Recent Progress in Understanding the Role of Monopoles in QCD

AR and E. Shuryak, Phys. Rev. D95 076019, Phys. Rev. D97 016010, and arXiv:1801.06922

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Gauge Topology 3: From Lattice to Colliders

ECT*, Trento, Italy

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Outline

Outline

Magnetic Scenario of QCD

Dual Superconductivity Model of the QCD Vacuum

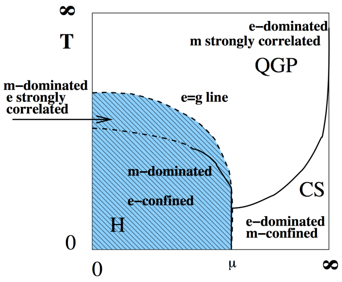
Nambu (1974), Mandelstam (1976), 't Hooft (1981)

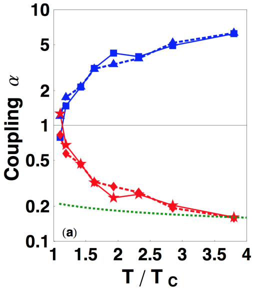
- Electric quasiparticles (quarks and gluons) and magnetic quasiparticles (monopoles, etc.)
- Confinement is due to the Bose-Einstein condensation (BEC) of magnetic quasiparticles
- Lattice studies have identified electric flux-tubes, monopole currents, and gauge-invariant magnetic field correlated with monopoles

  Koma, et al. (2003), Bornyakov, et al. (2003), etc.

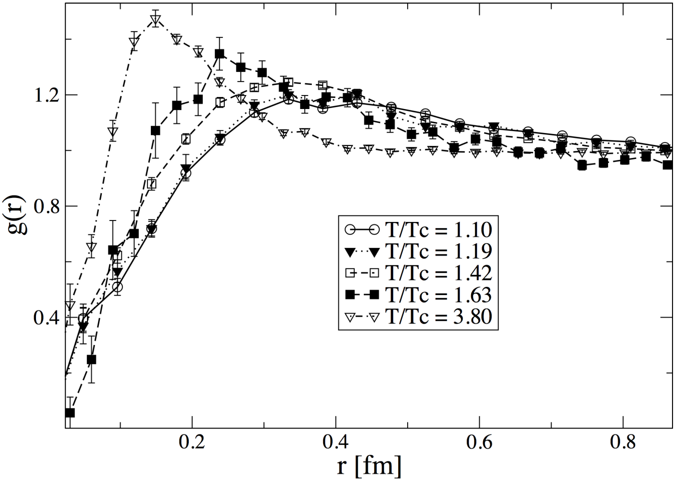
Magnetic Scenario of QCD

- Condition for making Dirac strings invisible: $\frac{eg}{4\pi} = \text{integer}$ Dirac (1931)
  - $\alpha_s = e^2/4\pi$ in QCD runs with T and μ
  - $\alpha_m = g^2/4\pi$ runs oppositely
- Classical studies of electric-magnetic plasma have seen this behavior of the coupling Liao and Shuryak (2007)





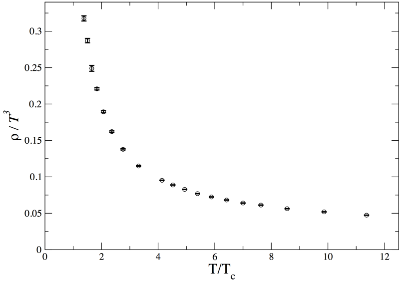
Monopoles on the Lattice: SU(2)



D'Alessandro and D'Elia (2007)

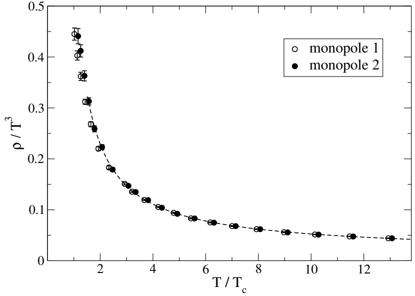
Monopoles on the Lattice

Density: $\rho_m/T^3 \sim \log(T)^{-3} \rightarrow$ Monopoles important near T_c



SU(2)

D'Alessandro and D'Elia (2007)



SU(3)

Bonati and D'Elia (2013)

Outline

Goals

- Classical monopoles $\rightarrow$ Quantum monopoles (BEC)
  - Reproduce behavior of lattice monopoles without other degrees of freedom
  - Identify thermodynamic contribution of monopoles to QCD around T_c
- Quantify BEC critical temperature behavior of one- and two-component quantum Coulomb systems

Path-Integral Monte Carlo

- Density matrix at finite temperature from path integrals:
  Feynman (1953), Matsubara (1951)

$$\rho(x_i, x_j, \beta) = \int_0^\beta \mathcal{D}x(\tau) \exp\{-S_E[x(\tau)]\}$$

