# Topological Properties of $SU(3)$ Yang-Mills Theory with Double Trace Deformation

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Gauge Topology 3: from lattice to colliders.
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# Outline

# The Final Goal

Consider $SU(3)$ Yang-Mills theory with a compactified direction in the limit of small compactification radius.

In such limit semiclassical methods could be used to study properties that are intrinsically non-perturbative such as topology, confinement.

**PROBLEM** → If we squeeze too much the compactification radius the system undergoes a phase transition.

# Finite Temperature $SU(3)$

- We compactify a direction and we impose periodic boundary conditions on the gauge fields on that direction. We discretise the theory on a $\mathbb{R}^3 \times S^1$ space.
- The compactified direction has a finite extension $[0, L = N_t a]$ where $N_t$ is the number of sites and $a$ is the lattice spacing.
- This is equivalent to study the theory at finite temperature with the temperature $T$ given by

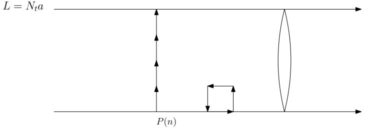
$$T = \frac{1}{L} = \frac{1}{N_t a}$$

- If we send $L \longrightarrow \infty$ we recover the limit of zero temperature.
- On the lattice we realise finite temperature using lattices with a temporal extension smaller compared to the spatial ones, e.g. $32^3 \times 8$.

# Spontaneous Symmetry Breaking of Centre Symmetry

- **PROBLEM:** when we consider $SU(3)$ at finite temperature with a small compactification radius the system undergoes a phase transition.
- The symmetry which is spontaneously broken in this phase transition is centre symmetry.
- A centre symmetry transformation consists of multiplying all the temporal links at a given time slice by the same element of the centre of the gauge group, which is, for $SU(3)$, $Z_3$.

$$U(t_0, \vec{x}) \to zU(t_0, \vec{x}) \qquad z \in Z_3$$




The order parameter of this phase transition is Polyakov loop

$$P(\vec{n}) = \frac{1}{N}\left(\prod_{t=1}^{N_t} U_4(\vec{n}, t)\right)$$

# A possible Solution: the Deformed Theory

The phase transition separates the low-$T$ regime from the high-$T$ one.

It is not possible to perform semiclassical methods in the high-$T$ regime (small compactification radius) and obtain information on low-$T$ properties: the two phases are not analytically connected.

The aim of the deformation is to restore centre symmetry even for small values of the compactification radius.

There are also other possibilities to restore centre symmetry, such as adding adjoint fermions (talk of M. Schifman).

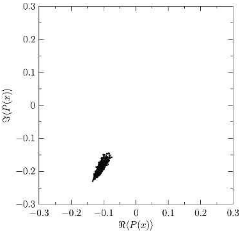
# The deformation

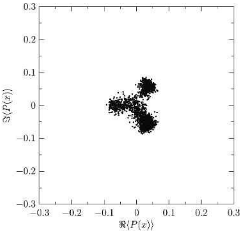
The discretised $SU(3)$ Yang-Mills action used in the simulations is

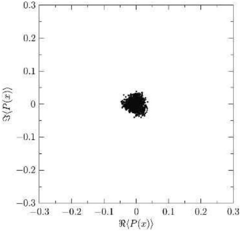
$$S^{\text{def}} = \beta \sum_{\mu>\nu} \left[ 1 - \mathrm{Re}\left( \mathrm{tr}\left( \Pi_{\mu\nu} \right) \right) \right] + \underbrace{h \sum_{\vec{n}} \left| \mathrm{tr}(P(\vec{n})) \right|^2}_{\text{deformation}}$$

where $\Pi_{\mu\nu}$ is the plaquette operator in the $\mu$-$\nu$ plane and $P(\vec{n})$ is Polyakov loop. The deformation was proposed by M. Unsal and L. Yaffe [ArXiv:0803.0344].

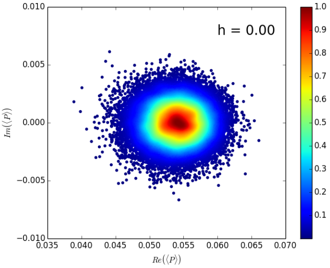
- The deformation disadvantages gauge configurations with $\langle P \rangle \neq 0$.
- We choose the parameter $h$ in order to restore centre symmetry.

