Unequal Rapidity Correlators in the Dilute Limit of JIMWLK

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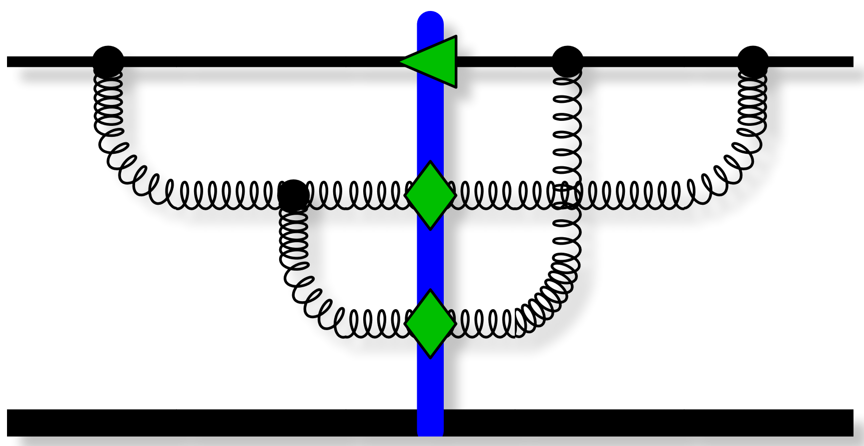

Probing QCD at the high energy frontier,
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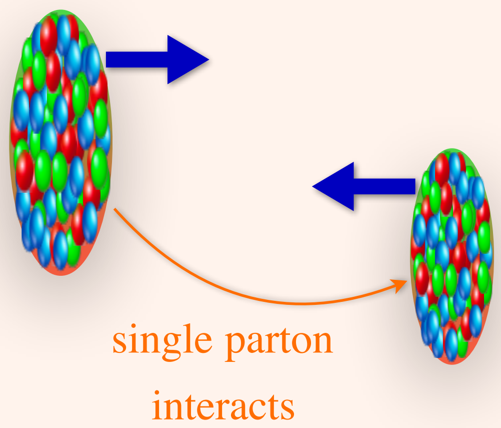
Context

Ultra-relativistic heavy ion collision:



- Small x, large $Q_s \Rightarrow$ saturation
- Target nucleus: Colour Glass Condensate
- Eikonal approximation: parton is colour-rotated, no transverse momentum kick

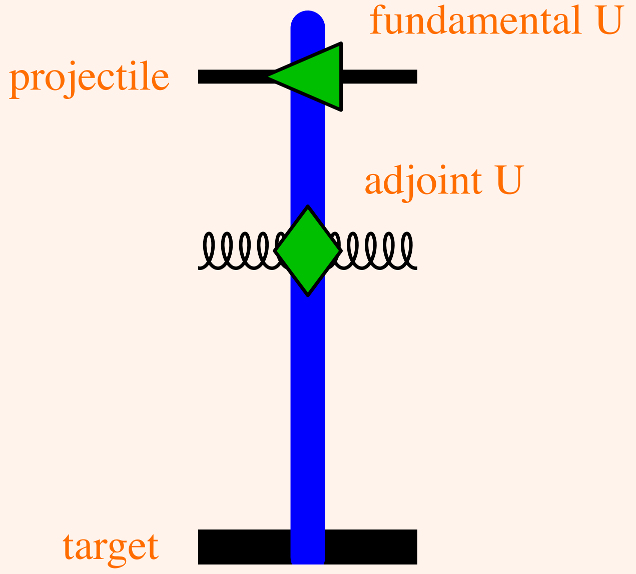
- Interaction described by Wilson line

$$U_{\boldsymbol{x}}^{\dagger} := P \exp \left\{ ig \int dx^{+} \alpha_{\boldsymbol{x}}^{a}(x^{+}) t^{a} \right\}$$

