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Dilute-dense collisions at high energies:

Lipatov's high energy effective action and the Color Glass Condensate formalism

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OUTLINE

1. Lipatov's high energy effective action
2. A new type of gluon-gluon-reggeized gluon vertex
3. The reggeized gluon and the shockwave
4. Leading order B-JIMWLK evolution
5. Conclusion

theoretical descriptions in the high energy limit:

2 alternatives

a) t-channel perspective ~ BFKL
(unintegrated gluon densities)

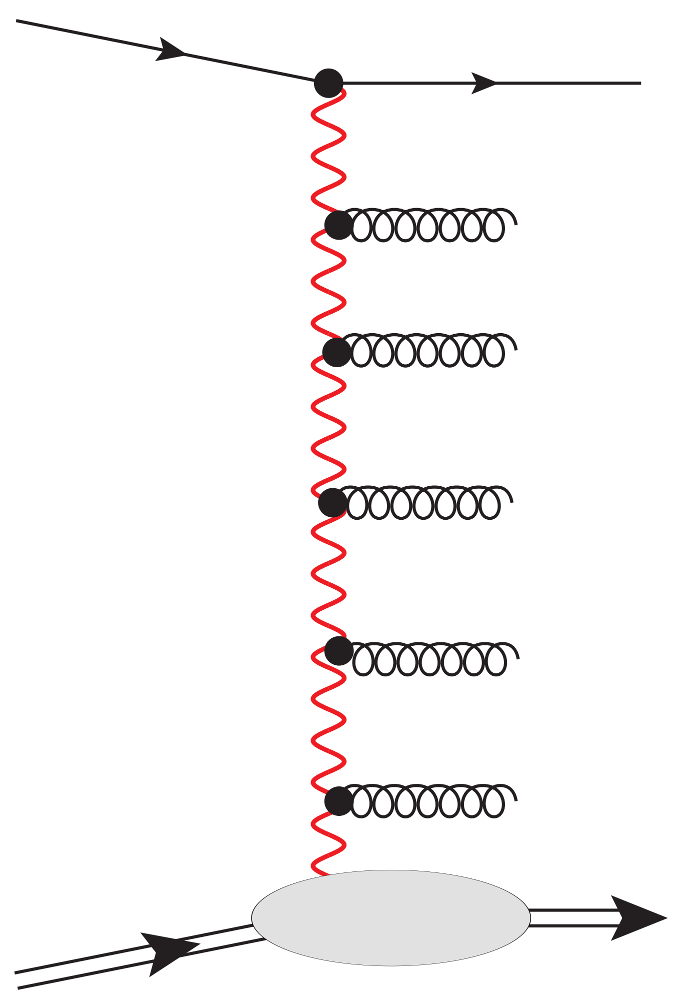
based on

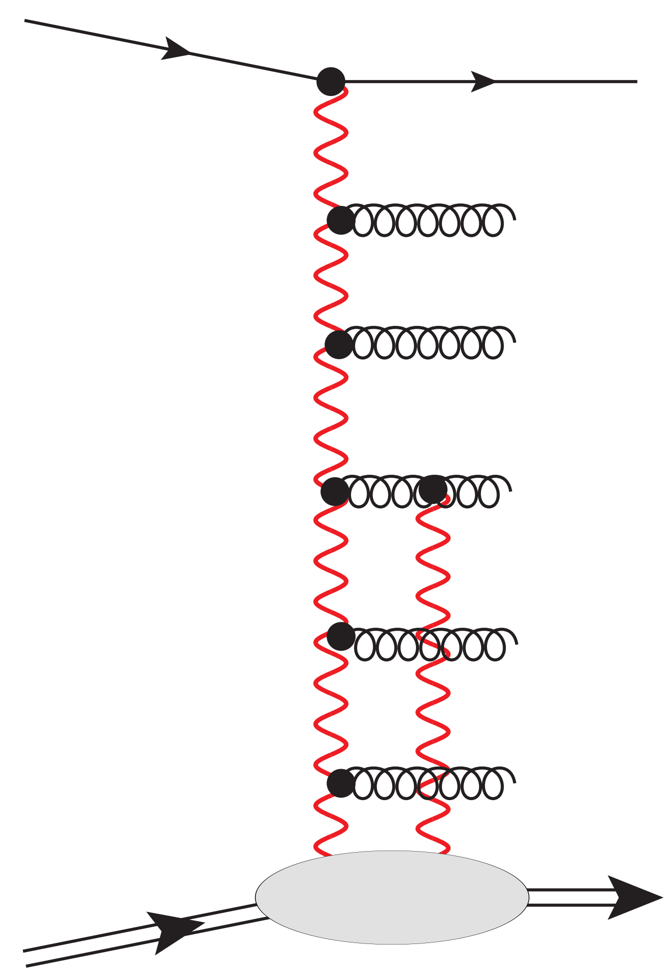
- gluon reggeization

$$\boxed{\mathcal{A}(s,t) \sim s^{1+\omega(t)}}$$

- effective (gauge invariant) production vertices

= original derivation of BFKL evolution





theoretical descriptions in the high energy limit:

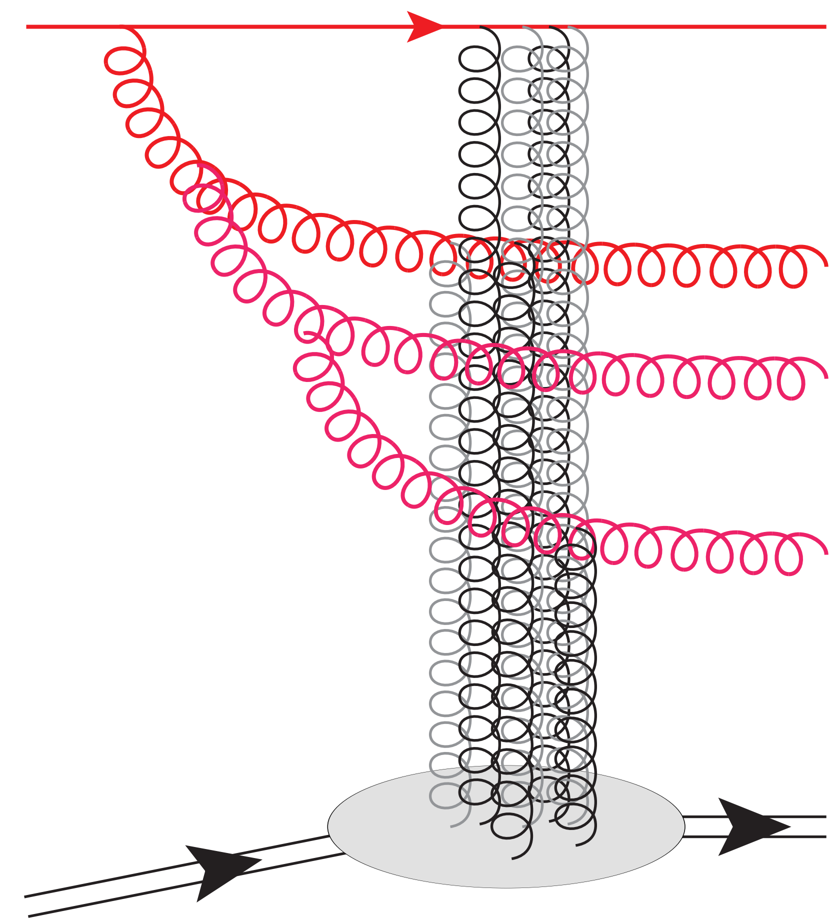
2 alternatives

a) t-channel perspective ~ BFKL
   (unintegrated gluon densities)

b) s-channel perspective ~ dipole picture

based on

- propagators which resum strong gluing background field to all orders
- path integral average over background field configurations



basis of derivation of JIMWLK/BK evolution equation

relation non-trivial

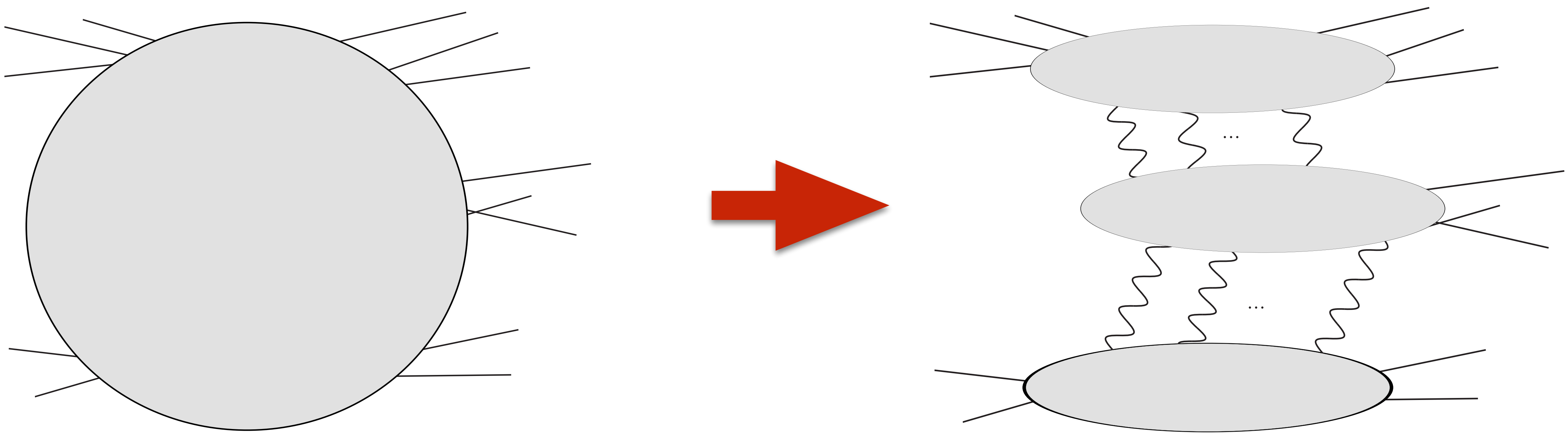
- gluon Regge trajectory *ω(t)* re-obtained relatively late from JIMWLK evolution [Caron-Huot, 1309.6521].
- first: BFKL evolution in coordinate space = low density limit of JIMWLK/BK evolution [Jalilian-Marian, Kovner, Leonidov, Weigert, NPB 504 415 (1997)]
- later on: BKP evolution, triple Pomeron vertex, … [Bartels, Lipatov, Vacca, hep-ph/0404110] [Chirilli, Szymanowski, Wallon,1010.0285] [Ayala, Cazaroto, Hernandez, Jalilian-Marian; 1408.3080]
- in general: very similar structure, but immediate one-to-one correspondence not directly obvious

essential difficulty

- traditional BFKL calculations: high energy limit of QCD scattering amplitudes
- heavy use of unitarity & renormalizability of theory, analyticity of scattering amplitudes + *s*-channel bootstrap
- similar for extensions (BKP, triple Pomeron vertex)
- Color Glass Condensate formalism/Balitsky-JIMWLK evolution = an effective theory $\leftrightarrow$ QCD correlators in presence of a background field (often in light-cone gauge)

an action formalism for reggeized gluons: Lipatov's high energy effective action [Lipatov; hep-ph/9502308]

basic idea:



correlator with regions localized in rapidity, significantly separated from each other

factorize using auxiliary degree of freedom = the reggeized gluon

- idea: factorize QCD amplitudes in the high energy limit through introducing a new kind of field: *the reggeized gluon $A_\pm$* (conventional QCD gluon: v_μ)

kinematics (strong ordering in light-cone momenta between different sectors): $\partial_+ A_-(x) = 0 = \partial_- A_+(x).$