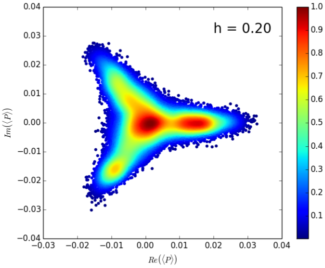


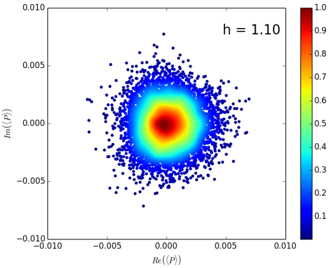




- J. Myers and M. Ogilvie [ArXiv:0707.1869] performed numerical simulation of $SU(3)$ with double trace deformation.
- They performed simulations on a $24^3 \times 4$ lattice ($\beta_c \approx 5.69$) using $\beta = 6.5$ and $h = 0.05,\ 0.08,\ 0.11$.



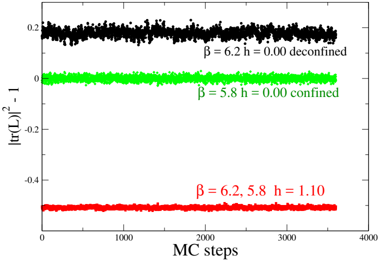




- Density plot of Polyakov loop points for different values of the deformation $h$ with fixed bare coupling $\beta = 6.2$ on a $32^3 \times 8$ lattice.
- The system starts deep into the deconfined phase and then centre symmetry is restored.

# Adjoint Polyakov Loop

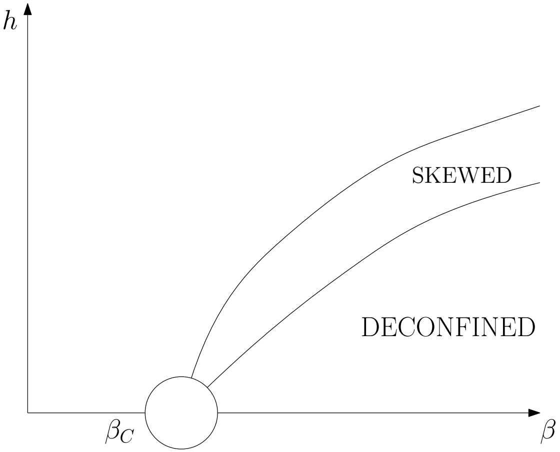
- The adjoint Polyakov loop gives information about how centre symmetry is recovered with the deformation.




When the deformation is switched on, the adjoint Polyakov loop is negative → **Different with respect to the usual confined phase.**

To check other possible differences between the two phases we considered the **topological properties.**

# Qualitative Phase Diagram



# $\theta$ Term

We consider the presence of a non-zero $\theta$ parameter in the $SU(3)$ Lagrangian

$$\mathcal{L}_\theta = \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) - i\theta \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

where

$$q(x) = \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

is the topological charge density and $Q = \int d^4x q(x)$ is the topological charge.

- The $\theta$ term breaks explicitly parity and time reversal.
- Experimental bounds are quite stringent, $\theta \lessapprox 10^{-10}$.
- The $\theta$ term enters in different aspects of hadron phenomenology, such as the solution of the $U(1)_A$ problem.
- The $\theta$ dependence is an intrinsically non perturbative characteristic of the theory.

# Free Energy Expansion

The free energy is defined as

$$F(\theta, T) = -\frac{1}{\mathcal{V}} \ln \left( \int [dA] \exp\left\{ -\int_0^{\frac{1}{T}} dt \int d^3x \mathcal{L}_\theta \right\} \right)$$

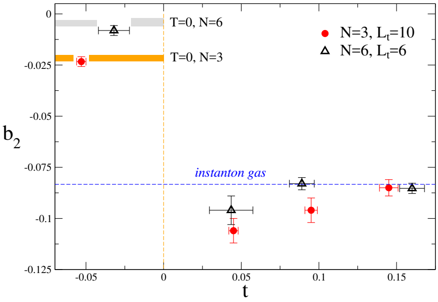
where $\mathcal{V}$ is the Euclidean $4D$ volume.

