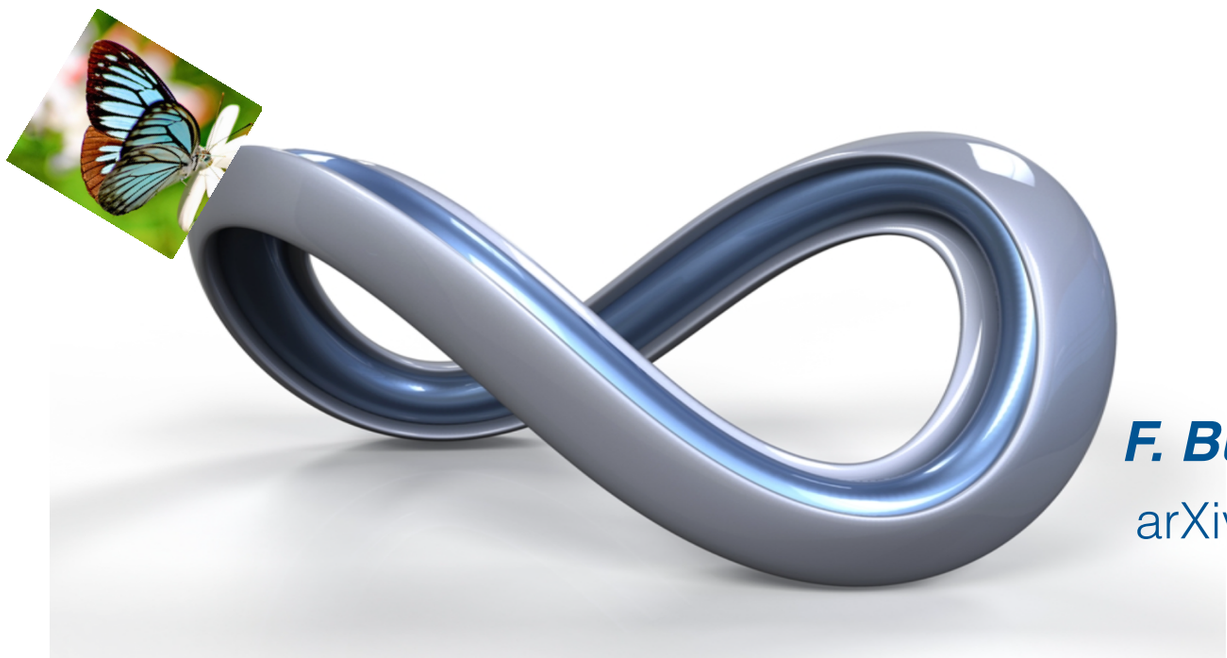
Chiral symmetry and topology in QCD with Nf = 2+1+1



Maria Paola Lombardo

INFN Istituto Nazionale di Fisica Nucleare

cost EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

F. Burger, E-M. Ilgenfritz, A. Trunin, MpL

arXiv:1805.06001 & work in progress

Gauge Topology 3: from Lattice to Colliders

ECT* Trento May 29 2018

Plan

Lattice QCD with twisted mass Wilson Fermions

Nf=2 $\longrightarrow$ Nf=2+1+1

Chiral symmetry and the (pseudocritical) region

Topology

Results
Topology&Axions
Topology@Colliders?

QCD Symmetries, lattice and the real world

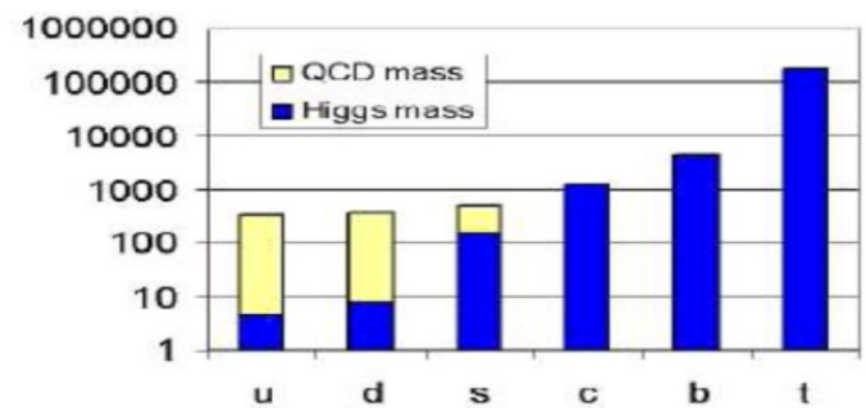
High T

$SU(N_f)_R \times SU(N_f)_L \times U_A(1)$

$SU(N_f)_V$

Low T

c,b,t do not participate in the chiral dynamics around the critical temperature. Lattice simulations **around Tc** are then performed with up,down,strange quarks – Nf = 2+1



	SU(N)XSU(N)	$U_A(1)$
Staggered	Remnant U(1)	Broken
Wilson	Broken	Broken
Domain Wall	Exact (for L $\to\infty$)	Exact (for L $\to\infty$)
Overlap	Exact	Exact
Wilson twisted	As good as staggered	Broken

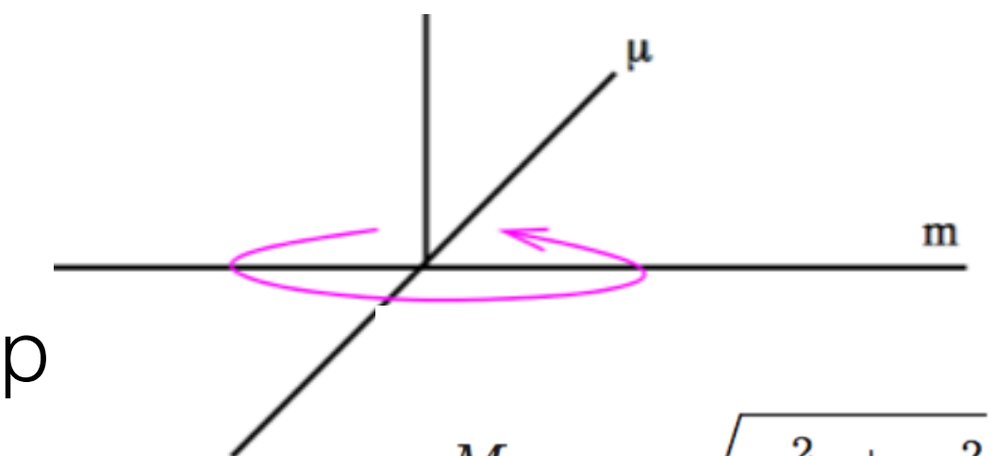
Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

A twisted mass term in flavor space:

$$i\mu\tau_3\gamma_5 \text{ for two degenerate light flavors}$$

is added to the standard mass term in the Wilson Lagrangian



$$M_{\rm inv} = \sqrt{m_0^2 + \mu_q^2}$$

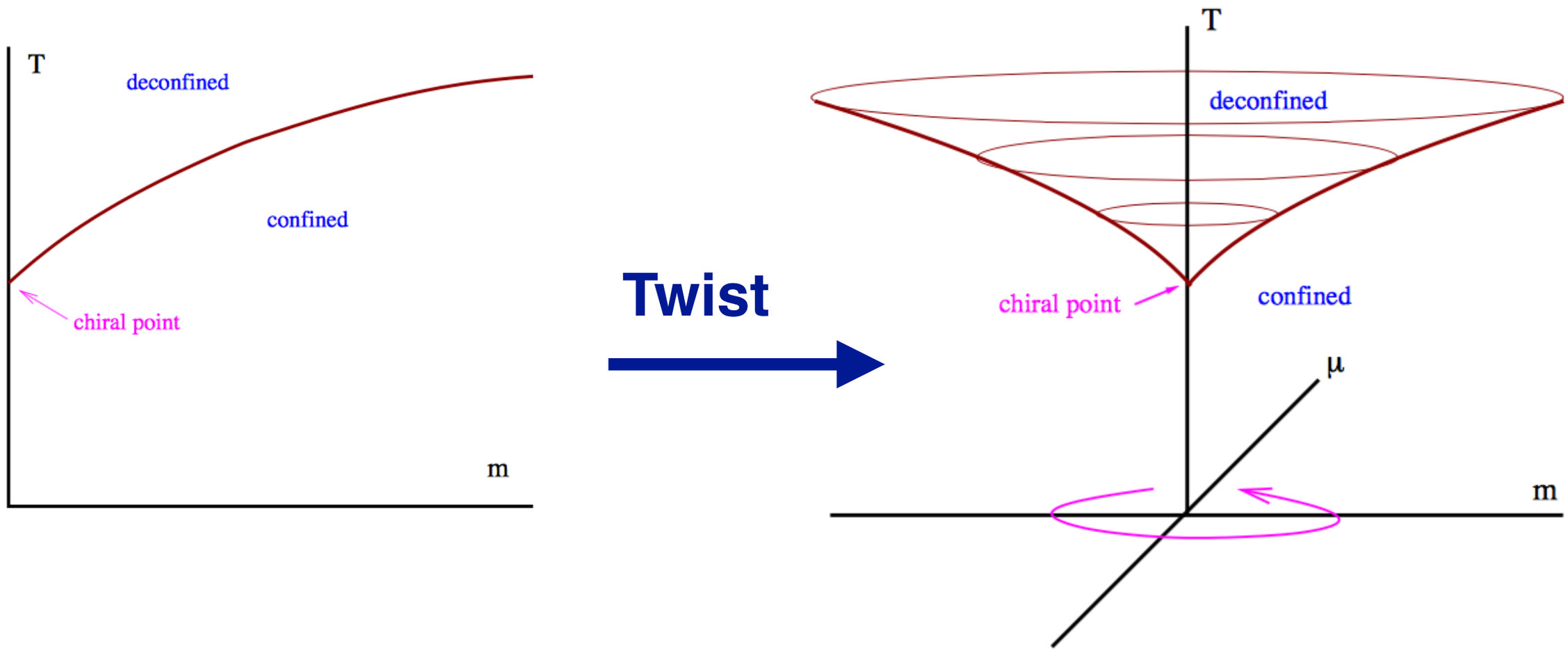
Consequences:

-simplified renormalization prop
-automatic O(a) improvement
-control on unphysical zero modes

Successful phenomenology at T=0

ETMC collaboration 2003—

Continuum scenario at finite temperature



$$M_{\rm inv} \;=\; \sqrt{m_0^2 \;+\; \mu_q^2} \qquad ; \qquad \tan(\omega) \;=\; \frac{\mu_q}{m_0}$$

On the lattice: special choice

maximal twist ($\omega = \frac{\pi}{2}$): $\quad m_{\mathrm{R}} = Z_m(m - m_{\mathrm{cr}}) = 0$

$\rightarrow$ **automatic** $\mathcal{O}(a)$ improvement[1]

$\longrightarrow$ good **renormalization** properties

$$[S^0]_{\mathrm{R}} = Z_{S^0}\left[[S^0]_{\mathrm{QCD}} + \frac{c_S(g_0^2)}{a^3}\right] + \cdots$$

Twist

$$[S^0]_{\mathrm{R}} = Z_P\left[[P^3]_{\mathrm{tmQCD}} + \frac{\mu_q c_P(g_0^2)}{a^2}\right] + \cdots$$

Nf = 2+1+1 Wilson fermions with a twisted mass term

Frezzotti Rossi 2003

two 'twisted' mass terms in flavor space:

$$i\mu\tau_3\gamma_5 \text{ for two degenerate light flavors}$$
$$i\mu_\sigma\tau_1\gamma_5 + \tau_3\mu_\delta \text{ for two heavy flavors}$$

are added to the standard Wilson Lagrangian

Consequences as for Nf=2

-simplified renormalization properties
-automatic O(a) improvement
-control on unphysical zero modes

Successful phenomenology at T=0

ETMC collaboration 2012—

Why Nf = 2 +1 +1 ?

