Three plays in the multi-Regge limit:

Jet production, tampering with the infrared & complexity in the Odderon

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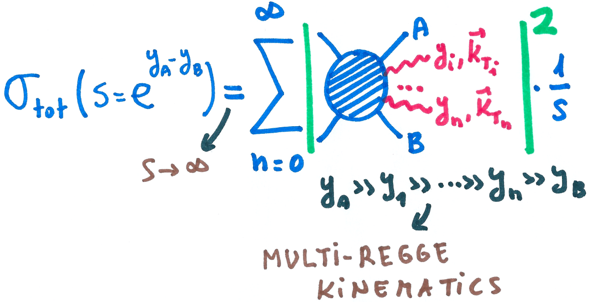
Seminar at ECT* workshop program, May 24th 2018.
Probing QCD at the high energy frontier

1. Multi-Regge limit
2. Monte Carlo event generator BFKLex
3. Collinear double logs
4. Infrared Effects
5. Solution of BKP equation
6. CHY amplitudes in Sudakov representation

Regge theory preludes QCD. Pomeron in terms of quarks & gluons?

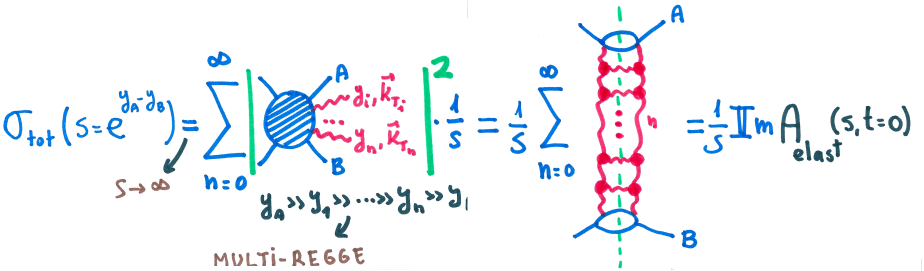
Perturbation theory with large scale $Q > \Lambda_{\mathrm{QCD}} \rightarrow \alpha_s(Q) \ll 1$.

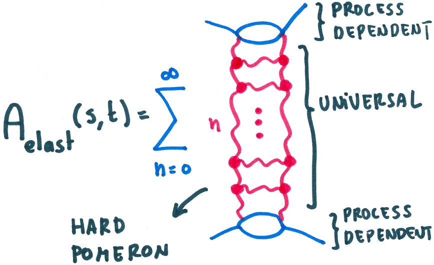
$s \gg t, Q^2 \rightarrow \alpha_s(Q) \log\left(\frac{s}{t}\right) \sim \mathcal{O}(1)$. Resummation needed.



$$\sigma_{\rm tot}^{\rm LL} = \sum_{n=0}^{\infty} \mathcal{C}_n^{\rm LL} \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \ldots \int_{y_B}^{y_{n-1}} dy_n = \sum_{n=0}^{\infty} \frac{\mathcal{C}_n^{\rm LL}}{n!} \underbrace{\alpha_s^n \left(y_A - y_B\right)^n}_{\rm LL}$$

Multi-Regge linked to elastic amplitudes via optical theorem:





New degree of freedom $= g_R$ ("Reggeized" gluon)

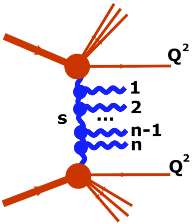
Pomeron= Bound state of 2 g_R

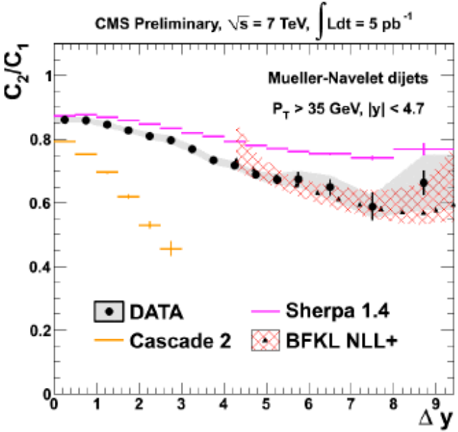
2-dimensional interaction Hamiltonian

New observable proposed (ASV)[2006] (Schwennsen,ASV)[2007]

as ideal to pin down BFKL

$$\mathcal{R}_{2,1} = \frac{<\cos(2\theta)>}{<\cos(\theta)>}$$





Confirmed in 2013

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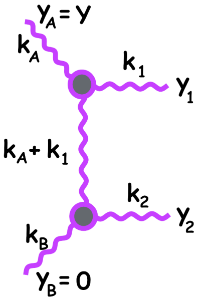
With Grigorios Chachamis

Effective Feynman rules:

Gluon Regge trajectory: $\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2}$

Modified propagators in the t-channel:

$$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$$

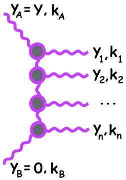


$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$

$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right) y_2}$$

$$\sigma(Q_1, Q_2, Y) = \int d^2\vec{k}_A d^2\vec{k}_B \underbrace{\phi_A(Q_1, \vec{k}_A)\,\phi_B(Q_2, \vec{k}_B)}_{\text{PROCESS}-\text{DEPENDENT}} \underbrace{f(\vec{k}_A, \vec{k}_B, Y)}_{\text{UNIVERSAL}}$$

$$f\left(\vec{k}_A, \vec{k}_B, Y\right) = \sum_n \left| \quad \right|^2$$

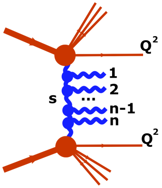


$$= e^{\omega(\vec{k}_A)Y}\left\{\delta^{(2)}\left(\vec{k}_A - \vec{k}_B\right) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i \frac{\theta\left(k_i^2 - \lambda^2\right)}{\pi k_i^2}\right.$$

