Lattice calculation of sphaleron diffusion rate in gluodynamics

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Gauge Topology 3: from Lattice to Colliders

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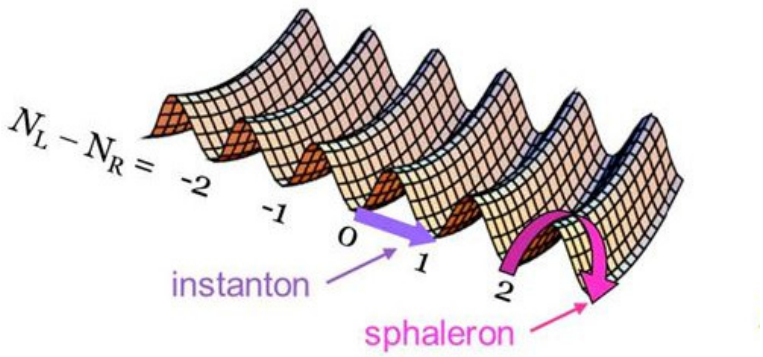
Topology changing transitions in YM

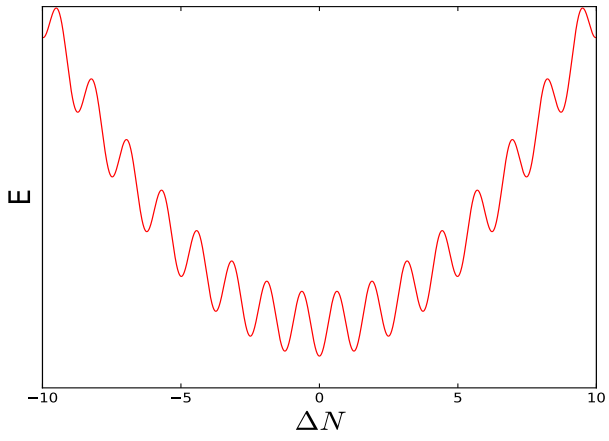


$$\Gamma_{CS} = \lim_{t\to\infty} \frac{\langle \Delta N_{CS}^2 \rangle}{Vt}$$

$$\Delta N_{CS} = Q_4 = \int d^4x q(x)$$

$$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

Sphaleron diffusion rate and chiral charge



- EW theory: baryon number n_B violation
- QCD: quark chirality non-conservation
- Fluctuation-dissipation theorem

$$\frac{dQ_5}{dt} = -CQ_5 \frac{\Gamma_{CS}}{2T}$$

- C depends on the fermionic part of the theory

Sphaleron diffusion rate

Weak coupling

$$\Gamma_{CS} \sim \alpha^5 \log 1/\alpha \, T^4$$

"Desperate extrapolation" to finite α (semiclassical field theory):

$$\Gamma_{CS} \sim 30 \alpha_s^4 T^4$$

G.D. Moore and M.Tassler, *The Sphaleron Rate in SU(N) Gauge Theory*, JHEP 1102 (2011) 105, arXiv:1011.1167

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Holography

- $N = 4SYM$: $\Gamma = \frac{(g^2 N)^2}{256\pi^3} T^4$

  D.T. Son, A.O. Starinets, *Minkowski space correlators in AdS / CFT correspondence: Recipe and applications*, JHEP 0209 (2002) 042, arXiv: hep-th/0205051

- Improved holographic QCD (large-N_c): $\Gamma(T_c)/T_c^4 > 1.64$

  U. Gursoy et al., *The Chern-Simons Diffusion Rate in Improved Holographic QCD*, JHEP 1302 (2013) 119, arXiv:1212.3894

Sphaleron diffusion rate and lattice

Topological charge density correlator $G(t) = \langle \int d^3x q(0) q(\bar{x}, t) \rangle$

$$\Gamma_{CS} = -\lim_{\omega \to 0} \frac{2T}{\omega} \mathrm{Im}[G^R(\omega, \vec{k} = 0)] = -2\pi T \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$$G_E(t) = \int_0^\infty \rho(\omega) \frac{\cosh \omega(\beta/2 - t)}{\sinh \omega\beta/2} d\omega$$

Similar to

- shear and bulk viscosity N. Astrakhantsev, V. Braguta, AK, 2017, 2018 H.B. Meyer, 2007
- electric conductivity G. Aarts et al., 2014
- heavy quark diffusion O. Kaczmarek, 2014

Topological charge density and Wilson flow

Wilson flow

$$\partial_\tau A_\mu(\tau, t, x) = -\frac{\partial S}{\partial A_\mu}$$

Topological charge density and Wilson flow

Wilson flow

$$\partial_\tau A_\mu(\tau, t, x) = -\frac{\partial S}{\partial A_\mu}$$

- $\partial_\tau q^\tau = \partial_\rho \omega^\tau_\rho$, $\partial_\tau Q^\tau = 0$
- $\langle q^\tau(x) q^\tau(0) \rangle = \langle q^{\tau=0}(x) q^{\tau=0}(0) \rangle + O_\tau(a^2) + O(\tau)$
- GF gives renormalized $q(x)$

