Bayesian Inferences of Nuclear EOS from Heavy-Ion Reactions, Giant Resonances and Neutron Stars

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- Incompressibility, skewness and kurtosis of symmetric nuclear matter from central collisions and giant resonances of heavy nuclei
- EOS parameters from observations of canonical neutron stars using gravitational waves and x-rays by LIGO/VIRGO+NICER
- High-density symmetry energy from radii of massive neutron stars observed by NICER and future STROBE-X satellites

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Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

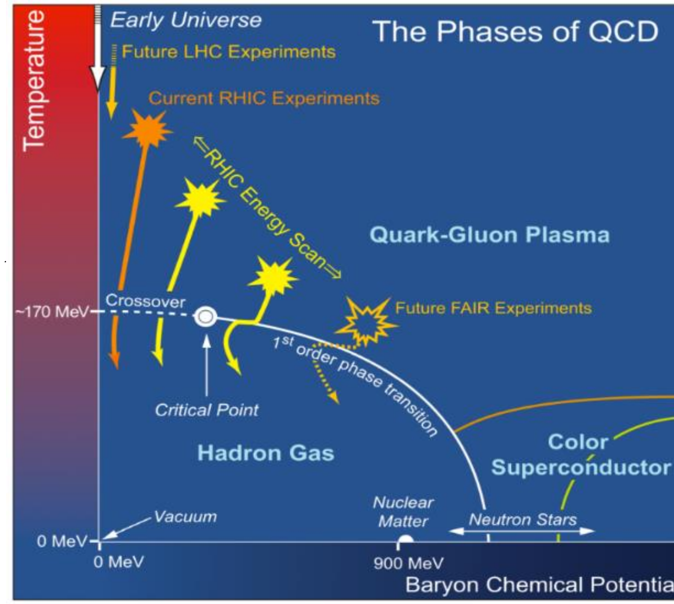
symmetry energy

Isospin asymmetry

$$E(\rho_n,\rho_p)=E_0(\rho_n=\rho_p)+E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2+o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter

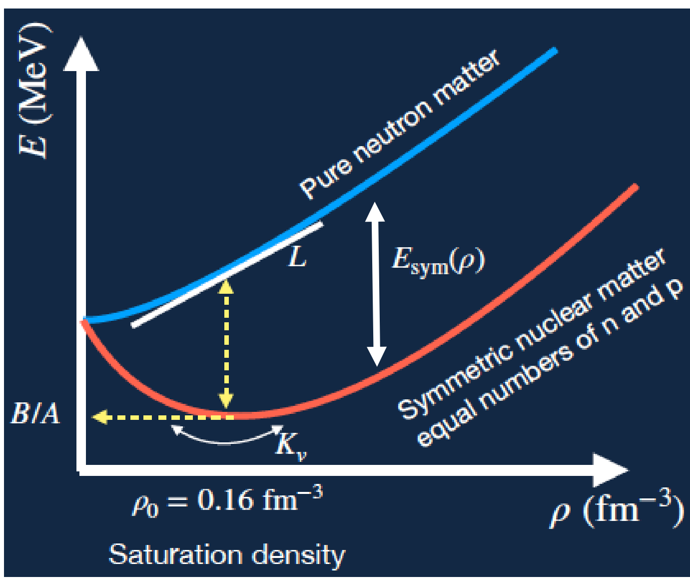


density

New opportunities

Isospin asymmetry

$\delta=(\rho_n-\rho_p)/\rho$



Isospin effects in observables of structures & collisions of neutron stars & heavy nuclei

$$E_{\rm sym}(\rho)=E_{\rm sym}(\rho_i)+\int_{\rho_i}^{\rho}\frac{P_{\rm PNM}(\rho_v)-P_{\rm SNM}(\rho_v)}{\rho_v^2}d\rho_v$$

Fundamental Microphysics Theories underlying each term in the EOS , what …, why …., where …how

Experimental and Observational Macrophysics underlying each observable and phenomenon, what …, why …., where …how

Empirical parameterizations especially useful for meta-modeling of EOS

Transport model simulations of heavy-ion collisions, energy density functionals for nuclear structures, Bayesian inferences of EOS, properties of neutron stars, waveforms of gravitational waves, ….

$$E(\rho,\delta) = E_0(\rho) + E_{\rm sym}(\rho)\cdot\delta^2$$

Assuming no hadron-quark phase transition

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{24}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4,$$

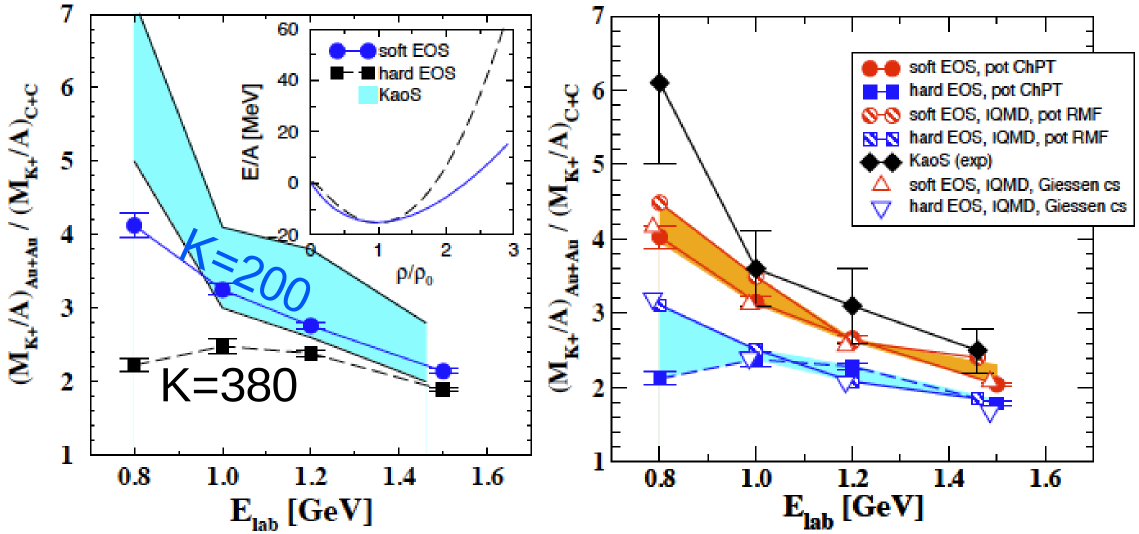
$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho}{\rho_0}-1\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho}{\rho_0}-1\right)^2 + \frac{J_{\rm sym}}{162}\left(\frac{\rho}{\rho_0}-1\right)^3 + \mathcal{O}\left[\left(\frac{\rho}{\rho_0}-1\right)^4\right]$$

Near the saturation density ρ_0, they are Taylor expansions, appropriate for structure studies. Just parameterizations when applied to heavy-ion collisions and the core of neutron stars

"Current" status of the restricted EOS parameter space:

Low density: $K_0 = 240 \pm 20$, $E_{\rm sym}(\rho_0) = 31.7 \pm 3.2$ and $L = 58.7 \pm 28.1$ MeV
High density: $-400 \leq K_{\rm sym} \leq 100$, $-200 \leq J_{\rm sym} \leq 800$, and $-800 \leq J_0 \leq 400$ MeV

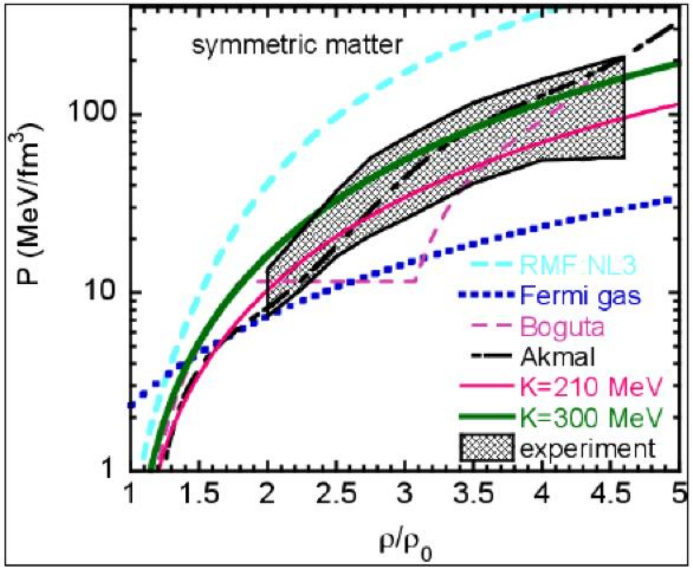
Constraints on K from Kaon production in heavy-ion collisions

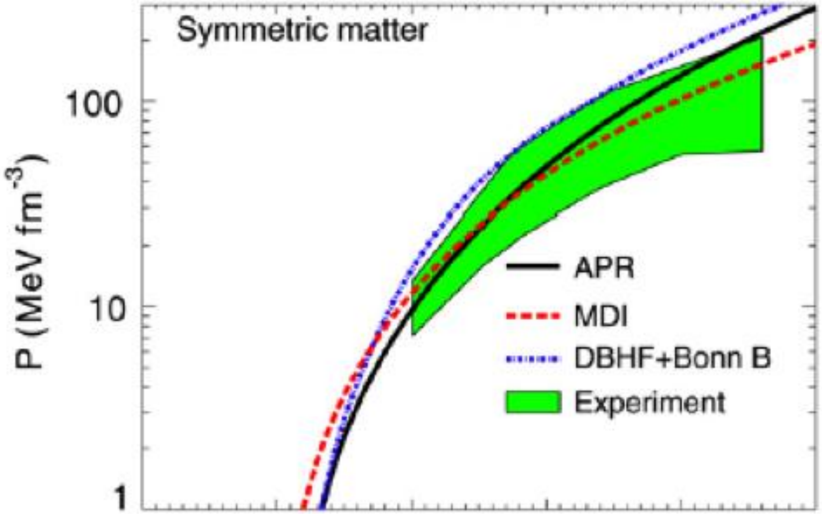


C. Fuchs et al., PRL 86 (2001) 1974
C. Hartnack et al., PRL 96 (2006) 012302

Indication: the incompressibility K should be less than 200 MeV

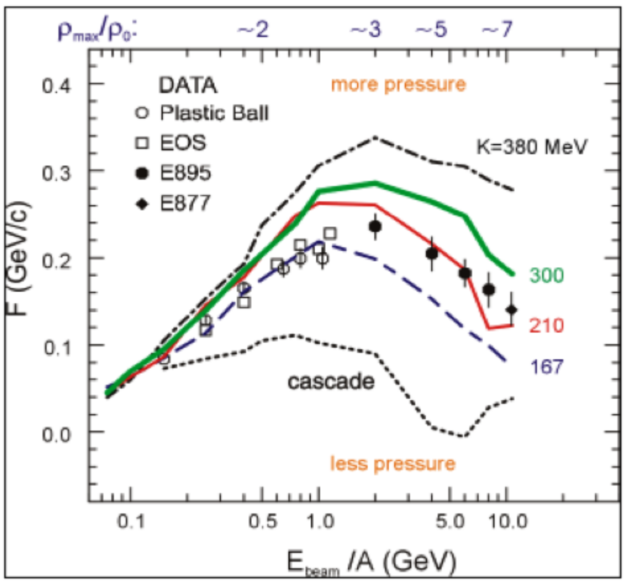
Danielewicz P. I., Lacey R and Lynch W G 2002 *Science* 298 1592

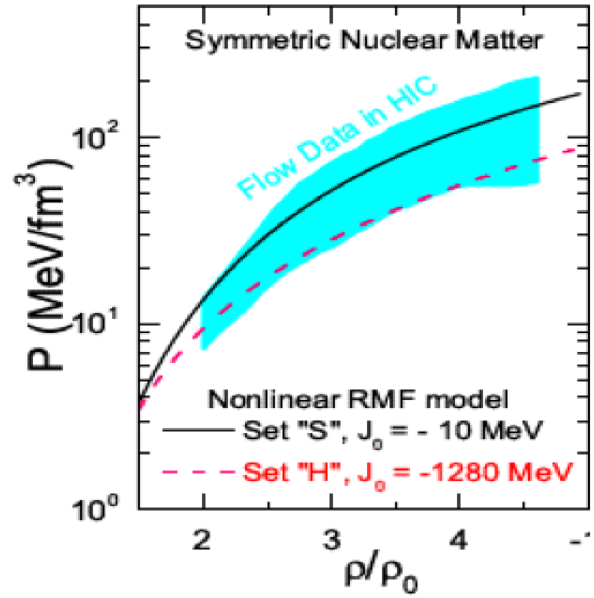




P. G Krastev, Bao-An Li, A. Worley, PLB 668, 1 (2008)

Flow favors K=210-300 MeV





BJ Cai, LW Chen, NST 28 (12), 1-9 (2017)

Bayesian inference of high-density SNM EOS parameters from heavy-ion reaction data

	K_0	J_0	Z_0
Av	235	−200	−146
σ	30	200	1728
Min	145	−800	−5330
Max (3σ)	325	400	5038

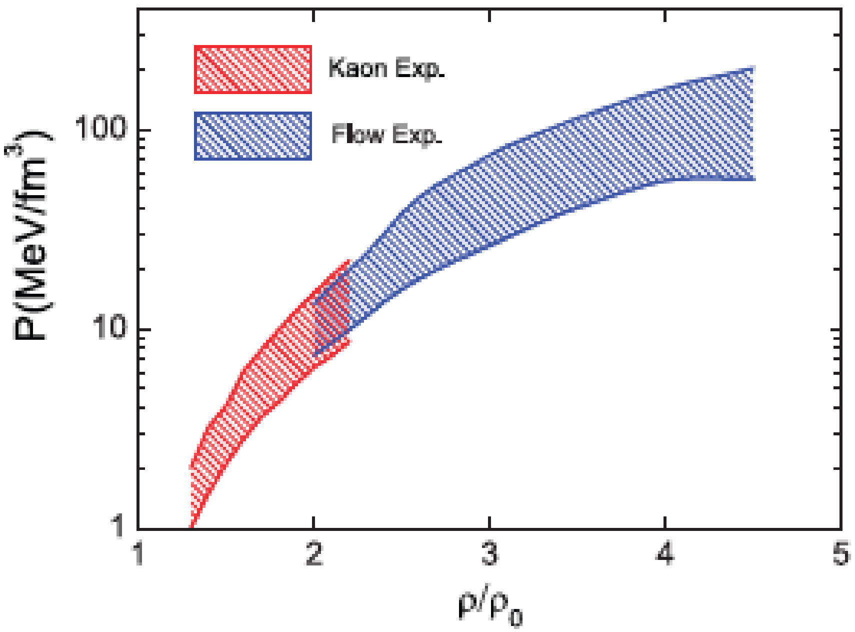
Prior ranges of SNM EOS parameters based on theories and data available

Margueron J, Hoffmann C R and Gulminelli F 2018 PRC 97, 025805 and 025806

Antic S, Chatterjee D, Carreau T and Gulminelli F 2019 J. Phys. G: Nucl. Part Phys. 46 065109

The pressure in symmetric nuclear matter

$$P(\rho) = \rho^2 \frac{\mathrm{d}E_0(\rho)}{\mathrm{d}\rho} = \frac{\rho^2}{\rho-\rho_0}\left[K_0\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \frac{Z_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^4\right].$$



Constraints on the EOS of symmetric nuclear matter from heavy-ion collisions

Danielewicz P. l., Lacey R and Lynch W G 2002 *Science* 298 1592

Fuchs C 2006 *Prog. Part. Nucl. Phys.* 56 1

Lynch W G *et al* 2009 *Prog. Part. Nucl. Phys.* 62 427

Reaching the stationary state in the Markov Chain Monte Carlo (MCMC) process

Bayes' theorem:

Prior distribution

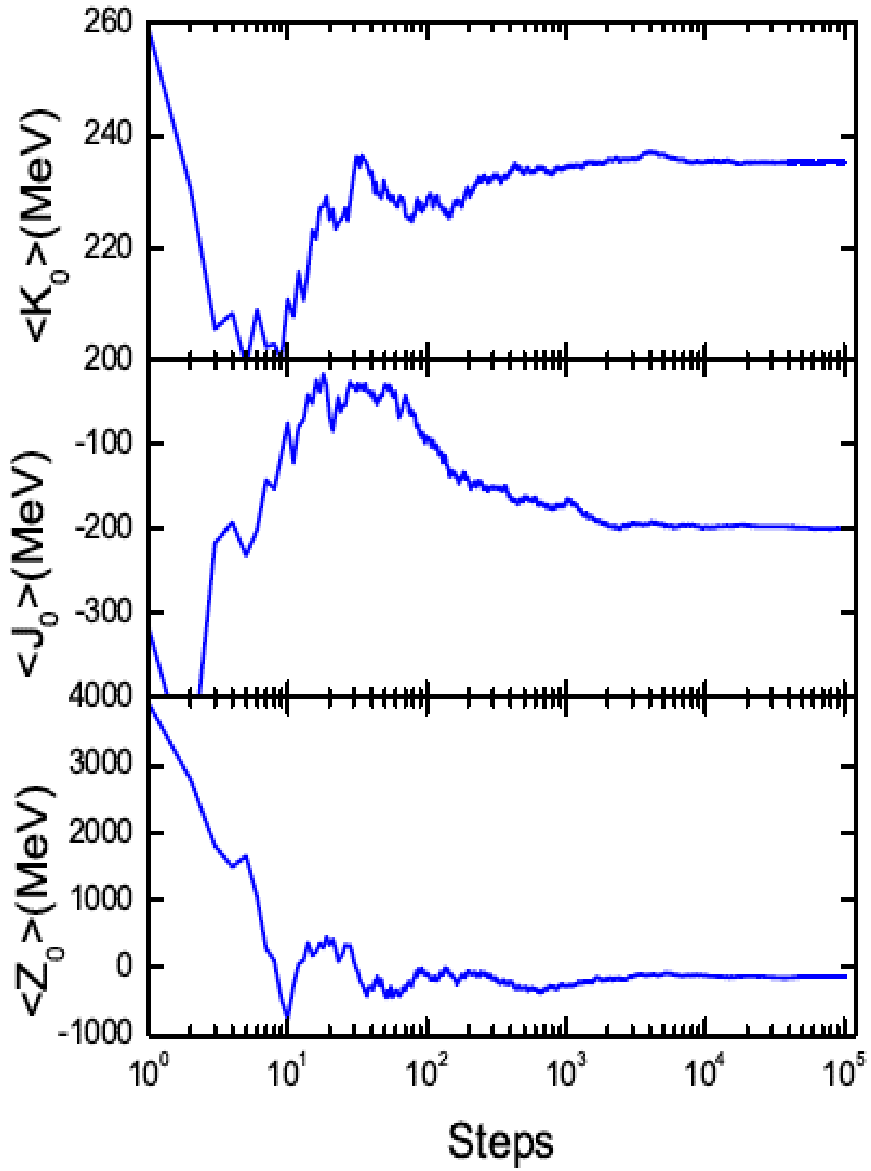
$$P(\mathcal{M}(K_0,J_0,Z_0)|D) = CP(D|\mathcal{M}(K_0,J_0,Z_0))P(\mathcal{M}(K_0,J_0,Z_0)),$$

Likelihod function

Calculations data

$$P[D|\mathcal{M}(K_0,J_0,Z_0)] = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{D,j}} \exp\left[-\frac{(P_{\mathrm{th},j}-P_{D,j})^2}{2\sigma_{D,j}^2}\right]$$

Metropolis-Hastings algorithm
in deciding each step



Bayesian inference of the SNM EOS parameters from the empirical pressure as a function of density “data”

