Effects of beyond-mean-field correlations and finite temperature on the nuclear monopole response

Elena Litvinova

Western Michigan University

NSCL

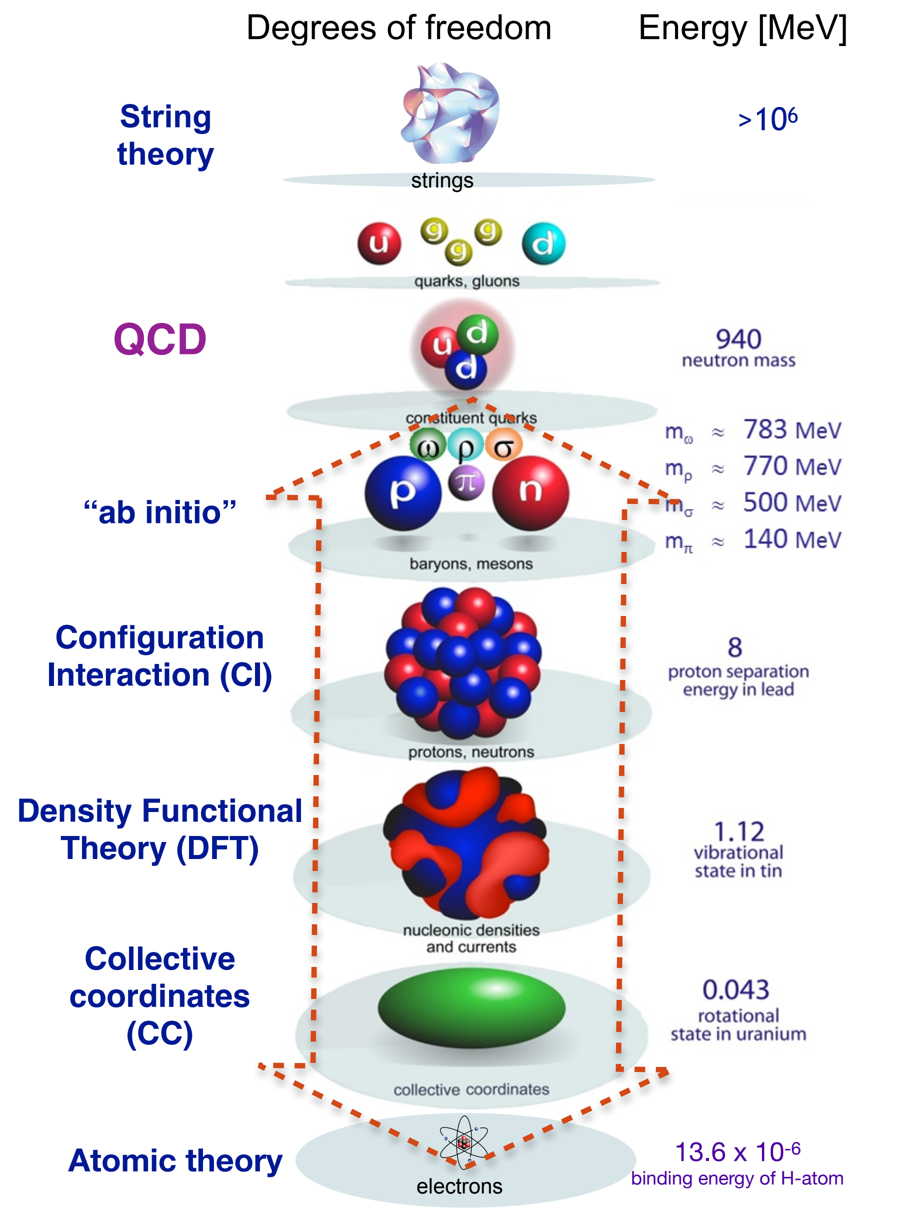
MICHIGAN STATE UNIVERSITY

NSF CAREER Award

ECT Workshop "Advances on giant nuclear monopole excitations and applications to multimessenger astrophysics"*

ECT Trento, July 11-15, 2022*

Hierarchy of energy scales and nuclear many-body problem



- ***The major conflict:***

Separation of energy scales => effective field theories

vs

The physics on a certain scale is governed by the next higher-energy scale

Hamiltonian:

$$H = K + V$$

center of mass

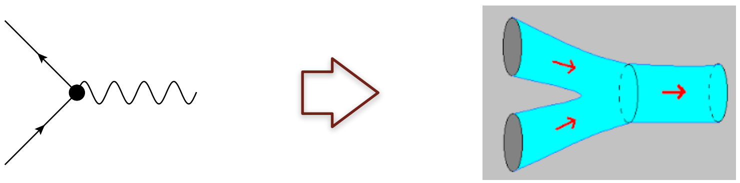
internal degrees of freedom: next energy scale

Standard Model:

free propagation and interaction, singularities & renormalizations

String theory:

merging strings NO "Interaction"



- ***Possible solution:***

Keep/establish connections between the scales via emergent phenomena

The Equation of Motion (EOM) method

- *Generates EOM's for time-dependent field operators and correlation functions, i.g., in-medium propagators.*
- *Propagators are linked directly to observables.*
- *Two-time (one fermion and two-fermion) propagators are most relevant ones for nuclear physics applications.*
- *Interaction kernels: static (short-range correlations) + dynamical (long-range correlations).*
- *The exact EOM's for the propagators are coupled into an N-body equation hierarchy via dynamical kernels.*
- *Practical implementations: full or partial decoupling via various approximations.*

EOM method:

- *D. J. Rowe, Rev. of Mod. Phys. 40, 153 (1968).*
- *P. Schuck, Z. Phys. A 279, 31 (1976).*
- *S. Adachi and P. Schuck, NPA496, 485 (1989).*
- *P. Danielewicz and P. Schuck, NPA567, 78 (1994)*
- *J. Dukelsky, G. Roepke, and P. Schuck, NPA 625, 14 (1995).*
- *P. Schuck and M. Tohyama, PRB 93, 165117 (2016).*
- *P. Schuck et al., Phys. Rep. 929, 1 (2021).*

Nuclear physics implementations beyond (Q)RPA: 2p2h, 3p3h

- *Nuclear field theory, NFT (P.F. Bortignon, R. Broglia, G. Colo, Milano-Copenhagen; V. Tselyaev, S. Kamerdzhiev et al.,St. Petersburg)*
- *Quasiparticle-phonon model, QPM (V.G. Soloviev et al., Dubna; V. Ponomarev, TU-Darmstadt)*
- *Multiphonon approach (N. Lo Iudice, G. De Gregorio et al., Naples & Prague)*
- *Self-consistent Green functions (W. Dickhoff, C. Barbieri, V. Soma, T. Duguet et al.)*
- *Second RPA, SRPA (C. Yannouleas, P. Chomaz, S. Drozdz, P. Papakonstantinou et al.)*
- *Relativistic NFT (E.L., P. Ring, P. Schuck, C. Robin, H. Wibowo, Y. Zhang)*

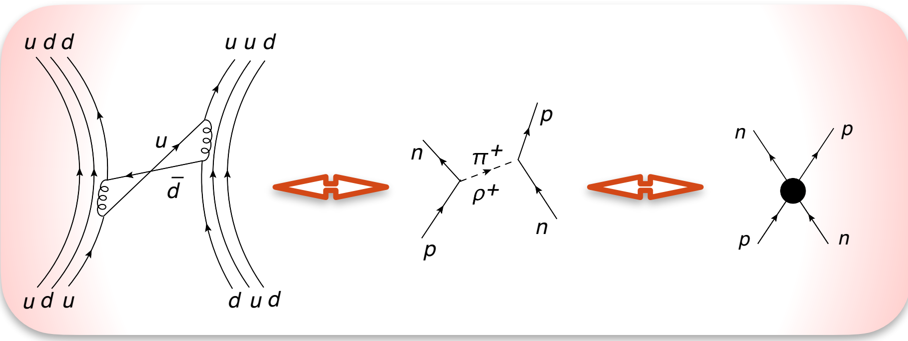
The underlying mechanism of NN-interaction : meson exchange and EFTs

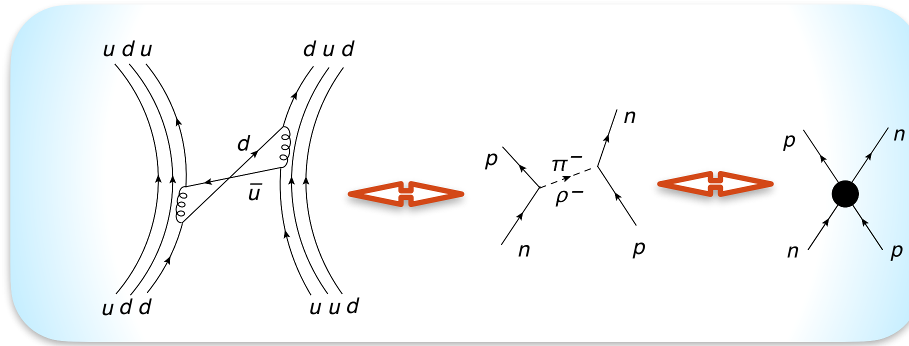
Charged mesons {π, ρ}:

Quantum Chromodynamics (QCD, high energy)

Quantum Hadrodynamics (QHD, intermediate energy)

Nuclear Structure (NS, low energy)





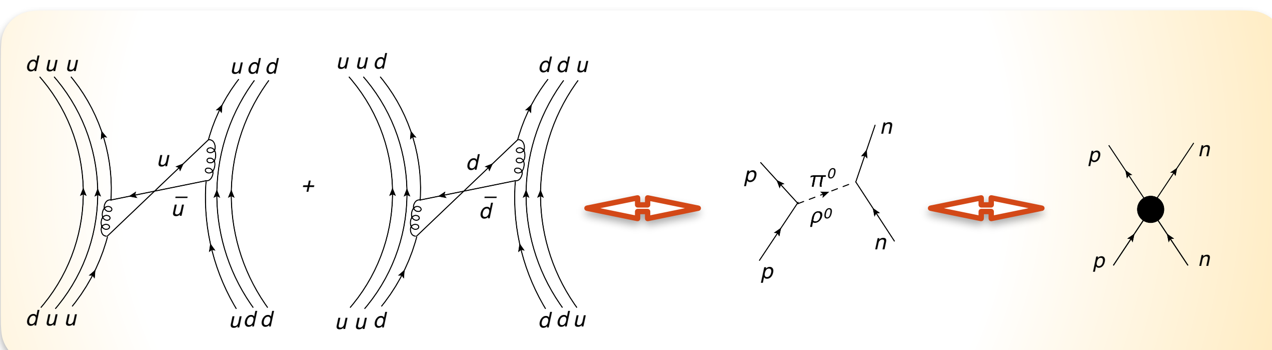
Neutral mesons {σ, ω, π, ρ}:

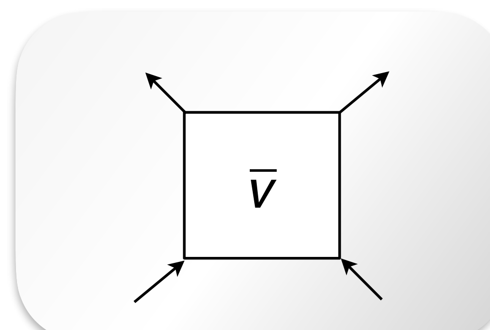
Generic interaction: model-independent, ALL channels included:

QCD

QHD

NS



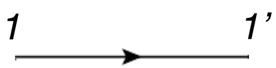


A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} t_{12}\psi^{\dagger}{}_1\psi_2 + \frac{1}{4}\sum_{1234} \bar{v}_{1234}\psi^{\dagger}{}_1\psi^{\dagger}{}_2\psi_4\psi_3$$

***Hamiltonian**, non-relativistic or relativistic, extendable to 3-body etc.*

$$G_{11'}(t-t') = -i\langle T\psi_1(t)\psi^{\dagger}_{1'}(t')\rangle$$



Single-particle propagator

Fourier image: observables

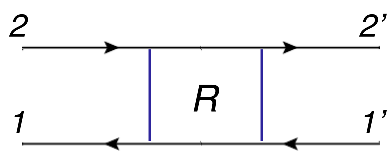
$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n \eta_{1'}^{n*}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \sum_m \frac{\eta_1^{m*}\eta_{1'}^{m}}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta}$$

$$\eta_1^n = \langle 0|\psi_1|n^{(N+1)}\rangle, \qquad \eta_1^m = \langle m^{(N-1)}|\psi_1|0\rangle$$

***Residues** - spectroscopic (occupation) factors*

***Poles** - single-particle energies*

$$R_{12,1'2'}(t-t') = -i\langle T(\psi^{\dagger}_1\psi_2)(t)(\psi^{\dagger}_{2'}\psi_{1'})(t')\rangle$$



Particle-hole (ph) response function

Fourier image: observables

$$R_{12,1'2'}(\omega) = \sum_\nu \left[\frac{\rho^{\nu}_{21}\rho^{\nu*}_{2'1'}}{\omega - \omega_\nu + i\delta} - \frac{\rho^{\nu*}_{12}\rho^{\nu}_{1'2'}}{\omega + \omega_\nu - i\delta}\right]$$

$$\rho^{\nu}_{12} = \langle 0|\psi^{\dagger}_2\psi_1|\nu\rangle$$

***Residues** - transition densities*

***Poles** - excitation energies*

Exact equations of motion (EOM) for binary interactions: one-body problem

$$G_{11'}(t-t') = -i\langle T\psi_1(t)\psi^\dagger_{1'}(t')\rangle$$

***EOM**: Dyson Equation*

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \qquad (*) \qquad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

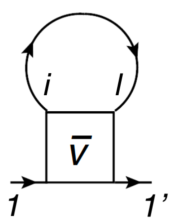
Free propagator → $G^{(0)}$

Irreducible kernel (Self-energy, exact): → $\Sigma(\omega)$

Instantaneous term (Hartree-Fock incl. "tadpole")
Short-range correlations

$$\Sigma^{(0)}_{11'} = -\delta(t-t')\langle[[V,\psi_1],{\psi^\dagger}_{1'}]_+\rangle$$

$$= -\sum_{jl} \bar{v}_{1j1'l}\,\rho_{lj} =$$

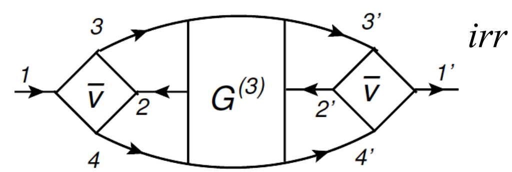


t-dependent (dynamical) term
Long-range correlations

$$\Sigma^{(r)}_{11'}(t-t') = -i\langle T[\psi_1,V](t)[V,{\psi^\dagger}_{1'}](t')\rangle^{irr}$$

$$= -\frac{1}{4}\sum_{234}\sum_{2'3'4'} \bar{v}_{1234}\,G^{irr}(432',23'4')\,\bar{v}_{4'3'2'1'}$$

$$= -\frac{1}{4}$$



Mean field, where $\rho_{ij} = -i\lim_{t=t'-0} G_{ij}(t-t')$ *is the full solution of (*): **includes the dynamical term!***

The self-energy and the one-body density
are fully determined by the bare (antisymmetrized) interaction
and by the three-body correlation function

