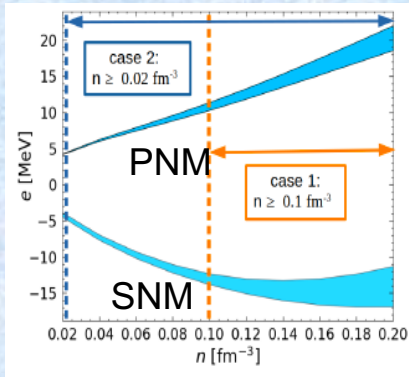
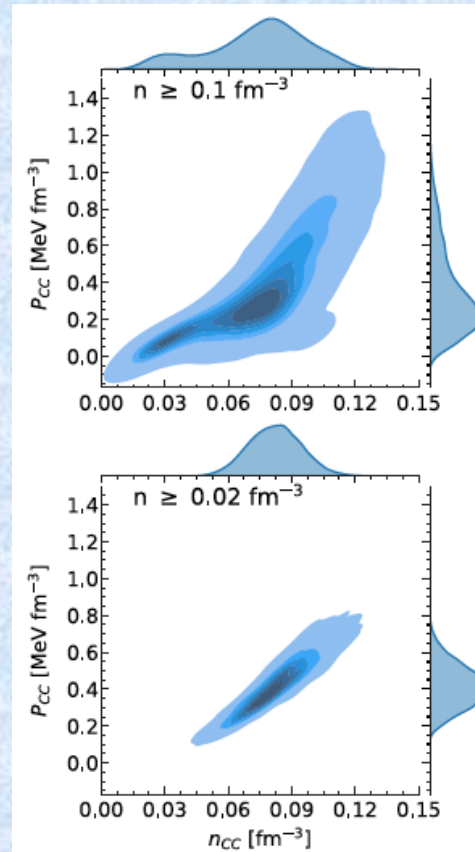
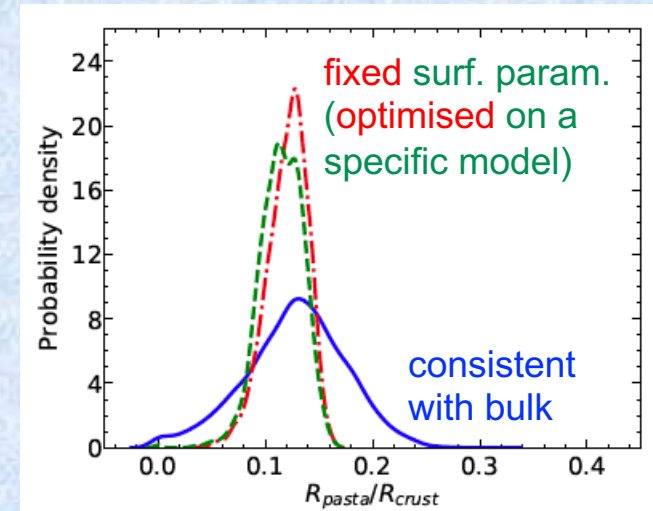


Fig. courtesy of H. Dinh Thi



## SURFACE TERMS



Dinh Thi et al., A&A 654, A114 (2021)  
Dinh Thi et al., EPJA 57, 296 (2021)

- importance of low-density EoS
- importance of consistent calculation of the surface terms





# Conclusions and open questions (NS)

- ❖ Dependence of predictions on the functional (in a complex way)
- ❖ High-order parameters also important for NS modelling
- ❖ Static properties: if GR  $\rightarrow$  possible “extraction” of EoS (with uncertainties)

but :

- ✗ Even if  $T = 0$  approx for mature NSs, description of phase transition challenging
- ✗ Other ingredients play a role
  - $\rightarrow$  low-density EoS
  - $\rightarrow$  surface terms (in neutron-rich nuclei)

 ✓  $K_{\text{sat}}$  not the (only) key parameter

 ✓ need of (low-density) constraints from ab-initio theory  
✓ need of experiments on neutron-rich nuclei to determine/extract different parameters (e.g. skins  $\rightarrow L_{\text{sym}}$ ; GMR in asymmetric matter  $\rightarrow K_{\tau, \text{sym}, \dots}$ )





*Thank you*