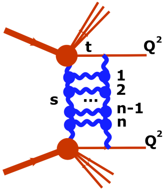
$$\left.\times \int_0^{y_{i-1}} dy_i e^{\left(\omega\left(\vec{k}_A + \sum_{l=1}^{i}\vec{k}_l\right) - \omega\left(\vec{k}_A + \sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A + \sum_{l=1}^{n}\vec{k}_l - \vec{k}_B\right)\right\}$$

BFKLex: Monte Carlo implementation of full NLO BFKL

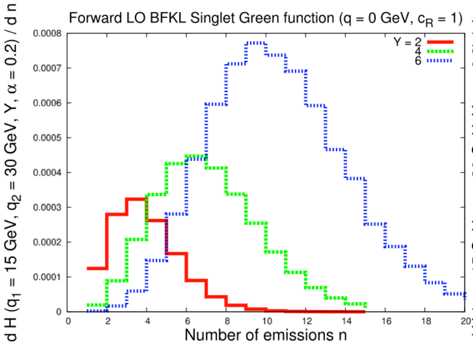
Jet production, tampering with the infrared & complexity in the Odderon

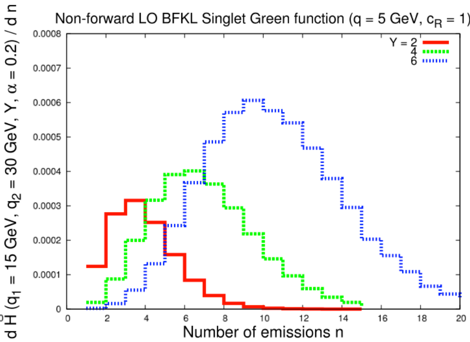


Cut Pomeron: Number of emissions?

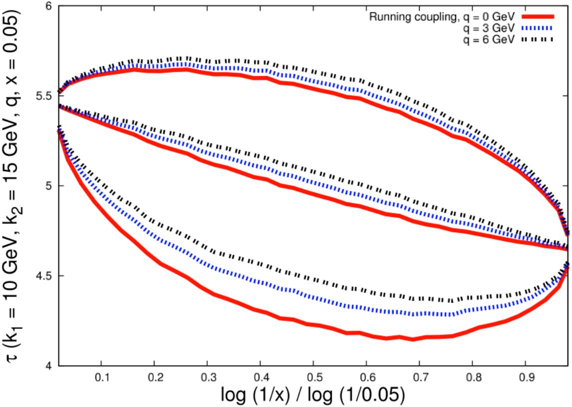


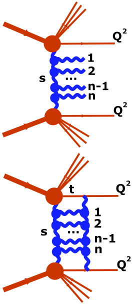
Pomeron: Number of rungs?





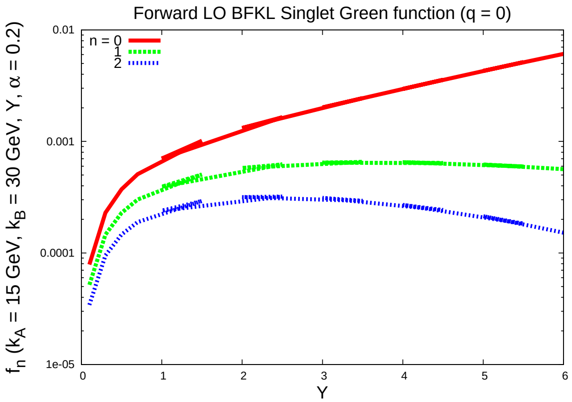
Reggeized (virtual) gluon $|p_T|$ at a given rapidity?





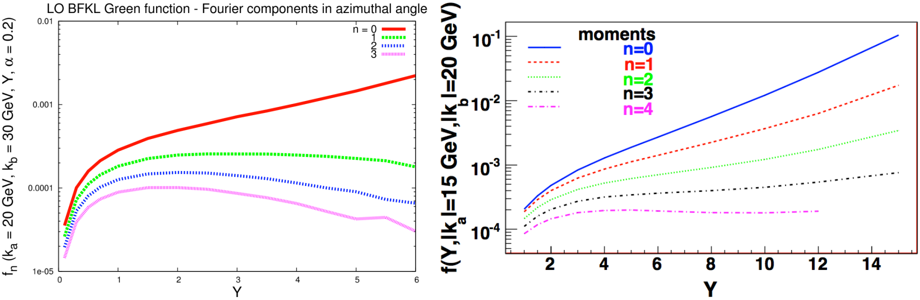
Growth with energy? Depends on the azimuthal angle Fourier component:

$$f_n\left(|\vec{k}_A|, |\vec{k}_B|, Y\right) = \int_0^{2\pi} \frac{d\theta}{2\pi} f\left(\vec{k}_A, \vec{k}_B, Y\right) \cos\left(n\,\theta\right)$$