- Enters cross sections through correlators $\left\langle \ldots \right\rangle_{Y}$

e.g. dipole $\left\langle \frac{\mathrm{tr}\left\{ U_{\boldsymbol{x}} U_{\boldsymbol{y}}^{\dagger} \right\}}{N_c} \right\rangle$

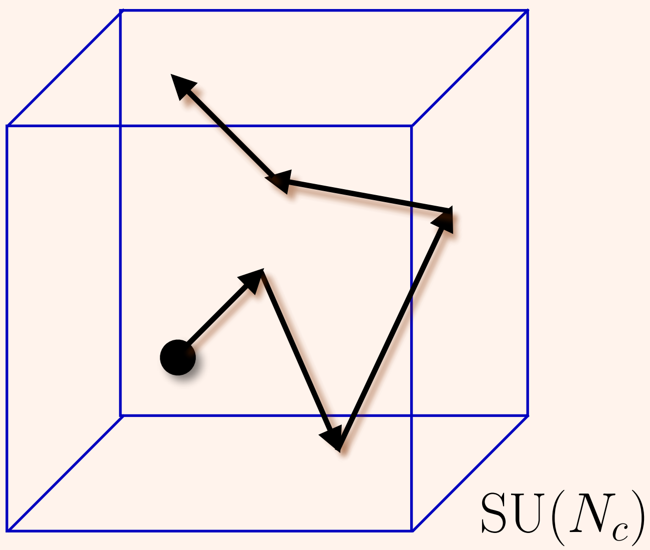
Diagrammatic notation:



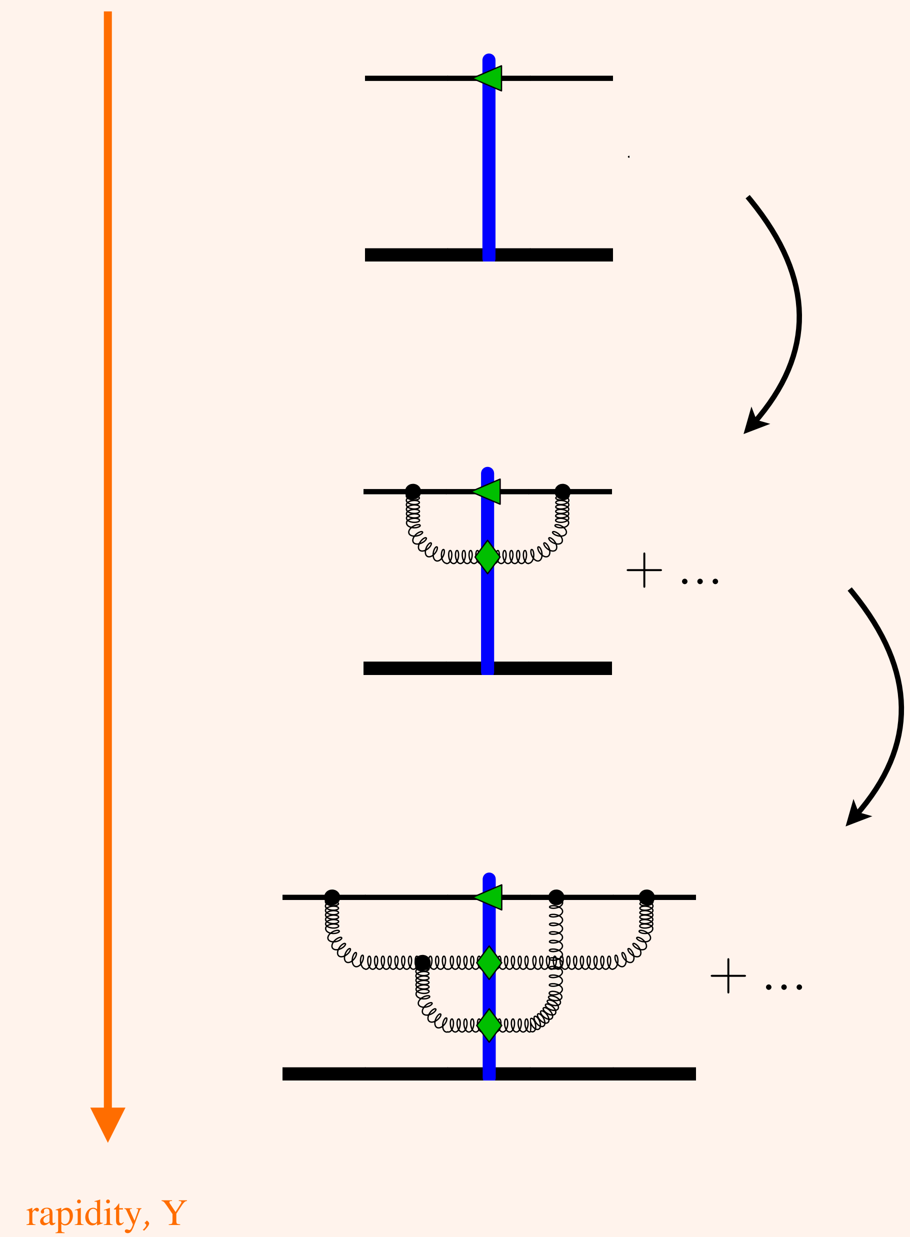
Evolution of Wilson Line

Stochastic, diffusive process

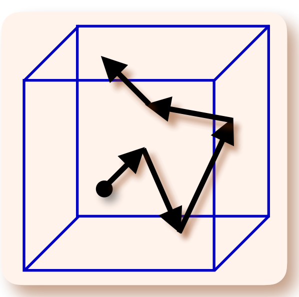
in space of Wilson lines:



JIMWLK evolution:



JIMWLK Equation



Fokker-Planck form:

$$\frac{d}{dY}\left\langle \ldots \right\rangle_Y = -H_{\mathrm{JIMWLK}} \left\langle \ldots \right\rangle_Y$$

$$H_{\mathrm{JIMWLK}} := -\frac{\alpha_s}{2\pi^2} \int d^2 z \, \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left(L^a_{\boldsymbol{x}} - U^{\dagger ab}_{\boldsymbol{z}} R^b_{\boldsymbol{x}} \right) \left(L^a_{\boldsymbol{y}} - U^{\dagger ac}_{\boldsymbol{z}} R^c_{\boldsymbol{y}} \right)$$

$$\tilde{\mathcal{K}}_{\boldsymbol{xzy}} := \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

$$L^a_{\boldsymbol{u}} := -ig\left(U_{\boldsymbol{u}} t^a\right)_{\alpha\beta} \frac{\delta}{\delta U_{\boldsymbol{u},\alpha\beta}}, \qquad R^a_{\boldsymbol{u}} := -ig\left(t^a U_{\boldsymbol{u}}\right)_{\alpha\beta} \frac{\delta}{\delta U_{\boldsymbol{u},\alpha\beta}}$$

Langevin form:

- Lesser known

Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469], Kovner & Lublinsky [JHEP 0611 (2006) 083], Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067], Hatta & Iancu [JHEP 1608 (2016) 083]

- Better suited to numerical studies
- Useful for unequal rapidity correlators

Langevin JIMWLK

Discretise rapidity interval: $Y - Y_A = \epsilon N$ $\mathbb{Z} \ni N \to \infty, \epsilon \to 0$ $n \in \{0, 1, ..., N\}$

Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469]

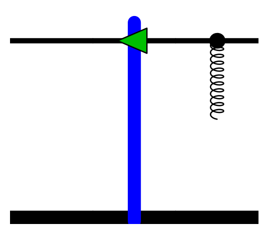
Langevin equation for Wilson line:

$$U^\dagger_{\boldsymbol{x},n+1} = e^{i\epsilon g \alpha^L_{\boldsymbol{x},n}} U^\dagger_{\boldsymbol{x},n} e^{-i\epsilon g \alpha^R_{\boldsymbol{x},n}}$$

