Covariant studies of monopole modes

F. Mercier, A. Bjelcic, T.Niksic, J.-P. Ebran, E. Khan, D. Vretenar, PRC 103, 024303 (2021)

F. Mercier, J.-P. Ebran, E. Khan, PRC 105, 034343 (2022)

Elias Khan

EDF method

- EDF: many-body system mapped into the **one-body density** and its powers, gradient

$$\rho_0(\mathbf{r}) = \rho_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau)$$

$$\rho_1(\mathbf{r}) = \rho_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau)\,\tau$$

$$\mathbf{s}_0(\mathbf{r}) = \mathbf{s}_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma}$$

$$\mathbf{s}_1(\mathbf{r}) = \mathbf{s}_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma}\,\tau$$

$$\mathbf{j}_T(\mathbf{r}) = \tfrac{i}{2}(\nabla' - \nabla)\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$

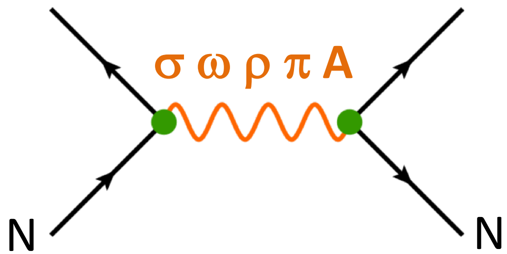
$$\mathcal{J}_T(\mathbf{r}) = \tfrac{i}{2}(\nabla' - \nabla)\otimes \mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$

$$\tau_T(\mathbf{r}) = \nabla\cdot\nabla'\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$

$$\mathbf{T}_T(\mathbf{r}) = \nabla\cdot\nabla'\,\mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}$$

- **Most general** antisymmetrised product of nucleonic wavefunctions
- **Not any a priori assumption** on the nucleons' wave function
- **Correlations** beyond the mean-field effectively included by the EDF
- Investigate nuclear structure on the **whole nuclear chart**
- **Relativistic**: the depth of the central potential is **consistently predicted**

Relativistic EDF in nuclei



$$\mathcal{L}_{int} = -g_\sigma(\rho_v)\bar{\psi}\sigma\psi - g_\omega(\rho_v)\bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho(\rho_v)\bar{\psi}\gamma_\mu\vec{\rho}^\mu.\vec{\tau}\psi - \frac{f_\pi(\rho_v)}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\vec{\pi}.\vec{\tau}\psi - e\bar{\psi}\gamma_\mu A^\mu\psi$$

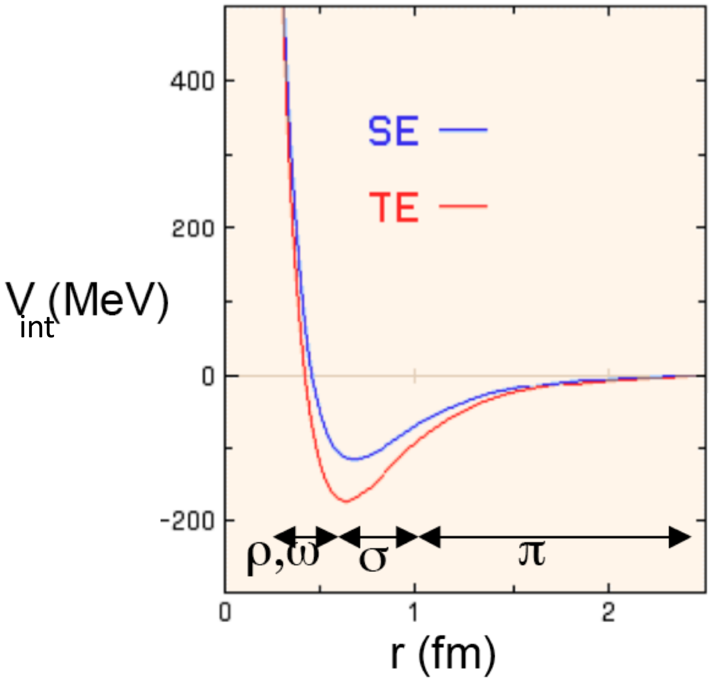
EDF [ρ ; σ,ω,ρ,π,A]

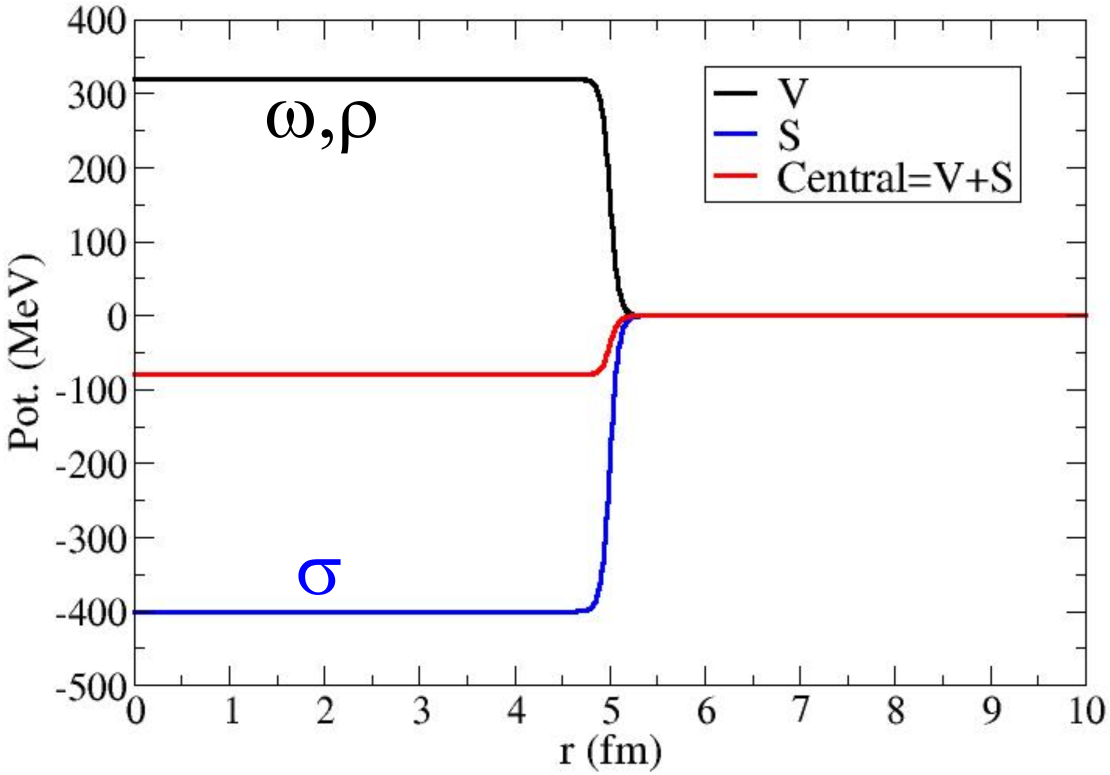
$$\left\{ \boldsymbol{p}\frac{1}{2\tilde{M}(r)}\boldsymbol{p} + W(r) + V_{ls}(r)\boldsymbol{l}.\boldsymbol{s} \right\} \varphi_i = \varepsilon_i^{NR}\varphi_i$$

$$W(r) = [V+S](r)$$

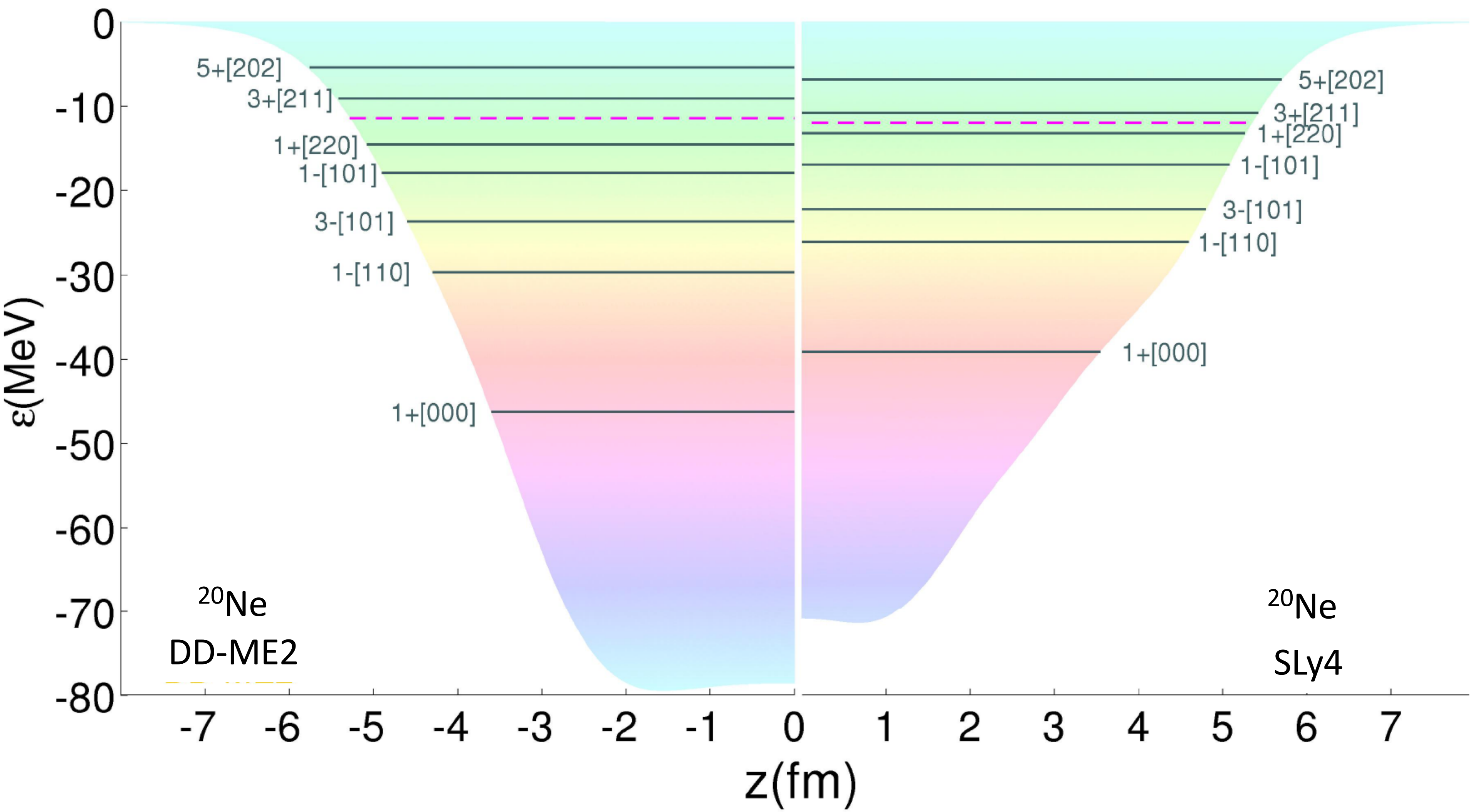
$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V-S)$$