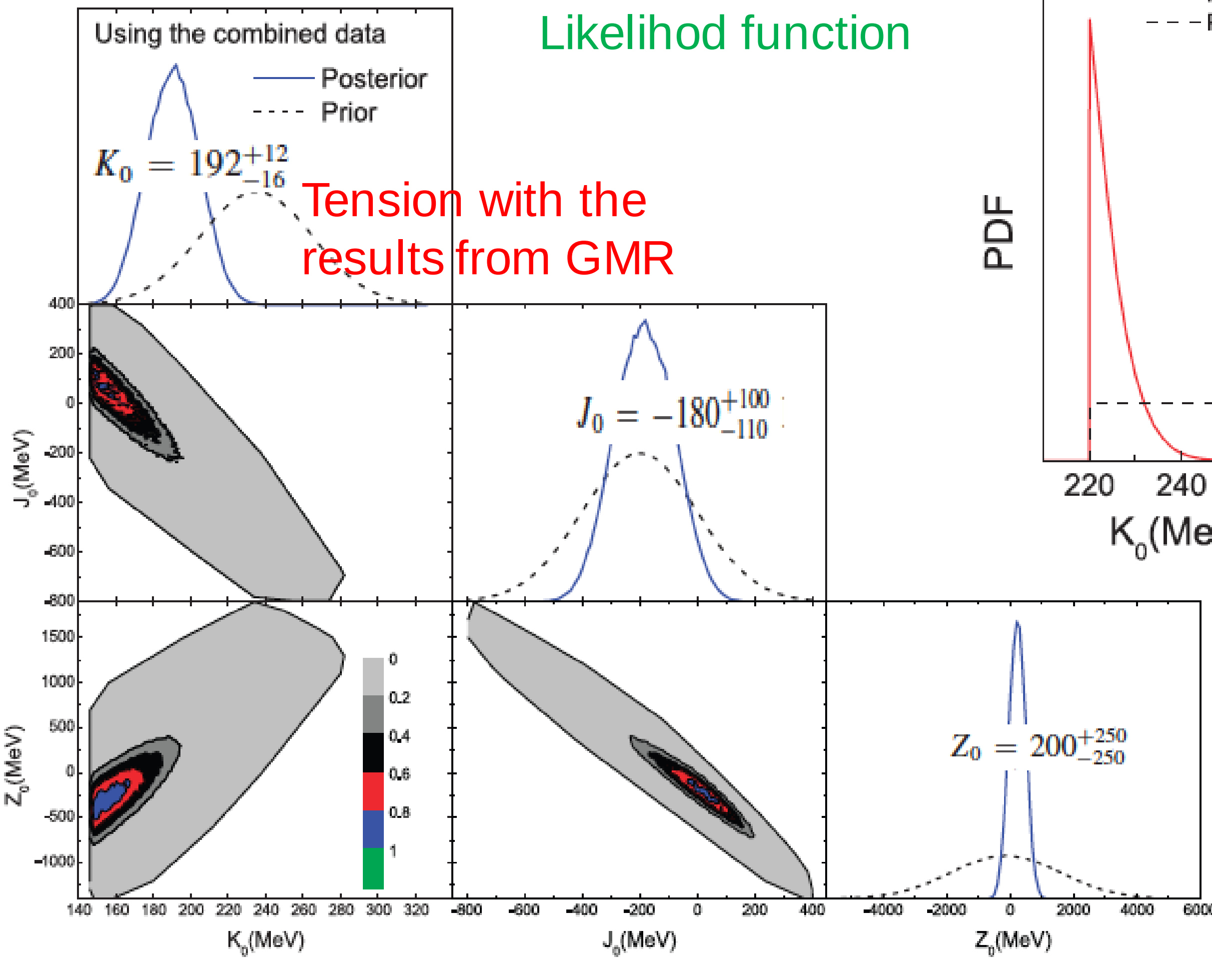
Wen-Jie Xie and Bao-An Li
JPG 48, 025110 (2021)

Posterior PDF: $P(\mathcal{M}(K_0, J_0, Z_0)|D) = CP(D|\mathcal{M}(K_0, J_0, Z_0))P(\mathcal{M}(K_0, J_0, Z_0)),$

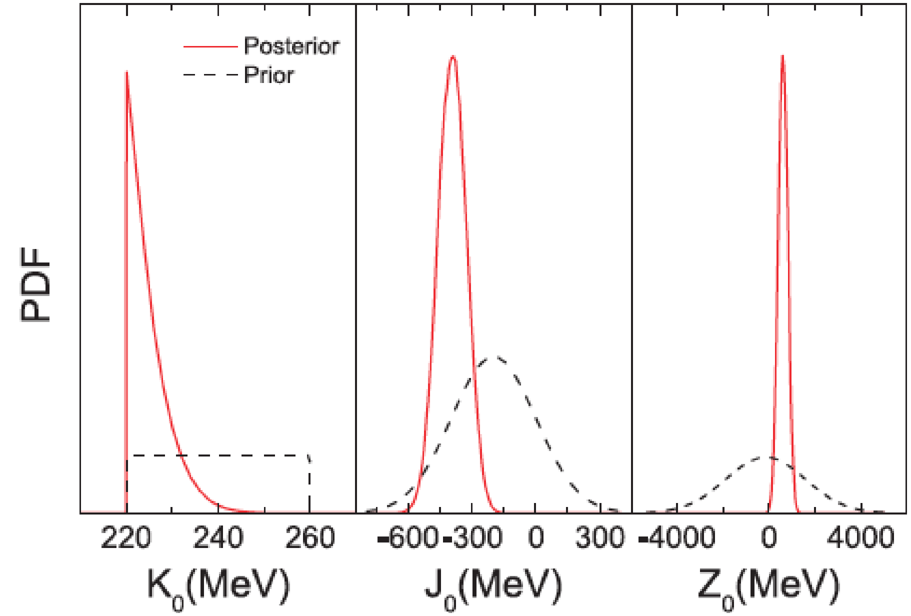
$$P[D|\mathcal{M}(K_0, J_0, Z_0)] = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{D,j}} \exp\left[-\frac{(P_{\mathrm{th},j} - P_{D,j})^2}{2\sigma_{D,j}^2}\right]$$

Likelihod function

Prior PDF (Probability Dist. Func.)



Tension with the results from GMR



Kaon production and elliptical flow data from GSI favors a K_0 smaller than the fiducial value from ISGMR data: 235±30 MeV

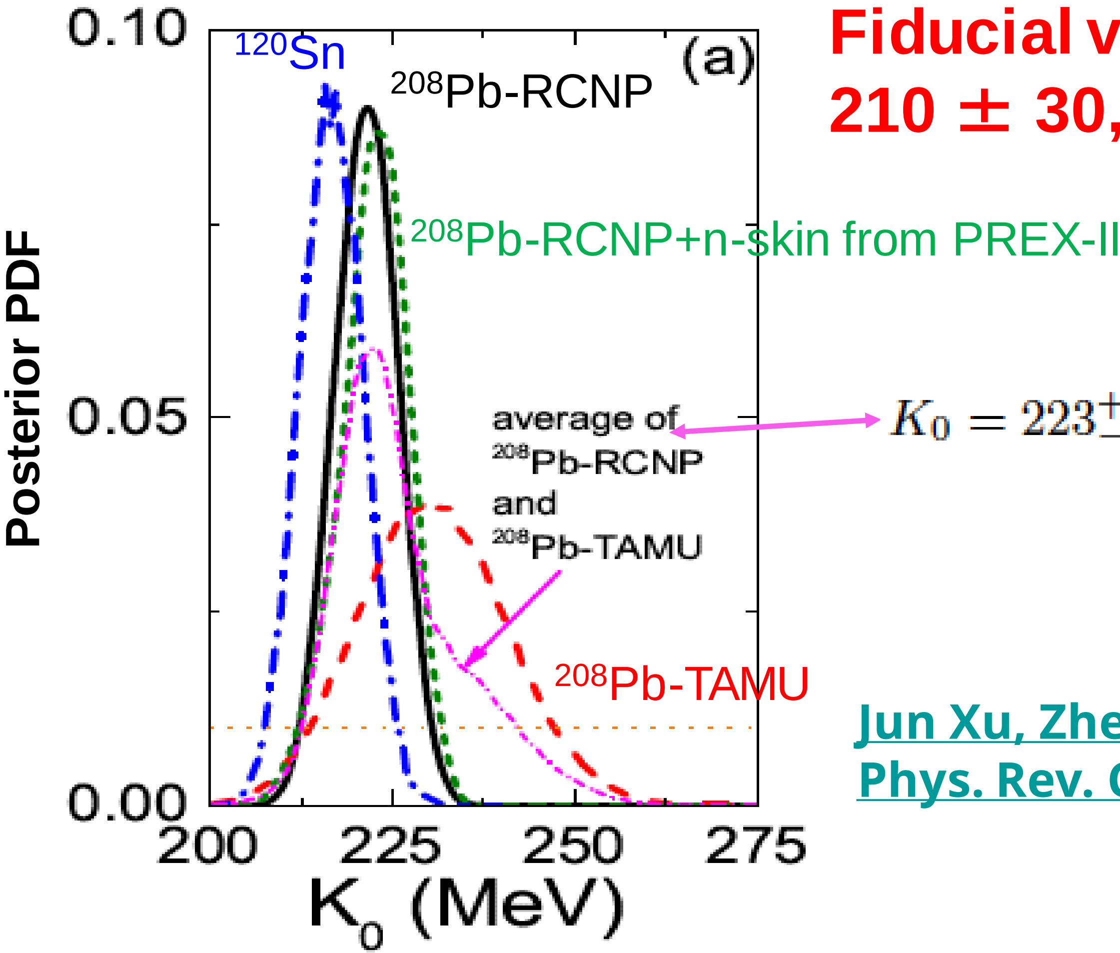
A. Le Fe`vre et al., NPA945, 112 (2016).

Y.J. Wang et al., PLB778, 207 (2018)

Bayesian inference of K_0 from combined data of centroid energy and electrical Polarizability of IVGDR, n-skin, and centroid energy of ISGMR using SHF+RPA

G. Colò, L. Cao, N. Van Gia, and L. Capelli, Comput. Phys.Commun. 184, 142 (2013).

	E_{-1} (MeV)	α_D (fm^3)	Δr_{np} (fm)	E_{ISGMR} (MeV)	E_b (MeV)	R_c (fm)
^{208}Pb-TAMU	13.46 ± 0.10	19.6 ± 0.6	0.170 ± 0.023	14.17 ± 0.28	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{208}Pb-RCNP	13.46 ± 0.10	19.6 ± 0.6	0.170 ± 0.023	13.9 ± 0.1	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{208}Pb-RCNP-PREXII	13.46 ± 0.10	19.6 ± 0.6	0.283 ± 0.071	13.9 ± 0.1	$-7.867452 \pm 3\%$	$5.5010 \pm 3\%$
^{120}Sn	15.38 ± 0.10	8.59 ± 0.37	0.150 ± 0.017	15.7 ± 0.1	$-8.504548 \pm 3\%$	$4.6543 \pm 3\%$

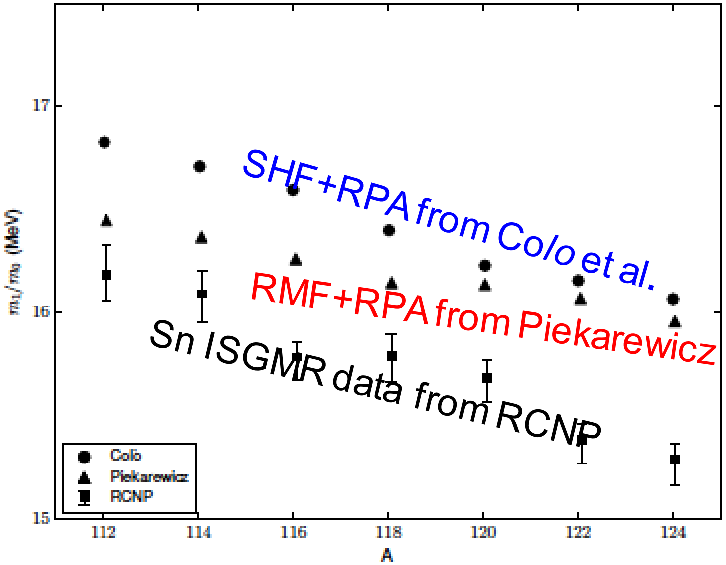


Fiducial value of K_0 since 1980: 210 ± 30, 235 ± 30, 220-260 MeV

$K_0 = 223^{+7}_{-8}$ MeV at 68% confidence level

Jun Xu, Zheng Zhang and Bao-An Li, Phys. Rev. C104, 054324 (2021)

The soft-tin ``puzzle'': both relativistic and non-relativistic models that well describe the ISGMR of ^{208}Pb can NOT reproduce the data of Sn isotopes



Two relatively recent reviews:
U. Garg and G. Colo, PPNP 101, 55 (2018)

X. Roca-Maza and N. Paar, PPNP 101, 96 (2018)

Fiducial value of K_0 since 1980: 210 $\pm$ 30 MeV

J. P. Blaizot, Phys. Rep. 64, 171 (1980).

My possibly biased and offending (I am very sorry!) observations:

The community "consensus": Using microscopic models to first describe well all data of finite nuclei before using them to calculate the incompressibility K_0 (K_∞) of infinite matter

The reality: such models do NOT exist, none of the available models can describe all data. Moreover, not all data for the same nuclei from different labs are consistent

The status quo: the fiducial value of K_0 (K_∞) has been the same for over 40 years

The need: the 30-40 MeV error bar for K_0 (K_∞) is too big for many purposes in NuP & AsP
(BUT it is actually much better than the situation for the symmetry energy parameters)

The challenge: **How to break the status quo?**

Differential analysis of incompressibility in neutron-rich nuclei

Bao-An Li and Wen-Jie Xie, Phys. Rev. C 104, 034610 (2021)

$$K_A \approx K_\infty(1+cA^{-1/3}) + K_\tau \delta^2 + K_{\rm Cou} Z^2 A^{-4/3}$$

J. P. Blaizot, Phys. Rep. **64**, 171 (1980).

Using K_A of any two nuclei (Z_1,A_1) and (Z_2,A_2)

$$K_\tau = \left[\frac{K_{A_1}}{S_1} - \frac{K_{A_2}}{S_2} - K_{\rm Cou}\left(\frac{Z_1^2 A_1^{-4/3}}{S_1} - \frac{Z_2^2 A_2^{-4/3}}{S_2}\right)\right] \Big/ \left(\frac{\delta_1^2}{S_1} - \frac{\delta_2^2}{S_2}\right),$$

$$K_\infty = \left[\frac{K_{A_1}}{\delta_1^2} - \frac{K_{A_2}}{\delta_2^2} - K_{\rm Cou}\left(\frac{Z_1^2 A_1^{-4/3}}{\delta_1^2} - \frac{Z_2^2 A_2^{-4/3}}{\delta_2^2}\right)\right] \Big/ \left(\frac{S_1}{\delta_1^2} - \frac{S_2}{\delta_2^2}\right),$$

$$S_i = 1 + cA_i^{-1/3}$$

$$c \approx -1.2 \pm 0.12 \quad K_{\rm Cou} \approx -5.2 \pm 0.7\ {\rm MeV}$$

S. K. Patra, M. Centelles, X. Viñas, M. Del Estal, Phys. Rev. C **65**, 044304 (2002).
H. Sagawa et al., Phys. Rev. C **76**, 034327 (2007).

Errors:

$$\sigma_{K_\tau} \approx \sqrt{\sigma_{K_{A_1}}^2 + \sigma_{K_{A_2}}^2} \Big/ \left|\delta_1^2 - \delta_2^2\right|,$$

$$\sigma_{K_\infty} \approx \sqrt{(\delta_2^2 \cdot \sigma_{K_{A_1}})^2 + (\delta_1^2 \cdot \sigma_{K_{A_2}})^2} \Big/ \left|\delta_1^2 - \delta_2^2\right|$$

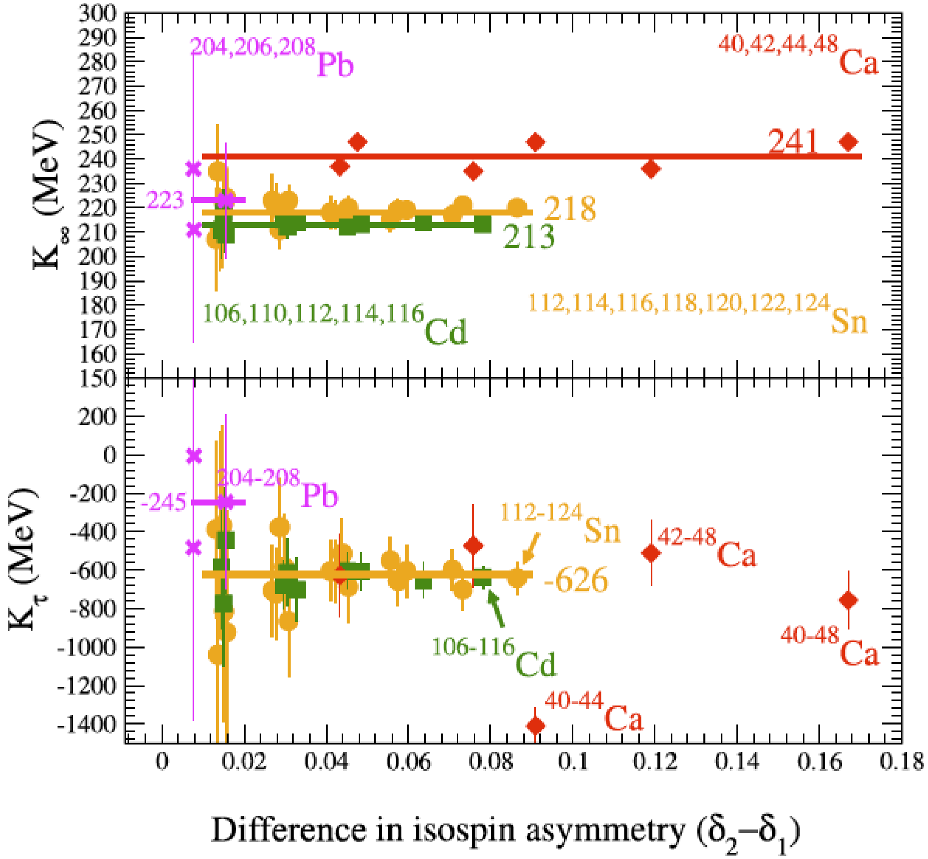
$$K_A = \left(\frac{E_{\rm ISGMR}}{\hbar c}\right)^2 Mc^2 < r^2 >$$

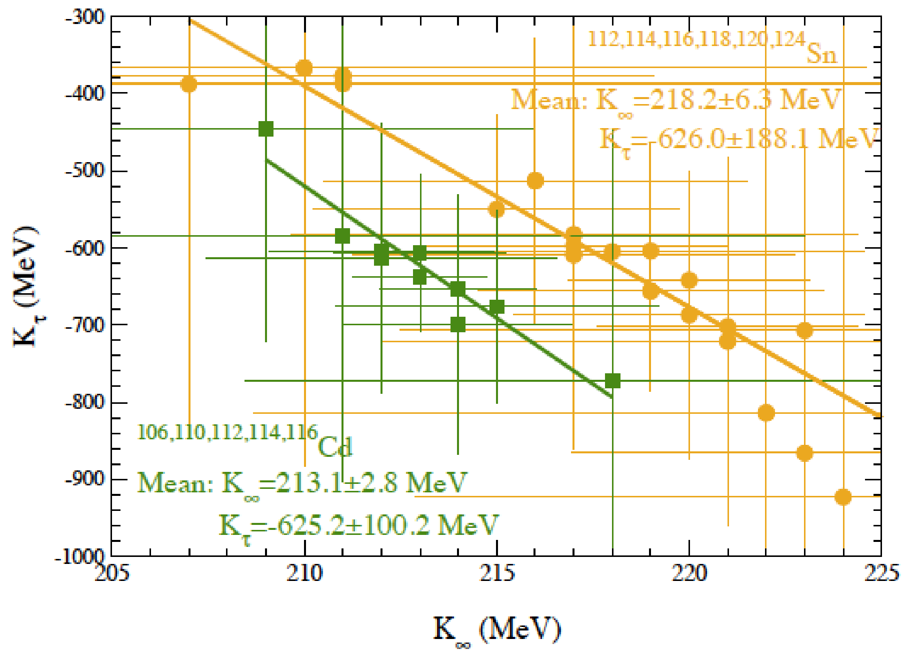
U. Garg and G. Colo, PPNP 101, 55 (2018)

