

# A scalar $SO(3)$ -theory and its topological excitations

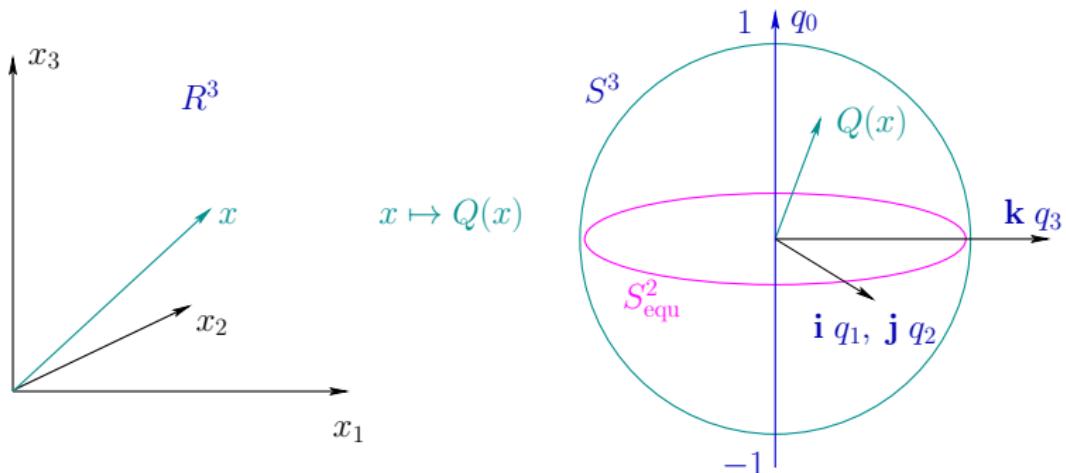
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in cooperation with

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Alexander Kobushkin, Mario Pitschmann, Lukas Rachbauer,  
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# Calorons in $SU(2)$

a field of Polyakov loop matrices  $Q(\vec{x})$ ,  $L(\vec{x}) = \text{Tr } Q(\vec{x})$   
covering  $S^3 \cong SU(2) \cong \text{unit quaternions}$



$$R^3 \rightarrow S^3, \quad x \mapsto Q(\vec{x}) = q_0 + \mathbf{i} q_1 + \mathbf{j} q_2 + \mathbf{k} q_3 = q_0 - i \vec{q} \vec{\sigma}$$
$$\mathbf{i} = -i\sigma_1, \mathbf{j} = -i\sigma_2, \mathbf{k} = -i\sigma_3$$

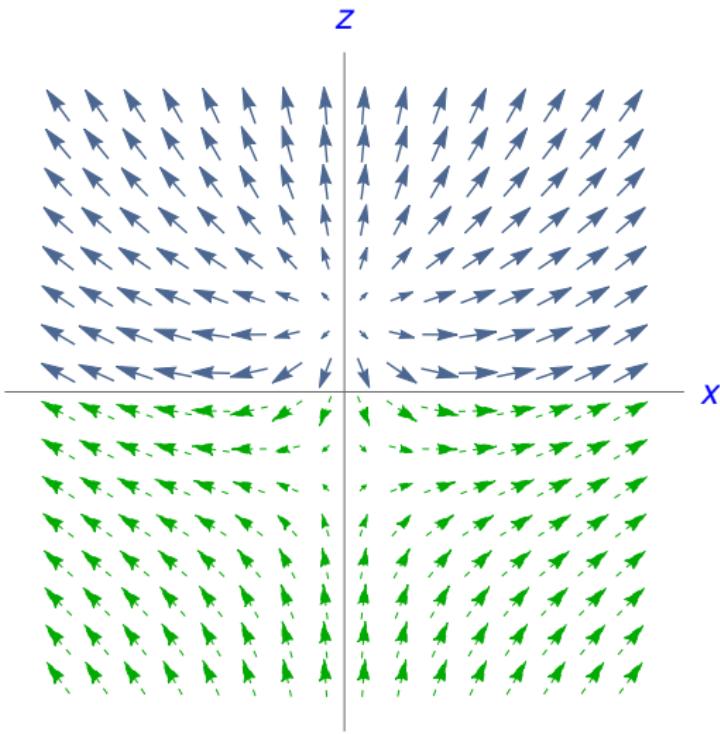
(an)holonomy = vacuum with broken symmetry, e.g.  $Q(\infty) = -i\sigma_3$

$$Q(\vec{x}) = q_0(\vec{x}) - i\vec{q}(\vec{x})\vec{\sigma}$$

$$q_0^2 + \vec{q}^2 = 1$$

$\vec{q}(\vec{x})$ -field

$$q_0 > 0$$



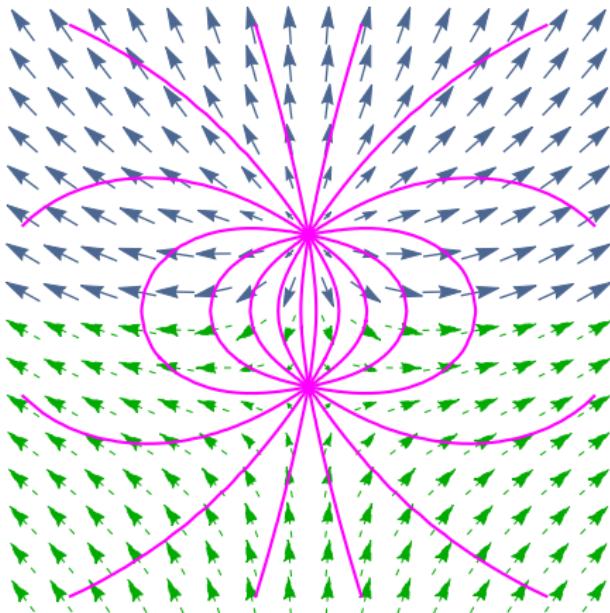
$$q_0 < 0$$

$$Q(\vec{x}) = q_0(\vec{x}) - i\vec{q}(\vec{x})\vec{\sigma} = \cos \alpha(\vec{x}) - i\vec{\sigma}\vec{n}(\vec{x}) \sin \alpha(\vec{x})$$

connect points with  $\vec{n} = \text{const.}$

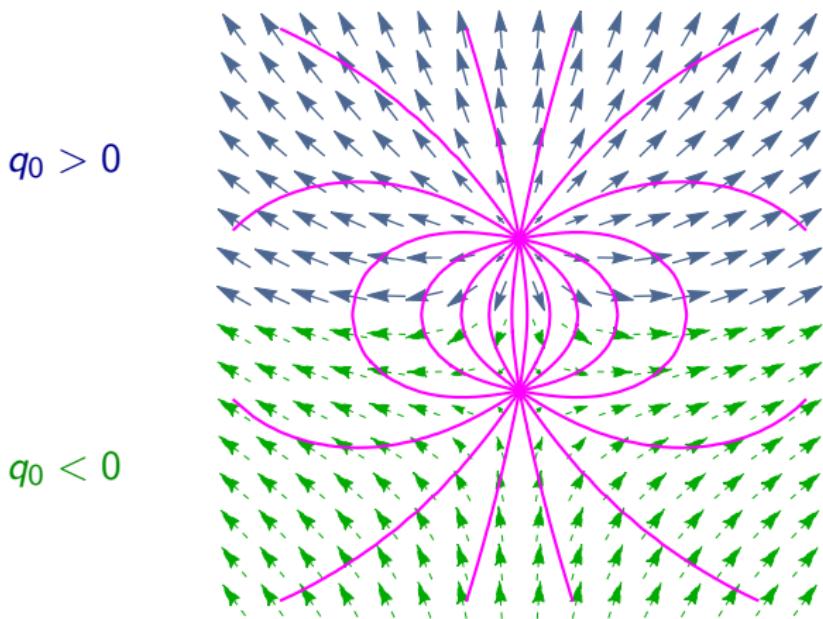
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a dipole field  $\Rightarrow$  NO SINGULARITY