V and S potentials

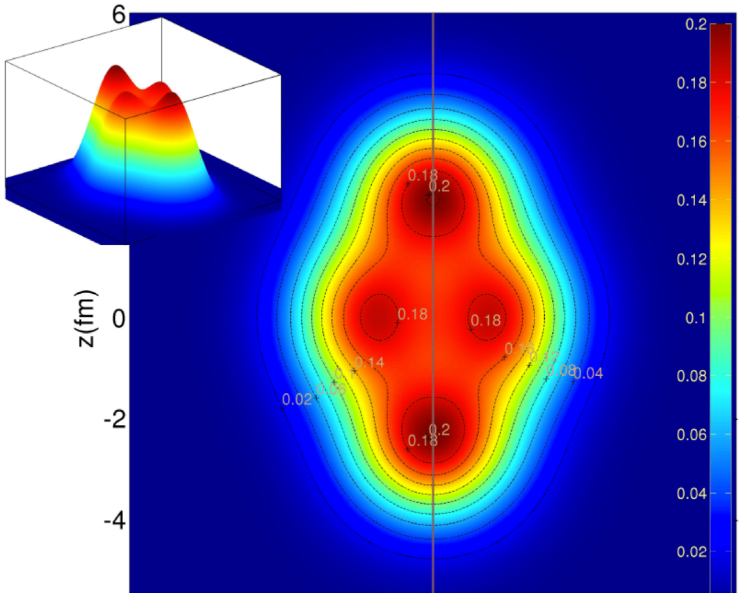




A way to vary the depth of the potential

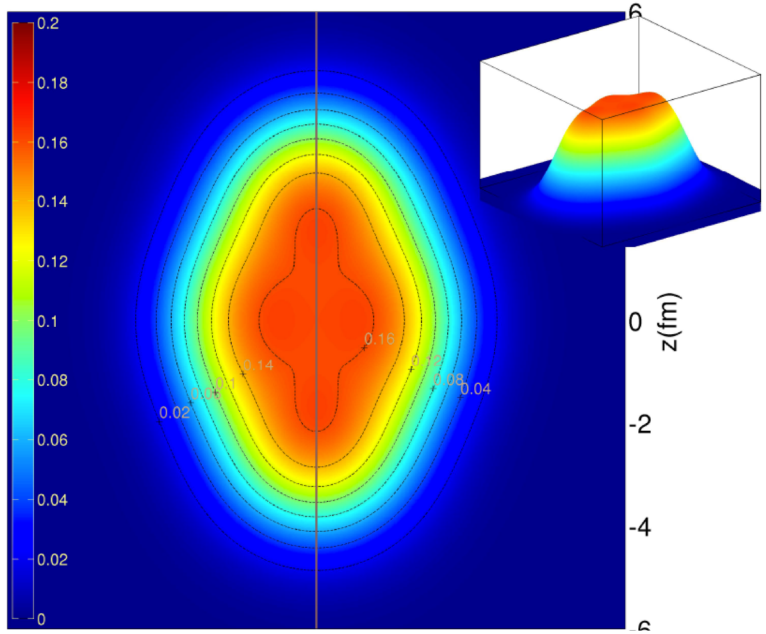


Clusterisation

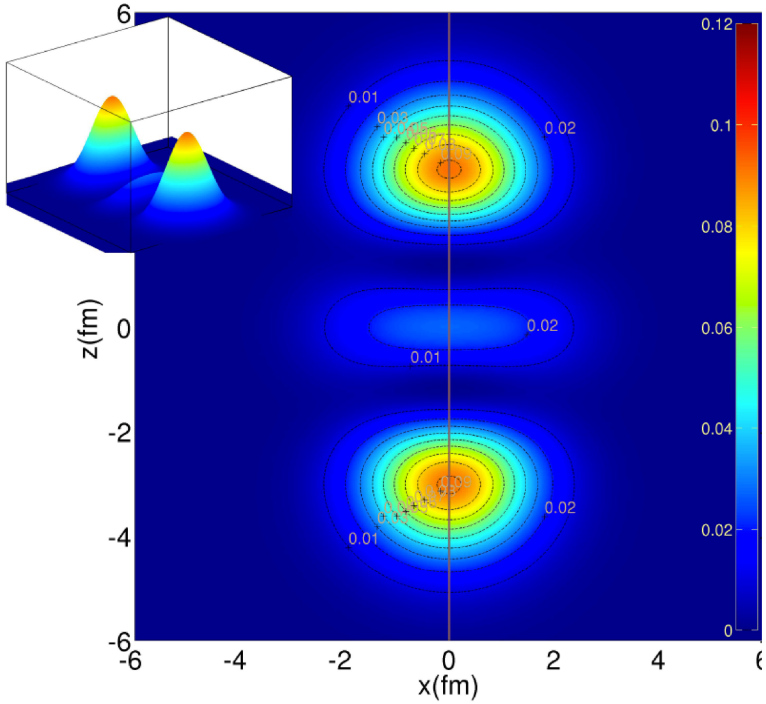


^{20}Ne

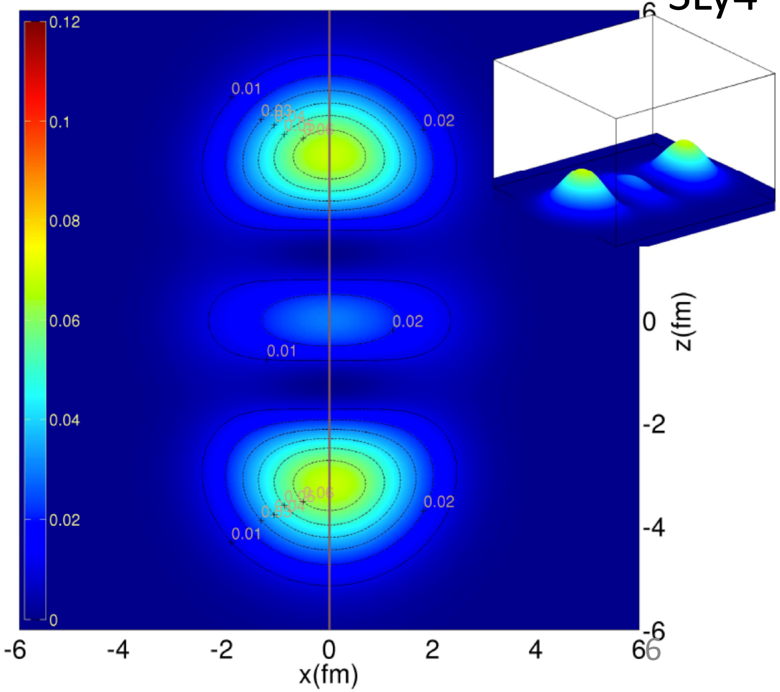
Total density



DDME2

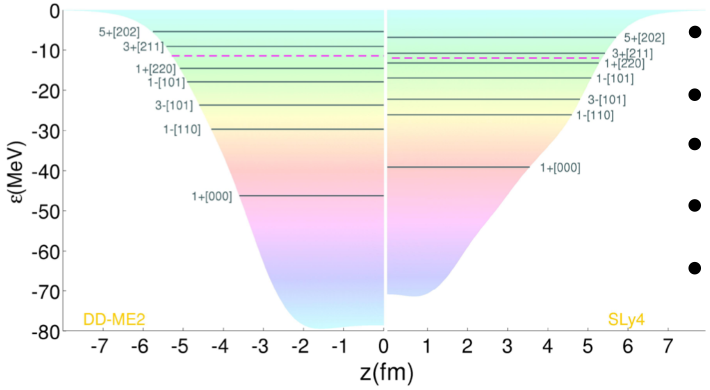


Last occupied state

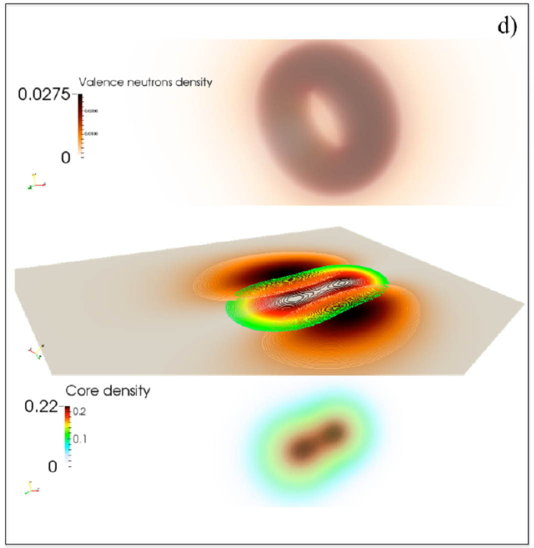


SLy4

Origins of nuclear clustering



- Depth of the confining potential
- Heavy vs. Light nuclei
- Deformation / excitation energy
- Density
- Neutron excess

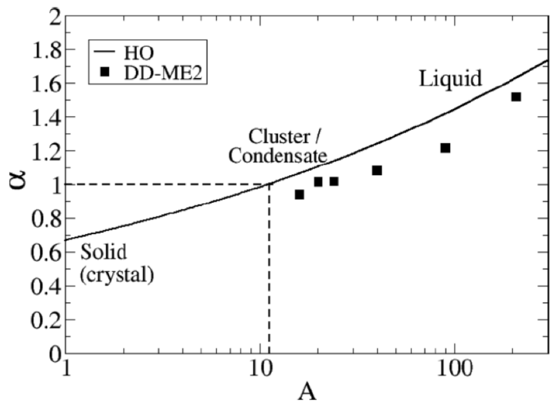


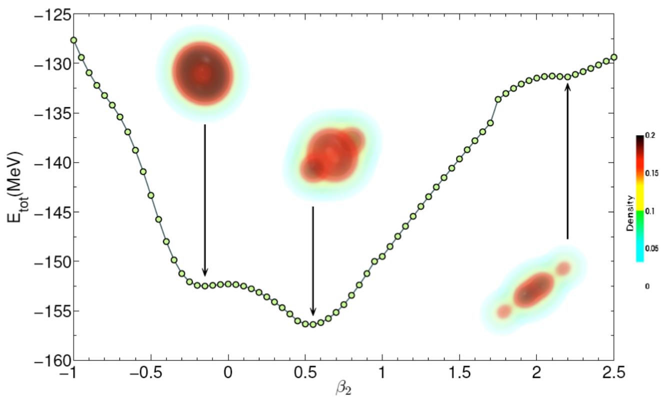
J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341

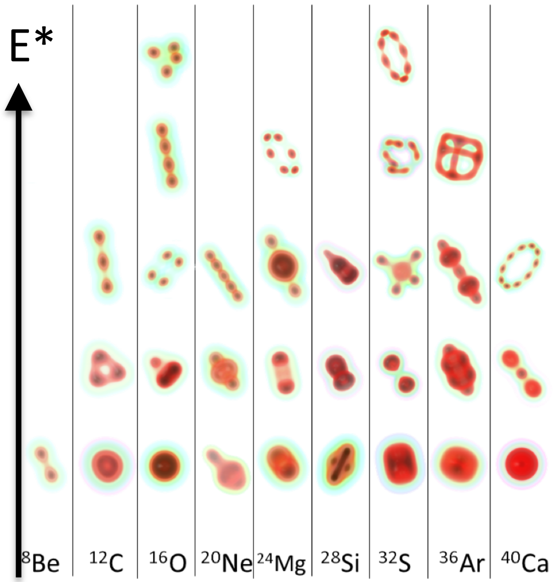
PRC87(2013)044307

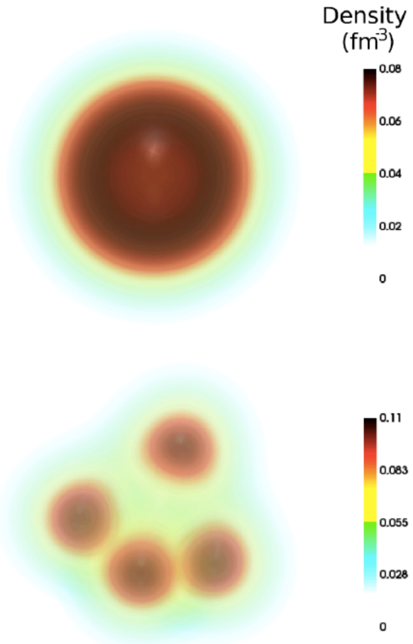
PRC90(2014)054329

PRC89(2014)031303(R)