Nucleus	K_A (MeV)	Reference
^{40}Ca	144.46 ± 0.33	[17]
^{42}Ca	139.00 ± 1.09	
^{44}Ca	137.36 ± 0.66	
^{48}Ca	131.90 ± 4.13	
^{106}Cd	127.84 ± 0.86	[14]
^{110}Cd	124.59 ± 0.86	
^{112}Cd	123.59 ± 0.77	
^{114}Cd	120.95 ± 1.24	
^{116}Cd	118.96 ± 0.86	
^{112}Sn	131.86 ± 1.53	[12, 13]
^{114}Sn	129.45 ± 1.64	
^{116}Sn	127.11 ± 1.53	
^{118}Sn	126.39 ± 1.54	
^{120}Sn	125.45 ± 1.63	
^{122}Sn	121.33 ± 1.54	
^{124}Sn	120.17 ± 1.62	
^{204}Pb	136.93 ± 1.99	[15]
^{206}Pb	137.44 ± 1.99	
^{208}Pb	136.44 ± 1.99	

All from RCNP Garg et al

Bao-An Li and Wen-Jie Xie,
Phys. Rev. C 104, 034610 (2021)



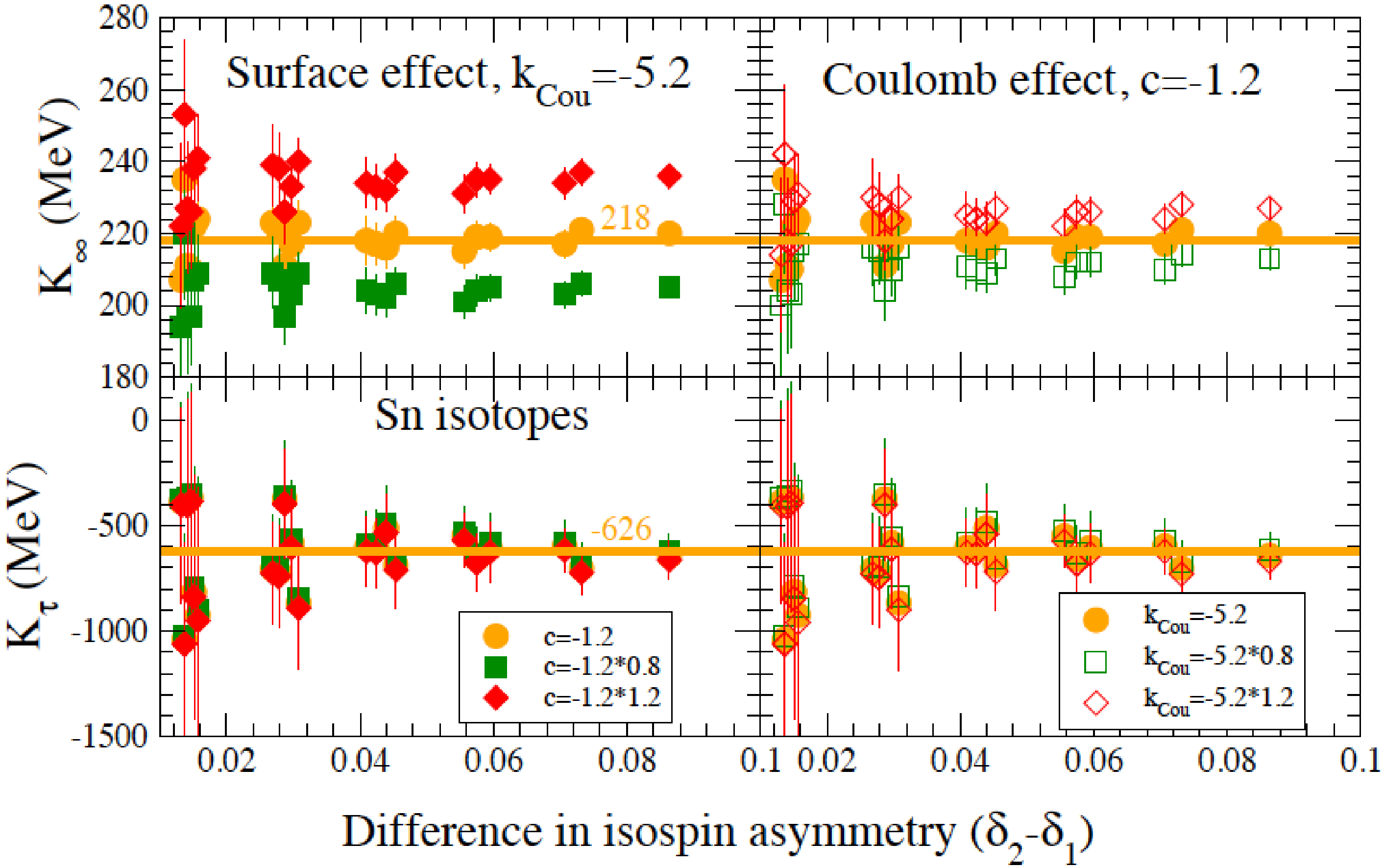


(1) Pb has only 3 points and all with large errors, needs more data for Pb isotopes
(2) Pb appears harder because its K_τ is much higher while its K_∞ is consistent with Sn
(3) Pairs with the largest difference in isospin asymmetry is the best (smallest error)
(4) Ca isotopes are probably too light for the liquid drop formula to work properly
(5) Sn looks normal than Pb compared to other nuclei

Soft Sn or Hard Pb puzzle?

J. Piekarewicz and M. Centelles, PRC 79, 054311 (2009)
Elias Khan, PRC 80, 011307 and 057302 (2009)

Surface and Coulomb effects



(1) Varying the surface (c) and/or Coulomb (k_{cou}) by 40% has little effect on K_τ

(2) Varying the surface (c) parameter by 40% leads to 210 < K_∞ < 240 MeV

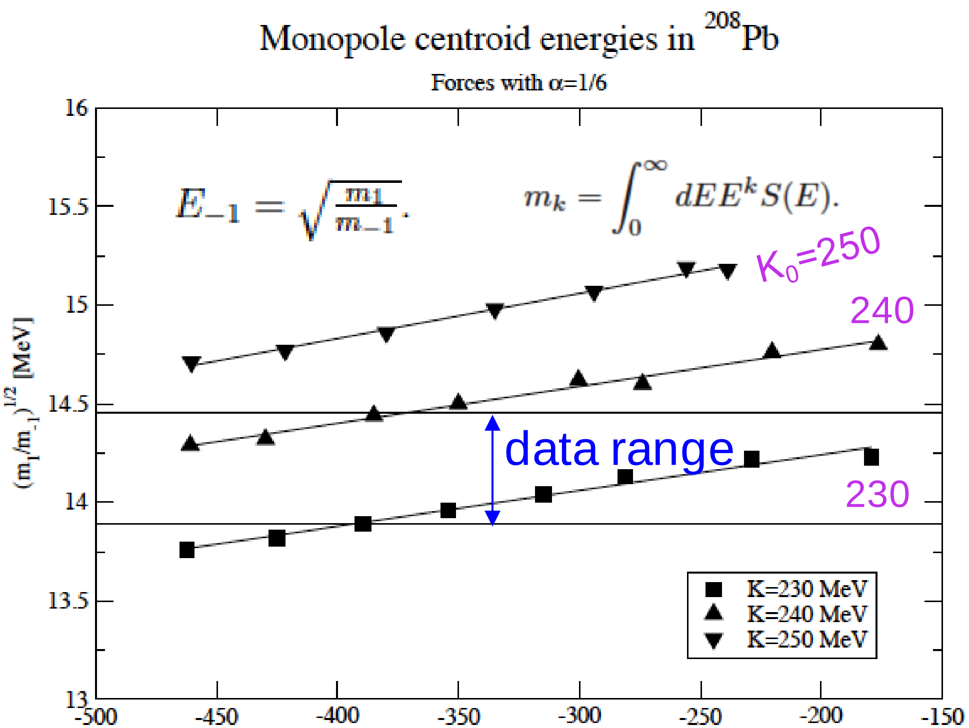
Bao-An Li and Wen-Jie Xie, Phys. Rev. C 104, 034610 (2021)

The main source of the remaining uncertainty of K_0 (K_∞) from giant resonances:

Its correlation with the poorly known isospin dependent term K_τ in the incompressibility K_A of finite nuclei

$$K_A \approx K_\infty(1 + cA^{-1/3}) + K_\tau \delta^2 + K_{\rm Cou} Z^2 A^{-4/3}$$

J. Colo, N. Van Giai, J. Meyer, K. Bennaceur, and P. Bonche, PRC 70, 024307 (2004).



Along the saturation line of infinite isospin-asymmetric matter

$$\rho_{\rm sat}(\delta) = \left[1 - \frac{3L}{K_0}\delta^2 + \left(\frac{3K_{\rm sym}L}{K_0^2} - \frac{3L_{\rm sym,4}}{K_0} - \frac{3J_0L^2}{2K_0^3}\right)\delta^4 + O(\delta^6)\right]\rho_0. \quad (22)$$

$$K_{\rm sat}(\delta) \equiv 9\rho_{\rm sat}^2 \frac{\partial^2 E(\rho,\delta)}{\partial \rho^2}\bigg|_{\rho=\rho_{\rm sat}} \approx K_0 + K_{\rm sat,2}\delta^2 + O(\delta^4)$$

$$K_{\rm sat,2} = K_{\rm sym} - 6L - J_0L/K_0.$$

L.-W. Chen, B.-J. Cai, C. M. Ko, B.-A. Li, C. Shen, and J. Xu, Phys. Rev. C **80**, 014322 (2009).

the K_τ term $K_{\rm sym} - 6L - J_0L/K_0.$

Along the saturation line of finite nuclei:

X. Roca-Maza and N. Paar, PPNP 101, 96 (2018)

$$\rho_{\rm eq} \approx \rho_0 + \frac{3\rho_0}{K_0}\left(1 - LI^2 + 2a_S A^{-1/3} - a_C \frac{Z^2}{A^{4/3}}\right)$$

I=δ

Fundamental physics underlying nuclear symmetry energy

Single-nucleon (Lane) potential in isospin-asymmetric matter: A. M. Lane, Nucl. Phys. 35, 676 (1962).

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\cdot\delta + U_{sym2}(k,\rho)\cdot\delta^2 + o(\delta^3)$$

Hugenholtz-Van Hove (HVH) theorem:

$$E_F = \frac{d\xi}{d\rho} = \frac{d(\rho E)}{d\rho} = E + \rho\frac{dE}{d\rho} = E + P/\rho$$

N.M. Hugenholtz, L. Van Hove, Physica 24 (1958) 363.

$$E_{sym}(\rho) = \frac{1}{3}\frac{k_F^2}{2M} + \frac{1}{2}U_{sym,1}(\rho,k_F) + \frac{k_F}{6}\left(\frac{\partial U_0}{\partial k}\right)_{k_F} - \frac{1}{6}\frac{k_F^4}{2M^3}$$

S. Fritsch, N. Kaiser, W. Weise, Nuclear Phys. A 750 (2005) 259.

Using the K-matrix theory:

K.A. Brueckner, J. Dabrowski, Phys. Rev. B 134 (1964) 722.
J. Dabrowski, P. Haensel, Phys. Lett. B 42 (1972) 163;

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho,k_F),$$

$$m_0^*(\rho,k) = \frac{m}{1+\frac{m}{\hbar^2 k}\frac{\partial U_0(\rho,k)}{\partial k}},$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F),$$

C. Xu, B.A. Li, L.W. Chen and C.M. Ko, NPA 865, 1 (2011); R. Chen et al., PRC 85, 024305 (2012).

Neutron-proton effective mass splitting: $m^*_{n-p} \approx 2\delta\frac{m}{\hbar^2 k_F}\left[-\frac{dU_{sym,1}}{dk} - \frac{k_F}{3}\frac{d^2U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2$

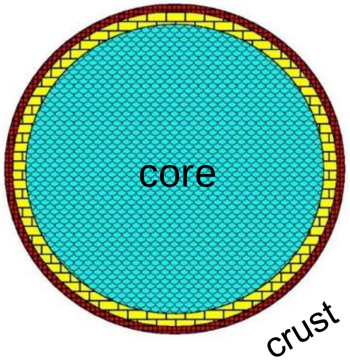
Bao-An Li, Bao-Jun Cai, Lie-Wen Chen and Jun Xu,
Progress in Particle and Nuclear Physics 99 (2018) 29–119

How does the symmetry energy affect neutron star properties?

(1) The proton fraction x is determined by the $E_{sym}(\rho)$ through charge neutrality and beta-equilibrium conditions:

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$

Critical for the cooling mechanism of protoneutron stars and associated neutrino emissions, appearance of hyperons, kaon condensation, baryon resonances…..



(2) The pressure in the npe matter at beta equilibrium:

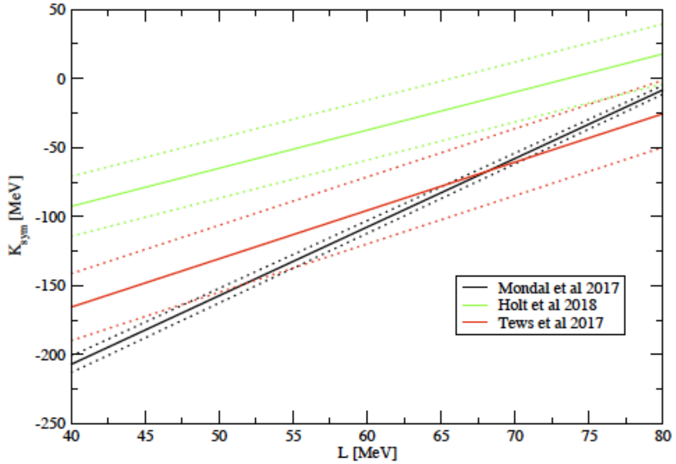
$$P(\rho,\delta) = \rho^2[\frac{dE_0(\rho)}{d\rho} + \frac{dE_{\rm sym}(\rho)}{d\rho}\delta^2] + \frac{1}{2}\delta(1-\delta)\rho E_{\rm sym}(\rho)$$

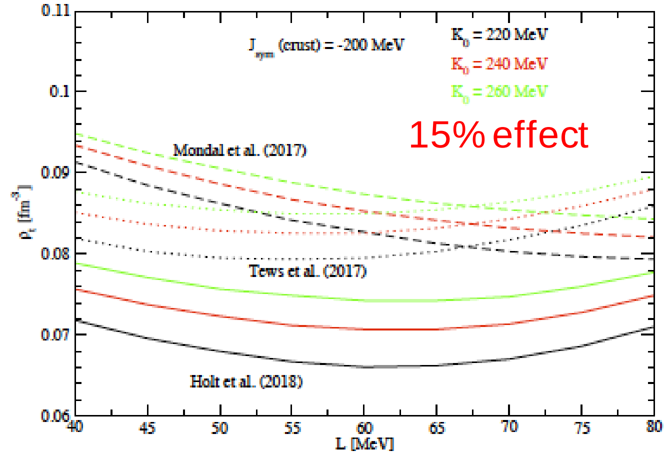
(3) The crust-core transition density and pressure is determined by setting the incompressibility of neutron star matter =0 (speed of sound becomes imaginary):

$$K_\mu = \rho^2\frac{d^2E_0}{d\rho^2} + 2\rho\frac{dE_0}{d\rho} + \delta^2\left[\rho^2\frac{d^2E_{sym}}{d\rho^2} + 2\rho\frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1}(\rho\frac{dE_{sym}}{d\rho})^2\right] = 0$$

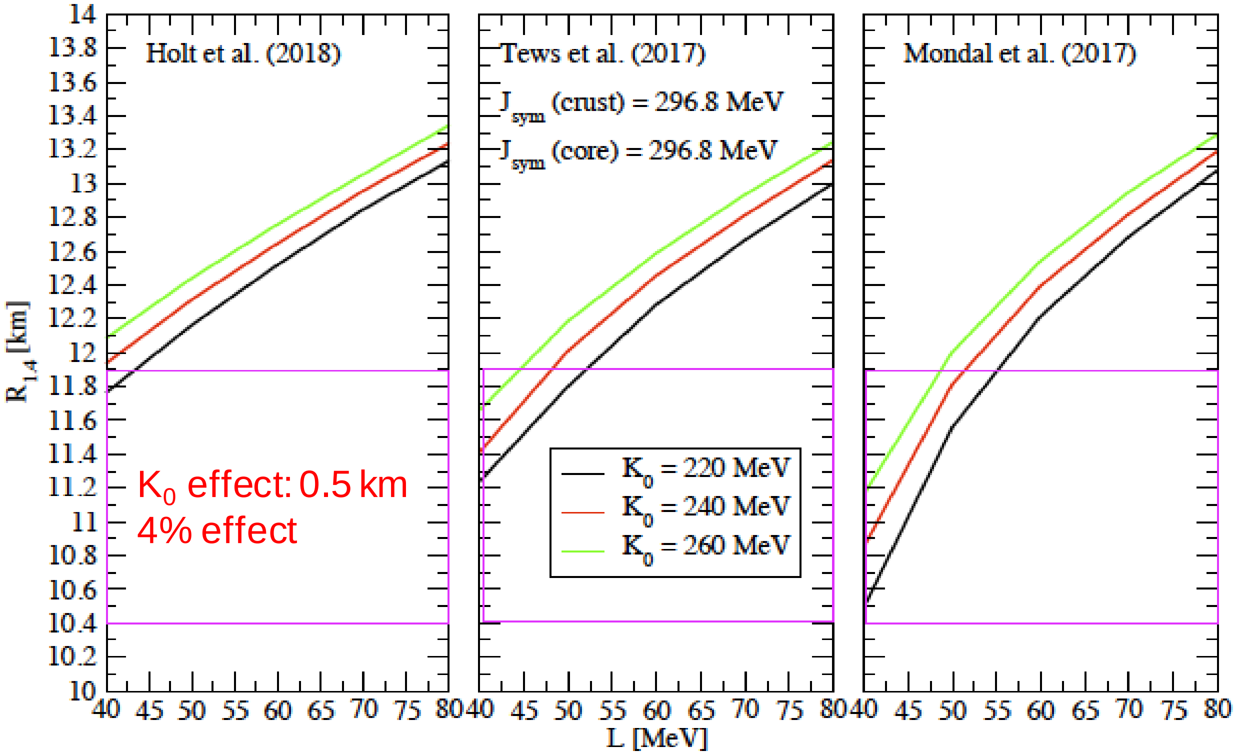
J. Lattimer & M. Prakash, Phys. Rep., 442, 109 (2007)

