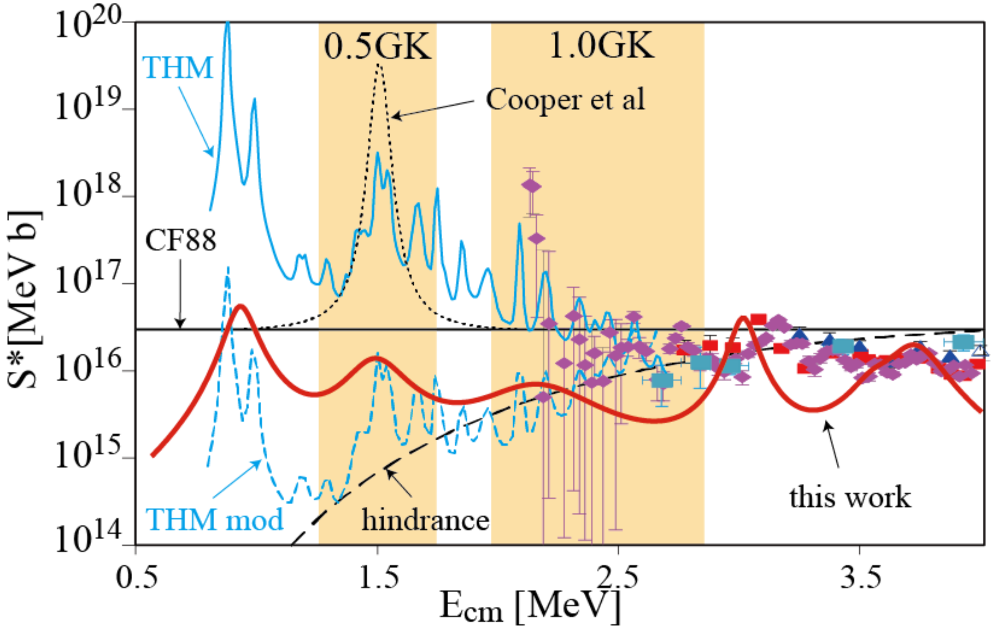
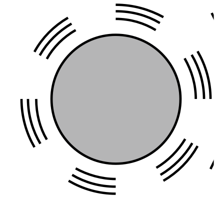
Low energy monopole resonances: A novel approach to the astrophysical fusion reactions

M. Kimura (RIKEN), Y. Taniguchi(Kagawa college)

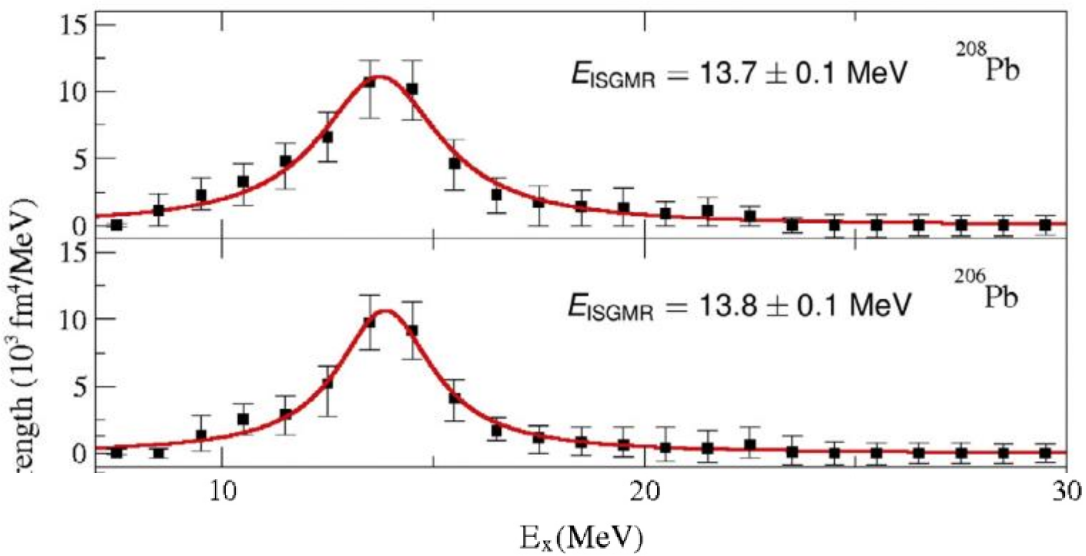


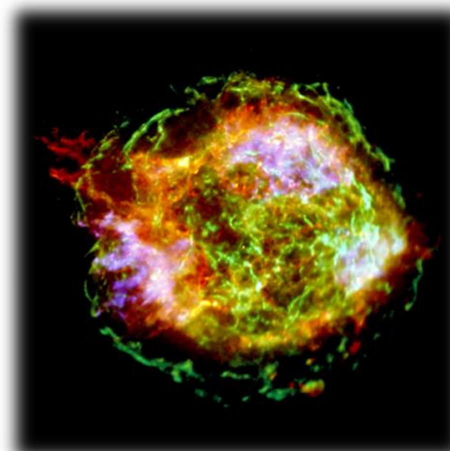
○ GMR as a probe for the nuclear incompressibility

- GMR is the vibration of nuclear density (breathing mode)



⇨ Energy of GMR is closely related to the incompressibility of finite nucleus and infinite nuclear matter





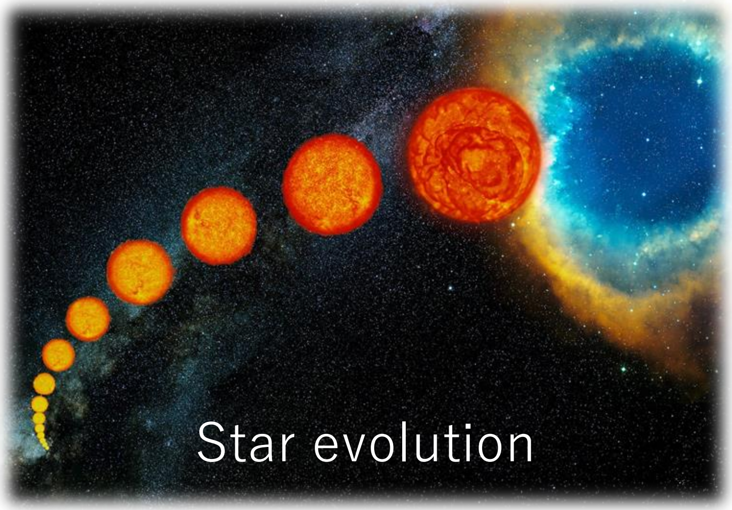
Core collapse supernova

e.g.) J.P. Blaizot, Phys. Rep. 64, 171 (1980).   U. Garg and G. Colò, PPNP101, 55 (2018).

The other aspect of the monopole transition

In this talk, I'll discuss the relation between monopole transition and astrophysical fusion reactions such as $^{12}C+^{12}C$, $^{12}C+^{16}O$, ···

$^{12}C+^{12}C$ fusion reaction significantly impacts the evolution of massive stars and X-ray superbusts.



Monopole transition is a novel probe to identify the resonances (cluster resonances) that govern these fusion reaction