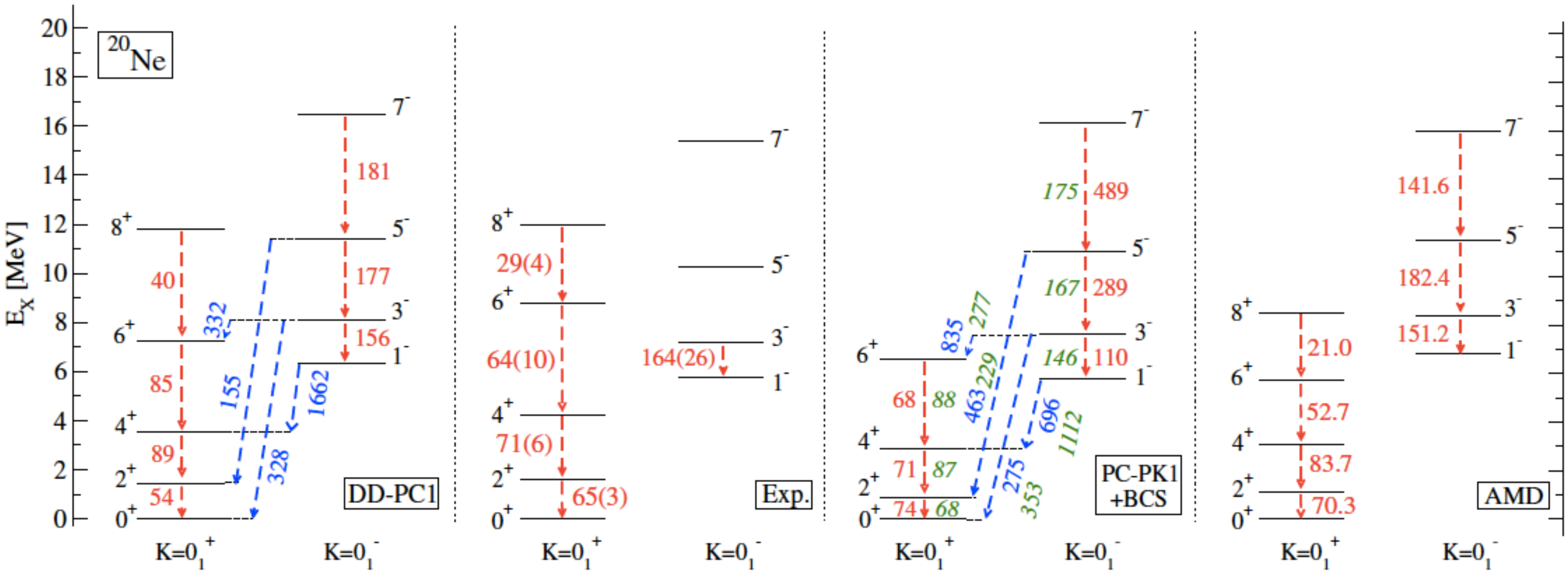






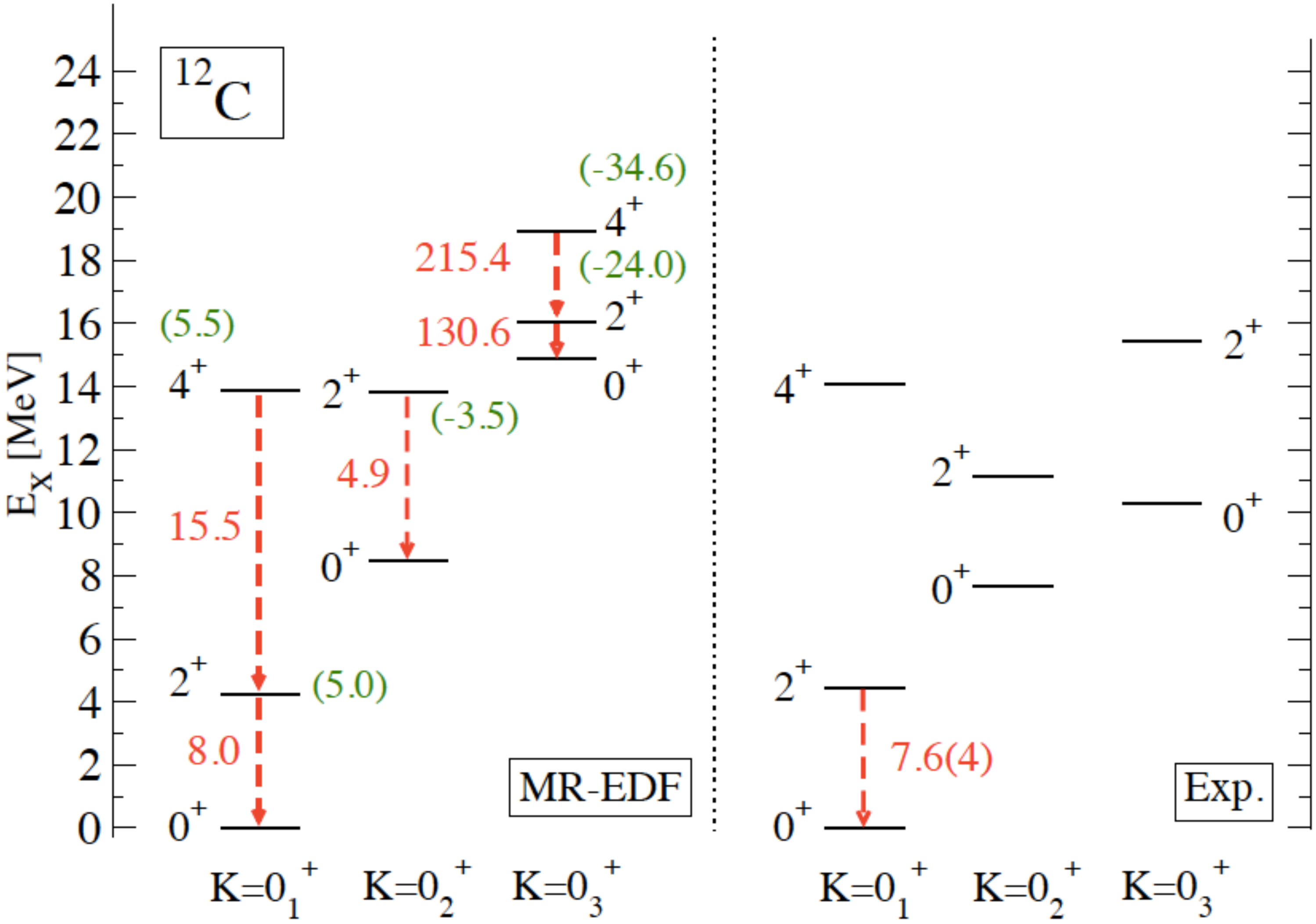


Comparison with the data



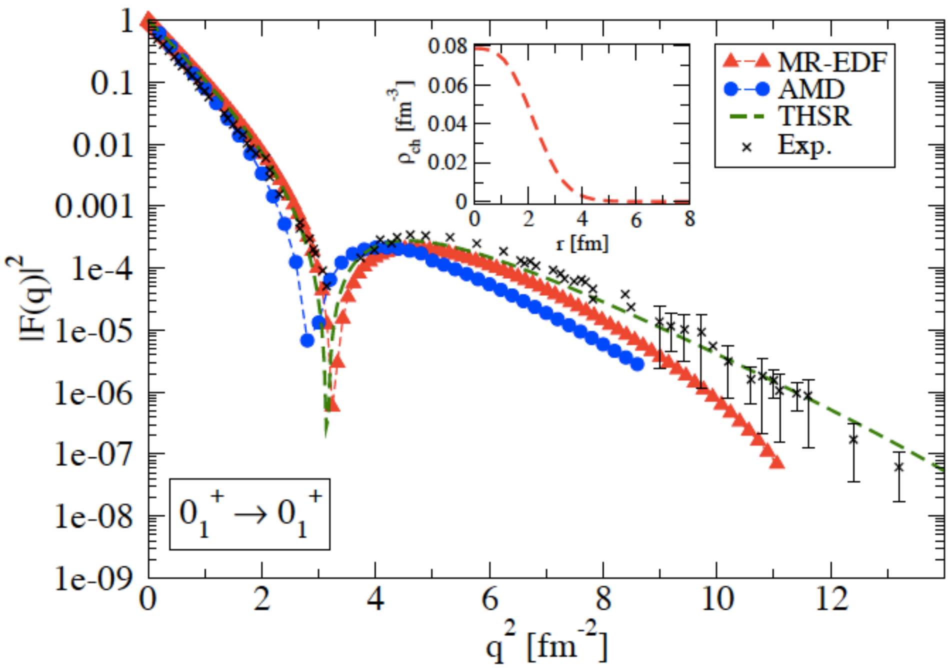
P. Marevic , J.-P. Ebran, E. Khan, T. Niksic , and D. Vretenar, PRC 97, 024334 (2018)

Comparison with exp. on ^{12}C

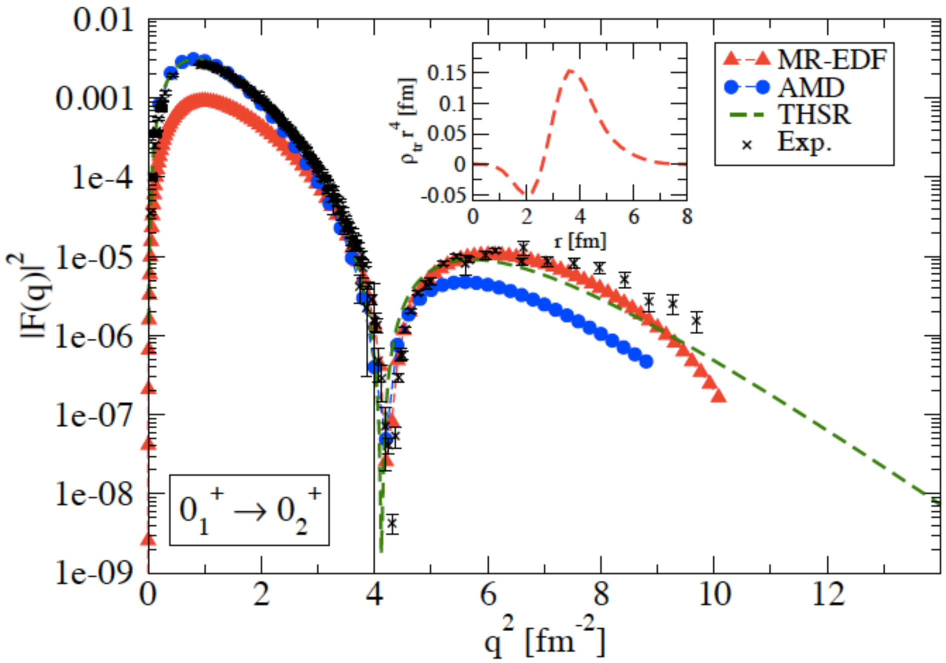


P. Marevic , J.-P. Ebran, E. Khan, T. Niksic , and D. Vretenar, PRC 99, 034317 (2019)

Comparison with exp. on ^{12}C



g.s.