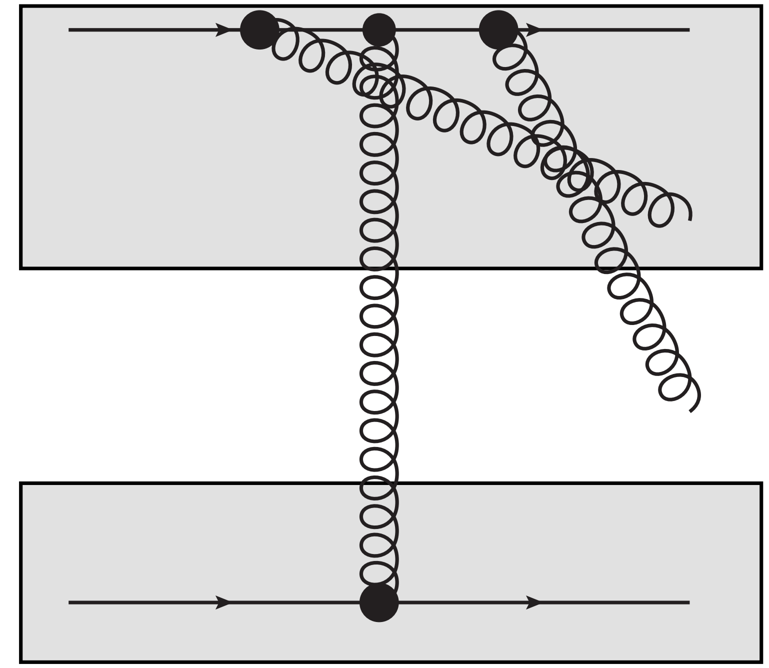
unconventional underlying idea:

- reggeized gluon globally charged under SU(NC) $A_\pm(x) = -it^a A^a_\pm(x)$

- but invariant under local gauge transformation

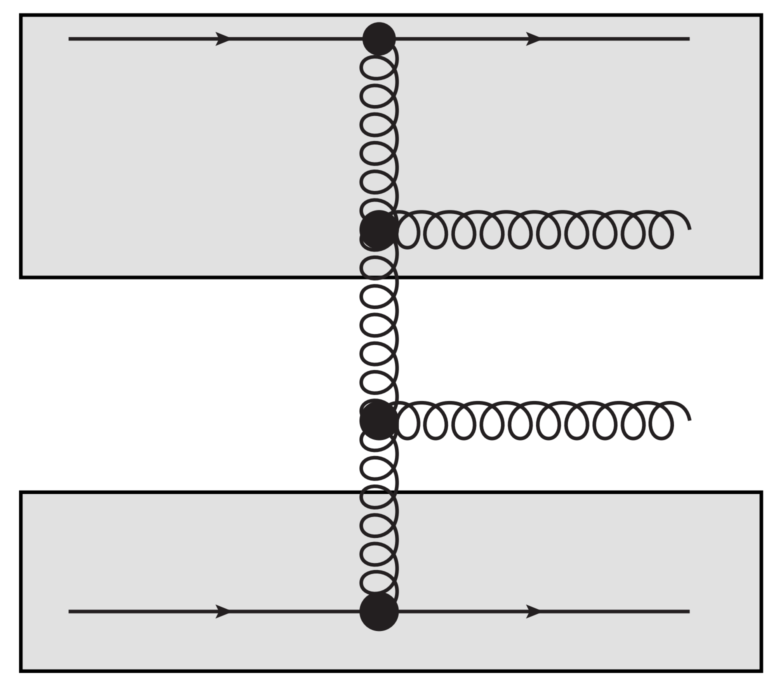
$$\delta_{\mathrm{L}} v\mu = \frac{1}{g}[D_\mu, \chi_L] \qquad \text{vs.} \qquad \delta_{\mathrm{L}} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

→ gauge invariant factorization of QCD correlators underlying idea:



- integrate out specific details of (relatively) fast +/- fields
- description in sub-amplitude local in rapidity: QCD Lagrangian + universal eikonal factor

$$T_{\pm}[v_{\pm}] = -\frac{1}{g}\partial_{\pm}\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^{\pm}} dx'^{\pm} v_{\pm}(x')\right)$$



- effective field theory for *each* local rapidity cluster

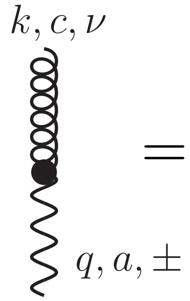
$$\boxed{S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}}$$

$$\boxed{S_{\text{ind.}} = \int \mathrm{d}^4x \left\{\mathrm{tr}\left[\left(T_{-}[v(x)] - A_{-}(x)\right)\partial_{\perp}^2 A_{+}(x)\right] + \left[\text{"} + \text{"} \leftrightarrow \text{"} - \text{"}\right]\right\}.}$$

eikonal factor = special

$$T_{\pm}[v_{\pm}] = -\frac{1}{g}\partial_{\pm}\mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^{\pm}} dx'^{\pm} v_{\pm}(x')\right)$$