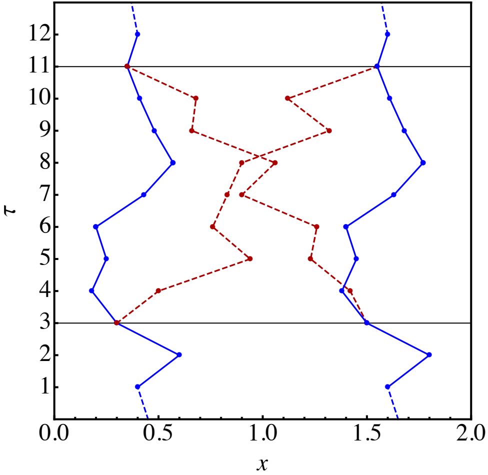
- Partition function: $Z = \mathrm{Tr}[\rho] \to$ periodic paths
- Discretize paths and sample configurations using Markov chain Monte Carlo (MCMC)
  - The configuration weight is given by the Euclidean action,

$$\pi(\{\vec{x}\}) = e^{-S_E(\{\vec{x}\})}$$

  - Sample the partition function $\to$ thermodynamics

Path-Integral Monte Carlo

- The imaginary-time paths of bosons can be permuted



Finding T_c: Permutation Cycles

- Measure $n_k(T)$, the probability of finding a particle in a k-permutation cycle as a function of temperature
- Densities of k-cycles go as

$$\rho_k(T) \equiv n_k(T)/k = \exp(-\hat{\mu}(T)k) \times f(k)\,,$$

where $f(k)$ is some decreasing function $\sim 1/k^{\alpha}$

- Critical temperature is where permutation cycles are no longer exponentially suppressed, i.e. $\hat{\mu}(T) = 0$

D'Alessandro, D'Elia, and Shuryak (2010)

Finding T_c: Superfluid Fraction

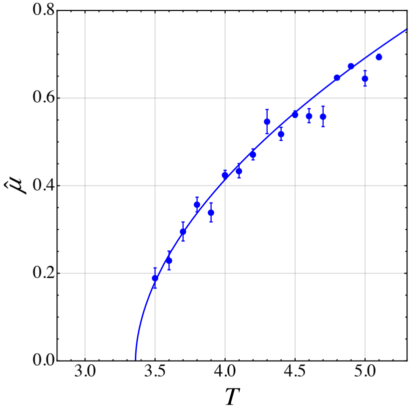
- The superfluid fraction of a condensate is related to the spatial winding number distribution

$$\frac{\rho_s}{\rho} = \frac{\langle W^2 \rangle}{2\lambda\beta N}$$

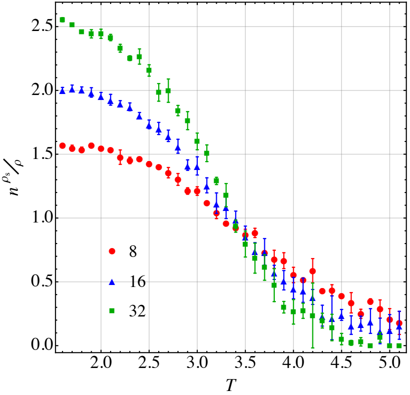
- Thermodynamic limit $\rightarrow$ expect the superfluid fraction to go from 0 (above T_c) to 1 (below T_c)
- Study finite-size scaling of this quantity $\rightarrow$ determine critical point by the intersection of the superfluid fraction lines

Pollock and Ceperley (1987)

Finding T_c: Examples



Permutation-cycle calculation



Finite-size scaling of the superfluid fraction

Study of Quantum Coulomb Systems

- Find the dependence of T_c on α, the coupling, with interaction potential

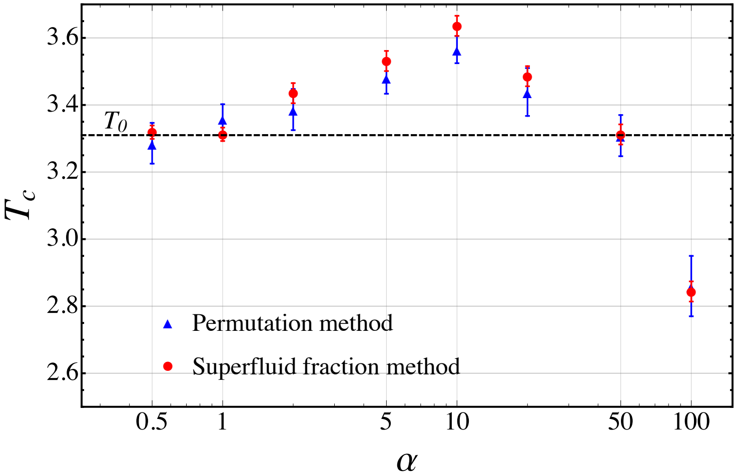
$$V_{\text{int}}(r_{ij}) = \alpha \frac{q_i q_j}{r_{ij}}$$

- Critical temperature of an ideal Bose gas

$$T_0 = \left(\frac{2\pi\hbar}{m k_b}\right)\left(\frac{n}{\zeta\left(\frac{3}{2}\right)}\right)^{2/3}$$

- $|q| = m = n = 1 \rightarrow$ scale is fixed by the temperature T
- Ideal gas BEC temperature in these units $T_0 = 3.3125$