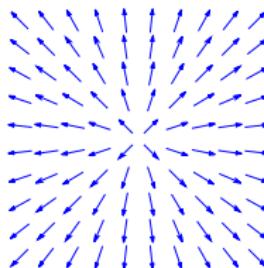
# Dirac monopoles

two types of singularities in Dirac description:  $A_\mu(x)$

- ▶ Dirac string
- ▶ Center singularity

one type of singularities in Wu-Yang description:  $\vec{n}(x) = \frac{\vec{x}}{|\vec{x}|}$

- ▶ Center singularity



no singularities in SU(2)-field description:  $Q(\vec{x})$

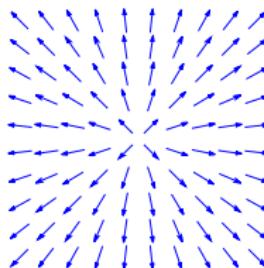
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- ⇒ the above singularities are Field singularities
- ⇒ singularities of inappropriate fields

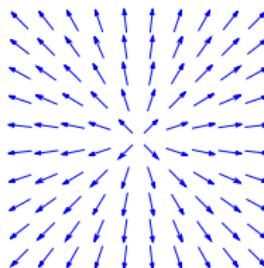
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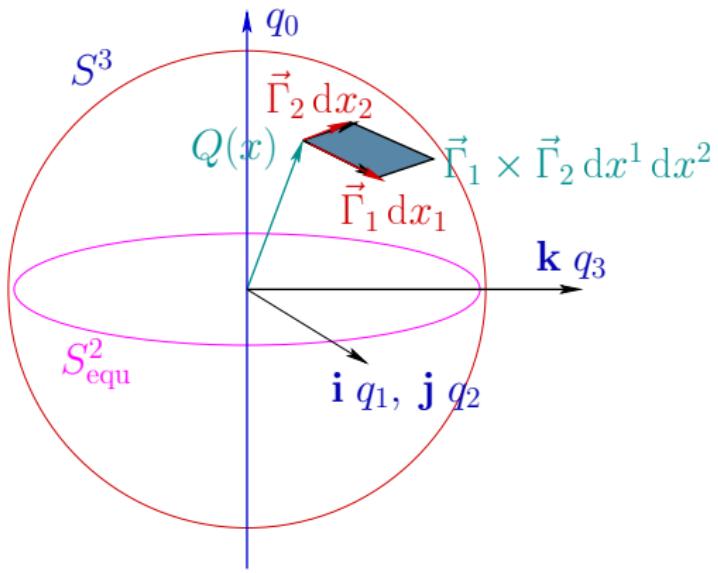
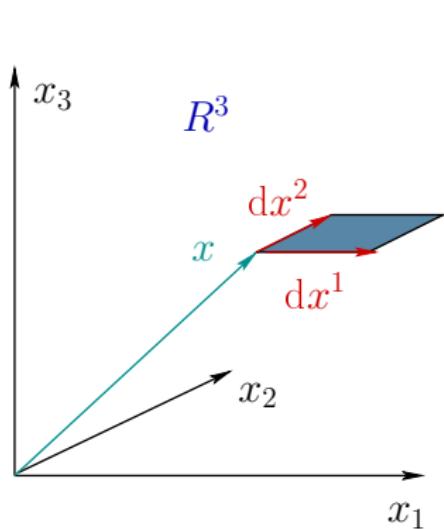
- ⇒ the above singularities are Field singularities
- ⇒ singularities of inappropriate fields

observe: monopole charges are quantised

# How describe dynamics of monopoles

with  $SU(2)$  scalar field. We relate

- ▶ vector field  $A_\mu(x) \rightarrow$  connection  $\vec{\Gamma}_\mu(x)$ ,  $\partial_\mu Q(x)Q^\dagger(x) =: -i\vec{\Gamma}_\mu(x)\vec{\sigma}$
- ▶ tensor field  $F_{\mu\nu}(x) \rightarrow$  curvature  $\vec{R}_{\mu\nu} := \vec{\Gamma}_\mu(x) \times \vec{\Gamma}_\nu(x)$



# Maurer-Cartan equation

from

$$\partial_\mu \partial_\nu Q(x) = \partial_\nu \partial_\mu Q(x)$$

we get the Maurer-Cartan equation

$$\partial_\mu \vec{\Gamma}_\nu - \partial_\nu \vec{\Gamma}_\mu - 2 \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu = 0$$

Interpretation:  $\vec{\Gamma}_\mu(x)$  can be derived from  $Q(x)$ ,  
12 dof can be derived from 3 dof

Therefore

$$\vec{R}_{\mu\nu} := \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu = \partial_\mu \vec{\Gamma}_\nu - \partial_\nu \vec{\Gamma}_\mu - \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$$

and  $\vec{A}_\mu = 2 \vec{\Gamma}_\mu$  is a trivial connection

$\vec{R}_{\mu\nu}$  is gauge covariant (gauge = choice of tangential basis on  $\mathbb{S}^3$ )

# Lagrangian and Hobart-Derrick theorem

$$\mathcal{L}_{\text{curv}} = -\frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} \quad \text{Skyrme term}$$

scaling argument

scale  $x \rightarrow \lambda x$ ,

$$\int d^3x \mathcal{L}_{\text{curv}} \rightarrow \frac{1}{\lambda} \int d^3x \mathcal{L}_{\text{curv}}$$

no stability of monopoles  $\rightarrow$  monopoles dissolve

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Hobart-Derrick theorem: compressing term needed

$$\mathcal{L}_{\text{kin}} = \kappa (\vec{\Gamma}_\mu)^2, \quad \int d^3x \mathcal{L}_{\text{kin}} \rightarrow \lambda \int d^3x \mathcal{L}_{\text{kin}}$$

Skyrme model:  $\mathcal{L} = \mathcal{L}_{\text{curv}} - \mathcal{L}_{\text{kin}}$

stable solitons = Skyrmions

short range interaction

monopoles get infinite energy  $\Rightarrow$  NO monopoles

# Potential $\Lambda(Q)$

term without derivatives  $\Rightarrow \int d^3x \Lambda(Q) \xrightarrow{x \rightarrow \lambda x} \lambda^3 \int d^3x \Lambda(Q)$

$$\Lambda(Q(\infty)) = 0$$

$$Q(\vec{x}) = \cos \alpha(\vec{x}) - i \vec{\sigma} \vec{n}(\vec{x}) \sin \alpha(\vec{x})$$

$$\alpha(\infty) = \frac{\pi}{2}, \quad Q(\infty) = -i \vec{\sigma} \vec{n}, \quad \vec{n} = \frac{\vec{x}}{|\vec{x}|} \quad \text{Dirac (Wu-Yang) monopole}$$