This is a distinct feature of BFKL

BFKL CCFM



All CCFM projections grow with energy, not in BFKL - 1102.1890

Observables only sensitive to $n > 0$ single out BFKL

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We can extend the formalism to include collinear regions

$$f = e^{\omega\left(\vec{k}_A\right)Y}\left\{\delta^{(2)}\left(\vec{k}_A-\vec{k}_B\right)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i\frac{\theta\left(k_i^2-\lambda^2\right)}{\pi k_i^2}\right.$$
$$\left.\times\int_0^{y_{i-1}}dy_i e^{\left(\omega\left(\vec{k}_A+\sum_{l=1}^{i}\vec{k}_l\right)-\omega\left(\vec{k}_A+\sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A+\sum_{l=1}^{n}\vec{k}_l-\vec{k}_B\right)\right\}$$

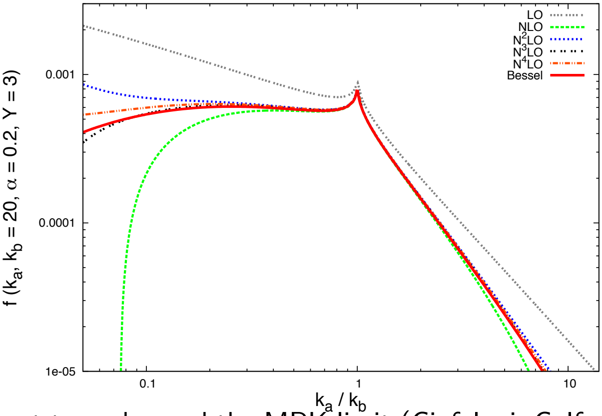
Key at NLL: $\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)\underbrace{-\frac{\bar{\alpha}_s}{4}\ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}_{\rm NLL}$

Resum it to all orders (SV-0507317):

$$\theta\left(k_i^2-\lambda^2\right)\to\theta\left(k_i^2-\lambda^2\right)+\sum_{n=1}^{\infty}\frac{(-\bar{\alpha}_s)^n}{2^n n!(n+1)!}\ln^{2n}\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)$$

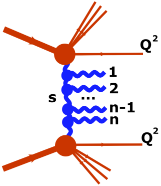
It corresponds to a Bessel function $J_1\left(\sqrt{2\bar{\alpha}_s\ln^2\left(\frac{\vec{k}_A^2}{\left(\vec{k}_A+\vec{k}_i\right)^2}\right)}\right)$

$$\sigma(Q_1, Q_2, Y) \quad = \quad \int d^2\mathbf{k}_a d^2\mathbf{k}_b \, \phi_A(Q_1, \mathbf{k}_a) \, \phi_B(Q_2, \mathbf{k}_b) \, f(\mathbf{k}_a, \mathbf{k}_b, Y)$$



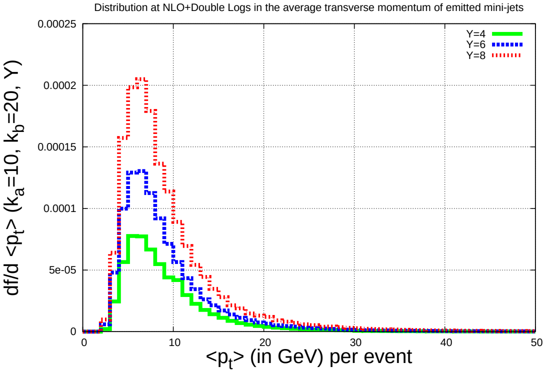
Important to go beyond the MRK limit (Ciafaloni, Colferai, Salam, Stasto).

For BFKL domain we need "δ-like" impact factors $\phi_{A,B}$ & $Q_1 \simeq Q_2$.



Average transverse momentum of emitted mini-jets?

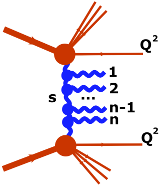
$$\langle p_t \rangle = \frac{1}{N} \sum_{i=1}^{N} |k_i|$$

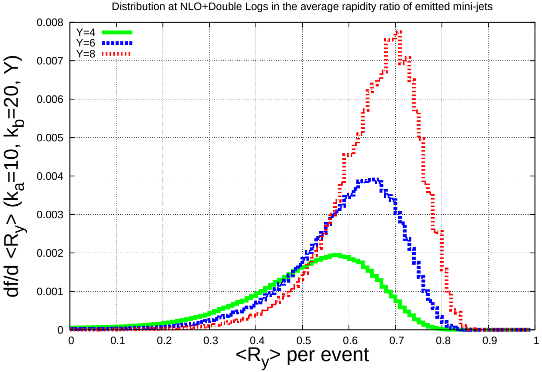


Average rapidity separation among emitted mini-jets?

$$\langle \mathcal{R}_y \rangle = \frac{1}{N+1} \sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}} \quad \simeq 1 + \frac{\Delta}{Y} \ln \frac{\Delta}{Y}$$

if $Y \simeq N\Delta$ in MRK and $Y \gg \Delta$





Higher $\langle \mathcal{R}_y \rangle_{\rm max}$ for higher energies: $\Delta_{\rm LO} \simeq 0.62$, $\Delta_{\rm LO+DLs} \simeq 0.81$

Lower mini-jet multiplicity when including higher order corrections

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With Douglas ROSS (Southampton)

BFKL equation with running coupling $\bar{\alpha}(t) = 1/(\bar{\beta}t)$:

$$\frac{\partial}{\partial Y}\mathcal{G}(Y,t_1,t_2) = \frac{1}{\sqrt{\bar{\beta}t_1}}\int dt\,\mathcal{K}(t_1,t)\frac{1}{\sqrt{\bar{\beta}t}}\mathcal{G}(Y,t,t_2)$$