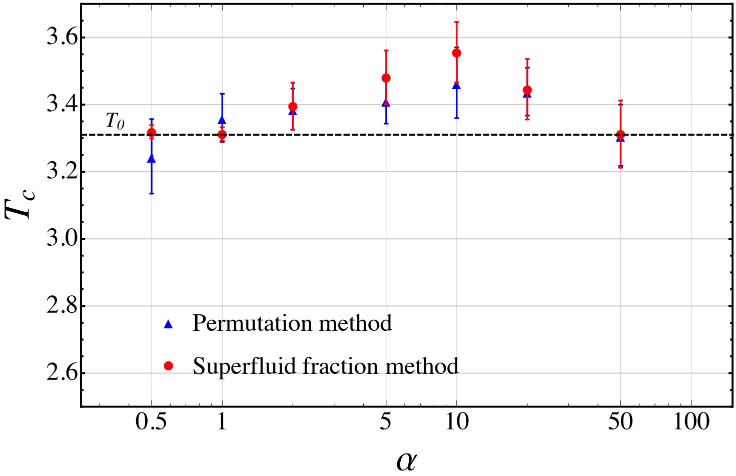
One-Component Coulomb Bose Gas: T_c



Same behavior seen by varying density of hard spheres Grüter, et al. (1997)

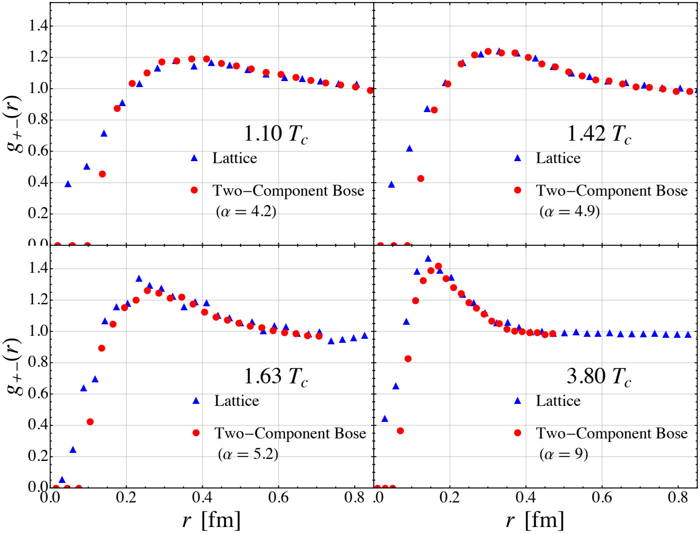
Analytically explained by M. Holzmann, G. Baym, J. P. Blaizot, and F. Laloe (2001)

Two-Component Coulomb Bose Gas: T_c



Same qualitative behavior as one-component case, but the increase in T_c at moderate coupling is slightly smaller

Matching Radial Distribution Functions



Fixing Parameters

- Scaling relation for α, the simulation coupling, to match lattice results:

$$\alpha(T/T_c) \approx T_c \times \rho^{1/3}(T/T_c)$$

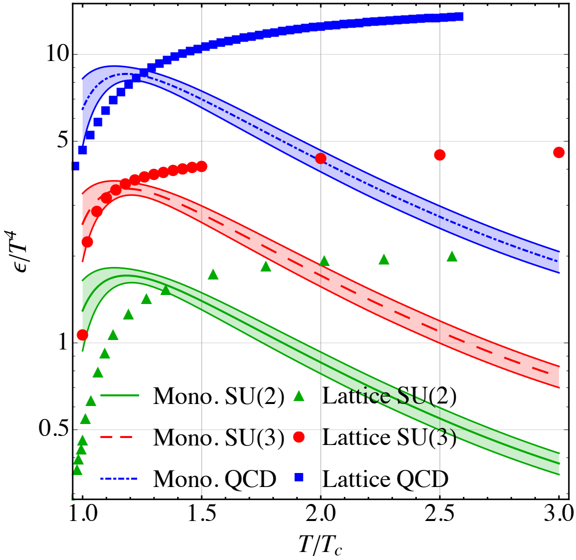
- $\rho_m(T)$ is the monopole density (in fm^{-3})
- One unit of length in our simulations is $\rho^{-1/3}(T)$ (in fm)
- $T_c \approx 3.45$ in our units of temperature

Equation of State

- Studies on the lattice found a large entropy density and trace anomaly at 170-250 MeV
- May not all be from quarks and gluons $\rightarrow$ other contributions to the thermodynamics of QCD-like theories at Tc (hadrons, instantons, etc.)
- What portion of thermodynamics comes from monopoles?

Wuppertal-Budapest Collab. (2013)
HotQCD Collab. (2014)
etc.