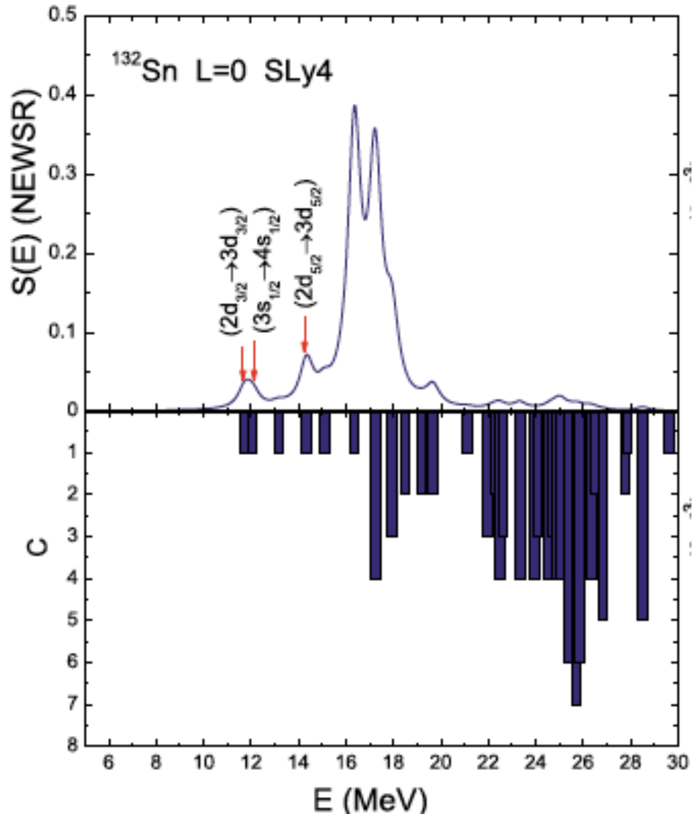


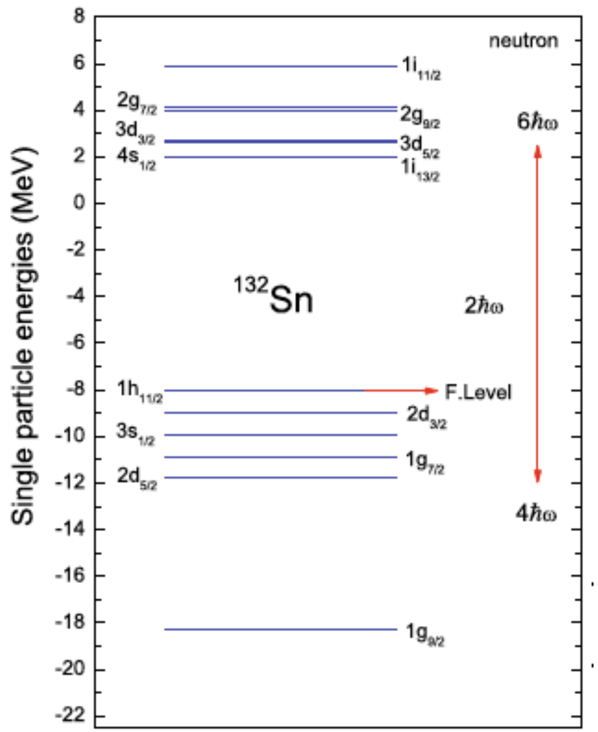
Hoyle

Questions

- Is there a soft monopole strength ?
- Are there specific cluster modes of excitation ?
- What is the interplay between cluster, neutron excess and deformation ?
- GMR vs. soft modes vs. cluster modes vs pairing modes ?

Soft monopole is non-collective

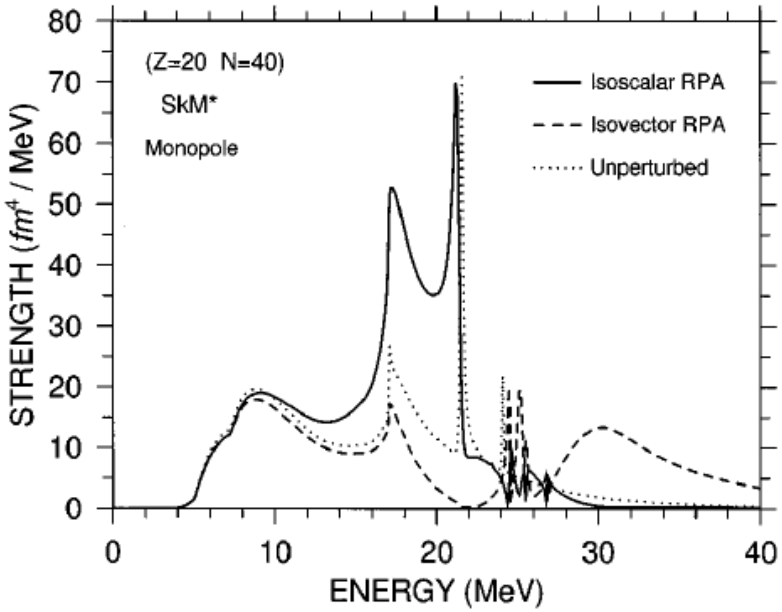




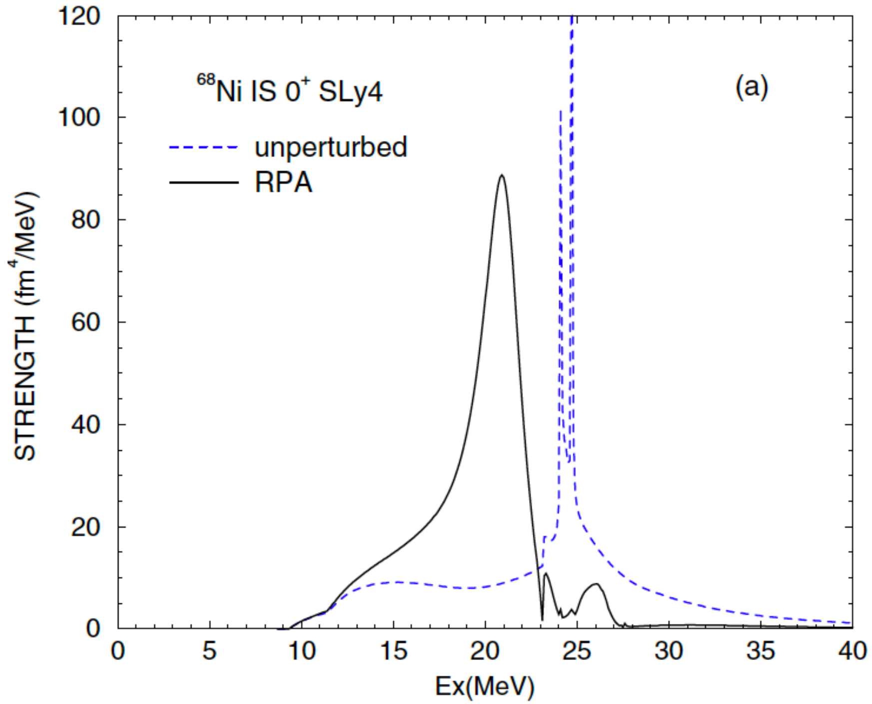
Probe for the single-particle spectrum ?

E. Yuksel, E. Khan and K. Bozkurt, EPJA 49, 124 (2013)

Continuum effects



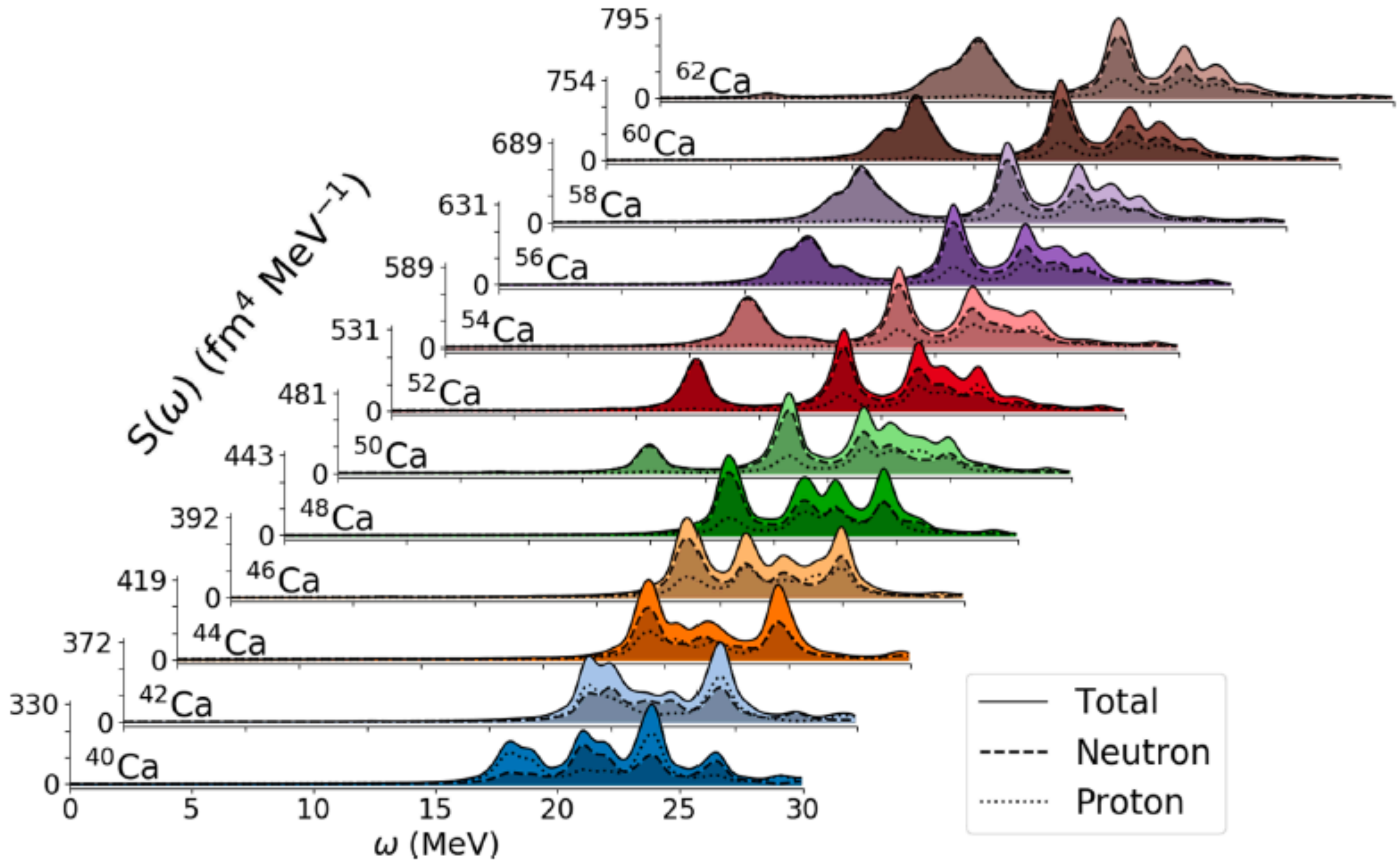
I. Hamammoto, H. Sagawa and X.Z. Zhang, PRC 56, 3121 (1997)