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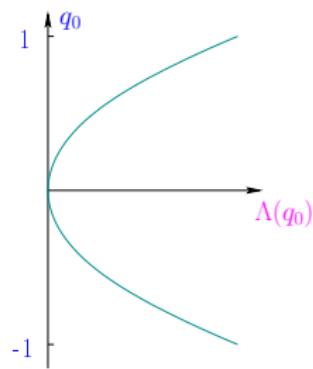
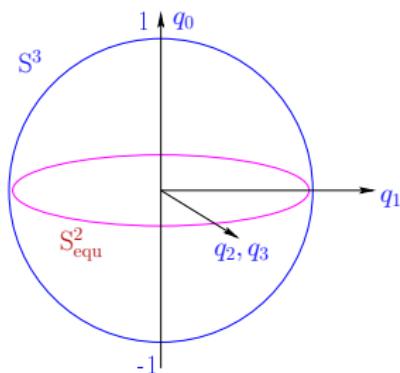
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Dirac (Wu-Yang) monopole



$$\Lambda(q_0) = \frac{q_0^{2m}}{r_0^4}, \quad m = 1, 2, 3, \dots \quad \text{scale } r_0, \rho = r/r_0$$

$$\mathcal{L} = \mathcal{L}_{\text{curv}} - \Lambda = -\kappa \left( \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \Lambda \right)$$

Consequences: two-dimensional degeneracy of vacuum

# Stable minima of energy (topological Solitons)

- ▶ hedgehog ansatz:  $\vec{n}(x) = \frac{\vec{r}}{r}$ ,  $x = (ct, \mathbf{r})$
- $Q(x) = \cos \alpha(x) + i \vec{\sigma} \vec{n}(x) \sin \alpha(x), \quad \text{with} \quad \alpha = \alpha(\rho), \quad \rho = r/r_0$
- ▶ minimisation of energy leads to non-linear differential equation

$$\partial_\rho^2 \cos \alpha + \frac{(1 - \cos^2 \alpha) \cos \alpha}{\rho^2} - m \rho^2 \cos^{2m-1} \alpha = 0$$

- ▶ solution for  $m = 3$

$$\alpha(\rho) = \text{atan}(\rho).$$

- ▶ energy of soliton  $E = \kappa \frac{\pi^2}{r_0}$
- ▶ compare with monopoles?

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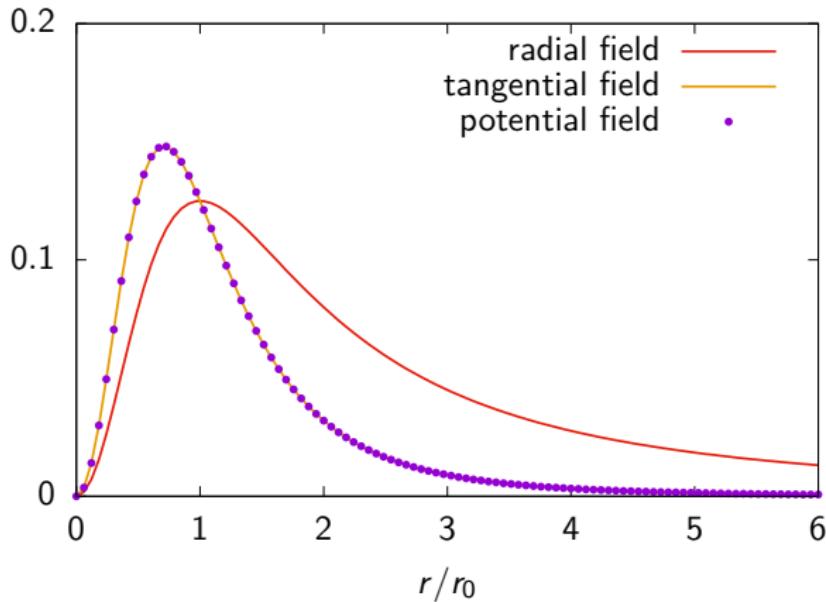
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$$\kappa = \frac{\alpha_f \hbar c}{4\pi} = 0.116 \text{ MeV fm}, \quad m_e c^2 = 0.511 \text{ MeV}, \quad r_0 = 2.21 \text{ fm}$$

# Energy densities

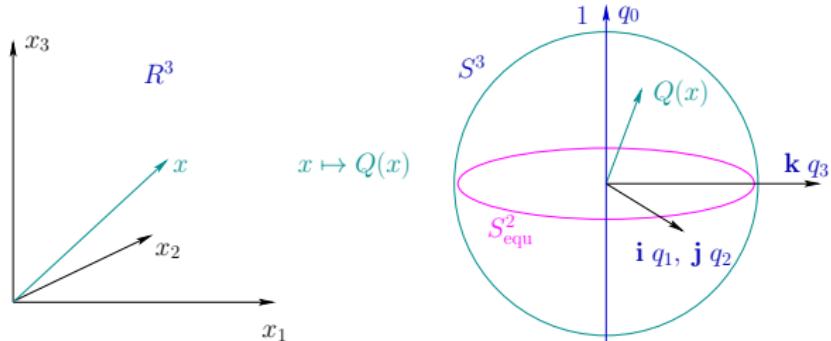
$$\mathcal{L} = -\kappa \left( \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{q_0^6}{r_0^4} \right), \quad q_0(\rho) = \cos \alpha(\rho) = \frac{1}{\sqrt{1+\rho^2}}$$

radial energy densities



particle and field are indistinguishable

# Topological quantum numbers



$$\begin{array}{ccc} \text{Volume element on } \mathbb{R}^3 & \mapsto & \text{Volume element on } S^3 \\ dr d\vartheta d\varphi & \mapsto & \vec{\Gamma}_r (\vec{\Gamma}_\vartheta \times \vec{\Gamma}_\varphi) dr d\vartheta d\varphi \end{array}$$

Topological charge  $\mathcal{Q} \equiv \text{number of coverings of } S^3$

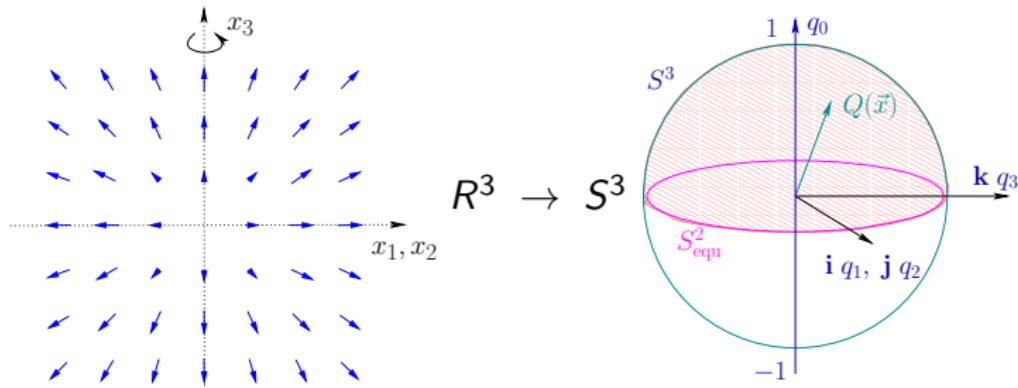
$$\mathcal{Q} = \frac{1}{V(S^3)} \int_0^\infty dr \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi \vec{\Gamma}_r (\vec{\Gamma}_\vartheta \times \vec{\Gamma}_\varphi)$$

$$V(S^3) = \int_{S^2} d^2n \int_0^\pi d\alpha \sin^2 \alpha = 2\pi^2$$

Topological charge is conserved:  $\frac{d\mathcal{Q}(t)}{dt} = 0$

Hedge-hog configuration:  $|\mathcal{Q}| = \frac{1}{2}$

# Spin, a topological quantum number



Field configuration  $Q(\mathbf{r})$  of unit charge covers hemisphere of  $S^3$ ,  $s = \frac{1}{2}$ .