Equation of State



Outline

Goals

- Monopoles are related to instanton-dyons, which are known to play a role in chiral symmetry breaking
  (see E. Shuryak's earlier talk)
    - Analytic solution for QCD monopole unknown $\rightarrow$ use a model for which we have analytic solutions to study role in chiral symmetry breaking
- Find the Dirac eigenvalue spectrum from fermionic zero modes of BPS monopoles in the Georgi-Glashow model as a proof of concept

Georgi-Glashow Model with Fermions

- Lagrangian of the Georgi-Glashow model

$$\mathcal{L} = -\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + \frac{1}{2}(D_\mu\phi)_a(D^\mu\phi)_a - \frac{1}{4}\lambda(\phi_a\phi_a - v^2)$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\epsilon_{abc}A_b^\mu A_c^\nu$$

$$(D^\mu\phi)_a = \partial^\mu\phi_a + g\epsilon_{abc}A_b^\mu\phi_c$$

- Fermion part of the Lagrangian

$$\mathcal{L}_{\mathcal{F}} = i\bar{\psi}_n\gamma^\mu(D_\mu\psi)_n - Gg\bar{\psi}_n\tau^a_{nm}\psi_m\phi_a$$

with G a constant, $\tau^a = \sigma^a/2$, and

$$(D_\mu\psi)_n = \partial^\mu\psi_n - ig\tau^a_{nm}A_a^\mu\psi_m$$

Monopole Solution/Ansatz

The 't Hooft-Polyakov monopole solution has the form

$$A_a^0 = 0\,,$$
$$A_a^i = \epsilon^{aij}\hat{r}_j \frac{A(r)}{g}$$
$$\phi_a = \hat{r}_a \frac{\phi(r)}{g}$$

Monopole Solution/Ansatz

The equations of motion from the pure-gauge Lagrangian are

$$
\begin{aligned}
0 = \frac{2}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}A}{\mathrm{d}r}\right) - \frac{2}{r}\frac{\mathrm{d}A}{\mathrm{d}r} + \frac{2}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(rA) \\
- \frac{6}{r^2}A - \frac{6g}{r}A^2 - 2g^2A^3 - \phi\left(\frac{2g}{r} + 2g^2A\right)
\end{aligned}
$$

$$
\begin{aligned}
0 = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\phi}{\mathrm{d}r}\right) - \frac{2}{r^2}\phi - \frac{4g}{r}A\phi \\
- 2g^2A^2\phi - 2U'(|\phi|^2)\phi
\end{aligned}
$$

with boundary conditions

$$
\left(r^2\frac{\mathrm{d}A}{\mathrm{d}r} + 2rA\right)\Bigg|_{r=0} = 0
$$

$$
\left(r^2\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)\Bigg|_{r=0} = 0
$$

Zero Modes of Monopoles

- Dirac equation for the fermion field

$$\Big[-i\vec{\alpha}\cdot\vec{\partial}\delta_{nm} + \frac{1}{2}A(r)\sigma^a_{nm}(\vec{\alpha}\times\vec{\hat{r}})_a + \frac{G\phi(r)}{2}\sigma^a_{nm}\hat{r}_a\beta\Big]\psi_m = E\psi_n$$

$n, m = 1, 2$ are the isospin indices, σ^a are the Pauli matrices,

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \beta = -i\begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}$$

Jackiw and Rebbi (1976)

Zero Modes of Monopoles

- Wavefunctions decomposed into upper and lower components

$$\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

- Using the 't Hooft - Polyakov anzatz and the resulting equations of motion, the zero-energy solutions are

$$\begin{aligned} \psi^-_{lm} &= 0\,, \\ \psi^+_{lm} &= N \exp\left(\int_0^r \mathrm{d}r' \left[A(r') - \frac{1}{2} G\phi(r')\right]\right) \\ &\quad \times (s^+_l s^-_m - s^-_l s^+_m) \end{aligned}$$

Jackiw and Rebbi (1976)

Quark Hopping Matrix and Chiral Symmetry Breaking

- Chiral "hopping matrix"

$$\mathbf{T} = \begin{pmatrix} 0 & iT_{ij} \\ iT_{ji} & 0 \end{pmatrix}$$

Schäfer and Shuryak (1998)

- T_{ij}s defined as the matrix element

$$T_{ij} \equiv \langle i| - i\not{D} \, |j\rangle$$

- Density of zero eigenvalues is proportional to the magnitude of the chiral condensate

Banks and Casher (1980)

Evaluating T_{ij}

- Using the Dirac operator and the zero modes, we can evaluate the matrix elements

$$
\begin{aligned}
T_{ij} &= \langle\psi_i|x\rangle \langle x| - i\not{D} |y\rangle \langle y|\psi_j\rangle \\
&= \int \mathrm{d}^3x \psi^\dagger_{kn}(x - x_i)(-i\not{D})\psi_{lm}(x - x_j) \\
&= \int \sum_m \mathrm{d}^3x \psi^\dagger_{km}(x - x_i)[-i\vec{\alpha} \cdot \vec{\partial}]^{kl} \psi_{lm}(x - x_j)
\end{aligned}
$$

- The operator is block off-diagonal, so the equation can be separated into two

$$
T_{ij} = \int \mathrm{d}^3x \xi^\dagger(x - x_i)[-i\vec{\sigma} \cdot \vec{\partial}]\psi(x - x_j)
$$

$$
T_{ji} = \int \mathrm{d}^3x \psi^\dagger(x - x_i)[-i\vec{\sigma} \cdot \vec{\partial}]\xi(x - x_j)
$$

