Adiabatic Continuity and Spectral Conspiracy in Adjoint QCD

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Gauge Topology 3:
from Lattice to Colliders,
Trento

- QCD is **the** theory of hadrons (discovered early 1970s)

✷ It is rich, self-consistent and beautiful

- The most interesting phenomena are at strong coupling

- Solving theories at strong coupling is notoriously difficult, even classical (cf. turbulence)

- 50 years in search of sol'n. Only islands of knowledge

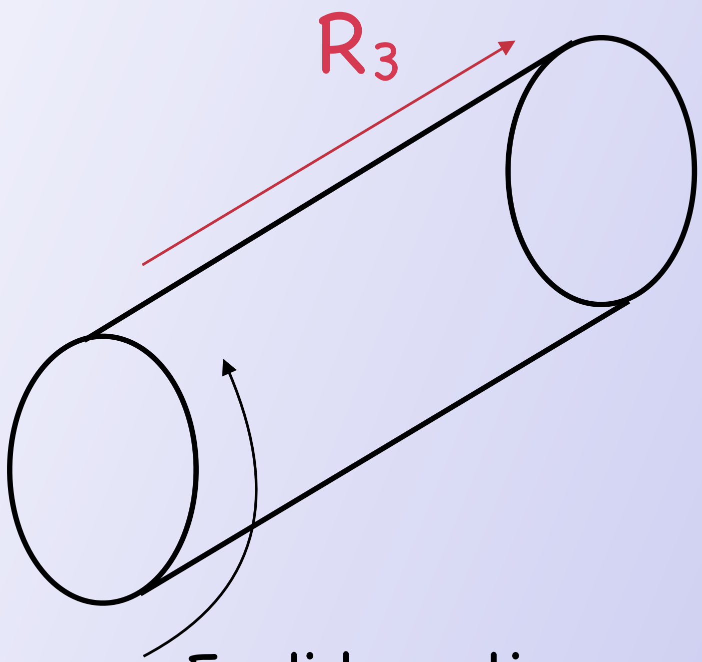
Promises:

Instantons 1974;
Large-N_c 1976;
OPE & Heavy quark expansions 1979-1995;
Supersymmetry: 1983-1994;
Seiberg-Witten model 1994-…;
Holography (ADS/QCD) 1998-2000's;

Every time – euphoric hopes, but alas…

No complete sol'n;
Still, not in vain!
Each finding is a discovery of a new island of knowledge!

A journey to one island began from A. Polyakov:
Thermal Yang-Mills



Euclidean time τ

$R_4 \rightarrow R_3 \times S_1$

Large $r(S_1)$ return to R_4

Circumference of time direction $\tau = \beta = 1/T$

Periodic b.c.

Polyakov line

$$\Omega = \mathrm{P}\exp\left\{ i\int_0^\beta d\tau A_0(\tau,\vec{x}) \right\}$$

$$A_0 \equiv A_0^a T^a\,, \qquad\qquad A_0(0,\vec{x}) = A_0(\beta,\vec{x})$$

In the vacuum no x dependence,

Ω is a unitary N$\times$N matrix, $\boxed{\Omega\ \Omega^{+} = 1}$

Upon diagonalization Ω = diag$\{e^{i\delta_1}, e^{i\delta_2}, \ldots, e^{i\delta_N}\}$

Polyakov's criterion of confinement

$$\mathrm{Tr}\ \Omega = 0$$

$$\Omega = \mathrm{diag}\{e^{i\delta_1}, e^{i\delta_2}, \ldots, e^{i\delta_N}\}$$

Large T (small β) ➔ weak coupling, PT (quasiclassics) is applicable if

$$\beta \ll \Lambda^{-1}, \qquad T \gg \Lambda$$

Small T (large β) ➔ strong coupling, all phases oscillate like crazy, each eigenvalue averages to 0!

Center-symmetric phase

What is the center of a group?

- The center of a group is the set of elements in the group which commute with all the elements of the group.
- For SU(Nc) the center is Z_{Nc}. Namely C is an element of the center iff $C=z_i\times\mathbf{1}$, with z_i^{Nc} equal 1.

- What is a center transformation in a gauge theory?

$$A_\mu \to A'_\mu = U A_\mu U^\dagger - U\partial_\mu U^\dagger$$

$$U(\tau+\beta) = CU(\tau)$$

- Since, it seems to act so much like a gauge transformation, how do we know it is physical?
  - There exist gauge invariant (i.e. physical) observables which are NOT invariant under center transformation!
  - The Polyakov loop is the simplest example.