M. Ce, G. P. Engel, L. Giusti, *Non-Gaussianities in the topological charge distribution of the SU(3) Yang–Mills theory*
Phys.Rev. D92 (2015) no.7, 074502, arXiv:1506.06052

Lattice details

- $SU(3)$ Gluodynamics + Gradient flow
- No multilevel algorithm
- Clover gluonic discretization of $F\tilde{F}$
- Continuum extrapolation
- Scale setting using GF (reference scale ω_0):
  M.Kitazawa, T.Iritani, M.Asakawa, T. Hatsuda, H.Suzuki, *Equation of State for SU(3) Gauge Theory via the Energy-Momentum Tensor under Gradient Flow*
  Phys.Rev. D94 (2016) no.11, 114512, arXiv:1610.07810

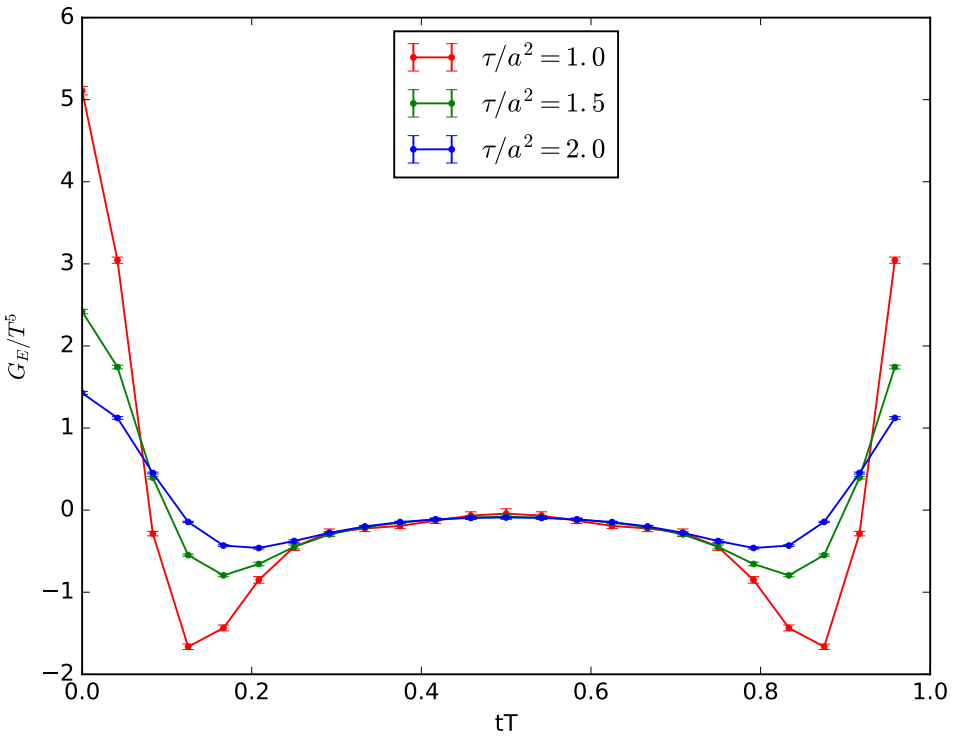
$T/T_c = 1.24$	N_{conf}	β	$T/T_c = 1.70$	N_{conf}	β
12×36^3	5474	6.4934	12×36^3	6663	6.7269
16×48^3	8081	6.7084	16×48^3	9207	6.9494
20×64^3	4732	6.88	20×64^3	2897	7.1265
24×70^3	1765	7.0232	24×70^3	1757	7.2738
16×32^3	9108	6.7084			

Procedure

1. Generate configurations for one temperature and various lattice steps
2. Perform GF on each configuration
3. Measure $G_q(t, \tau, a)$
4. Extrapolate $G_q(t, \tau) = \lim_{a \to 0} G_q(t, \tau, a)$
5. Extrapolate $G_q(t) = \lim_{\tau \to 0} G_q(t, \tau)$
6. Spline interpolation, if needed
7. Use $G_q(t)$ to invert integral relation and extract Γ_{CS}

How does the correlator $G_q(t, \tau, a)$ look like?