Evaluating T_{ij}

- Contracting indices,

$$T_{ij} = 2i \int \mathrm{d}^3\vec{x}\tilde{\xi}(\vec{x} - \vec{x}_i)\partial_x\tilde{\psi}(\vec{x} - \vec{x}_j)$$

$$T_{ji} = 2i \int \mathrm{d}^3\vec{x}\tilde{\psi}(\vec{x} - \vec{x}_i)\partial_x\tilde{\xi}(\vec{x} - \vec{x}_j)$$

where $\tilde{\psi}$ and $\tilde{\xi}$ are the radial part of the wavefunctions

BPS Monopoles

- The only analytic monopole solution is the Bogomolnyi-Sommerfeld-Prasad (BPS) solution for $\lambda = 0$ in the Lagrangian

$$
\begin{aligned}
A_a^0 &= 0 \\
A_a^i &= \epsilon^{aij} \frac{r_j}{gr^2}(1 - K(\zeta)) \\
\phi_a &= \frac{r_a}{gr^2} H(\zeta)
\end{aligned}
$$

where

$$H(\zeta) = \zeta \coth(\zeta) - 1$$

$$K(\zeta) = \frac{\zeta}{\sinh(\zeta)}$$

$$\zeta = gvr$$

BPS Monopoles and the Hopping Matrix

- The monopole zero mode, up to normalization,

$$\tilde{\psi} = \frac{1}{2}(gvr)^{\frac{G}{2}+1} \coth\left(\frac{gvr}{2}\right) \sinh^{-\frac{G}{2}}(gvr)$$

- The hopping matrix element equation for monopole-to-antimonopole:

$$\begin{aligned}
T_{ij}(r_0) =& 2i \int \mathrm{d}^3\vec{x}\tilde{\xi}(|r - r_0|)\partial_x\tilde{\psi}(r) \\
=& 2iN^2 \int \mathrm{d}^3\vec{x} \left(-\frac{x}{8r^2}\right) (gv)^{G+2} r^{\frac{G+2}{2}} |r - r_0|^{\frac{G+2}{2}} \\
& \times \coth\left(\frac{1}{2}gvr\right) \sinh^{-\frac{G}{2}-1}(gvr) \\
& \times \coth\left(\frac{1}{2}gv|r - r_0|\right) \sinh^{-\frac{G}{2}}(gv|r - r_0|) \\
& \times (-(G + 2)\sinh(gvr) + gGvr\cosh(gvr) + 2gvr)
\end{aligned}$$

Finding the Dirac Eigenvalues

1. Take monopole mass from D'Alessandro, D'Elia and Shuryak (2010) to numerically evaluate T_{ij} for temperature T
2. Evaluate the evolution operator U, defined as time-ordered integral of the hopping matrix over the Matsubara circle, for configurations of monopoles from path-integral study at temperature T
3. Apply appropriate boundary conditions

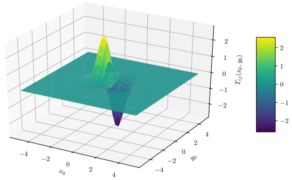
$$U = \oint_\beta d\tau e^{iH\tau} = -\mathbb{1}$$

to find the Dirac eigenvalues

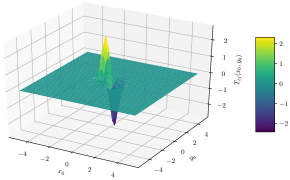
$$\omega_{i,n} = \left(n + \frac{1}{2}\right)\frac{2\pi}{\beta} - \lambda_i$$

where λ_is are the eigenvalues of the hopping matrix $\mathbf{T}$

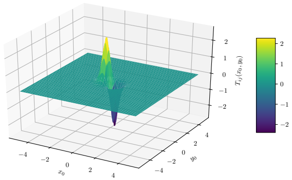
Evaluation of $\mathrm{Im}(T_{ij}(r_0))$ for r_0 in the xy plane for different temperatures.



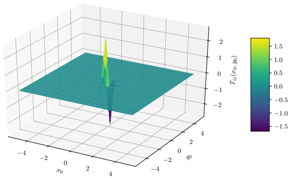
(a) $T = 1T_c$



(b) $T = 1.05T_c$

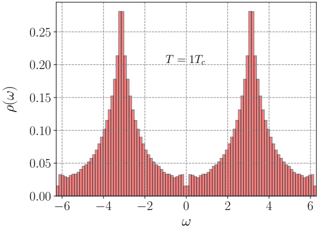


(c) $T = 1.1T_c$

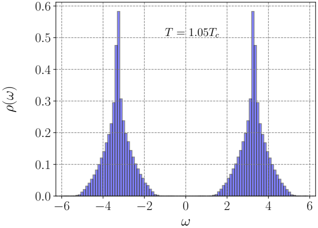


(d) $T = 1.2T_c$

Dirac Eigenvalue Spectra



(a) $T = 1T_c$



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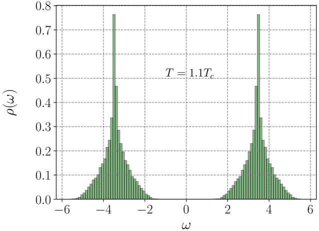