Indeed, $\Omega \rightarrow U(0)\,\Omega\,U^{+}(\beta)$

Polyakov loop is NOT invariant under non-trivial center transformations! Tr Ω is the order parameter!

Tr Ω=0 → center symm. OK; Tr $\Omega \neq 0$ → CS broken

Gross-Pisarsky-Yaffe effective potential for Tr Ω in YM

$$V_{\mathrm{eff}} = -\frac{2}{\pi^2 L^4} \sum_{n \geq 1} \frac{1}{n^4} \left[|\mathrm{tr}\Omega^n|^2 - 1 \right]$$

$$\downarrow$$

All eigenvalues clump say, at δ=0, Tr $\Omega \neq 0$, hence CS broken

DECONFINEMENT, phase transition at $\beta \sim 1/\Lambda$

Nothing you calculate at large T (small β) is useful in the strong coupling regime

Many years later: Ünsal and collaborators:

Can one change the theory at short distances in such a way that CS (and other appropriate symmetries) are preserved, and there is no phase transition on the way to large distances?

The answer is YES!

A couple of ways ensuring smooth journey were found, of which I like one particular better than others because it is quite physical: add Weyl (Majorana) fermions in the adjoint representation. Planar equivalence with AS quarks
(just quarks at N=3)

N_f Weyl fermions: What is expected on R_4?

N_f =1 ☞ Supersymmetry (SYM) /No PT potential

N_f =2 ☞ $SU(2)_{flavor} \rightarrow U(1)_{flavor}$, $Z_{2NN\_flavor} \rightarrow Z_2$

....

N_f =3 ☞ probably similar

N_f =4 ☞ ???

N_f =5 ☞ Banks-Zaks conformal regime

N_f =6 ☞ loss of asymptotic freedom!

If $\beta \neq 0$ and large, add center symmetry

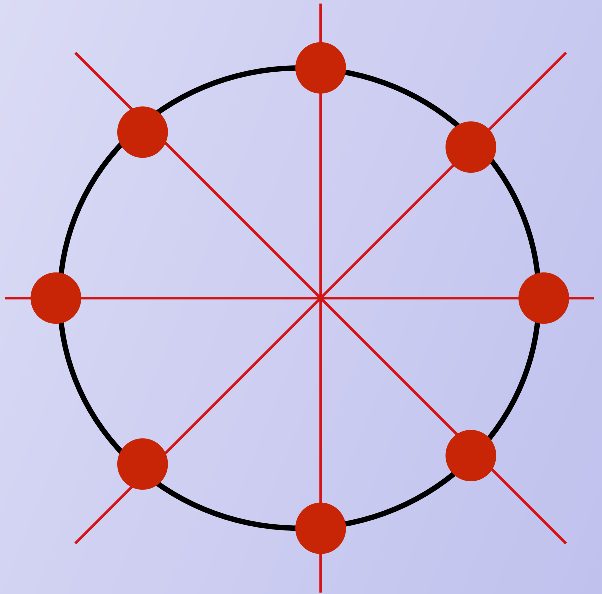
$R_4 \to R_3 \times S_1$, β small, quasiclassical domain

Ünsal and collaborators ☞ periodic b.c. for fermions (spatial)

$$V_{\rm eff}(\Omega) = \frac{2(N_F-1)}{\pi^2 L^4} \sum_{n\geq 1} \frac{1}{n^4} \left[|{\rm tr}\Omega^n|^2 - 1 \right]$$

The overall sign of $V_{\rm eff}$ changes if $N_f = 2, \ldots$

$e^{i\delta_1}, e^{i\delta_2}, \ldots, e^{i\delta_N}$



Tr $\Omega = 0$

CS → OK

The same refers to prvs works with fundamentals (Ü+MS)

$SU(2)_{\rm flavor}$ unbroken 😭

No smooth journey

What's to be done? (if anything)

$$\psi(x_3 + \beta) = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \psi(x_3)$$

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} .$$

SU(2)$_{\text{flavor}}$ → U(1)$_{\text{flavor}}$ explicitly 😀 ☞ $F = Q_{U(1)_F}$

Euclidean path integral computes a twisted partition function

$$\tilde{Z}(L, \varphi) = \mathrm{tr}(-1)^F e^{-\beta \widehat{H}} e^{i\varphi \widehat{Q}_{U(1)_F}}$$

Smooth journey achieved in the interval $0<|\varphi|< 2\pi/3N$!

$$V_{\text{eff}}(\Omega;\varphi) = -\frac{2}{\pi^2 L^4}\sum_{n\geq 1}\frac{1}{n^4}\left[|\text{tr}\Omega^n|^2 - 1\right] + \frac{2}{\pi^2 L^4}\sum_{n\geq 1}\frac{2\cos(n\varphi)}{n^4}\left[|\text{tr}\Omega^n|^2 - 1\right]$$

$$\downarrow$$

To preserve center symmetry we need $|\varphi| < \dfrac{2\pi}{3N}$

What about the Hagedorn phase transition?

