$U_A(1)$ breaking from the lattice and its topological origin

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Outline

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The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may affect the order of phase transition for $N_f = 2$ [Pisarski & Wilczek, 83].

- In model QFT with same symmetries as QCD, it is not possible to quantify the $U_A(1)$ effects in observables.

- Need lattice studies with fermions having exact chiral/flavour symmetry + reproduce exactly anomaly on the lattice.

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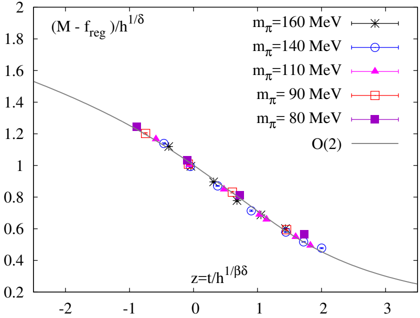
Why is it important?

- $m_{u,d} << \Lambda_{QCD}$, chiral symmetry drives phase transition at $\mu_B \to 0$
- The singular part of free energy should show critical scaling $\to$ hints of criticality from lattice studies [BI-BNL collaboration, 09].



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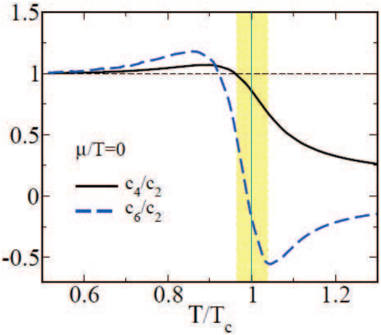
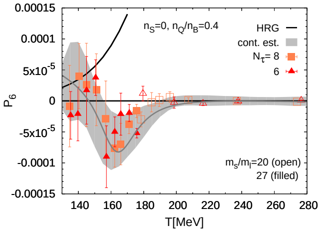
- Criticality at $\mu = 0$ changes on whether $U_A(1)$ is effectively restored [Pelissetto & Vicari, 13, Nakayama & Ohtsuki, 14].
    - $O(4)$ critical exponents for $U_A(1)$ broken
    - $U(2) \times U(2)$ if $U_A(1)$ effectively restored
- Effects should be visible in higher order fluctuations measured in the experiments [Karsch & Redlich, 11, Bielefeld-BNL-CCNU collaboration, 1701.04325]





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- Could affect the EoS relevant for anomalous hydrodynamics with chiral imbalance?
- Softening of η' mass near freezeout? [Grahl & Rischke, 14,15]
- Consequences for the critical end-point at finite μ_B?
- Lattice QCD can answer such questions from first principles.
- The microscopic constituents responsible for it may also be responsible for characteristic T dependence of topological susceptibility.

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The major issues with the lattice studies so far

- Finite volume effects $\rightarrow$ ensure presence of topological objects in a box.
- Most studies done with lattice fermions with reasonably good remnant of continuum chiral symmetry + explicitly broken $U_A(1)$ which is restored in the continuum limit [S. Chandrasekharan, 96, H. Ohno et. al 12, V. Dick et. al., 15].
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Observables sensitive to $U_A(1)$ breaking..

- Not an exact symmetry $\rightarrow$ no order-parameter $\rightarrow$
- Important to look at all point correlation functions between axial
- Atleast for the integrated 2 point correlators [Shuryak, 94]

$$\chi_\pi - \chi_\delta = \int d^4x \left[\langle i\pi^+(x) i\pi^-(0) \rangle - \langle \delta^+(x)\delta^-(0) \rangle \right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator [Cohen, 95, Hatsuda & Lee, 95]

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}\ ,\ \langle \bar{\psi}\psi \rangle \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{2m_f\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- Chiral symmetry restored: $\lim_{m_f\to 0} \lim_{V\to\infty} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ restored.