Tc

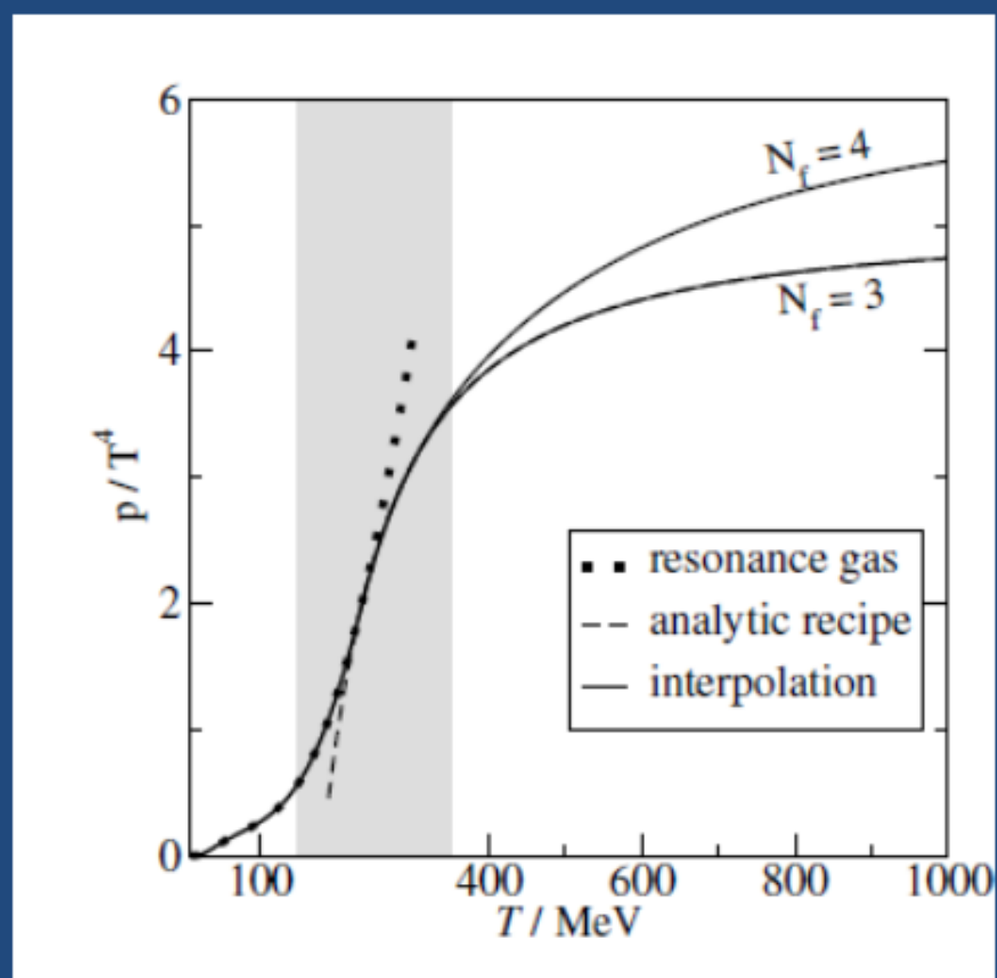
≈200MeV

340 –380 MeV
RHIC AuAu
200 GeV

420-480 MeV
LHC
2.76 TeV

500- 600MeV
LHC hot spots
2.76 TeV

1 GeV
LHC
7 TeV



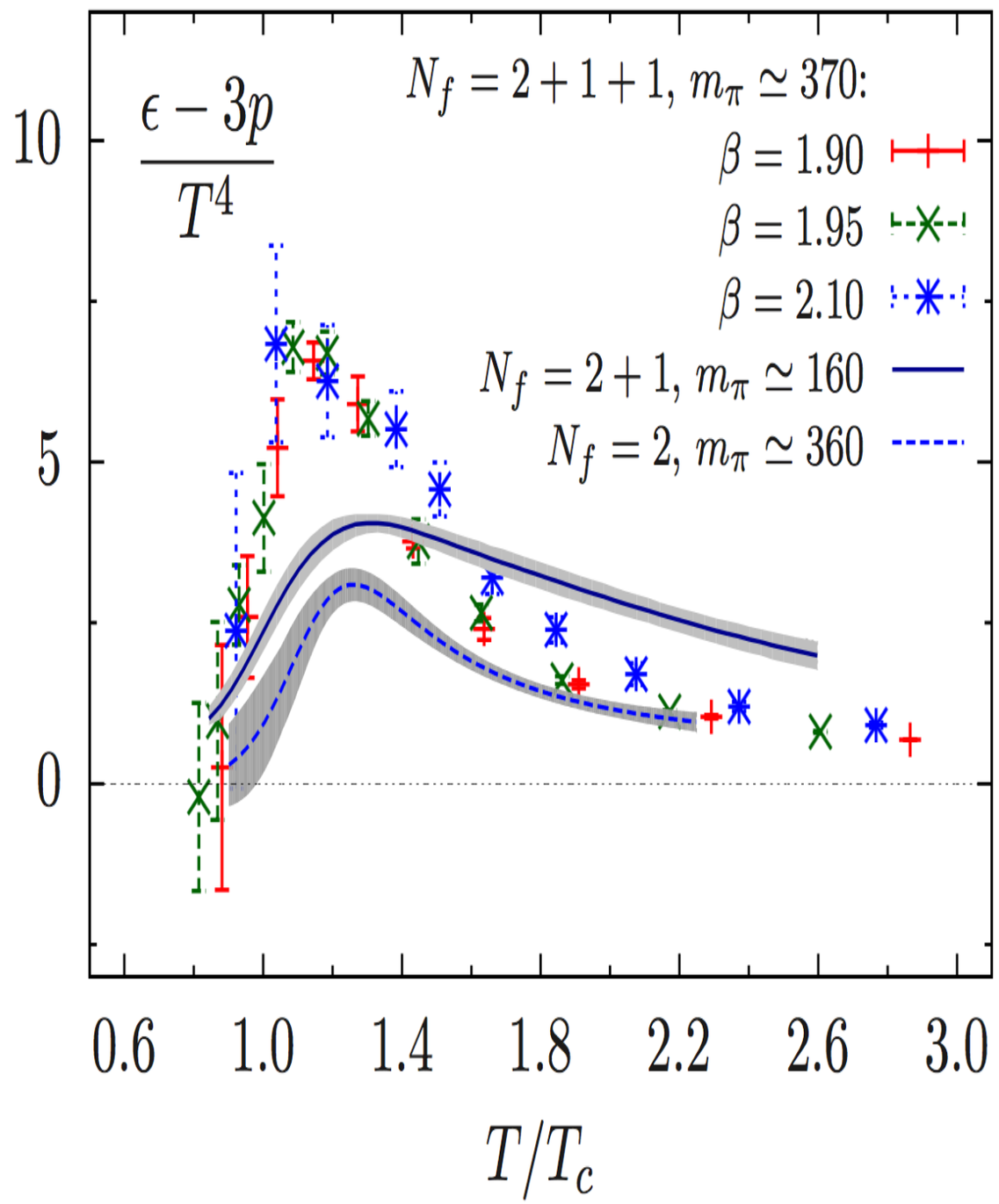
Quark Gluon Plasma @ Colliders

Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

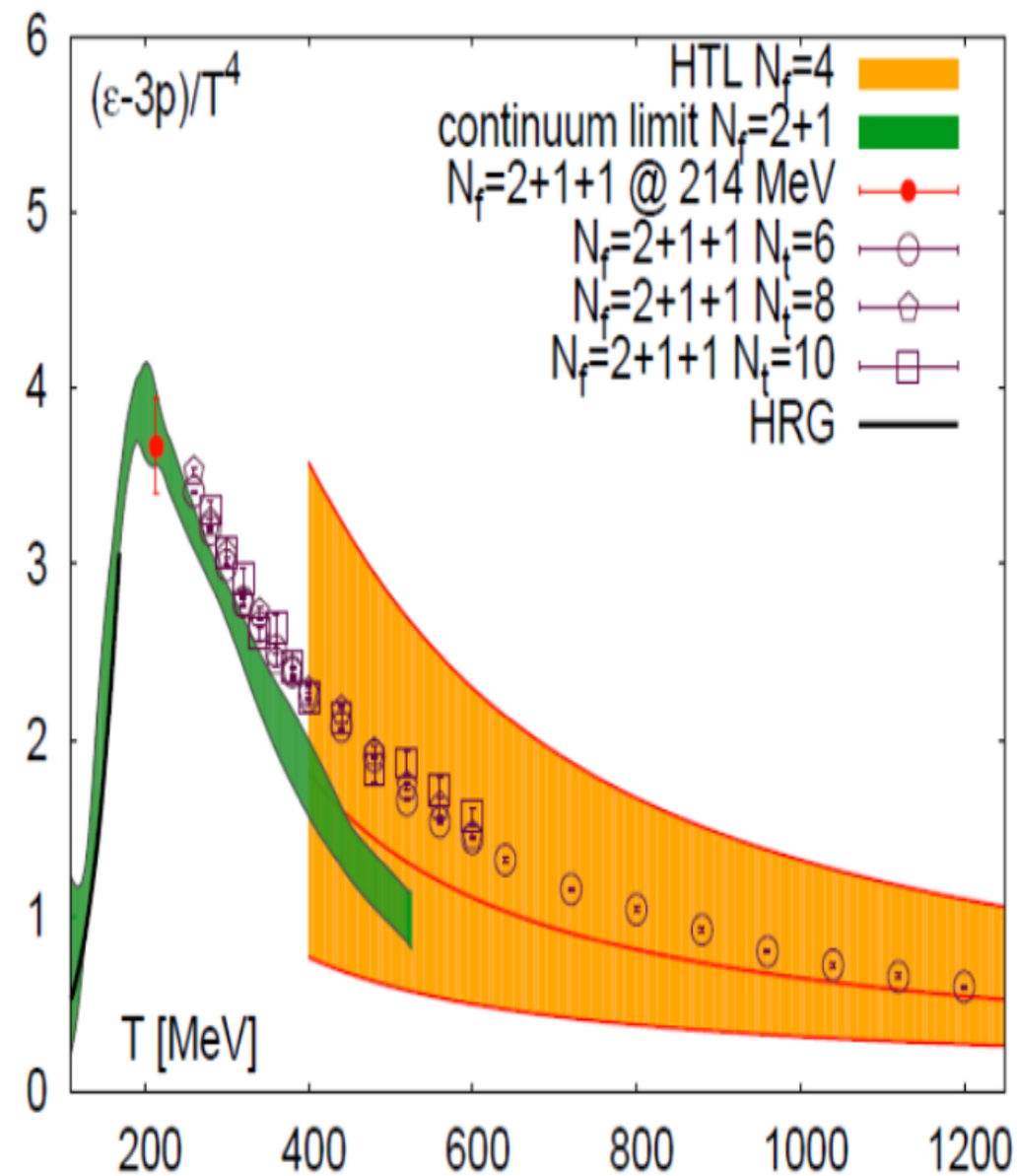
Laine Schroeder 2006

Trace anomaly: effects of a dynamical charm

Tmft



Wuppertal-Budapest



Staggered

Nf = 2 +1+1 Setup

Fixed varying scale

For each lattice spacing we explore a range of temperatures 150MeV — 500 MeV by varying Nt

We repeat this for three different lattice spacings following ETMC T=0 simulations.

Advantages: we rely on the setup of ETMC T=0 simulations. Scale is set once for all.

Disadvantages: mismatch of temperatures - need interpolation before taking the continuum limit

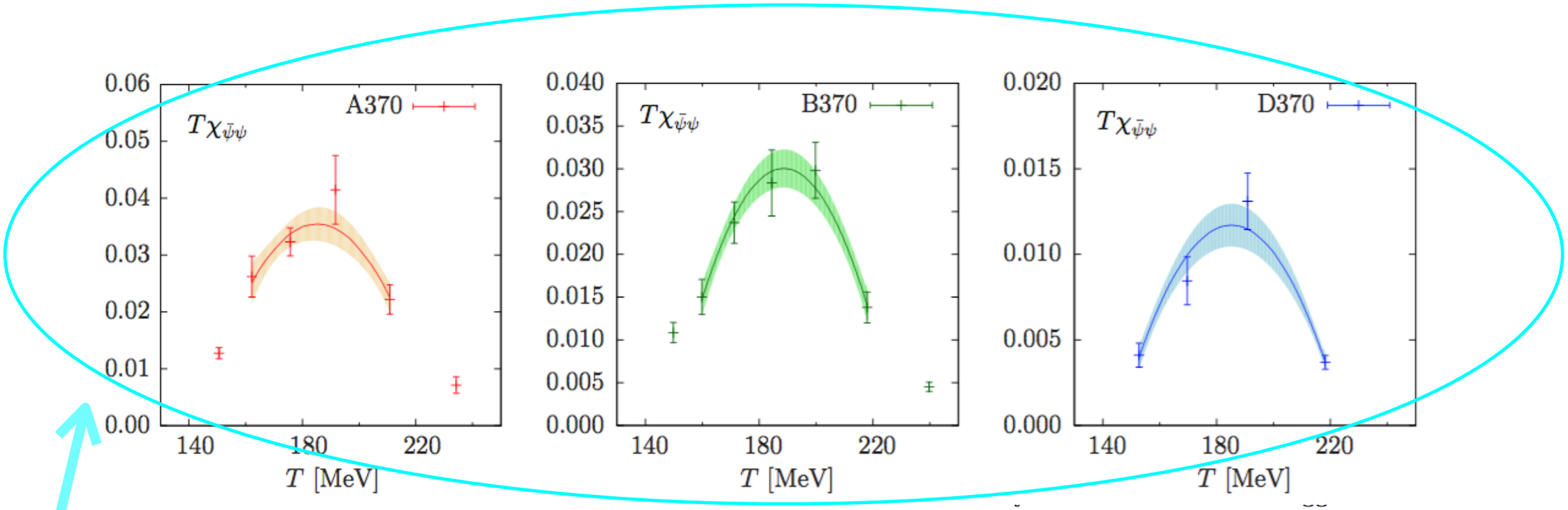
$T=0$ (ETMC) nomenclature	β	a [fm] [6]	N_σ^3	N_τ	T [MeV]	# confs.
A60.24	1.90	0.0936(38)	24^3	5	422(17)	585
				6	351(14)	1370
				7	301(12)	341
				8	263(11)	970
				9	234(10)	577
				10	211(9)	525
				11	192(8)	227
				12	176(7)	1052
			32^3	13	162(7)	294
				14	151(6)	1988
B55.32	1.95	0.0823(37)	32^3	5	479(22)	595
				6	400(18)	345
				7	342(15)	327
				8	300(13)	233
				9	266(12)	453
				10	240(11)	295
				11	218(10)	667
				12	200(9)	1102
				13	184(8)	308
				14	171(8)	1304
				15	160(7)	456
				16	150(7)	823
D45.32	2.10	0.0646(26)	32^3	6	509(20)	403
				7	436(18)	412
				8	382(15)	416
				10	305(12)	420
				12	255(10)	380
				14	218(9)	793
				16	191(8)	626
			40^3	18	170(7)	599
			48^3	20	153(6)	582

Four pion masses

Number of flavours	$m_{\pi^\pm}$
$N_f = 2+1+1$	210
	260
	370
	470
$N_f = 2$	360
	430

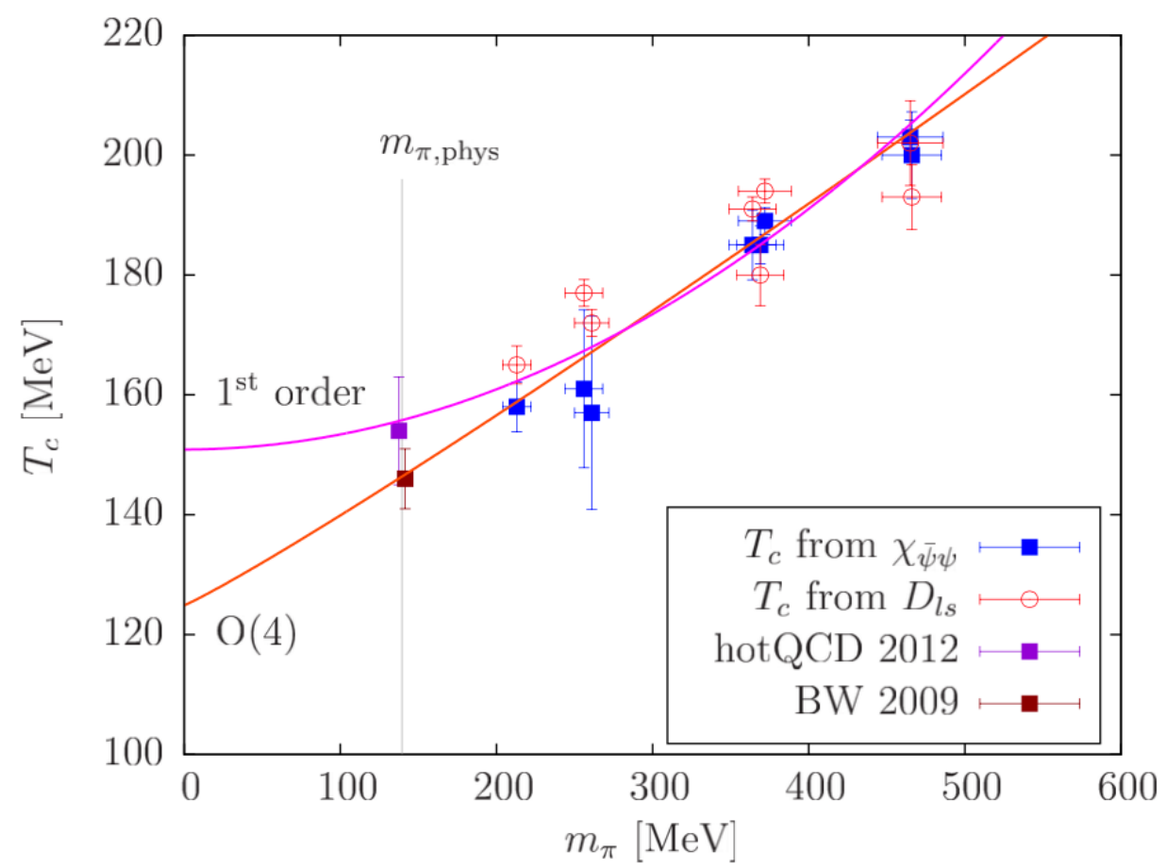
Overview of Chiral observables
Nf 2 + 1 +1

Outcome: twisted mass ok; and the results confirm that a dynamical charm does not contribute around Tc



a [fm]	m_π [MeV]	T_χ [MeV]	T_Δ [MeV]	$T_{\rm deconf}$ [MeV]
0.065	213	158(1)(4)	165(3)(1)	176(8)(8)
0.094	261	157(8)(14)	172(2)(1)	188(6)(1)
0.082	256	161(13)(2)	177(2)(1)	192(9)(2)
0.094	364	185(5)(3)	191(2)(0)	202(3)(0)
0.082	372	189(2)(1)	194(2)(0)	201(6)(0)
0.065	369	185(1)(3)	180(5)(1)	193(13)(2)
0.094	466	200(4)(6)	193(5)(2)	205(4)(2)
0.082	465	203(2)(2)	202(7)(1)	212(6)(1)

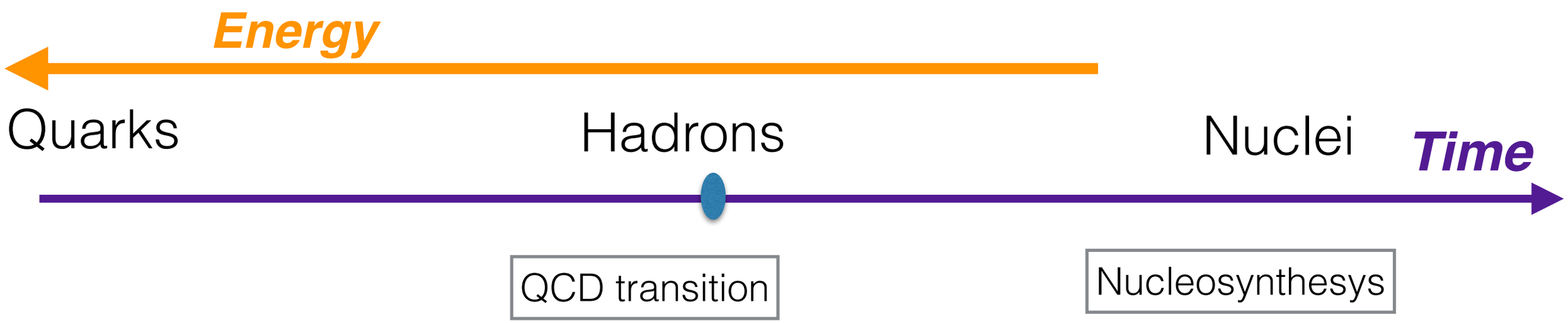
spacing effects below statistical errors



Topology

Topology and $U_A(1)$

Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential



Chiral symm. breaking
Confinement:

Chiral perturbation theory +
Potential models
=
Hadron spectrum

Hadron cosmology:

Origin of mass

Almost all hadrons can be described taking into account chiral symmetry breaking and confining potential