Equation of motion (EOM) for the particle-hole response

Particle-hole response (correlation function):

$$R^{(ph)}_{12,1'2'}(t-t') = -i\langle T(\psi^\dagger_1\psi_2)(t)(\psi^\dagger_{2'}\psi_{1'})(t')\rangle$$

spectra of excitations, masses, decays, …

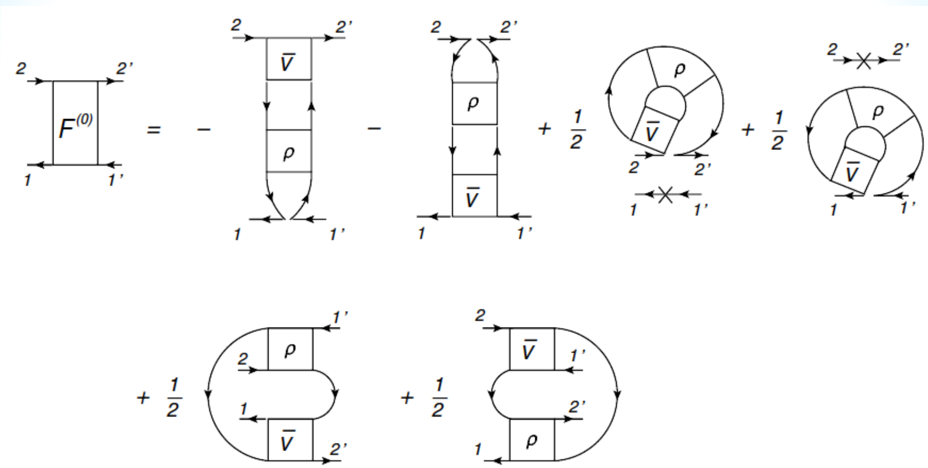
***EOM**: Bethe-Salpeter-Dyson Eq.*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Free propagator ($R^{(0)}$)

Irreducible kernel (exact): (F)

Instantaneous term ("bosonic" mean field):
Short-range correlations

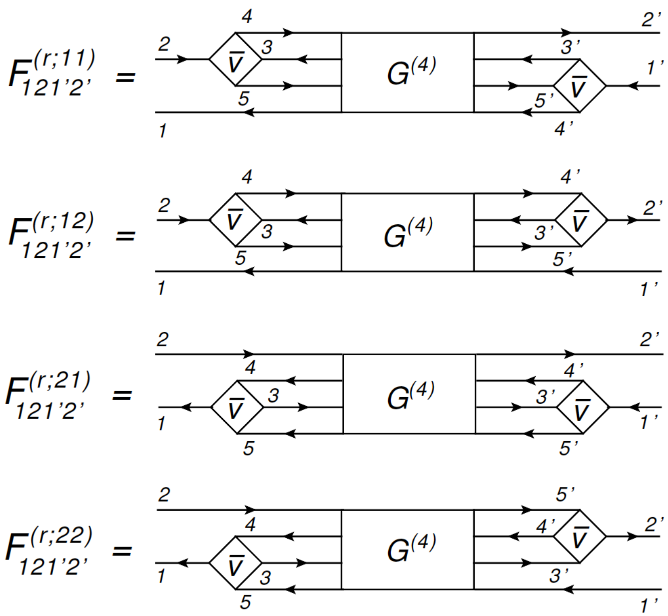


Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i \lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

*contains the full solution of (**) **including the dynamical term!***

t-dependent (dynamical) term:
Long-range correlations



$$F^{(r)}_{12,1'2'}(t-t') = \sum_{ij} F^{(r;ij)}_{12,1'2'}(t-t')$$

Non-perturbative treatment of two-point $G^{(n)}$ in the dynamical kernels

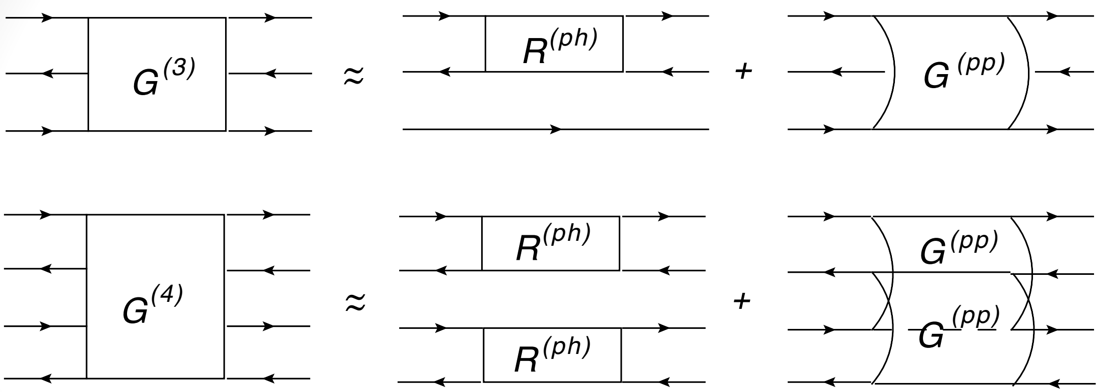
- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy. $N_{Equations} = N_{Particles}$ & Coupled !!!*

Truncation on two-body level

- ***Non-perturbative solution:***
  Cluster decomposition

✦ $G^{(3)} = G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(1)} + \Xi^{(3)}$

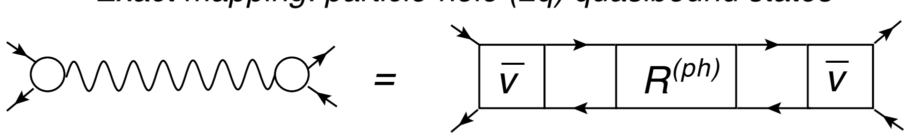
✦ $G^{(4)} = G^{(1)} G^{(1)} G^{(1)} G^{(1)} + G^{(2)} G^{(2)} + G^{(3)} G^{(1)} + \Xi^{(4)}$

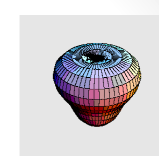


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

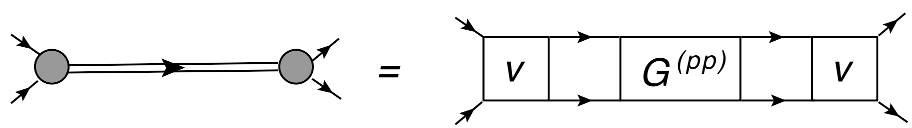
Exact mapping: particle-hole (2q) quasibound states

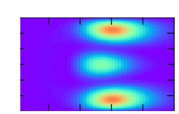
Emergence of effective "particles" (phonons, vibrations):





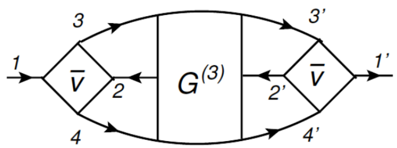
Emergence of superfluidity:



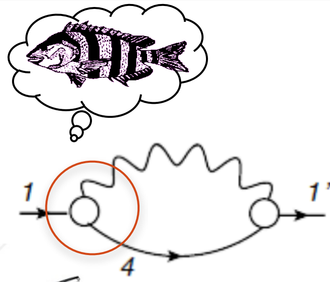


Emergence of effective degrees of freedom

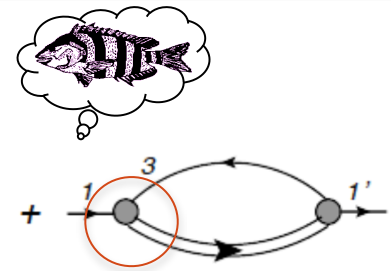
Dynamical self-energy:



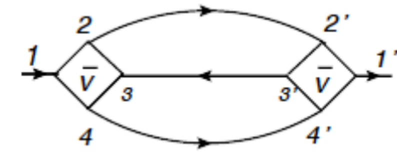
≈



+



+



"Fish" diagrams

Second-order

Quasiparticle-vibration coupling (QVC)

Emergent phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom

$$H = \sum_{12} h_{12}\psi^\dagger_1\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi^\dagger_1\psi^\dagger_2\psi_4\psi_3$$

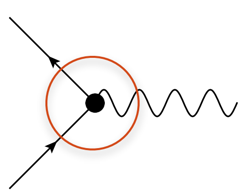
"Ab-initio"

$$H = \sum_{12} \tilde{h}_{12}\psi^\dagger_1\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q^\dagger_\lambda Q_{\lambda'} + \sum_{12\lambda}\Big[\Theta^\lambda_{12}\psi^\dagger_1 Q^\dagger_\lambda\psi_2 + h.c.\Big]$$

Effective

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:

Possibly derivable?

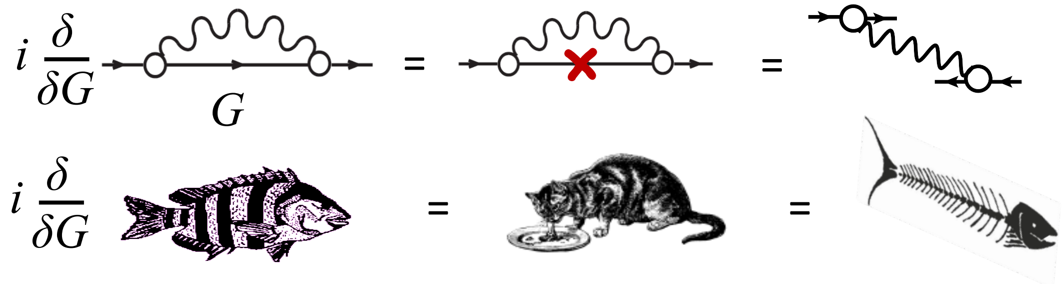


$$\gamma, g, W^\pm, Z^0$$

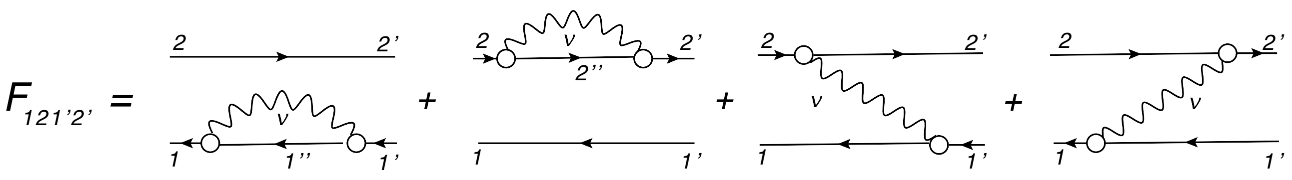
E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

Dynamical kernel of particle-hole propagator (response)

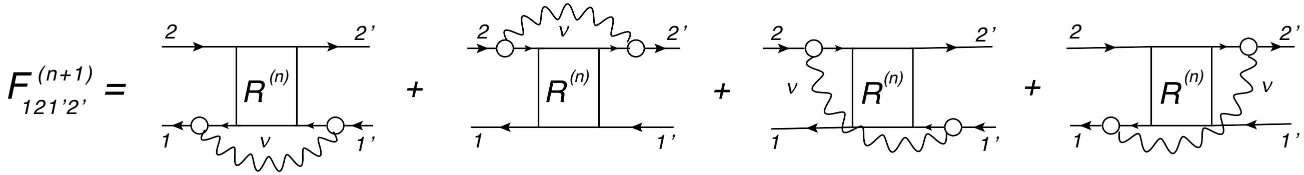
Induced (exchange) terms:
Consistency condition



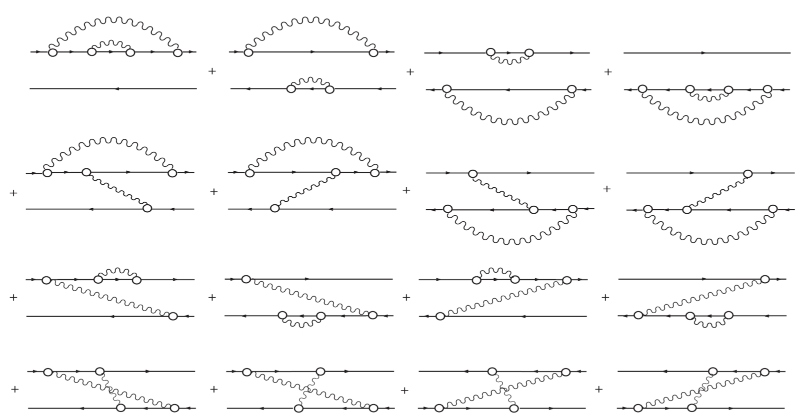
Leading approach
(this work):

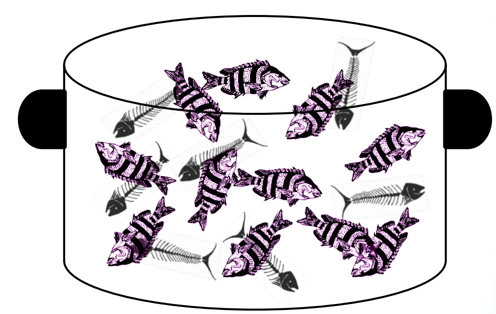


Iterated kernel:

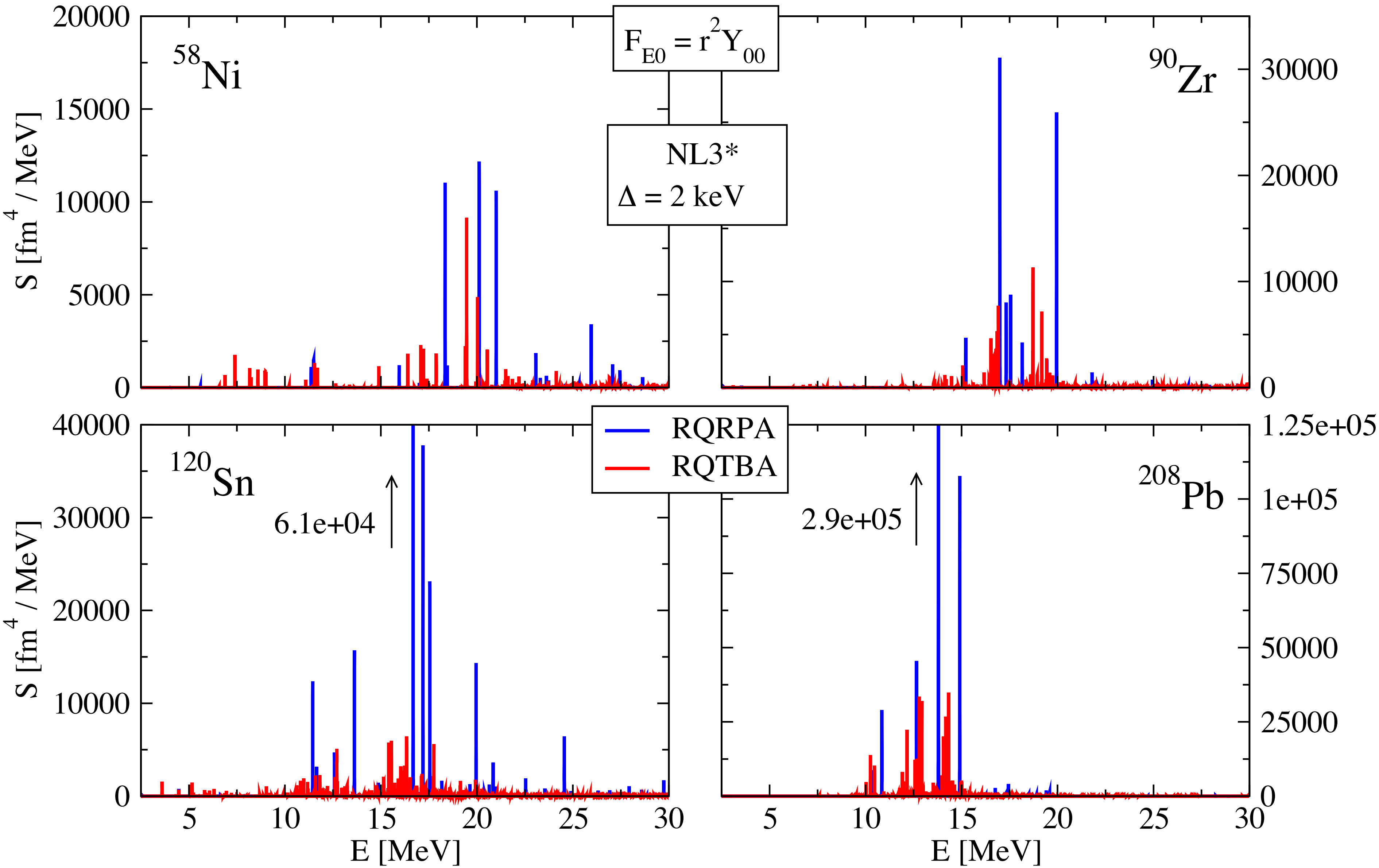


"Nested"
configurations

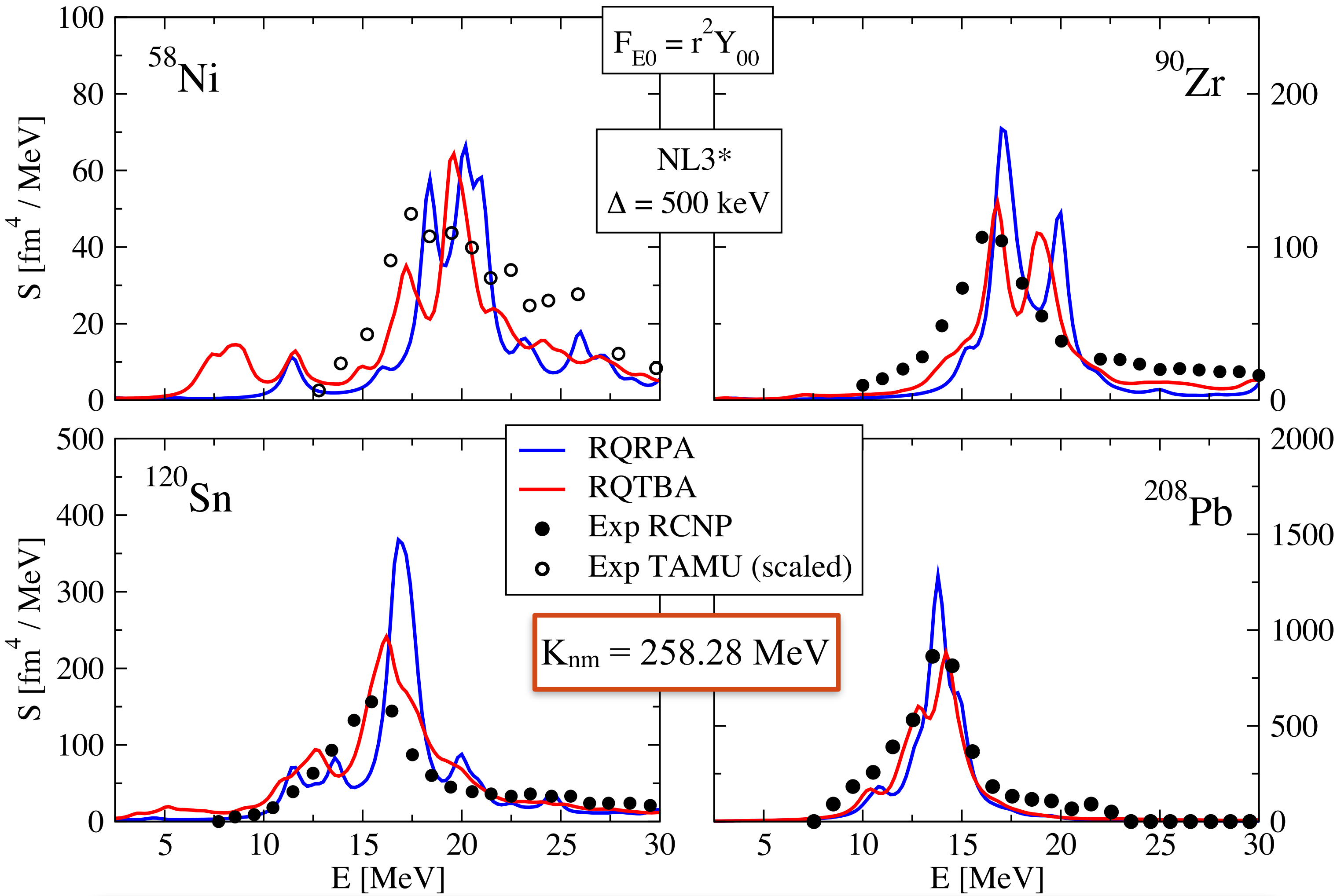




The "anatomy" (fine structure) of the ISGMR



Isoscalar giant monopole resonance (ISGMR) in medium-mass and heavy nuclei: comparison to data

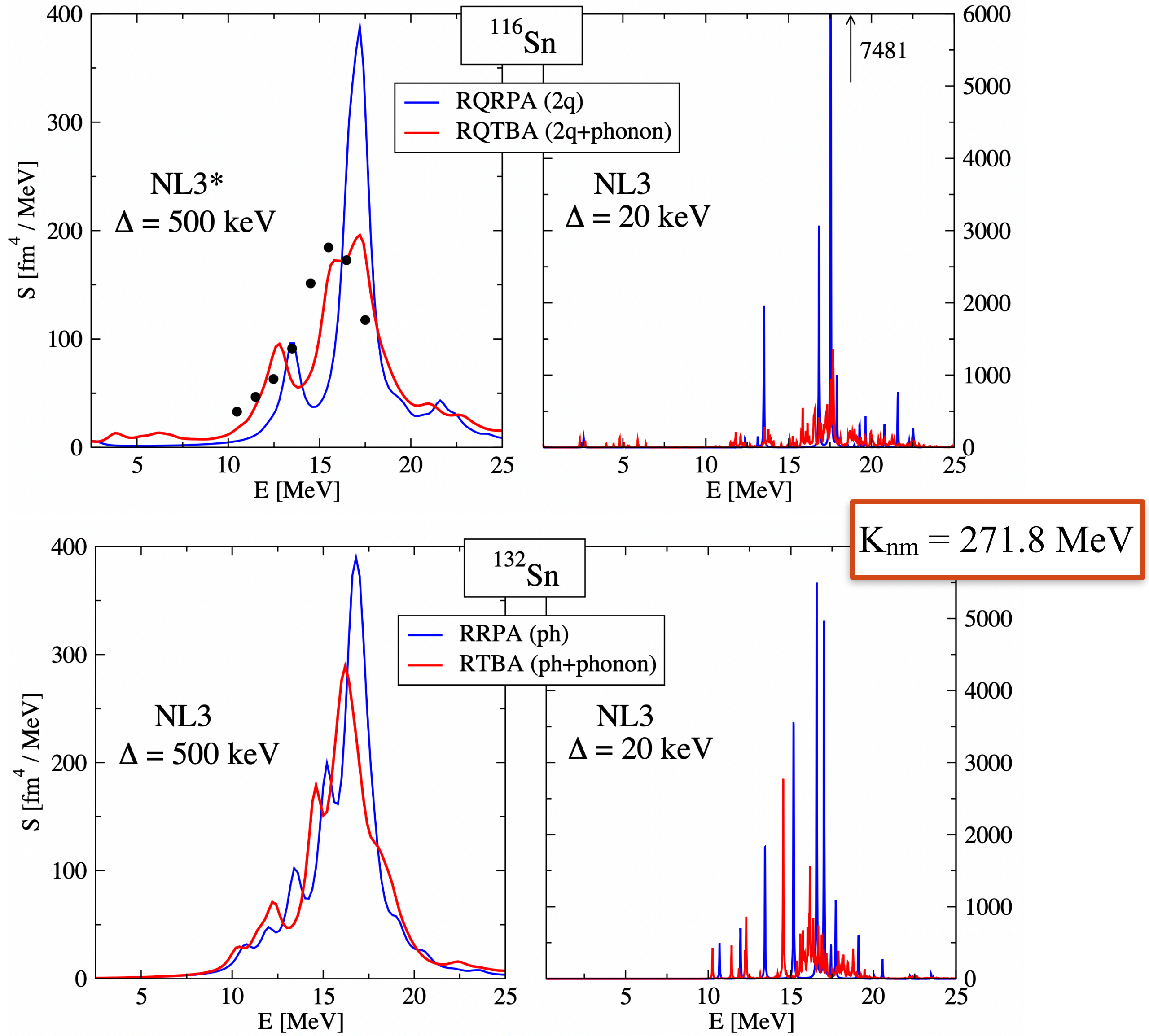


- *Δ is consistent with the experimental resolution: Δ = Γ/2*
- *Phonon subspace of RQTBA (2q+phonon): J^{π} = 2^+, 3^-, 4^+, 5^-, 6^+ below 15 MeV*
- *Further improvable by extending the phonon subspace*

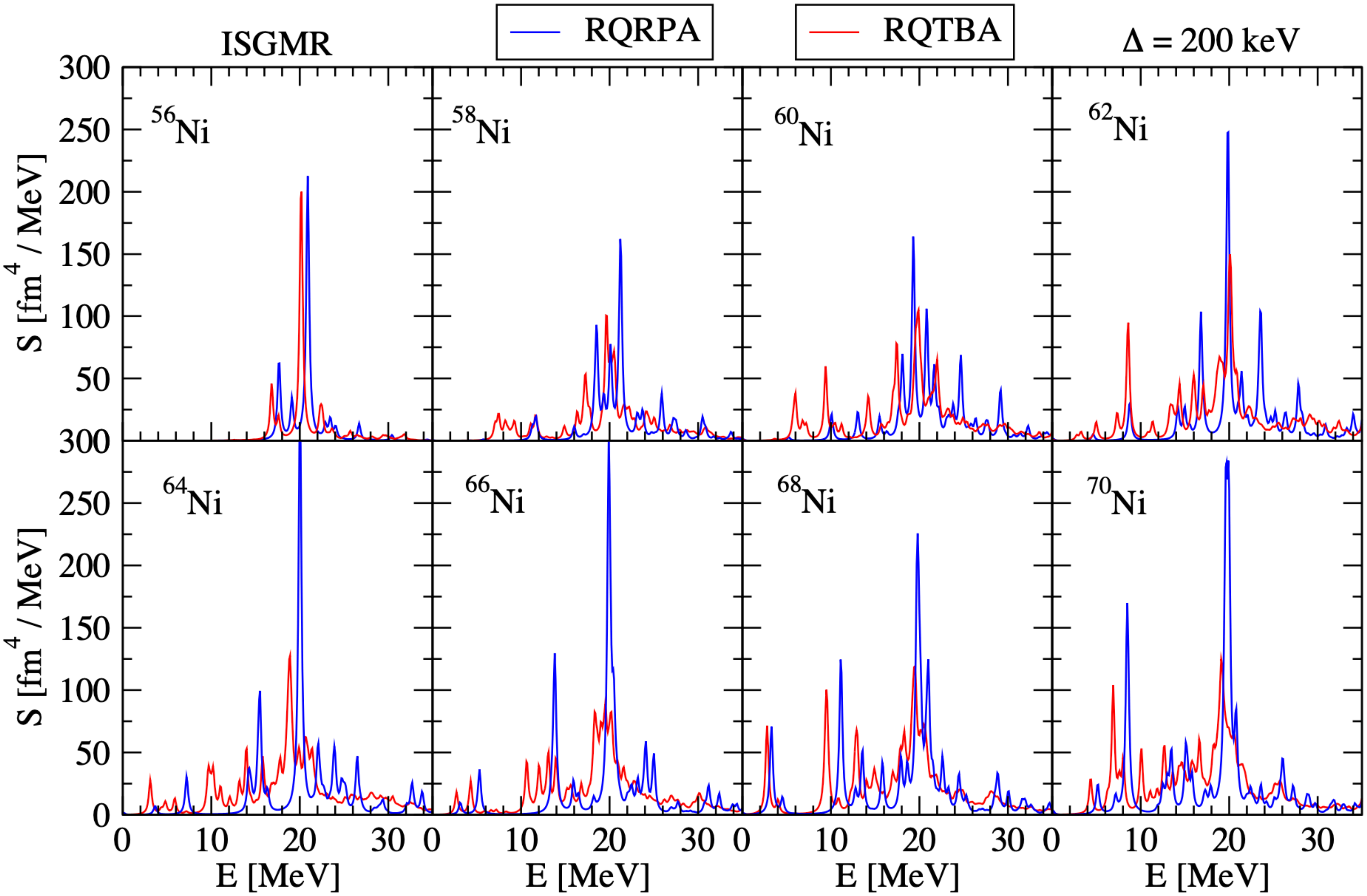
ISGMR in tin: open-shell vs closed-shell

NL3 vs NL3?*
A similar downward shift
in the open-shell Sn

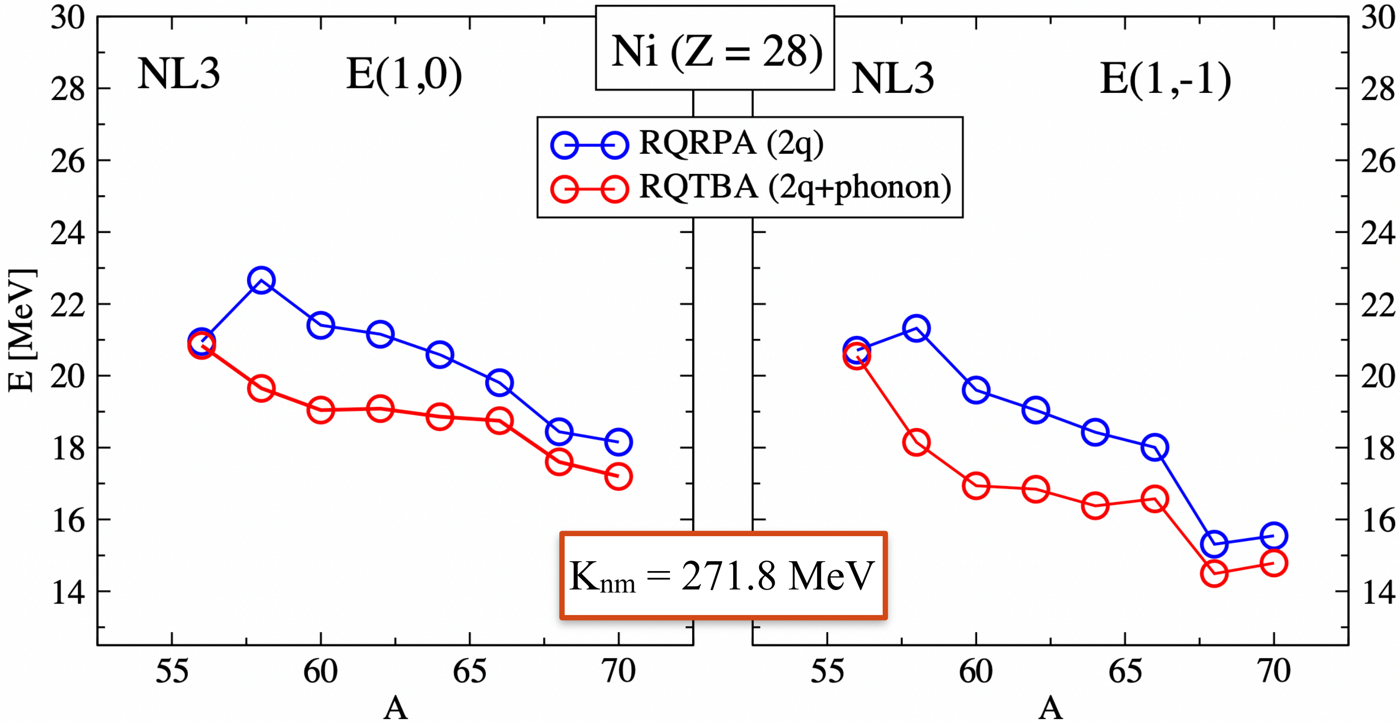
Ready for comparison
with data



ISGMR systematics in nickel isotopes



ISGMR systematics in nickel isotopes: the centroids



"Softness" increases:

- *with the neutron number*
- *with correlations beyond QRPA (q)PVC*

$$K_A = 4\langle r^2\rangle_0^2 \frac{d^2}{d\langle r^2\rangle^2}\left(\frac{E}{A}\right)_{\langle r^2\rangle=\langle r^2\rangle_0} = \frac{m\langle r^2\rangle_0^2}{A\hbar^2}\frac{m_1}{m_{-1}}$$

Effective nuclear compression modulus (incompressibility)

$$E_c \equiv \sqrt{\frac{m_1}{m_{-1}}}$$

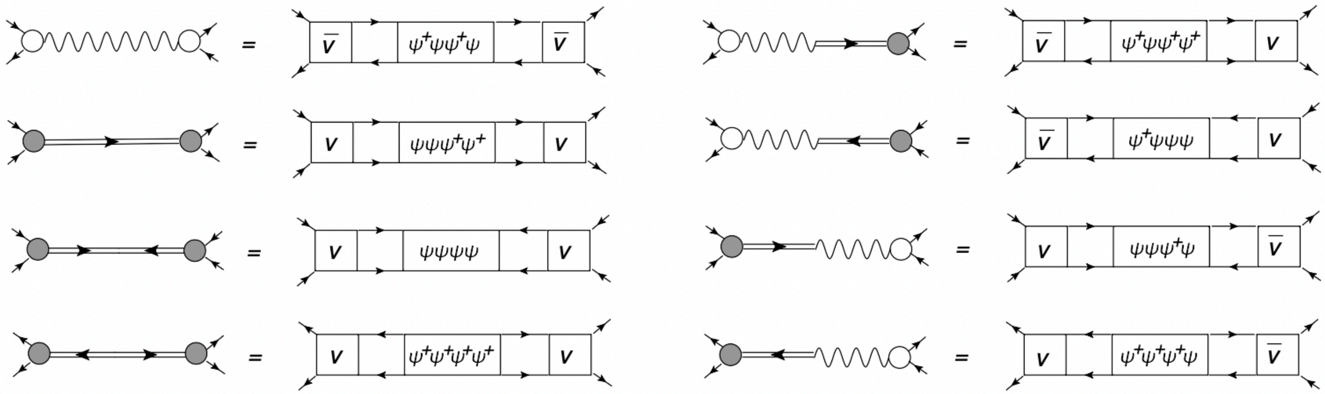
$$E_{ISGMR} = \sqrt{\frac{\hbar^2 A K_A}{m\langle r^2\rangle_0}}$$

U. Garg, G. Colò, Progress in Particle and Nuclear Physics 101 (2018) 55–95

Deformed nuclei

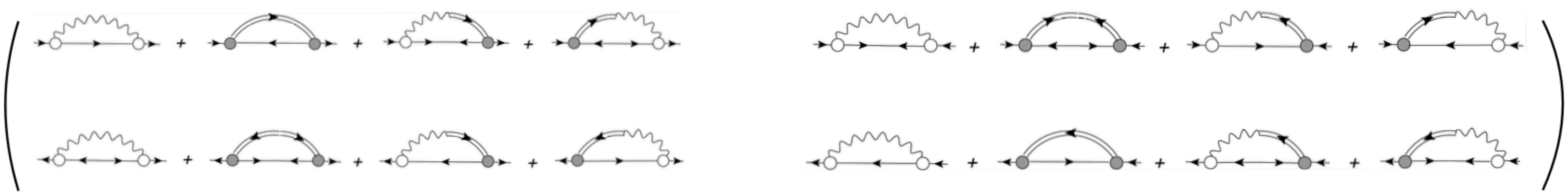
Superfluid dynamical kernel: adding particle-number violating contributions

Mapping on the QVC in the canonical basis



Quasiparticle dynamical self-energy (matrix):
normal and pairing phonons are unified

$\hat{\Sigma}^r =$



Cf.: Quasiparticle static self-energy (matrix) in HFB

$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^*_{11'} & -\tilde{\Sigma}^T_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Transformation to quasiparticle basis

Bogolyubov transformation:

$$\psi_1 = \sum_\nu (U_{1\nu}\alpha_\nu + V^*_{1\nu}\alpha^\dagger_\nu), \qquad \psi^\dagger_1 = \sum_\nu (V_{1\nu}\alpha_\nu + U^*_{1\nu}\alpha^\dagger_\nu)$$

$$G^{(+)}_{\nu\nu'}(\varepsilon) = \sum_{12} \begin{pmatrix} U^\dagger_{\nu 1} & V^\dagger_{\nu 1} \end{pmatrix} \hat{G}_{12}(\varepsilon) \begin{pmatrix} U_{2\nu'} \\ V_{2\nu'} \end{pmatrix}$$

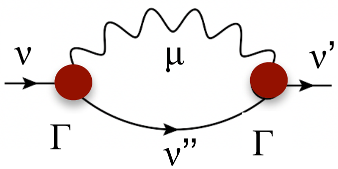
$$G^{(-)}_{\nu\nu'}(\varepsilon) = \sum_{12} \begin{pmatrix} V^T_{\nu 1} & U^T_{\nu 1} \end{pmatrix} \hat{G}_{12}(\varepsilon) \begin{pmatrix} V^*_{2\nu'} \\ U^*_{2\nu'} \end{pmatrix}$$

Propagator becomes diagonal

Dyson Eqs. decouple for η=1 and η=-1: Eq. for η=-1 is redundant

$$G^{(\eta)}_{\nu\nu'}(\varepsilon) = \tilde{G}^{(\eta)}_{\nu\nu'}(\varepsilon) + \sum_{\mu\mu'} \tilde{G}^{(\eta)}_{\nu\mu}(\varepsilon)\Sigma^{r(\eta)}_{\mu\mu'}(\varepsilon)G^{(\eta)}_{\mu'\nu'}(\varepsilon)$$

$$\Sigma^{r(+)}_{\nu\nu'}(\varepsilon) = \sum_{\nu''\mu}\Big[\frac{\Gamma^{(11)\mu}_{\nu\nu''}\Gamma^{(11)\mu*}_{\nu'\nu''}}{\varepsilon - E_{\nu''} - \omega_\mu + i\delta} + \frac{\Gamma^{(02)\mu*}_{\nu\nu''}\Gamma^{(02)\mu}_{\nu'\nu''}}{\varepsilon + E_{\nu''} + \omega_\mu - i\delta}\Big]$$



HFB basis

Dynamical self-energy: acquires the same form as the non-superfluid one!

Superfluid quasiparticle-vibration coupling (QVC) vertices:

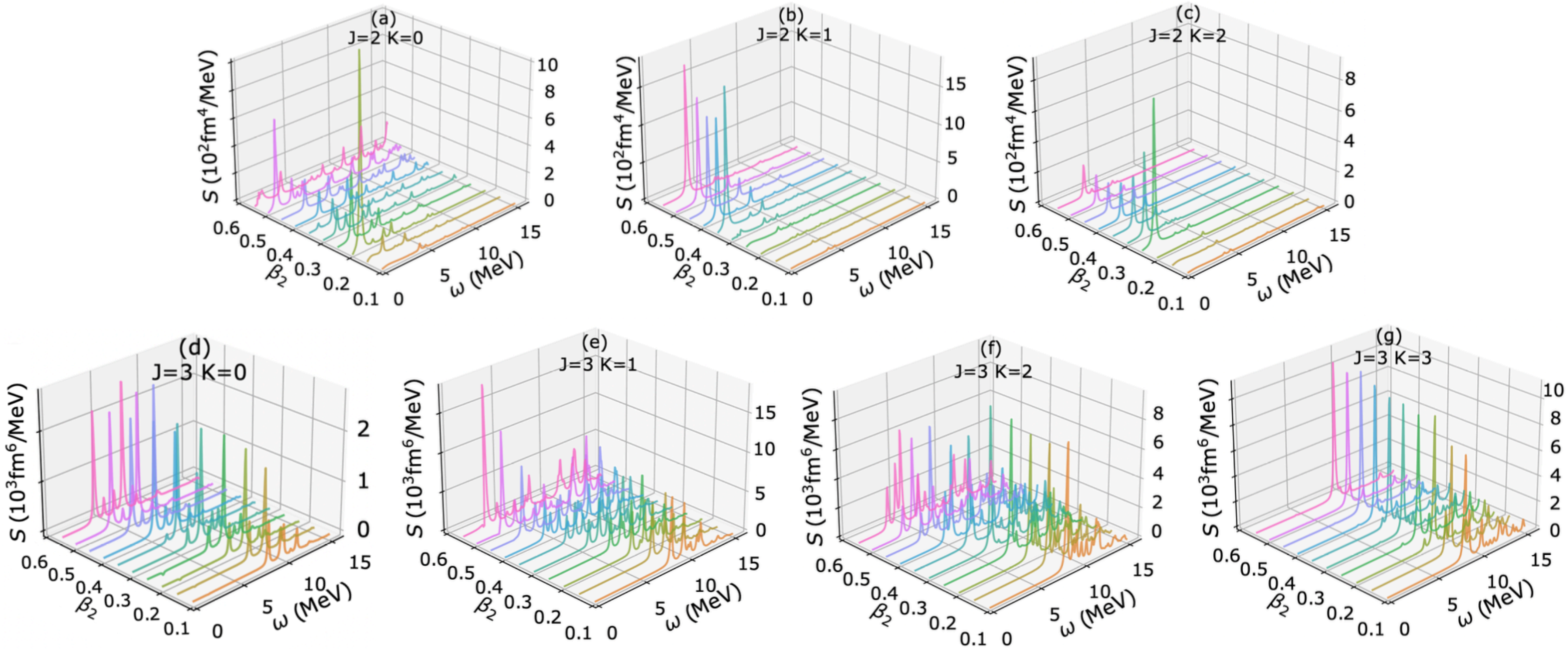
$$\Gamma^{(11)\mu}_{\nu\nu'} = \sum_{12}\Big[U^\dagger_{\nu 1}g^\mu_{12}U_{2\nu'} + U^\dagger_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - V^\dagger_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - V^\dagger_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

$$\Gamma^{(02)\mu}_{\nu\nu'} = -\sum_{12}\Big[V^T_{\nu 1}g^\mu_{12}U_{2\nu'} + V^T_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - U^T_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - U^T_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

E.L., Y. Zhang, PRC 104, 044303 (2021)

The phonon spectrum in ^{38}Si and QVC

(i) Relativistic meson-nucleon Lagrangian + (ii) Relativistic Hartree-Bogoliubov (RHB) + (iii) Quasiparticle random phase approximation (QRPA): $J = 2^+ - 5^-$, $K = [0,J]$. Finite amplitude method (FAM): A. Bjelčić et al., CPC 253, 107184 (2020). Relativistic DD-PC1 interaction.



(iv) QVC vertex extraction:

$$\Gamma^{(ij)\varkappa}_{\nu\nu'} = \lim_{\delta\to 0}\sqrt{\frac{\delta}{\pi S(\omega_\varkappa)}}\,\mathrm{Im}\Big(\delta\mathcal{H}^{(ij)}_{\nu\nu'}(\omega_\varkappa + i\delta)\Big)$$

Variation of the HFB Hamiltonian at the QRPA pole

(v) Dyson Eq. solution

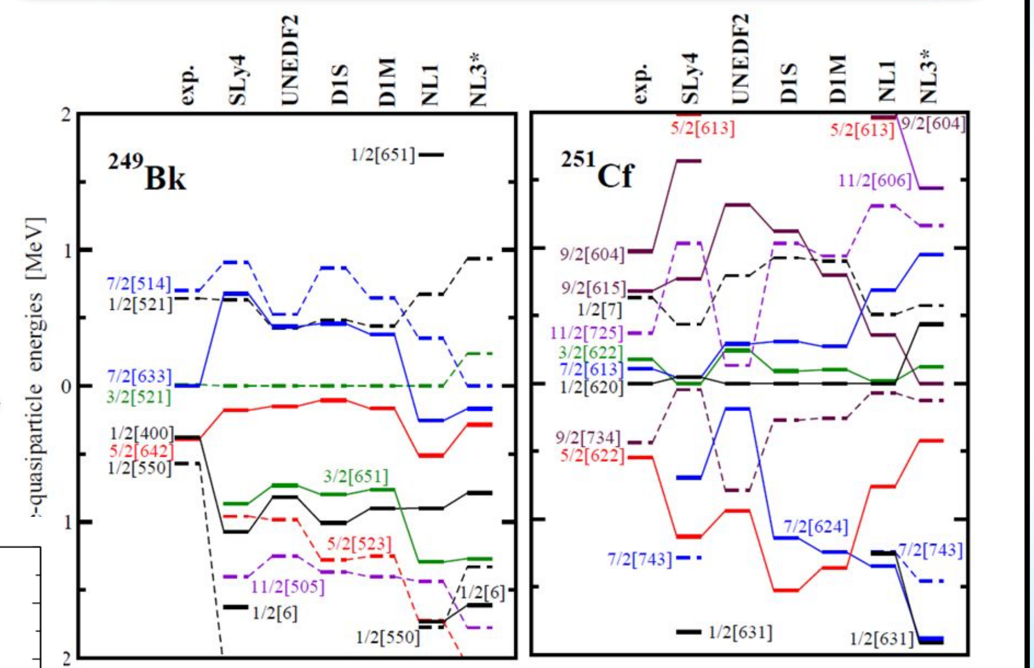
[E.L., Y. Zhang, PRC 104, 044303 (2021)]

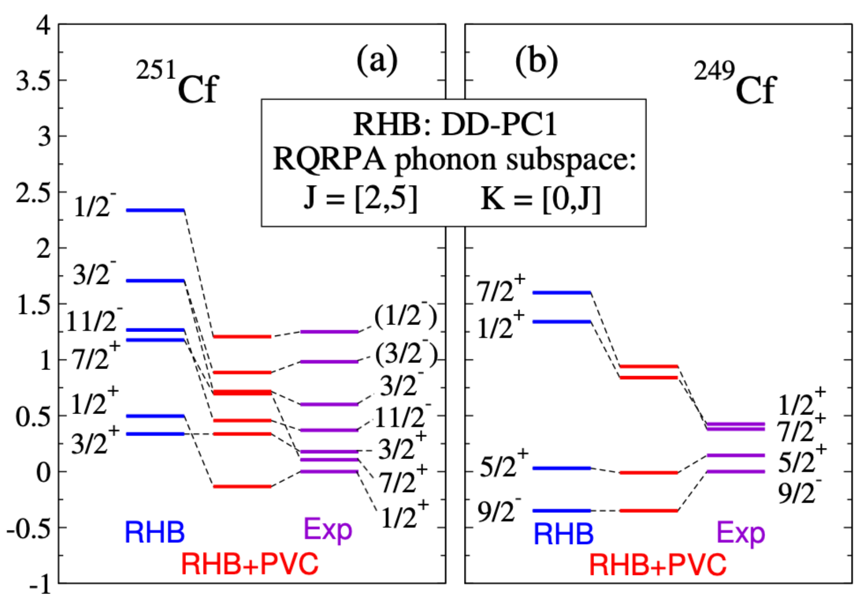
Single-particle states in ^{249,251}Cf

Deformed one-quasiparticle states: covariant and non-relativistic mean-field calculations vs experiment:

A. Afanasjev et al.: Long-standing problem of the description of single-particle states in deformed nuclei.

Systematic studies for ^{249}Bk and ^{251}Cf in the mean-field approximation:





Beyond mean field: RHB+QVC calculations. Dominant fragments in ^{251}Cf and ^{249}Cf.