- Chiral symmetry restored $+U_A(1)$ broken if

$$\lim_{\lambda\to 0} \rho(\lambda, m_f) \to \delta(\lambda) m_f^\alpha\ , 1 < \alpha < 2.$$

Spectral density of Dirac operator at finite T: Analytics

- Very little known. Only recently there are interesting results [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in m^2, λ, look at chiral Ward identities of n-point function of scalar & pseudo-scalar currents.
- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in these sectors for upto 6-point functions.
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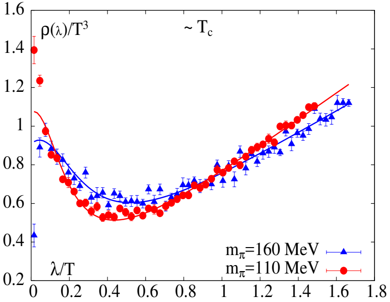
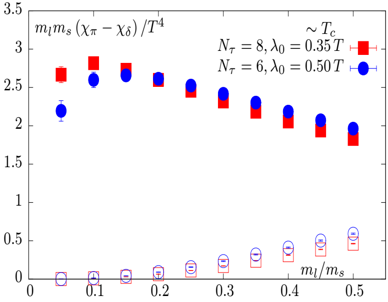
Results for QCD with staggered quarks

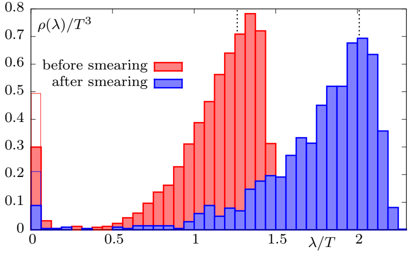
- D_{ov} has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of D_{ov} related to topological structures of the underlying gauge field. [Hasenfratz, Laliena & Niedermeyer, 98].
- Used overlap as valence operator to probe the infrared spectrum of Highly Improved Staggered Quarks(HISQ).
- $U_A(1)$ broken near T_c and near-zero modes primarily responsible for it.

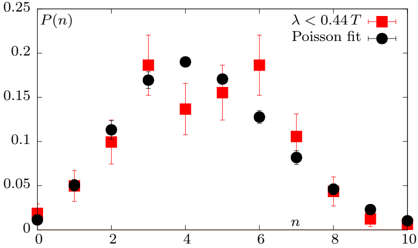




QCD medium at 1.5 T_c

- HYP smearing [Hasenfratz & Knechtli, 02] expected to eliminate dislocations





- Smearing does not eliminate the near zero modes.
- At 1.5 T_c, QCD medium is a dilute gas of small instantons
  $r = 0.23$ fm, $\rho = 0.15\text{fm}^{-4}$

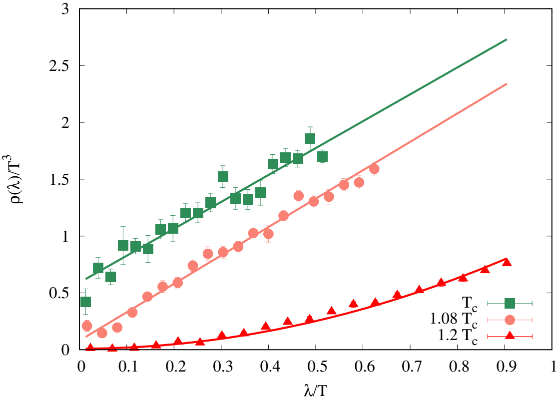
Numerical details

- Möbius domain wall fermions on 5D hypercube with $N = 32$ sites along each spatial 4-dim, $N_5 = 16$ and $N_\tau = 8$ sites along temporal dim. We also have results with staggered (HISQ) fermions.
- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, a is the lattice spacing.
- Box size: $m_\pi \ V^{1/3} > 4$
- 2 light+1 heavy flavour
- Input m_s physical ≈ 100 MeV and $m_s/m_l = 27, 12$ $\Rightarrow m_\pi = 135, 200$ MeV. [Columbia-BNL-LLNL, 13,14].
- The sign function and chiral symmetry maintained as precise as 10^{-10}.