Eigenfunctions in semi-classical approximation:

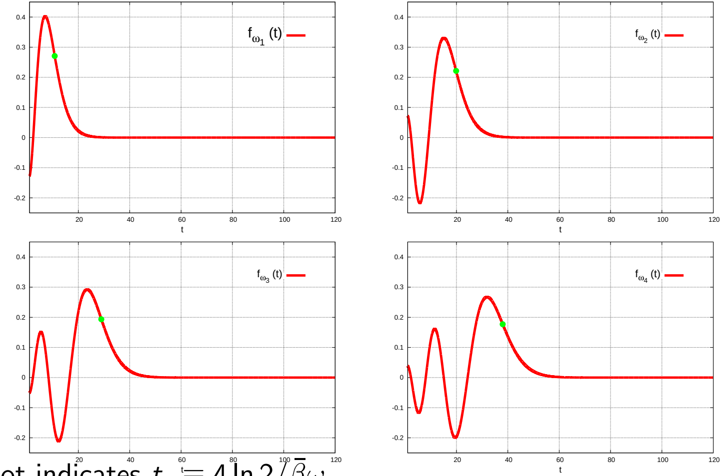
$$\int dt'\frac{1}{\sqrt{\bar{\beta}t}}\mathcal{K}(t,t')\frac{1}{\sqrt{\bar{\beta}t''}}f_\omega(t') = \omega f_\omega(t)$$

$$f_\omega(t) = \frac{|z_\omega(t)|^{1/4}}{\sqrt{\bar{\alpha}(t)\chi'(\nu_\omega(t))}}Ai\left(z_\omega(t)\right)$$

where $\nu_\omega(t) = \chi^{-1}(\bar{\beta}\omega t)$

$$z_\omega(t) = -\left(\frac{3}{2}\int_t^{4\ln 2/\bar{\beta}\omega} dt'\nu_\omega(t')\right)^{2/3}$$

Discrete eigenfunctions when fixing phase of oscillations at t=1



Dot indicates $t_c \equiv 4 \ln 2 / \bar{\beta}\omega$

$$\mathcal{G}(Y, t, t') = \sum_n f_{\omega_n}(t) f^*_{\omega_n}(t') e^{\omega_n Y}$$

Partial wave, with $z(t) \equiv \left(\frac{\bar{\beta}\omega}{14\zeta(3)}\right)^{1/3}\left(t - \frac{4\ln 2}{\bar{\beta}\omega}\right)$, is

$$\mathcal{G}_\omega(t_1, t_2) = \frac{\pi}{4}\frac{\sqrt{t_1 t_2}}{\omega^{1/3}}\left(\frac{\bar{\beta}}{14\zeta(3)}\right)^{2/3} Ai\left(z(t_1)\right) Bi\left(z(t_2)\right) \theta\left(t_1 - t_2\right) + t_1 \leftrightarrow t_2$$

Not unique (Lipatov), can add any solution of homogeneous eqn. with the same UV behavior but now in the IR

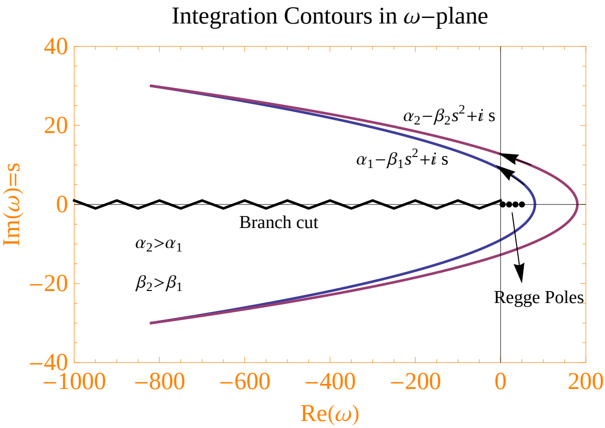
$$Bi(z) \to Bi(z) + Ai(z)\cot\left(\eta - \frac{2}{3}\sqrt{\frac{\bar{\beta}\omega}{14\zeta(3)}}\left(\frac{4\ln 2}{\bar{\beta}\omega} - t_0\right)^{3/2}\right)$$

introduces ∞ ω-poles when argument $= n\pi$

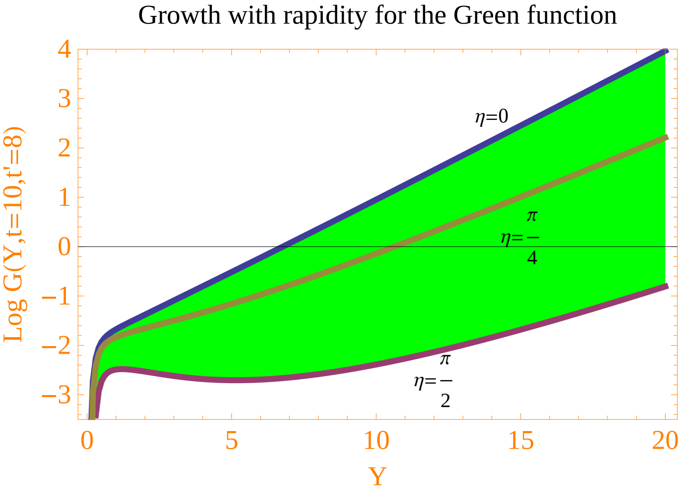
t_0 is the IR value of t at which we fix the phase to be η