The spectroscopic factors are quenched even stronger than in spherical nuclei. Can this be measured?

Y. Zhang et al., PRC 105, 044326 (2022)

Extended FAM (preliminary):

QVC vertex extraction:

$$\Gamma^{(ij)\varkappa}_{\nu\nu'} = \lim_{\delta\to 0}\sqrt{\frac{\delta}{\pi S(\omega_\varkappa)}}\,\mathrm{Im}\Big(\delta\mathcal{H}^{(ij)}_{\nu\nu'}(\omega_\varkappa + i\delta)\Big)$$

Variation of the HFB Hamiltonian at the QRPA pole

Generalized FAM (FAM-QVC)

QVC

$$\delta\mathcal{R}^{(20)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{20}_{\mu\nu}(\omega) + \sum_{\mu'\nu'}\Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(20)}_{\mu'\nu'}(\omega) + F^{20}_{\mu\nu}}{\omega - E_\mu - E_\nu}$$

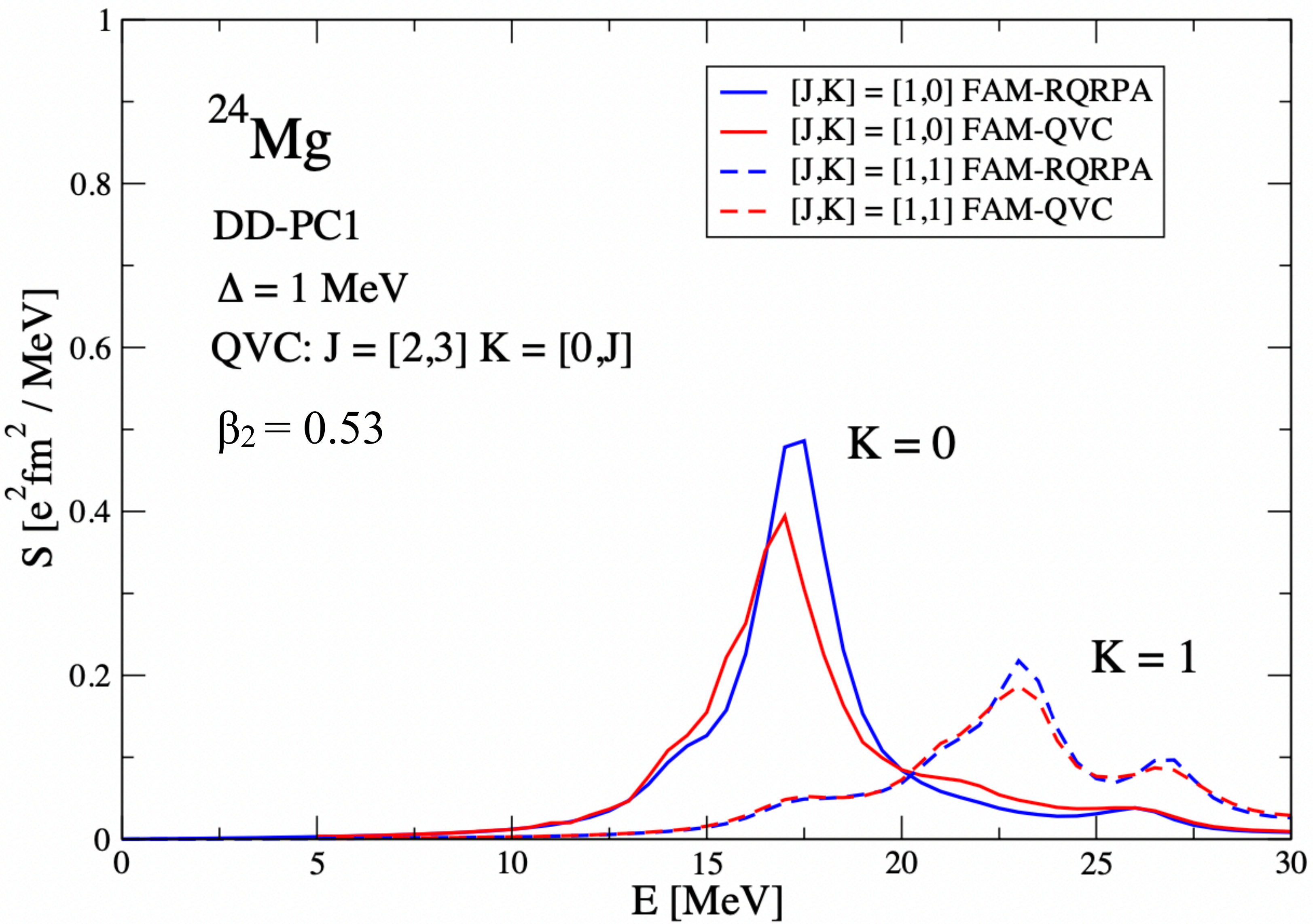
$$\delta\mathcal{R}^{(02)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{02}_{\mu\nu}(\omega) + \sum_{\mu'\nu'}\Phi^{(-)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(02)}_{\mu'\nu'}(\omega) + F^{02}_{\mu\nu}}{-\omega - E_\mu - E_\nu}.$$

QVC amplitude:

$$\Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega) = \sum_n\Big[\delta_{\mu\mu'}\sum_{\nu''}\frac{\bar\Gamma^{(11)n}_{\nu''\nu}\bar\Gamma^{(11)n*}_{\nu''\nu'}}{\omega - E_\mu - E_{\nu''} - \omega_n} + \delta_{\nu\nu'}\sum_{\mu''}\frac{\Gamma^{(11)n}_{\mu\mu''}\Gamma^{(11)n*}_{\mu'\mu''}}{\omega - E_{\mu''} - E_\nu - \omega_n} - \\ - \frac{\Gamma^{(11)n}_{\mu\mu'}\bar\Gamma^{(11)n*}_{\nu\nu'}}{\omega - E_{\mu'} - E_\nu - \omega_n} - \frac{\Gamma^{(11)n*}_{\mu'\mu}\bar\Gamma^{(11)n}_{\nu'\nu}}{\omega - E_\mu - E_{\nu'} - \omega_n}\Big];$$

E.L., Y. Zhang, in progress (2022)

FAM-QVC (preliminary):



E.L., Y. Zhang, in progress (2022)

Finite-temperature response: the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^\dagger_1\psi_2)(t)\psi^\dagger_{2'}\psi_{1'})(t') > \;\rightarrow\; \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^\dagger_1\psi_2)(t)\psi^\dagger_{2'}\psi_{1'})(t') >_T$$

$$< ... > \equiv < 0|...|0 > \;\rightarrow\; < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$$

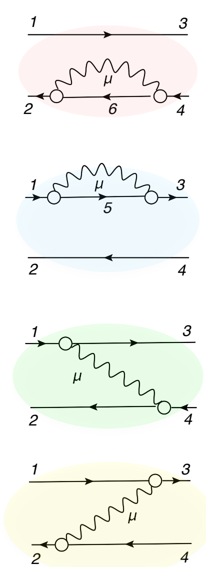
averages → *thermal averages*

Method: EOM for Matsubara Green's functions

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + $$
$$+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\big[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\big]\mathcal{R}_{3'4,4'3}(\omega,T)$$
$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

T > 0: *1p1h+phonon dynamical kernel:* *T = 0:*

$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)}\sum_{\mu;\eta_\mu=\pm1}\eta_\mu\Big[\delta_{13}\sum_6 \gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64}\times$$
$$\times\frac{\big(N(\eta_\mu\Omega_\mu)+n_6(T)\big)\big(n(\varepsilon_6-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_6-\eta_\mu\Omega_\mu} +$$
$$+\;\delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35}\times$$
$$\times\frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_5(T)\big)}{\omega-\varepsilon_5+\varepsilon_2-\eta_\mu\Omega_\mu} -$$
$$-\;\gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24}\times$$
$$\times\frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_3(T)\big)}{\omega-\varepsilon_3+\varepsilon_2-\eta_\mu\Omega_\mu} -$$
$$-\;\gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42}\times$$
$$\times\frac{\big(N(\eta_\mu\Omega_\mu)+n_4(T)\big)\big(n(\varepsilon_4-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_4-\eta_\mu\Omega_\mu}\Big],$$

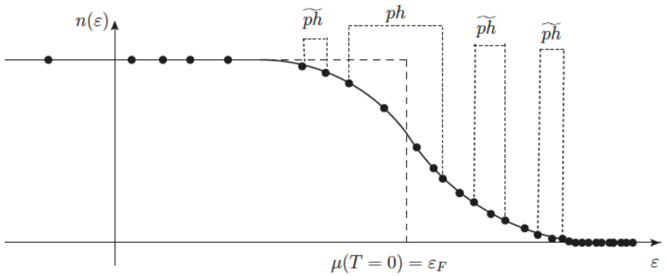


$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times$$
$$\times\Big[\delta_{13}\sum_6\frac{\gamma^\mu_{62}\gamma^{\mu*}_{64}}{\omega-\varepsilon_1+\varepsilon_6-\Omega_\mu} + \delta_{24}\sum_5\frac{\gamma^\mu_{15}\gamma^{\mu*}_{35}}{\omega-\varepsilon_5+\varepsilon_2-\Omega_\mu} - \frac{\gamma^\mu_{13}\gamma^{\mu*}_{24}}{\omega-\varepsilon_3+\varepsilon_2-\Omega_\mu} - \frac{\gamma^{\mu*}_{31}\gamma^\mu_{42}}{\omega-\varepsilon_1+\varepsilon_4-\Omega_\mu}\Big]$$

The role of the exponential factor: low-energy strength

$$S(E,T) = -\frac{1}{\pi} \lim_{\Delta \to +0} Im \, \langle V^{0\dagger} \mathcal{R}(E + i\Delta, T) V^0 \rangle$$

Thermal unblocking:

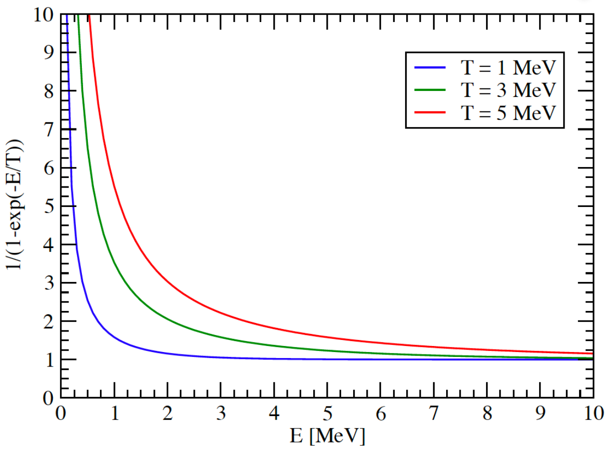


The final strength function at T>0:

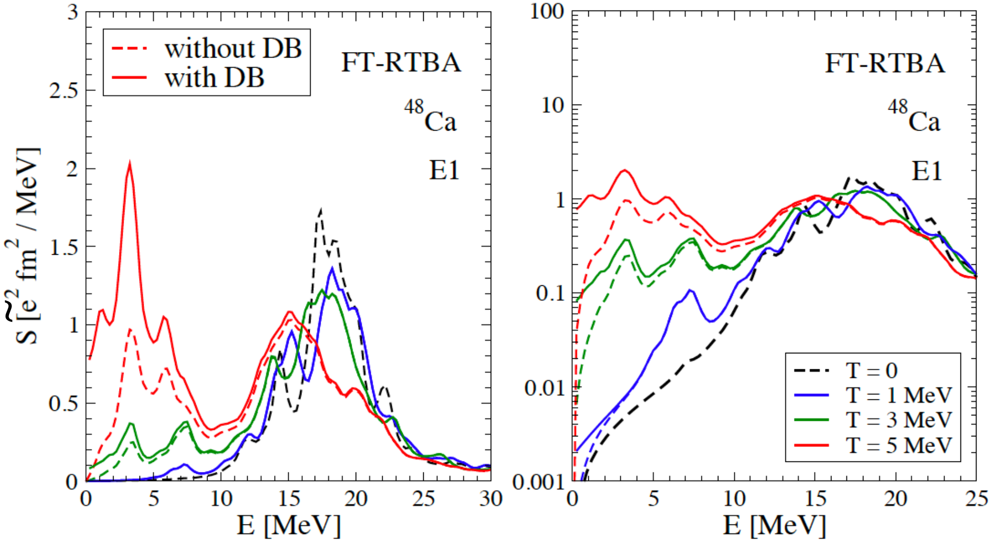
$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

$$\lim_{E \to 0} S(E,T) = 0$$

*The **generic** exponential factor:*



Dipole strength: absorption at T>0:



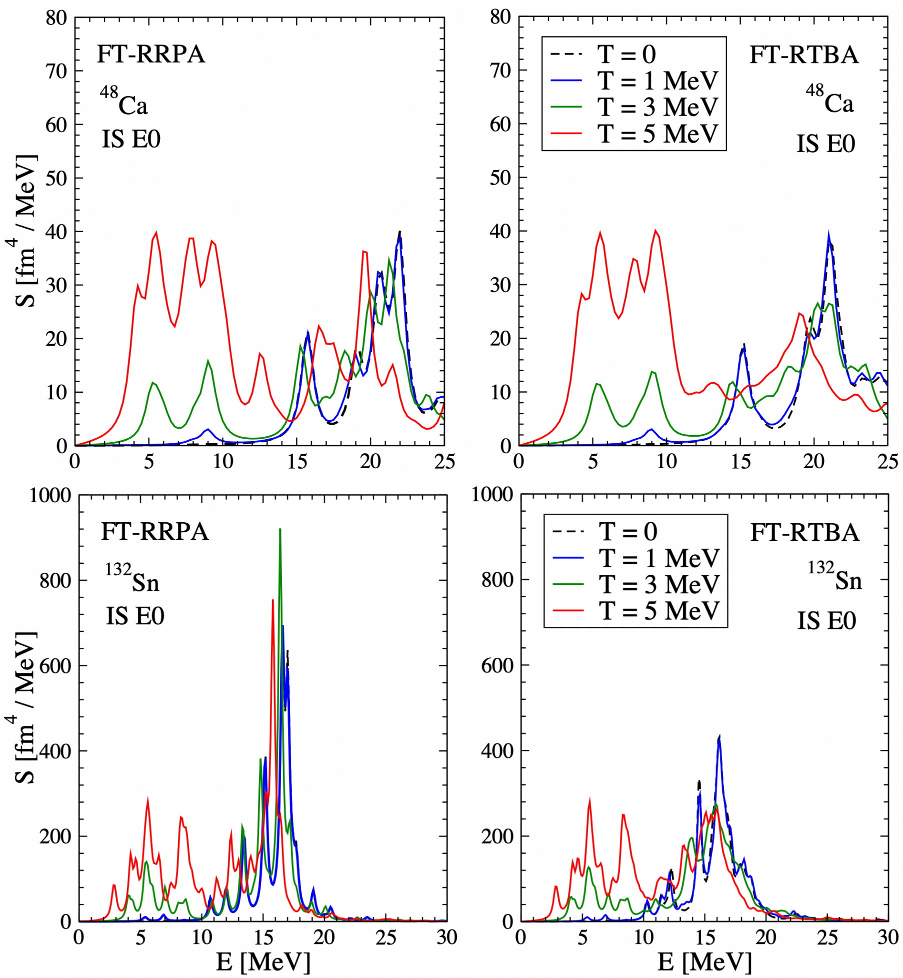
- *The exponential factor brings an additional enhancement in E<T energy region and provides the finite zero-energy limit of the strength (regardless its spin-parity)*

E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)

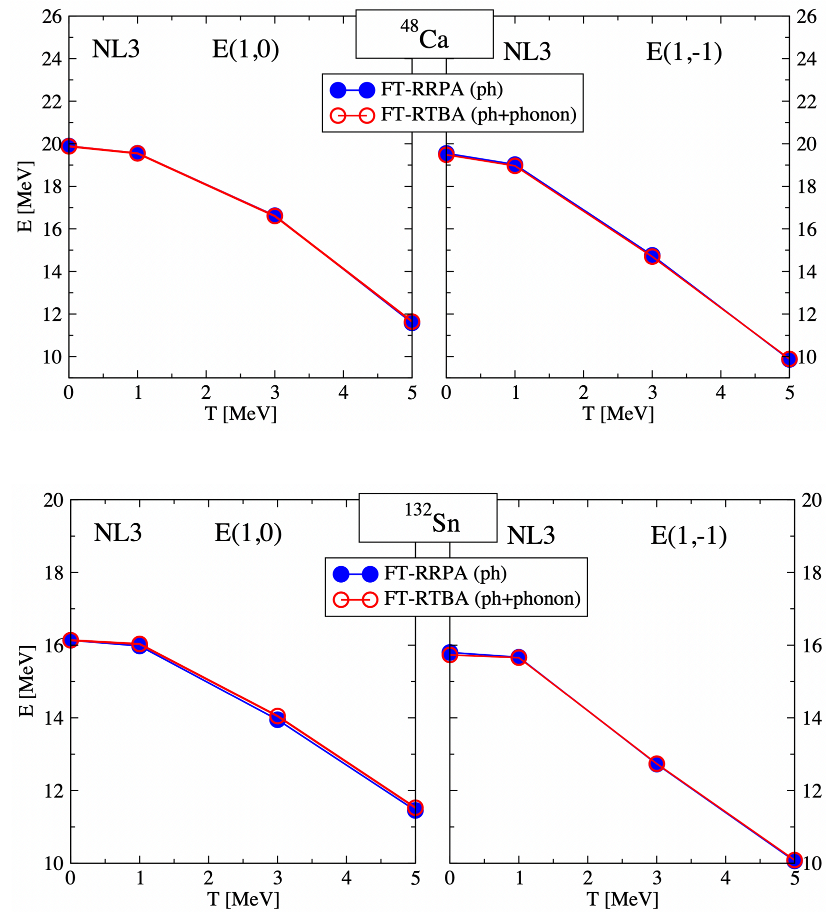
H. Wibowo, E.L., Phys. Rev. C 100, 024307 (2019)

Temperature evolution of the ISGMR

Strength distribution
(Exponential factor not included)



Centroids

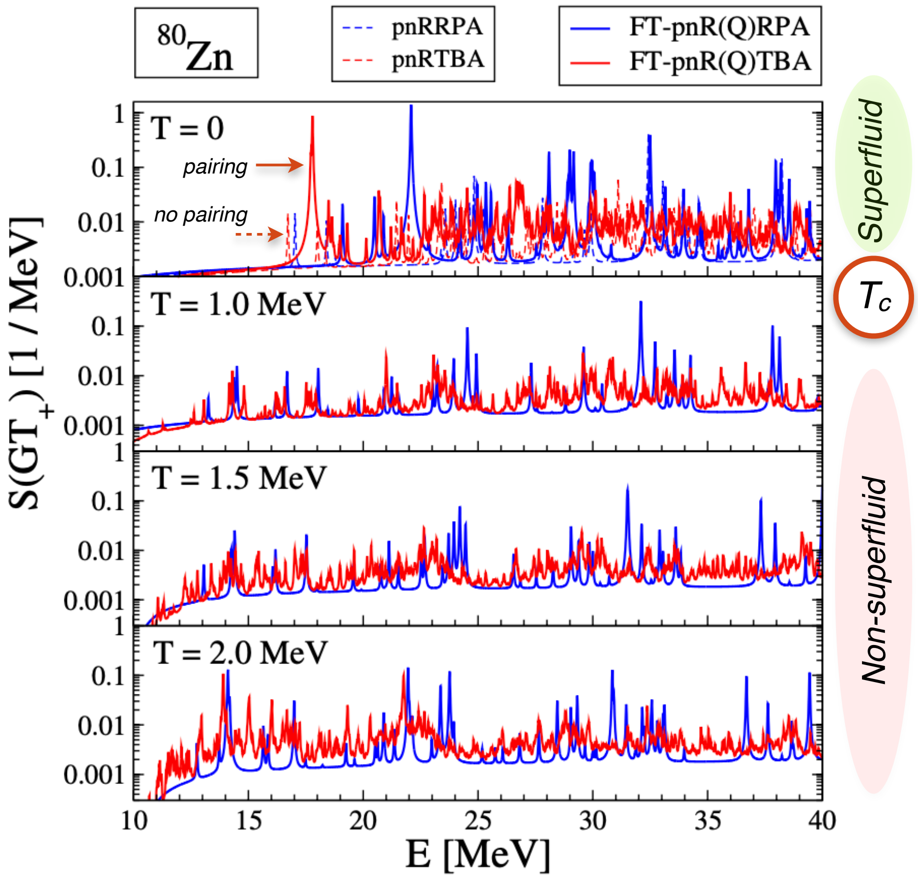


E.L., H. Wibowo, PRL 121, 082501 (2018)
E.L., H. Wibowo, EPJA 55, 223 (2019)

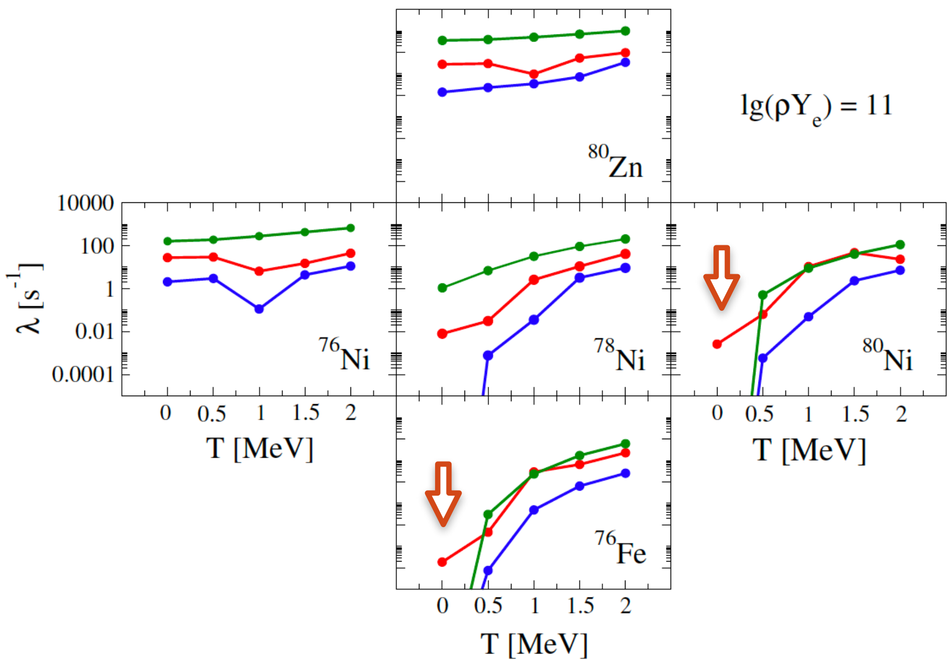
- *Major effect: softening as T grows*
- *Equation of State (EOS) to be modified*

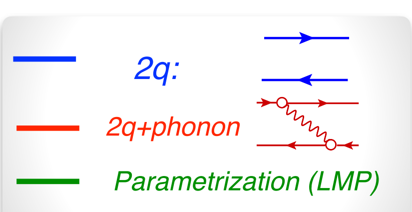
GT+ response and electron capture (EC) rates at T>0: the neighborhood of ^{78}Ni

GT+ response



Electron capture rates around ^{78}Ni





E.L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)
E.L., H. Wibowo, PRL 121, 082501 (2018)
E.L., C. Robin, PRC 103, 024326 (2021)

***Interplay** of superfluidity and collective effects in core-collapse supernovae:*

- *Amplifies the EC rates and, consequently,*
- *Reduces the electron-to-baryon ratio leading to lower pressure*
- *Promotes the gravitational collapse*
- *Increases the neutrino flux and effective cooling*
- *Allows heavy nuclei to survive the collapse*

Pairing gap (J=0) beyond BCS

Fermionic pair propagator:

$$G\,(12,1'2') = (-i)^2\langle T\psi(1)\psi(2)\psi^{\dagger}(2')\psi^{\dagger}(1')\rangle$$

$$iG_{12,1'2'}(\omega) = \sum_{\mu}\frac{\alpha^{\mu}_{21}\alpha^{\mu *}_{2'1'}}{\omega-\omega^{(++)}_{\mu}+i\delta} - \sum_{\varkappa}\frac{\beta^{\varkappa *}_{12}\beta^{\varkappa}_{1'2'}}{\omega+\omega^{(--)}_{\varkappa}-i\delta}$$

N+2   *N-2*

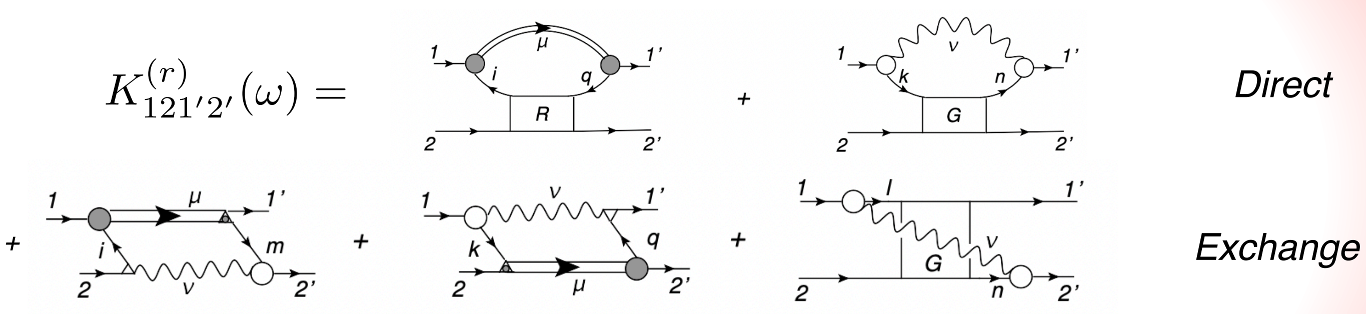
$$\alpha^{\mu}_{12} = \langle 0^{(N)}|\psi_2\psi_1|\mu^{(N+2)}\rangle$$

$$\beta^{\varkappa}_{12} = \langle 0^{(N)}|\psi^{\dagger}_2\psi^{\dagger}_1|\varkappa^{(N-2)}\rangle$$

E.L., P.Schuck, Phys. Rev. C 102, 034310 (2020)

Dynamical kernel ("minimal" truncation):

$$K^{(r)}_{121'2'}(\omega) =$$



EOM at ω = ω_s:

$$\alpha^{s}_{21} = \frac{1-n_1-n_2}{\omega_s-\tilde{\varepsilon}_1-\tilde{\varepsilon}_2}\frac{1}{4}\sum_{343'4'}\delta_{1234}K_{343'4'}(\omega_s)\alpha^{s}_{4'3'}$$

$$\Delta_1 = 2E_1\alpha^{s}_{1\bar{1}}$$

$\omega_s \sim 2\lambda$:

$$\Delta_1 = -\sum_{2}\mathcal{V}_{1\bar{1}2\bar{2}}\frac{\Delta_2}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2}\Big(K^{(0)}_{121'2'}+K^{(r)}_{121'2'}(2\lambda)\Big)$$

Formalism at T>0

Averages redefined:

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi_1^\dagger\psi_2)(t)\psi_{2'}^\dagger\psi_{1'})(t') > \quad\rightarrow\quad \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi_1^\dagger\psi_2)(t)\psi_{2'}^\dagger\psi_{1'})(t') >_T$$

Grand Canonical average: $< ... > \equiv < 0|...|0 > \quad\rightarrow\quad < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$

Matsubara imaginary-time formalism: temperature-dependent dynamical kernel

Direct:

$$\mathcal{K}^{(r;11)}_{121'2'}(\omega_n) = -\sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Theta^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1\big)$$
$$-\sum_{\nu\varkappa}\frac{\Theta^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1\big)\Big]$$

Exchange:

$$\mathcal{K}^{(r;12)}_{121'2'}(\omega_n) = \sum_{\nu'\nu''} w_{\nu'}w_{\nu''}$$
$$\times\Big[\sum_{\nu\mu}\frac{\Sigma^{\mu\nu;\nu'\nu''(+)}_{121'2'}}{i\omega_n - \omega_{\nu\nu'} - \omega^{(++)}_{\mu\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(++)}_{\mu\nu''})/T} - 1\big)$$
$$-\sum_{\nu\varkappa}\frac{\Sigma^{\varkappa\nu;\nu'\nu''(-)}_{121'2'}}{i\omega_n + \omega_{\nu\nu'} + \omega^{(--)}_{\varkappa\nu''}}\big(e^{-(\omega_{\nu\nu'}+\omega^{(--)}_{\varkappa\nu''})/T} - 1\big)\Big],$$