QCD Dirac spectrum at finite T

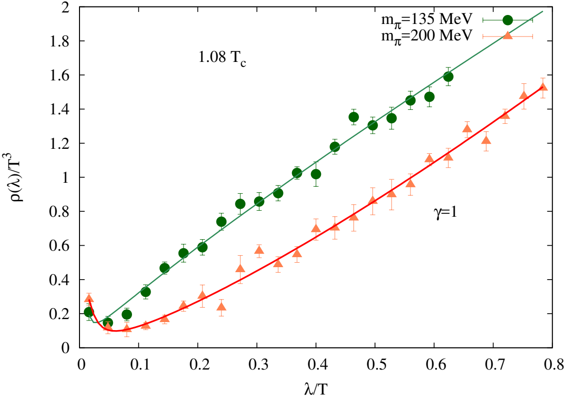
- General features: Near zero mode peak +bulk.
- No gap observed upto 1.2 T_c for physical quark mass

[V. Dick et. al. in prep, also 1602.02197].



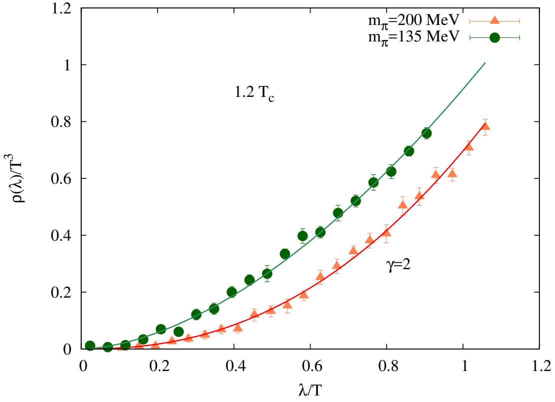
General Characteristics

- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$.
- Bulk rises linearly as λ near T_c.
- No gap even when quark mass reduced!



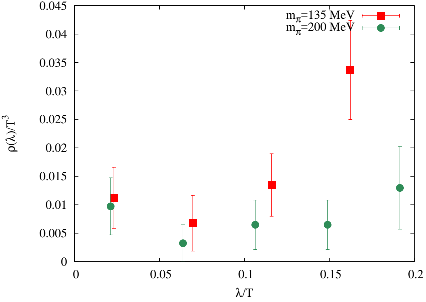
General Characteristics

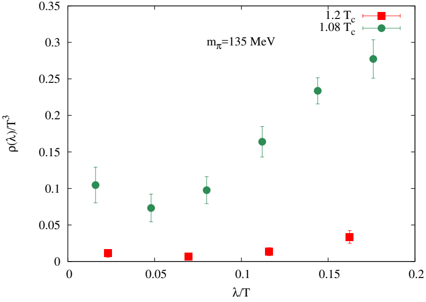
- The rise of the bulk is $\gamma \sim 2 \;\rightarrow$ Still not consistent with λ^3.
- Infrared modes becomes rarer with a small peak.



A closer look at the near-zero modes

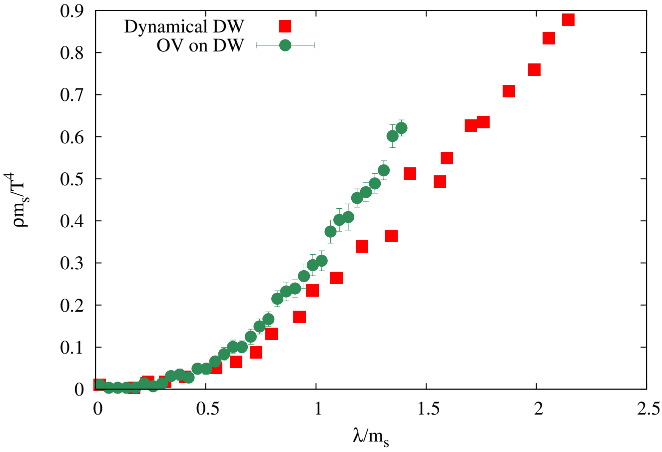
- The near-zero modes sensitive to the sea quark mass $\rightarrow$ sparse when m_π heavier but the peak survives!
- Falls by more than a third at $1.2\,T_c$.