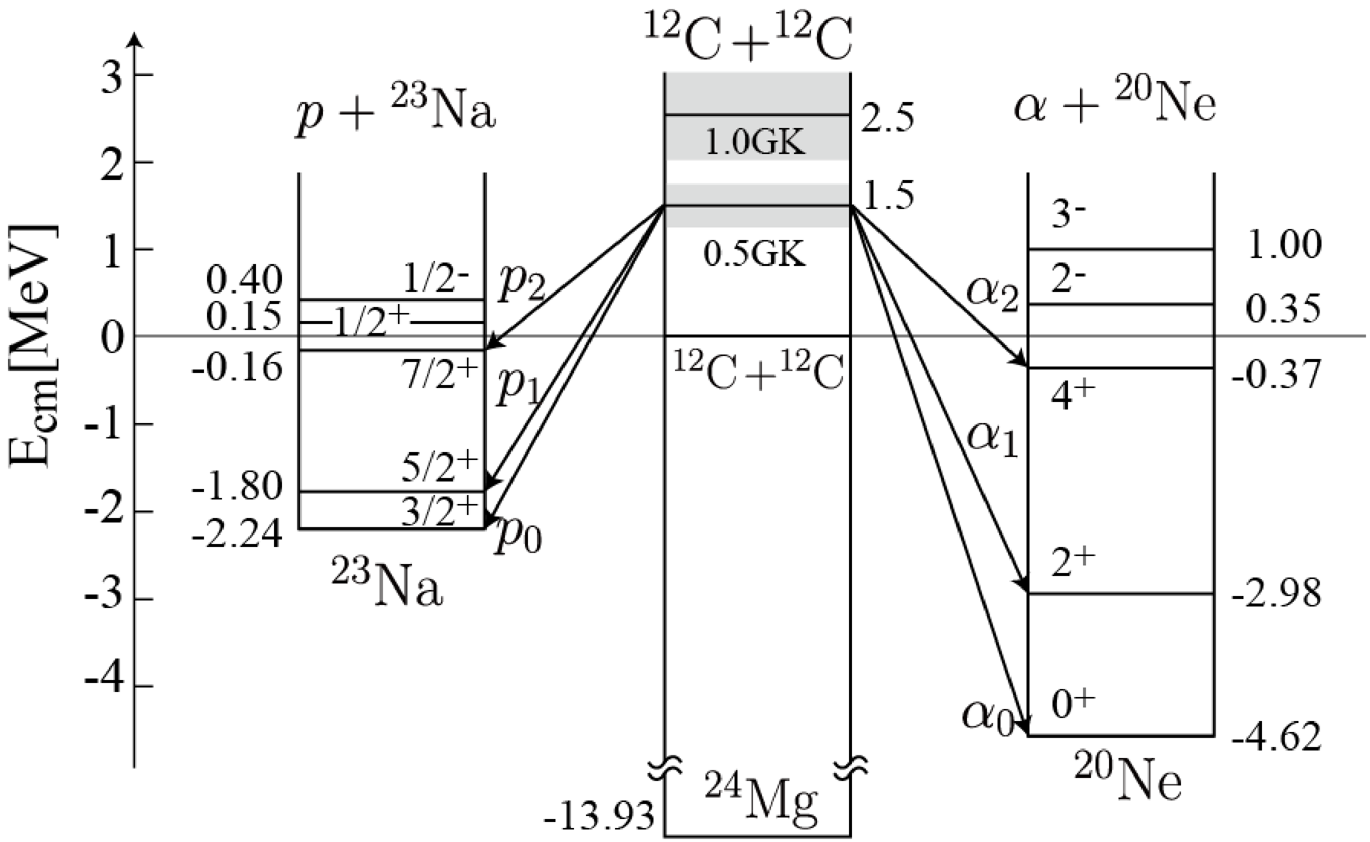
$^{12}C+^{12}C$ fusion reaction in the universe

The main reaction channels of $^{12}C+^{12}C$ reaction are

$$^{12}\mathrm{C} + ^{12}\mathrm{C} \rightarrow p + ^{23}\mathrm{Na} + 2.2\mathrm{MeV}$$
$$\rightarrow \alpha + ^{20}\mathrm{Ne} + 4.6\mathrm{MeV}$$

E_{cm}=1～3 MeV, E_{ex}=14～17MeV

The reaction energy is just below or overlaps with GMR of ^{24}Mg



$^{12}C+^{12}C$ fusion reaction: measured reaction rate (S-factor)

Due to the large Coulomb barrier, the direct measurement is difficult !

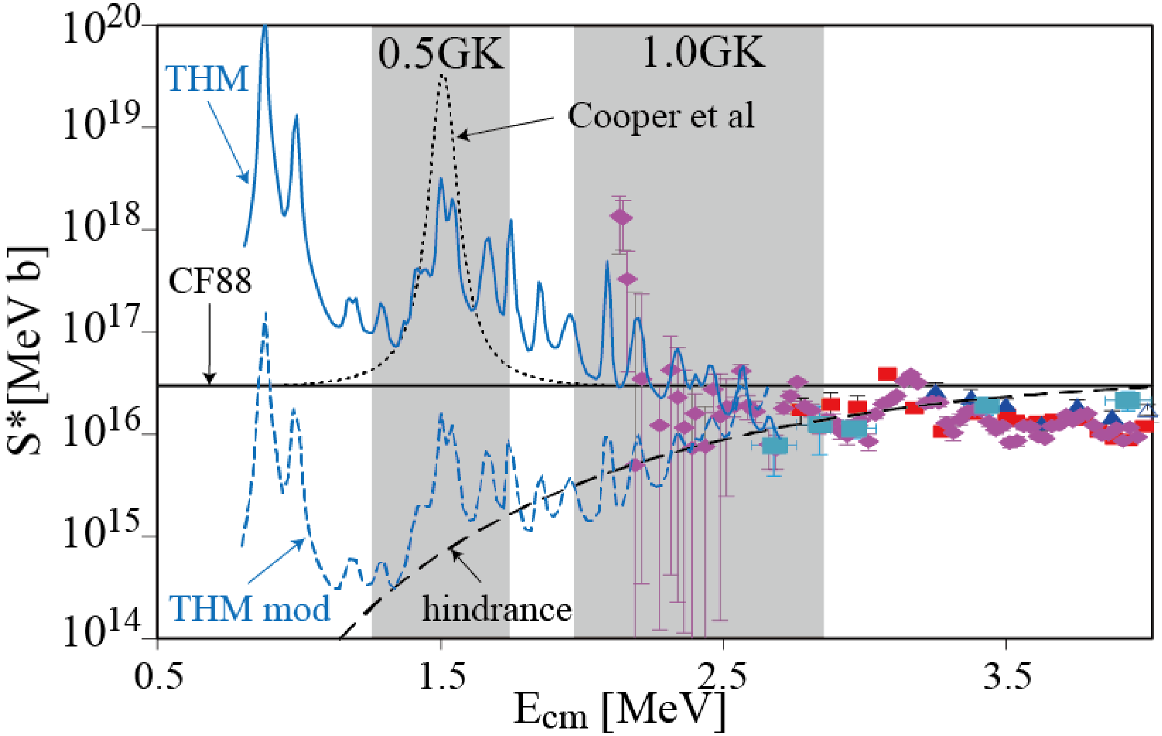
[1] Direct measurements are limited down to 2.3 MeV

[2] The extrapolated S-factor is used in the astronomical simulations

[3] Trojan Horse Method (THM) experiment suggested
the enhancement at low energy

[1]C. Beck et al, EPJA 56, 87 (2020).

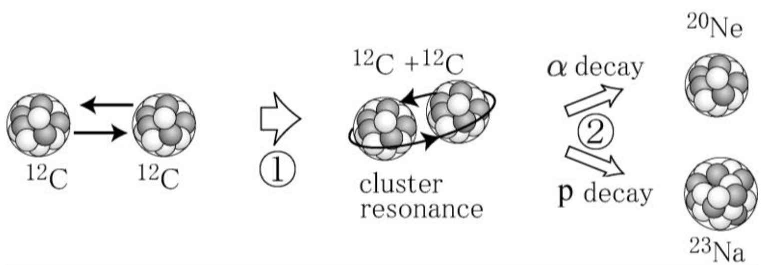
[3] A. Tumino et al: Nature 557, 687(2018).
$^{12}C(^{14}N,d)$ reaction



$^{12}C+^{12}C$ fusion reaction: measured reaction rate (S-factor)

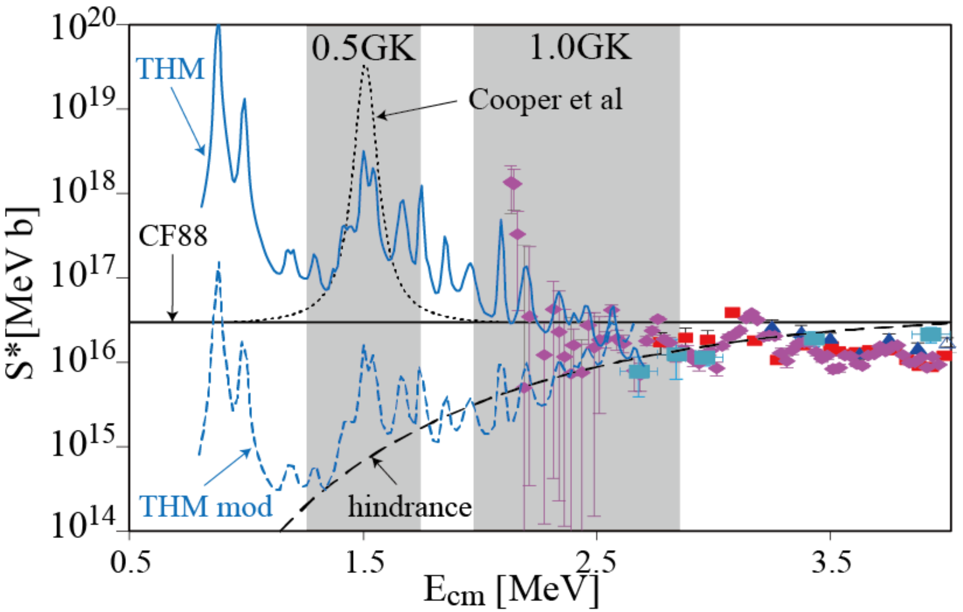
THM experiment suggests many (cluster-like) resonances with l =0 or 2 contributes to the $^{12}C+^{12}C$ fusion reaction