η has a non-perturbative origin

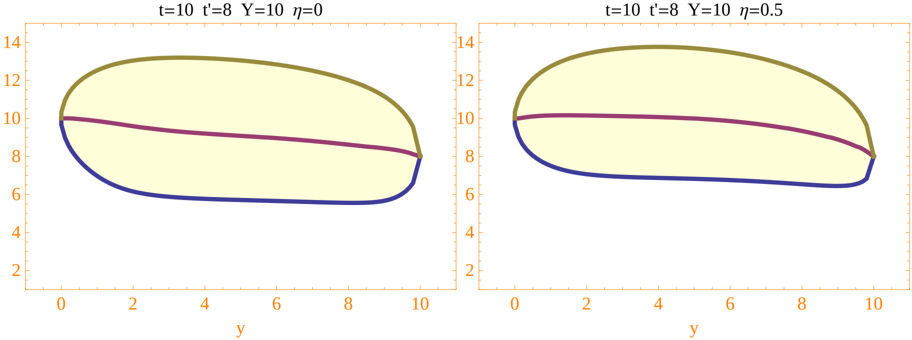
$$\mathcal{G}(Y, t_1, t_2) \;=\; \frac{1}{2\pi i}\int_{\mathcal{C}} d\omega\, e^{\omega Y}\mathcal{G}_\omega(t_1, t_2)$$



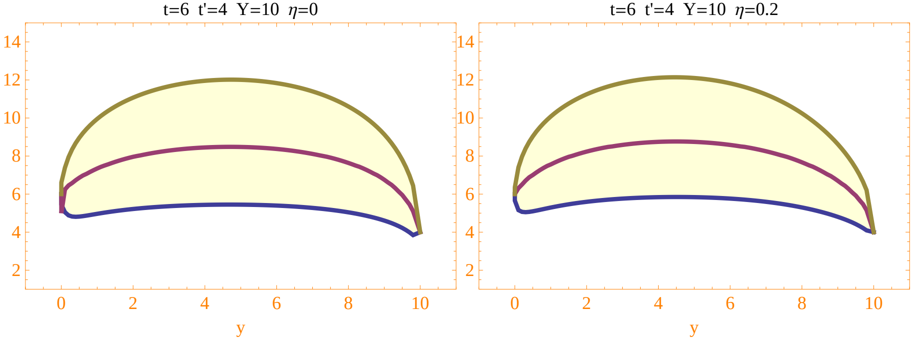
Integration takes into account the cut (from $z(t) < 0$) & Regge poles


Growth with rapidity for the Green function
Log G(Y,t=10,t'=8)
Y
0
5
10
15
20
4
3
2
1
0
−1
−2
−3
η=0
η=π/4
η=π/2

The diffusion into the IR is not modified for large external scales



For smaller external scales the diffusion into the IR is suppressed



If correct, this IR screening should be seen in exclusive observables.
η could be fixed by comparison to experimental data

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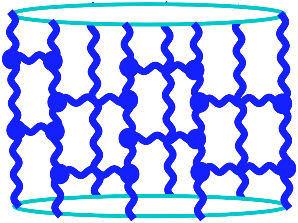
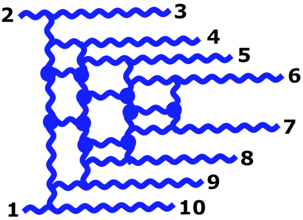
With Chachamis

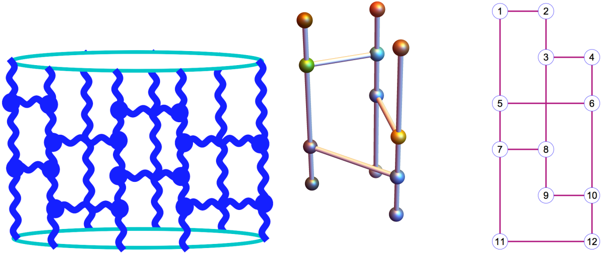
Monte Carlo integration can be applied in this case (Chachamis-ASV)
For high order amplitudes an open spin chain appears, which can be solved using MC





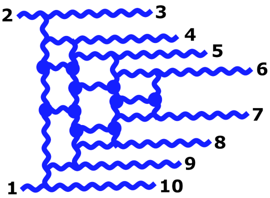
These are the most complicated contributions at higher order and with arbitrary number of legs.

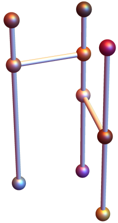
Associated Laplacian matrix $\boxed{L}$: Closed Chain

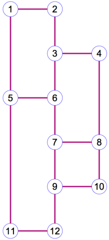


$$
\begin{pmatrix}
2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 3 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 3 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2
\end{pmatrix}
$$