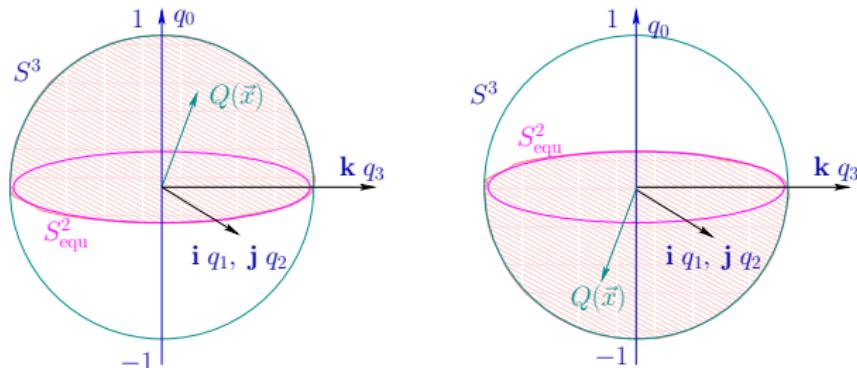
Spin quantum number  $s$

$$s := |\mathcal{Q}| = \left| \frac{1}{V(S^3)} \int_0^\infty dr \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi \vec{\Gamma}_r (\vec{\Gamma}_\vartheta \times \vec{\Gamma}_\varphi) \right|$$

Magnetic quantum numbers  $m_s = \pm 1/2$ : Upper and lower hemisphere

# $SO(3)$ versus $SU(2)$

Two hemispheres of  $S^3$



topologically different Hedge-hog configurations

Use rotational group  $SO(3)$

$\Rightarrow \pm\{Q(x)\}$  are identical,

$\Rightarrow$  interpretation: rotations of spatial Dreibein.

Calculations simpler with  $SU(2)$

no double counting of  $\pm\{Q(x)\}$ !

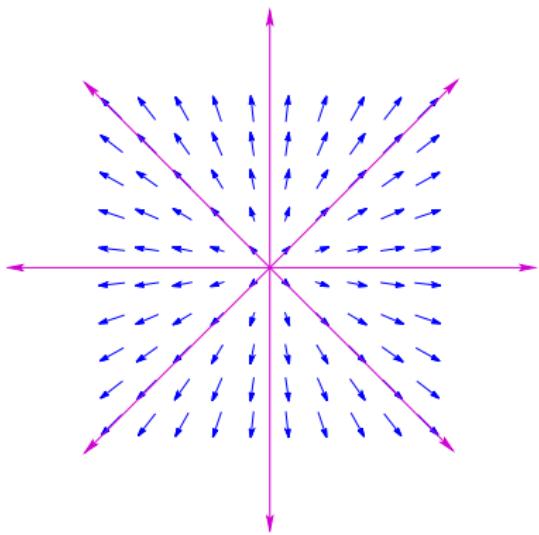
the above two configurations are identical

# Monopole is wired to surrounding space

flux lines  $\equiv$  lines of constant  $\vec{n}$ -field

flux lines  $\equiv$  strings

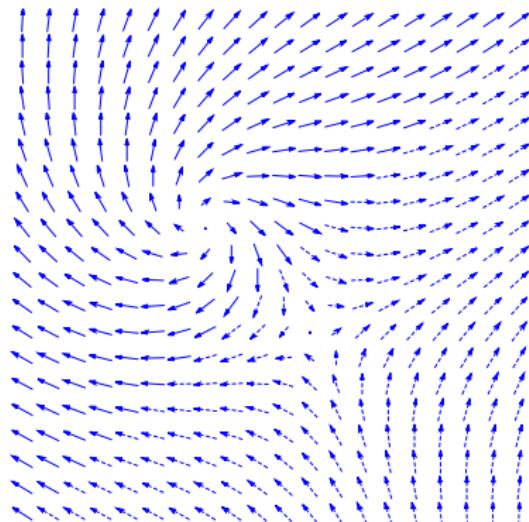
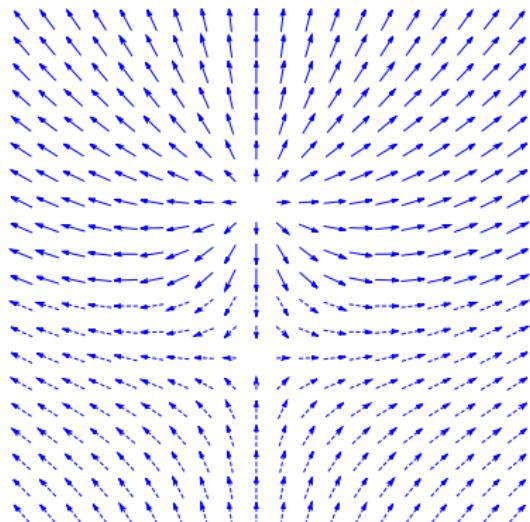
they connect the soliton with the surrounding,  
with other charges



after  $4\pi$ -rotation  
soliton configuration is restored  
a property of spin-1/2-particles

# Spin, an angular momentum

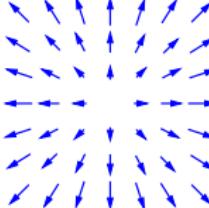
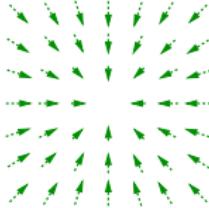
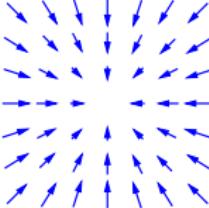
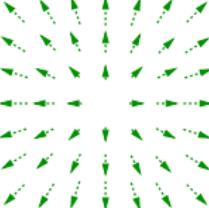
Symmetry broken vacuum,  $Q(\infty) = -i\sigma_3$ , Field at infinity is constant  
No rigid rotation possible



$S = 1$ , Charge Zero

# Types of solitons

up to global rotations

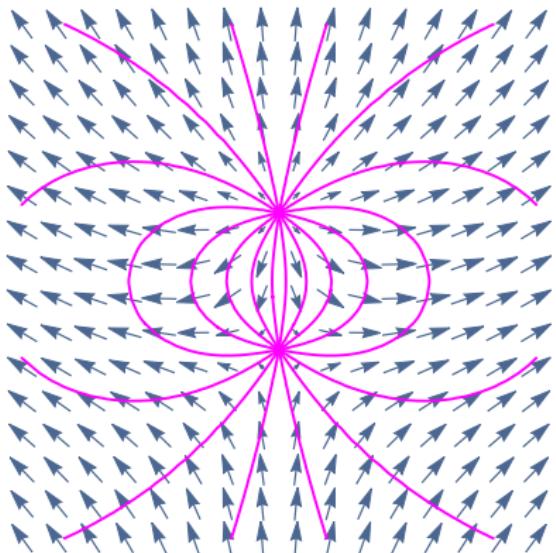
Transf.	1	$z$	$\Pi_n$	$z\Pi_n$
$\vec{n}$	$\vec{r}/r$	$-\vec{r}/r$	$-\vec{r}/r$	$\vec{r}/r$
$q_0$	$\geq 0$	$\leq 0$	$\geq 0$	$\leq 0$
$Q$	1	-1	-1	1
$\mathcal{Q}$	1/2	1/2	-1/2	-1/2
diagram				