The cluster resonances determine the reaction rates



① Enhance the cross section in order of magnitude

② Determine the reaction product



Goal of this study

- ○ Investigate the monopole or quadrupole resonances with cluster configurations in the energy region of Ex=14～17 MeV in ^{24}Mg
- ○ Propose a method to identify the resonances other than direct reactions

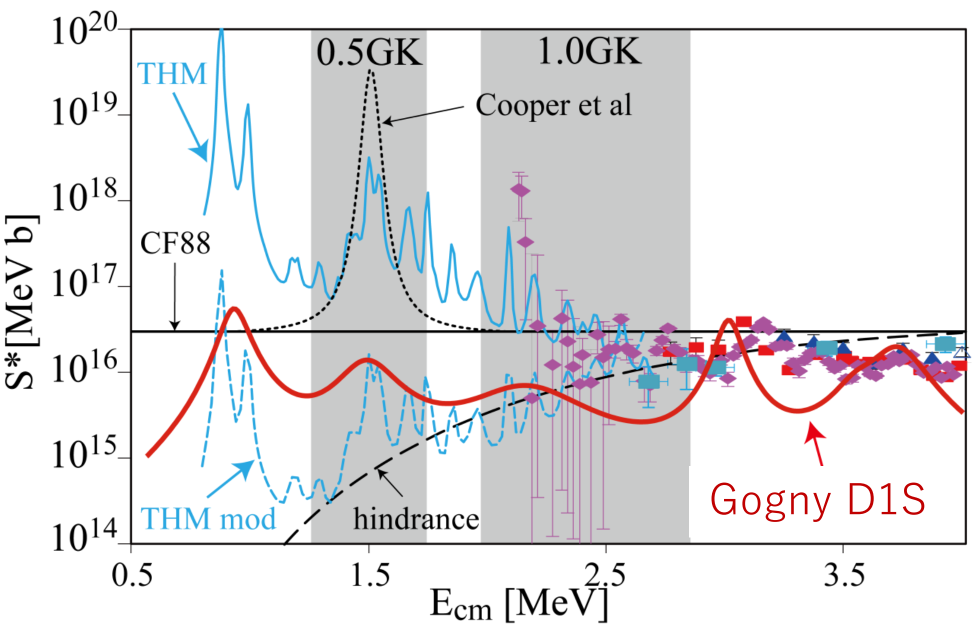
⇨ monopole (quadrupole) transitions from the ground state

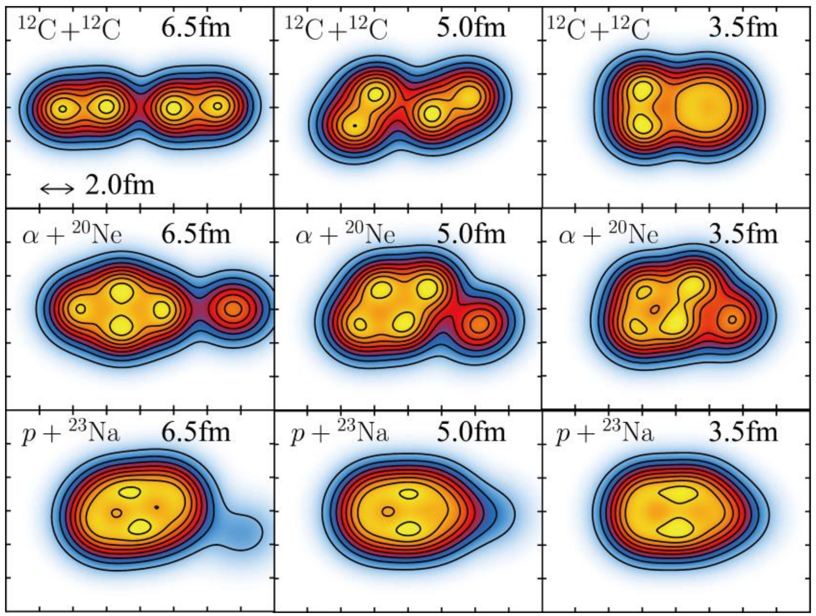
Study of the $^{12}C+^{12}C$ S-factor by antisymmetrized molecular dynamics(AMD)

Y. Taniguchi and M. Kimura: PLB 823, 136790 (2021).

We have investigated the cluster resonances in the energy region of Ex=14～17 MeV of ^{24}Mg

And estimated the astrophysical S*-factor for $^{12}C+^{12}C$ reaction from a microscopic nuclear model of antisymmetrized molecular dynamics (AMD)





Theoretical framework of AMD

Variational wave function: antisymmetrized product of nucleon wave packets

$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\Psi_{int} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}$$

$$\varphi_i(\boldsymbol{r}) \propto \exp\left\{-\nu_x\left(x-\frac{Z_{ix}}{\sqrt{\nu_x}}\right)^2 - \nu_y\left(y-\frac{Z_{iy}}{\sqrt{\nu_y}}\right)^2 - \nu_z\left(z-\frac{Z_{iz}}{\sqrt{\nu_z}}\right)^2\right\} \otimes \{a_i|\uparrow\rangle + b_i|\uparrow\rangle\} \otimes (|n\rangle \text{ or } |p\rangle)$$

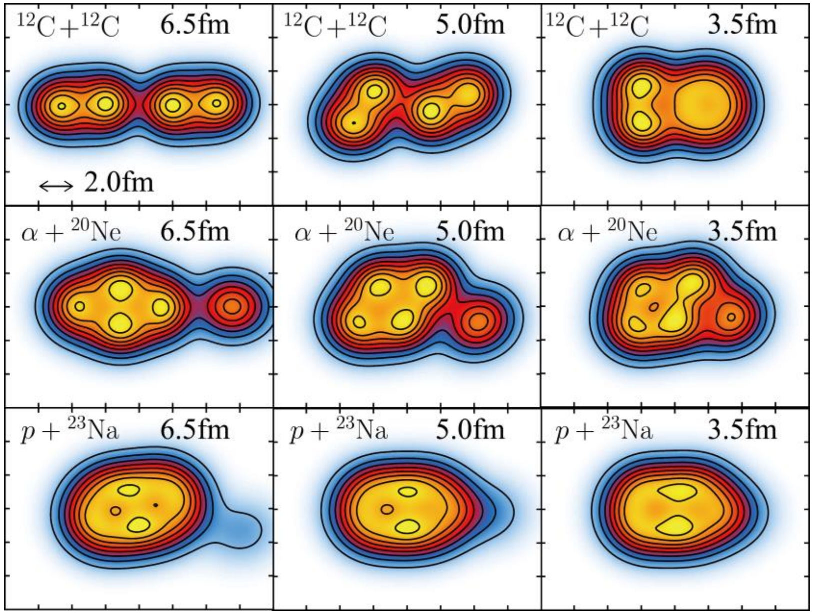
Each nucleon is described by localized Gaussians

Gaussian centroids Z_i describes mean position and momentum of each nucleon

⇨ It is straightforward to include all reaction channels, ^{12}C+^{12}C, α+^{20}Ne, p+^{23}Na

- In the practical calculation, parameters are determined by the energy variation in each reaction channel and at various nucleus-nucleus distances

- Rotation and polarization of nuclei in the reaction process are naturally described