$24 \times 70^3,\ T/T_c = 1.24$



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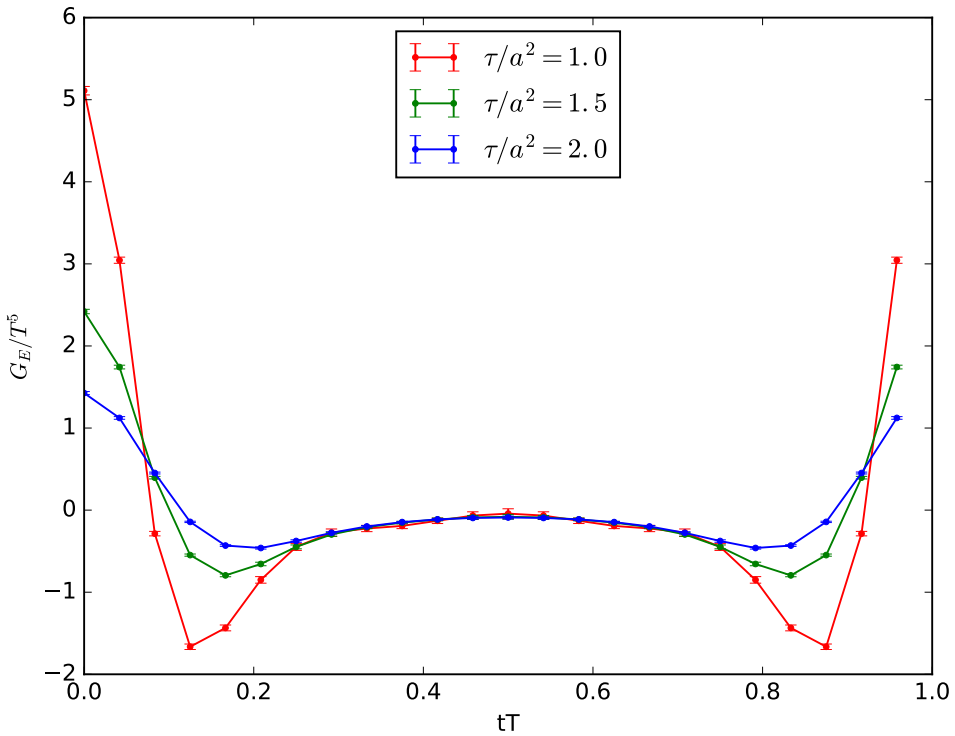
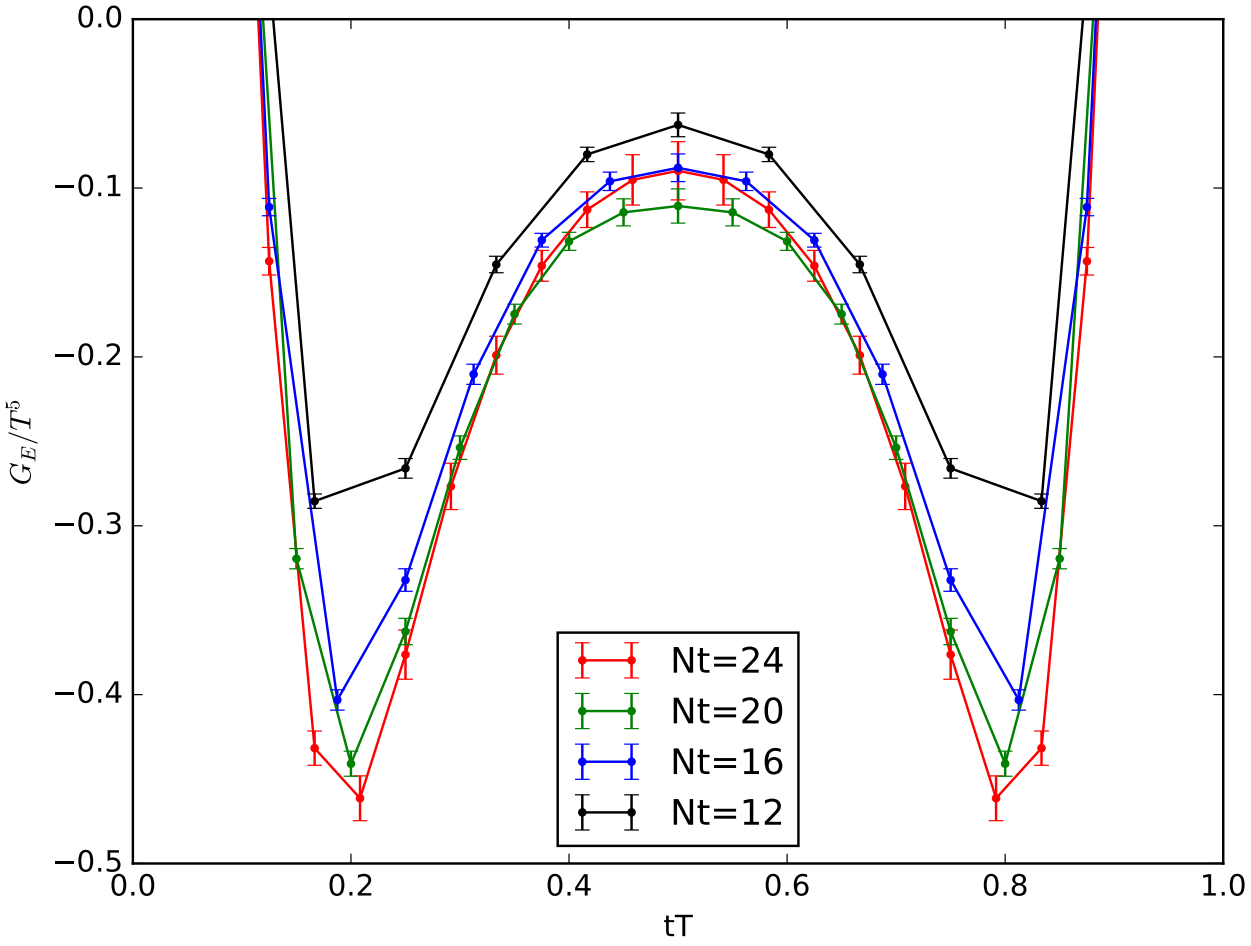