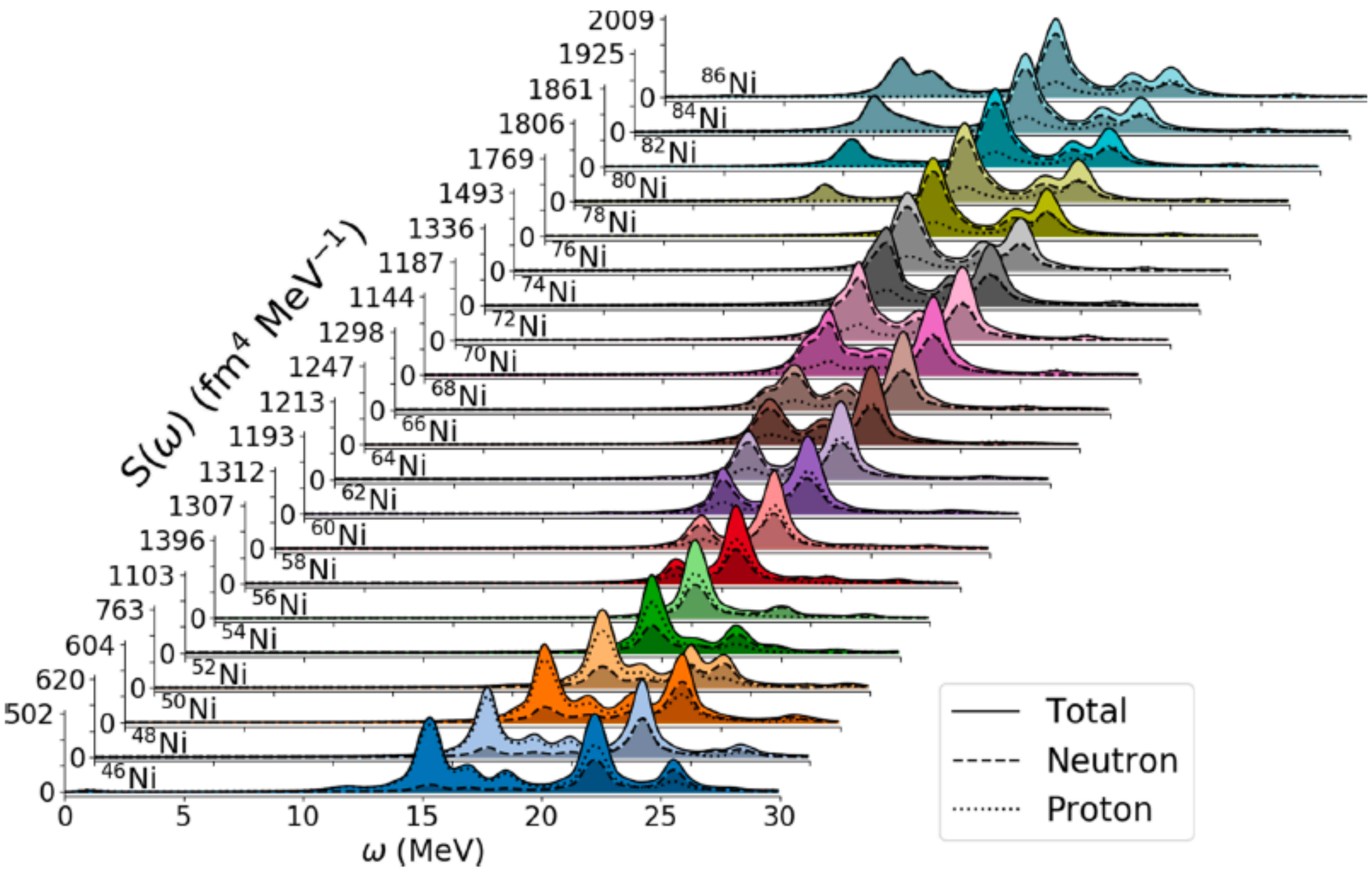
I. Hamammoto and H. Sagawa, PRC 90, 031302(R)(2014)

Role of the escape width ?
Role of the spreading width ? (see E. Litvinova+D. Gambacurta's talk)

Increase of the soft mode with n excess



Increase of the soft mode with n excess



Increase of the monopole strength around 15 MeV in ^{68}Ni

Cluster modes ?

$$\mathcal{L}_m = \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$E_{\mathrm{RHB}}[\hat{\rho},\hat{\kappa},\phi] = E_{\mathrm{RMF}}[\hat{\rho},\phi] + E_{\mathrm{pair}}[\hat{\kappa}].$$

^{20}Ne

$$E_{\mathrm{RMF}}[\hat{\rho},\phi] = \mathrm{Tr}[(-i\boldsymbol{\alpha}\nabla + \beta m)\hat{\rho}] + \sum_m \mathrm{Tr}[(\beta\Gamma_m\phi_m)\hat{\rho}] \pm \frac{1}{2}\sum_m \int d^3r[(\partial_\mu\phi_m)^2 + m_m^2].$$

$$\mathcal{L}_{\mathrm{int}} = -g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma^\mu\omega_\mu\psi - g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\vec{\rho}_\mu\psi - e\bar{\psi}\tfrac{1}{2}(1-\tau_3)\gamma^\mu A_\mu\psi,$$

$$V^{ph}_{kl'k'l} = \langle kl'|\hat{V}^{ph}|k'l\rangle = \frac{\delta^2 E_{\mathrm{RHB}}}{\delta\rho_{k'k}\delta\rho_{ll'}}$$

D. Peña Arteaga
(RQRPAz)

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix} = \Omega^{(\nu)}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix}$$

$$\langle 0|\hat{\mathcal{O}}|\nu\rangle = \sum_{kk'}\left(\mathcal{O}_{k'k}X^{(\nu)}_{kk'} + \mathcal{O}^*_{kk'}Y^{(\nu)}_{kk'}\right)(u_k v_{k'} + \tau v_k u_{k'})$$

Heavy calculations with QRPA

The Quasiparticle Finite Amplitude Method (QFAM)

Linear response to the external field F

$$(E_\mu + E_\nu - \omega)X_{\mu\nu}(\omega) + \delta H^{20}_{\mu\nu}(\omega) = -F^{20}_{\mu\nu}$$

$$(E_\mu + E_\nu + \omega)Y_{\mu\nu}(\omega) + \delta H^{02}_{\mu\nu}(\omega) = -F^{02}_{\mu\nu}$$

X(ω),Y(ω)

2 quasiparticle components of the induced Hamiltonian

$$\delta H^{20}(\omega) = U^\dagger \delta h(\omega) V^* + U^\dagger \delta\Delta^{(+)}(\omega) U^* - V^\dagger \delta\Delta^{(-)*}(\omega) V^* - V^\dagger \delta h^T(\omega) U^*$$

$$\delta H^{02}(\omega) = -V^T \delta h(\omega) U - V^T \delta\Delta^{(+)}(\omega) V + U^T \delta\Delta^{(-)*}(\omega) U + U^T \delta h^T(\omega) V$$

Perturbed fields

$$\delta h(\omega) = h[\delta\rho(\omega)],$$

$$\delta\Delta^{(+)}(\omega) = \Delta[\delta\kappa^{(+)}(\omega)],$$

$$\delta\Delta^{(-)}(\omega) = \Delta[\delta\kappa^{(-)}(\omega)],$$

Transition densities

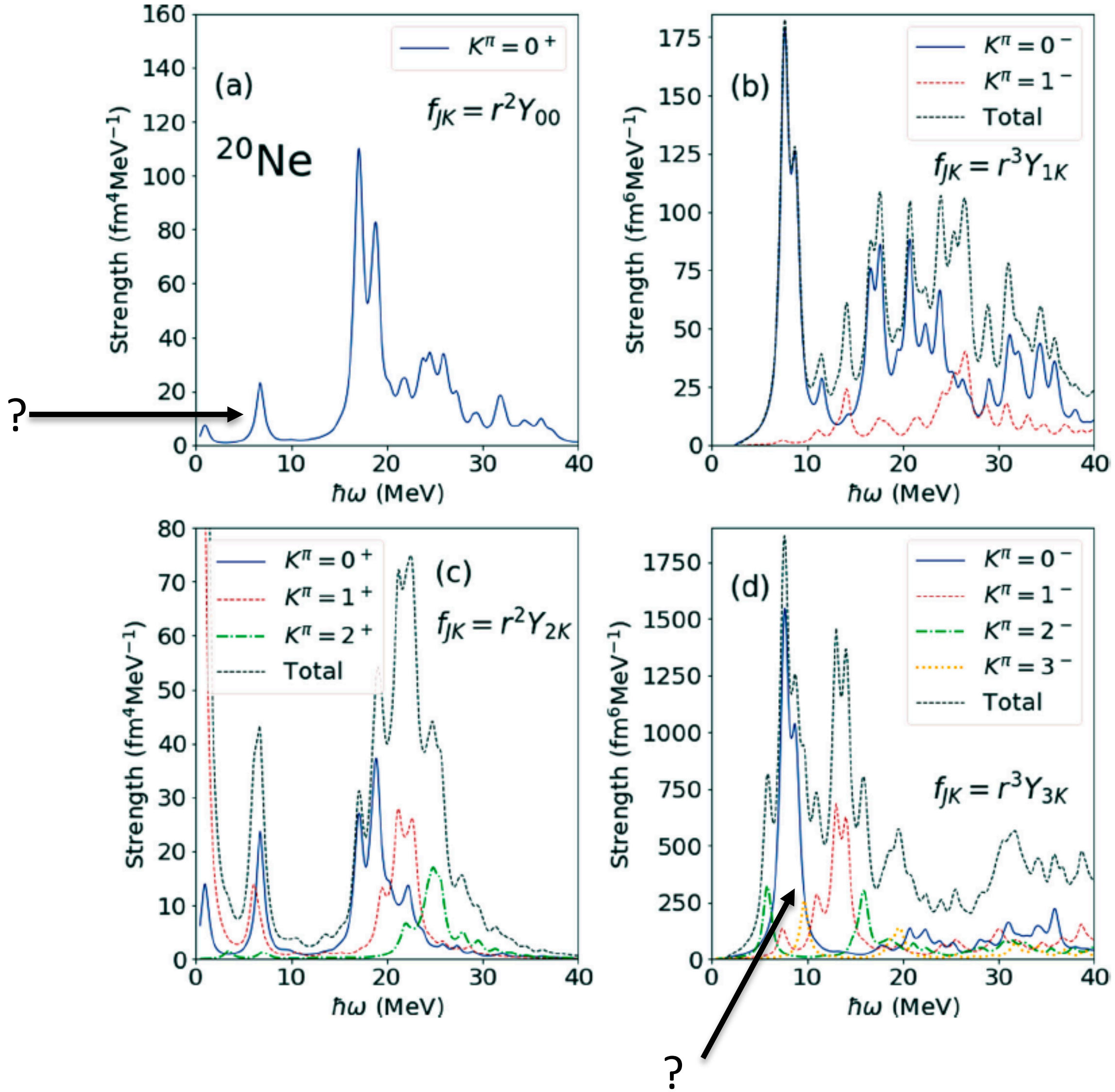
$$\delta\rho(\omega) = UX(\omega)V^T + V^*Y^T(\omega)U^\dagger,$$