Example: effects of K_0 and L on the crust-core transition density and $R_{1.4}$





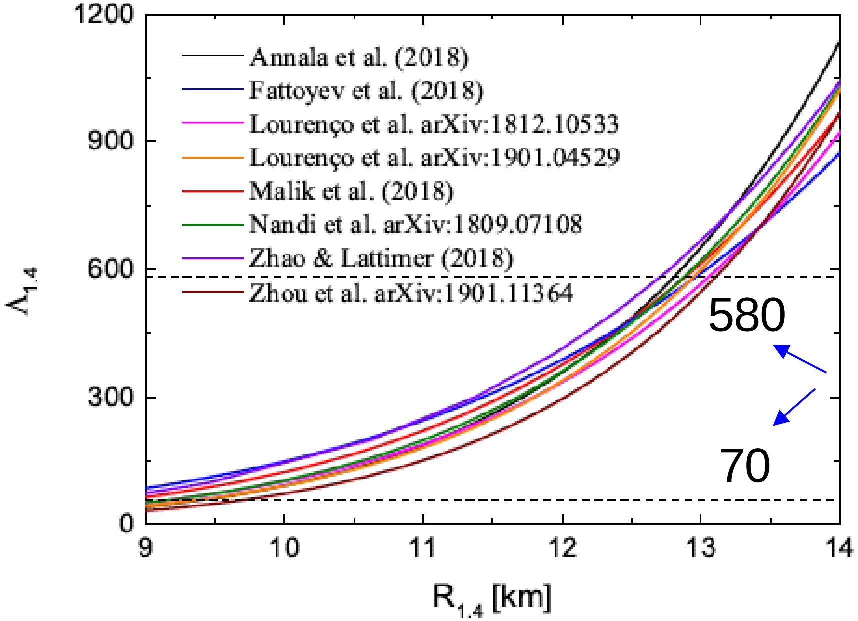
Crust-core transition density



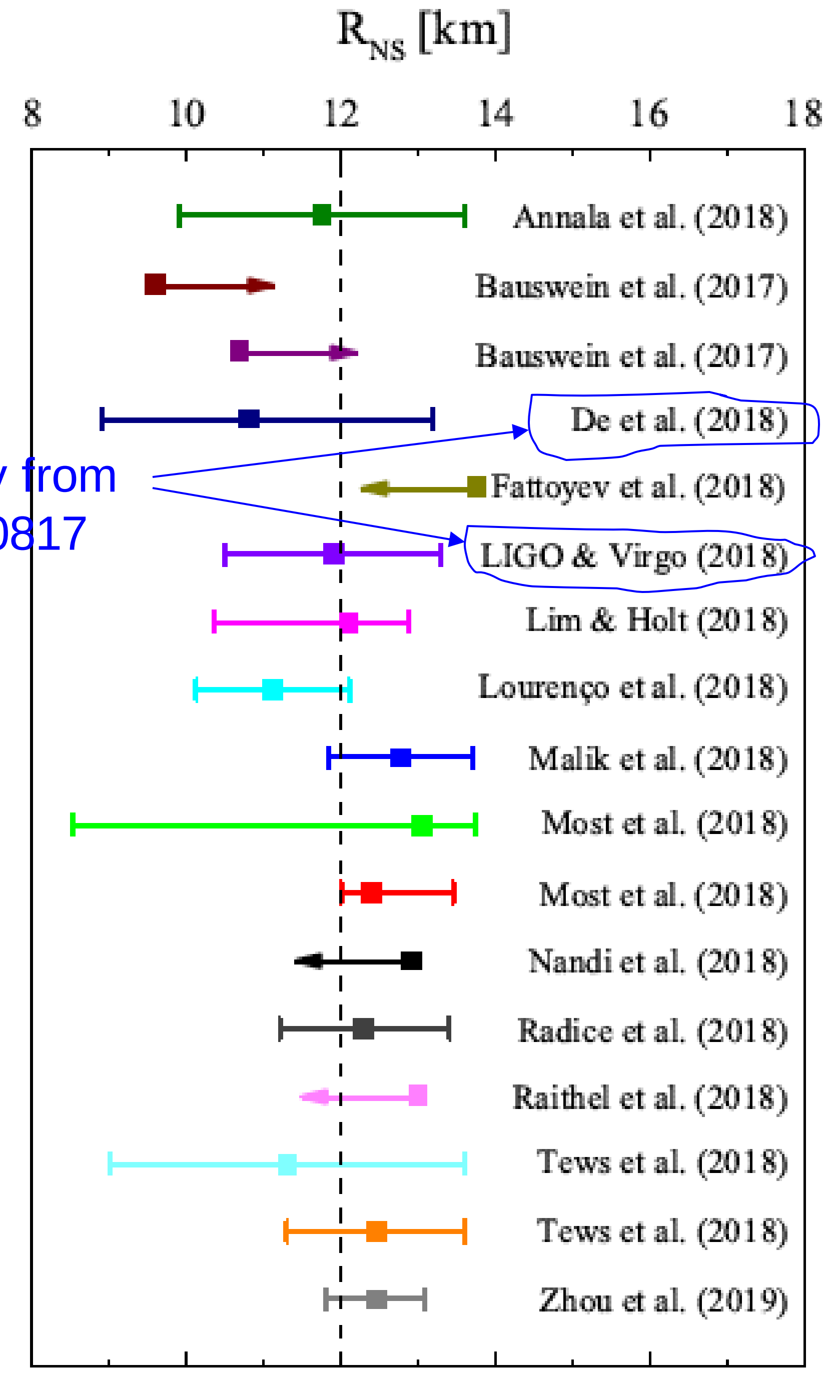
Radius of 1.4 solar mass neutron star

Bao-An Li & Macon Magno
PRC 102, 045807 (2020)

Tidal deformability and radius from GW170817



Directly from GW170817



$$\Lambda = \frac{2}{3}\frac{k_2}{\beta^5}$$

B.A. Li, P.G. Krastev, D.H. Wen and N.B. Zhang
Review Article, EPJA 55, 117 (2019)

Table 1. The radius $R_{1.4}$ data used in this work.

Wen-Jie Xie and Bao-An Li
APJ 883, 174 (2019)
APJ 899, 4 (2020)

Radius $R_{1.4}$ (km) (90% confidence level)	Source	Reference
$11.9^{+1.4}_{-1.4}$	GW170817	(Abbott et al. 2018)
$10.8^{+2.1}_{-1.6}$	GW170817	(De et al. 2018)
$11.7^{+1.1}_{-1.1}$	QLMXBs	(Lattimer & Steiner 2014)
$11.9 \pm 0.8, 10.8 \pm 0.8, 11.7 \pm 0.8$	Imagined case-1	this work
11.9 ± 0.8	Imagined case-2	this work

Posterior probability distribution $$P(\mathcal{M}|D) = \frac{P(D|\mathcal{M})P(\mathcal{M})}{\int P(D|\mathcal{M})P(\mathcal{M})d\mathcal{M}},$$ **(Bayes' theorem)**

Likelihood: $$P[D(R_{1,2,3})|\mathcal{M}(p_{1,2,\cdots 6})] = \prod_{j=1}^{3} \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{obs},j}} \exp\left[-\frac{(R_{\mathrm{th},j} - R_{\mathrm{obs},j})^2}{2\sigma^2_{\mathrm{obs},j}}\right],$$

Meta-modeling of nuclear EOS

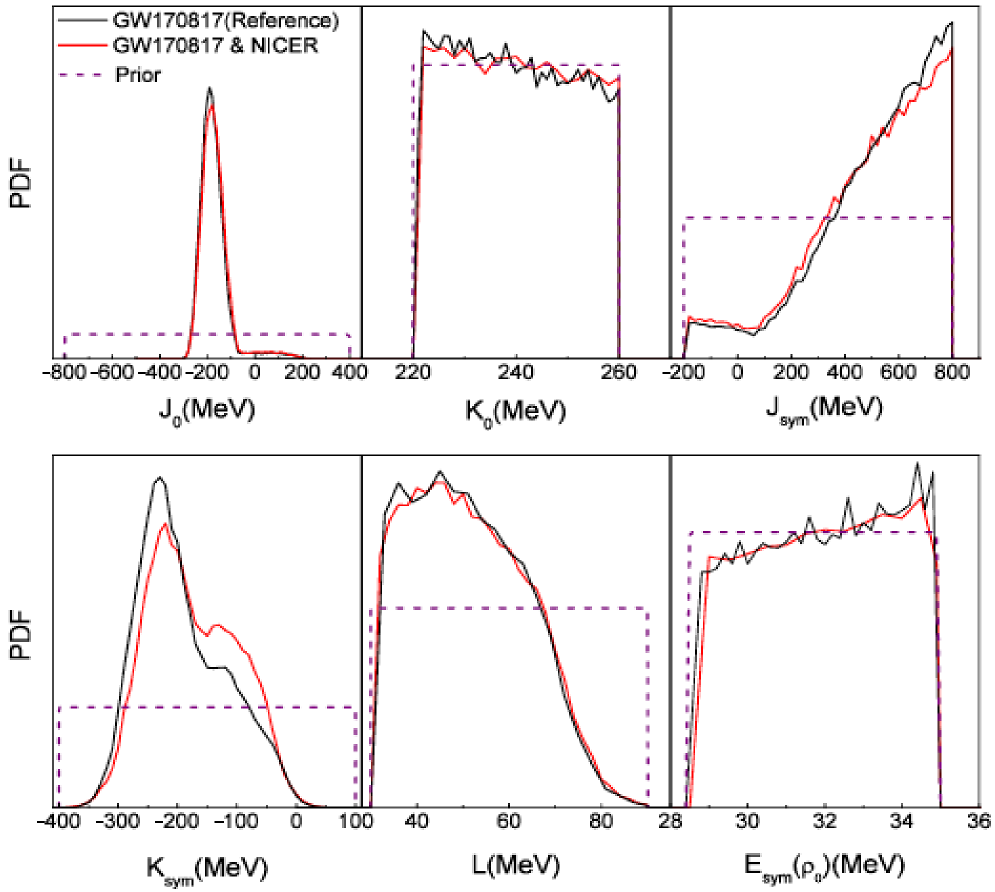
Uniform prior distribution P(M) in the ranges of

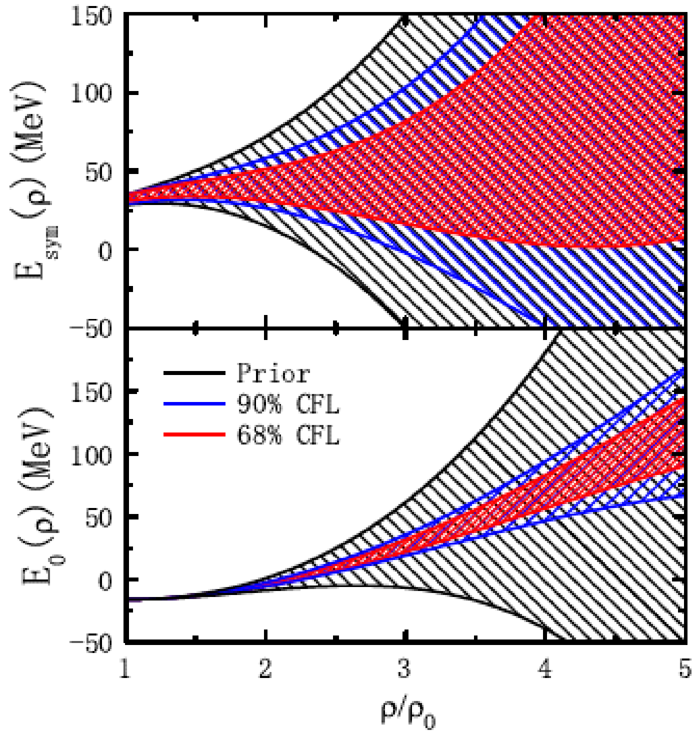
Table 2. Prior ranges of the six EOS parameters used

Parameters	Lower limit	Upper limit (MeV)
K_0	220	260
J_0	-800	400
K_{sym}	-400	100
J_{sym}	-200	800
L	30	90
$E_{\mathrm{sym}}(\rho_0)$	28.5	34.9

Bayesian inference of high-density E_{sym} from the radii $R_{1.4}$ of canonical neutron stars in 6D EOS parameter space

Posterior probability distribution function (PDF) of 6 EOS parameters from Bayesian analyses of GW170817 & NICER data for the canonical PSR J0030+0451



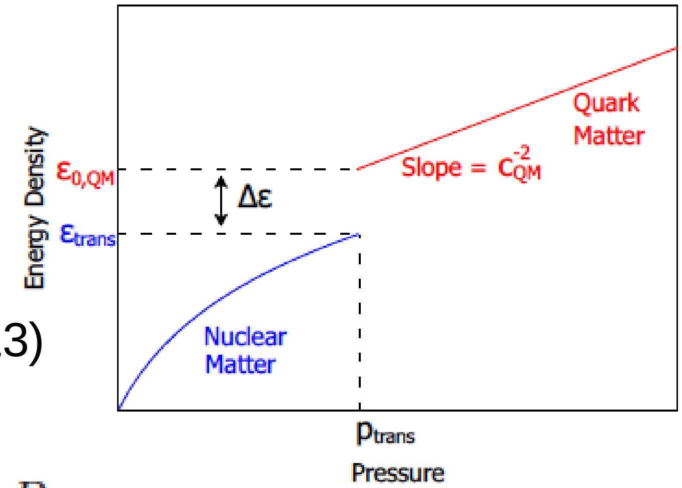


Wen-Jie Xie and Bao-An Li
APJ 883, 174 (2019)
APJ 899, 4 (2020)

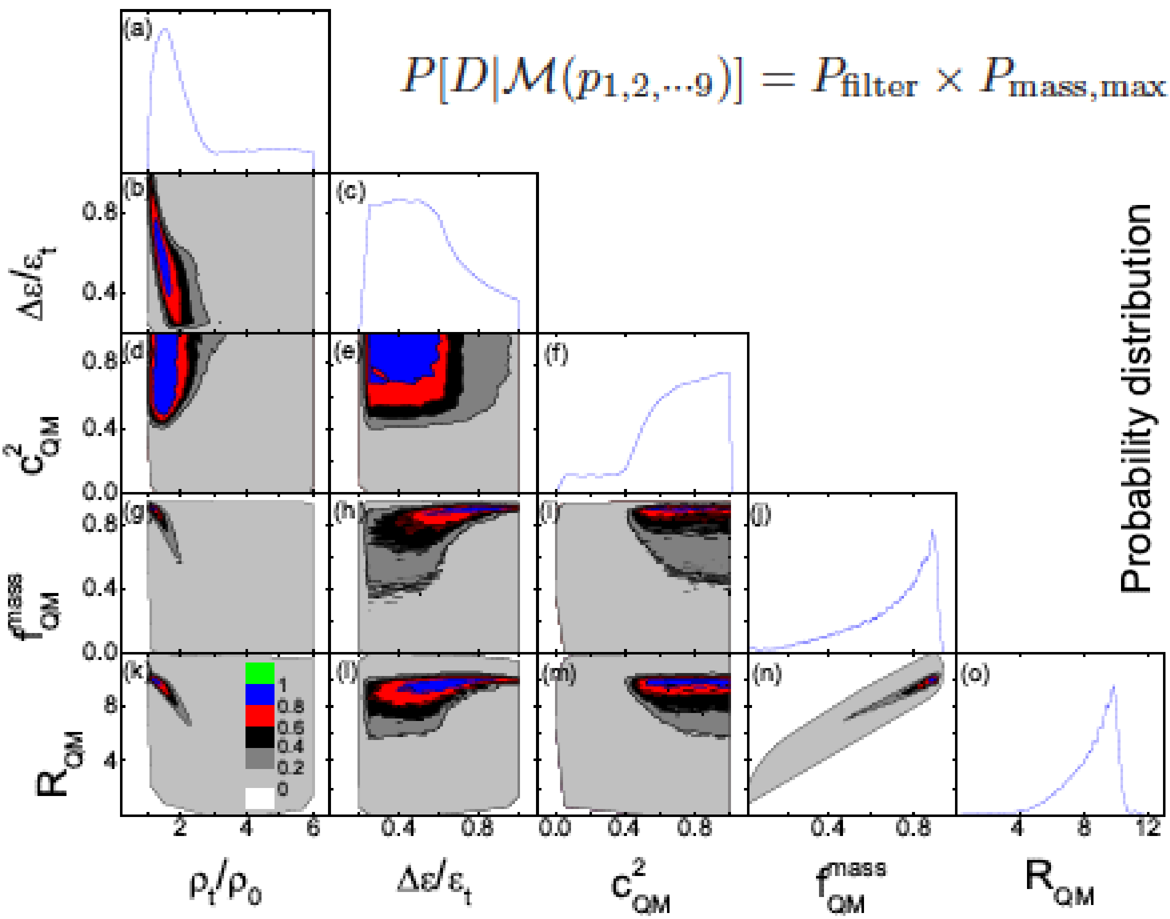
Effects of hadron-quark phase transition

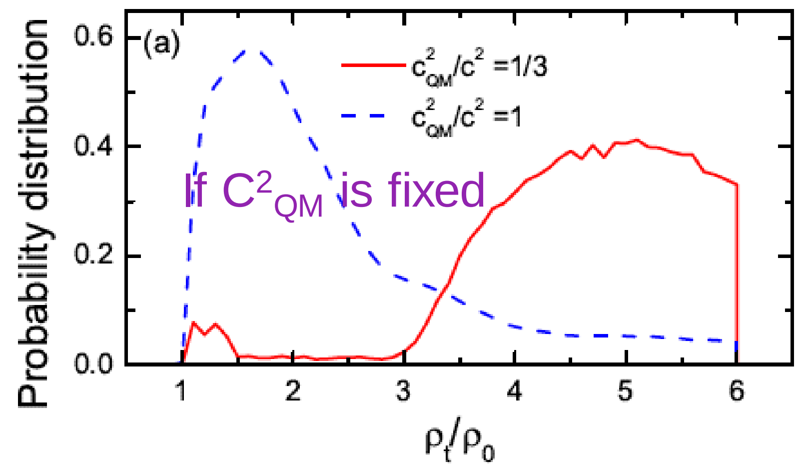
$$\varepsilon(p) = \begin{cases} \varepsilon_{\rm HM}(p) & p < p_t \\ \varepsilon_{\rm HM}(p_t) + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_t) & p > p_t \end{cases}$$

M. Alford, S. Han and M. Prakash, PRD 88, 083013 (2013)



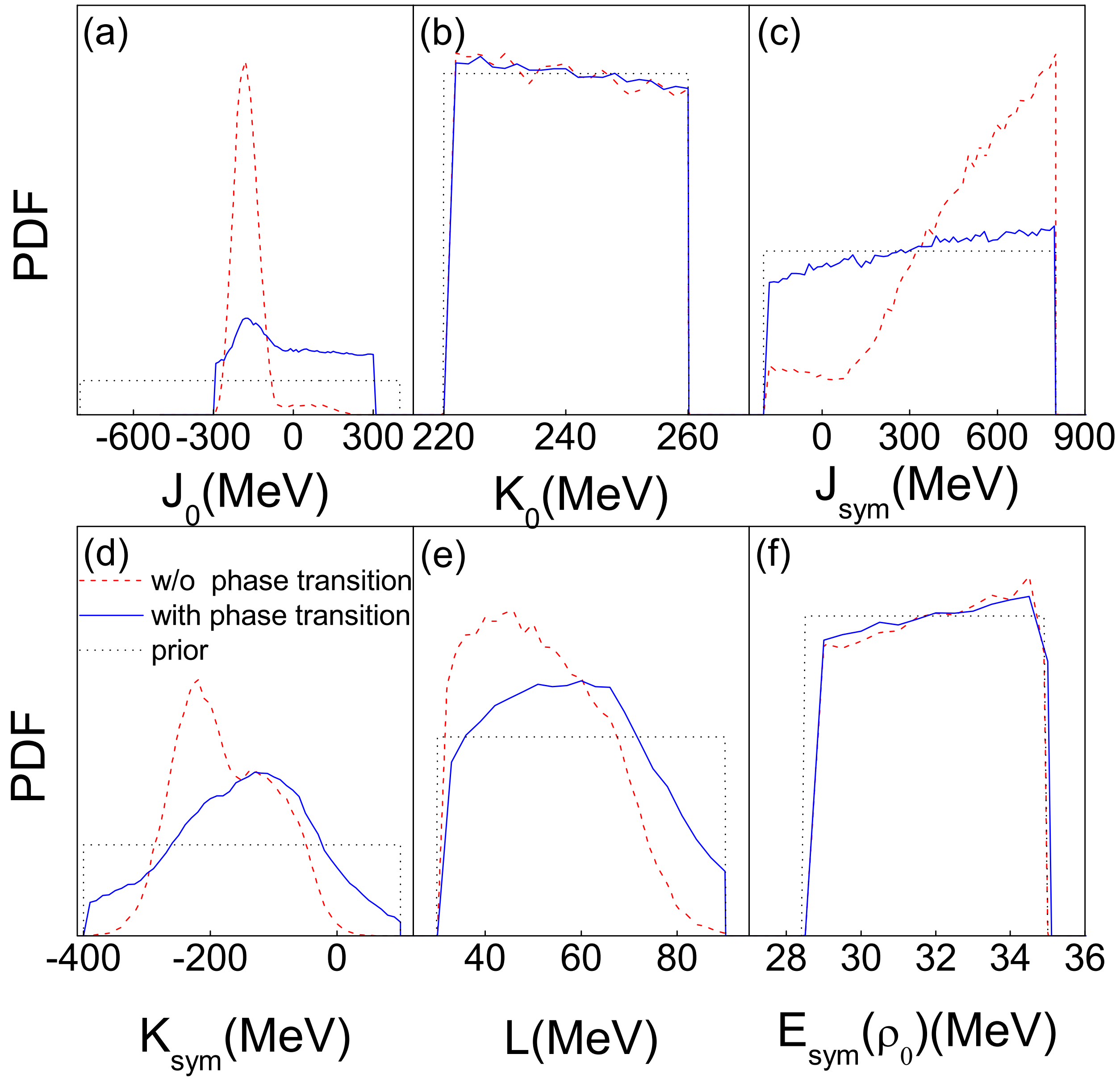
$$P[D|\mathcal{M}(p_{1,2,\cdots 9})] = P_{\rm filter} \times P_{\rm mass,max} \times P_{\rm radius}$$