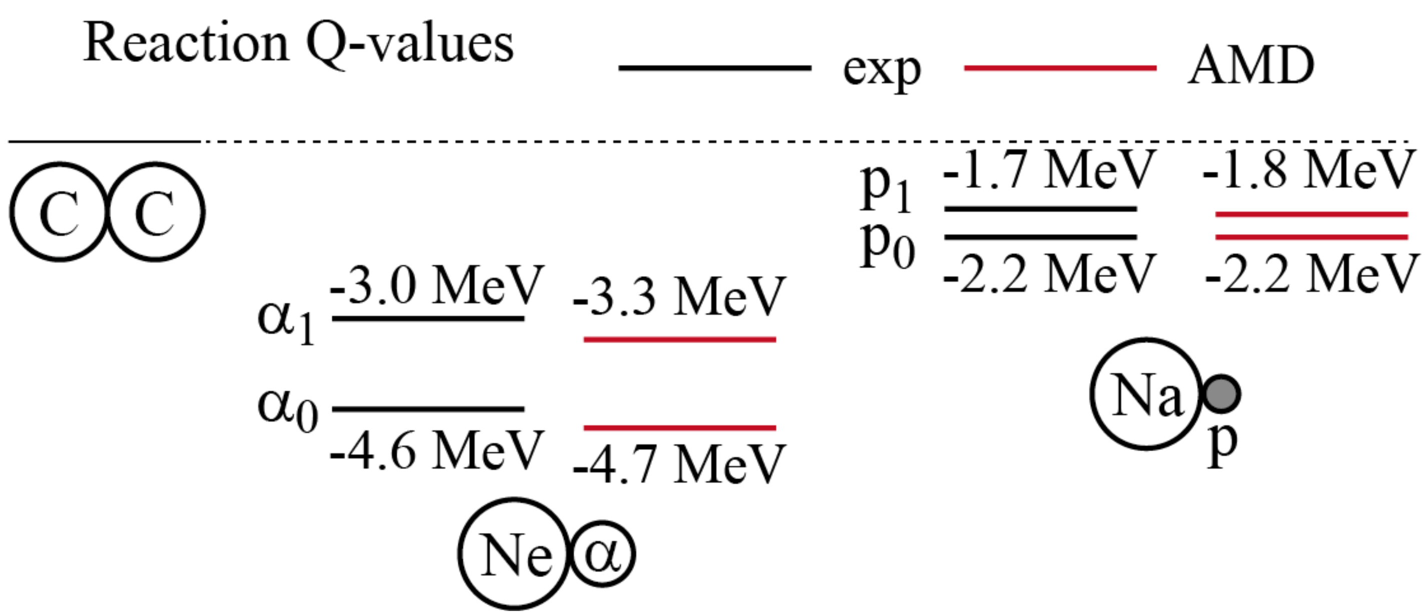


Theoretical framework of AMD

Microscopic Hamiltonian: Gogny D1S density functional

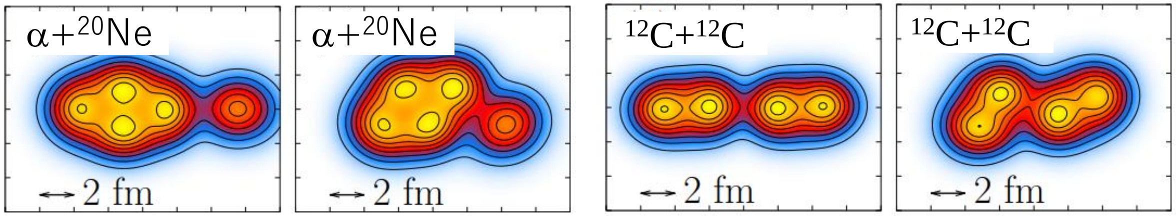
$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{GognyD1S}}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_{\mathrm{Coulomb}}(r_{ij})$$

This enables reasonable description of the reaction Q-values



Theoretical framework of AMD

Massive computing allows us to include all these reaction channels in a full-microscopic manner (dynamics of many nucleons)



○ All of the channel wave functions are projected to $l = 0$ or 2 and superposed.

○ The wave function is connected to the scattering boundary condition

- J-projection and superposition $$\Psi^{J\pm}_{\alpha} = \sum_{iK} c_{ik} \int d\Omega D^{J*}_{MK}(\Omega)\hat{R}(\Omega)\Psi^{\pm}_{\rm int},$$
- Diagonalize Hamiltonian $$\sum_{jK'} H_{iKjK'} c_{jK',\alpha} = E_{\alpha} \sum_{jK'} N_{iKjK'} c_{jK',\alpha},$$

$$H_{iKjK'} = \langle \Psi^{J\pm}_{MK}(\beta_i)|\hat{H}|\Psi^{J\pm}_{MK'}(\beta_j)\rangle, \quad N_{iKjK'} = \langle \Psi^{J\pm}_{MK}(\beta_i)|\Psi^{J\pm}_{MK'}(\beta_j)\rangle$$

⇨ The resonance energies and wave functions are calculated

Theoretical framework of AMD

We also estimate the decay widths within R-matrix approx.

Partial decay width $\Gamma = 2P_\ell(a)\dfrac{\hbar^2}{2\mu a}|ay_\ell(a)|^2$

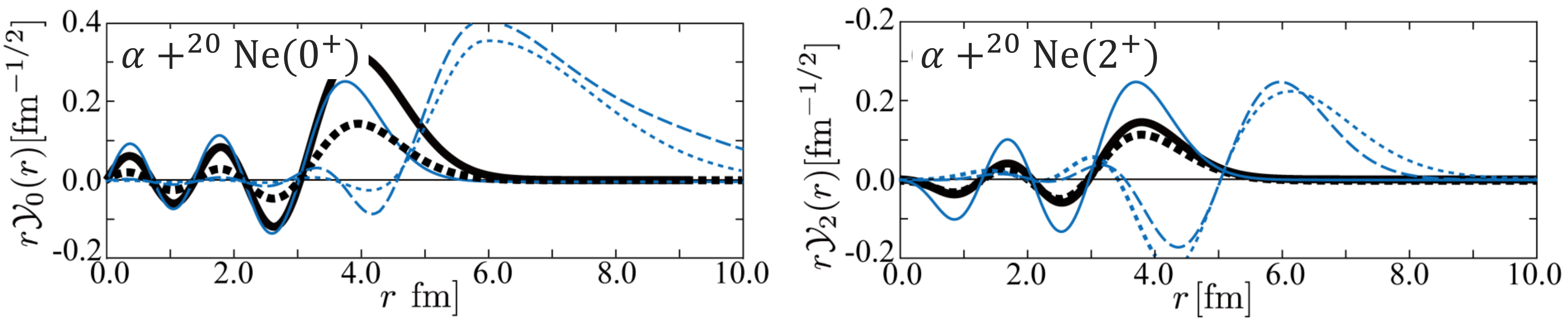
Coulomb penetration prob.

$$P_\ell(a) = \frac{ka}{F_\ell^2(ka) + G_\ell^2(ka)}$$

reduced width amp.
(overlap with decay channel)

$$ry_\ell(r) = r\sqrt{\frac{A!}{A_1!A_2!}}\langle Y_\ell(\hat{r})\phi_{A_1}\phi_{A_2}|\Psi\rangle$$

The Laplace expansion method enables the calculation of the decay to any channel. Y. Chiba and M.K., PTEP2017, 053D01 (2017)



Resonances in the $^{12}C+^{12}C$ reaction channel

Resonance parameters obtained by AMD calculation

TABLE I. The calculated energies in the unit of MeV, isoscalar transition matrix elements in the Weisskopf unit and RWAs of the $J^\pi = 0^+$ and 2^+ resonances. The dimensionless RWAs are multiplied by a factor of hundred. $l = 0$, 2 and 4 indicate the orbital angular momenta.

			$\theta^2_C \times 10^2$	$\theta^2_{\alpha_0} \times 10^2$		$\theta^2_{\alpha_1} \times 10^2$			$\theta^2_{p0} \times 10^2$			$\theta^2_{p_1} \times 10^2$		
J^π	E_R	$M(\mathrm{IS}\lambda)$	$l = J$	$l=0$	2	$l=0$	2	4	$l=0$	2	4	$l=0$	2	4
2^+	0.93	1.56	1.4	—	3.5	0.061	1.7	6.7	0.47	0.081	0.030	0.20	0.15	0.083
0^+	0.94	0.59	7.3	0.20	—	—	7.1	—	—	0.10	—	—	0.69	—
2^+	1.50	1.04	2.9	—	1.1	4.0	0.90	0.51	0.16	0.22	0.0012	0.012	0.098	0.005
2^+	2.18	0.51	3.4	—	1.0	1.0	0.19	3.4	3.3	0.12	0.0099	0.70	0.11	0.23
0^+	3.02	1.05	11	0.26	—	—	0.57	—	—	0.99	—	—	0.43	—
2^+	3.56	0.23	1.2	—	0.038	0.056	0.006	0.040	0.66	0.86	0.00089	0.029	0.79	0.041
2^+	3.73	0.41	8.3	—	0.10	0.066	0.10	0.88	0.24	0.72	0.028	0.043	0.67	0.089

Resonance energies

Decay widths and branches

$$\theta^2_{A_1+A_2}(a) = \frac{a}{3}\left|a y_l^{A_1+A_2}(a)\right|^2$$

○ The calculation reproduces some of known resonances (3.7 and 3.0 MeV)