$$\delta\kappa^{(+)}(\omega) = UX(\omega)U^T + V^*Y^T(\omega)V^\dagger,$$

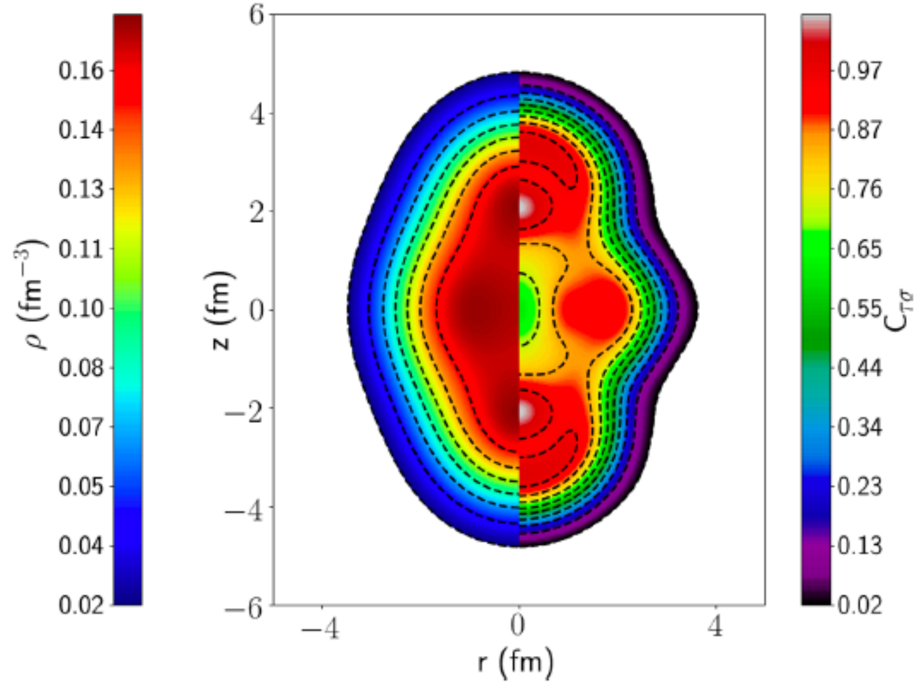
$$\delta\kappa^{(-)}(\omega) = UY^*(\omega)U^T + V^*X^\dagger(\omega)V^\dagger$$

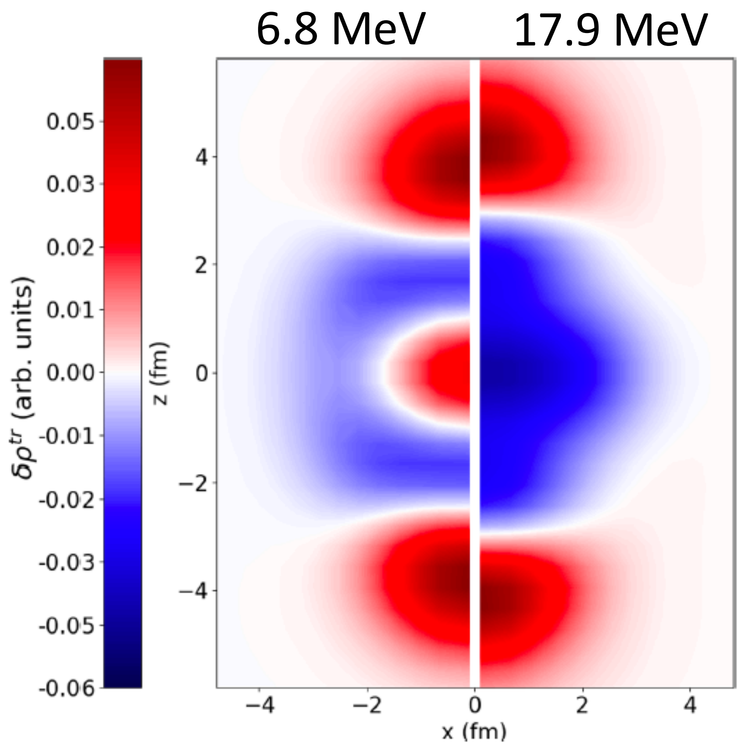
Here: DDPC-1 + quadrupole and octupole deformations

Strengths in N=Z nuclei



Cluster mode in ^{20}Ne

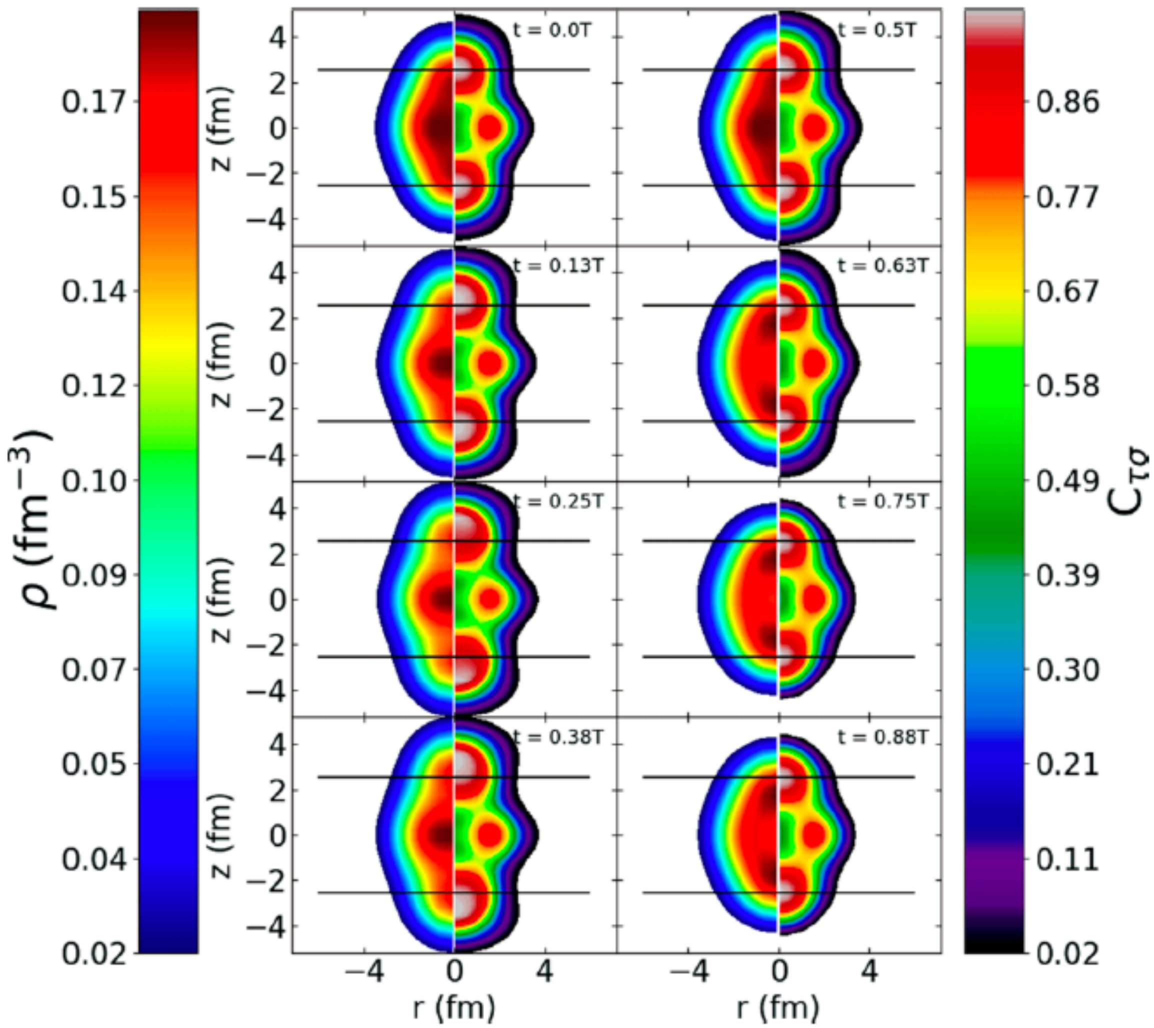




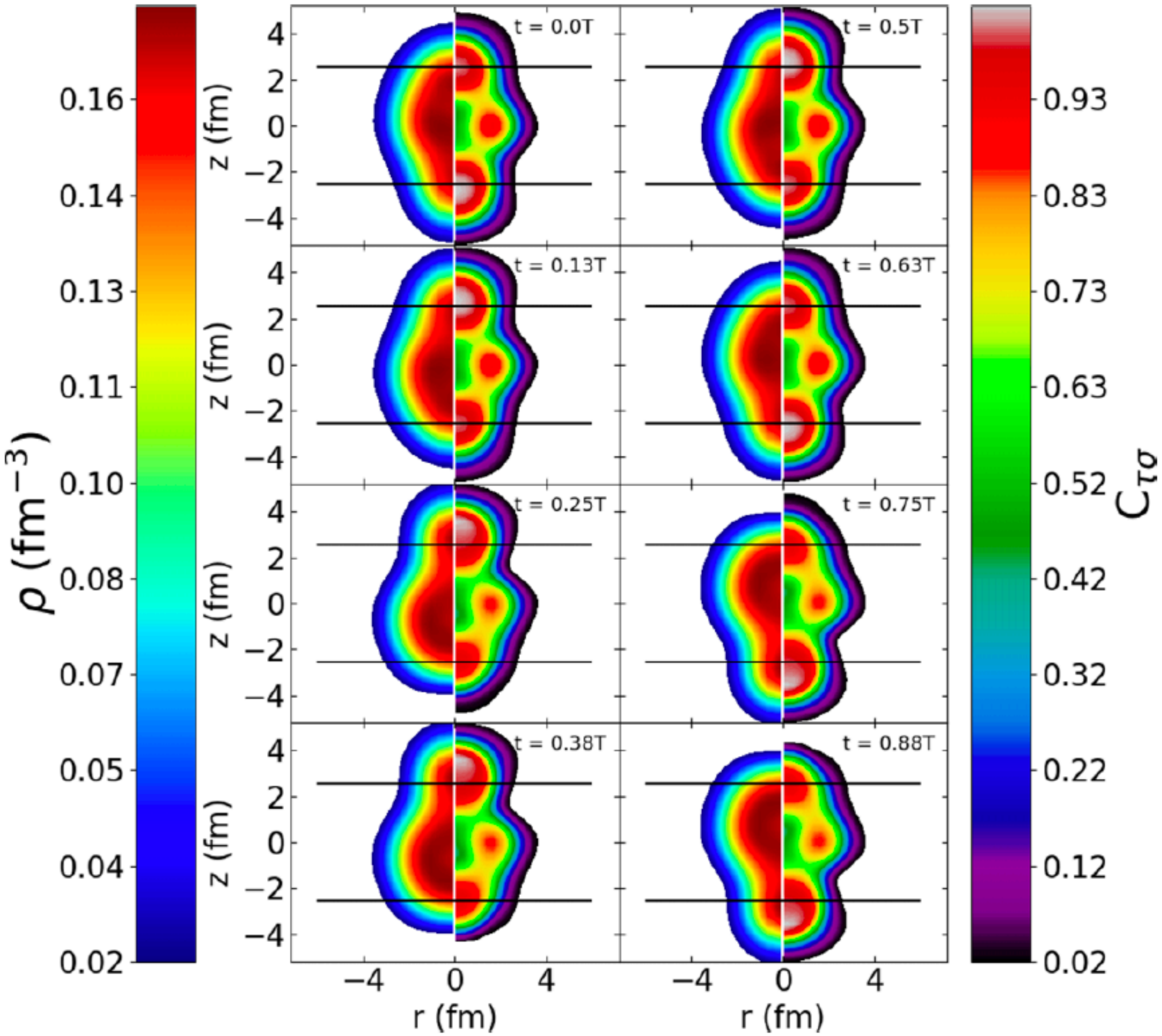
Clusters oscillations against the core

Depletion of the core (breathing mode)