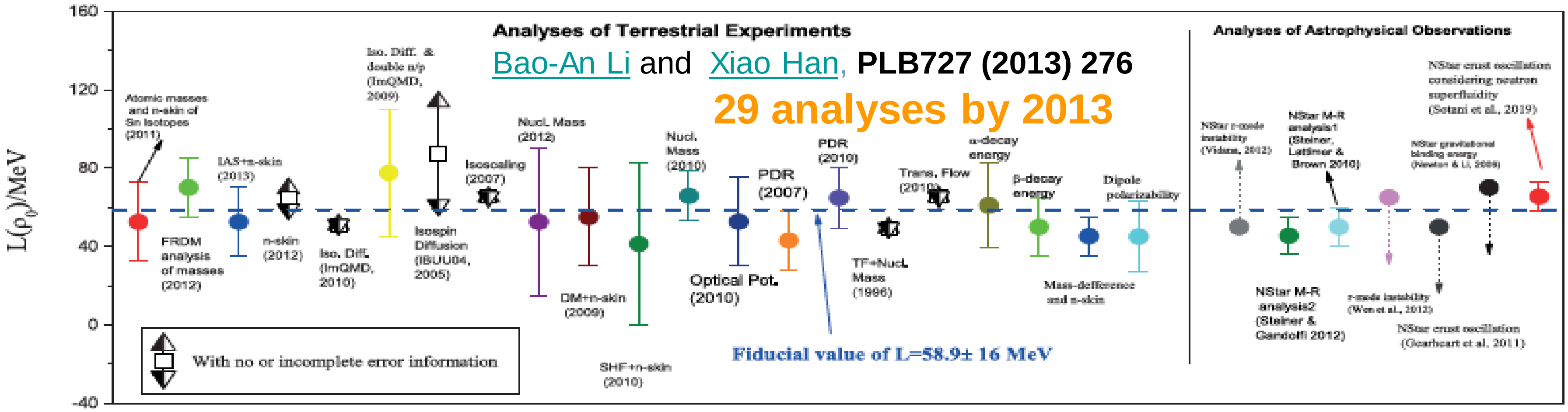


Wen-Jie Xie and Bao-An Li
PRC 103, 035802 (2021)

Nuclear EOS parameters inferred with and without the first-order PT are all in their currently known uncertainty ranges (i.e. indistinguishable)

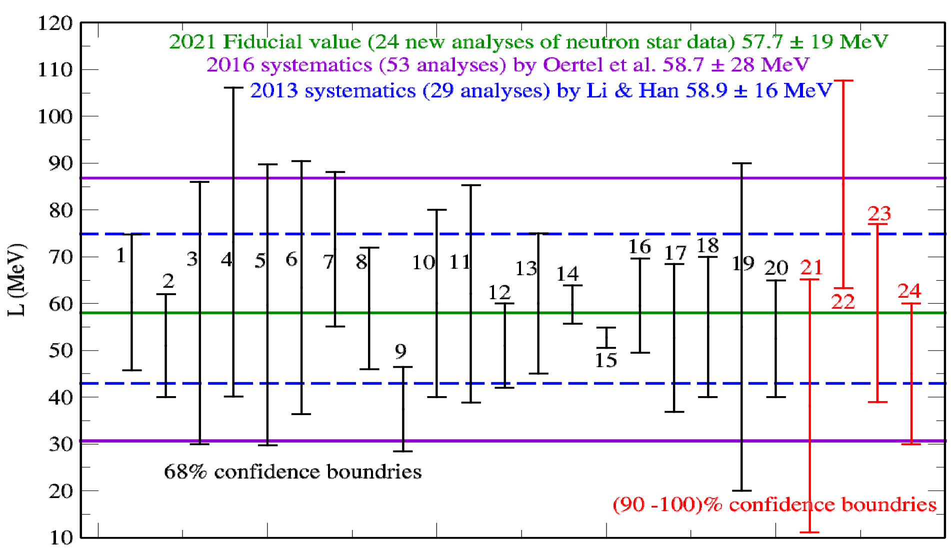


Is there a democratic principle in physics?



PREX2

Totally over 80 analyses so far



Bao-An Li et al. **Universe 7, 182 (2021)**

B.T. Reed, F. J. Fattoyev,
C. J. Horowitz, and J. Piekarewicz,
PRL 126, 17250 (2021)

L = 106 ± 37 MeV

Re-analysis of PREX2 & E-polarizability data
P.G. Reinhard, Xavier Roca-Maza,
and Witold Nazarewicz, PRL127, 232501 (2021).

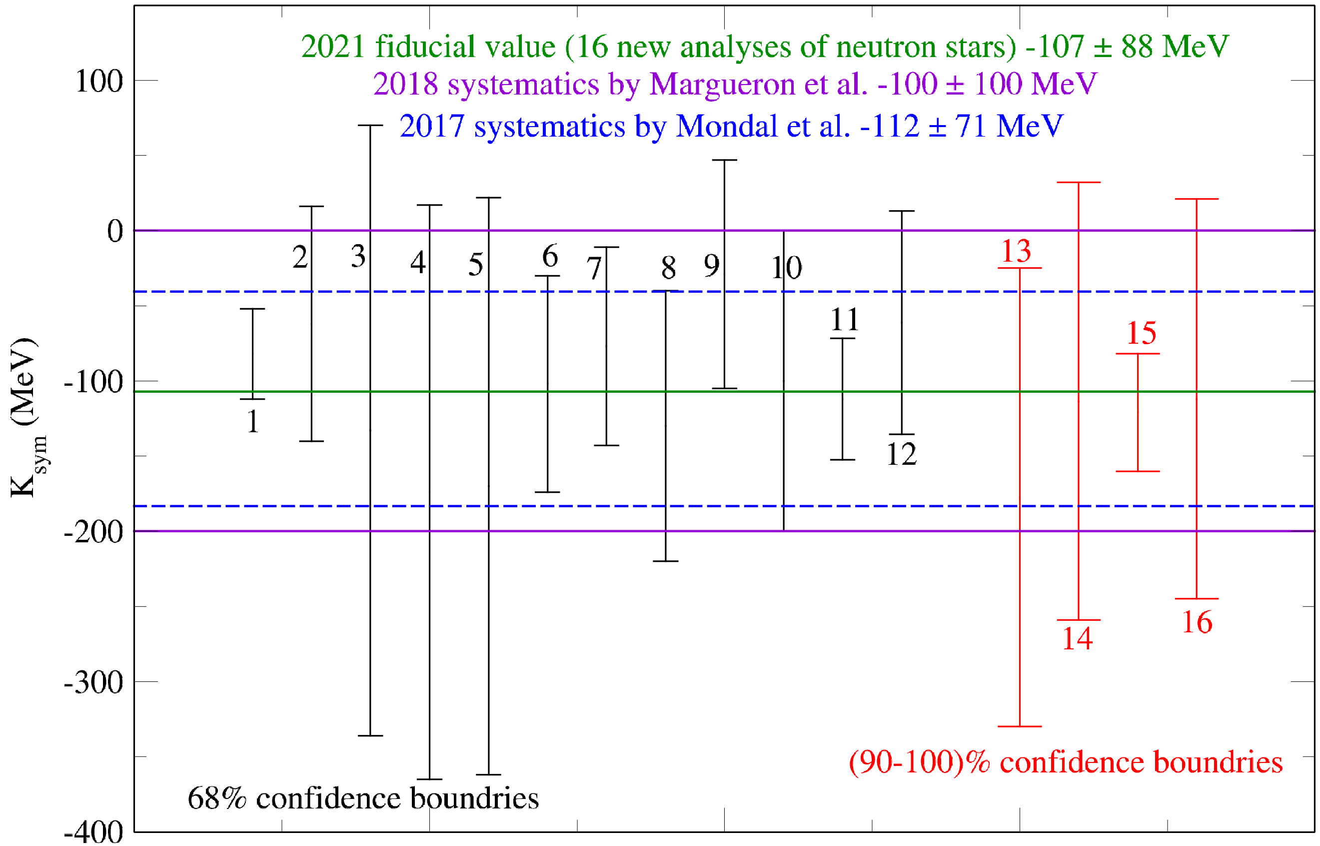
L = 54 ± 8 MeV,

D. Adhikari, **arXiv:2205.11593**

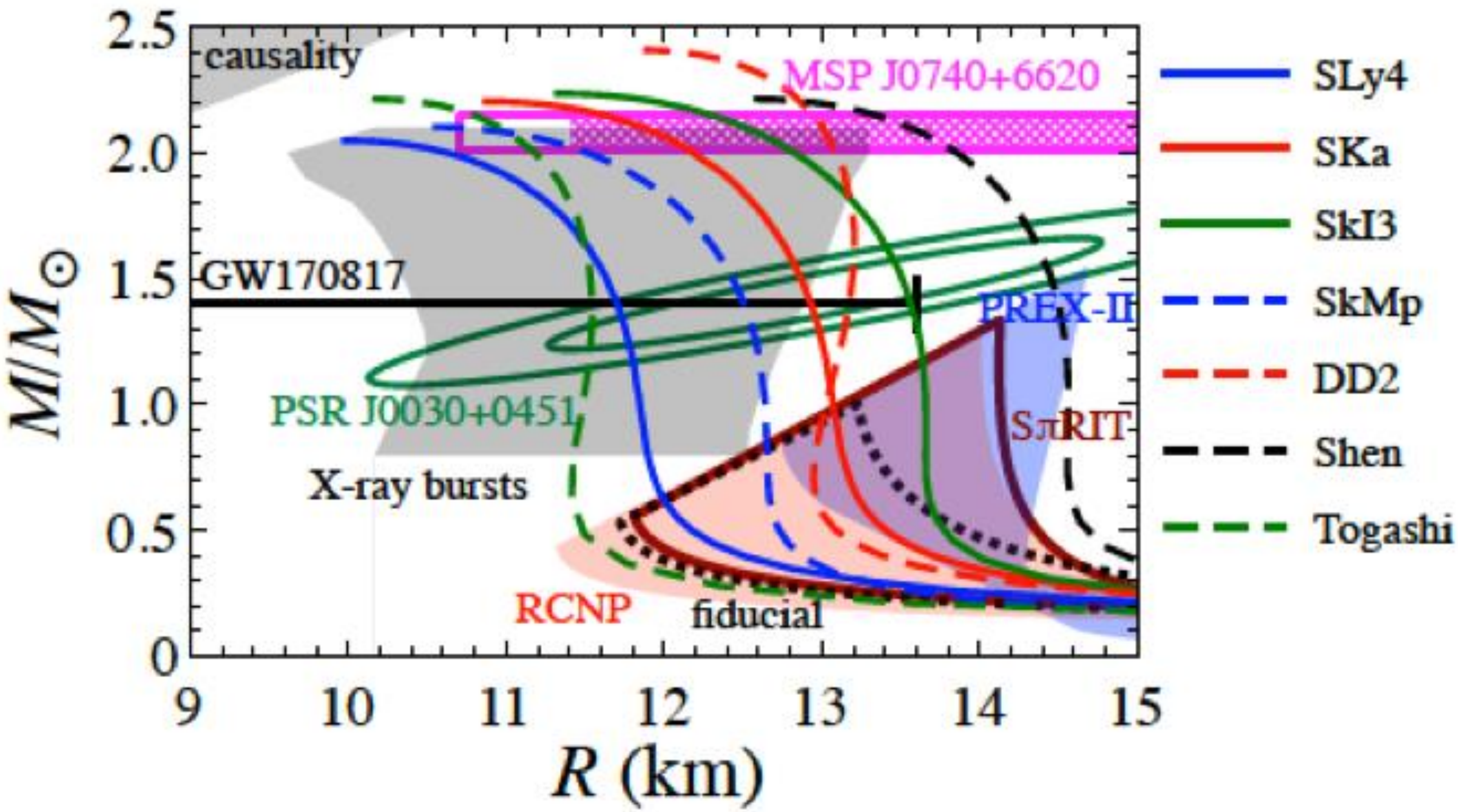
CREX L= -10 + 29.2-26.7 MeV
Z. Zhang and L.W. Chen,
ArXiv: 2207.03328

Curvature of the symmetry energy at saturation density

Bao-An Li, Bao-Jun Cai, Wen-Jie Xie, Nai-Bo Zhang, **Universe 7, 182 (2021)**



Currently available mass-radius observations of neutron stars



Hajime Sotani, Nobuya Nishimura, Tomoya Naito,
Progress of Theoretical and Experimental Physics, Volume 2022, Issue 4, 2022, 041D01

Solving the NS inverse-structure problems by calling the TOV solver within 3 Do-Loops: Given an observable-→ Find ALL necessary & sufficient EOSs

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{\boxed{J_0}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \quad (2.15)$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{\boxed{K_{sym}}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{\boxed{J_{sym}}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 \quad (2.16)$$

$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\cdot\delta^2$$

Fix the saturation parameters $E_0(\rho_0)$, $E_{sym}(\rho_0)$ and L at their most probable values currently known

Example:

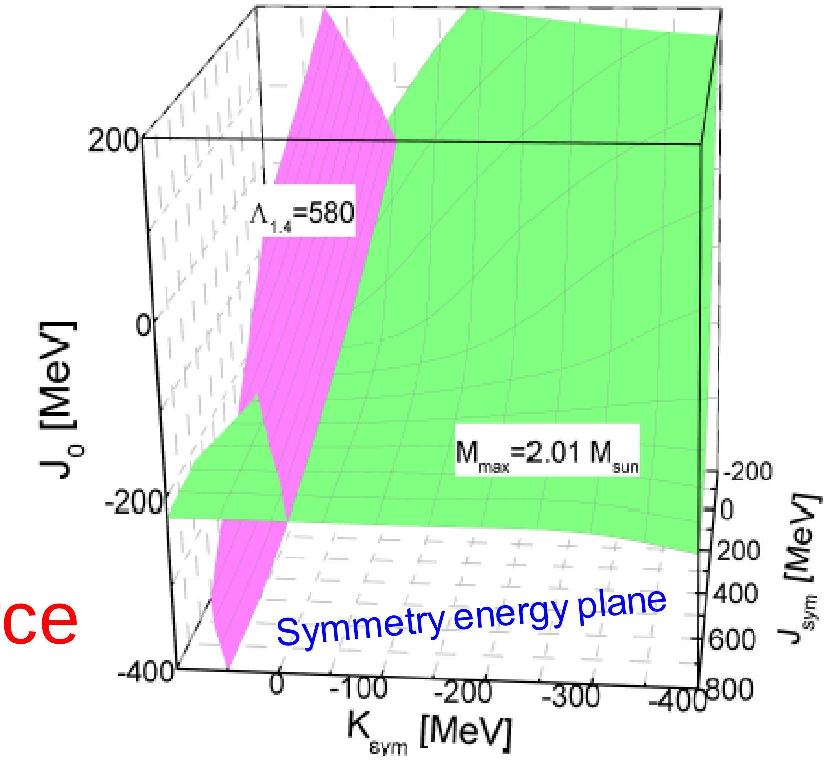
J_0=-189 is found given M_{max}=2.01M_{sun}

J_0 loop

TOV

at K_{sym}=-200 & J_{sym}=400 inside the K_{sym} and J_{sym} loops

Direct Inversion by Brute Force

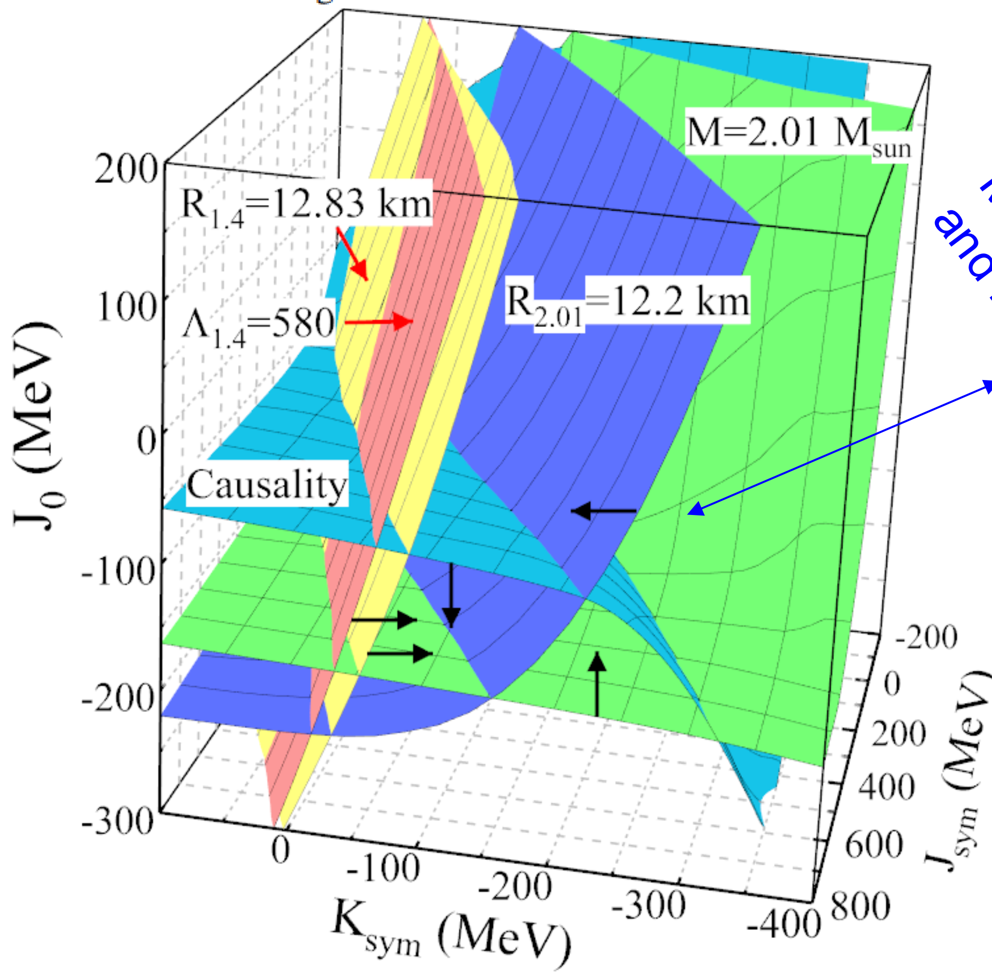


N.B. Zhang, B.A. Li and J. Xu, APJ 859, 90 (2018)