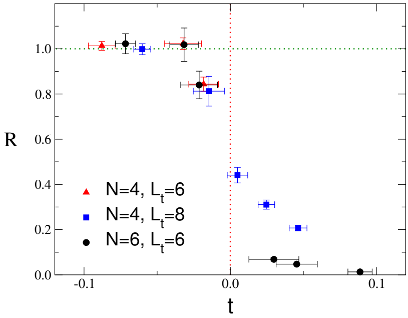
- We can parametrise the free energy as

$$f(\theta, T) = F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2 \left(1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right)$$

where $\chi(T)$ is the topological susceptibility. It is possible to relate $\chi(T)$ and the coefficients $b_{2j}$ to the distribution of the topological charge computed at $\theta = 0$, in particular

$$\chi = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}} \qquad b_2 = -\frac{1}{12\langle Q^2 \rangle_{\theta=0}} \left[ \langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle_{\theta=0}^2 \right]$$




$$R = \frac{\chi(T)}{\chi(T=0)}$$

- (left image) Bonati, D'Elia, Panagopoulos, Vicari [ArXiv:1301.7640] (right image) Del Debbio, Panagopoulos, Vicari [ArXiv: 0407068].
- Topological properties change from the low-$T$ and the high-$T$ regime.

# Discretisation of The Topological Charge

- In our simulations we will use the discretisation of the topological charge with definite parity

$$q_L(x) = -\frac{1}{2^9\pi^2} \sum_{\mu\nu\rho\sigma=\pm1}^{\pm4} \varepsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left[\Pi_{\mu\nu}\Pi_{\rho\sigma}\right]$$

- In the continuum limit $q_L(x)$ must be corrected by a renormalization factor $Z$ introduced by the lattice discretisation

$$q_L(x) \;\rightarrow\; a^4 Z q(x) + O(a^6)$$

- We remove UV fluctuation using the Cooling procedure.

# The Coefficient $b_2$: Numerical Problems

- The coefficient $b_2$ is given by

$$b_2 = -\frac{1}{12\langle Q^2\rangle}\left[\langle Q^4\rangle - 3\langle Q^2\rangle^2\right]$$

- This observable is very noisy, since we are trying to measure a fourth order momentum of a distribution, and a huge amount of statistics is required $\longrightarrow$ **Numerical problem**.
- A way out is to use the **imaginary $\theta$ method** to measure $b_2$. We perform numerical simulations at imaginary values of $\theta$ and then we exploit analytic continuation (talk C. Bonanno).

# The Imaginary $\theta$ Method

- We add to the Lagrangian an imaginary $\theta$ term

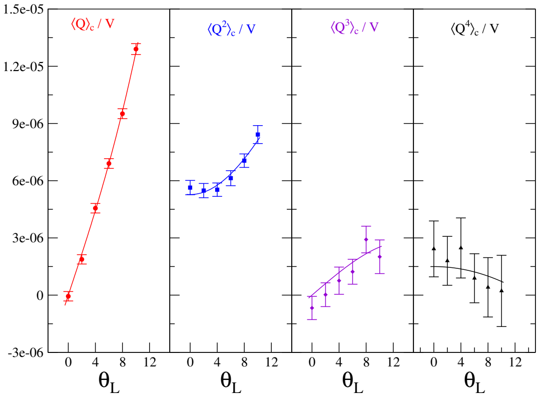
$$S^{\text{def},i\theta} = S_W + h\sum_{\vec{n}}\left|\text{tr}(P(\vec{n}))\right|^2 - \theta_L Q_L$$

where $Q_L$ is the discretisation of $Q$.

- We perform simulations using different values of $\theta_L$ and we obtain $\chi$, $b_2$ and $Z$ with a combined fit of the first four cumulants

$$\frac{\langle Q\rangle}{\mathcal{V}} = \chi Z\theta_L\left(1 - 2b_2 Z^2\theta_L^2 + 3b_4 Z^4\theta_L^4 + \cdots\right)$$
$$\frac{\langle Q^2\rangle_c}{\mathcal{V}} = \chi\left(1 - 6b_2 Z^2\theta_L^2 + 15b_4 Z^4\theta_L^4 + \cdots\right)$$
$$\frac{\langle Q^3\rangle_c}{\mathcal{V}} = \chi\left(-12b_2 Z\theta_L + 60b_4 Z^3\theta_L^3 + \cdots\right)$$
$$\frac{\langle Q^4\rangle_c}{\mathcal{V}} = \chi\left(-12b_2 + 180b_4 Z^2\theta_L^2 + \cdots\right)$$