○ A couple of resonances with Ecm< 2.2 MeV are obtained

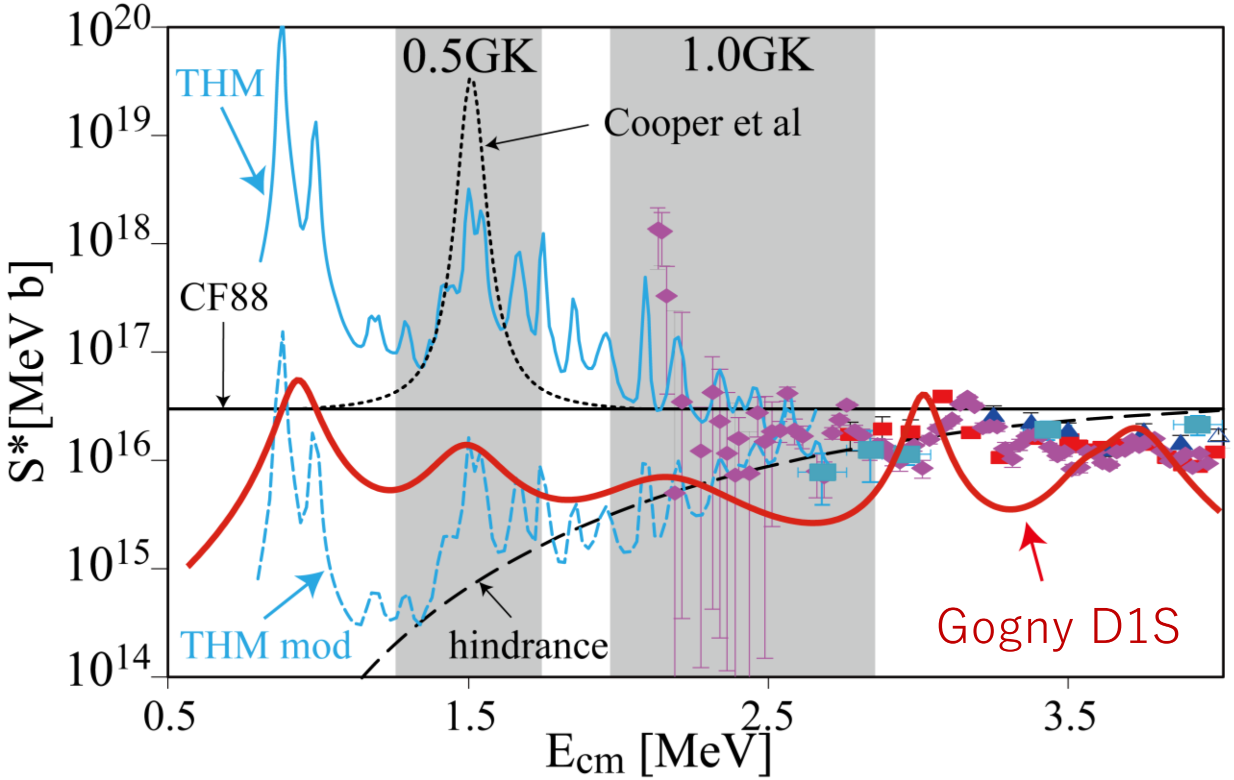
S-factor for $^{12}C+^{12}C$ fusion reaction

From resonance parameters, we estimated the reaction rate (Breit-Wigner formula, no interference of resonances)

BW cross section: $$\sigma(E) = \frac{\pi\hbar^2(2J+1)}{2\mu E}\frac{\Gamma_I \Gamma_F}{(E-E_R)^2+\Gamma^2/4},$$

S-factor: $$S^*(E) = E\sigma(E)\exp(2\pi\eta + 0.46\ \mathrm{MeV}^{-1}E),$$

[THM] A. Tumino et al: Nature 557, (2018) 687.
[THM mod] A. M. Mukhamedzhanov, D. Y. Pang and A. S. Kadyrov: Phys. Rev. C 99, (2019), 064618



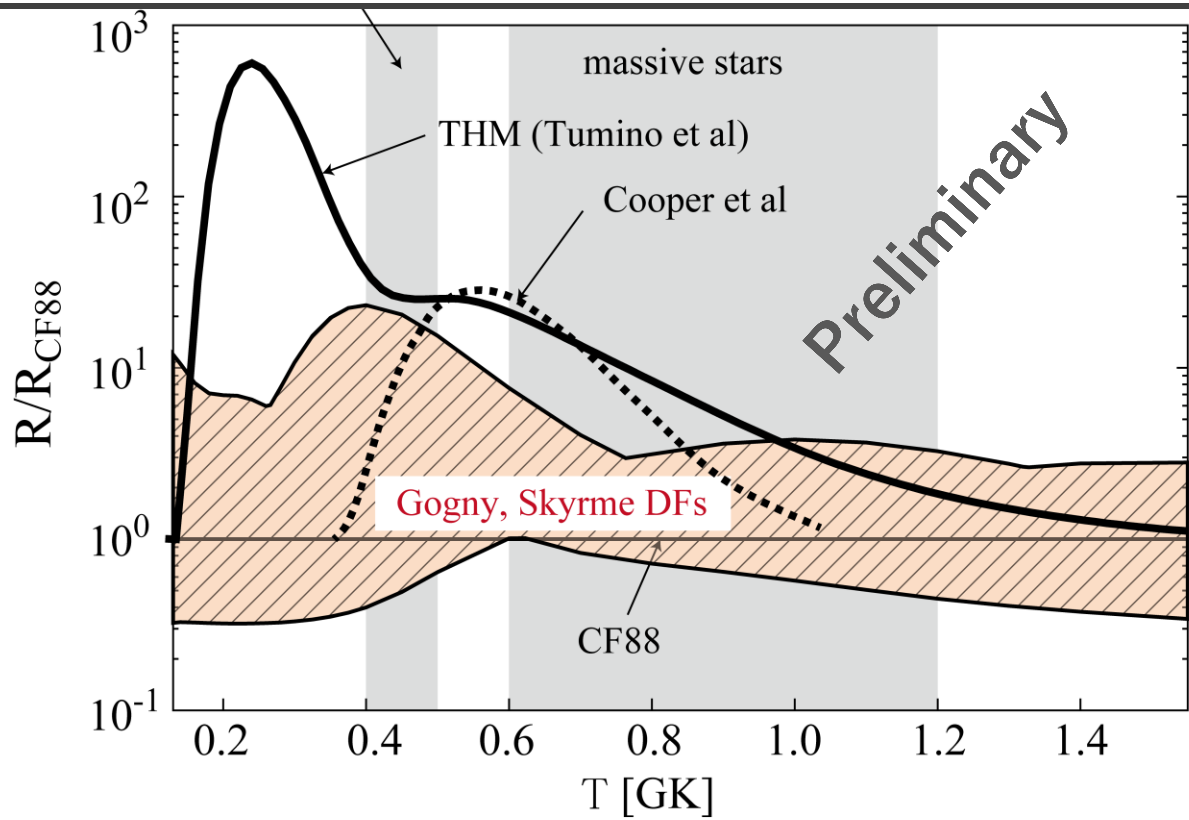
Reaction rate from microscopic nuclear model

Problem

How can we confirm the low-energy resonances (Ex=14-17 MeV) observed by THM exp. and predicted by nuclear model calc?