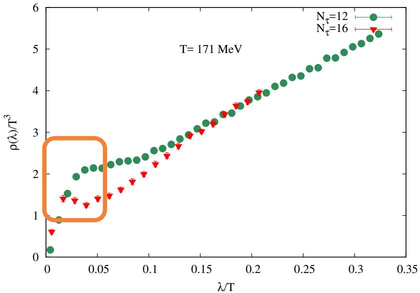
Comparing with earlier results

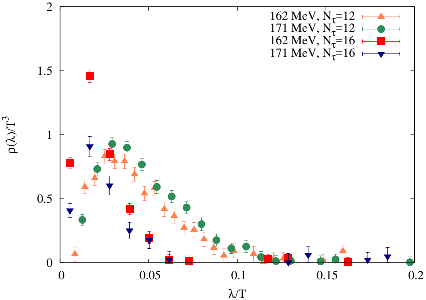
- The renormalized spectra of dynamical Domain wall fermions [Columbia-BNL-LLNL, 13] agrees very well with what we measured with the overlap.



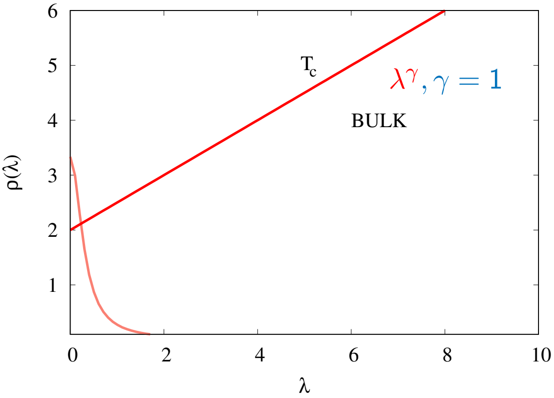
Comparing eigenspectra for different lattice fermions

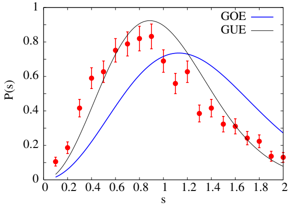
- Exponent characterizing the bulk spectra of staggered quarks(HISQ) consistent with domain wall fermions. [HotQCD collab. in prep.]
- The near-zero peak start appearing for finest lattice spacings even with staggered quarks → non-perturbative characteristic of QCD eigenvalue spectrum
- Suffer from strong finite volume effects [G. Cossu et. al, 13, A. Tomiya et. al, 15,16] due to which there has been serious debate on it!





Summary of eigenvalue spectrum at finite T





The bulk spectrum has level spacings characteristic from GUE in Random matrix theory

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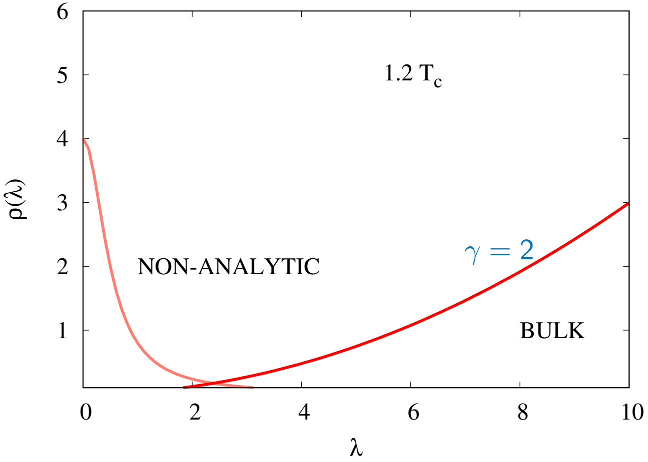