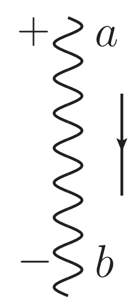
an ∞ # of gluons in the reggeized gluon



$$= \frac{-i}{2}\boldsymbol{q}^2\delta^{ac}(n^{\pm})^{\nu},$$

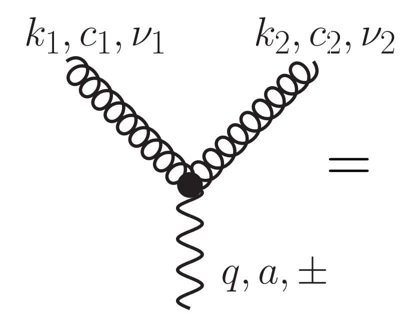
$$k^{\pm} = 0.$$

(a)



$$= \delta^{ab}\frac{2i}{\boldsymbol{q}^2}\ ,$$

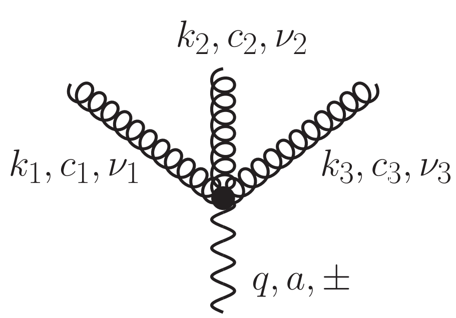
(b)



$$= \frac{g}{2}f^{c_1 c_2 a}\frac{\boldsymbol{q}^2}{k_1^{\pm}}(n^{\pm})^{\nu_1}(n^{\pm})^{\nu_2},$$

$$k_1^{\pm} + k_2^{\pm} = 0,$$

(c)



$$= \frac{ig^2}{2}\boldsymbol{q}^2\left(\frac{f^{a_3 a_2 e}f^{a_1 e a}}{k_3^{\pm}k_1^{\pm}} + \frac{f^{a_3 a_1 e}f^{a_2 e a}}{k_3^{\pm}k_2^{\pm}}\right)(n^{\pm})^{\nu_1}(n^{\pm})^{\nu_2}(n^{\pm})^{\nu_3},$$

$$k_1^{\pm} + k_2^{\pm} + k_3^{\pm} = 0.$$

“induced” vertices

(d)

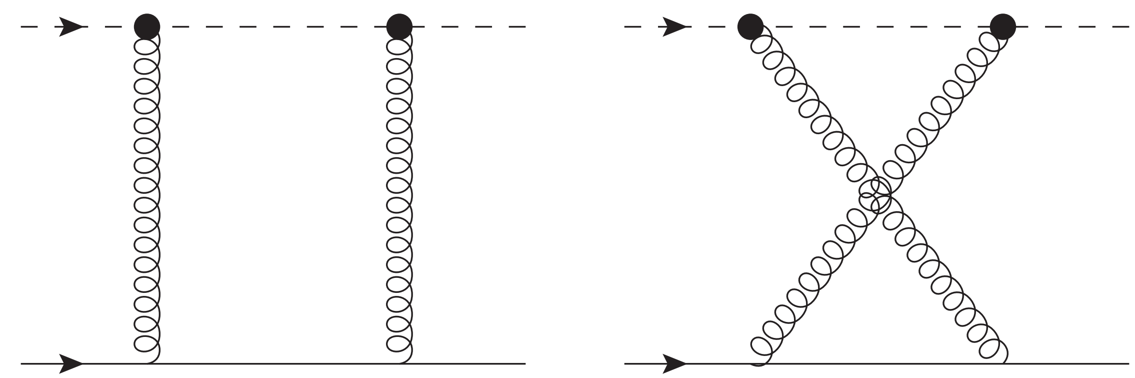
eikonal factor = special

$$T_\pm[v_\pm] = -\frac{1}{g}\partial_\pm \mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^\pm} dx'^{\pm} v_\pm(x')\right)$$

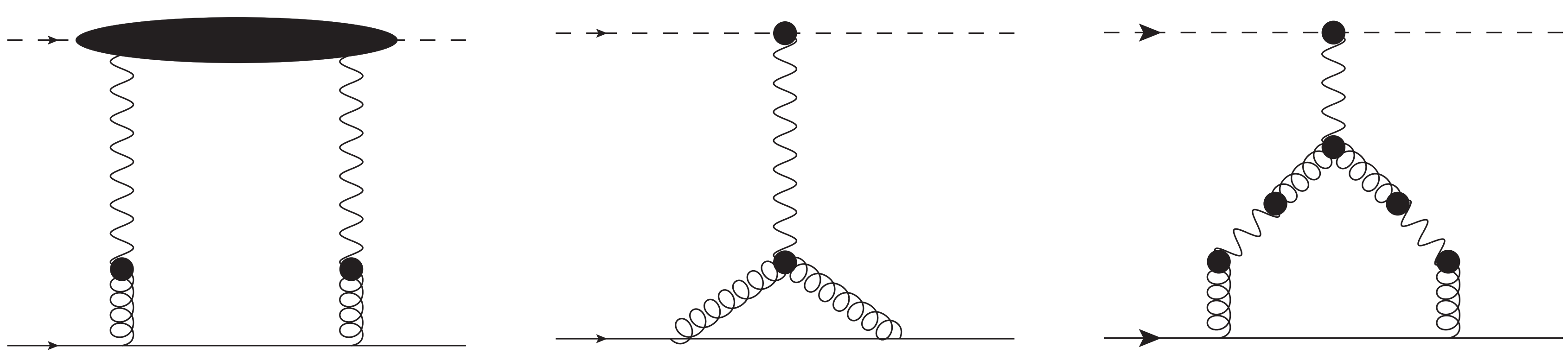
- Combination with reggeized gluon field in action → projected on SU(N_c) color octet
- real production: immediately purely anti-symmetric terms ~ SU(N) structure constants
- loop corrections: need pole-prescription → use explicit projection on purely anti-symmetric terms **[MH, 1112.4509]**
- symmetric terms: multiple reggeized gluon exchange

example: “scattering” of a Wilson line on a quark

- only multiple (=2) gluon exchange contr.
- QCD: 2 diagrams

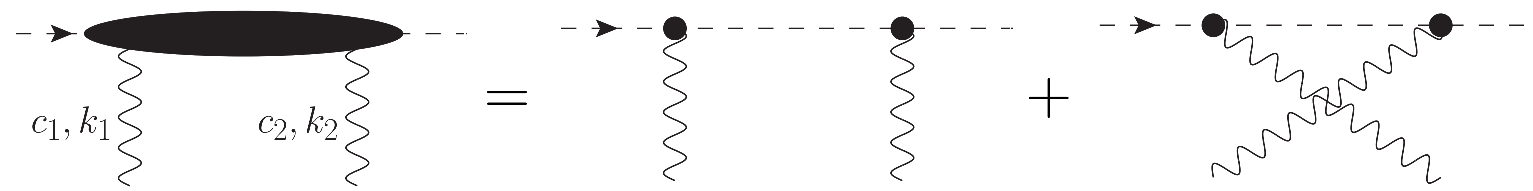


effective action:



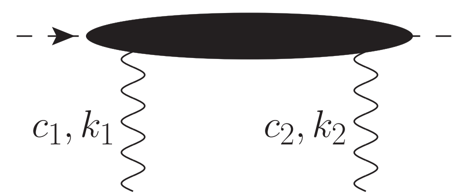
decompose Wilson line into 3 contributions (the last one =0)

coupling of 2
reggeized gluons
to a Wilson line



$$= igt^{c_2}\frac{i}{k_1^- + i\epsilon}igt^{c_1} + igt^{c_1}\frac{i}{k_2^- + i\epsilon}igt^{c_2}\,.$$

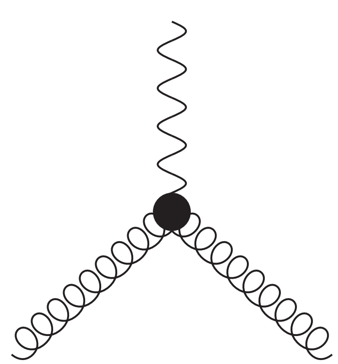
$$\int \frac{dk_1^-}{2\pi}\int \frac{dk_2^-}{2\pi}\, 2\pi\delta(k_1^- + k_2^-)$$



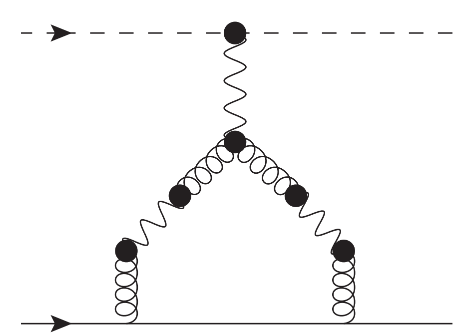
$$= (ig)^2 \cdot \frac{1}{2}\left(t^{c_1}t^{c_2} + t^{c_2}t^{c_1}\right)$$

$$= (ig)^2 S_2(12)\,.$$

symmetric color tensor
after integration

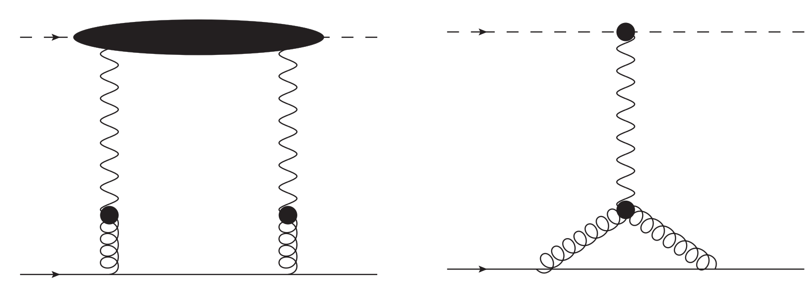


$\Rightarrow$

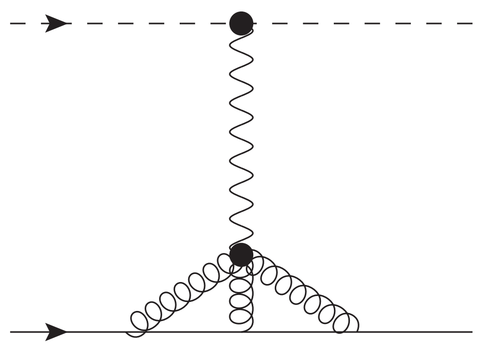


vanishes

by construction:
anti-symmetric
color



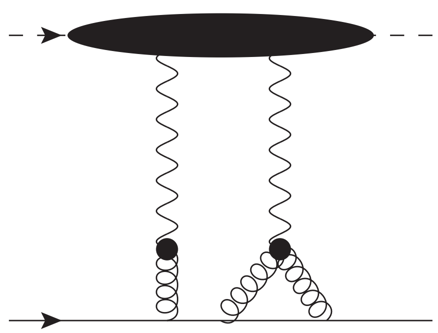
symmetric + anti-symmetric = complete

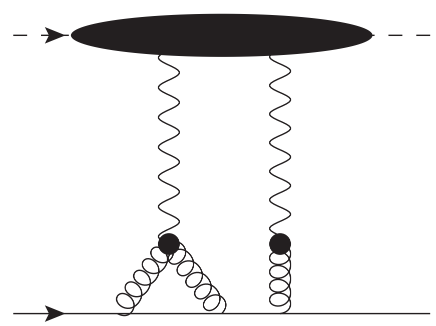


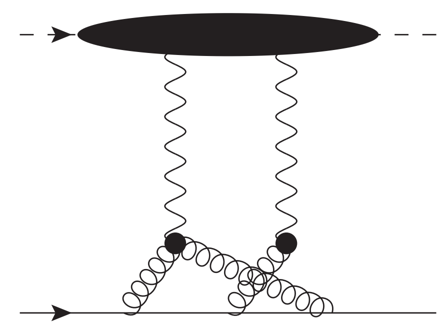
“[[3,2]1]” + “[[3,1],2]”
double anti-symmetric
(by construction)

$$[i, j] \equiv [t^{c_i}, t^{c_j}]$$

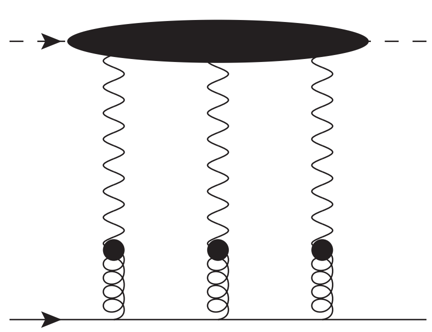
$$S_n(1 \ldots n) \equiv \frac{1}{n!} \sum_{i_1,\ldots,i_n} t^{c_{i_1}} \cdots t^{c_{i_n}}$$