In a similar situations (with periodic b.c.) it was suggested by Basar et al.

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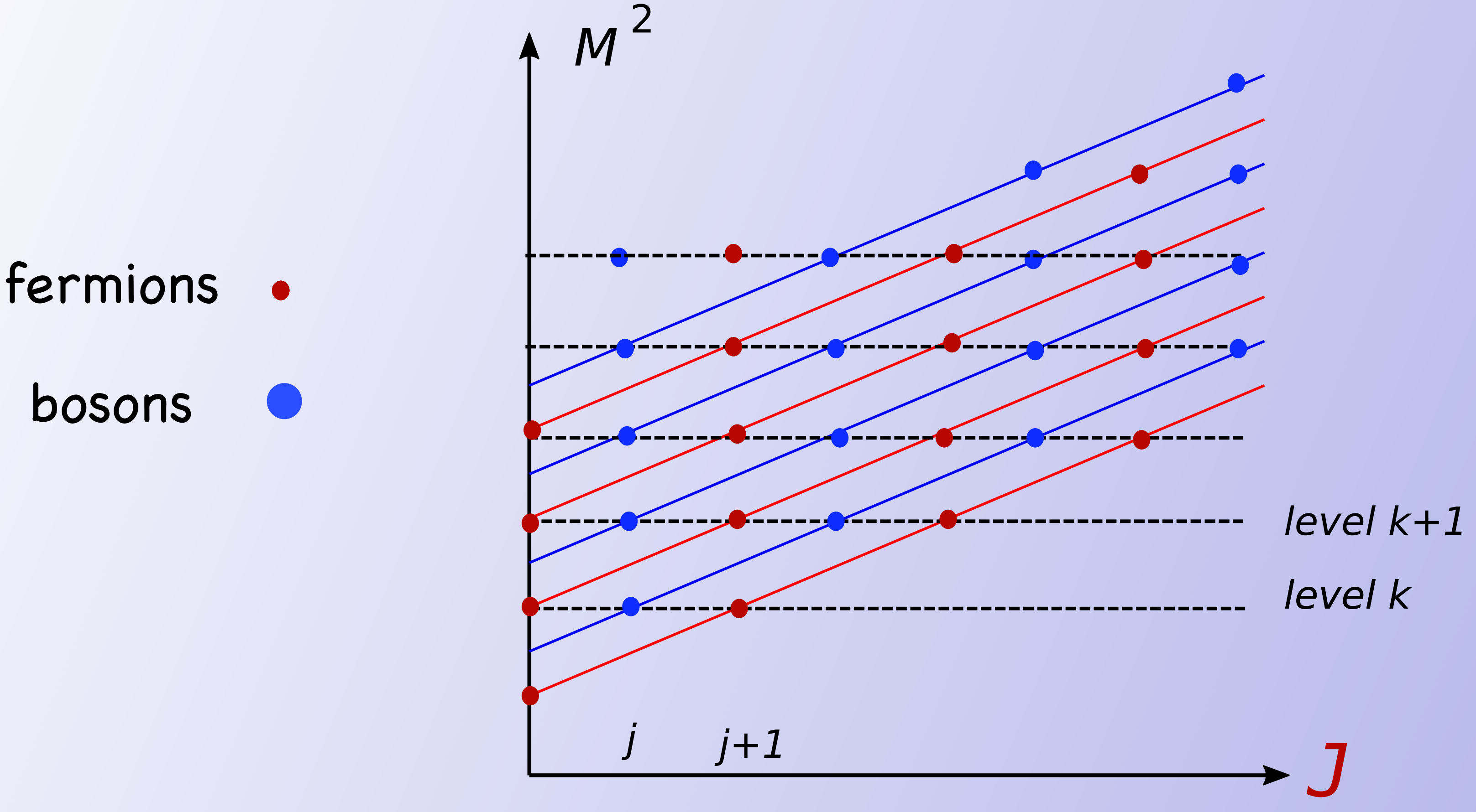
that a "supersymmetry" dynamically emerges even though the theory under consideration had no SUSY in the Lagrangian. This was totally counter-intuitive, in fact, counter-everything!