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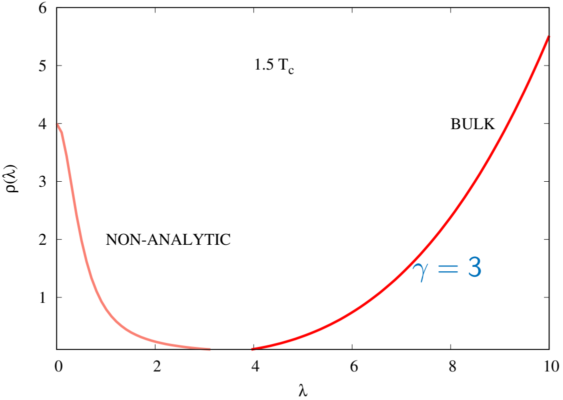
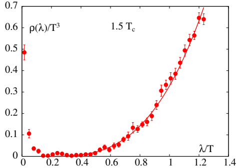
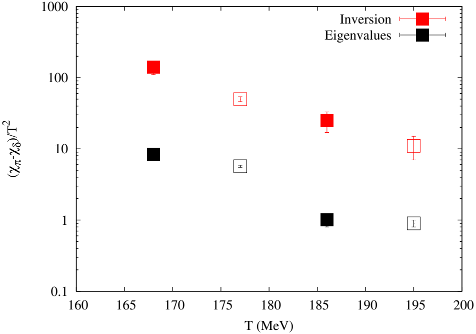




[V. Dick, et. al, 1502.06190, 1602.02197].

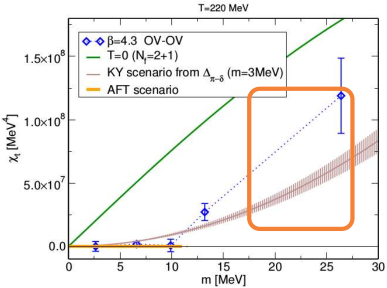
Fate of $U_A(1)$ near T_c

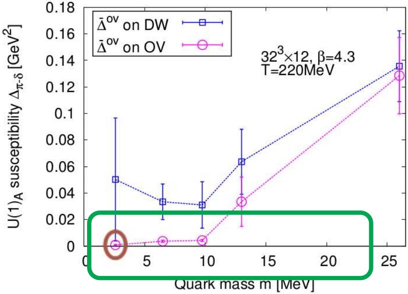
- Contribution to $U_A(1)$ breaking in 2-point correlation functions mainly come from small eigenvalues.
- First 50 eigenvalues produce most of the breaking obtained from inversion of the Domain wall Dirac operator with good chiral properties. [V. Dick, et. al, 1602.02197, Columbia-BNL-LLNL, 13,14].



Summary of independent lattice results from JLQCD

Reference: Y. Aoki, XQCD 2018





Outline

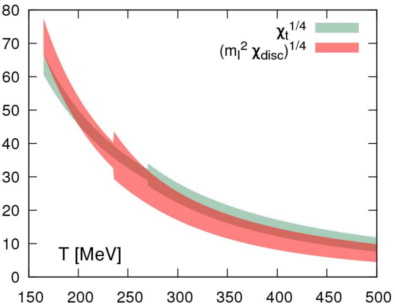
What are the constituents of the hot QCD medium?

- At $T = 0$, anomaly effects related to instantons [t'Hooft, 76].
- Near chiral crossover transition T_c, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model [Shuryak, 82].
- At $T >> T_c$, medium like a dilute gas of instantons [Gross, Pisarski & Yaffe, 81]. How high is the T?
- What is the medium made up of for $T_c \leq T \leq 2T_c$?