Answer

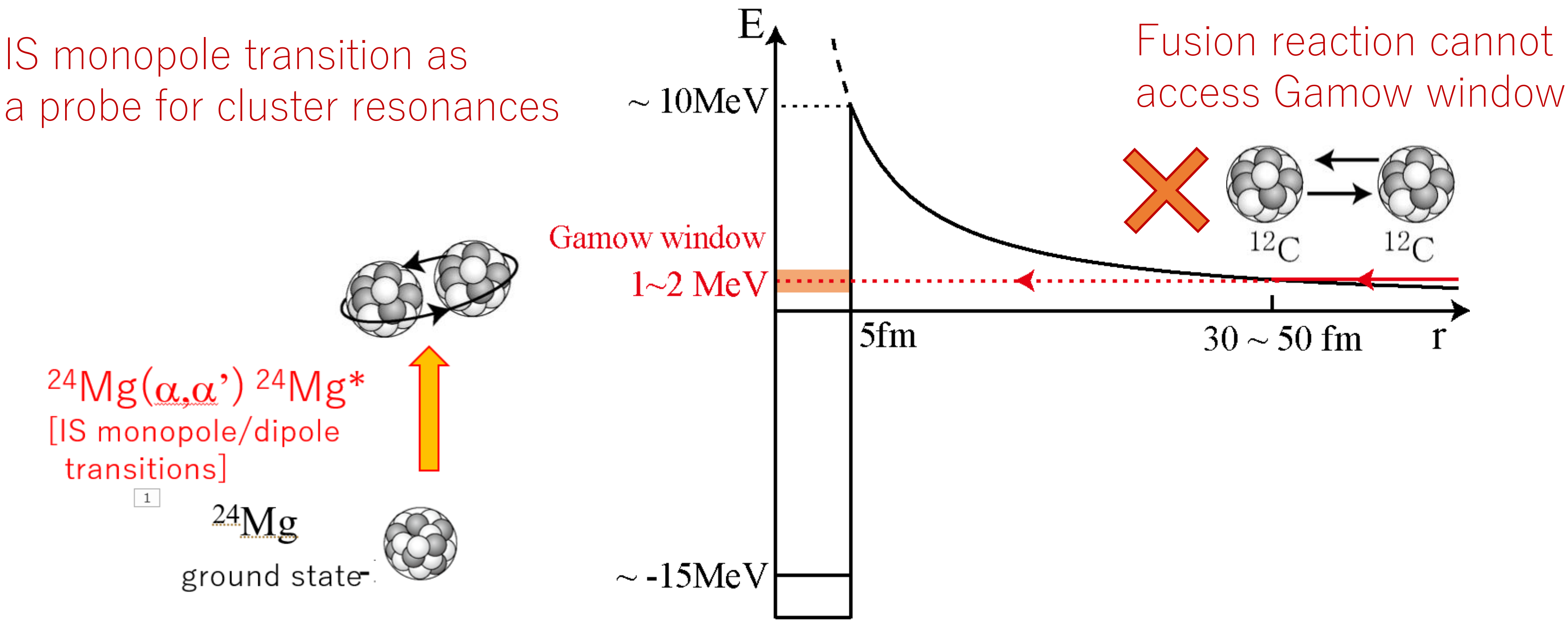
IS Monopole (Quadrupole) transition from g.s. to resonances



$^{12}C+^{12}C$ fusion reaction and molecular resonances

Question: We want to know where is cluster resonances

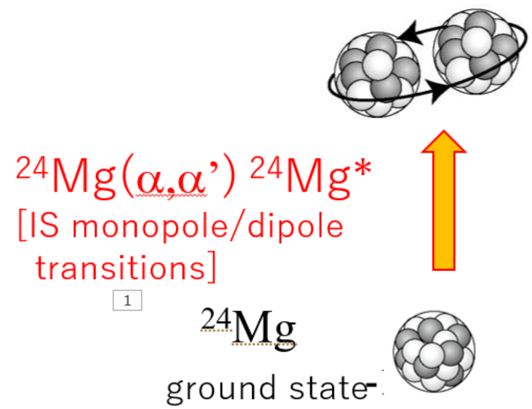
Answer: Use IS monopole transition as a probe for cluster resonances



This reaction bypasses the Coulomb barrier,
hence measurable.

The key issue

IS monopole transitions from g.s.
to cluster resonances are enhanced.



Why?

The mechanism was explained by Yamada et al. (PTP120, 1139)

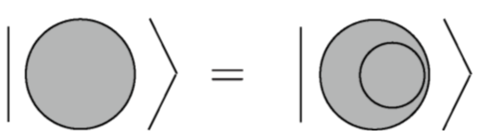
○ Bayman-Bohr theorem [Nucl. Phys. 9, 596 (1958/1959)]
An SU(3) shell model wave function is mathematically equivalent to a cluster wave function

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \mathcal{A}\{\underbrace{(0s)^4(0p)^{12}}_{^{16}\mathrm{O}\ \text{core}}(0d1s)^4\} = n\mathcal{A}\{\underbrace{R_{80}(r)Y_{00}(\hat{r})}_{\text{overlapping } \alpha \text{ and } ^{16}\mathrm{O} \text{ clusters}}\phi_\alpha\phi_{^{16}\mathrm{O}}\}\phi_{cm}(\boldsymbol{r}_{cm})$$

^{16}O core

overlapping α and ^{16}O clusters

shell model wave function
= completely overlapping clusters



Monopole transition as a probe for cluster resonances

The ordinary idea : Monopole operator excites the ground state to GMR

○ Shell model wave function

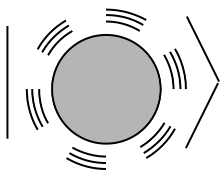
$$\Phi_{g.s.}(^{20}\mathrm{Ne}) = \mathcal{A}\{\,(0s)^4(0p)^{12}(0d1s)^4\,\}$$

○ Monopole operator

$$\mathcal{M}^{IS0}_{\mu} = \sum_{i=1}^{A}(\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2$$

○ Monopole operator yields 1p1h configurations

$$\mathcal{M}\Phi_{g.s.}(^{20}\mathrm{Ne}) = \mathcal{A}\{\,(0s)^4(0p)^{12}(1s0d)^3(2s)\,\} + \mathcal{A}\{\,(0s)^4(0p)^{12}(1s0d)^3(1d)\,\} + \cdots =$$



This is true, but let us think different

Monopole transition as a probe for cluster resonances

Think different: Monopole operator gives rise to clusters

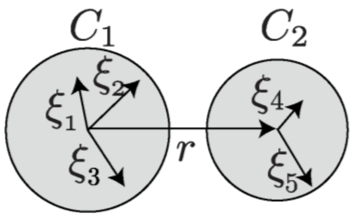
○ Bayman-Bohr theorem

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \mathcal{A}\{\,(0s)^4(0p)^{12}(0d1s)^4\,\} = n\mathcal{A}\{R_{80}(r)Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}\phi_{cm}(\boldsymbol{r}_{cm})$$

○ Cluster coordinate representation
of the monopole operator

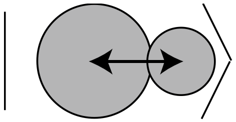
C_1, C_2 : masses of clusters
r : inter-cluster coordinate
ξ_i : internal coordinates of clusters

$$\mathcal{M}_\mu^{IS0} = \sum_{i=1}^{A}(\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2 = \sum_{i\in C_1}\xi_i^2 + \sum_{i\in C_2}\xi_i^2 + \frac{C_1C_2}{C_1+C_2}r^2$$



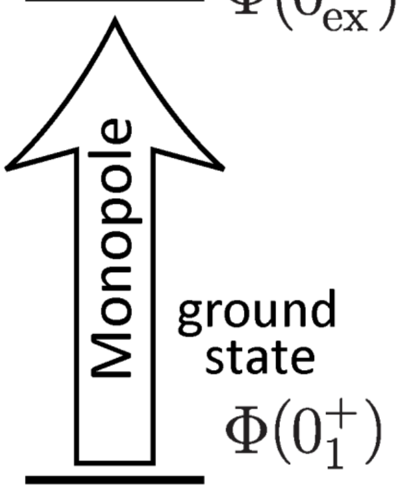
○ Monopole operator excites the inter-cluster motion

$$\Rightarrow\ \mathcal{M}_\mu^{IS0}\,\Phi_{g.s.}(^{20}\mathrm{Ne}) \simeq \sum_{N=N_0+2}^{\infty} f_N n_N \mathcal{A}\{R_{\boldsymbol{N}0}(r)Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

Excited 0^+ state 

Monopole transition as a probe for cluster resonances

Assuming that the ground state is a simple shell model state,
the transition matrix can be estimated analytically



Cluster estimate for ^{20}Ne (analytical)

$$M^{IS0} = \langle \Phi(0^+_{\rm ex}) | \mathcal{M}^{IS0} | \Phi(0^+_1) \rangle$$
$$= f_{N_0+2} \sqrt{\frac{\mu_{N_0}}{\mu_{N_0+2}}} \langle R_{N_0 0} | r^2 | R_{N_0+2 0} \rangle$$
$$\simeq 7.67\ f_{N_0+2} = 5.48\ \mathrm{fm}^2$$

Single particle estimate

$$M^{IS0}_{\rm WU} = \frac{3}{5}(1.2A^{1/3})^2 \simeq 6.37\ \mathrm{fm}^2$$

We note that similar argument also applies
to the isoscalar dipole and quadrupole transitions