E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

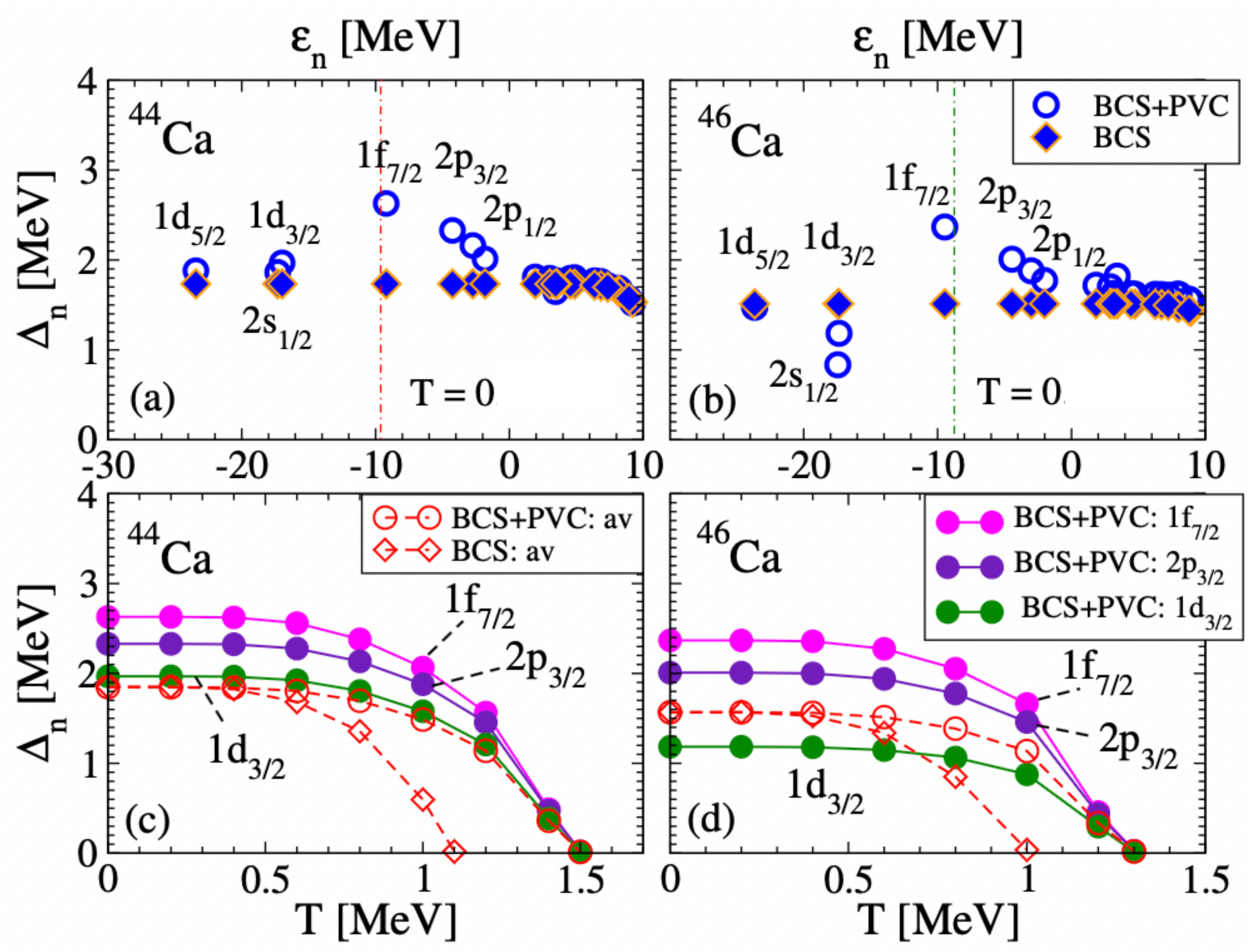
BCS-like gap Eq., but with non-trivial T-dependence in $K^{(r)}$:

$$f_1(T) = \frac{1}{\exp(E_1/T) + 1}$$

$$\Delta_1(T) = -\sum_2 \mathcal{V}_{1\bar{1}2\bar{2}}\frac{\Delta_2(T)(1 - 2f_2(T))}{2E_2}$$

$$\mathcal{V}_{121'2'} = \frac{1}{2}\Big(K^{(0)}_{121'2'} + K^{(r)}_{121'2'}(2\lambda)\Big)$$

Pairing gap at T = 0, T>0 and critical temperature



E.L., P.Schuck, Phys. Rev. C 104, 044330 (2021)

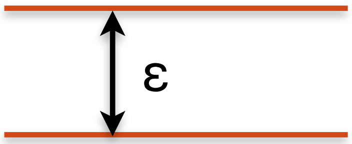
Are there theoretical limits on accuracy?

- Higher-rank configurations = higher accuracy? Can we quantify this? How accurately we can describe the observed spectra, in principle?
- Spectroscopic accuracy in nuclear structure: experiment (laser spectroscopy [eV], nuclear resonance fluorescence [keV]) … no standards for theory. ~100 keV?
- Chemical accuracy 1 kcal/mol = 0.043 eV is possible with the gold standard for quantum chemistry calculations, namely the canonical coupled cluster (CC) expansion truncated at the second order in the electronic excitation operator and including an approximate treatment of the triple excitations (CCSD(T), where S stands for single, D for double, and (T) for non-iterative triple) [P.J. Ollitrault et al, Phys. Rev. Res. 2, 043140, 2020]
- CCSD(T) includes up to (correlated) 3p3h configurations and scales as $O(N^7)$ with the number of degrees of freedom N of the model Hamiltonian.
- In nuclear structure, there are relatively rare calculations with (correlated) 3p3h configurations for medium-heavy nuclei (QPM, EOM/RQTBA^3, CC). The results are still not ideal.
- Is the problem in the underlying strong "forces", which are not weak and known with limited accuracy? Or the many-body methods? Likely both.
- Working with model (solvable) Hamiltonians allows one to solely focus on the many-body problem. Can be studied with quantum and hybrid algorithms on NISQ devices.

Lipkin Hamiltonian on quantum computer

Two-level Lipkin (Meshkov-Glick), LMG, Hamiltonian:

$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right) - \frac{w}{2}\left(\hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+\right)$$



Quasispin operators:

$$\hat{J}_z = \frac{1}{2}\sum_{p=1}^{N}\left(\hat{a}_{p,+}^\dagger \hat{a}_{p,+} - \hat{a}_{p,-}^\dagger \hat{a}_{p,-}\right),$$

N = 2j + 1

$$\hat{J}_+ = \sum_{p=1}^{N} \hat{a}_{p,+}^\dagger \hat{a}_{p,-} \text{ and } \hat{J}_- = \left(\hat{J}_+\right)^\dagger$$

Monopole excitations

Configuration complexity:

Excitation operator:

$$\hat{O}_n^\dagger = \sum_\alpha \sum_{\mu_\alpha}\left[X_{\mu_\alpha}^\alpha(n)\hat{K}_{\mu_\alpha}^\alpha - Y_{\mu_\alpha}^\alpha(n)\left(\hat{K}_{\mu_\alpha}^\alpha\right)^\dagger\right]$$

$$\hat{K}_{\mu_1}^1 = a_i^\dagger a_{j'} \qquad \hat{K}_{\mu_2}^2 = a_i^\dagger a_j^\dagger a_{j'} a_{i'} \qquad \ldots$$

M. Hlatshwayo, R. LaRose et al., arXiv:2203.01478, Phys. Rev. C (2022)

Lipkin Hamiltonian on quantum computer

The algorithm: *Variational Quantum Eigensolver (VQE) + quantum EOM (qEOM)*

- *VQE: a minimal encoding scheme is found (“J-scheme”) and implemented, based on the symmetry of the LMG Hamiltonian. Yields an accurate ground state l0>.*
- *qEOM generates efficiently the EOM matrix:*

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^* & \mathcal{A}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix} = E_{0n} \begin{bmatrix} \mathcal{C} & \mathcal{D} \\ -\mathcal{D}^* & -\mathcal{C}^* \end{bmatrix} \begin{bmatrix} X^n \\ Y^n \end{bmatrix}$$

$$\mathcal{A}_{\mu_\alpha \nu_\beta} = \langle 0| \left[\left(\hat{K}^{\alpha}_{\mu_\alpha} \right)^\dagger, \left[\hat{H}, \hat{K}^{\beta}_{\nu_\beta} \right] \right] |0\rangle$$

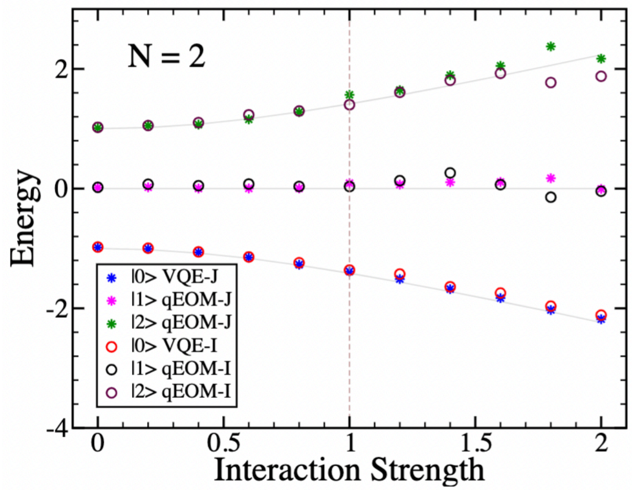
$$\mathcal{B}_{\mu_\alpha \nu_\beta} = - \langle 0| \left[\left(\hat{K}^{\alpha}_{\mu_\alpha} \right)^\dagger, \left[\hat{H}, \left(\hat{K}^{\beta}_{\nu_\beta} \right)^\dagger \right] \right] |0\rangle$$

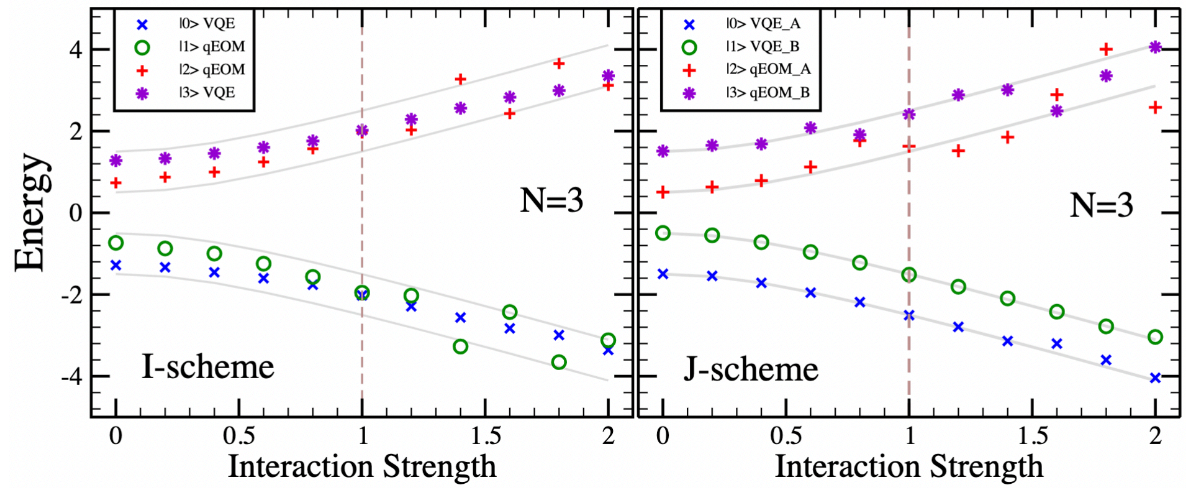
$$\mathcal{C}_{\mu_\alpha \nu_\beta} = \langle 0| \left[\left(\hat{K}^{\alpha}_{\mu_\alpha} \right)^\dagger, \hat{K}^{\beta}_{\nu_\beta} \right] |0\rangle$$

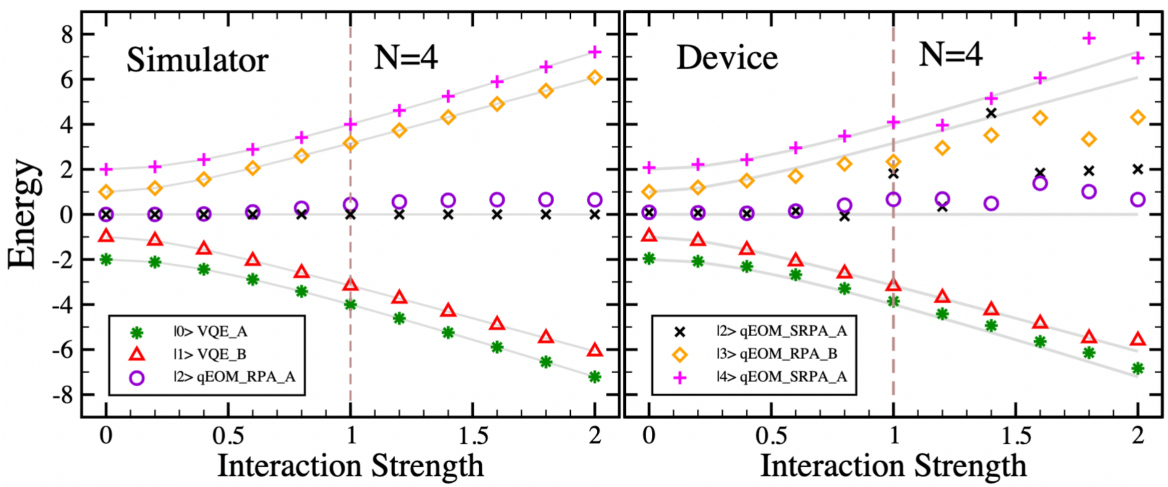
$$\mathcal{D}_{\mu_\alpha \nu_\beta} = - \langle 0| \left[\left(\hat{K}^{\alpha}_{\mu_\alpha} \right)^\dagger, \left(\hat{K}^{\beta}_{\nu_\beta} \right)^\dagger \right] |0\rangle .$$

M. Hlatshwayo, R. LaRose et al., arXiv:2203.01478, Phys. Rev. C (2022)

Lipkin Hamiltonian on quantum computer: hardware results







Conventions:

- *n_q = number of states*
- *N = number of particles*
- *ν = v /ε effective interaction strength*

- ***I-scheme:*** *individual spin basis, $n_q = 2^N$*

- ***J-scheme:*** *total spin basis (coupled form), symmetry: $n_q = N/2 + 1$*

Observations:

- *Higher-rank excitation ~ higher accuracy*
- *Stronger coupling ~ lower accuracy*
- *More particles ~ lower accuracy*
- *Less qubits ~ higher accuracy*

M. Hlatshwayo, R. LaRose et al., arXiv:2203.01478, Phys. Rev. C (2022)

Outlook

Summary:

- The nuclear field theory (NFT) is formulated and advanced in the Equation of Motion (EOM) framework, with the emphasis on emergent degrees of freedom.
- The emergent collective effects renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- Relativistic NFT is generalized to finite temperature and applied to neutral and charge-exchange response of medium-heavy nuclei as well as to the studies of nuclear superfluidity.
- The presented study of the monopole excitations suggest that correlations beyond mean field/QRPA are significant and enhanced in open-shell nuclei. Together with the finite-temperature effects, they have important astrophysical implications.