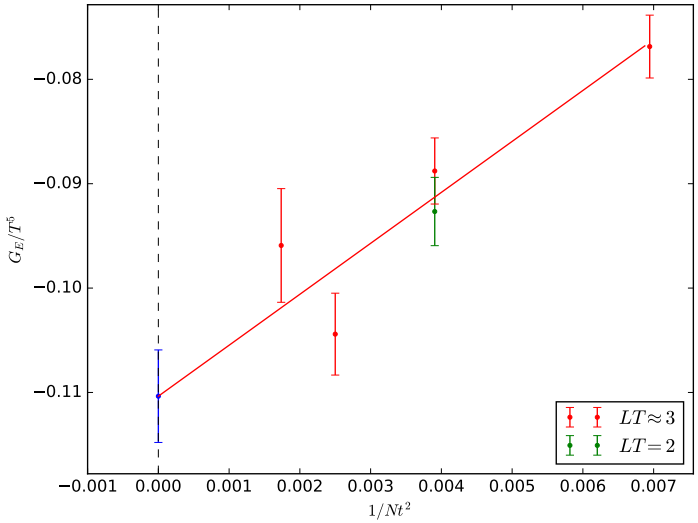
One should use points $a^2 \lessapprox \tau \lessapprox t^2/8$

a-extrapolation, $T/T_c = 1.24$

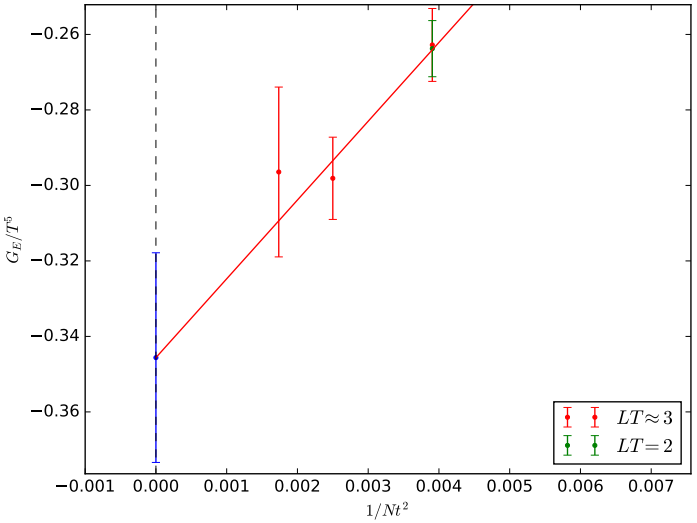


$\tau/a_{24}^2 = 2.0$

a-extrapolation, $T/T_c = 1.24$

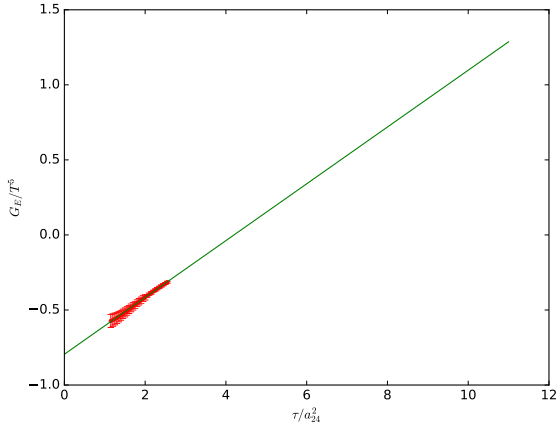


$$\tau/a_{24}^2 = 4$$
$$t/a_{24} = 12$$

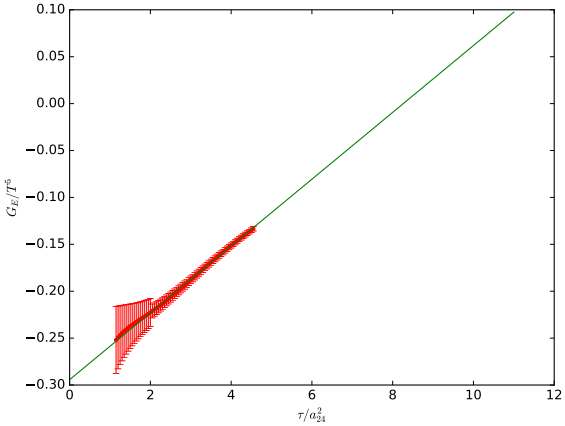


$$\tau/a_{24}^2 = 1.5$$
$$t/a_{24} = 7$$

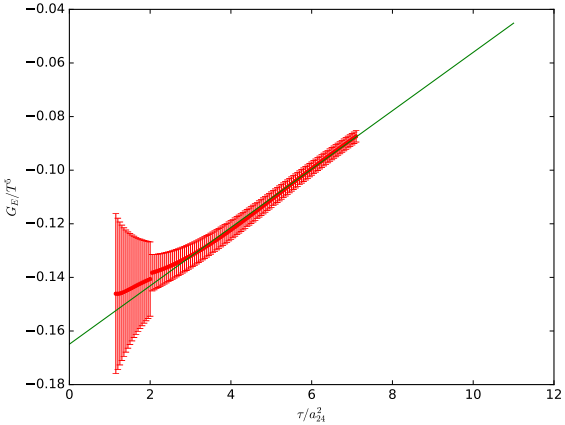
τ-extrapolation, $T/T_c = 1.24$



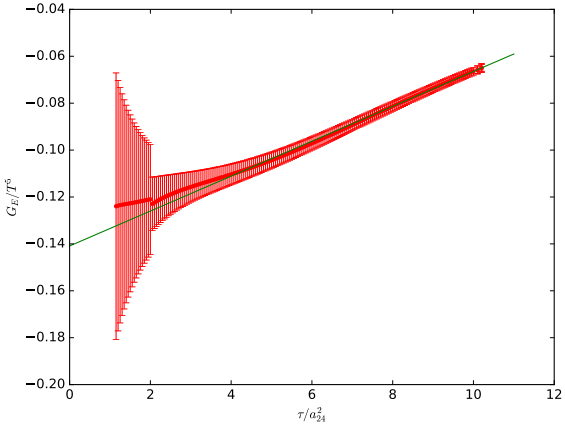
$t/a_{24} = 6$



$t/a_{24} = 8$

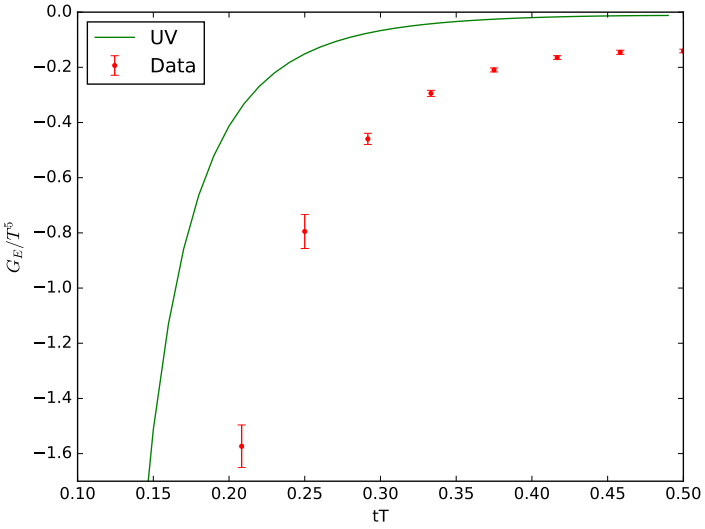


$t/a_{24} = 10$

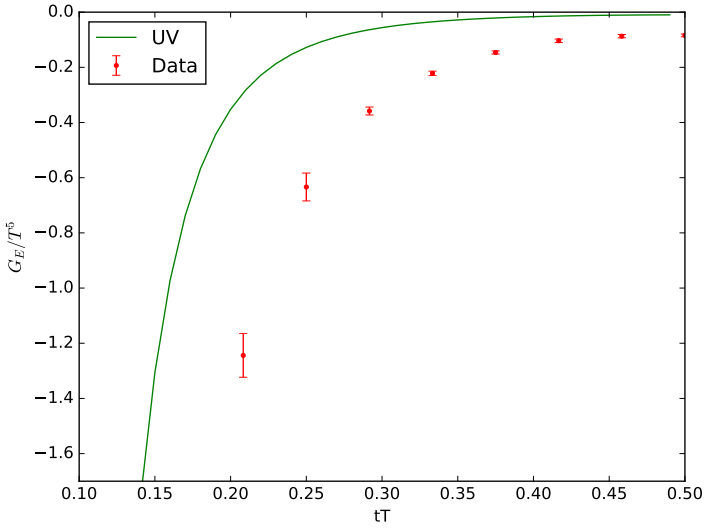


$t/a_{24} = 12$

Extrapolated correlator



$T/T_c = 1.24$



$T/T_c = 1.70$

Asymptotic freedom

$$\rho_q(\omega) = -\frac{d_A \alpha_s^2}{256\pi^4}\omega^4 \qquad G_E(t) = \int_0^\infty \rho(\omega)\frac{\cosh\omega(\beta/2 - t)}{\sinh\omega\beta/2}d\omega$$

N. Iqbal, H. B. Meyer, 0911 (2009) 029, arXiv:0909.0582

UV part seems to be small!

Backus-Gilbert method for the spectral function

Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{\cosh\left(\frac{\omega}{2T} - \omega x_i\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

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Let $b_i(\omega)$ be arbitrary functions. Define $\tilde{\rho}(\bar{\omega})$:

$$\tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

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How should we choose $b_i(\omega)$?
Minimize the width of the resolution function $\hat{\delta}(\bar{\omega}, \omega)$:

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$

$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

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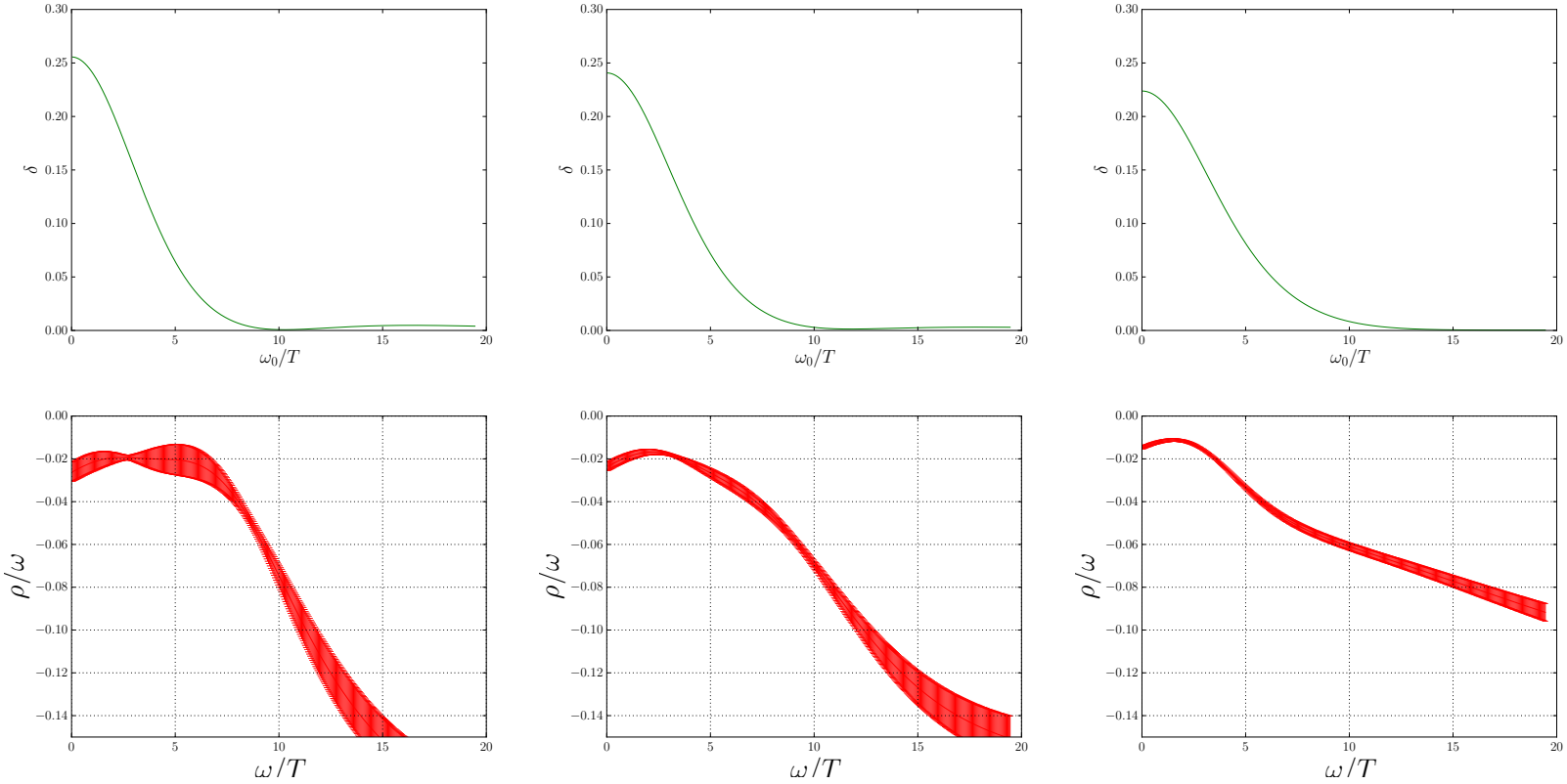
Regularization by the parameter λ: numerically more stable, but wider resolution function

$$W_{ij} \to \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method. $T/T_c = 1.24$