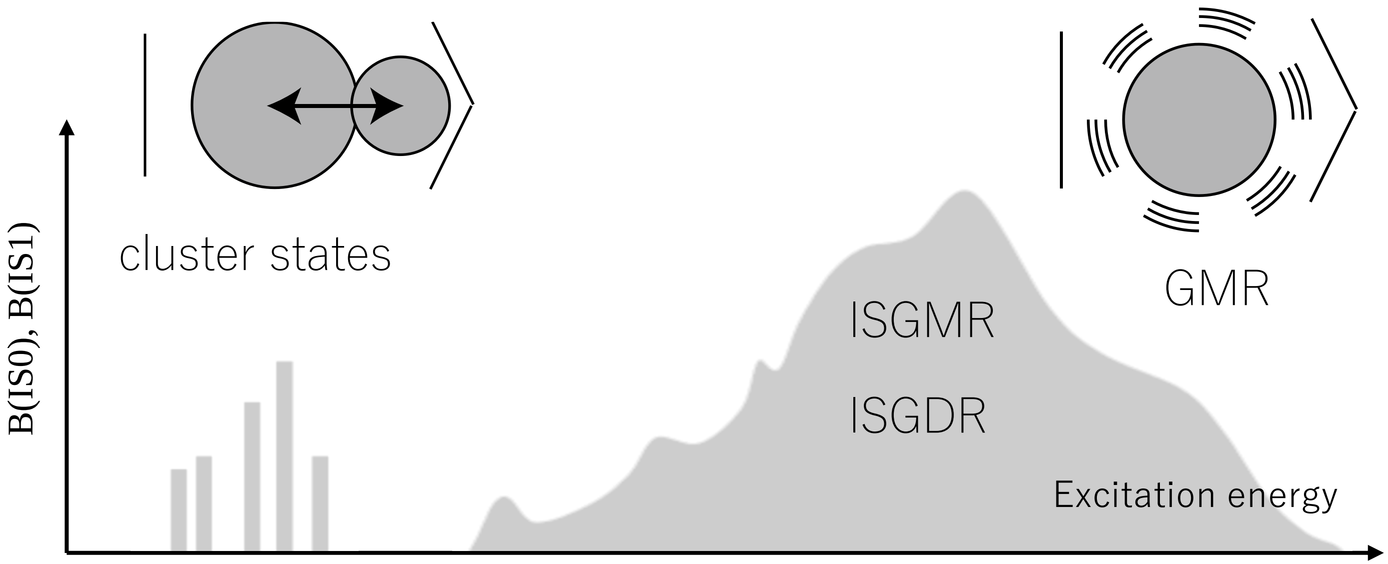
Y. Chiba, M. K., Y. Taniguchi, Phys. Rev. C 93, 034319 (2016)

Monopole transition as a probe for cluster resonances

In summary,
both GMR and cluster resonances can have enhanced monopole strength

◎ Collective excitation: GMR, ISGDR $E_x > 15$ MeV

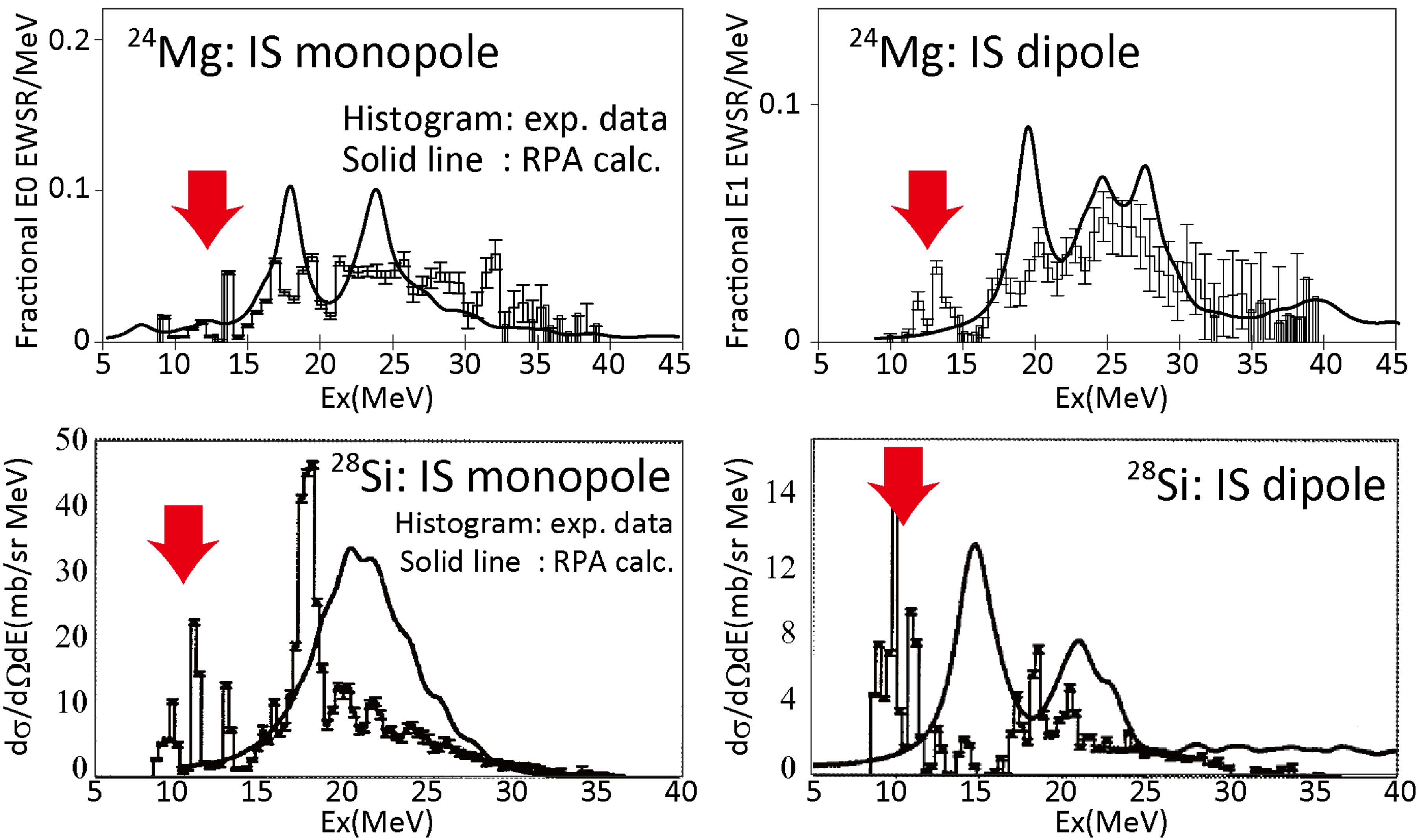
◎ Cluster excitation: $E_x < 15$ MeV (clusters may appear close to the decay threshold)



Monopole transition as a probe for cluster resonances

Experimental data

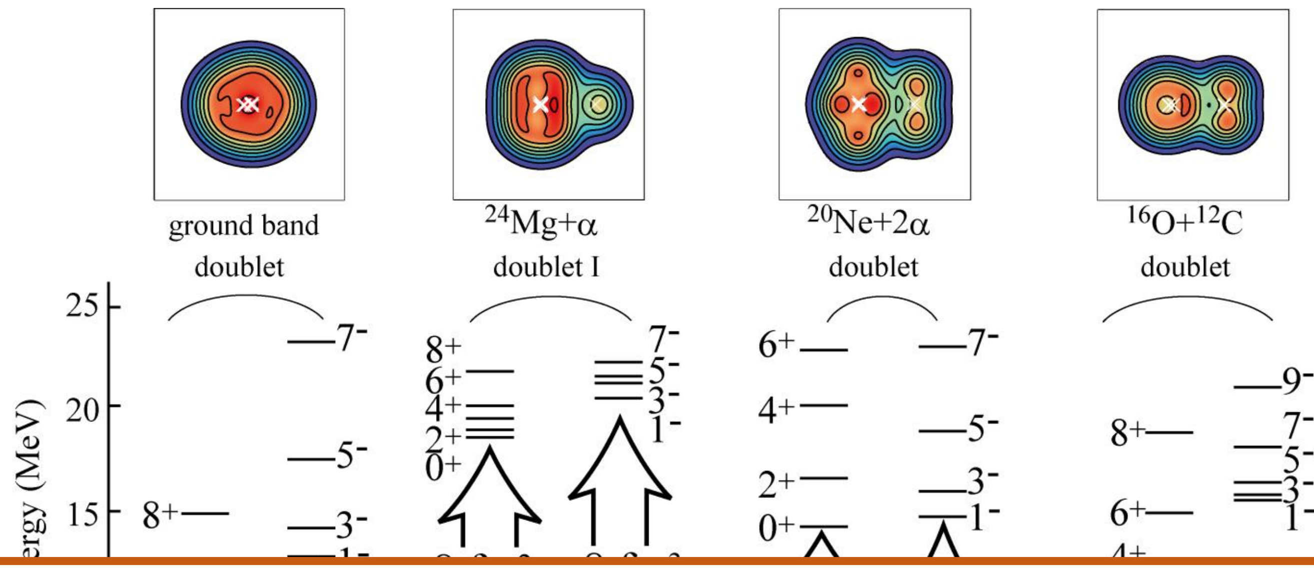
X. Chen et al.,PRC80, 014312 (2009). D. H. Young-Blood et al.,PRC65, 034302 (2002).