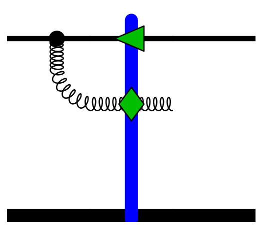
Colour rotations:

$$\alpha^L_{\boldsymbol{x},n} := \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n} t^a \qquad \alpha^R_{\boldsymbol{x},n} := \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b$$

$$\alpha^L_{\boldsymbol{x},0} U^\dagger_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},0} t^a \propto$$



$$\alpha^R_{\boldsymbol{x},0} U^\dagger_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},0} U^{\dagger ab}_{\boldsymbol{z},0} t^b \propto$$



Weizsäcker-Williams emission kernel: $\mathcal{K}^i_{\boldsymbol{xy}} := \frac{(\boldsymbol{x}-\boldsymbol{y})^i}{(\boldsymbol{x}-\boldsymbol{y})^2}$

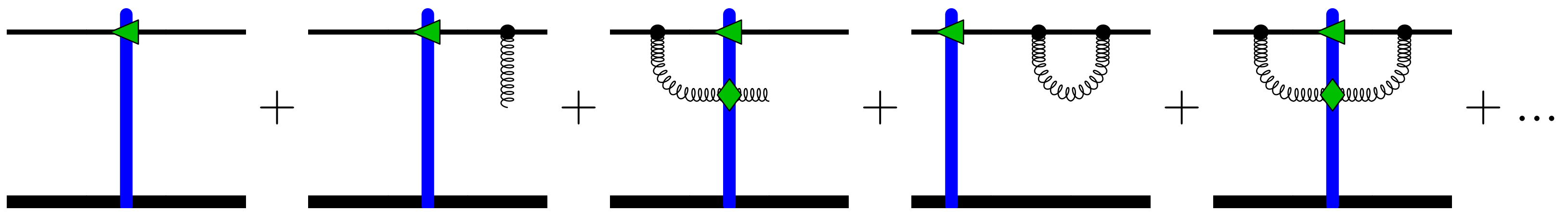
Local Gaussian white noise: $\left\langle \nu^{i,a}_{\boldsymbol{x},m} \right\rangle = 0$ $\left\langle \nu^{i,a}_{\boldsymbol{x},m} \nu^{jb}_{\boldsymbol{y},n} \right\rangle = \frac{1}{\epsilon} \delta^{ij} \delta^{ab} \delta_{mn} \delta_{\boldsymbol{xy}}$ $\nu^{i,a}_{\boldsymbol{x},m} \in \mathbb{R}$

Dilute Limit

Expand in ϵ :

$$U^\dagger_{\boldsymbol{x},n+1} = U^\dagger_{\boldsymbol{x},n} + \int_{\boldsymbol{z}} \left(\frac{ig}{\sqrt{4\pi^3}} \epsilon \nu^{i,a}_{\boldsymbol{z},n} \mathcal{K}^i_{\boldsymbol{xz}} - \frac{g^2}{4\pi^3} \epsilon \mathcal{K}^i_{\boldsymbol{xz}} \mathcal{K}^i_{\boldsymbol{xz}} t^a \right) \left(t^a U^\dagger_{\boldsymbol{x},n} - U^\dagger_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b \right) + \mathcal{O}(\epsilon^2)$$

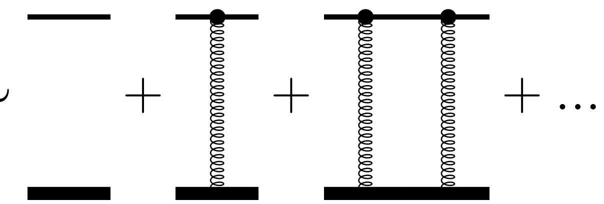
$U^\dagger_{\boldsymbol{x},1} \sim$



Expand in $\lambda_{\boldsymbol{x},n} = \lambda^T_{\boldsymbol{x},n}$ (where $\lambda_{\boldsymbol{x},n} \in \mathrm{su}(N_c)$)):

$$U^\dagger_{\boldsymbol{x},n} := P \exp \left\{ ig \int dx^+ \alpha^a_{\boldsymbol{x},\boldsymbol{n}}(x^+) t^a \right\} =: e^{i\lambda_{\boldsymbol{x},n}} = \mathbb{1} + i\lambda_{\boldsymbol{x},n} + \mathcal{O}(\lambda^2_{\boldsymbol{x},n})$$