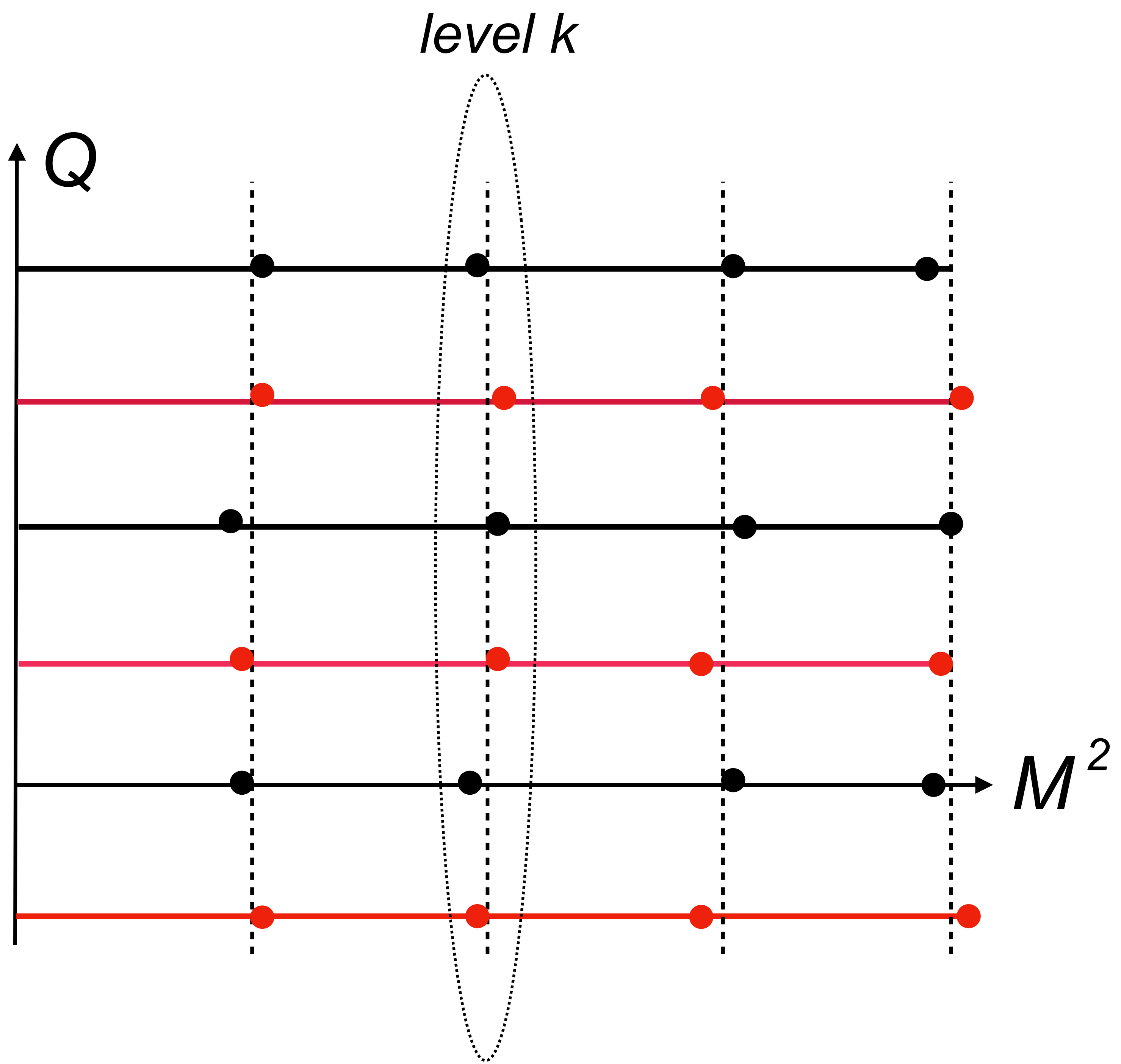
Now we see that this is impossible because the phase φ is not just zero

↓

Spectral conspiracies

The absence of all Hagedorn phase transitions in $\tilde{Z}(L,\varphi)$ requires much more contrived spectral conspiracies, non-local in mass, i.e. involving a large number of levels with different fermion numbers and masses. Indeed, the Hagedorn phase transitions must be absent in $\tilde{Z}(L,\varphi)$ for *any* value of φ from the interval $0<\varphi<2\pi(3N)^{-1}$. The conventional, "local" Bose-Fermi degeneracy by itself does not provide cancellations in $\tilde{Z}(L,\varphi) = \mathrm{tr}(-1)^F e^{-L\widehat{H}} e^{i\varphi\widehat{Q}_{U(1)_F}}$ because of different phase factors for the boson and fermion sates in the given pair.





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Adiabatic Continuity in Adjoint QCD

➜

Spectral Conspiracy

A strong qualitative result