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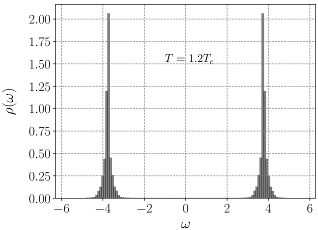
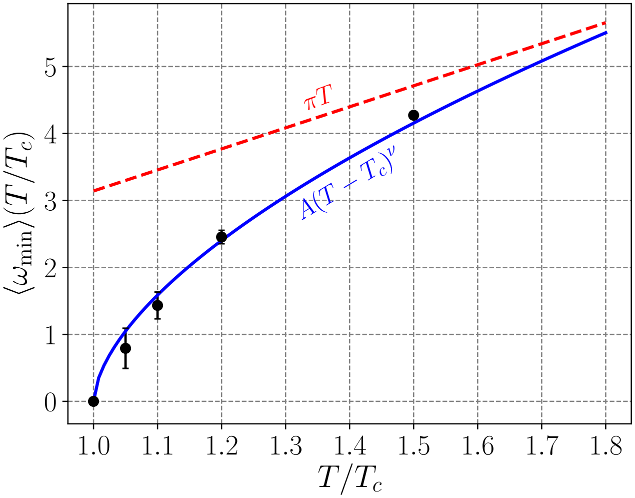


(d) $T = 1.2T_c$

Critical Scaling of Eigenvalue Gap



$\nu = 0.60$, roughly consistent with Ising model

Outline

Preliminaries: Jet Observables

We focus on two observables of heavy-ion collisions:

- The nuclear modification factor

$$R_{\mathsf{AA}}(p_\perp) = \frac{\mathrm{d}N^{\mathsf{AA}}/\mathrm{d}p_\perp}{\langle N_{\mathsf{coll}}\rangle \mathrm{d}N^{\mathsf{pp}}/\mathrm{d}p_\perp}$$

- The azimuthal anisotropy, v_2, from

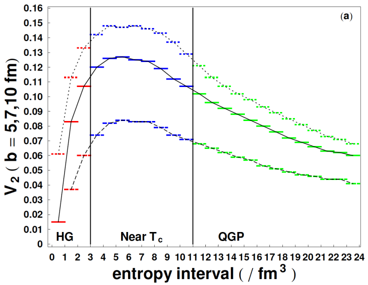
$$\frac{\mathrm{d}N}{\mathrm{d}p_\perp \mathrm{d}\phi} = \frac{1}{2\pi}\frac{\mathrm{d}N}{\mathrm{d}p_\perp}\left(1 + 2\sum_n v_n \cos(n(\phi - \Psi_n))\right)$$

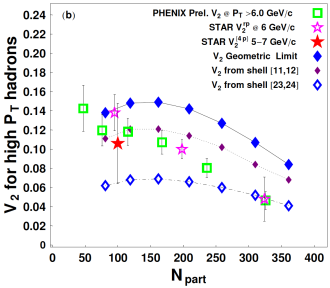
$R_{AA} \oplus v_2$ "puzzle"

- First RHIC data on the nuclear modification factor and azimuthal anisotropy in early/mid 2000s
    - Jet quenching models of the time under-predicted azimuthal anisotropy by approximately a factor of 2
- Improvements on jet quenching models to try to fix this discrepancy
    - Coupling of jets to flow Betz, Gyulassy (2014)
    - Event-by-event fluctuations Noronha-Hostler, Betz, Noronha, Gyulassy (2016)
    - Non-perturbative effects Liao, Shuryak (2008), Xu, Liao, Gyulassy (2015, 2016)
    - ...

$R_{AA} \oplus v_2$ "puzzle"

Shuryak (2007), Liao and Shuryak (2008)





Near T_c enhancement of jet quenching could explain large v_2

Goals

Introduce monopoles into the BDMPS jet quenching framework
$\rightarrow$ jet observables

- Study the effects of different monopole density parameterizations
- Study the effects of different background evolutions
- Study the energy dependence of jet quenching parameters/observables

BDMPS Jet Energy Loss

Baier, Dokshitzer, Mueller, Peigne, Schiff (1997, 1998)

- Energy loss:

$$-\frac{dE}{dz} \propto \hat{q} z$$

$$\hat{q}(z) \approx \rho(z) \int_0^{1/b^2} \mathrm{d}^2\vec{q}_\perp \vec{q}_\perp^{\,2} \frac{\mathrm{d}\sigma}{\mathrm{d}\vec{q}_\perp^{\,2}} (\vec{q}_\perp^{\,2}, z)\,,$$

where $\rho(z)$ is the density of scatterers.

- Screened Coulomb scattering centers:

$$V(q_\perp^2) = \frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}^2\vec{q}_\perp}(\vec{q}_\perp) = \frac{\mu^2}{\pi(q_\perp^2 + \mu^2(z))^2}$$

Cross Sections

Generic form of $\mathrm{d}\sigma/\mathrm{d}q_\perp^2$ is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp^2} = \frac{C}{(q_\perp^2 + \mu^2)^2}\,.$$

$$\frac{\mathrm{d}\sigma_{qq}}{\mathrm{d}q_\perp^2} = \frac{(4/3)^2 \pi \alpha_s^2(q_\perp^2)}{(q_\perp^2 + \mu_E^2)^2}$$