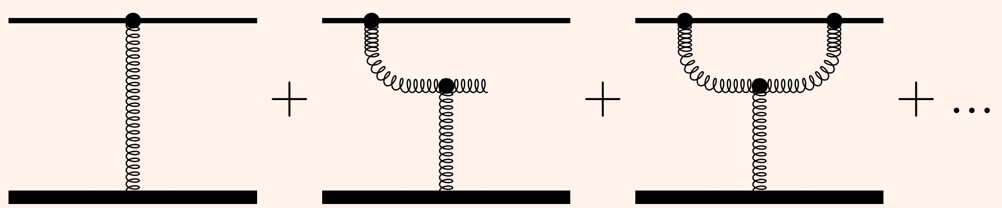
$\sim$



Langevin equation:

$$\lambda_{\boldsymbol{x},n+1} = \lambda_{\boldsymbol{x},n} + \int_{\boldsymbol{z}} \left(\frac{ig}{\sqrt{4\pi^3}} \epsilon \nu^{i,a}_{\boldsymbol{z},n} \mathcal{K}^i_{\boldsymbol{xz}} - \frac{g^2}{4\pi^3} \epsilon \mathcal{K}^i_{\boldsymbol{xz}} \mathcal{K}^i_{\boldsymbol{xz}} t^a \right) [t^a, \lambda_{\boldsymbol{x},n} - \lambda_{\boldsymbol{z},n}] + \mathcal{O}(\lambda^2) + \mathcal{O}(\epsilon^2)$$

$\lambda_{\boldsymbol{x},1} \sim$



Gluon Reggeization

Take average:

$$\frac{1}{\epsilon}\left\langle \lambda_{\boldsymbol{x},n+1} - \lambda_{\boldsymbol{x},n}\right\rangle = -\frac{g^2}{4\pi^3}\int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}}\mathcal{K}^i_{\boldsymbol{xz}}\left\langle t^a\left[t^a, \lambda_{\boldsymbol{x},n} - \lambda_{\boldsymbol{z},n}\right]\right\rangle$$

Fourier transform:

$$\lambda^a(p) = \int_{\boldsymbol{z}} e^{ip\cdot z}\lambda^a_{\boldsymbol{z}}$$

$$\frac{d}{dY}\lambda^a(p) = \alpha_g(p)\lambda^a(p) + \mathcal{O}(g^4\lambda^3)$$

$$\alpha_g(p) := \frac{\alpha_s C_A}{2\pi^2}\int_{\boldsymbol{z}}\frac{1}{z^2}\left(e^{ip\cdot z} - 1\right)$$

Caron-Huot [High Energ. Pays. (2015) 93]

- Same result from linearising Fokker-Planck JIMWLK and acting on single λ
- Regge pole behaviour: amplitude has power law behaviour on energy, i.e. amp $\propto s^{\alpha(p)}$
- $\alpha_g(p)$ is Regge trajectory

BFKL Recovered

Start with BK equation:

$$\frac{d}{dY}\left\langle \frac{\mathrm{tr}\left\{U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right\}}{N_c}\right\rangle = \frac{\alpha_s}{\pi^2}\frac{N_c}{2}\int_{\boldsymbol{z}} \tilde{\mathcal{K}}_{\boldsymbol{xyz}}\left\langle \frac{\mathrm{tr}\left\{U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{z}}\right\}}{N_c}\frac{\mathrm{tr}\left\{U_{\boldsymbol{z}}U^\dagger_{\boldsymbol{y}}\right\}}{N_c} - \frac{\mathrm{tr}\left\{U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right\}}{N_c}\right\rangle$$

$$\tilde{\mathcal{K}}_{\boldsymbol{xyz}} = \mathcal{K}^i_{\boldsymbol{xz}}\mathcal{K}^i_{\boldsymbol{xz}} + \mathcal{K}^i_{\boldsymbol{yz}}\mathcal{K}^i_{\boldsymbol{yz}} - 2\mathcal{K}^i_{\boldsymbol{xz}}\mathcal{K}^i_{\boldsymbol{yz}}$$