With an important exception

Quarks Hadrons Nuclei ***time***

QCD transition

Nucleosynthesys

Chiral symm. breaking
Confinement:

Chiral perturbation theory +
Potential models
=
Hadron spectrum

Heavy

Topology, η' and the $U_A(1)$ problem:

Solved if

The $U_A(1)$ symmetry $q \to e^{i\alpha\gamma_5} q$

would be broken by the (spontaneously generated) $\bar{q}q$:

the candidate Goldstone is the η'

too heavy!! (900 MeV)

BUT:
the divergence of the current $j_5^\mu = \bar{q}\gamma_5\gamma_\mu q,$

contains a mass independent term

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F\tilde{F}.$$

IF $$\frac{1}{32\pi^2}\int d^4x F\tilde{F} \neq 0$$

The $U_A(1)$ symmetry is **explicitly** broken

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c²)
Pion[6]	π^+	π^-	$u\bar{d}$	139.570 18 ±0.000 35
Pion[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.9766 ±0.0006
Eta meson[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a]	547.862 ±0.018
Eta prime meson[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a]	957.78 ±0.06
Kaon[12]	K^+	K^-	$u\bar{s}$	493.677 ±0.016
Kaon[13]	K^0	$\bar{K}^0$	$d\bar{s}$	497.614 ±0.024

Topology, η' and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$

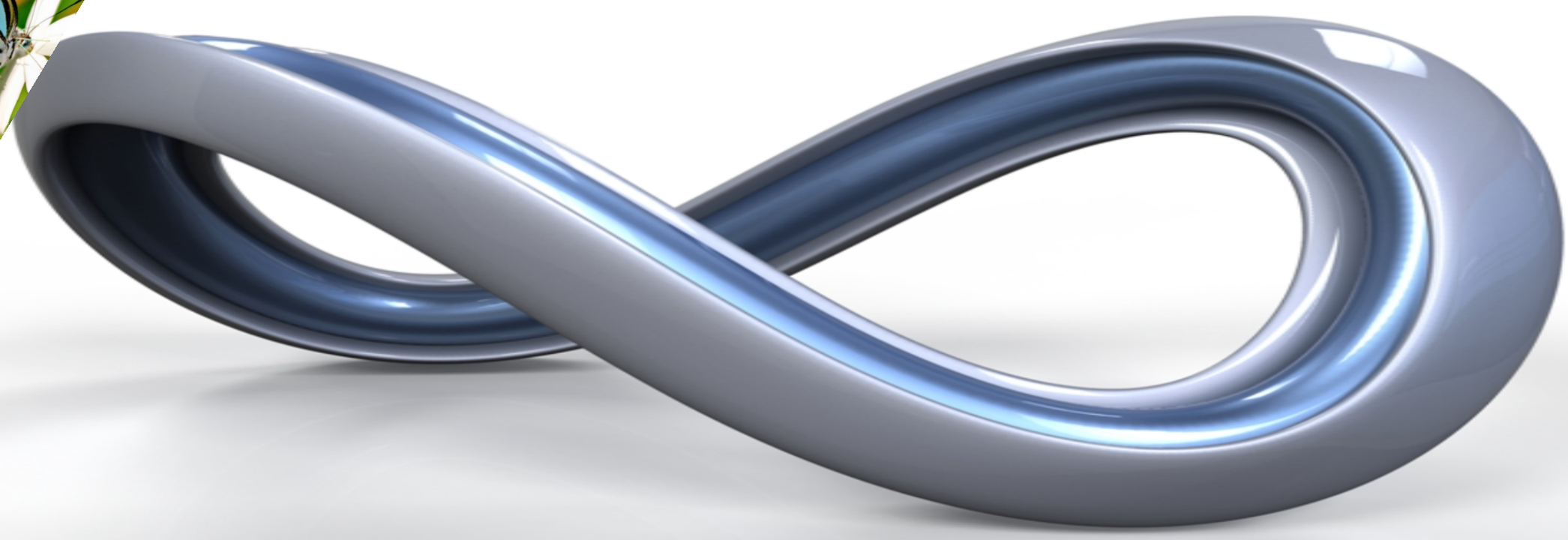
Gluonic definition

and

$$Q = n_+ - n_-$$

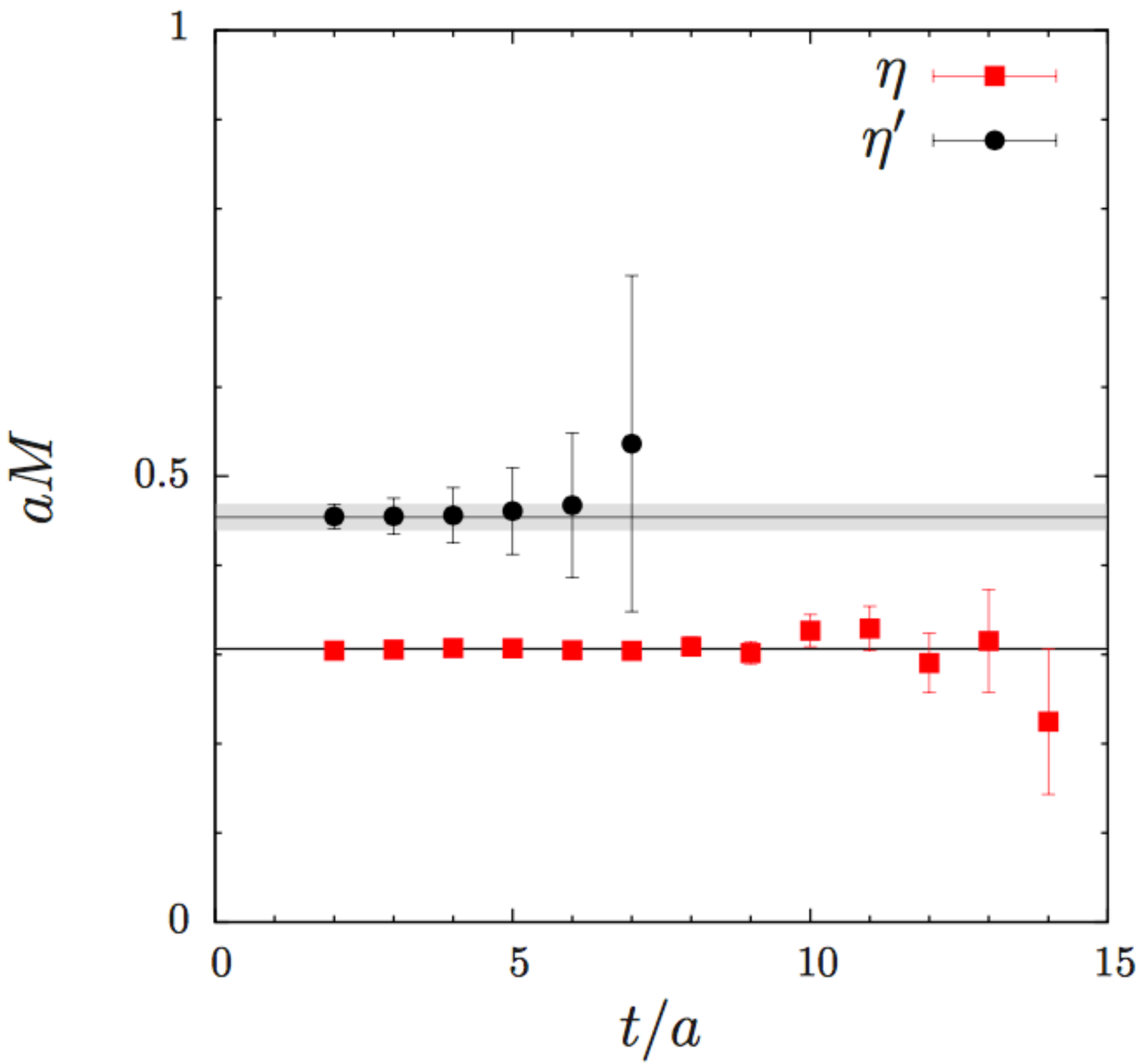
Fermionic definition

$F\tilde{F}$



Contemporary studies of η, η'

Topology at work at T=0!



Otnnad, Urbach, Michael (ETMC) 2013

$$M_\eta = 551(8)_{\text{stat}}(6)_{\text{sys}}\,.$$

$$M_{\eta'} = 1006(54)_{\text{stat}}(38)_{\text{sys}}(+61)_{\text{ex}}$$

OK!

Results on topology in hot QCD

see also C. Bonati and S. Sharma's talks

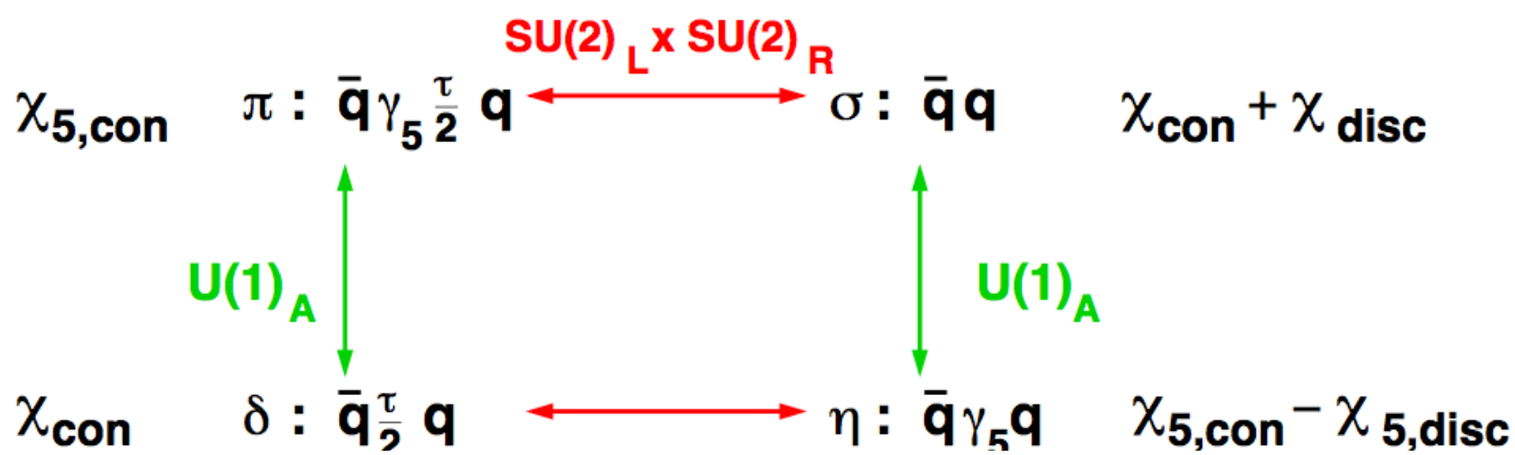
Topological and chiral susceptibility

Kogut, Lagae, Sinclair 1999
HotQCD, 2012

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{5,disc}$$

From the fermionic def.:

$$m \int d^4x \bar{\psi}\gamma_5\psi = Q_{top}$$

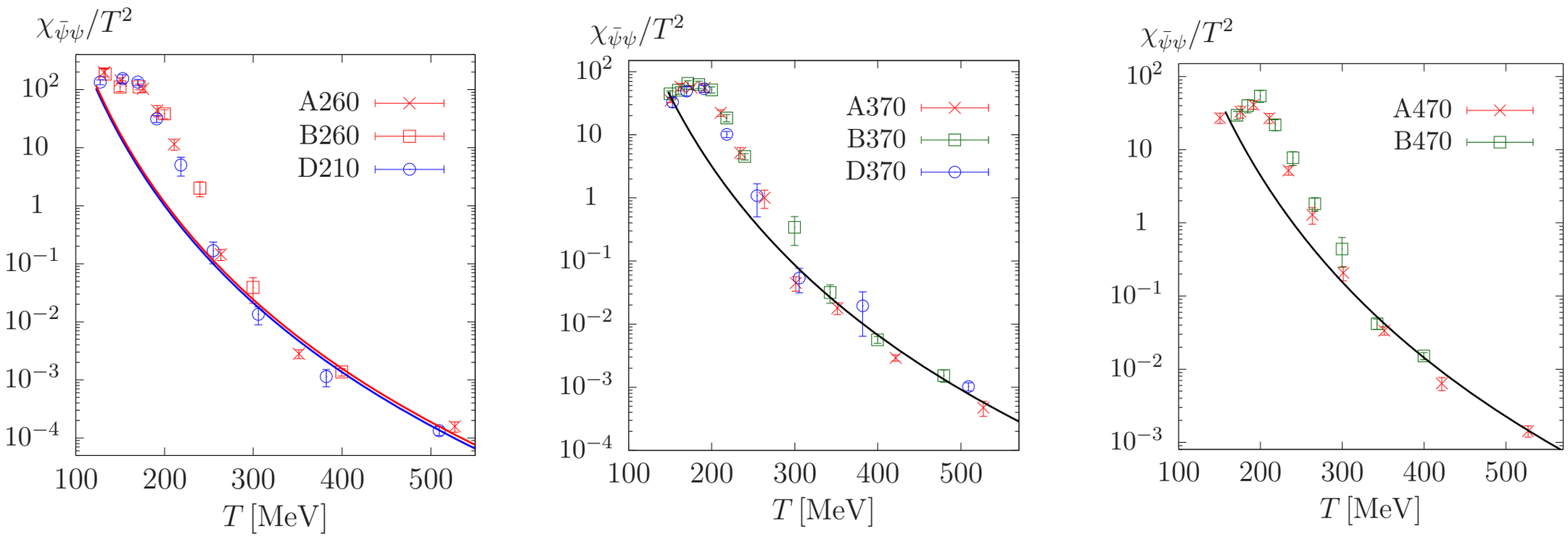


$$\chi_\pi - \chi_\delta = \chi_{\rm disc} = \chi_{5,\rm disc}\ , \qquad \text{for} \quad T \geq T_c\ ,\ m_l \to 0$$