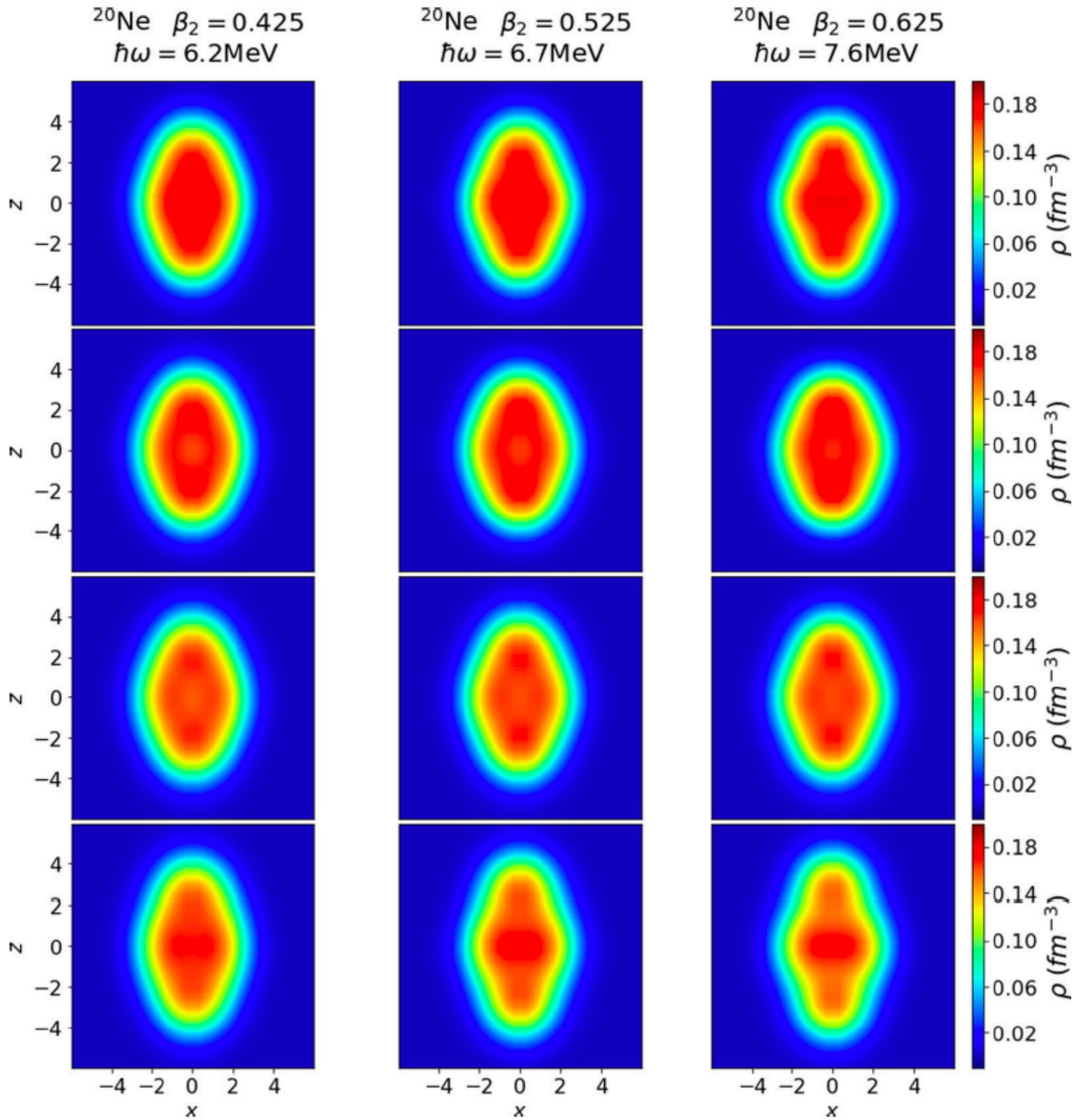
Density and localization function in ^{20}Ne (monopole response)



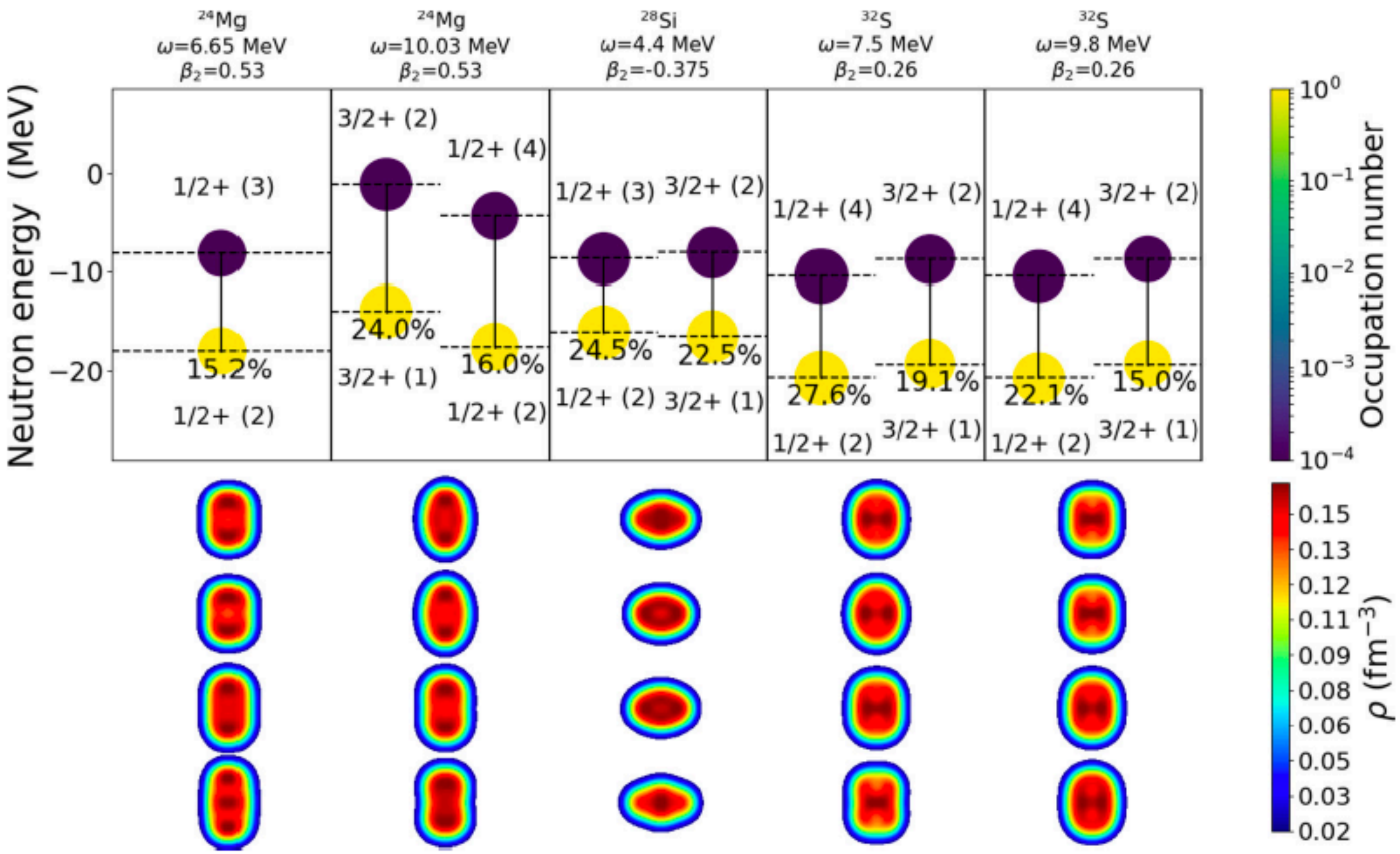
Density and localization function in ^{20}Ne (octupole response, K=0)