Write in terms of "BFKL pomeron":

$$\mathrm{tr}\left\{U_{\boldsymbol{x},n}U^\dagger_{\boldsymbol{y},n}\right\} \to (\lambda^a_{\boldsymbol{x},n} - \lambda^a_{\boldsymbol{y},n})^2$$

$$\frac{1}{\epsilon}\left\langle(\lambda^a_{\boldsymbol{x},n+1} - \lambda^a_{\boldsymbol{y},n+1})^2 - (\lambda^a_{\boldsymbol{x},n} - \lambda^a_{\boldsymbol{y},n})^2\right\rangle = -\frac{\alpha_s}{\pi^2}N_c\int_{\boldsymbol{z}}\tilde{\mathcal{K}}_{\boldsymbol{xyz}}\left\langle \lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{z},n} + \lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{y},n} - \lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{y},n} - \lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{z},n}\right\rangle$$

Hatta & Iancu [JHEP 1608 (2016) 083]

- Calculate left side from Langevin equation ⇒ BFKL recovered from Langevin formalism

Unequal Rapidities: Example

Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

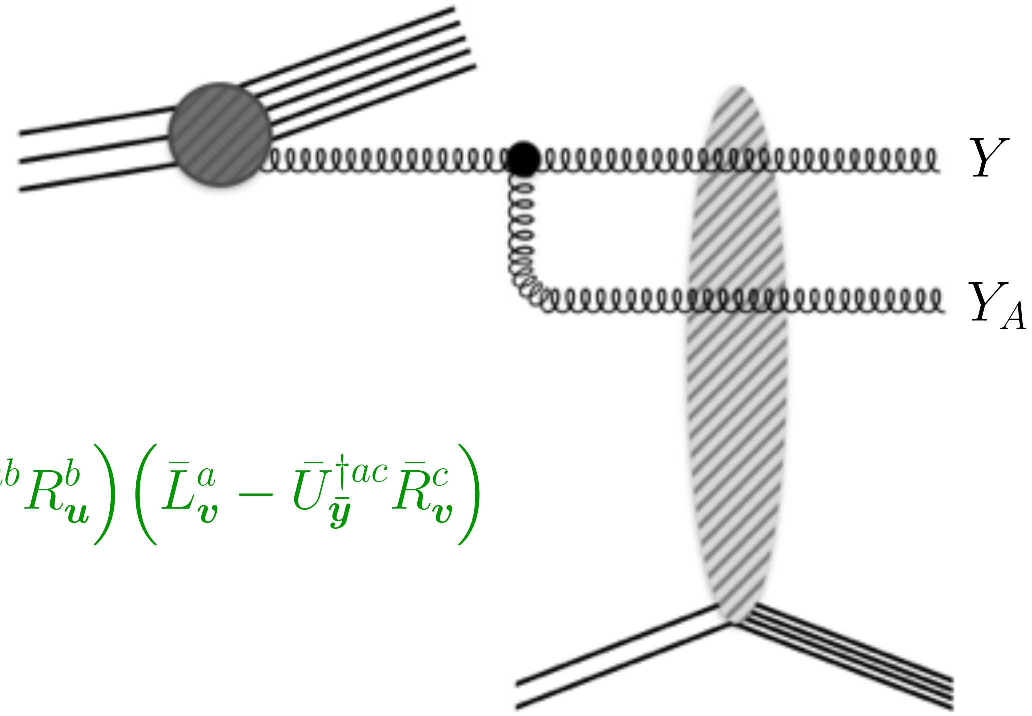
Inclusive two-gluon production:

$$\frac{d\sigma_{2g}}{dY\, d^2\boldsymbol{p}\, dY_A d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \times \left\langle H(\boldsymbol{k}_A) \left\langle \frac{\mathrm{tr}\left\{\bar{U}_{\bar{\boldsymbol{x}},A} U^{\dagger}_{\boldsymbol{x},A}\right\}}{(N_c^2-1)} \right\rangle_{Y-Y_A}\Bigg|_{\bar{U}_A = U_A} \right\rangle_{Y_A}$$

$$\langle \hat{\mathcal{O}} \rangle_{Y_A} := \int [DU][D\bar{U}] W_{Y_A}[U,\bar{U}]\mathcal{O}$$

$$\langle \hat{\mathcal{O}} \rangle_{Y-Y_A} := \int [DU\, D\bar{U}] W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A]\hat{\mathcal{O}}$$

$$H(\boldsymbol{k}_A) := \frac{1}{4\pi^3}\int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{i\boldsymbol{k}_A\cdot(\bar{\boldsymbol{y}}-\boldsymbol{y})}\int_{\boldsymbol{uv}} \mathcal{K}^i_{\boldsymbol{yu}}\mathcal{K}^i_{\bar{\boldsymbol{y}}\boldsymbol{v}} \left(L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{y}} R^b_{\boldsymbol{u}}\right)\left(\bar{L}^a_{\boldsymbol{v}} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}}} \bar{R}^c_{\boldsymbol{v}}\right)$$



$\Delta Y\, \alpha_s \geq 1$

Bilocal Langevin Equation

Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

Work in progress:

$$R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n+1} = e^{i\epsilon g \alpha^L_{\boldsymbol{x},n}} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} e^{-i\epsilon g \alpha^R_{\boldsymbol{x},n}} - \frac{i\epsilon g}{\sqrt{4\pi^3}} e^{i\epsilon g \alpha^L_{\boldsymbol{x},n}} U^\dagger_{\boldsymbol{x},n} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \Big[U_{\boldsymbol{z},n} \nu^i_{\boldsymbol{z},n} U^\dagger_{\boldsymbol{z},n}, U_{\boldsymbol{z},n} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{z},n} \Big]$$

$$\begin{aligned} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n+1} = {} & R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} \\ & + \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,b}_{\boldsymbol{z},n} \Big(t^b R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} - R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} U^{\dagger bc}_{\boldsymbol{z},n} t^c - U^\dagger_{\boldsymbol{x},n} U_{\boldsymbol{z},n} t^b R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{z},n} + U^\dagger_{\boldsymbol{x},n} U_{\boldsymbol{z},n} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{z},n} U^{\dagger bc}_{\boldsymbol{z},n} t^c \Big) \\ & - \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{xxz}} \Big(C_F R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} - t^b R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x},n} U^{\dagger bc}_{\boldsymbol{z},n} t^c - t^b U^\dagger_{\boldsymbol{x},n} U_{\boldsymbol{z},n} t^b R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{z},n} + t^b U^\dagger_{\boldsymbol{x},n} U_{\boldsymbol{z},n} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{z},n} U^{\dagger bc}_{\boldsymbol{z},n} t^c \Big) \end{aligned}$$

- Square of this, plus 3 other similar terms, appear in cross section
- Go to dilute limit to recover BFKL ladder diagrams from Langevin formalism

Summary

- JIMWLK equation governs rapidity evolution of classical target field
- Two formulations for dynamics - Langevin is lesser known
- Langevin JIMWLK is better for numerics and for unequal rapidity correlators
- Gluon reggeizes in the dilute limit under single gluon exchange between target and projectile
- Dilute limit gives familiar BFKL evolution
- Inclusive cross sections need Langevin equations for RU etc.
- Useful insight: how do BFKL ladder diagrams emerge from this formalism?