$$\chi_{top} =< Q_{top}^2 > /V = m_l^2 \chi_{disc}$$

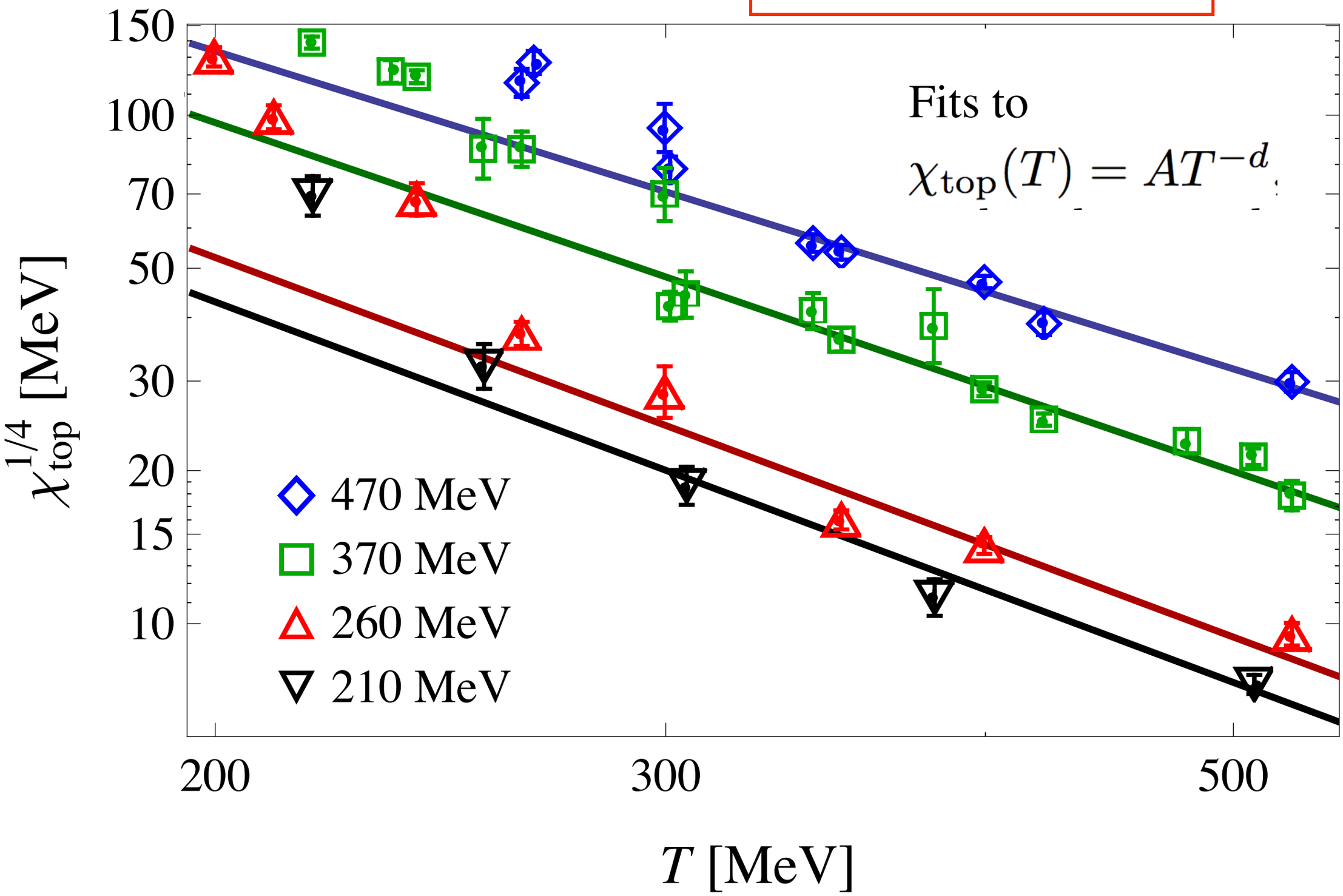
$$T > T_{U(1)_A} \simeq T_c$$

Chiral susceptibility

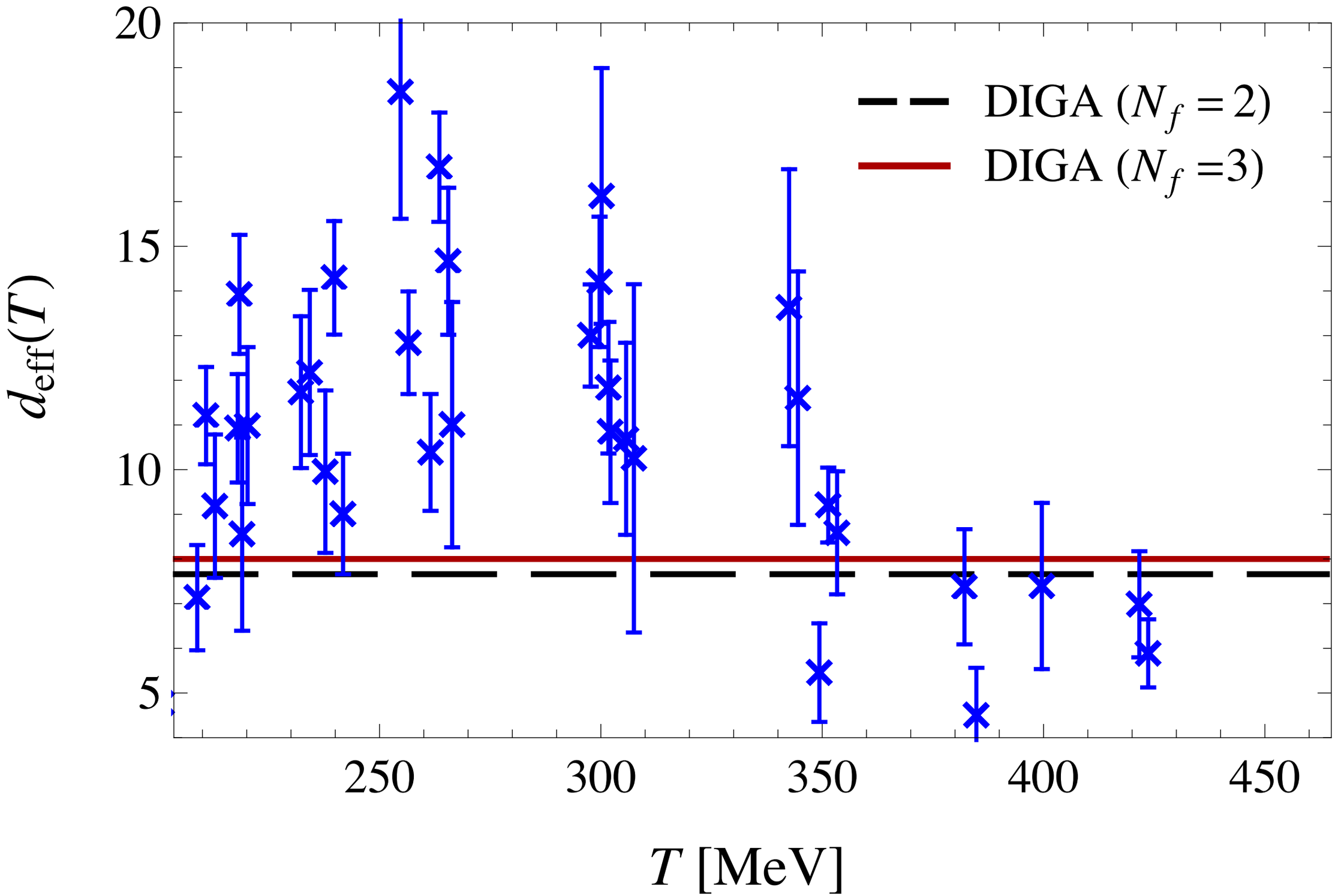


Within errors, no discernable spacing dependence

$$\chi_{\rm top} \equiv \frac{\langle Q^2 \rangle}{V} = m_l^2 \chi_{\cancel{5}}^{\rm disc}$$



$$d_{\rm eff}(T) = T\, d\log\chi_{\rm top}(T)/dT$$

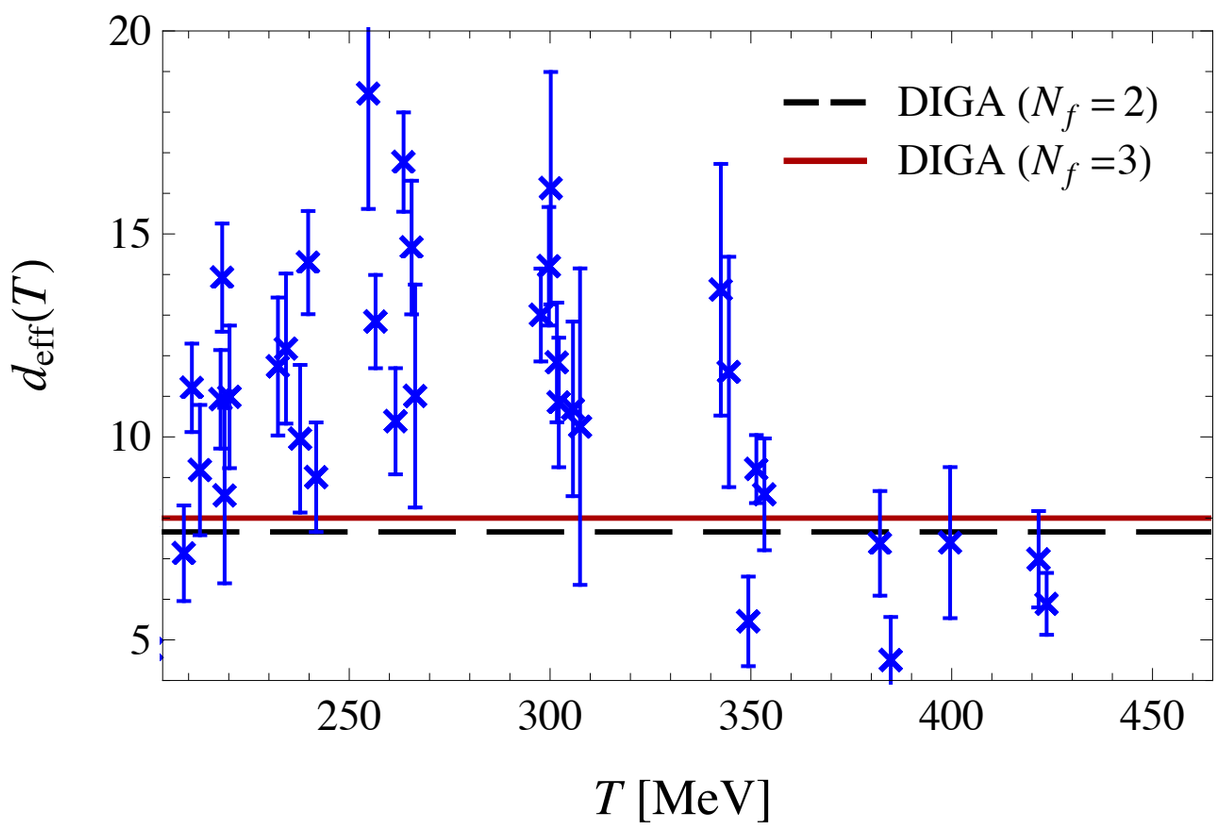


Power-law decay?

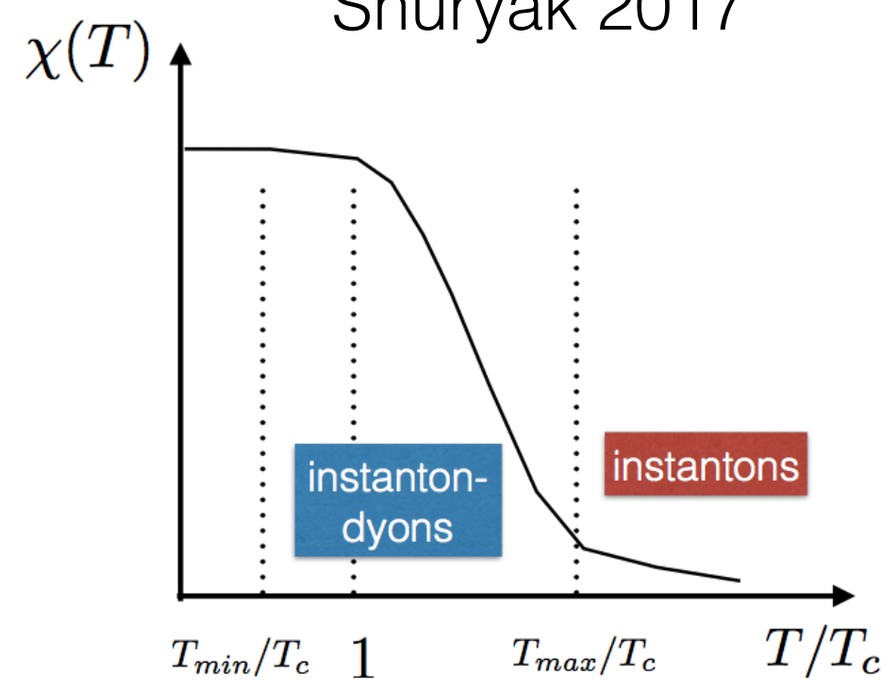
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $\quad d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T\frac{d}{dT}\ln\chi^{0.25}(T)$$



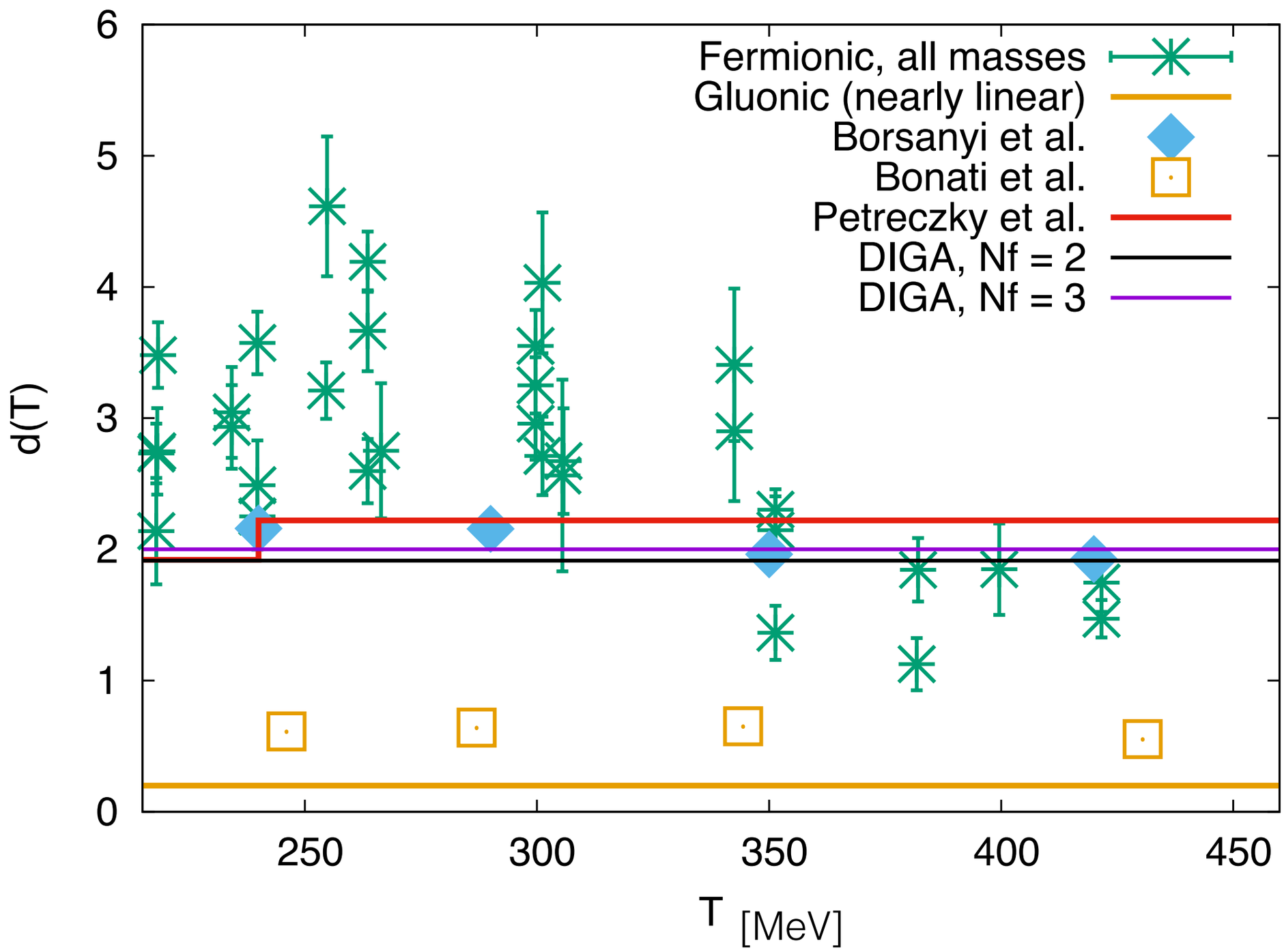
Possibly consistent
with instant -dyon?
Shuryak 2017