S_2(1[2,3]), S_2([1,2]3]),
S_2([1,3]2)
mixed symmetry



S_3(123) - symmetric

- pole-prescription for eikonal factor = minimal sector required for gauge invariance
- **universal:** commutator does NOT depend on the representation of Wilson line

Results obtained so far from Lipatov's effective action

- high energy evolution (gluon trajectory, BFKL kernel, …) = coefficient of rapidity singularities
- various studies address multiple reggeized gluon exchange → agreement with established results
- took a while, now we know how to use Lipatov's action for loop calculations (up to 2 loop)
  → correct, since agrees with high energy limit of exact QCD scattering amplitudes

[Braun, Salykin, 1702.04796], [Braun, Vyazovsky, Podnyakov, Salykin; 1306.3583, 1502.03152]; [Braun, Vyazovsky; 1601.03469], [Braun, Lipatov, Salykin, Vyazovski; 1103.3618]

[MH, 0908.2576]

[MH, Sabio Vera;1110.6741]

[Chachamis, MH, Madrigal, Sabio Vera; 1202.064, 1212.4992, 1307.2591]

[MH, Madrigal, Murdaca, Sabio Vera; 1404.2937, 1406.5625, 1409.6704]

[Bartels, Fadin, Lipatov,Vacca; 1210.0797]

Lipatov's effective action & the CGC formalism

- idea to compare both emerged very early

[Jalilian-Marian, Kovner, Leonidov, Weigert; NPB504, 415 (1997)]

- attempts mainly on the level of effective Lagrangians

[Hatta; hep-ph/0607126]

[Bondarenko, Lipatov,Pozdnyakov, Prygarin;1706.0027, 1708.05183]

[Bondarenko, Zubkov;1801.08066]

- here: pragmatic approach: compare results for scattering amplitudes & ***propagators***

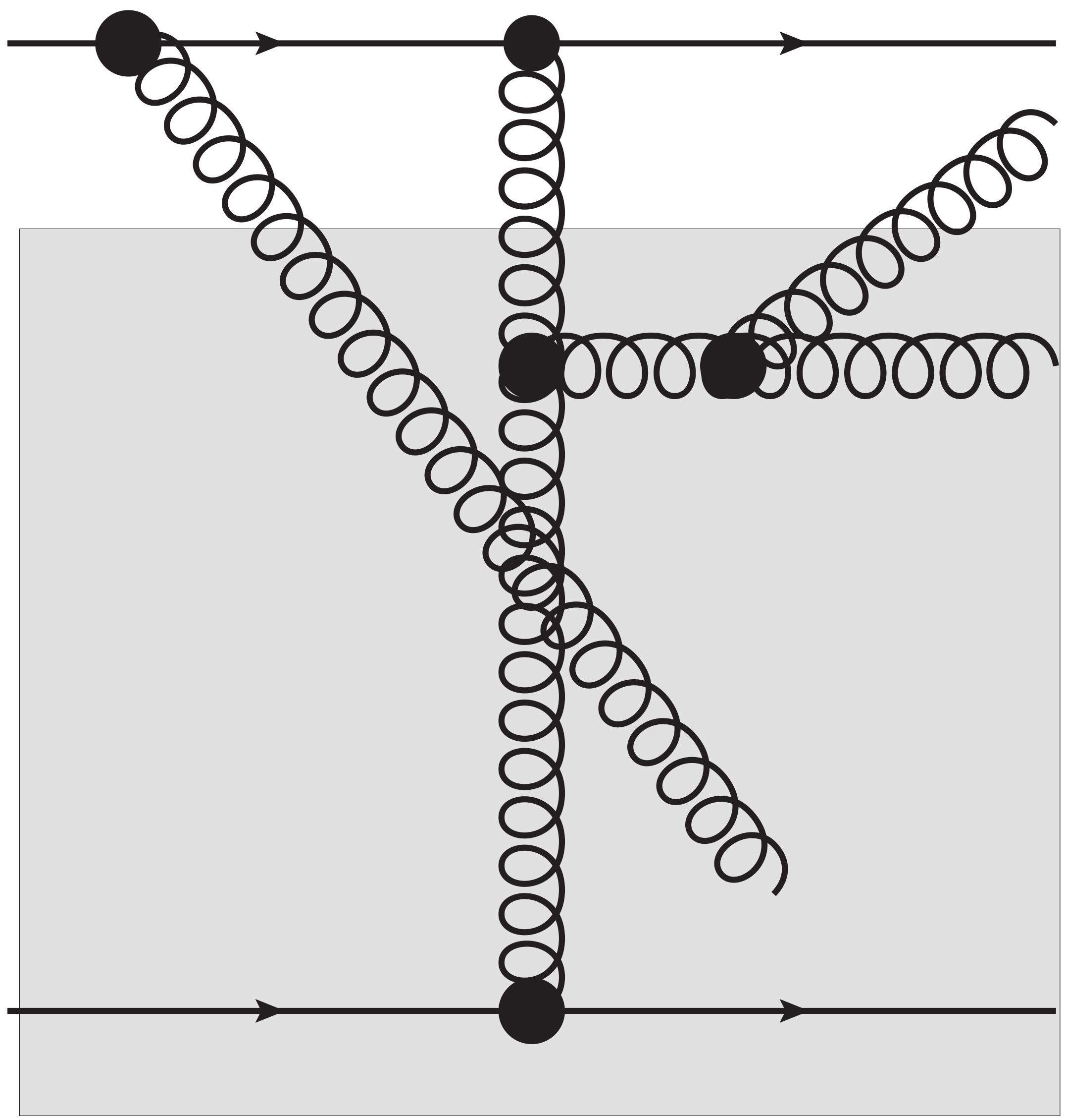
[MH, 1802.06755]

proposal: calculate the correlators for scattering of a perturbative (=dilute) projectile one a dense target