Crossing the soliton  $\rightarrow$  Rotation of spatial Dreibein by  $2\pi$ ,  
chirality  $\chi$  of this rotation given by sign of topological charge  $\mathcal{Q}$

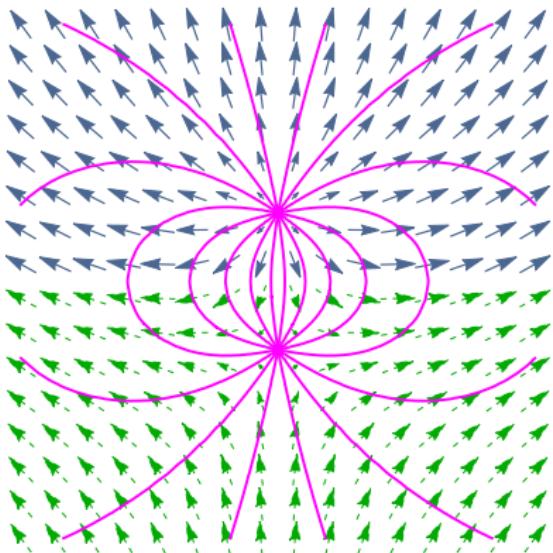
$$\mathcal{Q} = \chi \cdot S$$

T.D.Lee: Why does the mass violate chiral symmetry?

# Field lines of dipol

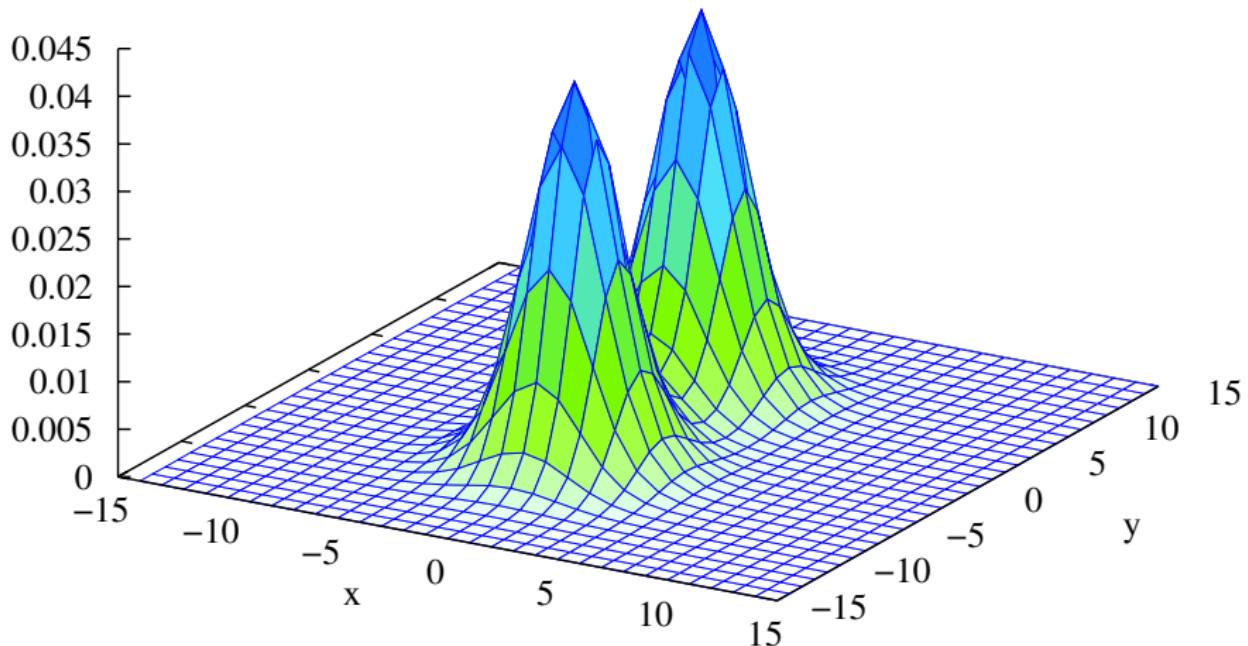


Left:  $S = 0$  configuration,  
field lines = lines of constant  $\vec{n}$ -field