THE ASTROPHYSICAL JOURNAL, 921:111 (11pp), 2021 November 10 https://doi.org/10.3847/1538-4357/ac1e8c

Impact of NICER's Radius Measurement of PSR J0740+6620 on Nuclear Symmetry Energy at Suprasaturation Densities

Nai-Bo Zhang[1] and Bao-An Li[2]



Power of simultaneous Measurements of mass and radius

NICER results :

Mass: $2.08 \pm 0.07\, M_\odot$

Radius: $13.7^{+2.6}_{-1.5}$ km (68%) (Miller et al. 2021) or $12.39^{+1.30}_{-0.98}$ km (Riley

Setting lower boundaries in the high-density E_{sym} plane using the radius and mass of J0740 from NICER

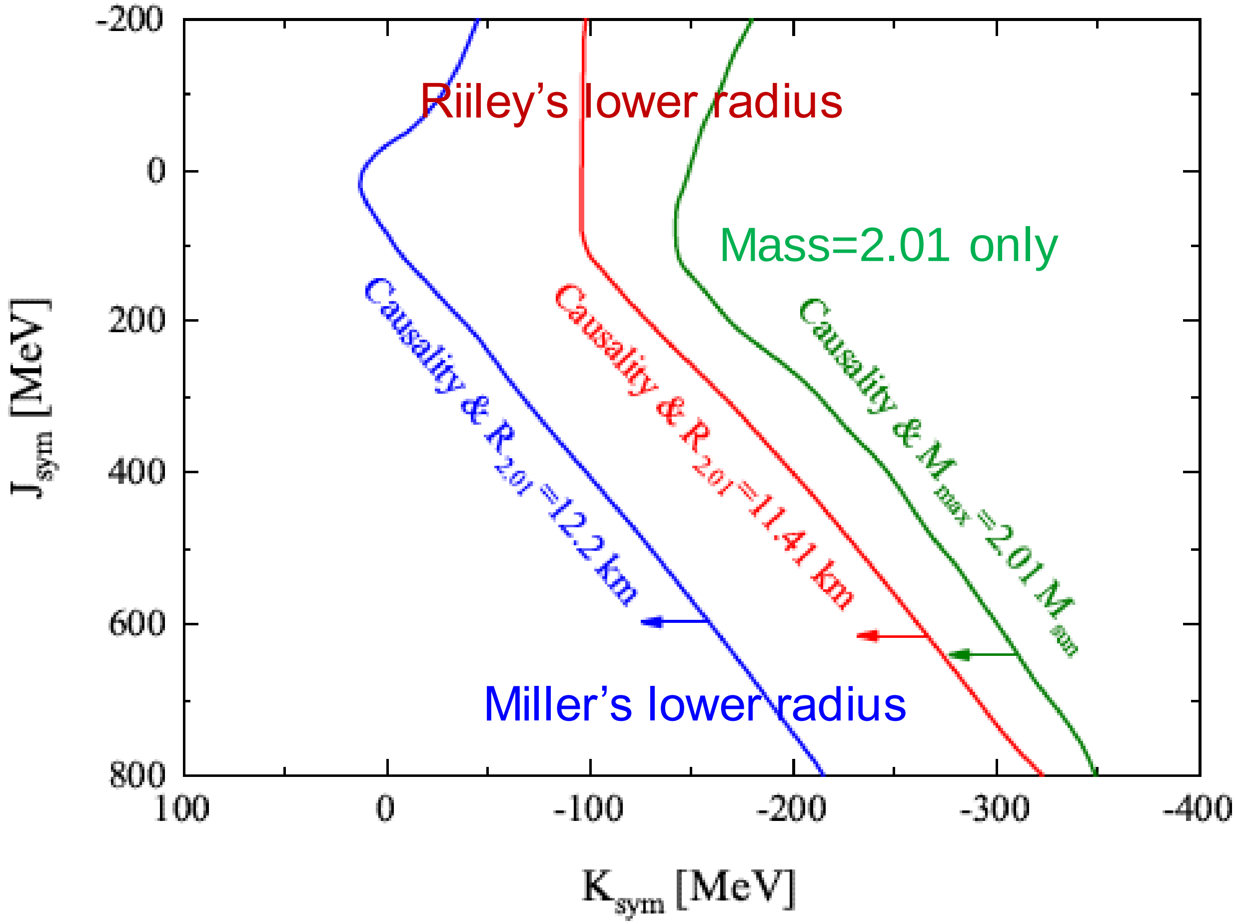
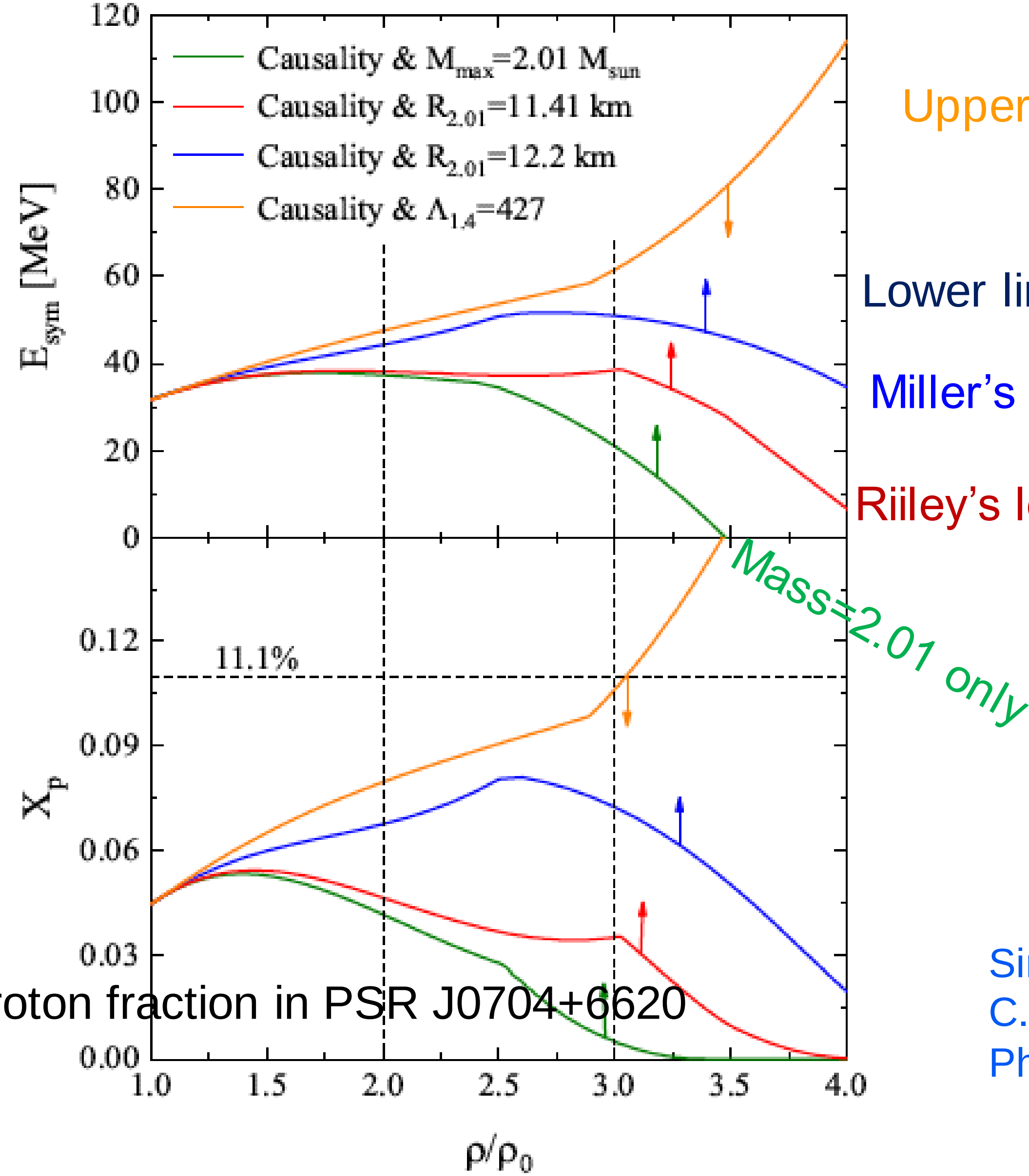


Figure 5. The projection of the crossline between surfaces of causality condition and $M_{\rm max} = 2.01\ {\rm M}_\odot$ (green line), $R_{2.01} = 11.41$ km (red line) and $R_{2.01} = 12.2$ km (blue line), respectively, on the $K_{\rm sym} - J_{\rm sym}$ plane for $L = 58.7$ MeV. The arrows point to the directions satisfying the corresponding constraints.

Impact of NICER's Radius Measurement of PSR J0740+6620 on Nuclear Symmetry Energy at Suprasaturation Densities,

Nai-Bo Zhang and Bao-An Li, APJ 921, 111 (2021)



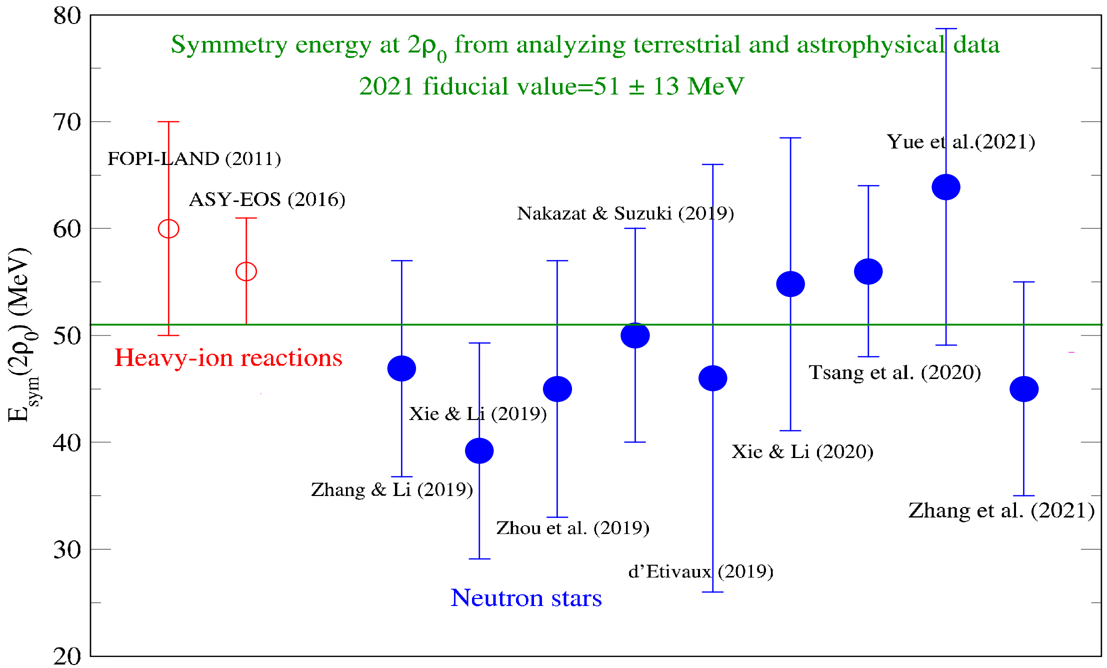
Upper limit on E_{sym} from GW170817

Lower limit on E_{sym} from PSR J0740+6620

Miller's lower radius

Riiley's lower radius

Similar conclusion by
C. Mondal and F. Gulminelli,
Phys. Rev. D 105, 083016 (2022).



Bao-An Li, Bao-Jun Cai, Wen-Jie Xie, Nai-Bo Zhang, **Universe 7, 182 (2021)**

Examples of theoretical predictions for $E_{sym}(2\rho_0)$:

(1) Chiral EFT, $E_{\rm sym}(2\rho_0) \approx 45 \pm 3$ MeV

C. Drischler, R. J. Furnstahl, J. A. Melendez, and D. R. Phillips, PRL**125**, 202702 (2020)

(2) Quantum Monte Carlo, $E_{\rm sym}(2\rho_0) \approx 46 \pm 4$

D. Lonardoni, I. Tews, S. Gandolfi, and J. Carlson, Phys. Rev. Research **2**, 022033(R) (2020)

(3) Relativistic BHF in full Dirac space: 51.6 MeV

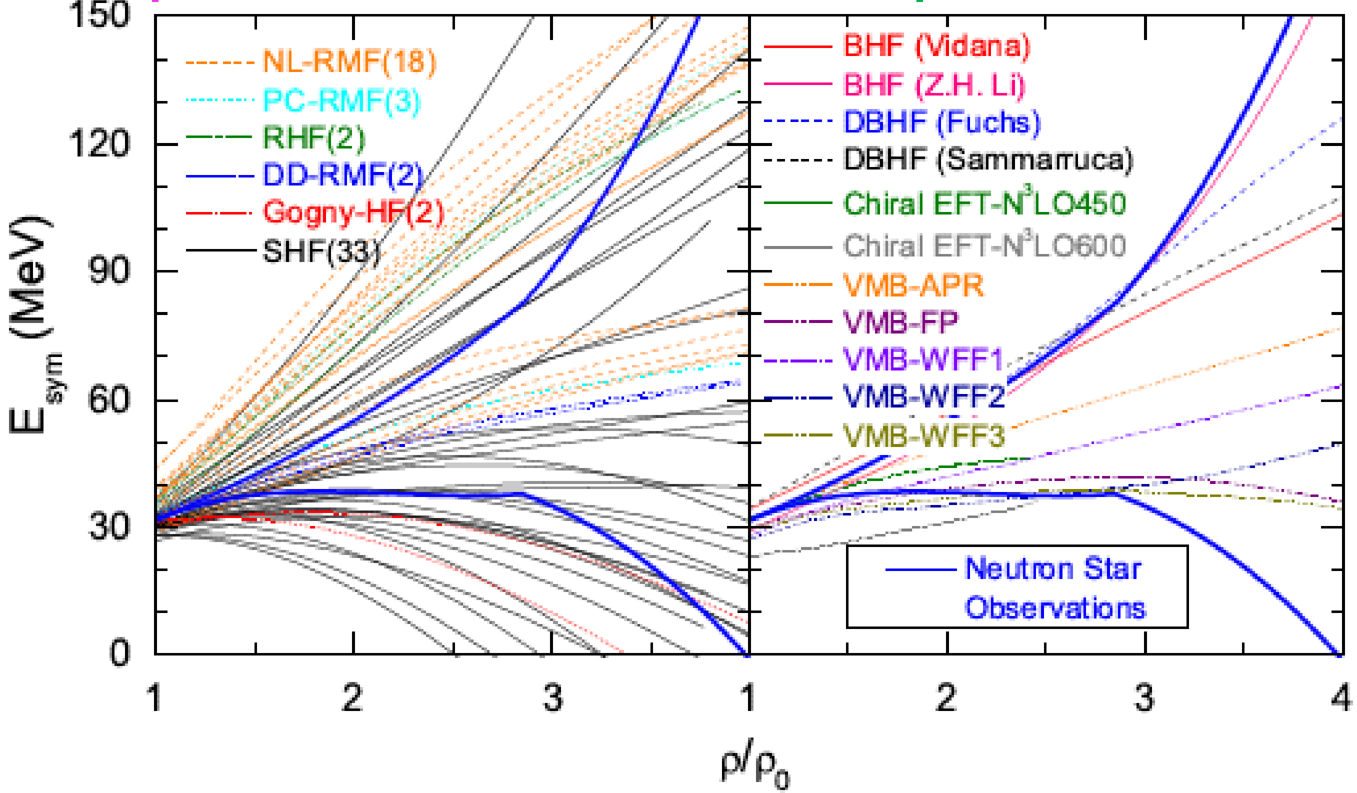
Sibo Wang, Hui Tong, Qiang Zhao, Chencan Wang, Peter Ring, Jie Meng, arXiv:2203.05397

(4) Relativistic BHF: ~ 53 MeV

Chencan Wang, Jinniu Hu, Ying Zhang, Hong Shen, arXiv:2203.03139

Phenomenological Models
60 examples

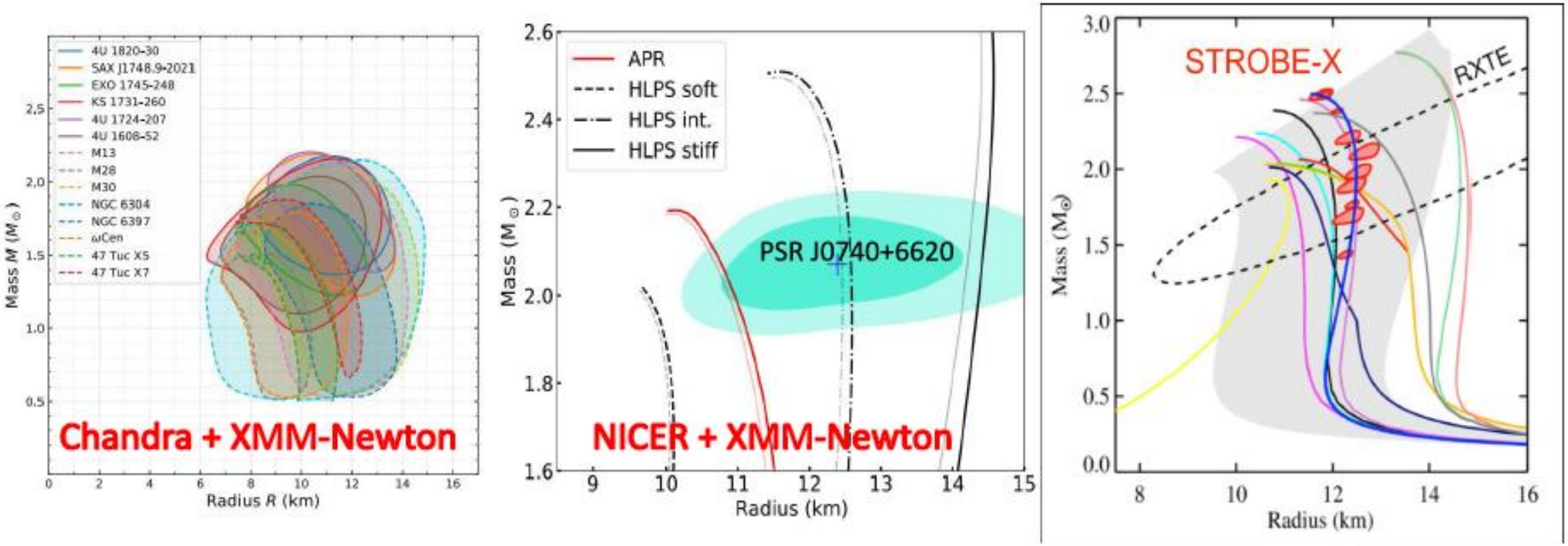
Microscopic & *ab initio* Theories
11 examples



- E_{sym} around $(1\text{-}2)\rho_0$ is most relevant for determining the radii of canonical neutron stars, existing $1.4M_{sun}$NS observations do NOT constrain much E_{sym} above $2\rho_0$ where SRC effects are important.