Associated Laplacian matrix L: Open Chain



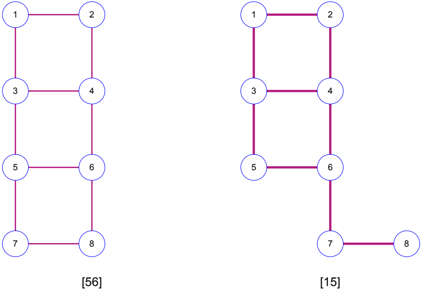




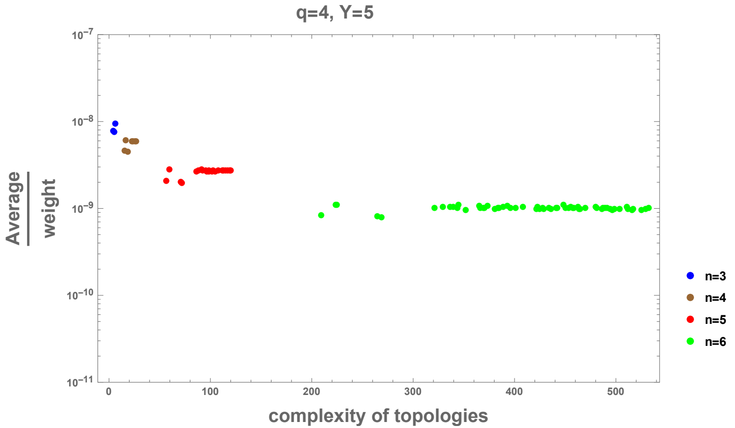
$$
\begin{pmatrix}
2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 3 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2
\end{pmatrix}
$$

Graph Complexity: Number of possible spanning trees
(Connected diagrams crossing all nodes with no loops)

Matrix Tree theorem by Kirchoff: determinant of any principal minor of L

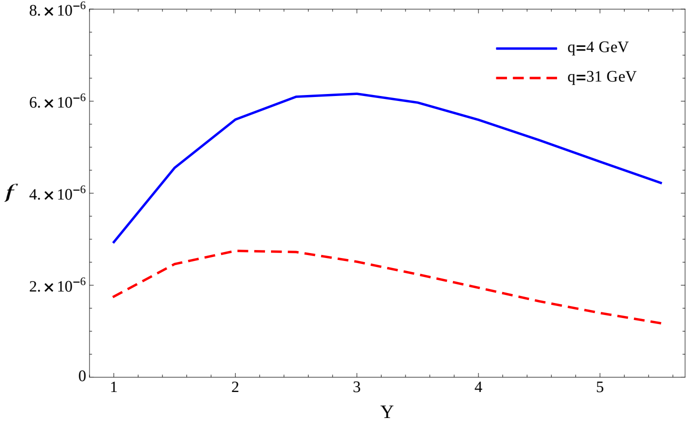


Average weight per complexity class for Reggeon webs.
Emerging scaling.



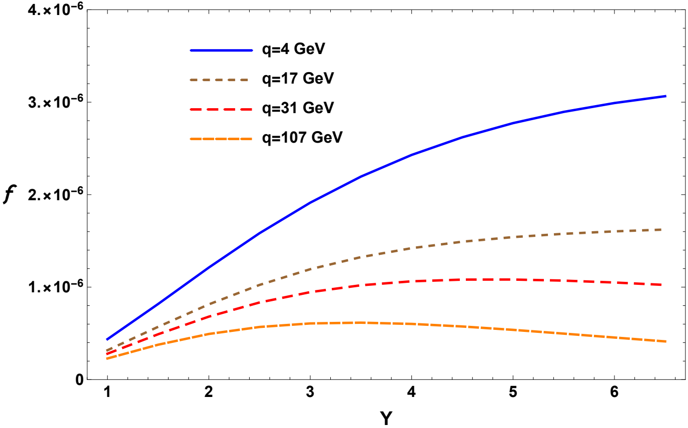
Due to integrability?

Our solution contains both Bartels-Lipatov-Vacca & Janik-Wosiek Odderons



This is the Odderon solution.

On-going work on phenomenological applications ...



This is the solution to the "adjoint Odderon"
(open spin chain) relevant in $N = 4$ SUSY amplitudes.

1. Multi-Regge limit
2. Monte Carlo event generator BFKLex
3. Collinear double logs
4. Infrared Effects
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6. CHY amplitudes in Sudakov representation

With Chachamis, Medrano & Vázquez-Mozo

Recent idea for amplitudes without Feynman diags. (Cachazo-He-Yuan):

$$\mathcal{A}_n = i\, g^{n-2} \int \frac{d^n\sigma}{\mathrm{Vol}[\mathrm{SL}(2,\mathbb{C})]} \sigma_{kl}\sigma_{lm}\sigma_{mk} \prod_{i\neq k,l,m} \delta\left(\sum_{j\neq i}^{n} \frac{2\, p_i \cdot p_j}{\sigma_{ij}}\right) I_L I_R$$

I_L carries the color traces $I_L = \sum_{\beta\in S_n/\mathbb{Z}_n} \frac{\mathrm{Tr}\left(T^{a_{\beta(1)}} T^{a_{\beta(2)}} \ldots T^{a_{\beta(n)}}\right)}{\sigma_{\beta(1)\beta(2)}\sigma_{\beta(2)\beta(3)}\cdots\sigma_{\beta(n)\beta(1)}}$.

$I_R \ = \mathrm{Pf}' M_n$ is the reduced Pfaffian of $M_n = \begin{pmatrix} M_A & -M_C^T \\ M_C & M_B \end{pmatrix}$ where

$$M_A^{ij} = \begin{cases} \frac{p_i\cdot p_j}{\sigma_{ij}} \\ 0 \end{cases},\ M_B^{ij} = \begin{cases} \frac{\epsilon_i\cdot \epsilon_j}{\sigma_{ij}} \\ 0 \end{cases},\ M_C^{ij} = \begin{cases} \frac{\epsilon_i\cdot p_j}{\sigma_{ij}} & \text{for} \quad i\neq j \\ -\sum_{k\neq i} \frac{\epsilon_i\cdot p_k}{\sigma_{ik}} & \text{for} \quad i = j \end{cases},$$

ϵ_i: i-th gauge boson polarization.$(Pf\ A)^2 = \det(A)$.
$\mathrm{Pf}' M_n$ removes k-th row, ℓ-th column, times $(-1)^{k+\ell}\sigma_{k\ell}^{-1}$.