Current and future developments:

- Deformed nuclei: correlations vs shapes; first results on quasiparticle states just released (Yinu Zhang et al.);
- HFB pairing: EOM for the response function; formulation underway;
- Toward an "ab initio" description: implementations with bare NN-interactions;
- Superfluid pairing at T>0 to extend the application range (r-process);
- Efficient algorithms for strong coupling regimes; quantum computing for increasing N and α (Manqoba Hlatshwayo);
- Relativistic EOM's, bosonic EOM's, hadron physics, neutron stars,…

Many thanks for collaboration and support:

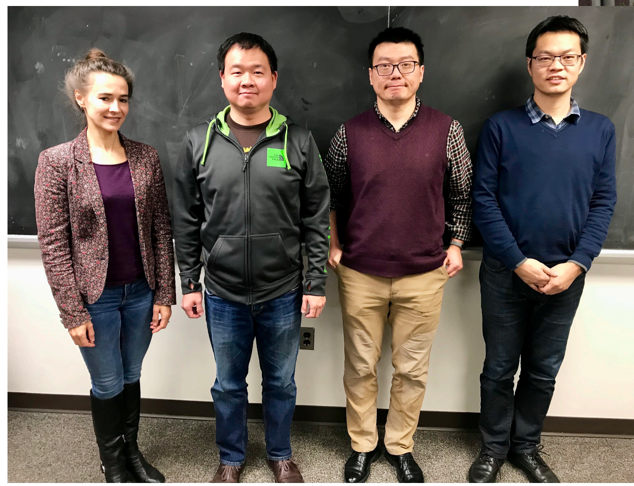
Yinu Zhang (WMU)
Manqoba Hlatshwayo (WMU)
Herlik Wibowo (AS Taipei)
Caroline Robin (U. Bielefeld & GSI)
Peter Schuck (IPN Orsay)
Peter Ring (TU München)
Tamara Niksic (U Zagreb)

US-NSF PHY-1404343 (2014-2018)

NSF CAREER PHY-1654379 (2017-2023)

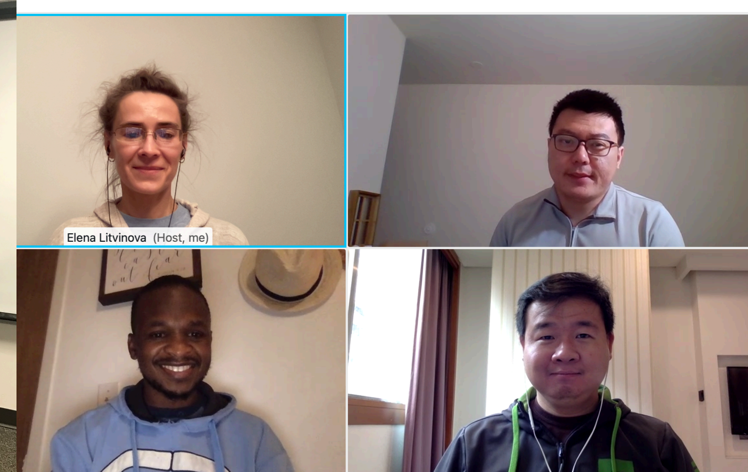
2017:

2018:

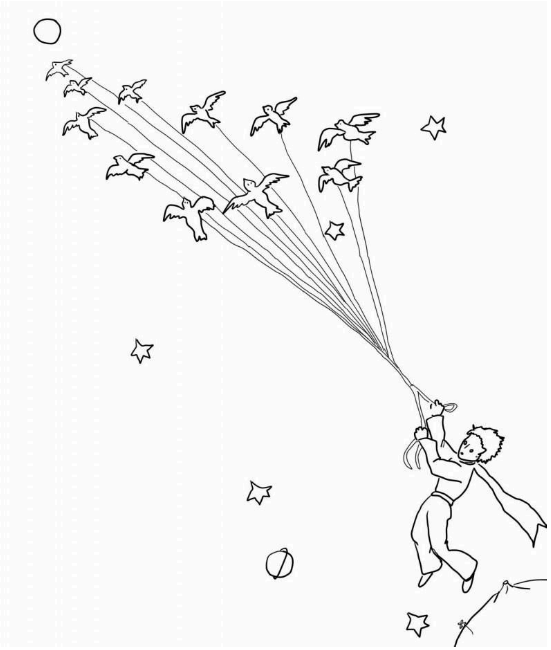


2019-2020:

2020-2022:

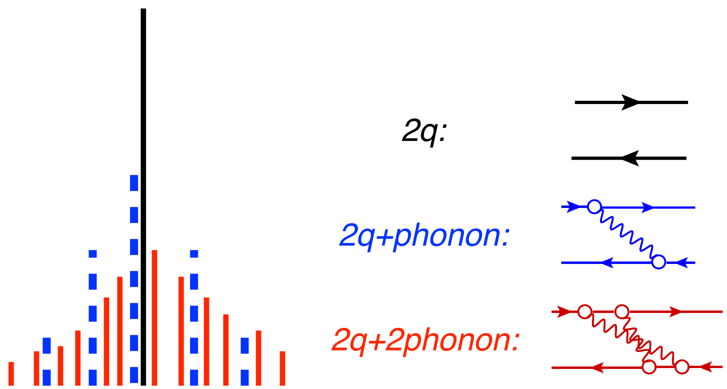


Thank you!

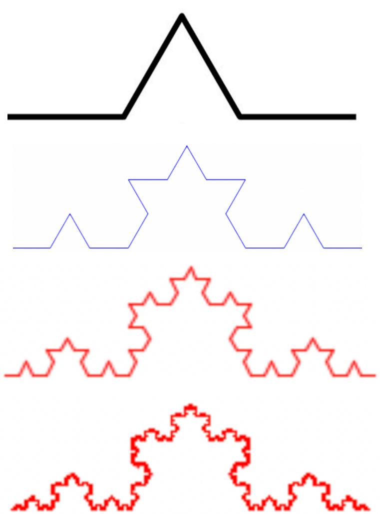


Excitation spectrum: Hierarchy of configuration complexity

Fragmentation mechanism



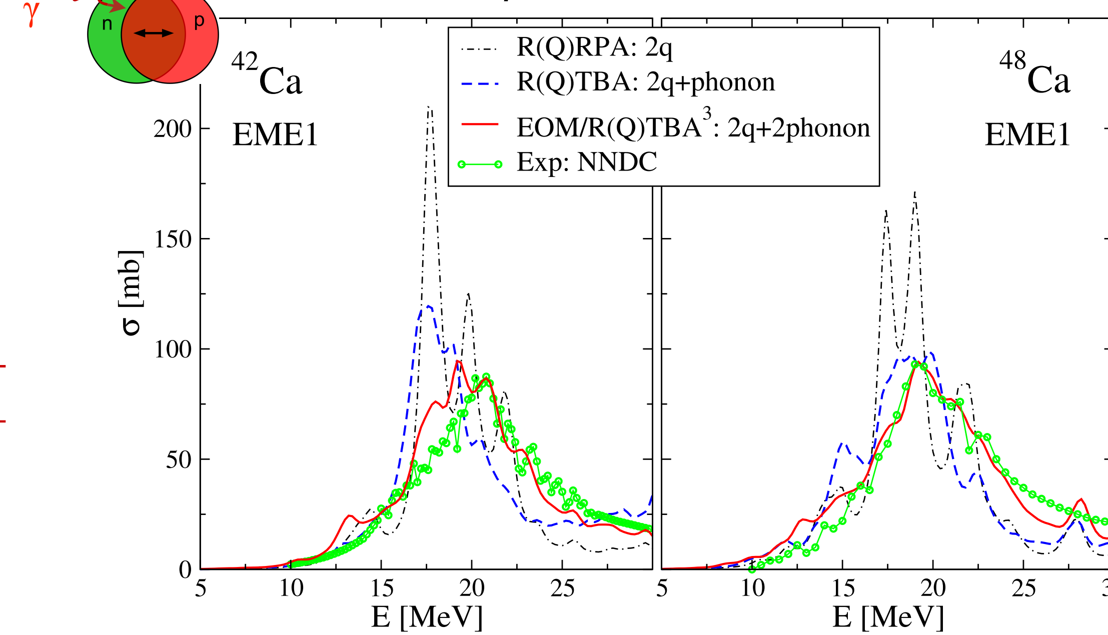
Fractals: Koch curve



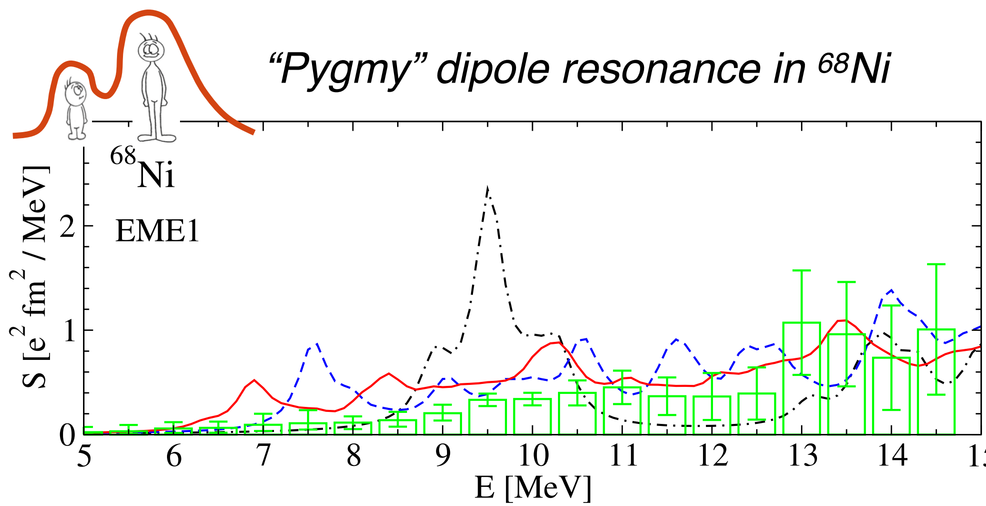
Gross structure

Fine structure

Giant dipole resonance in $^{42,48}Ca$

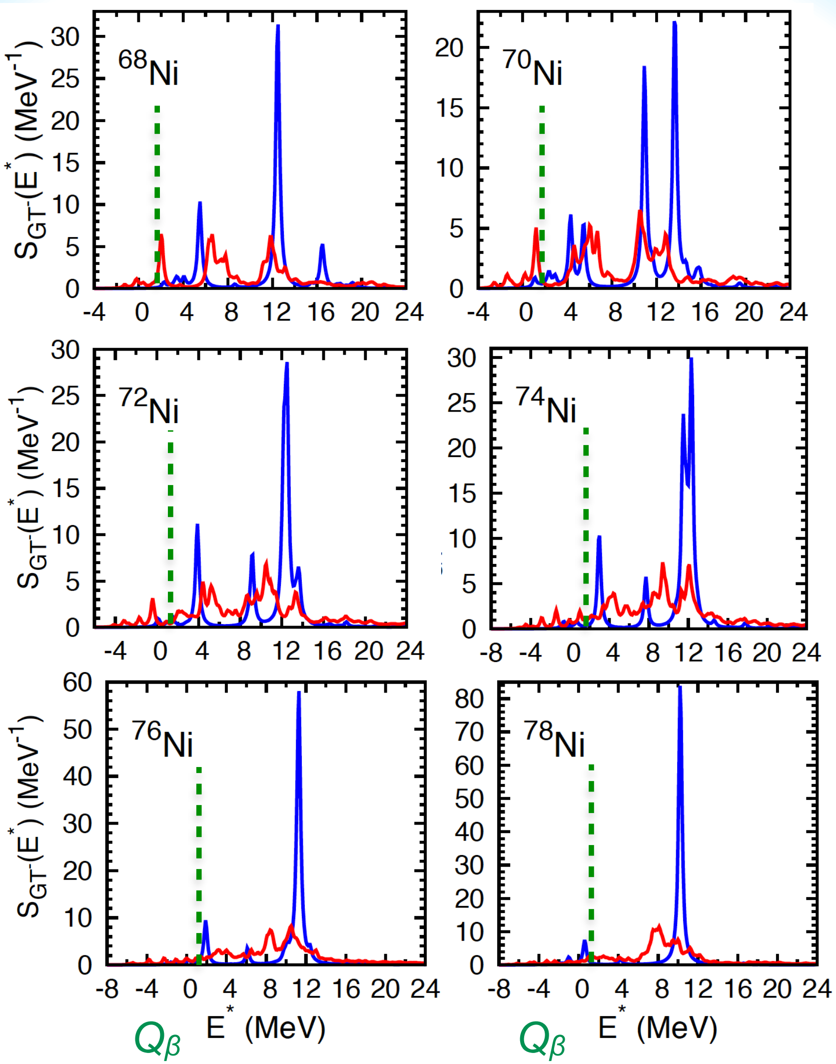


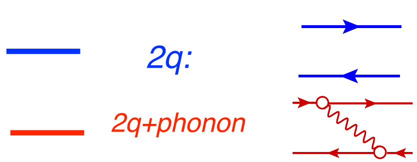
"Pygmy" dipole resonance in ^{68}Ni

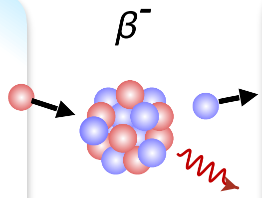


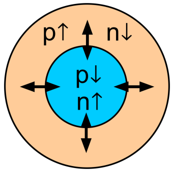
Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)

Spin-isospin excitations: Gamow-Teller resonance in neutron-rich nickel









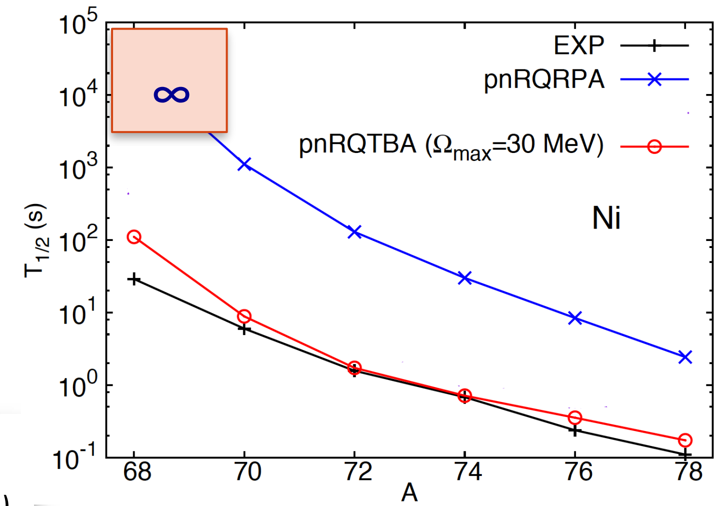
Dynamical pn-pairing

Excitation operator:

$$P = \sum_i \sigma^{(i)} \tau_-^{(i)}$$

β⁻ decay half-life

$$T_{1/2}^{-1} = \frac{g_A^2}{D} \int_{Q_\beta} f(Z, \Delta_{np} - E) S(E) dE$$



C. Robin, E.L., EPJA 52, 205 (2016)
C. Robin, E.L., PRL 123, 202501 (2019)