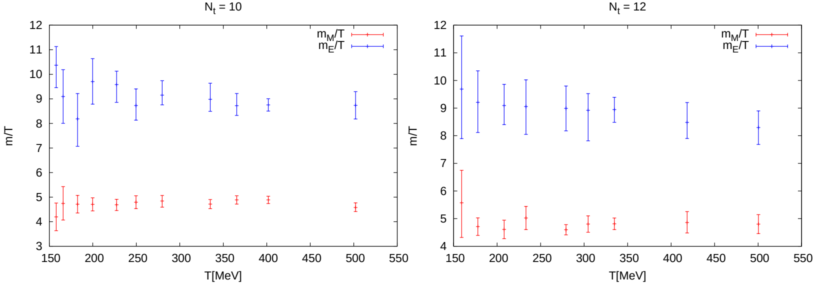
$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}q_\perp^2} = \frac{4\pi \alpha_s^2(q_\perp^2)}{(q_\perp^2 + \mu_E^2)^2}$$

$$\frac{\mathrm{d}\sigma_{gg}}{\mathrm{d}q_\perp^2} = \frac{9\pi \alpha_s^2(q^2)}{(q^2 + \mu_E^2)^2}$$

$$\frac{\mathrm{d}\sigma_{qm}}{\mathrm{d}q_\perp^2} = \frac{(4/3)\pi}{(q_\perp^2 + \mu_M^2)^2}$$

$$\frac{\mathrm{d}\sigma_{gm}}{\mathrm{d}q_\perp^2} = \frac{3\pi}{(q_\perp^2 + \mu_M^2)^2}$$

Screening Masses



$$m_E/T = 7.31$$
$$m_M/T = 4.48$$

Borsányi, Fodor, Katz, Pásztor, Szabó, Török (2015)

Monopole Density

- Two forms of monopole density
  - Direct lattice measurement: Bonati, D'Elia (2013)

$$\frac{\rho_m}{T^3} = \frac{3.66}{\log((1/0.163)T/T_c)^3}$$

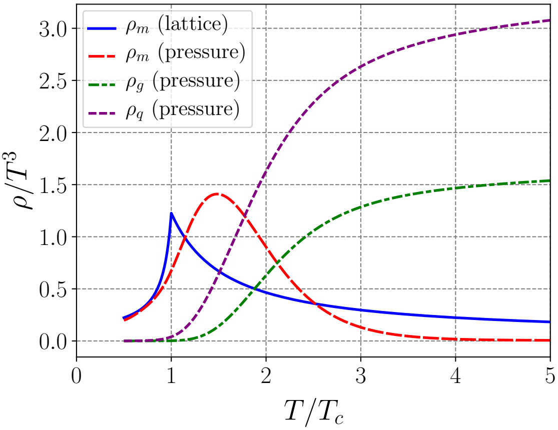
  - Polyakov line with EoS: Xu, Liao, Gyulassy (2015)

$$\rho_E(T) \propto c_q L(T) + c_g L^2(T)$$

$$\rho_M(T) \propto 1 - \rho_E(T)$$

Monopole Density

- Two forms of monopole density

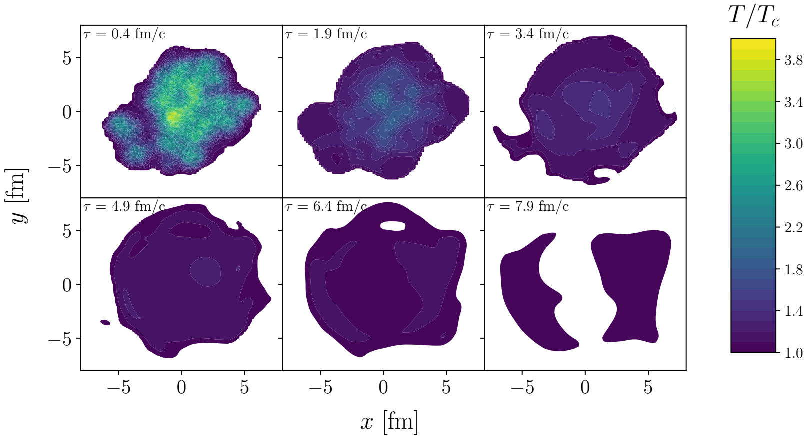


Background Medium

- Focus on 20-30% centrality AA collisions
- Different expanding-medium backgrounds:
  - Glauber-like smooth initial conditions with Bjorken (1+1)D expansion
  - Glauber-like smooth initial conditions with (2+1)D expansion with and without bulk viscosity
  - IP-Glasma initial conditions with (2+1)D expansion with and without bulk viscosity