# Example of the Fit Procedure



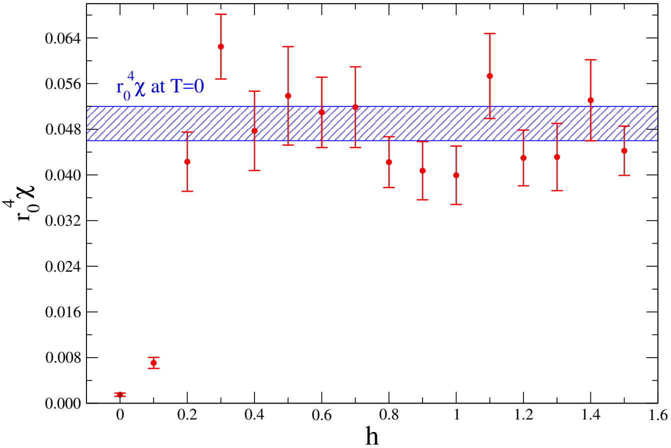

- $32^3 \times 8$ lattice, $\beta = 6.4$ and $h = 1.20$.

# Numerical Results

- We measured the topological susceptibility $\chi$ and the first coefficient $b_2$ in the deformed theory.
- We studied lattices with different temporal extensions and with different bare couplings $\beta$.

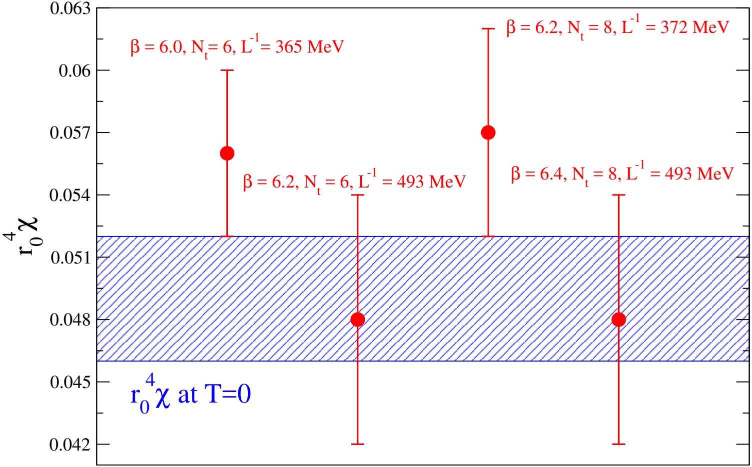
Are $\chi$ and $b_2$ compatible with the values measured in standard $SU(3)$ Yang-Mills theory at zero temperature?

# $r_0^4\chi$ vs $h$ on $32^3 \times 8$ lattice, $\beta = 6.4$

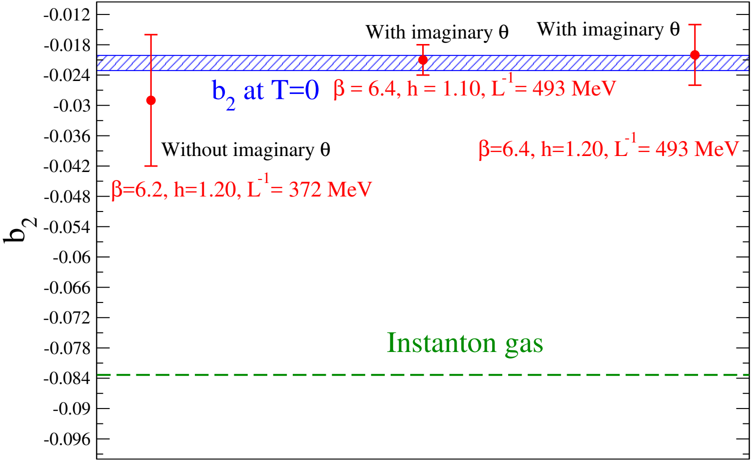



- $r_0$ is the Sommer parameter, used to fix the scale, and it is approximately 0.5 fm.
- We assumed that the deformation does not modify the lattice spacing.

# $r_0^4\chi$ on Different Lattices



# $b_2$ on Different lattices




- **No assumptions on the lattice spacing have been made.**

# Conclusions

- The **topological susceptibility** $\chi$ and the **coefficient** $b_2$ computed in the **deformed theory**, when centre symmetry is recovered, are compatible with the values obtained in **zero temperature** $SU(3)$ Yang Mills theory for the studied values of the compactification radius.
- It would be interesting to perform simulations with a smaller compactification length $L$.
  **Numerical problem** $\rightarrow$ lattices with large volumes are required to have $\langle Q^2 \rangle$ not too small.

# Thank you for your attention!