Effect of the deformation

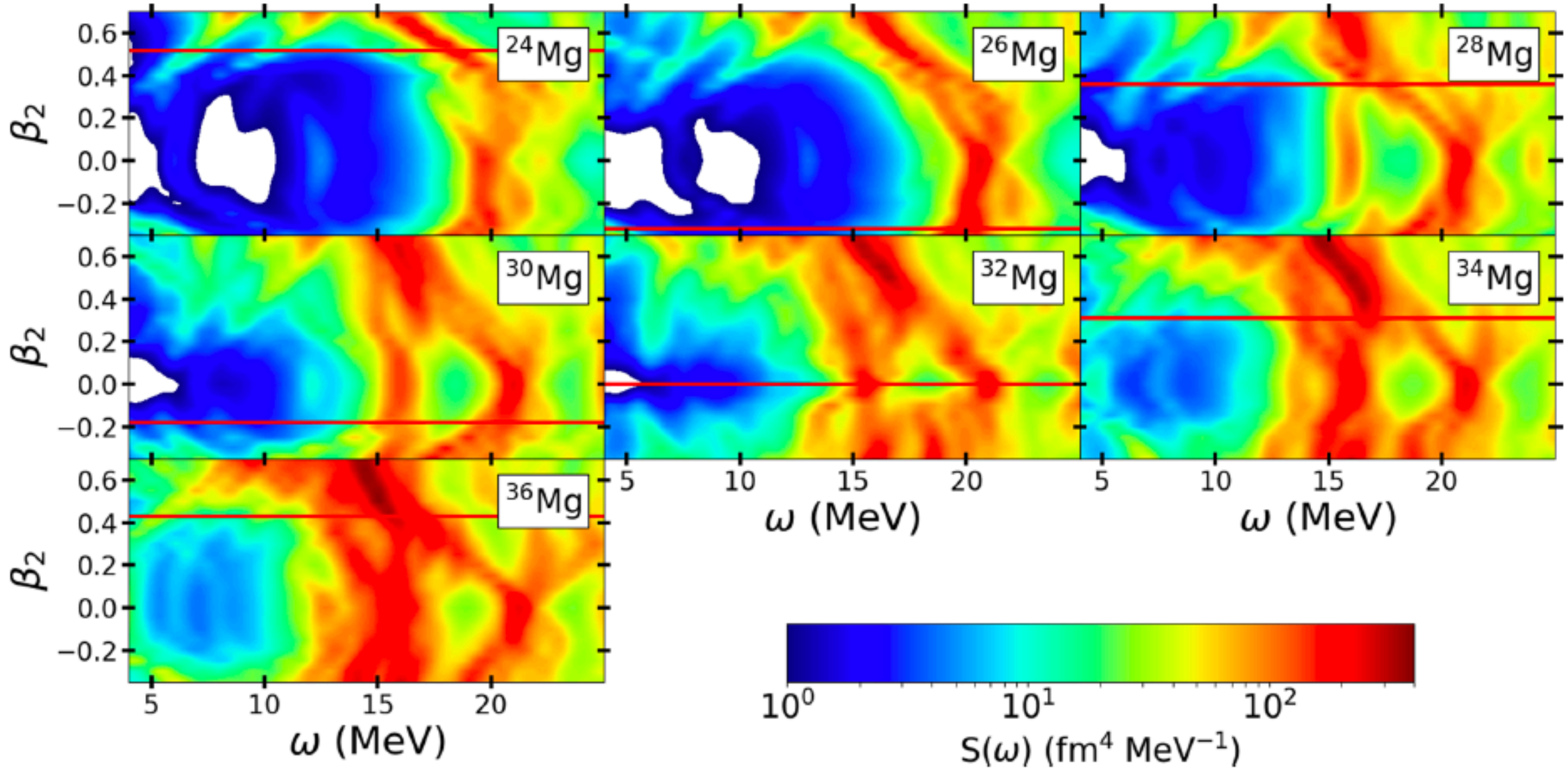


Non-collective soft monopole modes

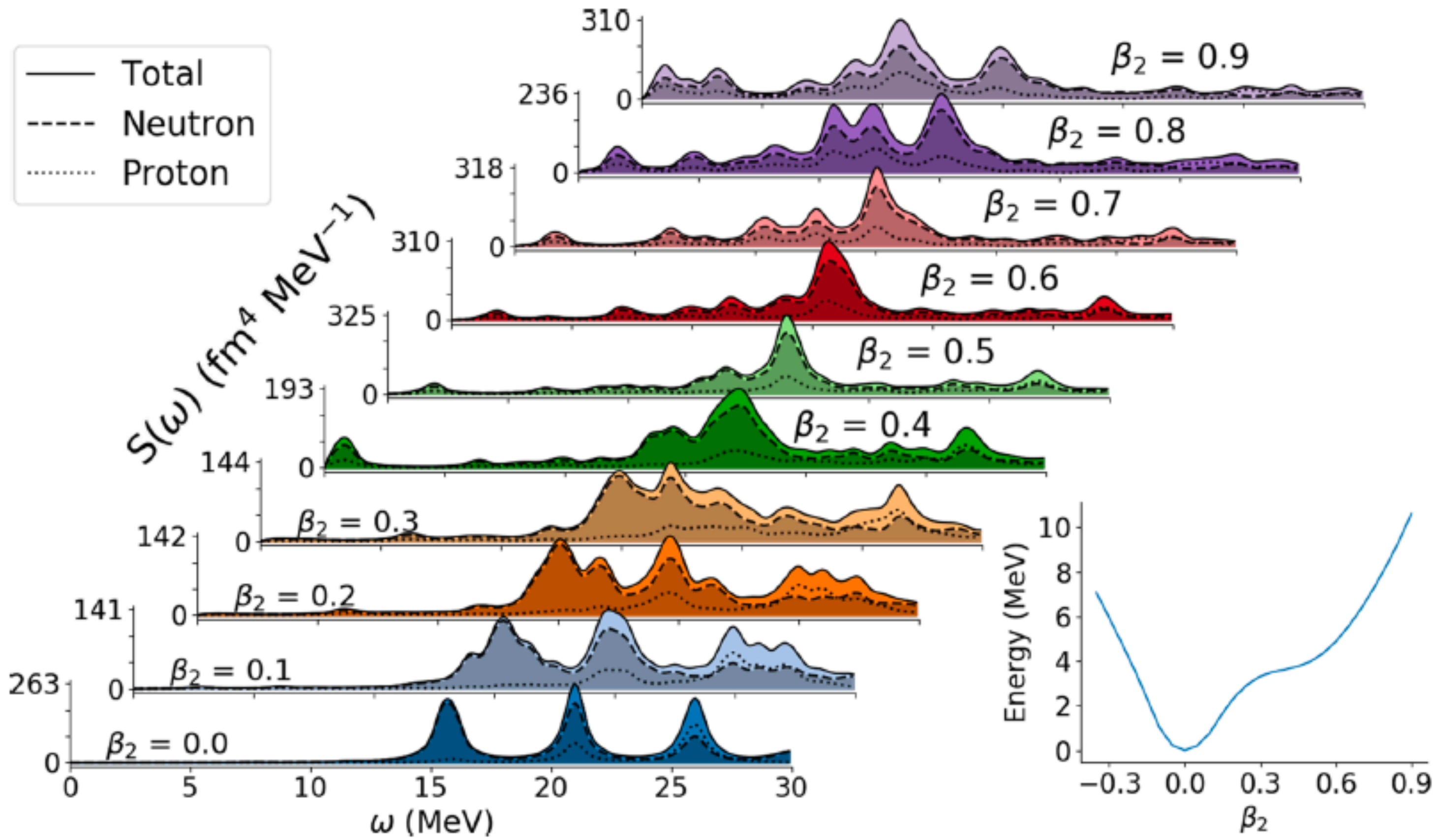


^{20}Ne: 1/2+ states from 1d5/2 substate to 2s1/2

Impact of neutron excess and deformation on the monopole response

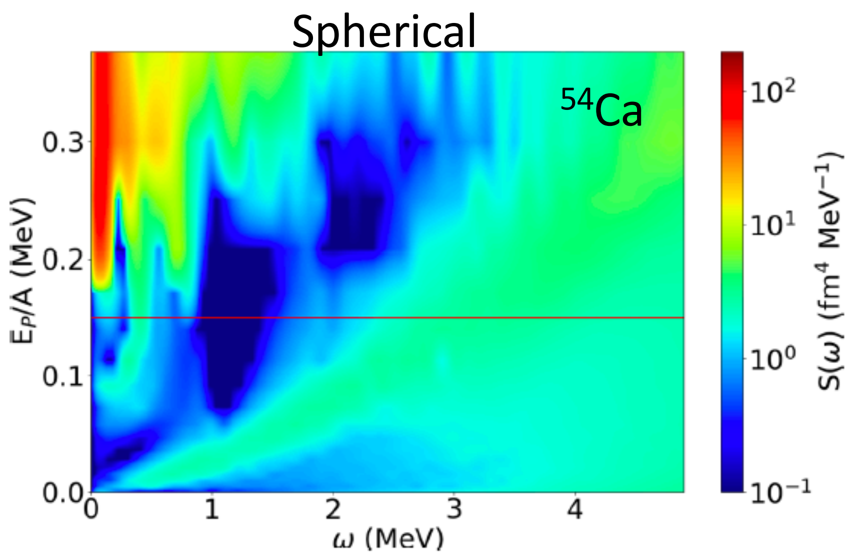


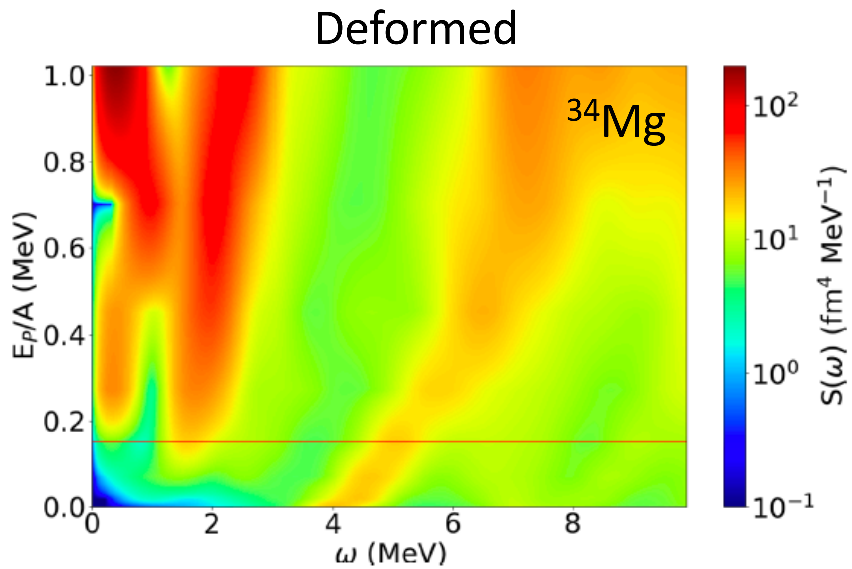
Monopole response in n-rich ^{32}Mg



- GMR (15 MeV) splitting vs. soft n mode (10 MeV) starts from β_2=0.6
- Cluster modes (5 MeV) starts from β_2=0.4

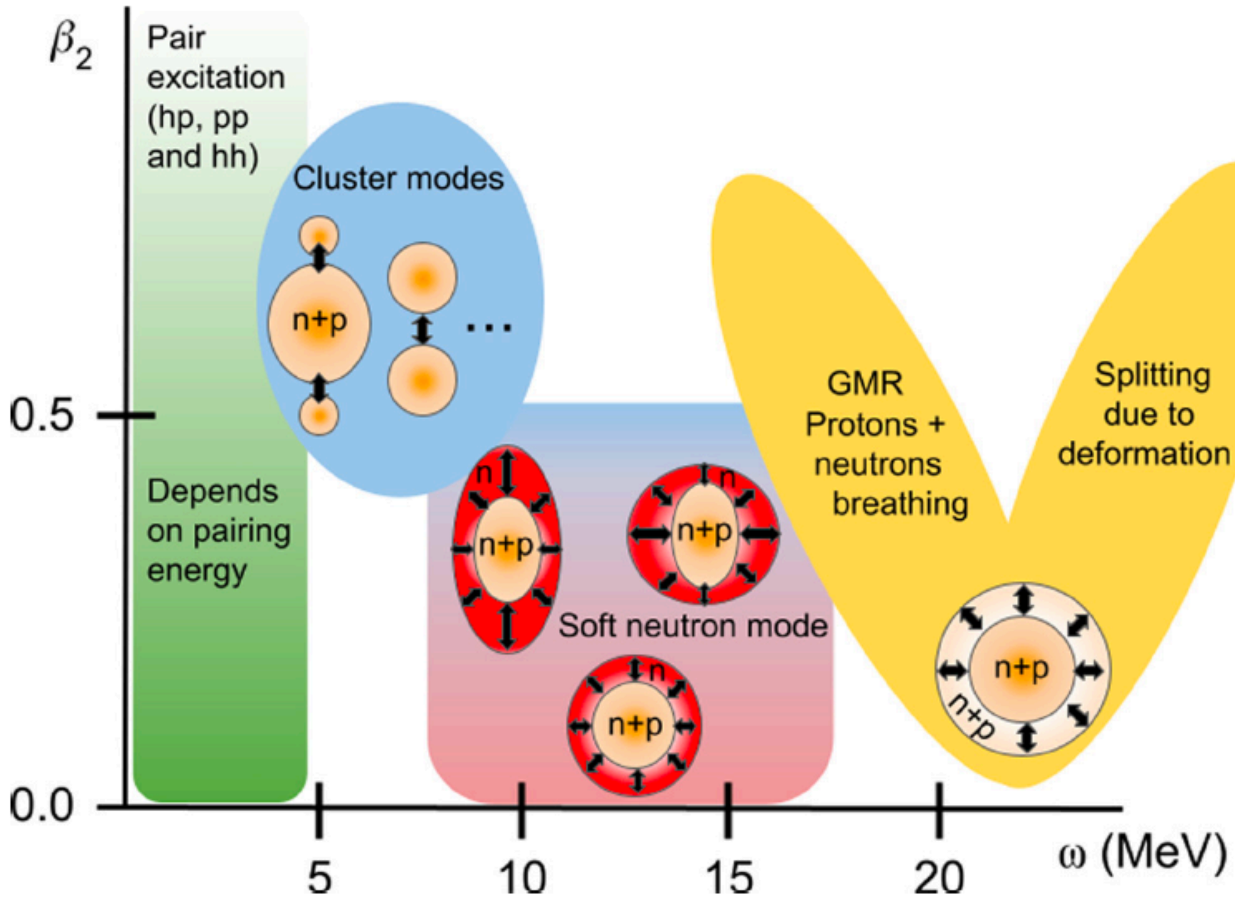
Pairing modes





Pairing modes occur below 2 MeV

Summary



β_2
0.5
0.0
Pair excitation (hp, pp and hh)
Depends on pairing energy
Cluster modes
n+p
n+p
n
Soft neutron mode
GMR Protons + neutrons breathing
Splitting due to deformation
n+p
5
10
15
20
ω (MeV)