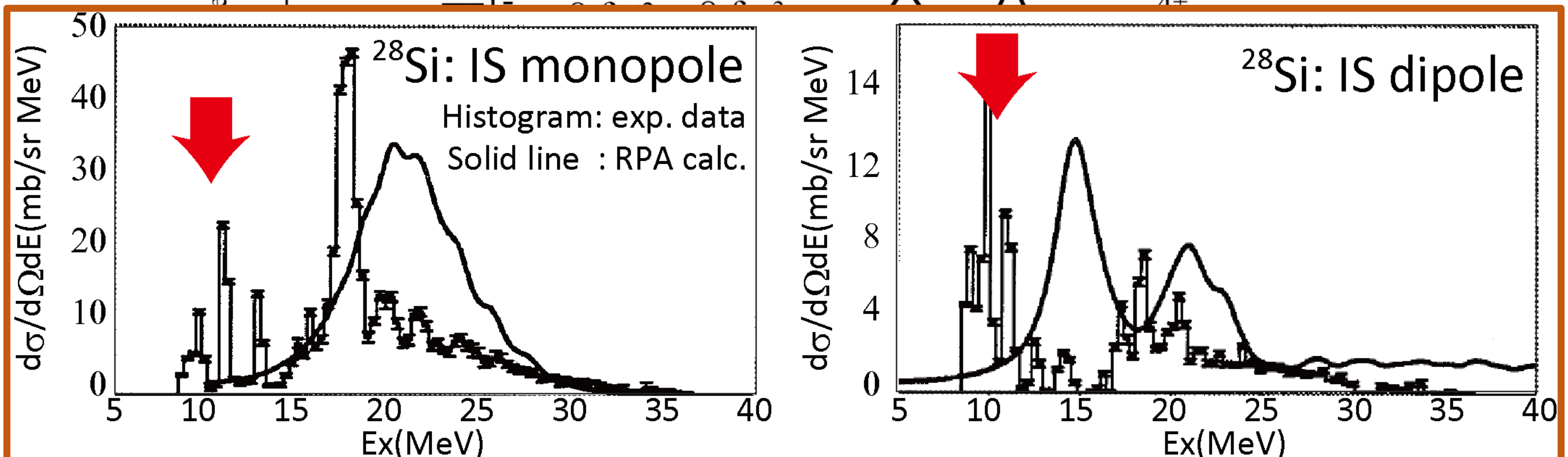


Monopole transition: The case of ^{28}Si

^{28}Si (α+^{24}Mg and ^{8}Be+^{20}Ne cluster resonances)

Y. Taniguchi, Y. Kanada-En'yo and M.K. PRC80, 044316 (2009).
Y. Chiba, M.K., and Y. Taniguchi, PRC (2019)





Monopole transition: The case of ^{24}Mg

Resonances in ^{12}C+^{12}C reaction channel obtained by AMD calculation

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0^+	0.94	0.59	7.3	0.20	—	—	7.1	—	—	0.10	—	—	0.69	—
2^+	1.50	1.04	2.9	—	1.1	4.0	0.90	0.51	0.16	0.22	0.0012	0.012	0.098	0.005
2^+	2.18	0.51	3.4	—	1.0	1.0	0.19	3.4	3.3	0.12	0.0099	0.70	0.11	0.23
0^+	3.02	1.05	11	0.26	—	—	0.57	—	—	0.99	—	—	0.43	—
2^+	3.56	0.23	1.2	—	0.038	0.056	0.006	0.040	0.66	0.86	0.00089	0.029	0.79	0.041
2^+	3.73	0.41	8.3	—	0.10	0.066	0.10	0.88	0.24	0.72	0.028	0.043	0.67	0.089

All resonances have IS monopole (quadrupole) transition strength as large as Weisskopf Unit.

These sharp resonances might be observed via ^{24}Mg(α,α') reaction *just below or embedded in the GMR (Ex∼15.0 MeV)*

Monopole transition: The case of ^{24}Mg

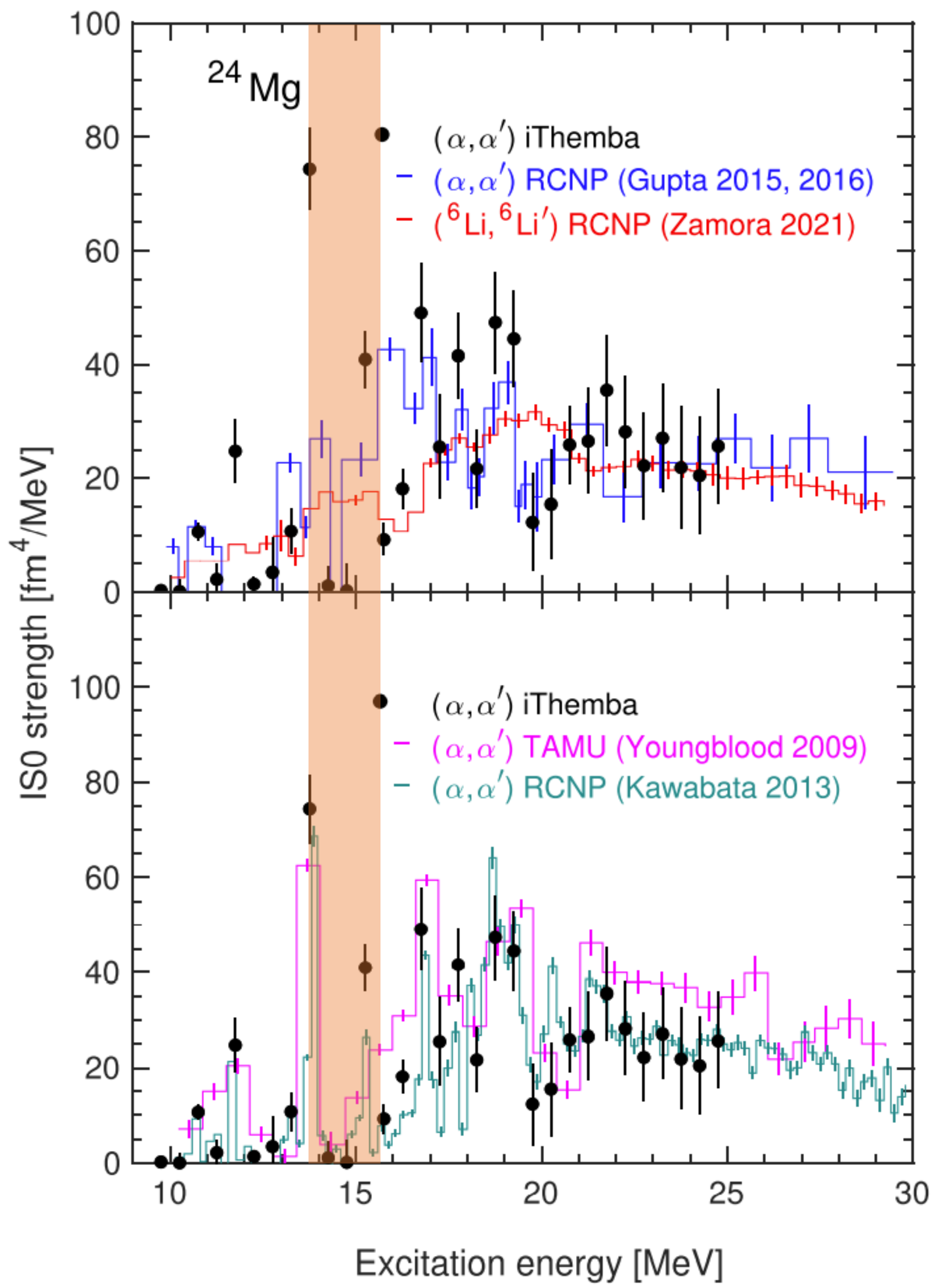
Data from RCNP & iThemba

Y. K. Gupta et al., PRC93, 044324(2016)
A. Bahini et al., PRC105, 024311 (2022)

A couple of sharp resonances look exist just below or in the GMR

We need more information
- Width of these resonances
- Their decay branch proton/alpha

We also need high-resolution Quadrupole data



Summary

I have focused on the other aspects of the monopole transition

Monopole transition as a probe for cluster resonances which determines the $^{12}C+^{12}C$ (and $^{12}C+^{16}O$, $^{16}O+^{16}O$) fusion reaction rate

High resolution $^{24}Mg(\alpha,\alpha')$ exp. can bypass the Coulomb barrier and directly access the cluster resonances