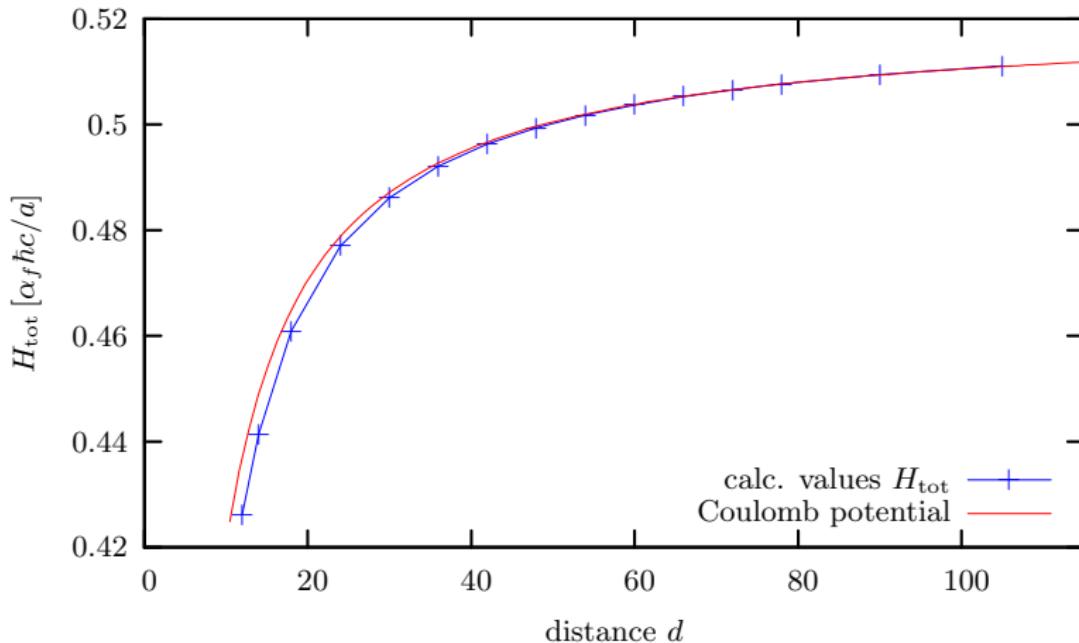


# Energy density

energy density at  $z = 0$

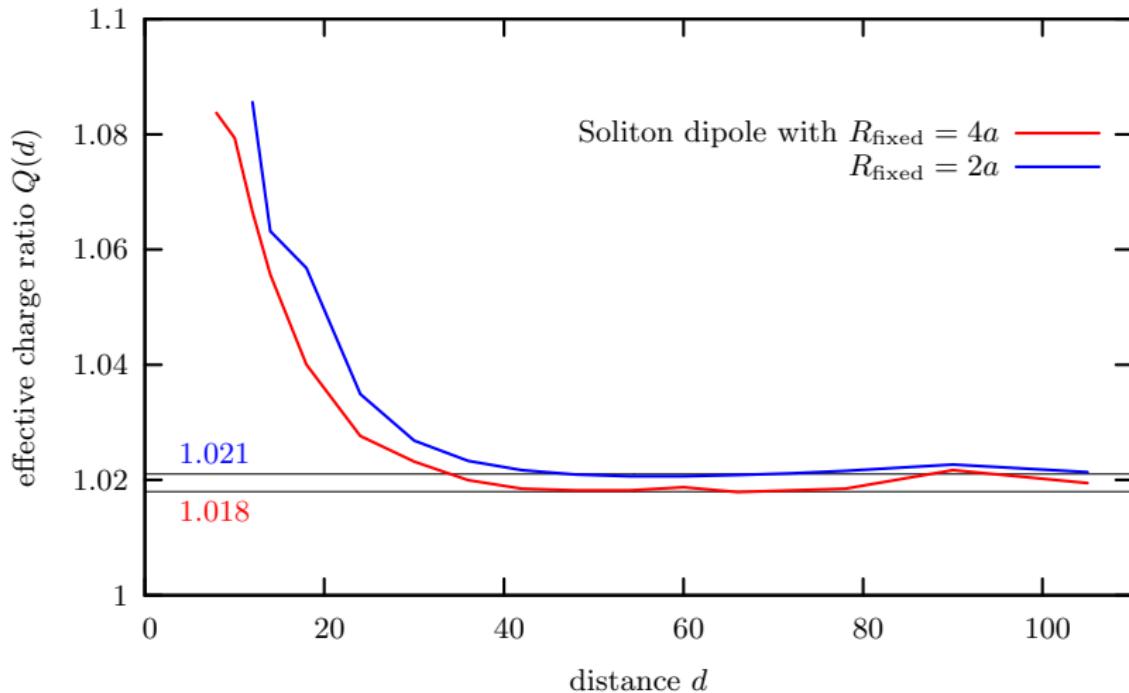


# Soliton-antisoliton-potential



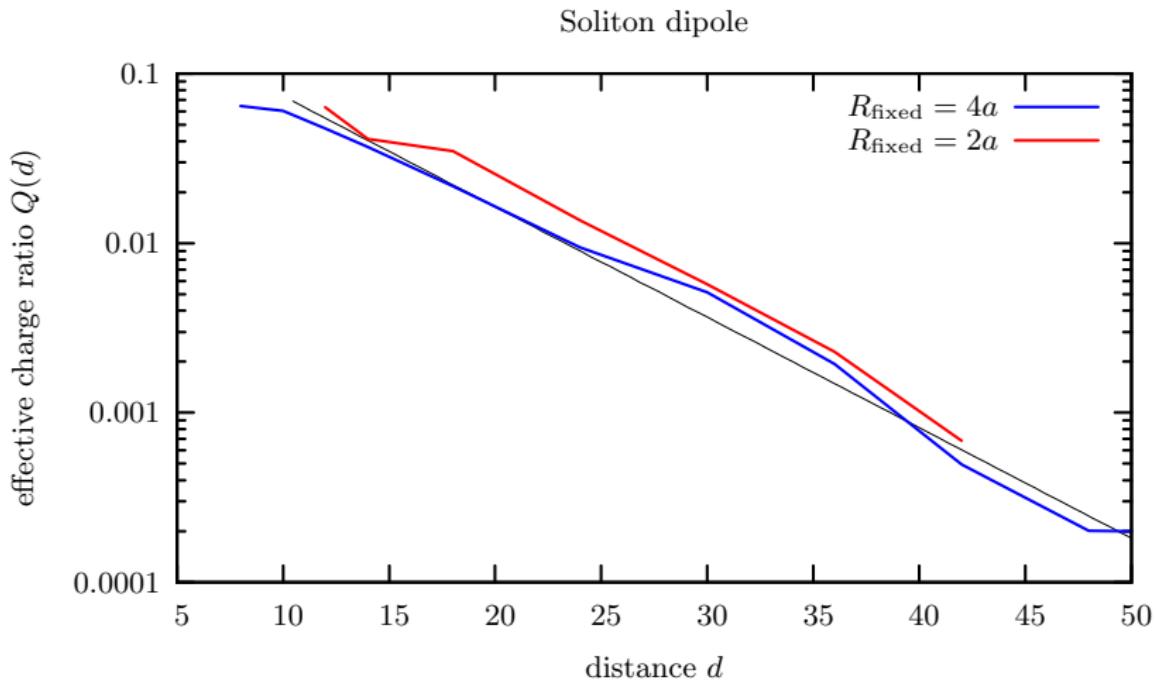
Total energy of a soliton pair for varying distance  $d = r/r_0$ .  
From diploma work of Dominik Theuerkauf (2016).

# Effective charge ratio



From diploma work of Dominik Theuerkauf (2016).

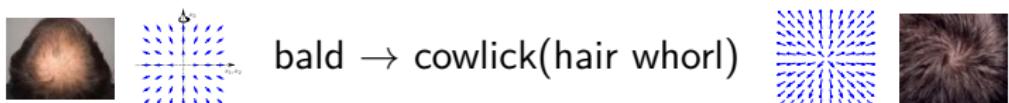
# Effective charge ratio



From diploma work of Dominik Theuerkauf (2016).

# Dirac (Electrodynamic) limit

artificial separation between particle and field



$r_0 \rightarrow 0 \iff q_0 = \cos \alpha = 0 \iff \alpha = \frac{\pi}{2}$   
Dual Dirac monopoles (dual Wu-Yang monopoles)  
soliton has singularity in the center

$$Q(x) = -i\vec{\sigma}\vec{n}(x), \quad \vec{\Gamma}_\mu(x) = \vec{n}(x) \times \partial_\mu \vec{n}(x), \quad \vec{R}_{\mu\nu}(x) = \partial_\mu \vec{n}(x) \times \partial_\nu \vec{n}(x)$$

$${}^*F_{\mu\nu}(x) = -\frac{e_0}{4\pi\varepsilon_0 c} \vec{R}_{\mu\nu} \vec{n} = -\frac{e_0}{4\pi\varepsilon_0 c} \vec{n}(x) [\partial_\mu \vec{n}(x) \times \partial_\nu \vec{n}(x)]$$

$$\mathcal{L}_{ED} = -\frac{1}{4\mu_0} {}^*F_{\mu\nu}(x) {}^*F^{\mu\nu}(x)$$

$$Q_{el}(S) = -\frac{e_0}{4\pi} \oint_{S(u,v)} du dv \vec{n} [\partial_u \vec{n} \times \partial_v \vec{n}]$$

world-lines of singularities

# Maxwell equations?

world-lines of singularities are closed

$$j^\mu = -\epsilon_0 c \sum_{i=1}^N \int d\tau_i \frac{dX^\mu(\tau_i)}{d\tau_i} \delta^4(x - X(\tau_i)) = (c\rho, \mathbf{j})$$