- know how to do this within the CGC-formalism (= use propagators which resum strong background field $b_+ \sim 1/g$ into Wilson lines)

- high energy effective action: study effective action for quasi-elastic scattering, assuming a strong reggeized gluon field $gA_+ \sim 1$

$$S_{\text{eff}}^{\text{q.e.}} = S_{\text{QCD}} + S_{\text{ind.}}^{\text{q.e.}} \qquad S_{\text{ind.}}^{\text{q.e.}} = \int d^4x \, \text{tr}\left(\{T_-[v] - A_-(x)\} \, \partial^2 A_+(x)\right]$$

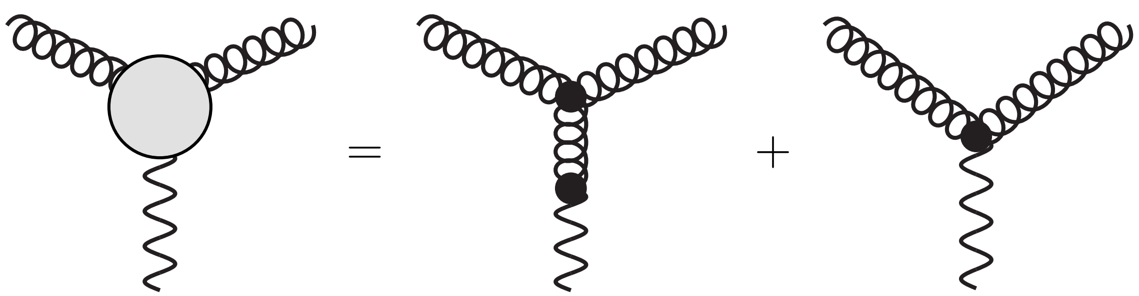


- quasi-elastic scattering = integrate out fields only from one side

- task: resum interaction of QCD fields with ∞ # of reggeized gluon fields

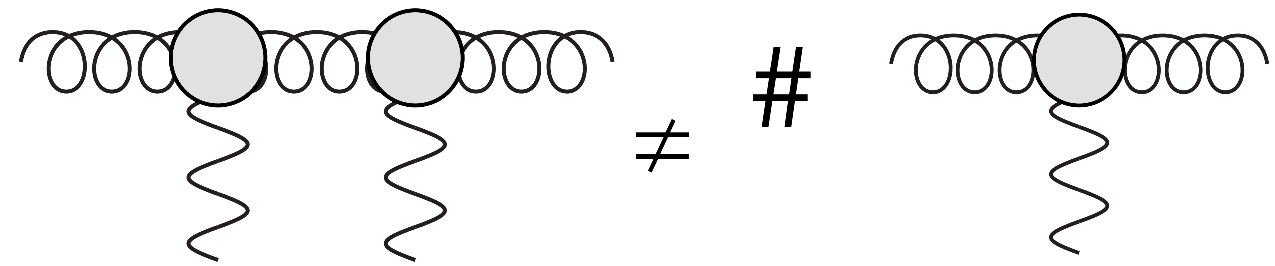
quarks: relatively straightforward

gluon: at first a mess ….

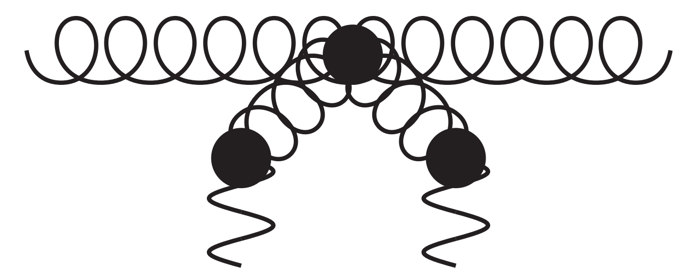


original idea: use GGR (gluon-gluon-reggeized gluon) vertex & iterate

essential problem:
no iteration/
"reggeization"



one term (internal momenta)
cancels against 4 gluon vertex



terms (external momenta):
vanish for real gluon

$$\sim \quad p^\mu (n^\pm)^\nu - \frac{p^2}{p^\pm}(n^\pm)^\mu (n^\pm)^\nu$$

off-shell: non-trivial cancelation against other diagrams
→ see *e.g.* calculation of Triple Pomeron Vertex

they way out:
a proposal made by Lipatov already in 1995

$$S_{\mathrm{ind.}} = \int \mathrm{d}^4 x \left\{ \mathrm{tr}\left[\left(T_-[v(x)] - A_-(x) \right) \partial_\perp^2 A_+(x) \right] + \left[" + " \leftrightarrow " - " \right] \right\}.$$

$$T_\pm[v] = = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \dots$$

gives direct transition gluon v_- to reggeized gluon A_+

get rid of it
through shift

$$v_\pm \to V_\pm = v_\pm + A_\pm$$

problem: gauge
transformation properties
are not consistent

$$\delta_{\mathrm{L}} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

$$\delta_{\mathrm{L}} V_\mu = \frac{1}{g}[D_\mu, \chi_L],$$

$$V^\mu(x) = v^\mu(x) + \frac{1}{2}(n_-)^\mu B_+[v_-]$$

better use a special parametrization of the gluon field

= sort of: a gauge rotation of the reggeized gluon field $A_\pm$

$$B_\pm[v_\mp] = U[v_\mp]A_\pm U^{-1}[v_\mp]$$

Wilson line operator

$$U[v_\pm] = \frac{1}{1+\frac{g}{\partial_\pm}v_\pm}$$

and its inverse …

$$U^{-1}[v_\pm] = 1 + \frac{g}{\partial_\pm}v_\pm$$

<u>program:</u>

- shift $$v^\mu \to V^\mu = v^\mu + \frac{1}{2}(n_-)^\mu\left(A_+ + [A_+, \frac{g}{\partial_-}v_-]\right) + \mathcal{O}(v_-^2).$$
- expand Lagrangian up to quadratic order in quantum fields $\boldsymbol{v}^\mu$, $\boldsymbol{\psi}$, but all orders in $gA_+ \sim 1$