Needs data from post-mergers with high-frequency gravitational waves, radii of massive NSs or messengers directly from the core, e.g., neutrinos

N.B. Zhang and B.A. Li, APJ 902, 38 (2020)



Fujimoto, Y., Fukushima, K., & and Murase, K., 2020, PRD 101, 054016

Riley, T. E., Watts, A. L., Ray, P. S., et al. 2021, ApJ, 918, L27

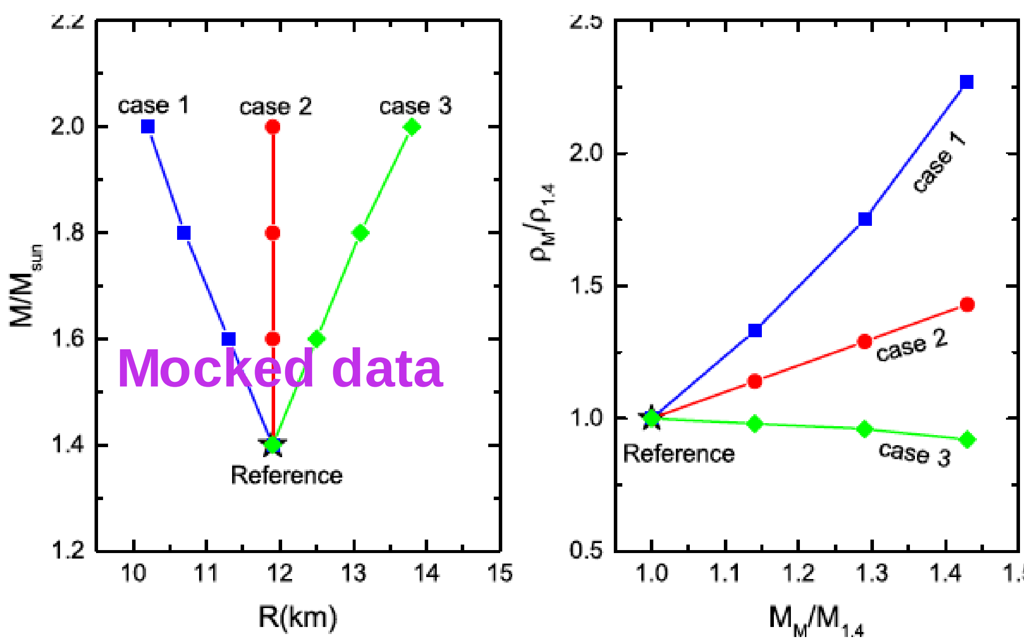
Ray, P. S., et al., 2019, arXiv:1903.03035v2,
STROBE-X: X-ray Timing and Spectroscopy on Dynamical Timescales from Microseconds to Years.

How can precise radii of massive neutron stars help?

Wen-Jie Xie and Bao-An Li
APJ 899, 4 (2020)

Table 1. Imagined massive NS radii at 90% confidence level

	$R_{1.4}$	$R_{1.6}$	$R_{1.8}$	$R_{2.0}$ (km)
Reference	11.9±1.4	(GW170817)		
case-1	11.9±1.4	11.3±1.4	10.7±1.4	10.2±1.4
case-2	11.9±1.4	11.9±1.4	11.9±1.4	11.9±1.4
case-3	11.9±1.4	12.5±1.4	13.1±1.4	13.8±1.4



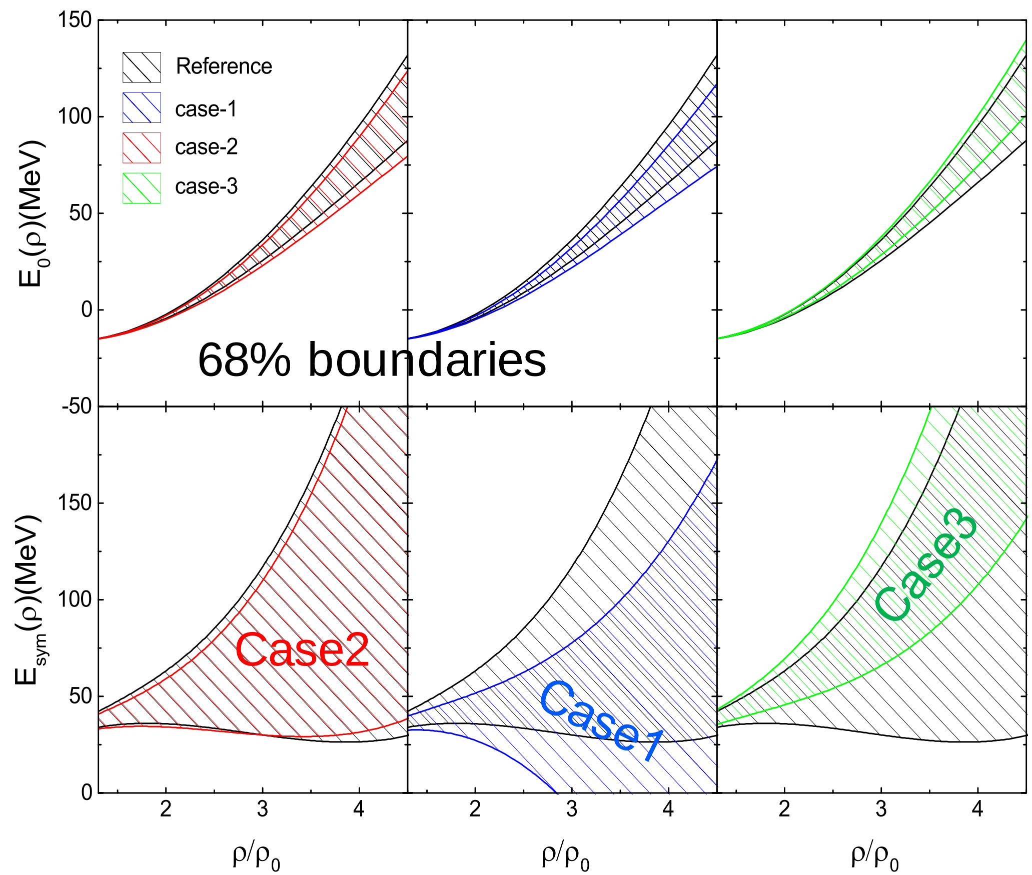
$$\rho_M/\rho_{1.4} \equiv (M/M_{1.4}) \cdot (R_{1.4}/R_M)^3$$

Figure 9. (**Left**) Representative mocked mass–radius correlations considered for massive NSs with

The predicted most probable values and 68% boundaries using the mocked neutron star radius data

Parameters (MeV)	200 MeV $\leq J_{sym} \leq$ 800 MeV Reference, case-1, case-2, case-3
J_0 :	$-165^{+55}_{-45}, -180^{+50}_{-50}, -170^{+60}_{-40}, -100^{+20}_{-70}$
K_0 :	$258^{+2}_{-24}, 222^{+24}_{-0}, 222^{+26}_{-0}, 260^{+0}_{-22}$
K_{sym} :	$-120^{+80}_{-100}, -110^{+30}_{-120}, -100^{+70}_{-90}, -30^{+80}_{-70}$
L :	$66^{+12}_{-20}, 38^{+18}_{-6}, 50^{+14}_{-14}, 70^{+12}_{-16}$

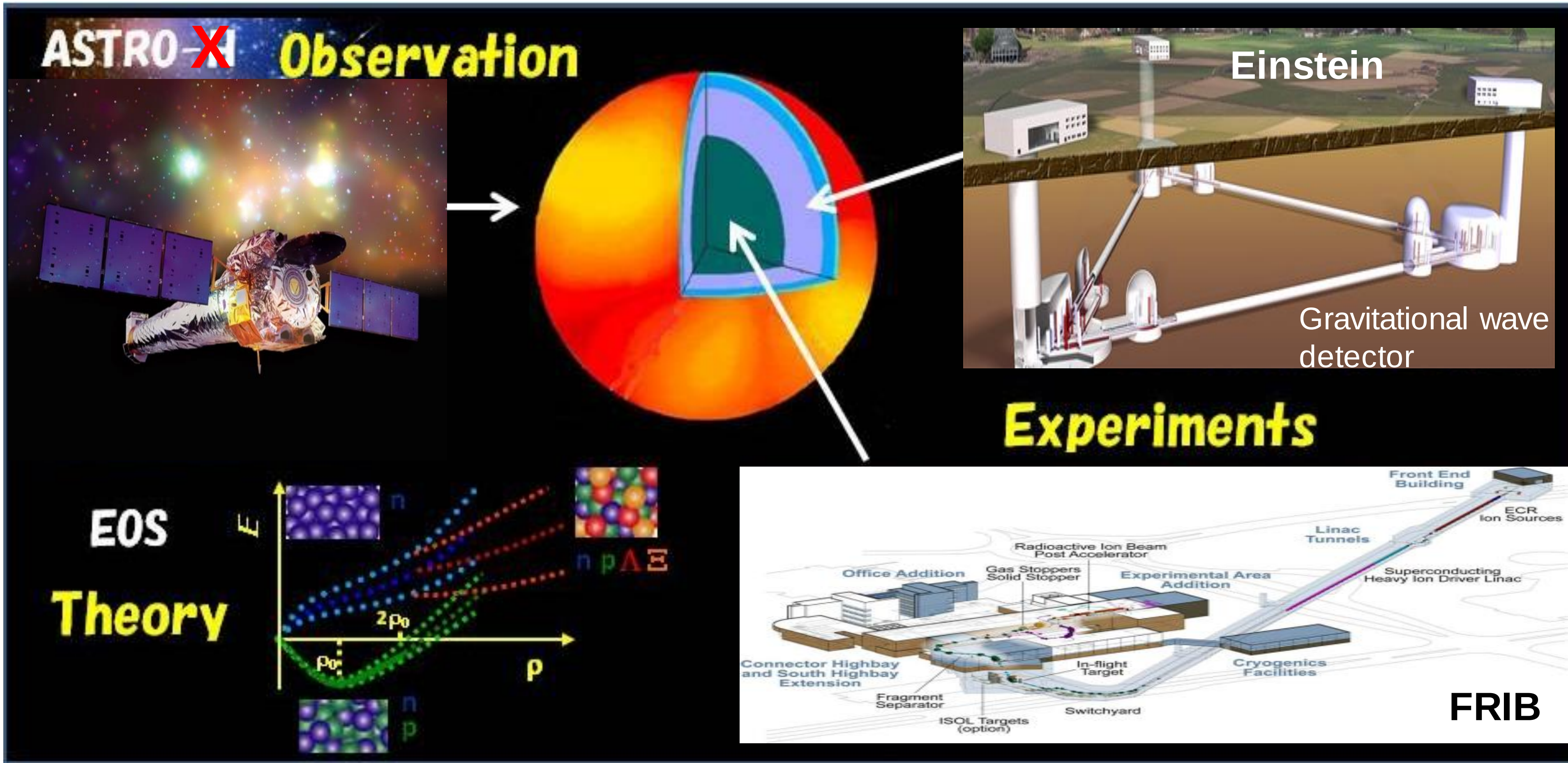
$E_{sym}(\rho)$ (MeV)	200 MeV $\leq J_{sym} \leq$ 800 MeV Reference, case-1, case-2, case-3
$E_{sym}(2\rho_0)$:	$54.8^{+8.4}_{-19}, 43.2^{+8.5}_{-15.9}, 50.5^{+9.4}_{-16.3}, 61.1^{+8.4}_{-15.4}$
$E_{sym}(3\rho_0)$:	$91.3^{+25.8}_{-61.2}, 52.2^{+25.6}_{-60.4}, 85.1^{+24.9}_{-55}, 114^{+25.8}_{-48}$



From Earth to Heaven: multi-messengers of nuclear EOS

(1) Significant progress has been made in constraining the EOS below $2\rho_0$

(2) Truly multi-messenger approach to probe the EOS of dense neutron-rich matter = astrophysical observations + terrestrial experiments + theories + …



Connecting the Skyrme-HF parameters with saturation properties of nuclear matter

Lie-Wen Chen, Che Ming Ko, Bao-An Li, Jun Xu, PRC 82, 024321,2010

$$E_0(\rho) = E_{\rm kin}^0 u^{2/3} + C u^{5/3} + \alpha u/2 + \beta u^\gamma/(\gamma+1),$$

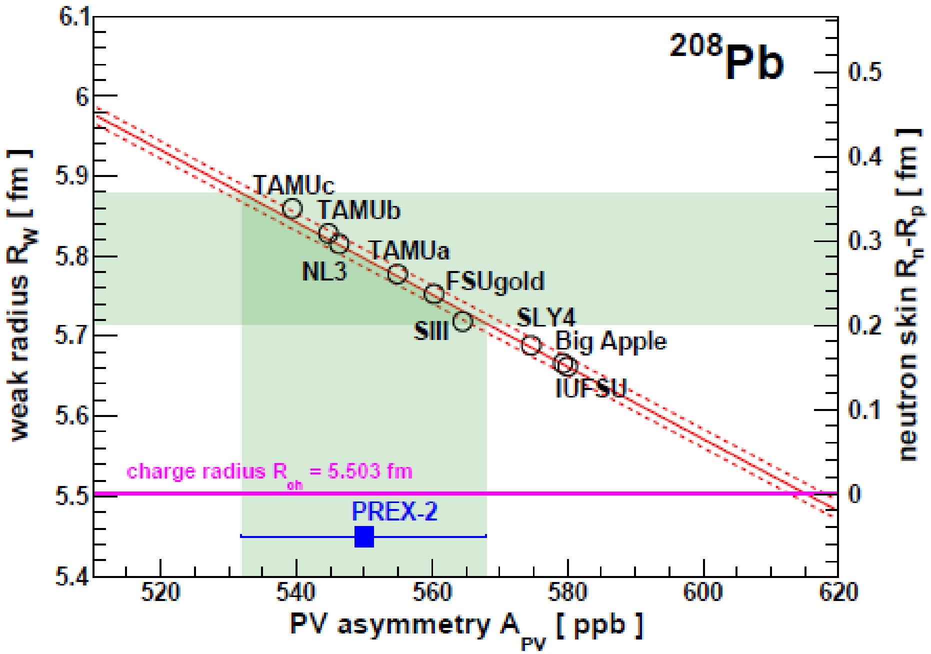
$$E_{\rm sym}(\rho) = E_{\rm sym}^{\rm kin}(\rho_0) u^{2/3} + D u^{5/3} + E_{\rm sym}^{\rm loc}(\rho)$$

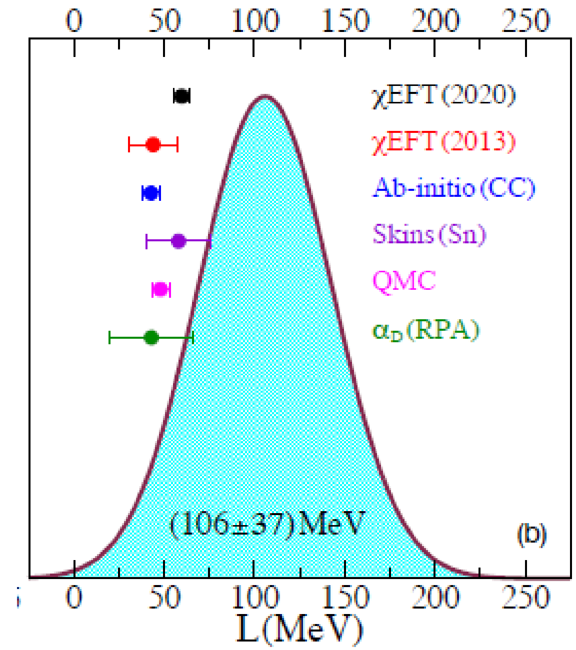
$$E_{\rm sym}^{\rm loc}(\rho) = (1-y) E_{\rm sym}^{\rm loc}(\rho_0) u + y E_{\rm sym}^{\rm loc}(\rho_0) u^\gamma$$

$$E_{\rm sym}^{\rm loc}(\rho) = (1-y) E_{\rm sym}^{\rm loc}(\rho_0) u + y E_{\rm sym}^{\rm loc}(\rho_0) u^\gamma$$

$$\begin{aligned}
t_0 &= 4\alpha/(3\rho_0) \\
x_0 &= 3(y-1)E_{\rm sym}^{\rm loc}(\rho_0)/\alpha - 1/2 \\
t_3 &= 16\beta/\left[\rho_0{}^\gamma(\gamma+1)\right] \\
x_3 &= -3y(\gamma+1)E_{\rm sym}^{\rm loc}(\rho_0)/(2\beta) - 1/2 \\
t_1 &= 20C/\left[9\rho_0(k_{\rm F}^0)^2\right] + 8G_S/3 \\
t_2 &= \frac{4(25C-18D)}{9\rho_0(k_{\rm F}^0)^2} - \frac{8(G_S+2G_V)}{3} \\
x_1 &= \left[12G_V - 4G_S - \frac{6D}{\rho_0(k_{\rm F}^0)^2}\right]/(3t_1) \\
x_2 &= \left[20G_V + 4G_S - \frac{5(16C-18D)}{3\rho_0(k_{\rm F}^0)^2}\right]/(3t_2) \\
\sigma &= \gamma - 1
\end{aligned}$$

Quantity	MSL0	Quantity	MSL0
t_0 (MeV·fm^3)	−2118.06	ρ_0 (fm^{-3})	0.16
t_1 (MeV·fm^5)	395.196	E_0 (MeV)	−16.0
t_2 (MeV·fm^5)	−63.9531	K_0 (MeV)	230.0
t_3 (MeV·fm$^{3+3\sigma}$)	12857.7	$m_{s,0}^*/m$	0.80
x_0	−0.0709496	$m_{v,0}^*/m$	0.70
x_1	−0.332282	$E_{\rm sym}(\rho_0)$ (MeV)	30.0
x_2	1.35830	L (MeV)	60.0
x_3	−0.228181	G_S (MeV·fm^5)	132.0
σ	0.235879	G_V (MeV·fm^5)	5.0
W_0 (MeV·fm^5)	133.3	$G_0'(\rho_0)$	0.42



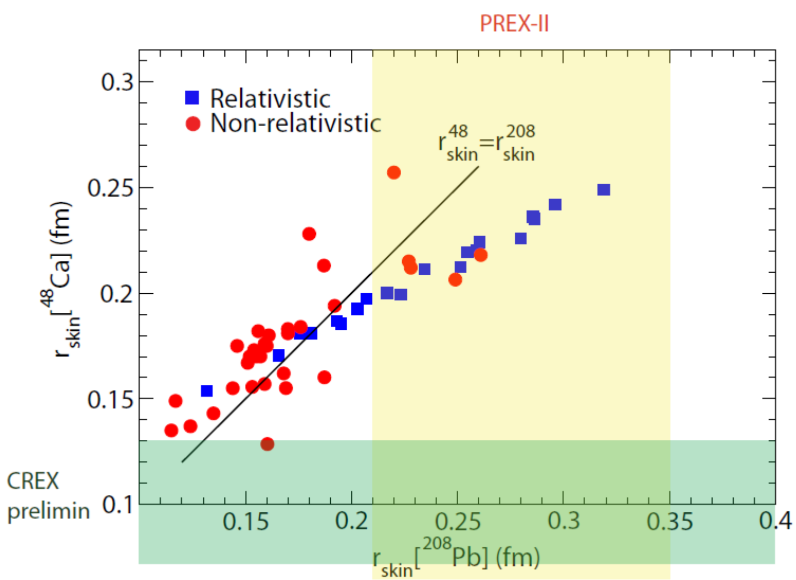


B.T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, PRL **126**, 17250 (2021)

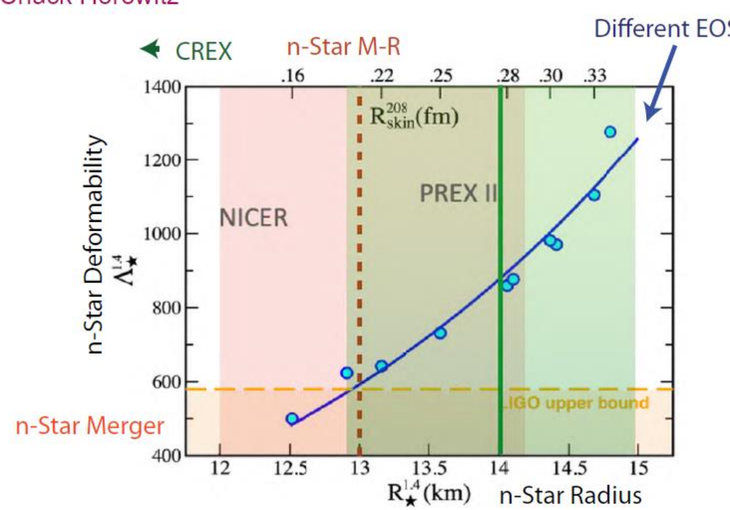
Parity Violation in *e*-Scattering: PREX & CREX