Gravity: $I_L = \mathrm{Pf}' M_n$, $I_R = \mathrm{Pf}' M_n$.

$s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j, \qquad \sigma_{ij} = \sigma_i - \sigma_j$

$$\mathcal{A}_n = i\, g^{n-2} \int \frac{d^n\sigma}{\mathrm{Vol}[\mathrm{SL}(2,\mathbb{C})]} \sigma_{kl}\sigma_{lm}\sigma_{mk} \prod_{i\neq k,l,m} \delta\left(\mathcal{S}_i(\sigma)\right) I_L I_R$$

Support of the integral on Scattering Equations: $\mathcal{S}_i(\sigma) \equiv \sum_{j\neq i}^{n} \frac{s_{ij}}{\sigma_{ij}} = 0$

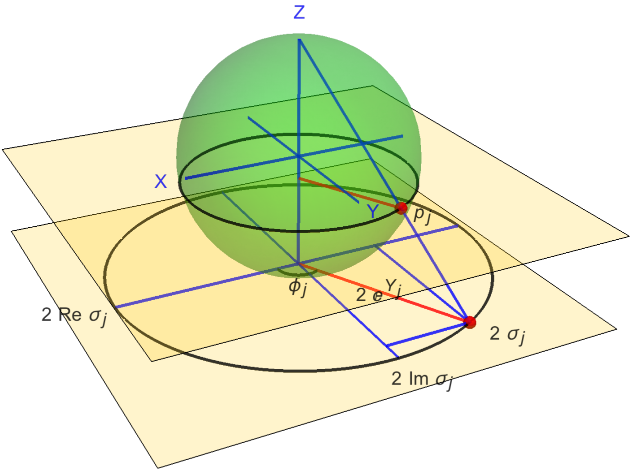
$(n-3)!$ solutions $(\sigma_1^{(i)}, \ldots, \sigma_n^{(i)})$ as n-punctured spheres.
It is a very complicated algebraic problem to find them.

Sudakov variables simplify the finding of solutions greatly.
(Chachamis, Medrano, SV, Vázquez-Mozo)

Parametrize on-shell momenta with energy & $\mathbb{S}^2$ stereographic coordinates:

$$p_j = \omega_j \left(1, \frac{\zeta_j + \bar{\zeta}_j}{1+\zeta_j\bar{\zeta}_j}, i\frac{\bar{\zeta}_j - \zeta_j}{1+\zeta_j\bar{\zeta}_j}, \frac{\zeta_j\bar{\zeta}_j - 1}{1+\zeta_j\bar{\zeta}_j}\right)$$

In dimension four this solution by Fairlie always exists $\sigma_j = \zeta_j = e^{Y_j + i\phi_j}$.



Geometric interpretation rapidity Y_j & azimuthal angle ϕ_j

$\sigma_j = \zeta_j = e^{Y_j + i\phi_j}$ on punctured sphere.

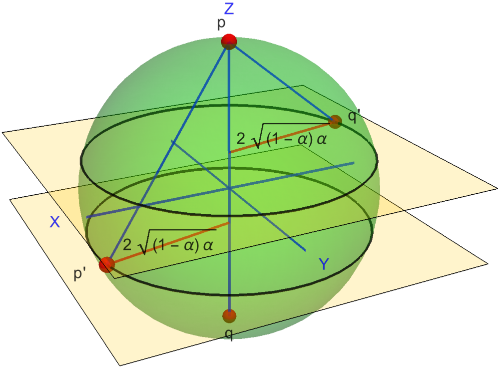
Take incoming momenta as $p = \frac{\sqrt{s}}{2}(1,0,0,1), \;\; q = \frac{\sqrt{s}}{2}(1,0,0,-1)$

Then $\sigma_p = \frac{e^{i\phi}}{\epsilon} \longrightarrow \infty, \;\; \sigma_q = -e^{Y_q + i\phi} = -\epsilon\, e^{i\phi} \longrightarrow 0$

Four-point amplitude with $p + q \to p' + q'$.

Introduce Sudakov representation

$$q_1 \equiv p - p' = \alpha\,(p - q) + \mathbf{q}_1, \quad \mathbf{q}_1 = q_1^{\perp}\,(0, \cos\theta_1, \sin\theta_1, 0)\,.$$



There is only one, (n-3)!, solution to the Scattering Equations:

$\sigma_p = \infty, \;\; \sigma_q = 0, \;\; \sigma_{p'} = -\frac{Q_1}{\alpha}, \;\; \sigma_{q'} = \frac{Q_1}{1-\alpha}$

$Q_j = \frac{q_j^{\perp}}{\sqrt{s}} e^{i\theta_j}, \;\; |Q_1|^2 = \alpha(1-\alpha)$

The four-point scalar amplitude is easy to calculate:

$\mathcal{A}_4^{\varphi^3} = \int dz_{p'} \frac{z_{pq}^2 z_{qq'}^2 z_{q'p}^2}{\left(z_{pq} z_{qq'} z_{q'p'} z_{p'p}\right)^2} \delta\left(\mathcal{S}_{p'}\right) = \int \frac{dz_{p'}}{(z_{p'} - \sigma_{q'})^2} \delta\left(\frac{s_{p'q}}{z_{p'}} - \frac{s_{p'q'}}{z_{p'} - \sigma_{q'}}\right)$