Electric charge is conserved, a topological quantum number

Gauß-law = inhomogeneous Maxwell equations

$$\frac{1}{2\mu_0} \oint_{\partial V} dx^\mu dx^\nu {}^*F_{\mu\nu} = \frac{1}{6} \int_V dx^\mu dx^\nu dx^\rho \epsilon_{\mu\nu\rho\sigma} j^\sigma$$

magnetic currents  $\stackrel{\wedge}{=} \text{homogeneous Maxwell equations}$

$$g^\mu = c \partial_\nu {}^*F^{\nu\mu} \quad \Leftrightarrow \quad \begin{cases} \rho_{\text{mag}} = \nabla \cdot \mathbf{B}, \\ \mathbf{g} = -\nabla \times \mathbf{E} - \partial_t \mathbf{B}, \end{cases}$$

equation of motion

$$\partial_\mu \vec{n} g^\mu = 0 \quad \Leftrightarrow \quad \begin{cases} \mathbf{B} \cdot \mathbf{g} = 0, \\ c^2 \mathbf{B} \cdot \rho_{\text{mag}} = \mathbf{g} \times \mathbf{E}. \end{cases}$$

solutions of Maxwell equations solve equations of motion

# Coulomb and Lorentz forces

Canonical energy-momentum tensor  $\Theta^\mu{}_\nu$  in electrodynamic limit

$$T^\mu{}_\nu(x) = -\frac{1}{\mu_0} {}^*F_{\nu\sigma}(x){}^*F^{\mu\sigma}(x) - \frac{1}{4\mu_0} {}^*F_{\lambda\sigma}(x){}^*F^{\lambda\sigma}(x) \delta^\mu_\nu$$

is symmetric

We split total force density

$$f_\nu = \partial_\mu \Theta^\mu{}_\nu = f_{\text{charges}}^\mu + \partial^\nu T^\mu{}_\nu = 0$$

interaction is a consequence of topology

Coulomb and Lorentz forces

$$\begin{aligned} f_{\text{charges}}^0 &= \frac{1}{c} \mathbf{j} \cdot \mathbf{E}, \\ \mathbf{f}_{\text{charges}} &= \rho \mathbf{E} + \mathbf{j} \times \mathbf{B}. \end{aligned}$$

# $U(1)$ gauge invariance

in color space, rotate all vectors with

$$\Omega(x) = e^{-i\vec{\omega}(x)\vec{L}} = e^{\vec{\omega}(x)\times}, \quad \vec{\omega} = 2\alpha\vec{n}$$

$$\vec{n}' = \Omega\vec{n} = (\vec{n}\vec{e}_\omega)\vec{e}_\omega + \sin\omega \vec{e}_\omega \times \vec{n} - \cos\omega \vec{e}_\omega \times (\vec{e}_\omega \times \vec{n})$$

$$\vec{\Gamma}_\mu = \vec{n} \times \partial_\mu \vec{n}$$

$$\Gamma_\mu = \vec{L}\vec{\Gamma}_\mu \mapsto \Gamma'_\mu = \vec{L}\vec{\Gamma}'_\mu = \Omega(\Gamma_\mu - i\partial_\mu)\Omega^\dagger$$

there is a rotational invariance around  $\vec{n}$ -direction

far-field of hedge-hog soliton: align radial electric field along z-axis

$$\vec{E}'_r = \frac{e_0}{4\pi\varepsilon_0} \frac{\vec{e}_3}{r^2}, \quad \frac{\vec{\Gamma}_\varphi(\vartheta)}{r \sin \vartheta} = \frac{1-\cos\vartheta}{r \sin \vartheta} \vec{e}_3, \quad \text{potential of Dirac monopole}$$

rotational invariance of local Dreibein around 3-axis

$U(1)$  gauge invariance is rotational invariance around  $\vec{n}$  axis.

define lines of constant  $\vec{n}$ -field, fibres.

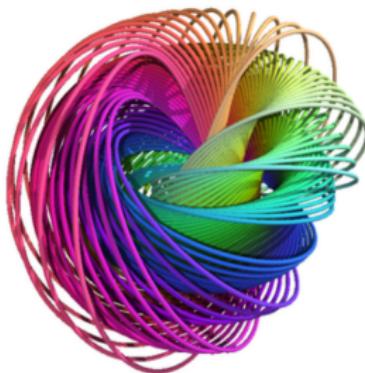
# Hopf map

map  $\mathbb{S}^3 \rightarrow \mathbb{S}^2$ ,  $q = (q_0, q_1, q_2, q_3) \mapsto n = (n_1, n_2, n_3)$

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \text{ and } n_1^2 + n_2^2 + n_3^2 = 1$$

circles of  $\mathbb{S}^3$  mapped to linked fibres of  $\mathbb{S}^2$

visualisation after stereographic projection:  $\mathbb{S}^2 \rightarrow \mathbb{R}^3$



$$n_1 = 2(q_0q_2 + q_1q_3)$$

$$n_2 = 2(q_2q_3 - q_0q_1)$$

$$n_3 = -q_1^2 - q_2^2 + q_3^2 + q_0^2$$

from: <http://nilesjohnson.net/etcetera.html>

**define fibres,  $\vec{n} = \text{const.} \in S^2$**

$$\mathbb{R}^3 \cup \infty \sim S^3, \quad \pi_3(S^2) = \mathbb{Z}$$

**linking fibres**

Gauß's linking number  $v$  (Verschlingungszahl)

Topological invariant, coverings of  $S^2$

$$v = \frac{1}{4\pi} \oint_{C_1} \oint_{C_2} \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \cdot (\mathrm{d}\mathbf{r}_1 \times \mathrm{d}\mathbf{r}_2)$$

linking number  $v$  can be positive or negative,

**left or right circulating Goldstone bosons**

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**conjecture:**

Gauß's linking number = Hopf number = **photon number**

of dual photon field  $\vec{C}_\mu = -\frac{e_0}{4\pi\varepsilon_0 c} \vec{\Gamma}_\mu = -\frac{e_0}{4\pi\varepsilon_0 c} \vec{n} \times \partial_\mu \vec{n}$

# relations to space-time?

- ▶  $SO(3)$ -field

rotation of spatial Dreibein,

3 degrees of freedom are the three Euler angles,  
crossing unit charge - rotation by  $2\pi$ .

- ▶ potential term  $\Lambda(x)$  - a cosmological function

cosmological constant from  $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = 0.69$

$$\rho_c c^2 = \frac{3H^2 c^2}{8\pi G_N} = 4,9 \frac{\text{GeV}}{\text{m}^3},$$

prediction of particle physics  $\rho_\Lambda c^2 = 1,82 \cdot 10^{121} \frac{\text{GeV}}{\text{m}^3}$ .

due to Derricks theorem 1/4 of mass due to  $\Lambda$ ,

15 nucleons per  $\text{m}^3 \rightarrow \Omega_\Lambda$ .

- ▶ transition from  $Q = 1$  and  $\Lambda = 1/r_0^4$  to  $\Lambda = 0$

release of energy density of

$$\alpha_f \hbar c / (4\pi r_0^4) = 4.8 \text{ keV/fm}^3 = 7.7 \cdot 10^{29} \text{ J/m}^3,$$

could be a mechanism for inflation.

# Summary

- ▶ Particles are topological solitons
- ▶ Mass is field energy
- ▶ Charge and spin are topological quantum numbers
- ▶ Integer multiples of elementary charge only
- ▶ Distinction between charges and their field is artificial
- ▶ Charges can be described by  $2\pi$ -rotations of space.
- ▶ Spin angular momentum as Eigen-angular momentum,  
Spin as a consequence of orbital motion.
- ▶ Two types of massless excitations,  $\vec{n}$ -waves = electromagnetic waves
- ▶ Pauli principle has topological origin
- ▶ U(1) gauge invariance =  
= rotational invariance of Dreibein around  $\vec{n}$
- ▶ Cosmological constant  $\Lambda \rightarrow$  Cosmological Function  $\Lambda(t, \vec{r})$
- ▶ Only 3 rotational degrees of freedom of space were used.
- ▶ Only Space and Time.

# Problems and speculations

## Conjectures

- ▶ Stable particles are stable solitons with topological quantum numbers.
- ▶ Only topological solitons don't escape our detectors?
- ▶ Waves escape our detectors.
- ▶ Waves disturb the pathes of particles -> Quantum Mechanics
- ▶ Particles get in resonance with waves -> interference  
(in analogy to Couder's experiments)
- ▶  $\alpha$ -waves and magnetic currents → dark matter?
- ▶ Potential  $\Lambda(t, \vec{r}) \rightarrow$  dark energy?
- ▶ Inflation: transition of vacuum  $Q = 1 \rightarrow Q = i\vec{n}\vec{\sigma}$ ?