$\lambda = 10^{-3}$ $\lambda = 10^{-4}$ $\lambda = 10^{-5}$



Backus-Gilbert method. $T/T_c = 1.70$

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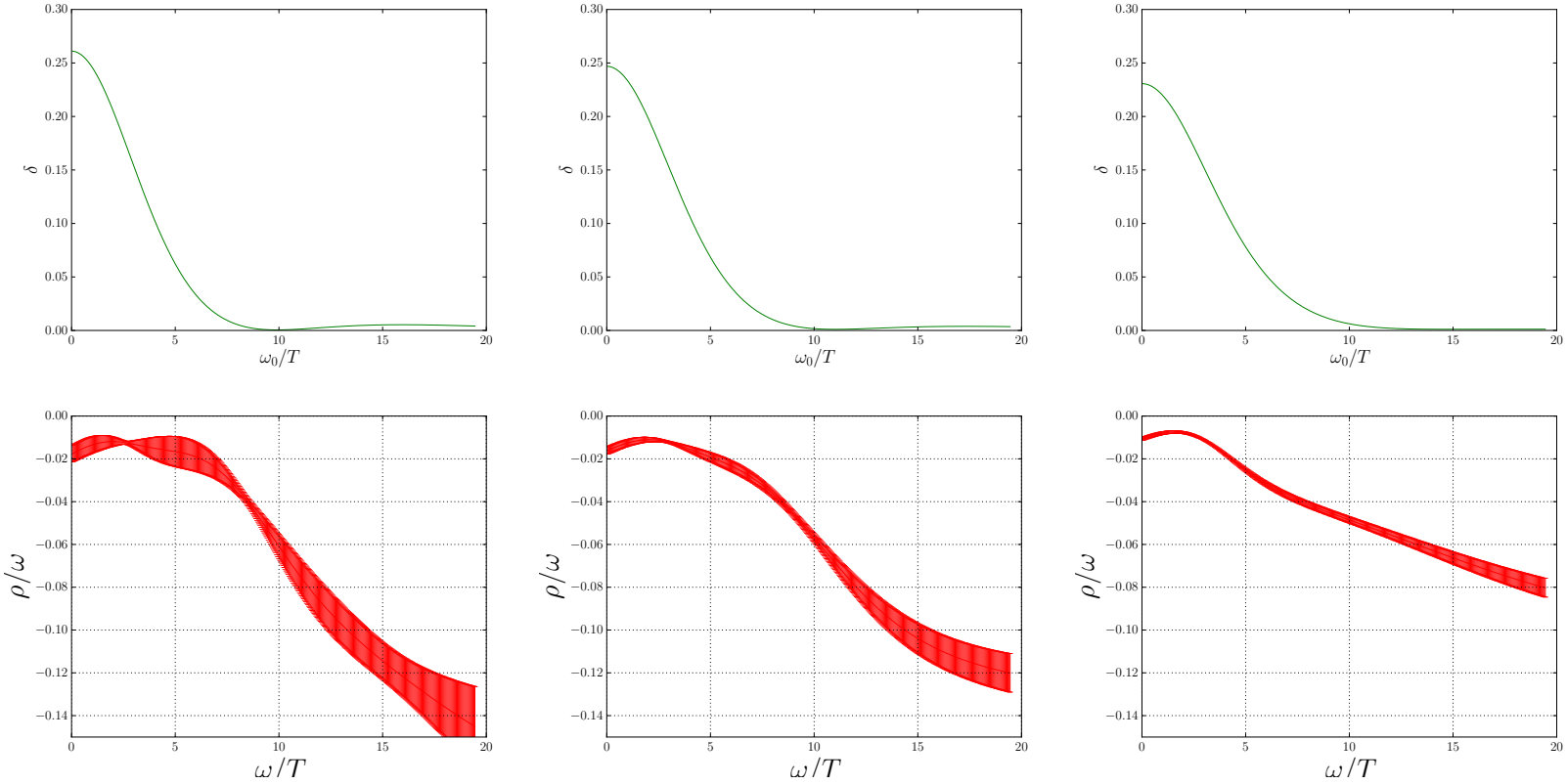
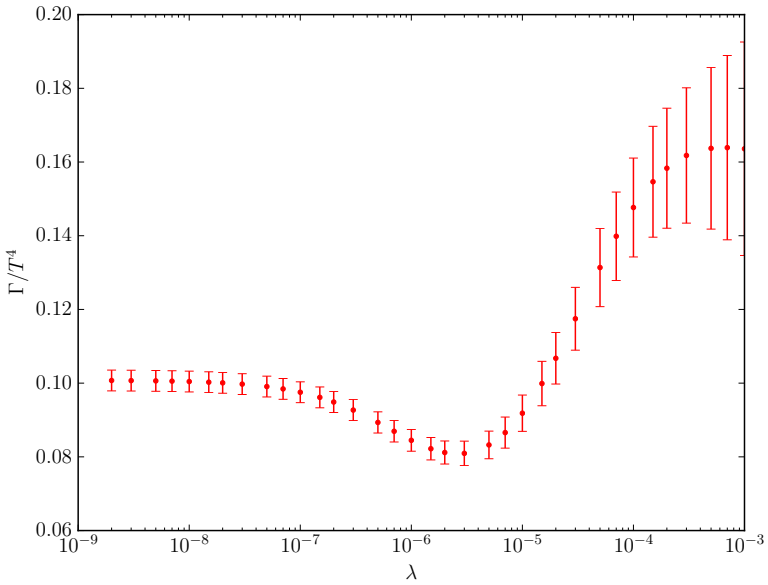


Backus-Gilbert. λ-dependence.



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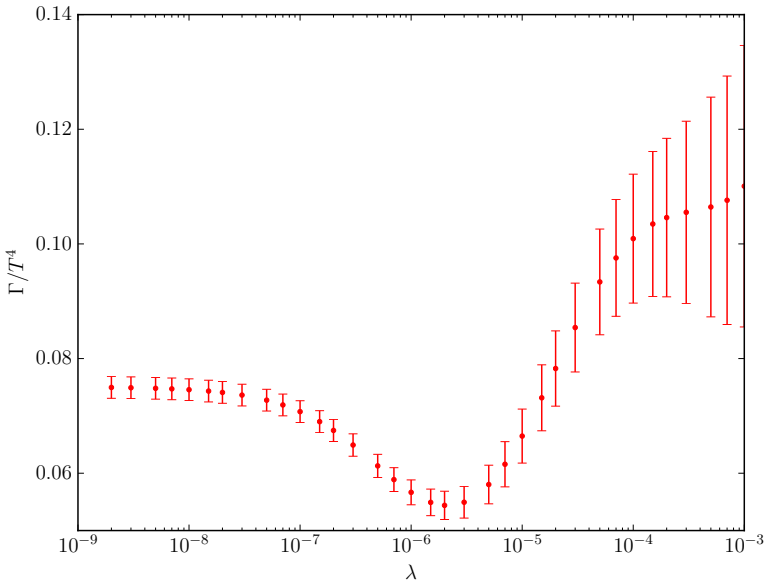

$T/T_c = 1.70$

We can estimate the systematic error of the method.