Background Medium

Example of the hydrodynamic evolution



Numerical Simulation

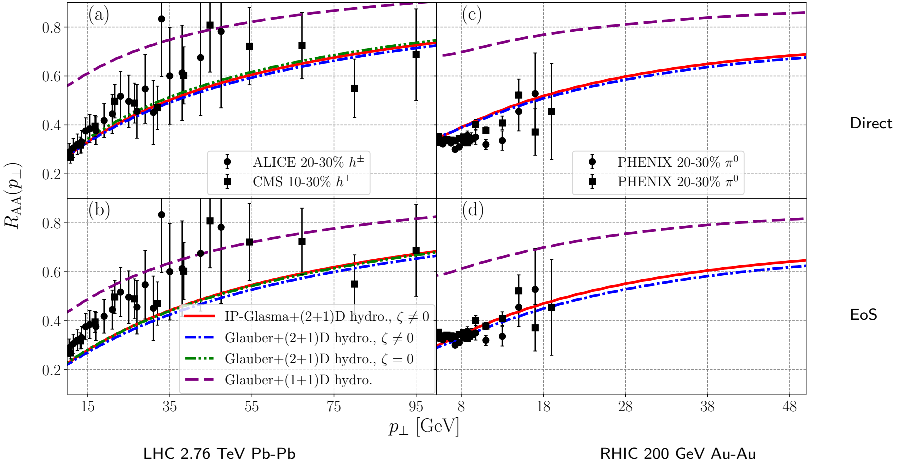
1. Jets created at $\tau = 0$ in the medium with probability proportional to energy density
2. Randomly oriented in azimuthal angle ϕ
3. Initial energy is sampled from power law spectra for quarks and gluons
4. Jet parton then traverses the (evolving) medium and loses energy
5. Fragmentation into pions / charged hadrons

Numerical Simulation

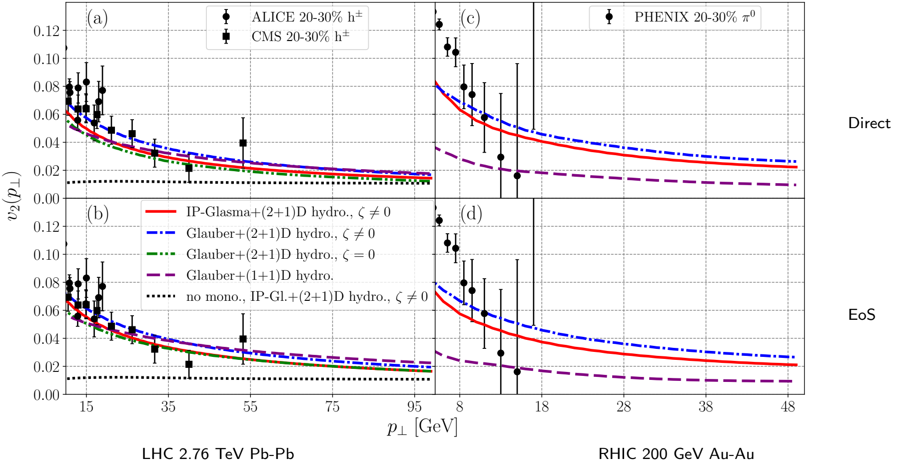
Energy loss given by

$$
\begin{aligned}
-\mathrm{d}E &= z\mathrm{d}z\frac{\alpha_s N_c}{12}\hat{q}(z,E) \\
&= z\mathrm{d}z\frac{\alpha_s N_c \pi C_p}{12}\left(\rho_q(z)\int_0^{q^2_{\mathsf{max}}}\mathrm{d}q^2\frac{(4/3)\alpha_s^2(q^2)}{(q^2+\mu_E^2(z))^2}\right. \\
&\quad +\rho_g(z)\int_0^{q^2_{\mathsf{max}}}\mathrm{d}q^2\frac{3\alpha_s^2(q^2)}{(q^2+\mu_E^2(z))^2} \\
&\quad \left. +\rho_m(z)C_{\mathsf{corr}}\int_0^{q^2_{\mathsf{max}}}\mathrm{d}q^2\frac{1}{(q^2+\mu_M^2(z))^2}\right)
\end{aligned}
$$

Nuclear Modification Factor R_{AA}

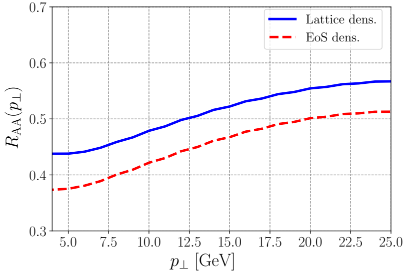


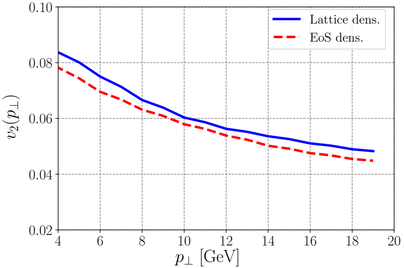
Azimuthal Anisotropy v_2



Predictions for the Beam Energy Scan

62.4 GeV Au-Au collisions, optical Glauber initial conditions





R_{AA} and v_2 of same magnitude as higher energy collisions

Outline

Summary I

- Found parameters of a two-component Coulomb quantum Bose gas that give an effective model of QCD monopoles
- Identified the contribution of monopoles to QCD equation of state
- Found the effect of coupling on the critical temperature of Coulomb quantum Bose gases

Summary II

- Evaluated Dirac eigenvalue spectra for configurations of BPS monopoles in the Georgi-Glashow model
- Non-zero eigenvalue density appears for configurations at T_c
- The critical scaling of the eigenvalue gap is consistent with the Ising model

Summary III

- BDMPS framework with monopoles can reproduce correct trends for experimental observables
  - Needs realistic hydrodynamic background, but event-by-event fluctuations not necessary
- Lower energy collisions should see similar v_2 and R_{AA} to higher energy collisions $\rightarrow$ monopole dominated
  - Can be probed in Beam Energy Scan