## Problems:

- ▶ non-topological magnetic currents
- ▶ excited states or  $\alpha$ -waves with non-zero mass
- ▶ perturbatively non-renormalisable
- ▶ quantum properties

# My suppositions

- ▶ a lot of geometry still hidden in physics
- ▶ physics is geometry and not algebra
- ▶ we should use algebra to investigate geometry

## General relativity:

Wheeler: “Matter tells space how to warp.  
And warped space tells matter to move.”

## Electro dynamics:

... Charges and electromagnetic fields tell space how to rotate.

Thanks

# Sine-Gordon model

field variable:  $\vartheta(x, t)$ ,  $e^{i\vartheta(x, t)} \in \mathbb{S}^1$

in natural units of length, time and energy

hamiltonian density:  $h(x, t) = \frac{1}{2} \left( \frac{\partial \vartheta}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \vartheta}{\partial x} \right)^2 + (1 - \cos \vartheta)$

degenerate vacua:  $h(x, t) \equiv 0$ ,  $\vartheta(x, t) = \text{const} = 2\pi n$ ,  $n \in \mathbb{Z}$

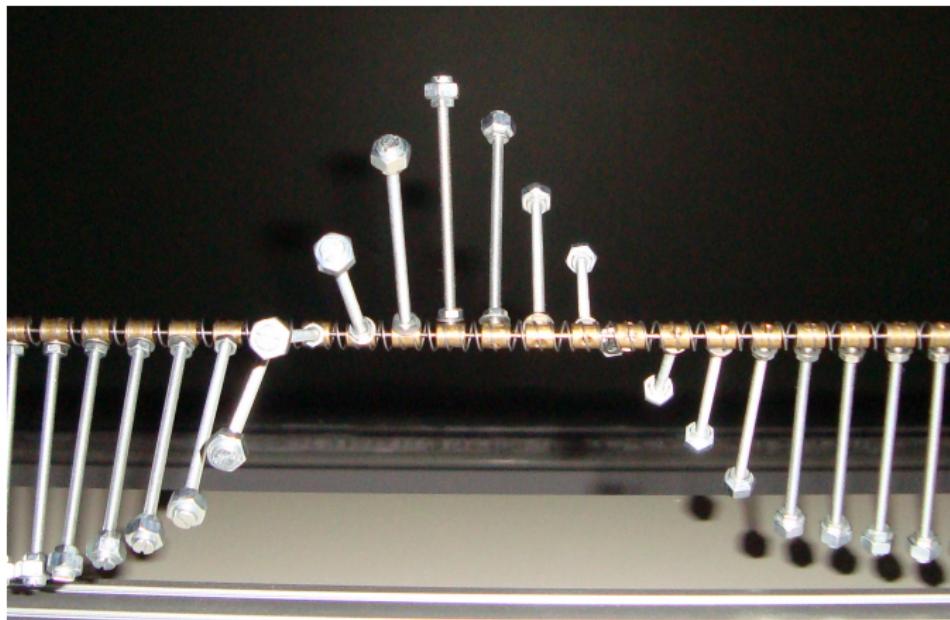
equation of motion a non-linear differential eq.:  $\partial_x^2 \vartheta - \partial_t^2 \vartheta = \sin \vartheta$

soliton solutions:  $\vartheta_{\beta\pm}(x, t) = 4 \arctan e^{\pm\gamma(x-\beta t)}$  with  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

soliton energy:  $E(\beta) = \int dx h(x, t) = 4(\beta^2 \gamma + \gamma + \frac{1}{\gamma}) = 8\gamma$

infinitely many time dependent solutions

...in the experiment



Picture: Gerald Pechoc

# Moving soliton?



Picture: Gerald Pecho

# Topological quantum number $Z$

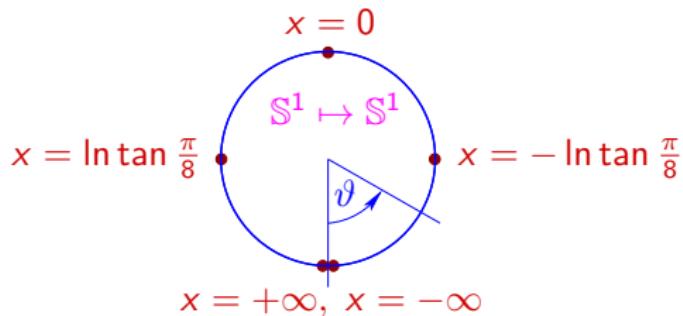
$x \in \mathbb{R}^1$

with boundary condition for  $\vartheta(x)$ :  $e^{i\vartheta(-\infty)} = e^{i\vartheta(+\infty)}$

$\mathbb{R}^1$  topologically equivalent to  $\mathbb{S}^1$

field  $\vartheta(x)$  is a map:  $\mathbb{R}^1 \mapsto \mathbb{S}^1 \xrightarrow{\text{boundary condition}} \mathbb{S}^1 \mapsto \mathbb{S}^1$

winding number  $Z$ ,  
soliton:  $Z = 1$



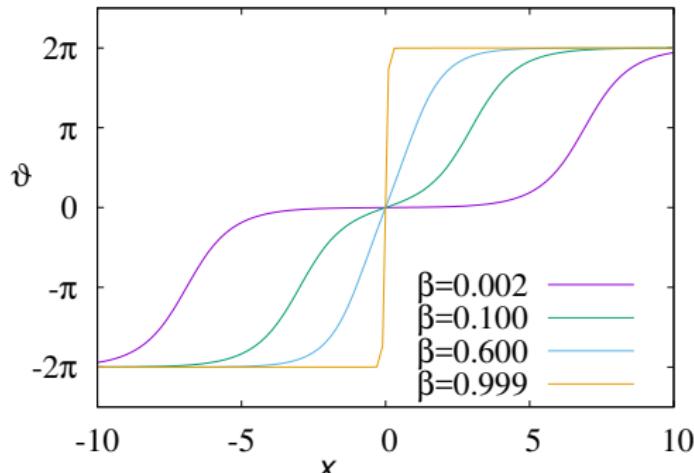
$Z$  is a particle minus antiparticle number

# Are electrons point-like?

Size of electrons:

- ▶ LEP accelerated electrons to 100 GeV,
- ▶ Resolution  $2 \cdot 10^{-18}$  m, ( $\hbar c \approx 200$  MeV fm = 200 GeV am),
- ▶ Electrons still point-like,
- ▶ but  $\alpha_f(0 \text{ GeV}) = \frac{1}{137.036} \rightarrow \alpha_f(90 \text{ GeV}) = \frac{1}{127}$ .

Sine-Gordon solitons have arbitrary small size:



Scattering of solitons:

$$\vartheta(x, t) = 4 \operatorname{atan} \frac{\beta \sinh(\gamma x)}{\cosh(\beta \gamma t)},$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

they have arbitrary small size

# Nice features of Sine-Gordon model

- ▶ solitons like particles with relativistic properties
- ▶ Mass is field energy
- ▶ particle number is a topological quantum number
- ▶ particle number is therefore integer
- ▶ forces are a consequence of the structure of particles
- ▶ distinction between field and particle is artificial

## Aim

Stable topological solitons,  
with long range (Coulombic) interaction,  
in 3+1D