Faster decrease before DIGA sets in

Comparisons with other results I :

$$\chi_{top}^{1/4} = aT^{-d(T)}$$



Mass dependence&rescaling

In the symmetric phase: Analyticity + symmetry

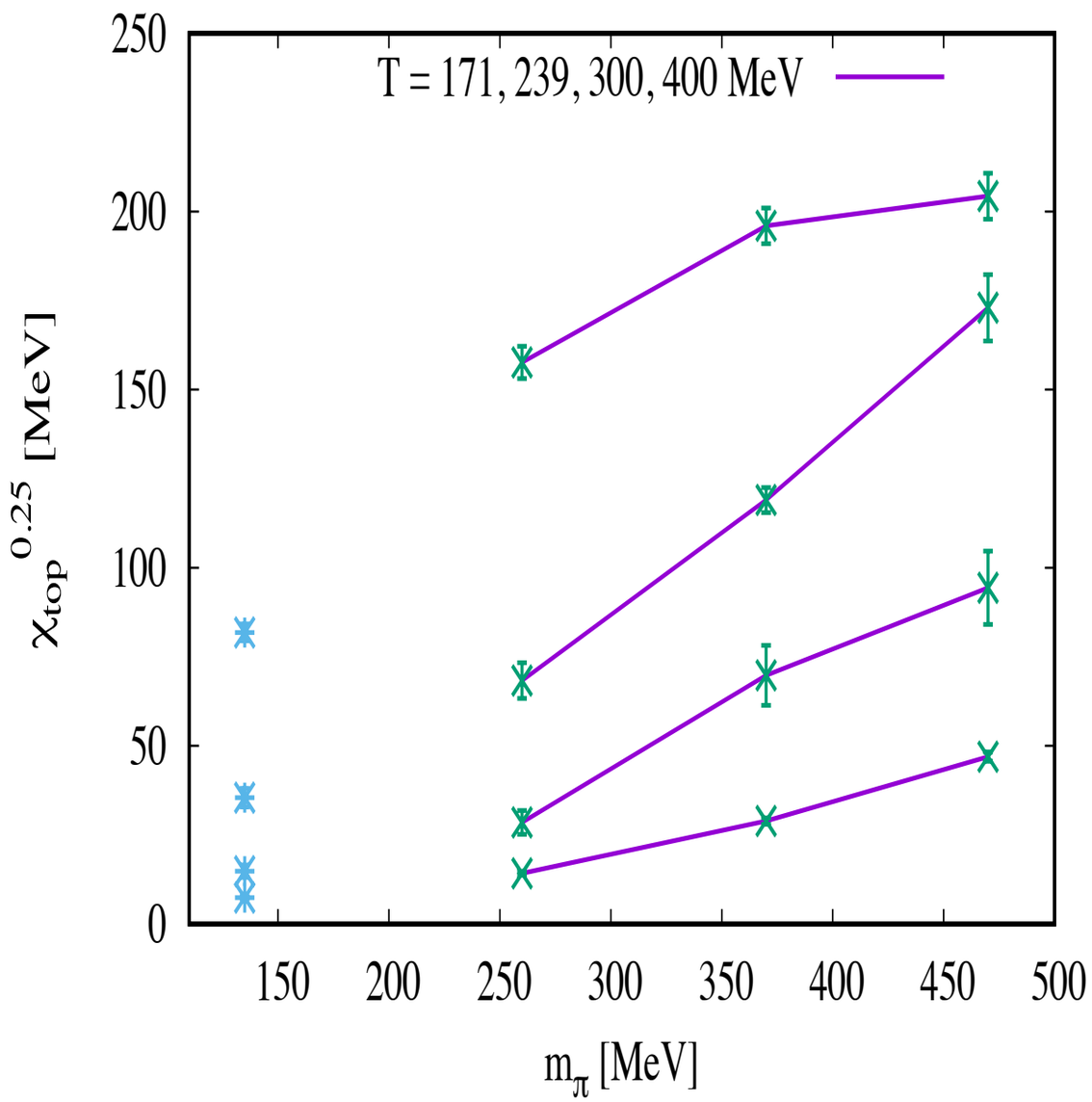
$$\langle\bar{\psi}\psi\rangle = \sum_{n=0} a_n m_l^{2n+1}$$

$$\chi = \frac{V}{T}\frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle \equiv \chi^{\rm disc}_{\bar{\psi}\psi} + \chi^{\rm conn}_{\bar{\psi}\psi} = \sum_{n=0} a_n m_l^{2n}.$$

$$\chi_{\rm top} = m_l^2 \chi^{\rm disc}_{\bar{\psi}\psi} = \sum_{n=0} a_n m_\pi^{4(n+1)}$$

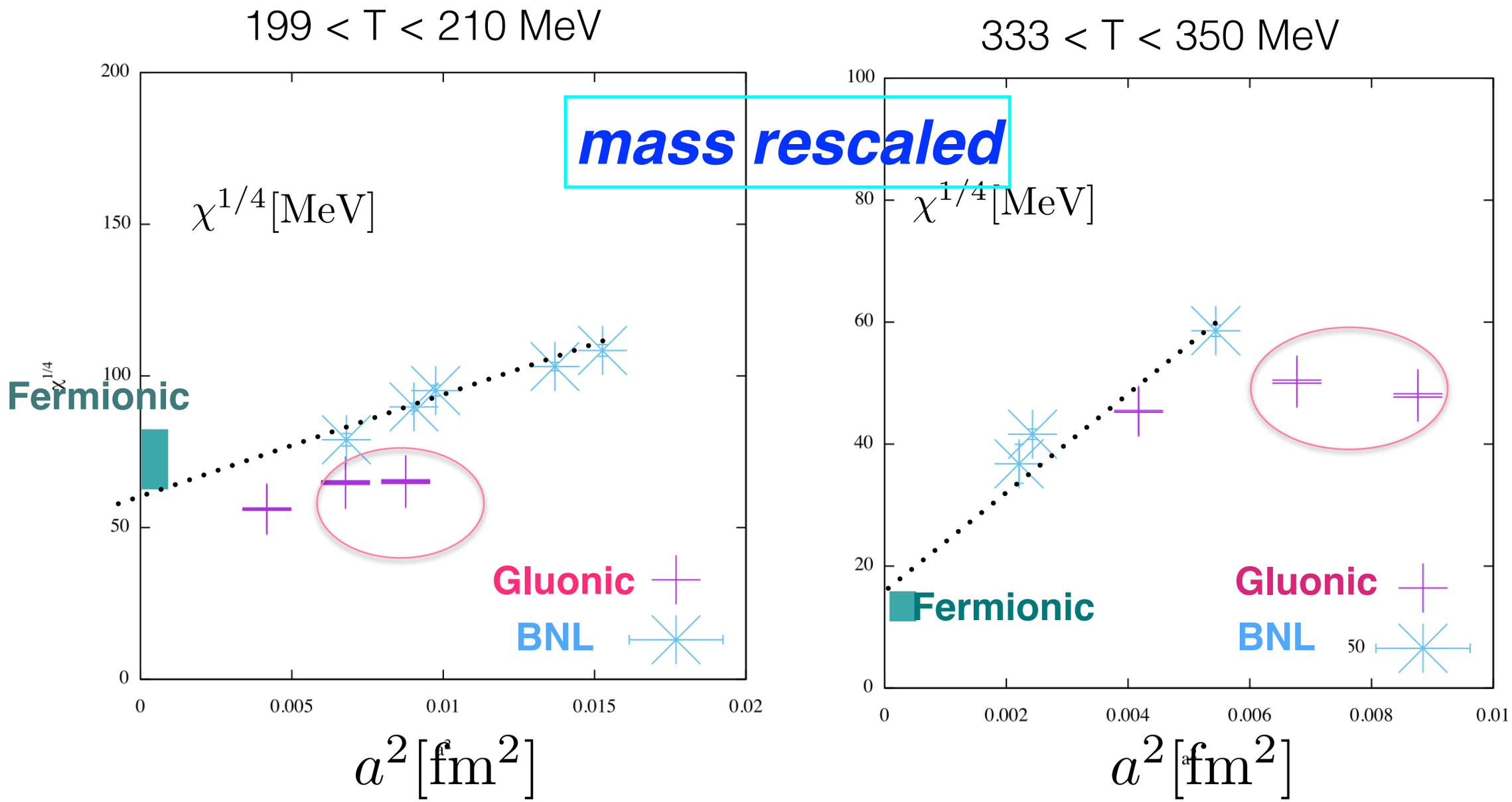
leading order: DIGA

$$\chi_{\rm top} \propto m_\pi^4$$

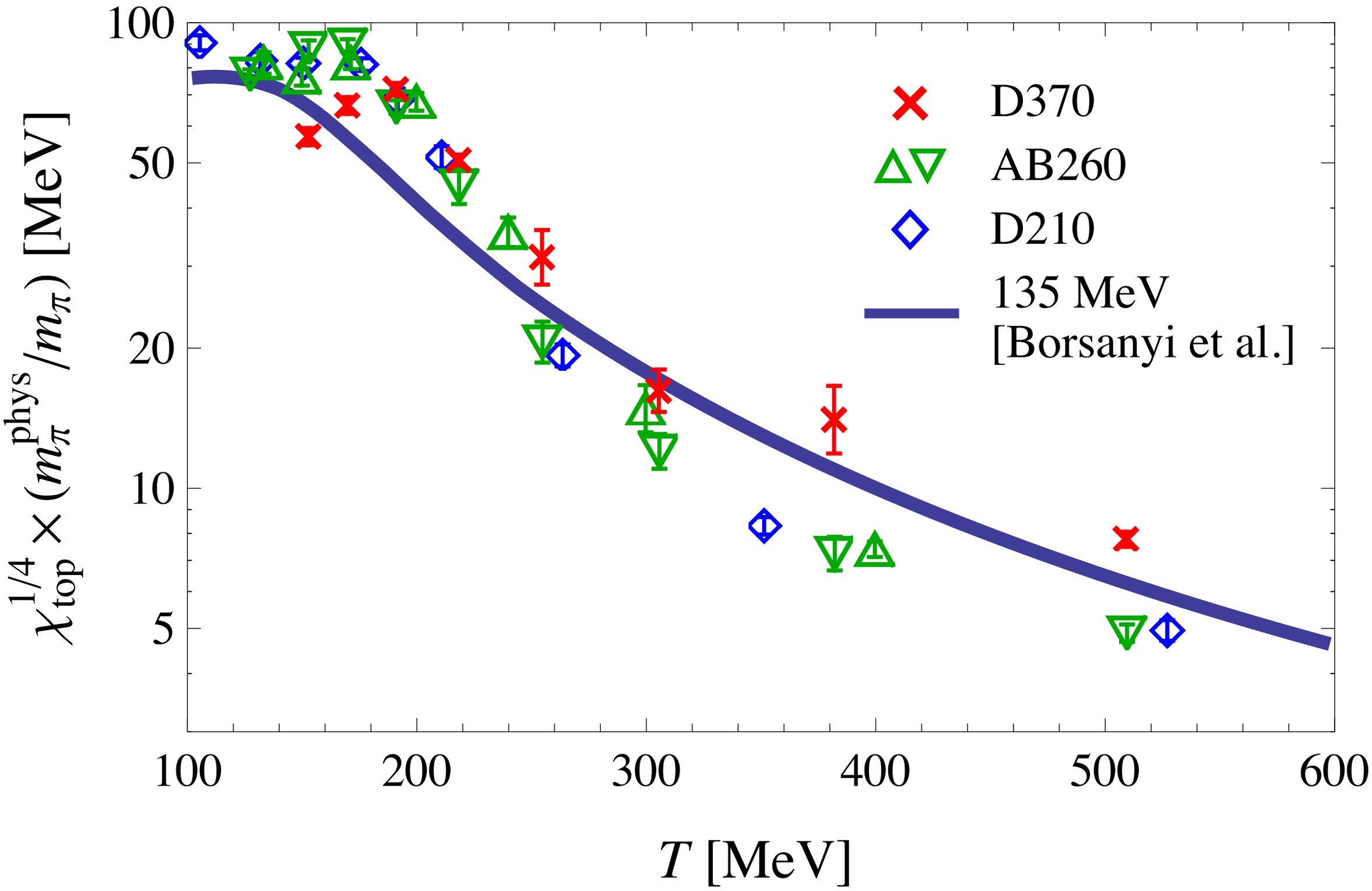


Comparison with other results II

numerical data courtesy *S. Sharma*



dotted lines to guide the eye

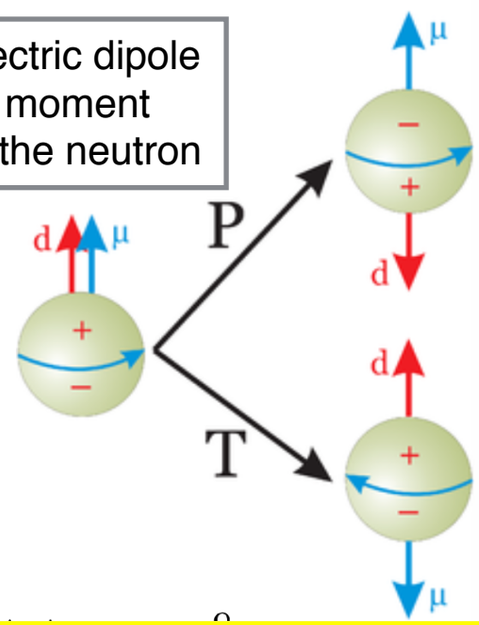

100
50
20
10
5
100
200
300
400
500
600
$\chi_{\rm top}^{1/4} \times (m_\pi^{\rm phys}/m_\pi)$ [MeV]
T [MeV]
D370
AB260
D210
135 MeV
[Borsanyi et al.]

Topology beyond topological susceptibility

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

electric dipole moment of the neutron



$$|d_n| < 2.9 \times 10^{-26} \; e\,\mathrm{cm}$$

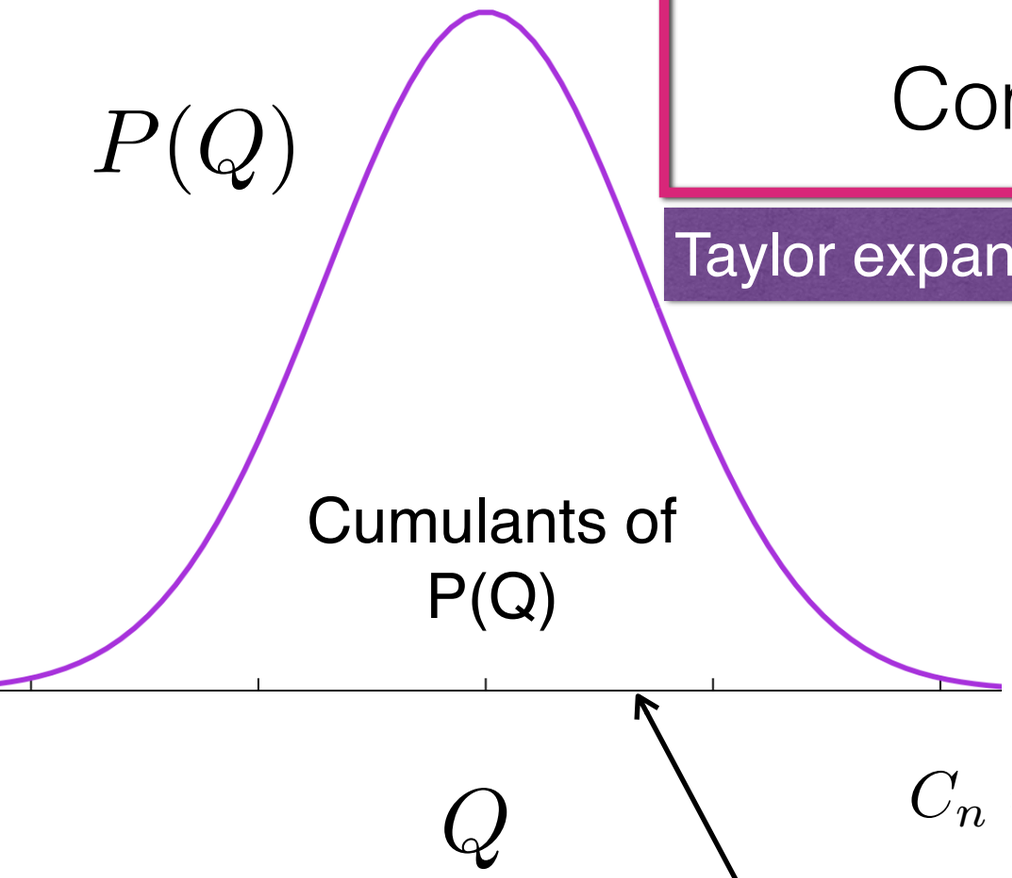
$$d_n(\theta) \sim e\,\theta \frac{m_u m_d}{(m_u + m_d)m_n^2} \Rightarrow |\theta| < 10^{-9}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

$$\left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T) = (< Q^2 > - < Q >^2)/V$$

Computing $F(\theta)$

Taylor expansion, and cumulants of the topological charge distribution



$$e^{-F(\theta)} = \langle e^{i\theta Q} \rangle$$

$$P_\nu = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta\nu} e^{-F(\theta)} \qquad Q = \nu$$

$$C_n = (-1)^{n+1} \frac{1}{V} \frac{d^{2n}}{d\theta^{2n}} F(\theta)\Big|_{\theta=0} \equiv < Q^{2n} >_{conn}$$

Taylor coefficients of

$$F(\theta) = V \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{(2n)!} C_n$$

$$P_\nu = \frac{e^{-\frac{\nu^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \left[1 + \frac{1}{4!} \frac{\tau}{\sigma^2} \mathrm{He}_4 \left(\nu/\sigma \right) \right]$$