Independent confirmation: Topological susceptibility

- Topological susceptibility measurement at high T on the lattice suffers from rare topological tunneling, lattice artifacts.
- Going towards continuum limit difficult due to freezing of topology.

- Fermionic observables [L. Giusti, G. C. Rossi, M. Testa, 0402027, HotQCD 1205.3535] shown to agree with standard definition of $\chi_t = \int d^4x \langle F\tilde{F}(x) F\tilde{F}(0)\rangle$ in the continuum even with staggered quarks. [P. Petreczky, H-P Schadler, SS, 1606.03145].
- Continuum extrapolated results now available for QCD!



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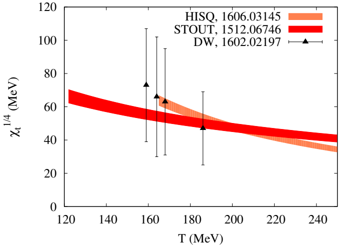
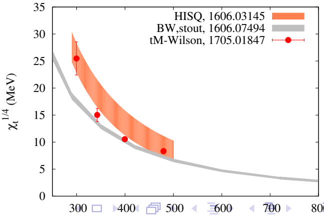


- Fit ansatz: $\chi_t^{1/4} = AT^{-b}$.
- $b = 0.9 - 1.2$ for $T < 250$ MeV from continuum extrapolated results with HISQ.
  [P. Petreczky, H-P Schadler, SS, 1606.03145].
  Agrees well with an independent study [Bonati et. al, 1512.06746] and with results with chiral fermions 1602.02197.
- Dilute gas prediction:
  $b = 2 - \frac{11N_c}{12} - \frac{2N_f}{12}$.

- $T > 300$ MeV: Continuum extrapolated $b = 1.85(15)$. Agreement with dilute instanton gas.
  Confirmed also in an independent study with reweighting techniques.
  [Borsanyi et. al, 1606.07494]
  Wilson type quarks with m rescaling agrees quite well
  [F. Burger et. al, 1705.01847, Y. Taniguchi et. al., 1611.02413]



More Diagnostics!

- Since θ is tiny, $F(\theta) = \frac{1}{2}\chi_t\theta^2\left(1 + b_2\theta^2 + ...\right)$.
  [L. D. Debbio, H. Panagopoulos, E. Vicari, 0407068]
- Strong non-Gaussianity in higher order expansions. Hints about existence of instanton-dyons? Hints observed in lattice studies
  [M. Ilgenfritz, M-Mueller Pruessker, et. al. 14, 15].
- Evident also from the T-dependence of χ_t [P. Petreczky, H-P Schadler, SS, 1606.03145]. New lattice techniques are being discussed to explore them.
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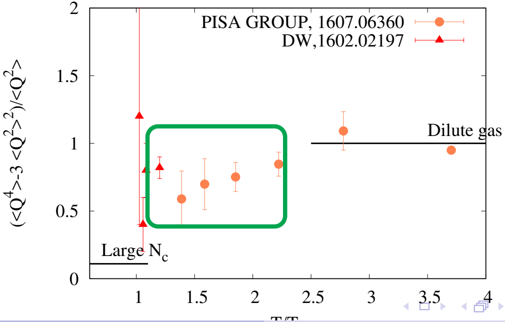

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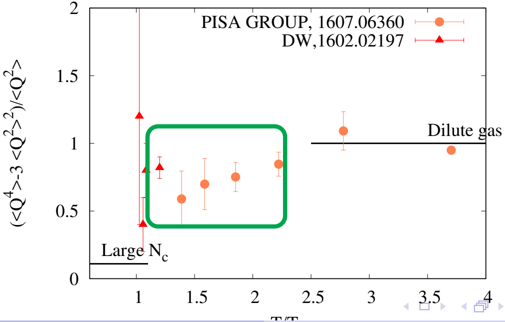