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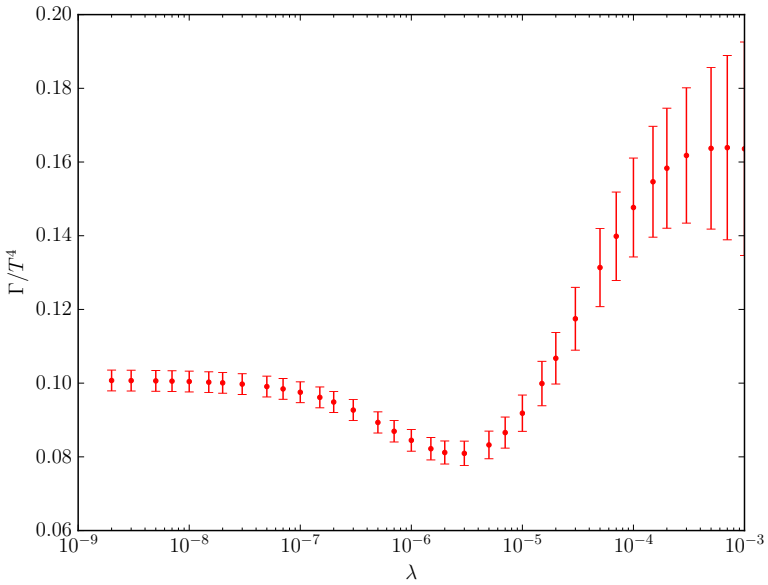

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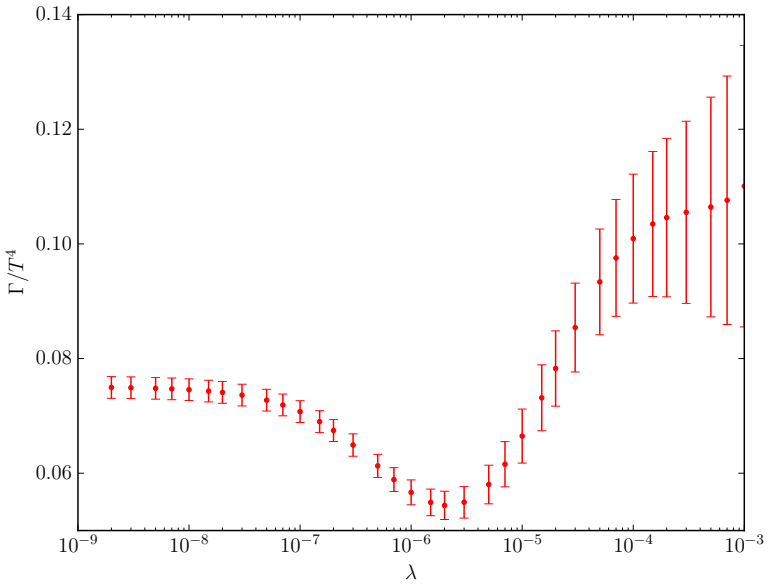

$T/T_c = 1.70$

We can estimate the systematic error of the method.
$\Gamma/T^4 =$

- $0.13 \pm 0.05,\ T/T_c = 1.24$
- $0.085 \pm 0.030,\ T/T_c = 1.70$

Results

$\Gamma_{CS}/T^4 =$

- 0.13 ± 0.05, $T/T_c = 1.24$, $\alpha_s((1-4)\pi T) \approx 0.15 - 0.27$
- 0.085 ± 0.030, $T/T_c = 1.70$, $\alpha_s((1-4)\pi T) \approx 0.14 - 0.22$

"Desperate extrapolation" to finite α (semiclassical field theory)

$$\Gamma_{CS}/T^4 \sim 30\alpha_s^4 =$$

$$0.016 - 0.16, \quad T/T_c = 1.24$$

$$0.012 - 0.08, \quad T/T_c = 1.70$$

G.D. Moore and M.Tassler, *The Sphaleron Rate in SU(N) Gauge Theory*, JHEP 1102 (2011) 105, arXiv:1011.1167

Conclusion

- Estimation of sphaleron diffusion rate in lattice gluodynamics
- Data is extrapolated to continuum
- $\Gamma_{CS}/T^4 =$
  - $0.13 \pm 0.05,\ T/T_c = 1.24$
  - $0.085 \pm 0.030,\ T/T_c = 1.70$
- The results are in agreement with

G.D. Moore and M.Tassler, *The Sphaleron Rate in SU(N) Gauge Theory*, JHEP 1102 (2011) 105, arXiv:1011.1167