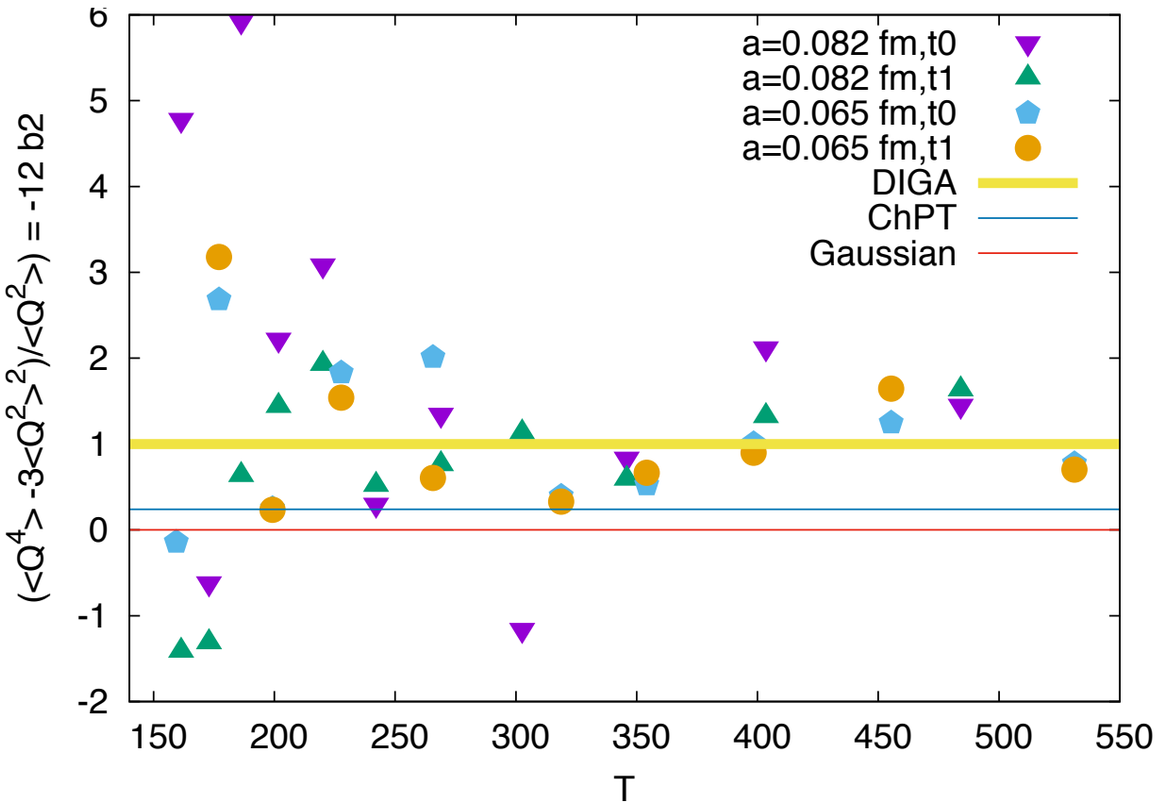
$\sigma^2 = VC_1$ and $\tau = C_2/C_1$ $\quad$ $P(Q)$ is Gaussian for $V \to \infty$

$F(\theta)$ is 'hidden' in $P(Q)$'s cumulants

Instanton potential - cumulants' ratio b2

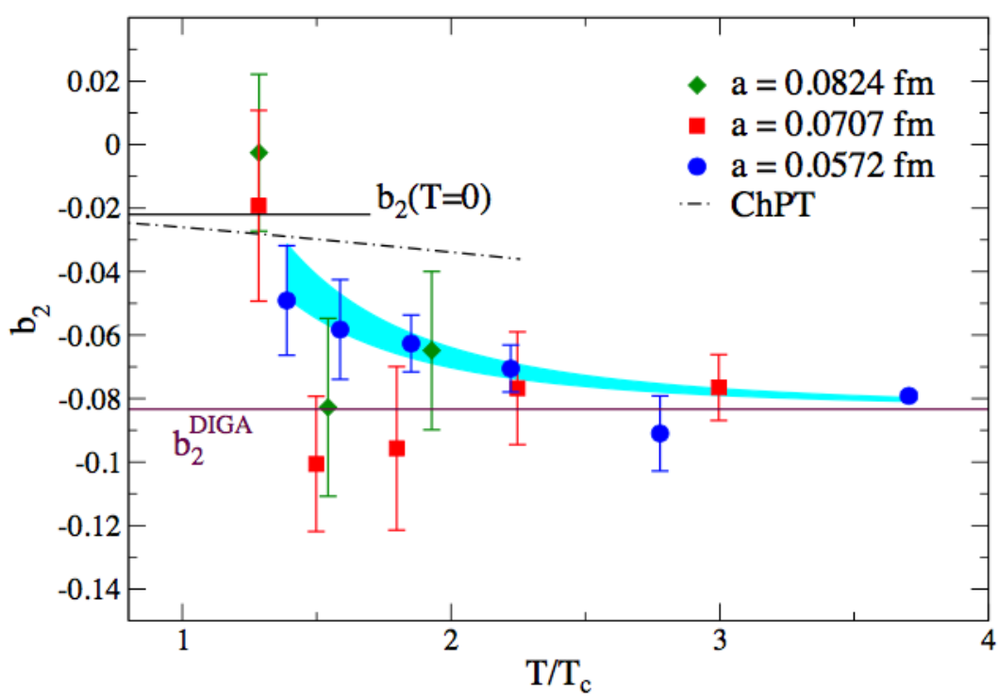
DIGA predicts

$$F(\theta,T) - F(0,T) = \chi(T)(1 - cos(\theta)) \longrightarrow b_2 = -1/12$$



b2 = -1/12

DIGA limit for T > 350 MeV



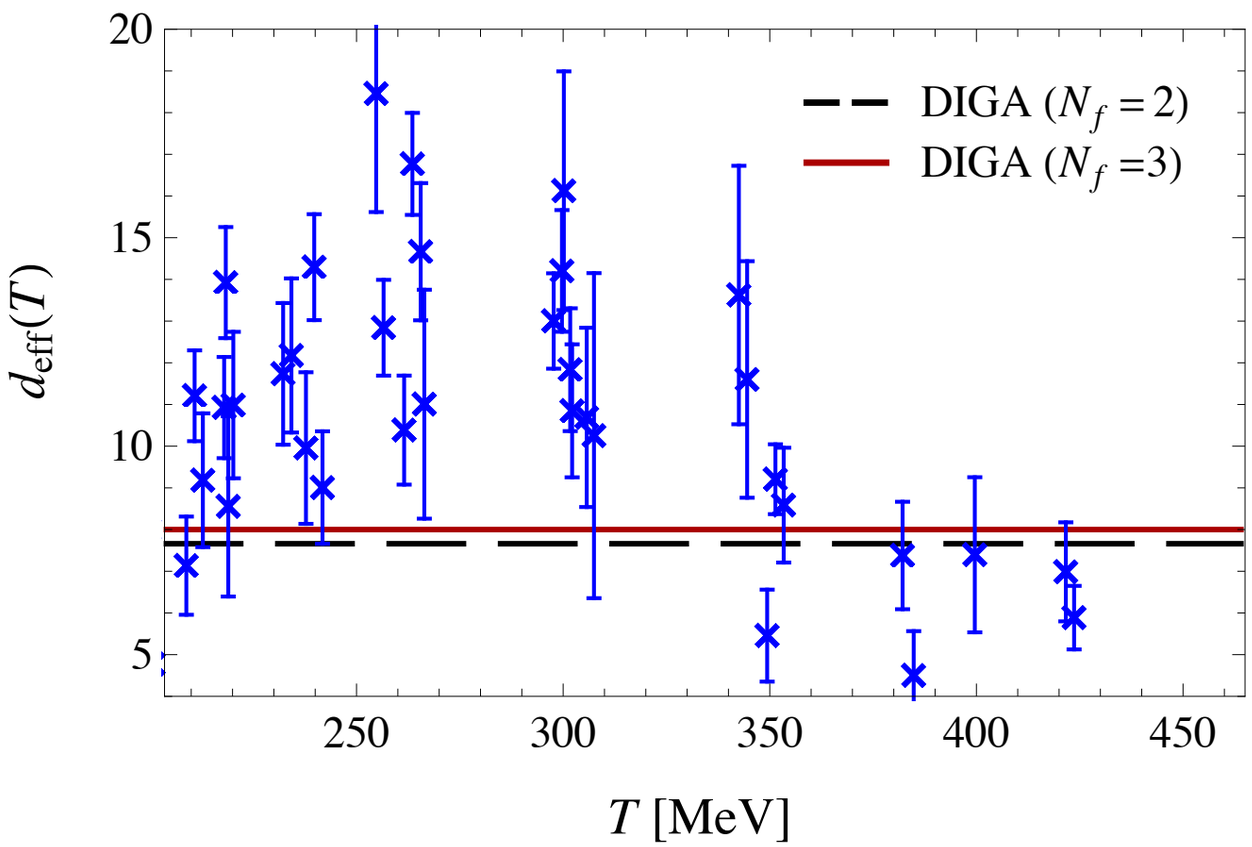
Consistent with Bonati et al.

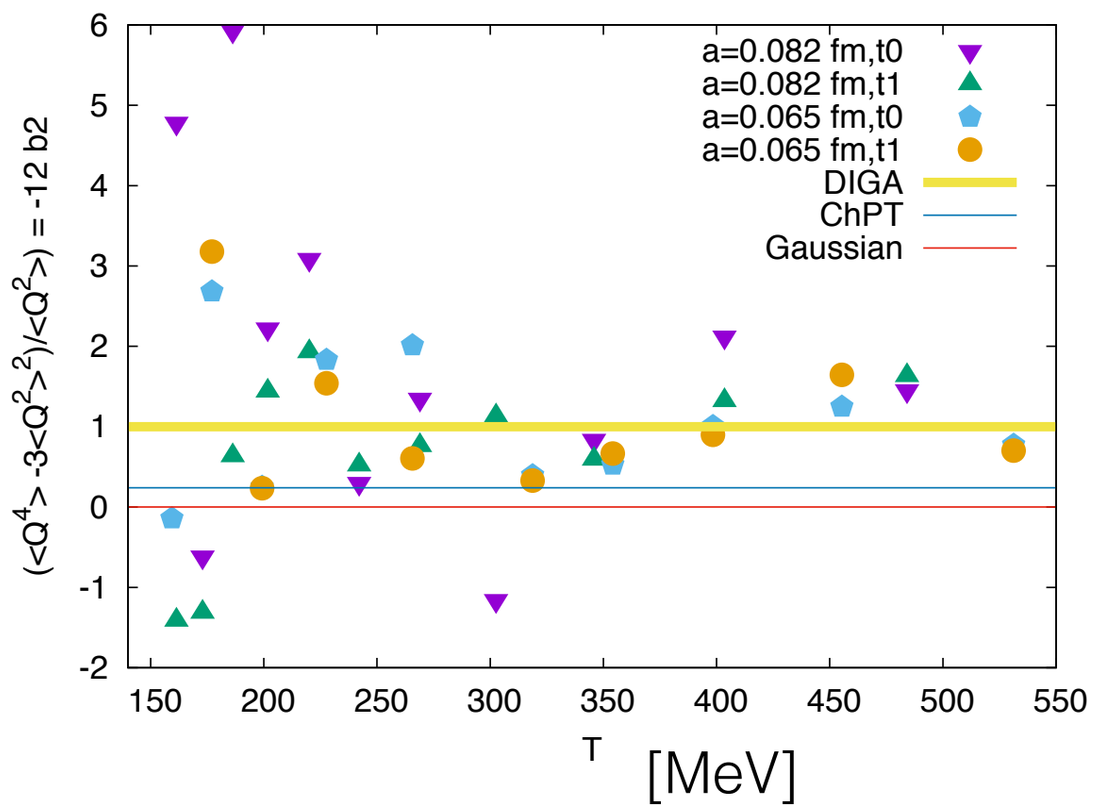
Effective exponent :

Same DIGA onset seen in b2 ≈ 350 MeV

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

Results for $F(\theta)$ coherent with d(T)





A window on Axions

Time from Big Bang

Temperature

Hubble parameter

$$H(T) \simeq T^2/M_P$$

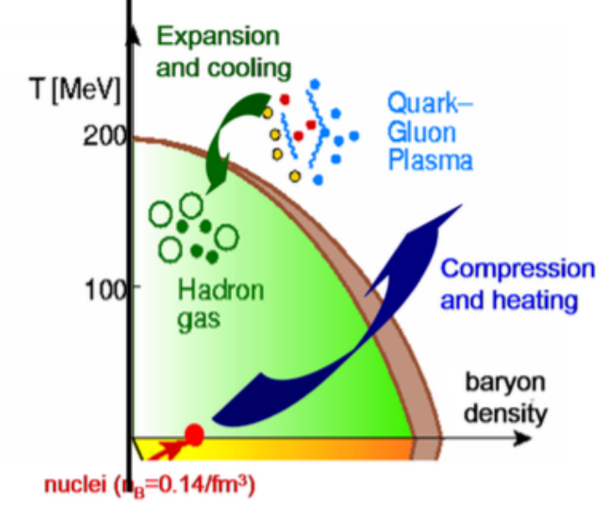
Axions's freezout

$$3H(T) = m_a(T)$$

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

Axions' mass and density today

Quark Gluon Plasma: Topology



Axions 'must' be there: solution to the strong CP problem

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

Postulate axions, coupled to Q:

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)$$

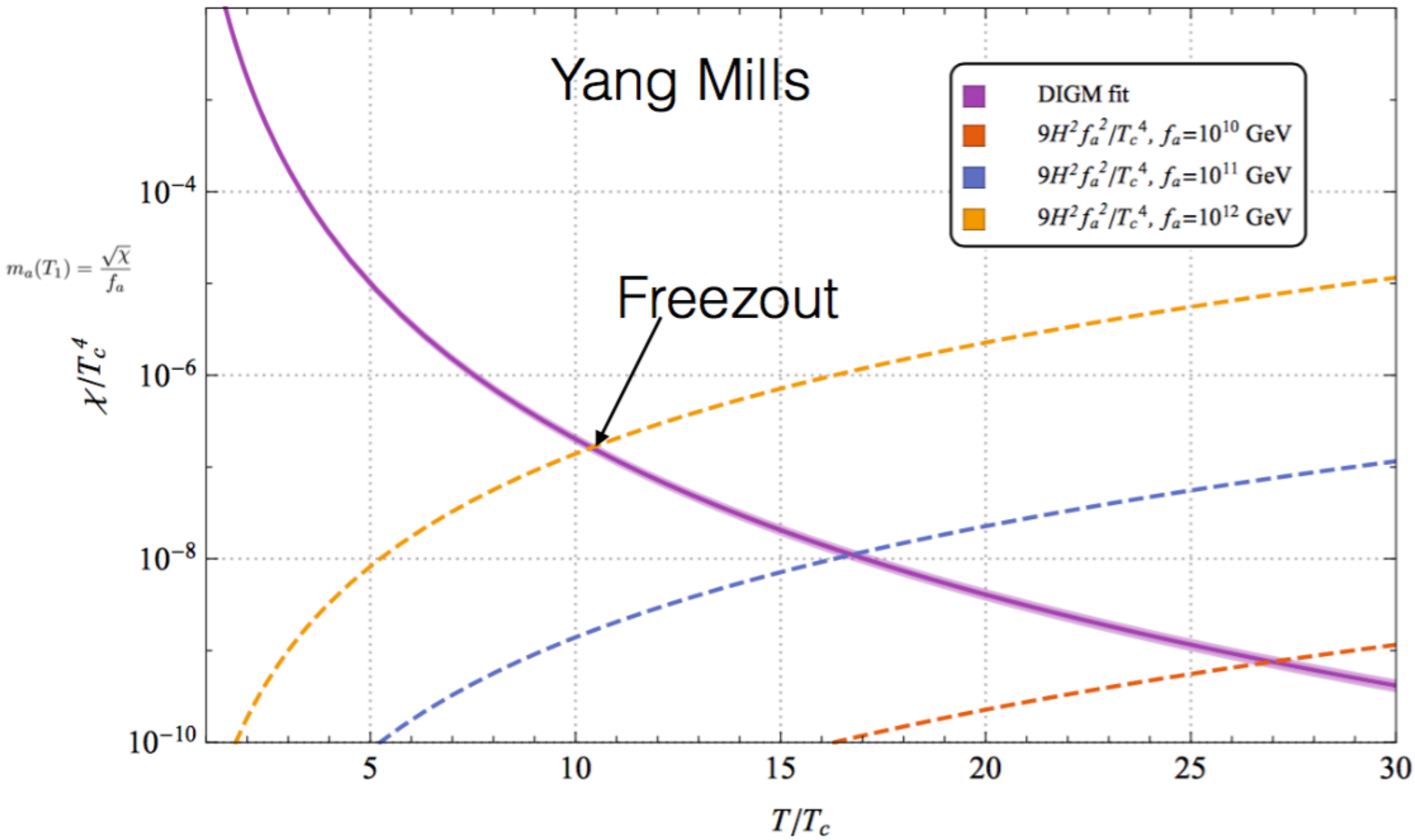
Axion potential

$$m_a^2(T)f_a^2 = \left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T), \quad f_A \gtrsim 4 \times 10^8 \text{ GeV}$$