There is Jacobian which is very important:

$$\mathcal{A}_4^{\varphi^3} = \int dz_{p'} \left[z_{p'} - \frac{Q_1}{1-\alpha}\right]^{-2} \frac{Q_1^2}{s\alpha^3(\alpha-1)} \delta\left(z_{p'} + \frac{Q_1}{\alpha}\right)$$

$$= \left[\frac{\alpha^2(1-\alpha)^2}{Q_1^2}\right] \left[\frac{Q_1^2}{s\alpha^3(\alpha-1)}\right] = \frac{(\alpha-1)}{s\alpha} = \frac{1}{s} + \frac{1}{t}$$

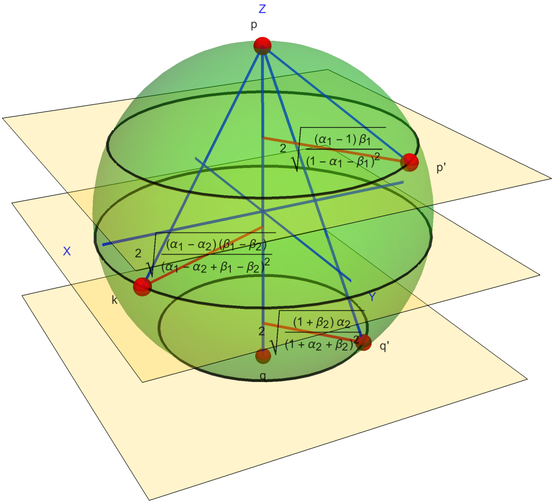
Five-point amplitude $p + q \to p' + k + q'$. Sudakov representation:

$$q_1 = p - p' = \alpha_1 p + \beta_1 q + \mathbf{q}_1,$$
$$q_2 = q' - q = \alpha_2 p + \beta_2 q + \mathbf{q}_2,$$
$$k = q_1 - q_2 = (\alpha_1 - \alpha_2)\, p + (\beta_1 - \beta_2)\, q + \mathbf{q}_1 - \mathbf{q}_2,$$

Scattering Equations with (n-3)!=2 complex conjugated solutions:

$$\sigma_{p'}^{(+)} = \sigma_{p'}^{(-)*} = \frac{Q_1 e^{-i\theta_2}}{\beta_1},$$
$$\sigma_{q'}^{(+)} = \sigma_{q'}^{(-)*} = \frac{Q_2 e^{-i\theta_2}}{1 + \beta_2},$$
$$\sigma_{k}^{(+)} = \sigma_{k}^{(-)*} = \frac{(Q_1 - Q_2) e^{-i\theta_2}}{\beta_1 - \beta_2}.$$

In Sudakov space the CHY approach is much simpler.



Punctures on the Riemann sphere for the five-particle amplitude

$$\mathcal{A}_5^{\varphi^3} = \int dz_{p'} dz_{q'}\, \delta\left(\mathcal{S}_{p'}\right) \delta\left(\mathcal{S}_{q'}\right) \frac{z_{pq}^2 z_{qk}^2 z_{kp}^2}{\left(z_{pq} z_{qq'} z_{q'k} z_{kp'} z_{p'p}\right)^2}$$

$$= \int dz_{p'} dz_{q'}\, \mathcal{J}^{-1} \delta\left(z_{p'} - \sigma_{p'}\right) \delta\left(z_{q'} - \sigma_{q'}\right) \frac{z_k^2}{z_{q'}^2 z_{q'k}^2 z_{kp'}^2} + \text{c.c.}$$

$$= \frac{2}{s^2} \text{Re} \left[\left(\frac{\sigma_{p'}}{\sigma_{q'}} \right) \frac{1}{L\widetilde{L} - R\widetilde{R}} \right]$$

$$= \frac{1}{s^2} \left[\frac{1}{\alpha_1 + \beta_1} - \frac{1}{\alpha_2 + \beta_2} + \frac{1}{(\alpha_1 + \beta_1)\beta_1} - \frac{1}{\beta_1 \alpha_2} + \frac{1}{\alpha_2(\alpha_2 + \beta_2)} \right]$$

where

$$L = \frac{\sigma_{p'k}}{\sigma_{p'q'}} \left[(\alpha_1 - 1) \frac{\sigma_{q'}}{\sigma_{p'}} + \beta_1 \frac{\sigma_{p'}}{\sigma_{q'}} \right], \quad R = \left(\frac{\sigma_{p'} \sigma_{p'k}}{\sigma_{p'q'}} \right) \frac{(1-\alpha_1+\alpha_2-\beta_1+\beta_2)(\alpha_1+\beta_1)}{(\alpha_1-\alpha_2+\beta_1)\sigma_{p'} - (1+\beta_2)\sigma_{q'}}$$

$$\widetilde{\mathcal{O}}\left(\alpha_1, \alpha_2, \beta_1, \beta_2, \theta_1 - \theta_2\right) = \mathcal{O}\left(1 - \alpha_2, 1 - \alpha_1, -1 - \beta_2, -1 - \beta_1, \theta_2 - \theta_1\right)$$

Factorization channels are encoded in the Jacobian ...

Three plays in the multi-Regge limit:

Jet production, tampering with the infrared & complexity in the Odderon

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