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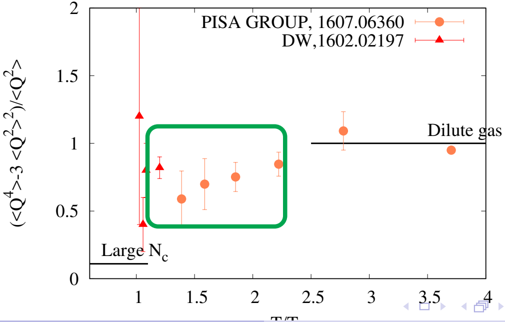
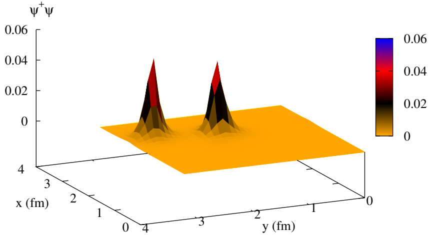
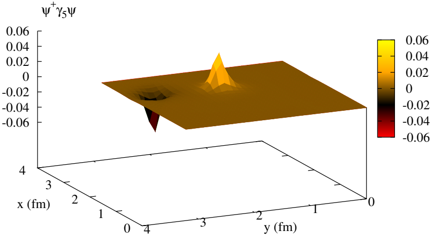


Near-zero modes at $\sim$ 200 MeV

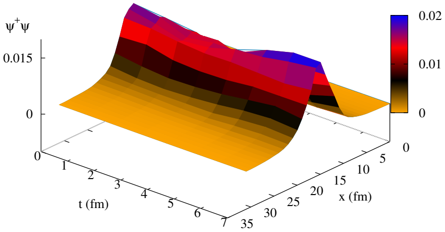


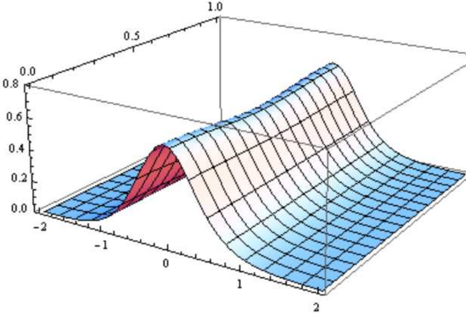


Near-zero modes of QCD Dirac operator at 1.5 T_c due to a weakly interacting instanton-antiinstanton pair!

Zero modes at $1.1\ T_c$

- We use twisted b.c. for the overlap on the thermal Domain wall fermion ensembles → detects the different instanton-dyons.
- The shape of the zero mode strongly depends on the separation between the instanton-dyons [R. Larsen, SS,Shuryak, in prep., More in Lattice 2018].





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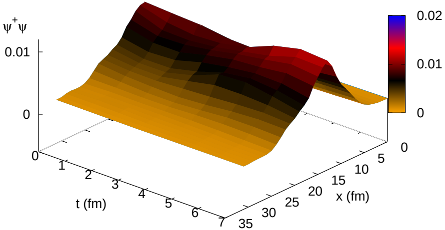

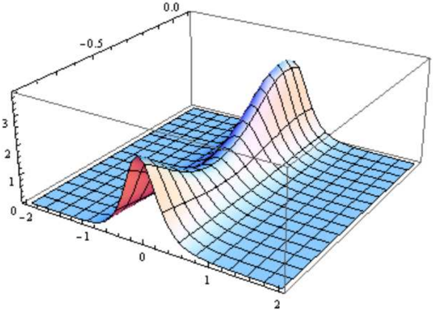

Outline

Summary

- On large volume lattice we found that $U_A(1)$ broken upto $T \leq 1.5\,T_c$.
- Infrared eigenvalues contribute dominantly to its breaking.
- Consists of near-zero+tail of the bulk modes. The latter quite robust insensitive to lattice cut-off effects.
- Near-zero modes require a careful study.
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