weakly coupled

Axion freezout : $3H(T) = m_a(T) = \sqrt{\chi(T)}/f_a$

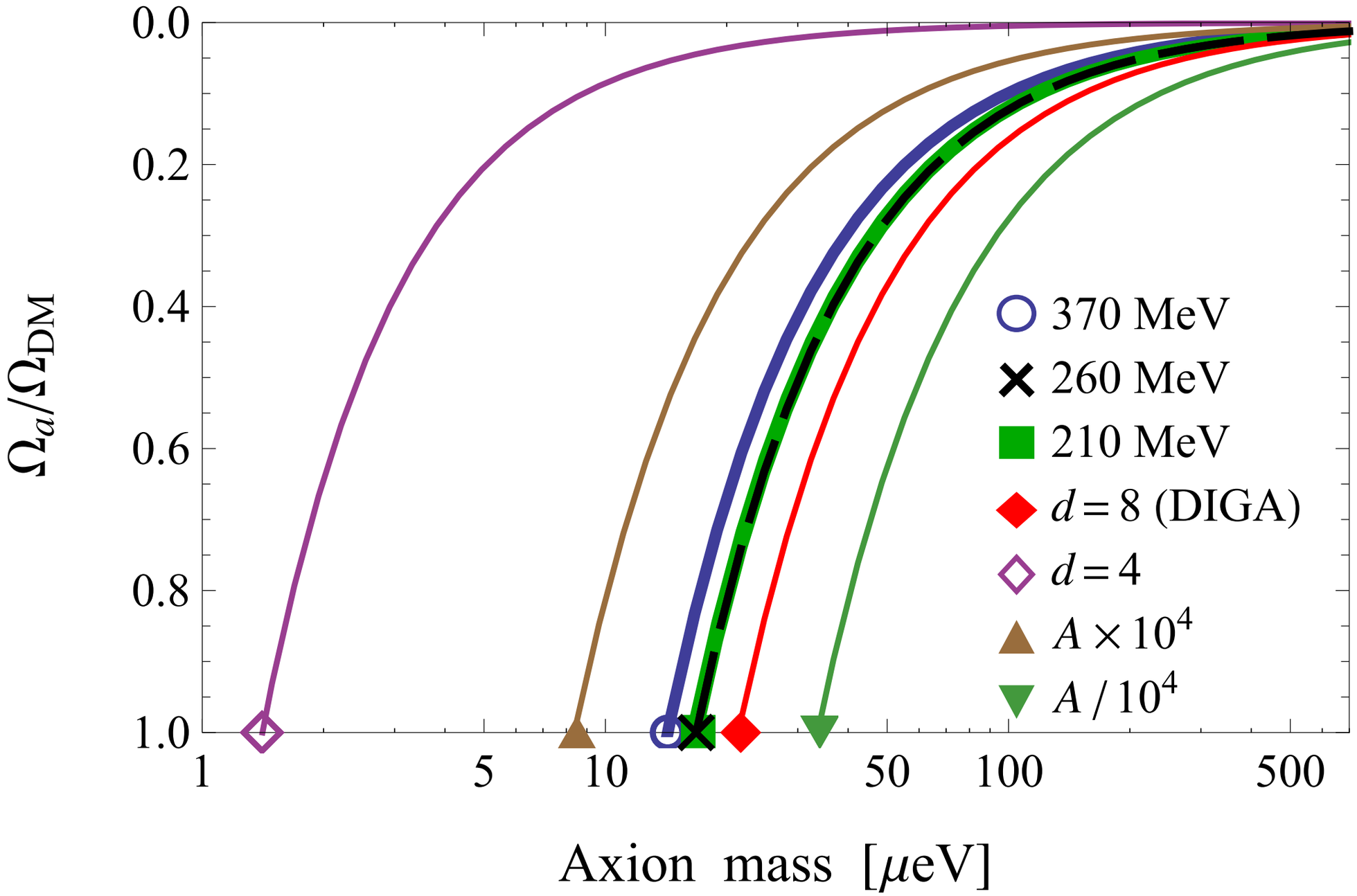
Berkowitz Buchoff Rinaldi 2015



Axion density at freezout controls axion density today

$$\rho_a(m_a) \propto m_a^{-\frac{3.053+d/2}{2.027+d/2}}$$

$$\Omega_a \equiv \rho_a/\rho_c$$



Summary

Method:

- Twisted mass Wilson fermions seem well suited for QCD thermodynamics, in particular for applications involving chiral symmetry and topology. A dynamical charm is important above 400 MeV.

Topology:

- In the sQGP region we observe a fast decrease of the topological susceptibility — faster than observed than others

- Above 400 MeV both the b2 cumulant and the exponent of the power law behavior of the topological susceptibility computed with fermionic method approach their DIGA value.

- Several discrepancies remain among results obtained by different groups, which have also impact on axion phenomenology and should be resolved.

A comment on experiments: →

What happens to topology in the Quark Gluon Plasma?

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Return of the prodigal Goldstone boson

J. Kapusta
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

D. Kharzeev
Theory Division, CERN, Geneva, Switzerland
and Fakultät für Physik, Universtät Bielefeld, Bielefeld, Germany

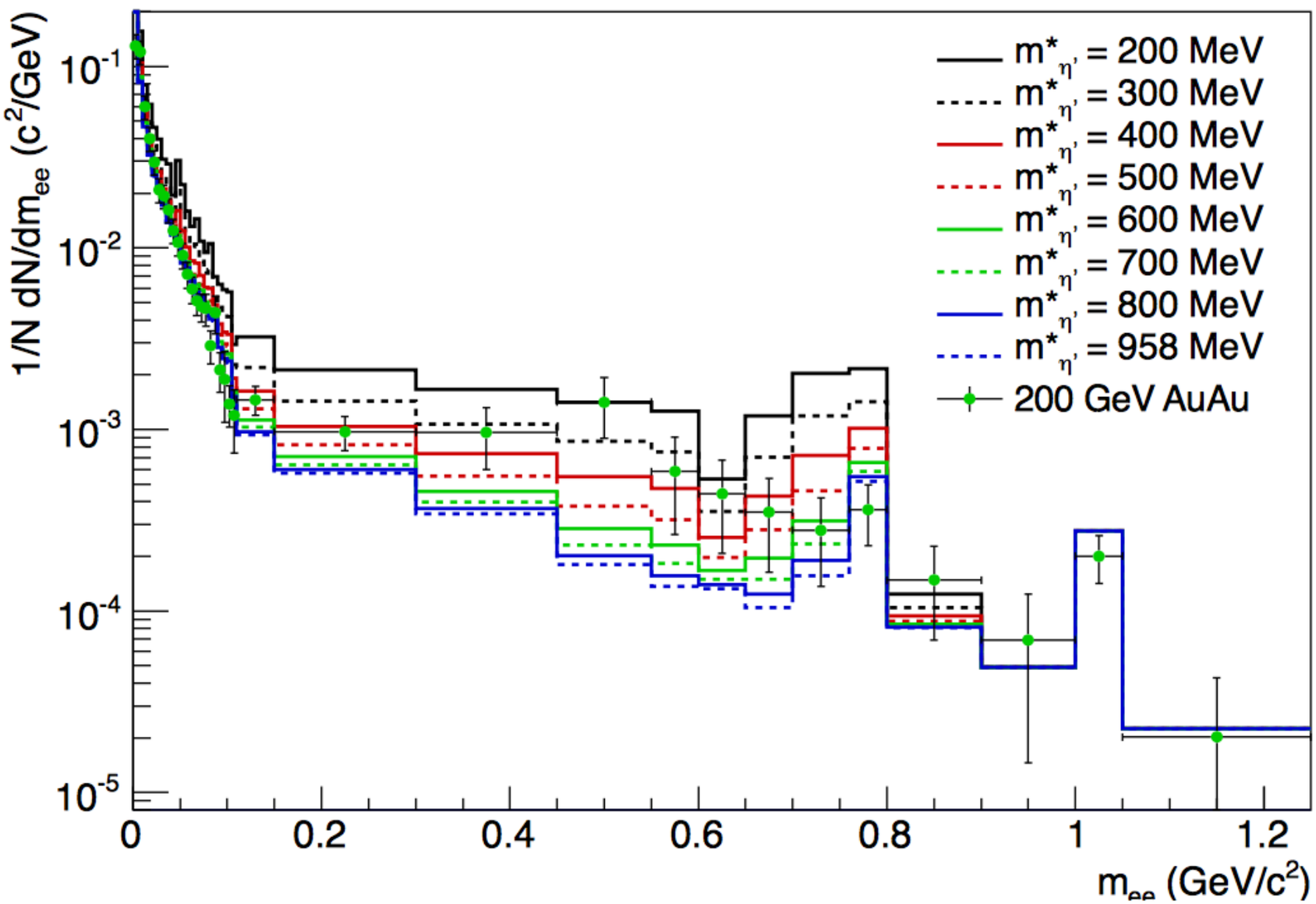
L. McLerran
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 14 July 1995)

We propose that the mass of the η' meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark-gluon plasma. We argue that the mass of the η' excitation in hot and dense matter should be small, and, therefore, that the η' production cross section should be much increased relative to that for pp collisions. This may have observable consequences in dilepton and diphoton experiments.

Indication of topology suppression in PHENIX

Effects of chain decays, radial flow and $U_A(1)$ restoration on the low-mass dilepton enhancement in $\sqrt{s_{NN}}$=200 GeV Au+Au reactions

Márton Vargyas[a,b,1], Tamás Csörgő[b,2], Róbert Vértesi[b,c,3]

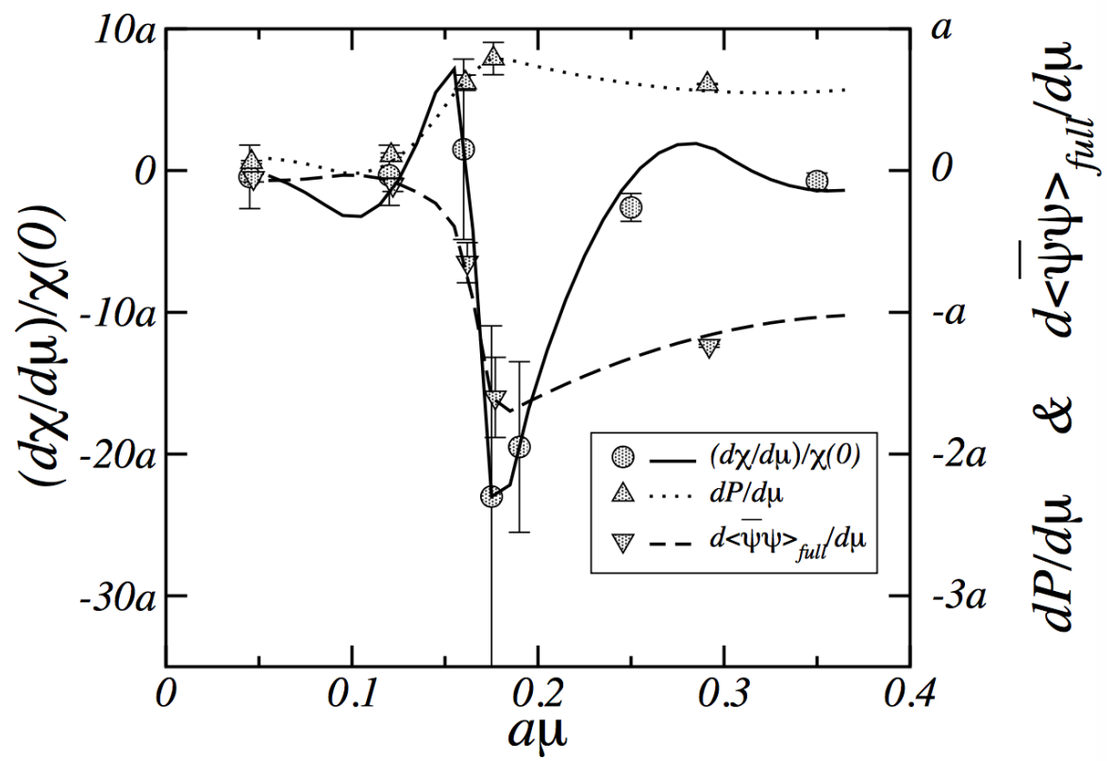


This is at finite density!

Some indication of the ‘return of the prodigal Goldstone boson’ from RHIC ?

No results as yet from LHC ?

Question to theory:
interplay of UA(1) and SU(N)XSU(N) symmetries in dense matter?



Alles, d’Elia, MpL 2006

new results should come soon at FAIR? talk by T. Galantchyuk at QM2018

Backup slides

Gradient flow

Lüscher, Lüscher Weisz

Evolve the link variables in a fictitious flow time:

$$\dot{V}_{x,\mu}(t) = -g_0^2 \left[\partial_{x,\mu} S_{\text{Wilson}}(V(t)) \right] V_{x,\mu}(t),$$

Monitor $\langle E \rangle = \frac{1}{2N_\tau N_\sigma^3} \sum_{x,\mu,\nu} \text{Tr}[F_{\mu\nu}(x) F^{\mu\nu}(x)]$ as a function of t

Stop flowing when $t^2 \langle E \rangle \big|_{t=t_0} = 0.3$

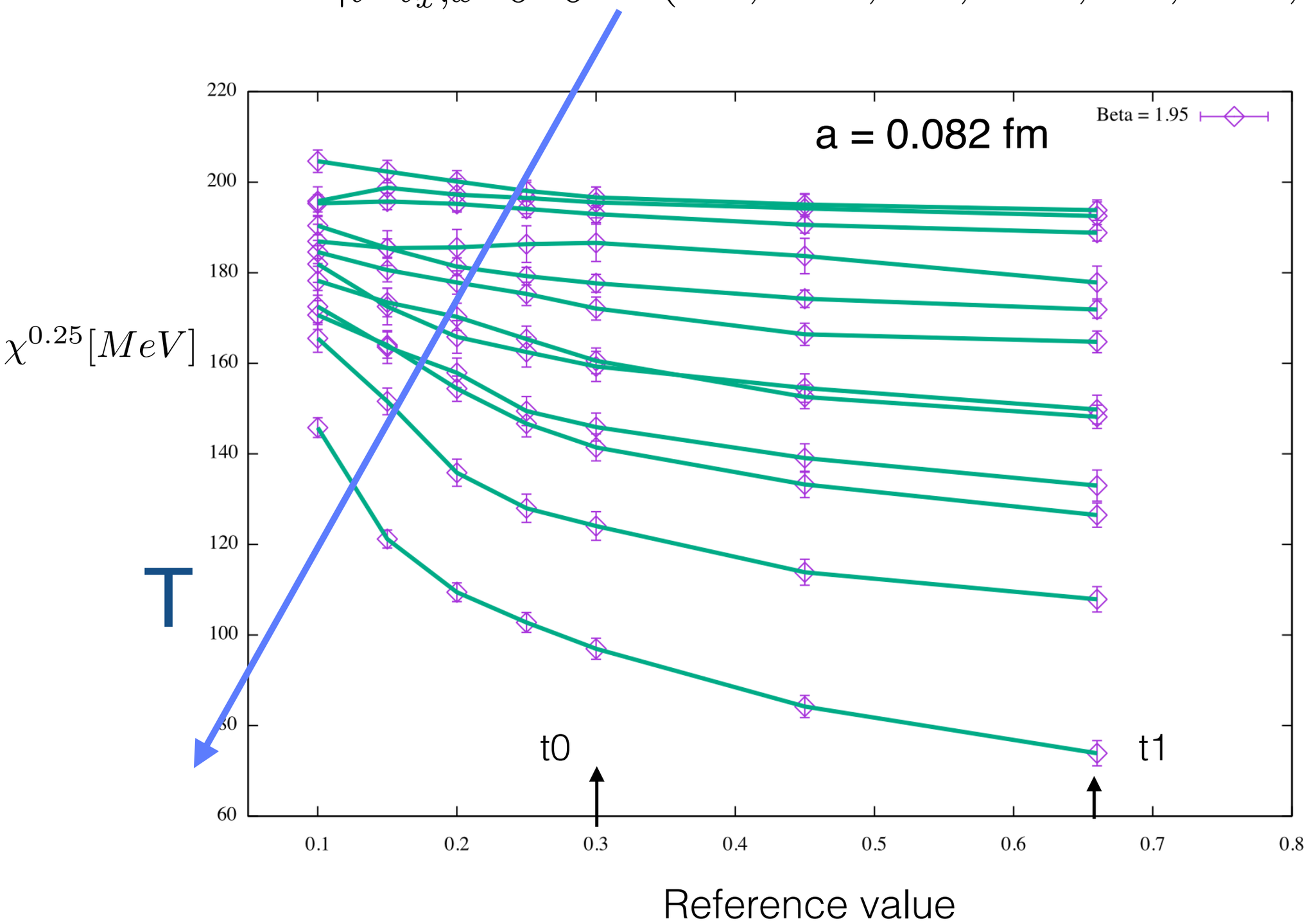
Observables $< O(t) >$ renormalized at $\mu = 1/\sqrt{8t}$

Continuum limit of $< O(t) >$ is independent on the chosen reference value

Caveat: note comments by Kanaya et al.