Adhikari *et al.*, PRL126(21)172502 + Palatchi@DNP'21

Heaven vs Earth



after Chuck Horowitz



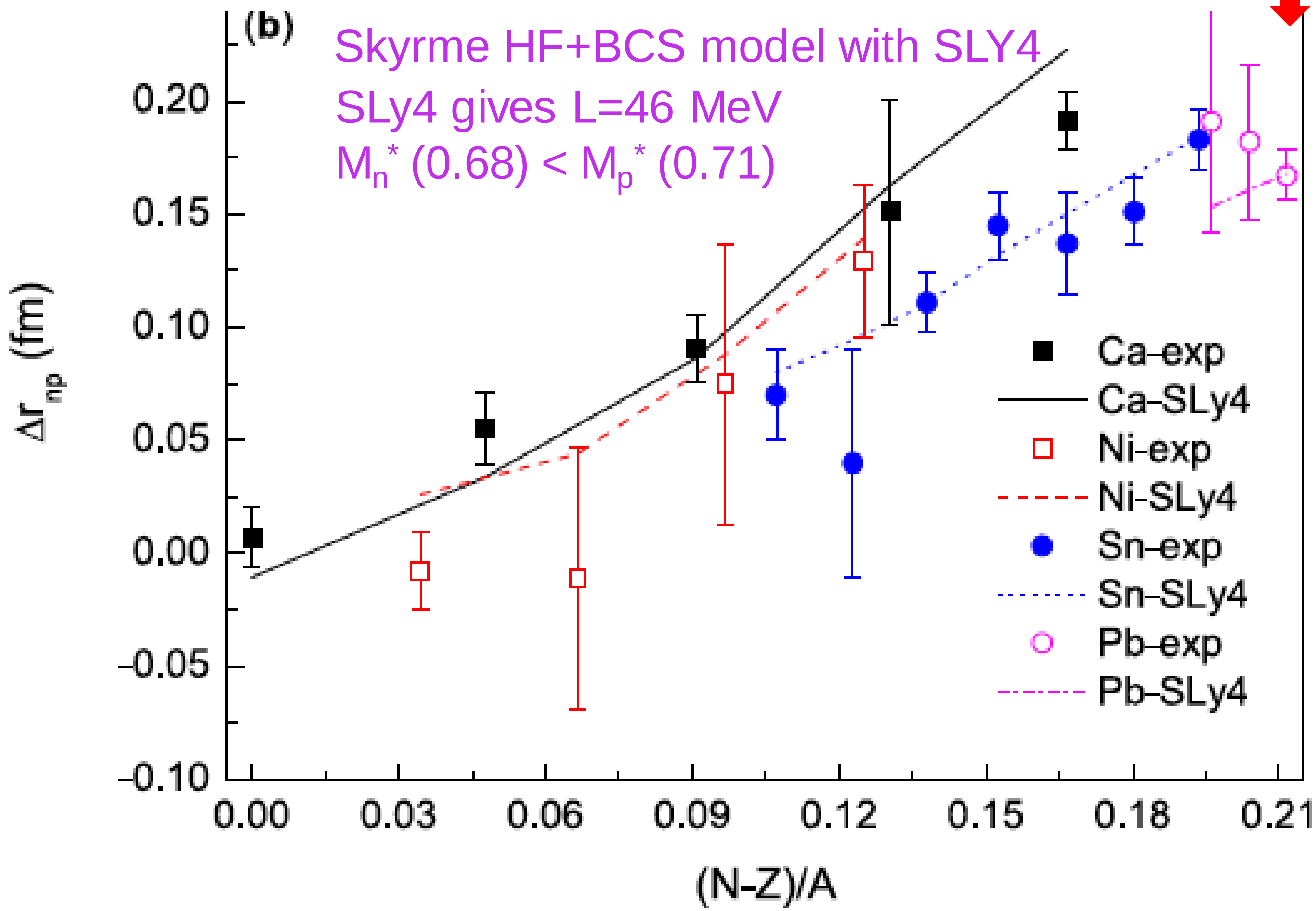
Tensions. . .

Systematic trends of neutron skin thickness versus relative neutron excess

J. T. Zhang,[1,2,3] X. L. Tu,[3,4,5,*] P. Sarriguren,[6] K. Yue,[3,†] Q. Zeng,[3,7] Z. Y. Sun,[3] M. Wang,[3]
Y. H. Zhang,[3] X. H. Zhou,[3] and Yu. A. Litvinov[3,8]

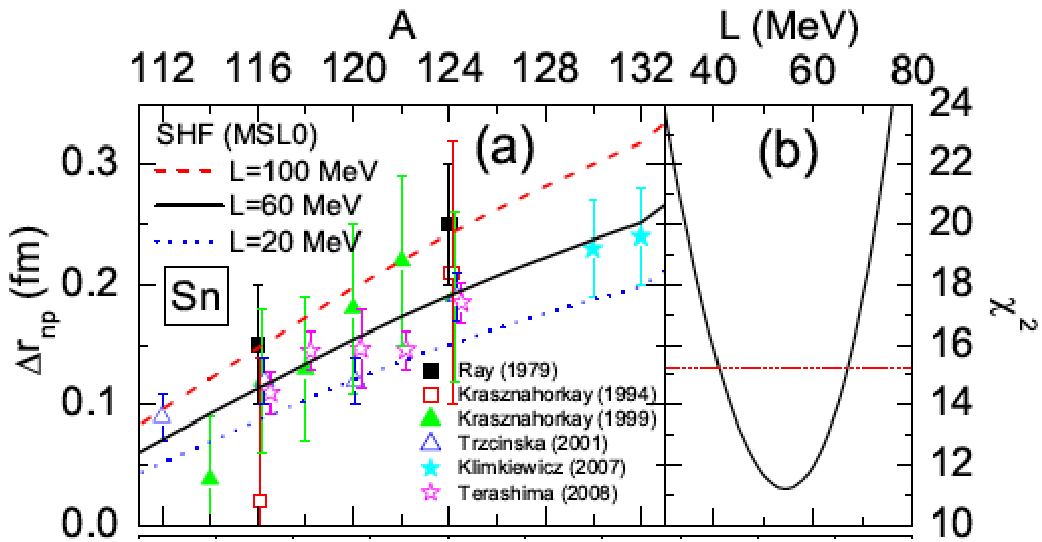
PHYSICAL REVIEW C **104**, 034303 (2021)

PREX2 gives
Δr_{np} =0.283 (0.071)
L=106 (37) MeV



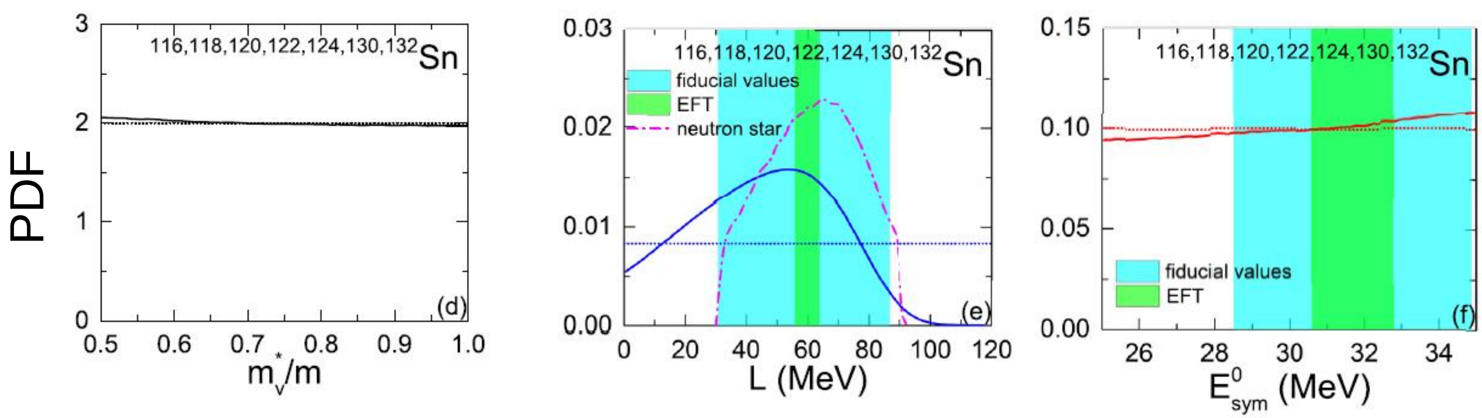
L from n-skin within SHF using the Modified-Skyrme-Like parameters

Lie-Wen Chen, Che Ming Ko, Bao-An Li, and Jun Xu, PRC**82**, 024321 (2010)



Bayesian inference of nuclear symmetry energy from measured and imagined neutron skin thickness in ^{116,118,120,122,124,130,132}Sn, ^{208}Pb, and ^{48}Ca

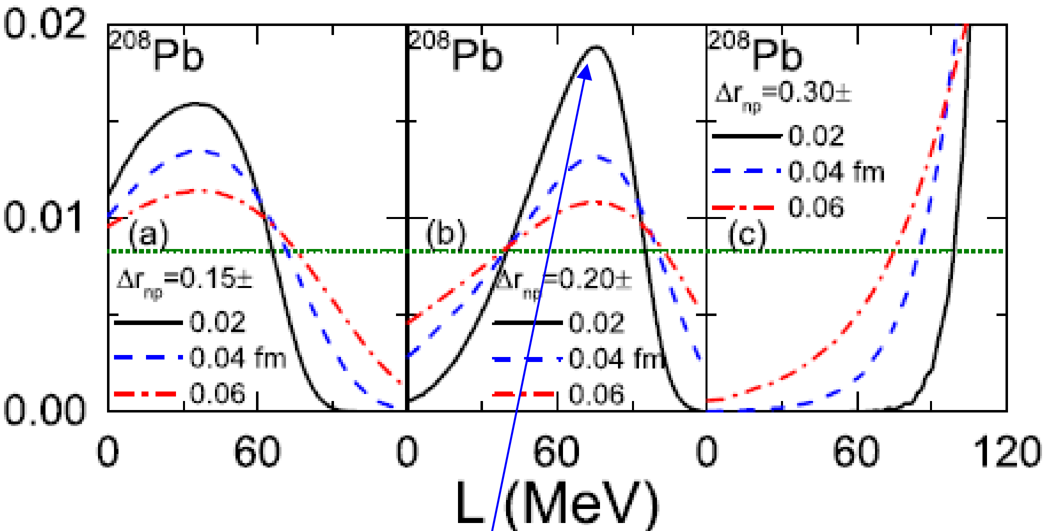
Jun Xu, Wen-Jie Xie and Bao-An Li, PRC 102, 044316 (2020)



To be consistent with the Sn data, n-star observations and chiral EFT

the predicted n-skin of ^{208}Pb and ^{48}Ca

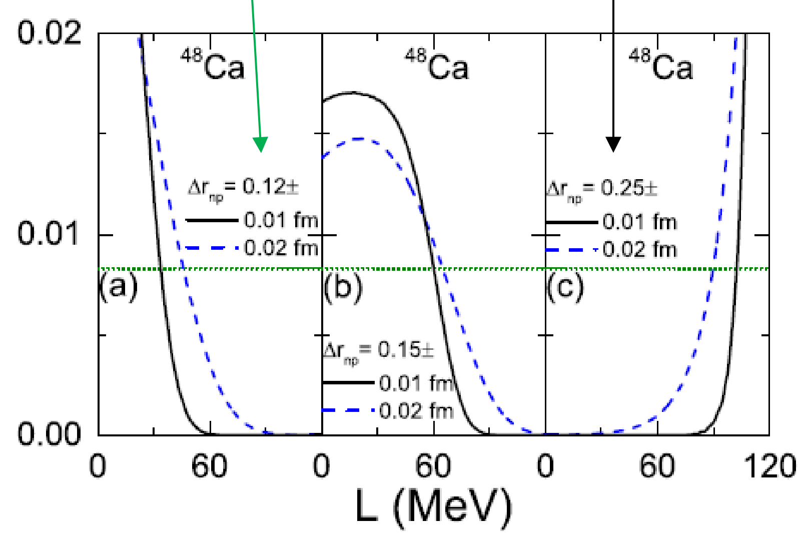
Jun Xu, Wen-Jie Xie and Bao-An Li, PRC 102, 044316 (2020)



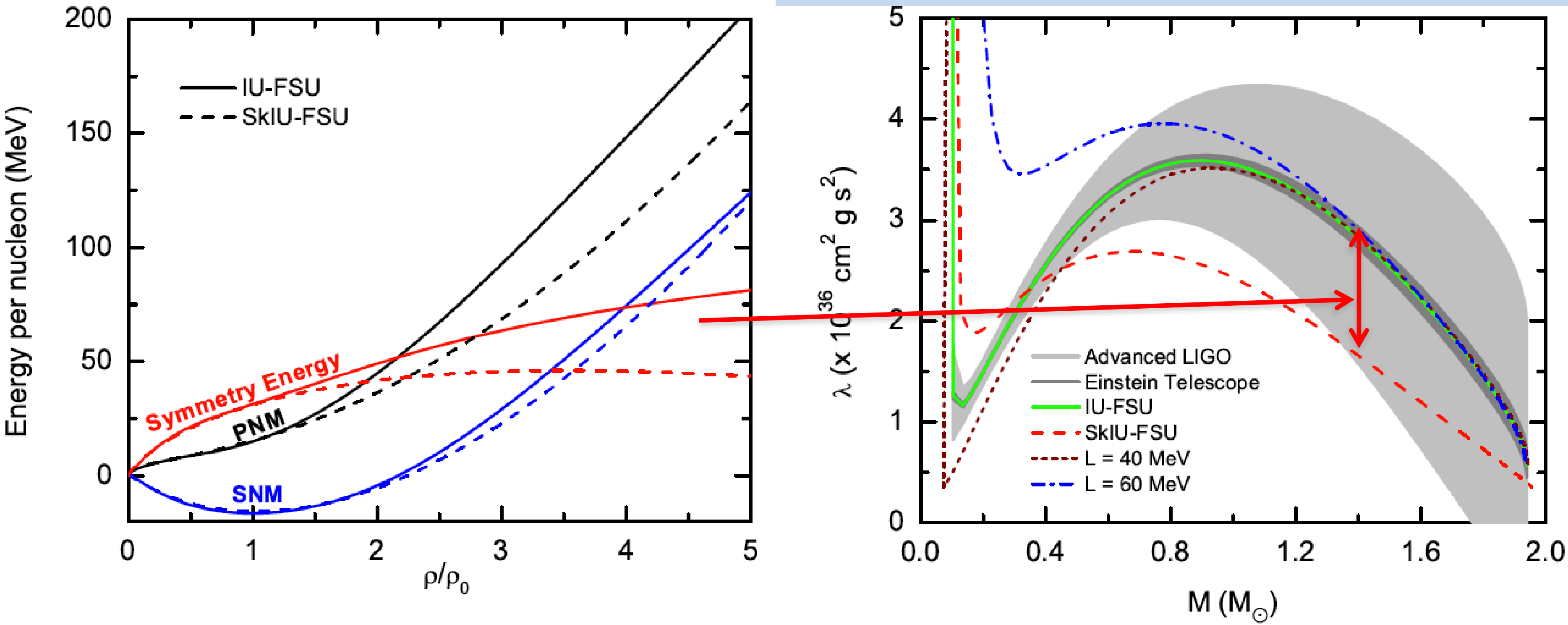
^{208}Pb has to have a n-skin about 0.18 fm with a precision less than 0.02 fm

Self-consistent Green's function

Coupled cluster model



Impact of high-density symmetry energy on the tidal deformability of neutron stars in mergers



F. Fattoyev, J. Carvajal, W.G. Newton and B.A. Li, PRC87, 15806 (2013)

The tidal deformation:

$$\lambda = 2k_2R^5/(3G)$$

Given a EOS, the Love number k_2 and radius R for a given mass m can be solved from the Tolman-Oppenheimer-Volkoff Eq. coupled with a differential Eq. for the strength of the perturbed time-time component of the metric

EOS of dense neutron-rich matter is a major scientific motivation of
(1) High-energy rare isotope beam facilities around the world
(2) Various x-ray satellites
(3) Various gravitational wave detectors

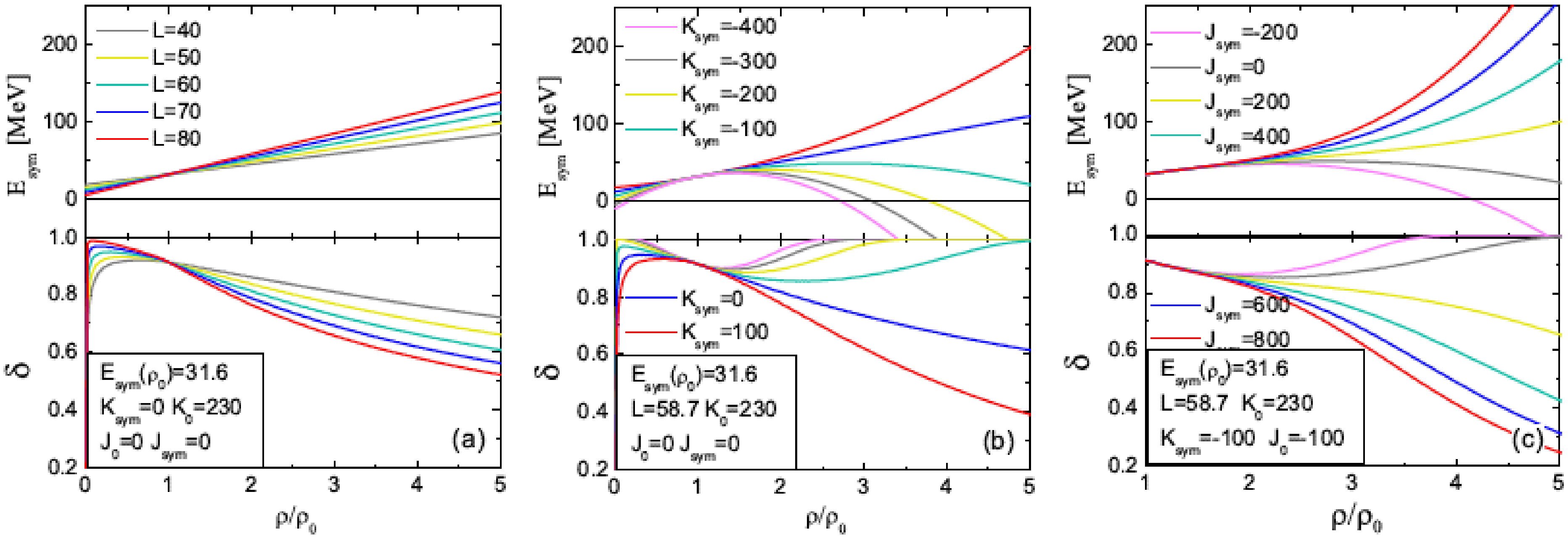
Among the promising observables of high-density symmetry energy:

- **<u>π^-/π^+ and n/p spectrum ratio</u>, neutron-proton differential flow and correlation function in heavy-ion collisions at intermediate energies**
- **Radii of neutron stars**
- **Neutrino flux of supernova explosions**
- **Tidal polarizability in neutron star mergers, strain amplitude of gravitational waves from deformed pulsars, frequency and damping time of neutron star oscillations**

B.A. Li, L.W. Chen and C.M. Ko, Phys. Rep. 464, 113 (2008)

EPJA, Vol. 50, No. 2 (2014)

How neutron rich the neutron stars can be?



Symmetry energy and the isospin asymmetry at beta equilibrium

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

The proton fraction x:

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$