Flowing towards the plateau

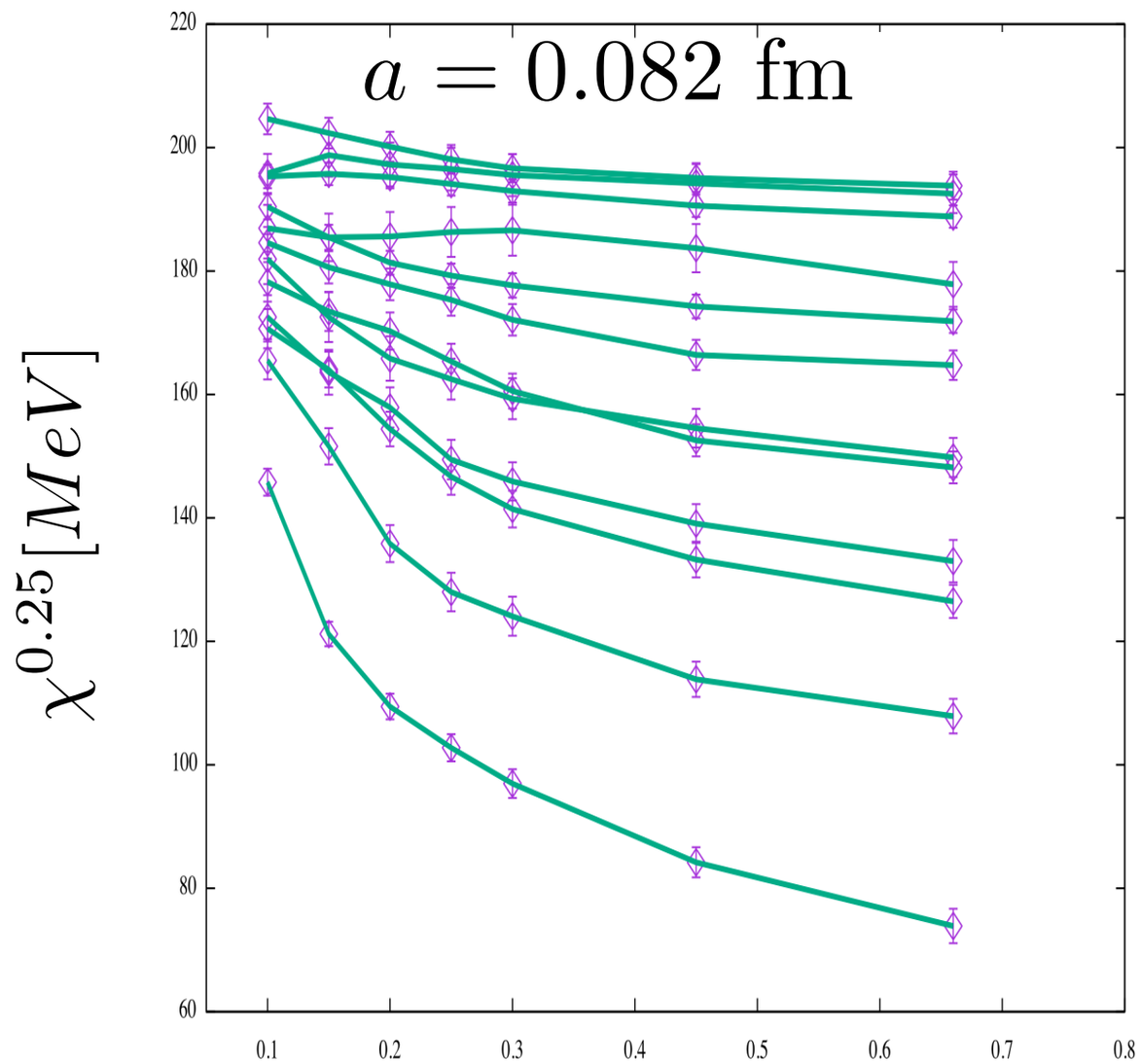
$$t^2 < E > |_{t=t_x, x=0-6} = (0.3, 0.66, 0.1, 0.15, 0.2, 0.25, 0.45)$$



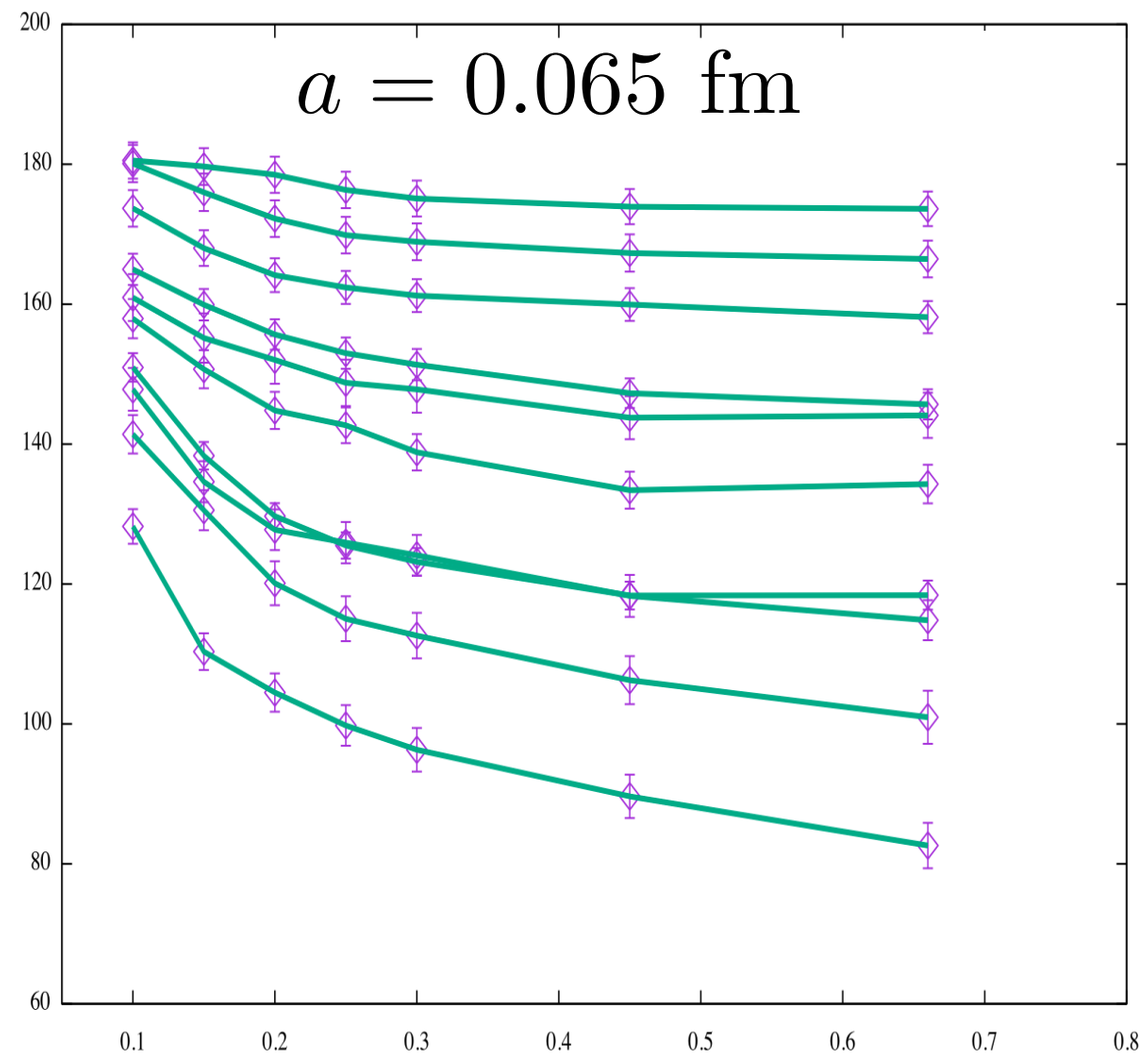
We focus on t(0.3) = t0 and t(0.66)= t1

On finer lattices, plateau is almost reached:

Gradient method coincides with cooling



Reference value



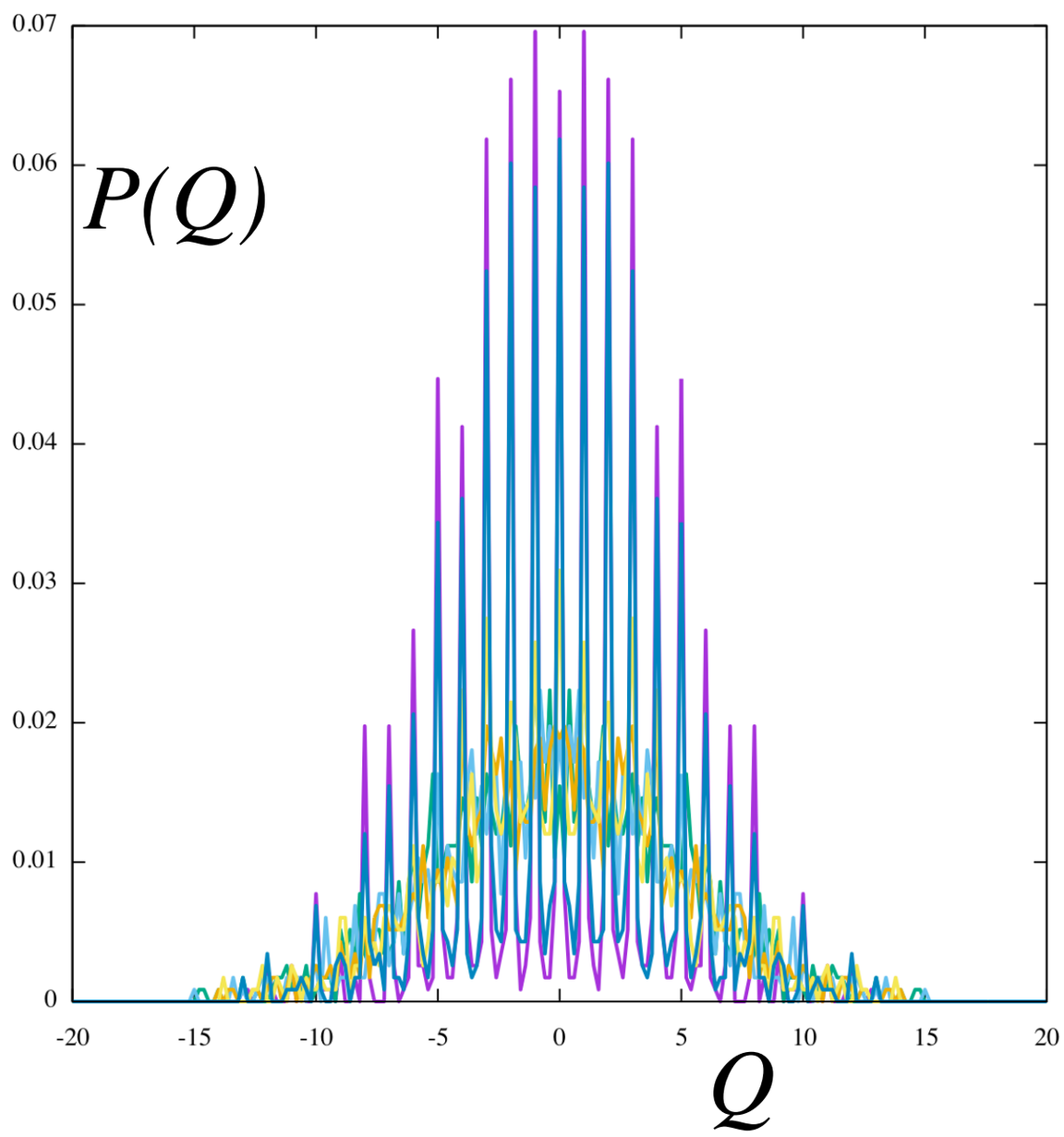
Reference value

Distribution of the topological charge $P(Q)$

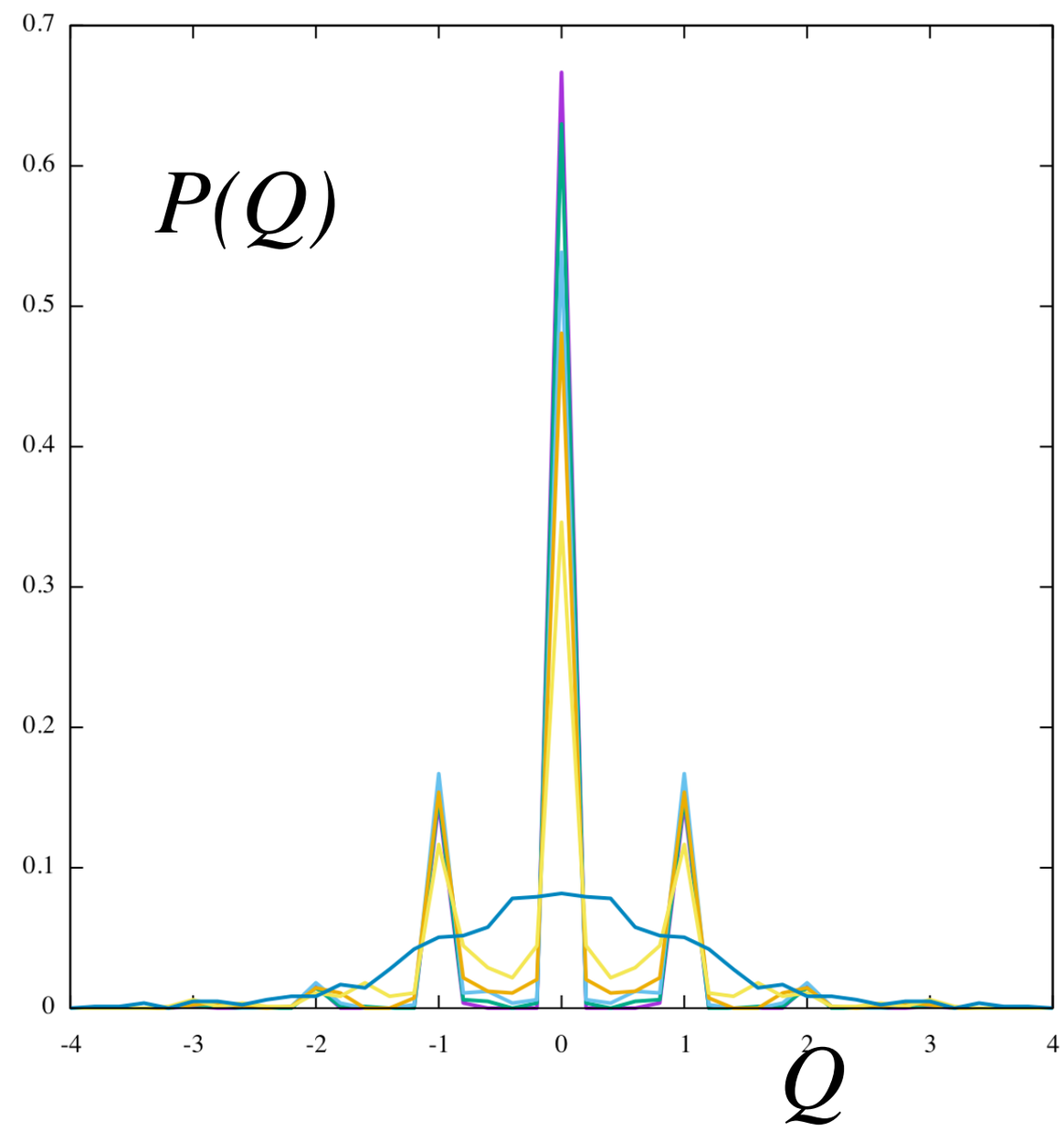
cluster around integers as cooling proceeds

(results for a = 0.06 fm)

T=153 MeV



T=253 MeV



Puzzling features: lack of mass dependence, slow decay