a) transformation properties

$$V^\mu(x) = v^\mu(x) + \frac{1}{2}(n_-)^\mu B_+[v_-]$$

shifted field transforms like gauge field → consistent transformation properties

$$\delta V_\pm = [D_\pm, \chi] + [gB_\pm, \chi] = [D_\pm + gB_\pm, \chi]$$

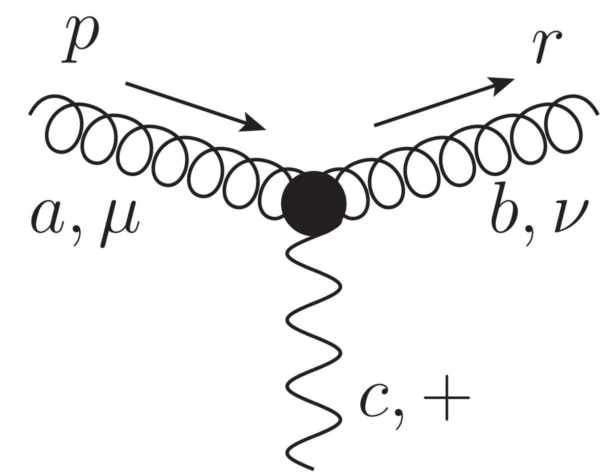
this is NOT the case for $v_\pm \to V_\pm = v_\pm + A_\pm$

since

$$\delta_{\mathrm{L}} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

$$\delta_{\mathrm{L}} V_\mu = \frac{1}{g}[D_\mu, \chi_L]$$

b) a new gluon-gluon-reggeized gluon vertex



$$\Gamma_+^{\nu\mu}(r,p) = p^+ g^{\mu\nu} - (n^+)^\mu p^\nu - (n^+)^\nu r^\mu + \frac{r \cdot p}{p^+}(n^+)^\mu (n^+)^\nu$$

- already written down by Lipatov in 1995
- good properties: current conservation

$$r_\nu \cdot \Gamma_+^{\nu\mu}(r,p) = 0 = \Gamma_+^{\nu\mu}(r,p) \cdot p_\mu$$

- properties Lipatov didn't like: violates for individual Feynman diagrams Steinmann relations

argue: shifted version of a theory which respects Steinmann relations → OK for physical observables

c) for us: important property

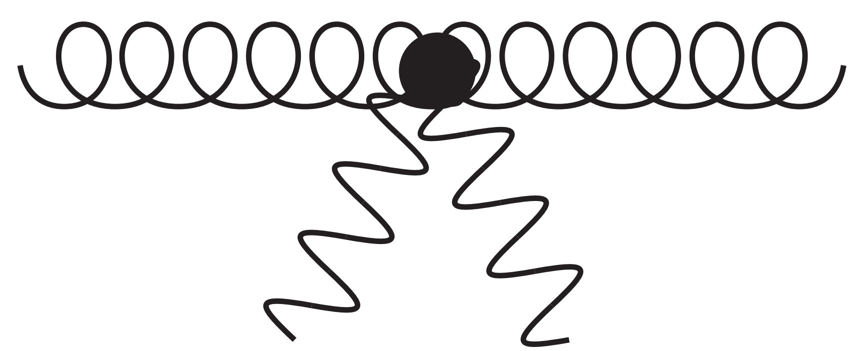
$$n_\nu^+ \cdot \Gamma_+^{\nu\mu}(r,p) = 0 = \Gamma_+^{\nu\mu}(r,p) \cdot n_\mu^+$$

$$\Gamma_+^{\nu\alpha}(r,k) \cdot (-g_{\alpha\alpha'}) \cdot \Gamma_+^{\alpha'\mu}(k,p) = -p^+ \Gamma_+^{\nu\mu}(r,p)$$

- n reggeized gluons = 1 reggeized gluon × factor ~ "reggeization"/iteration
- technical details aside: allows to sum up ∞ # of reggeized gluons into a Wilson line of reggeized gluons
- works also for other gauges than "Feynman gauge"

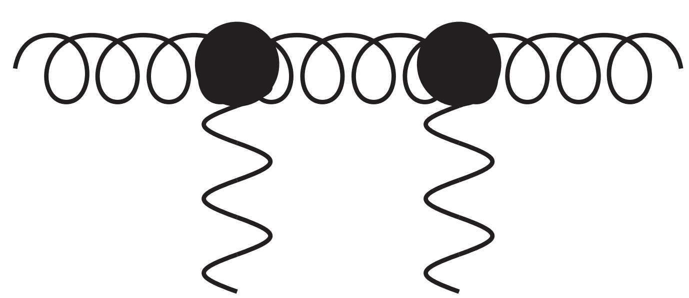
- interaction of *n* gluons with a reggeized gluon ($n>2$) is $O(g^2)$ → re-appears at NLO and beyond; for the time being not relevant (care about LO)

- 2 gluon + *n* reggeized gluons vanish



=0

- iteration works



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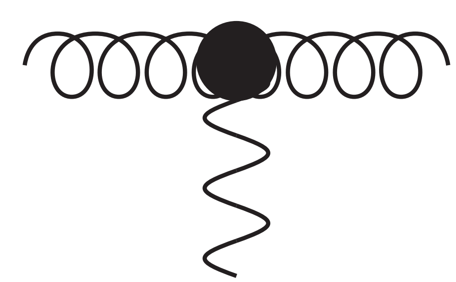
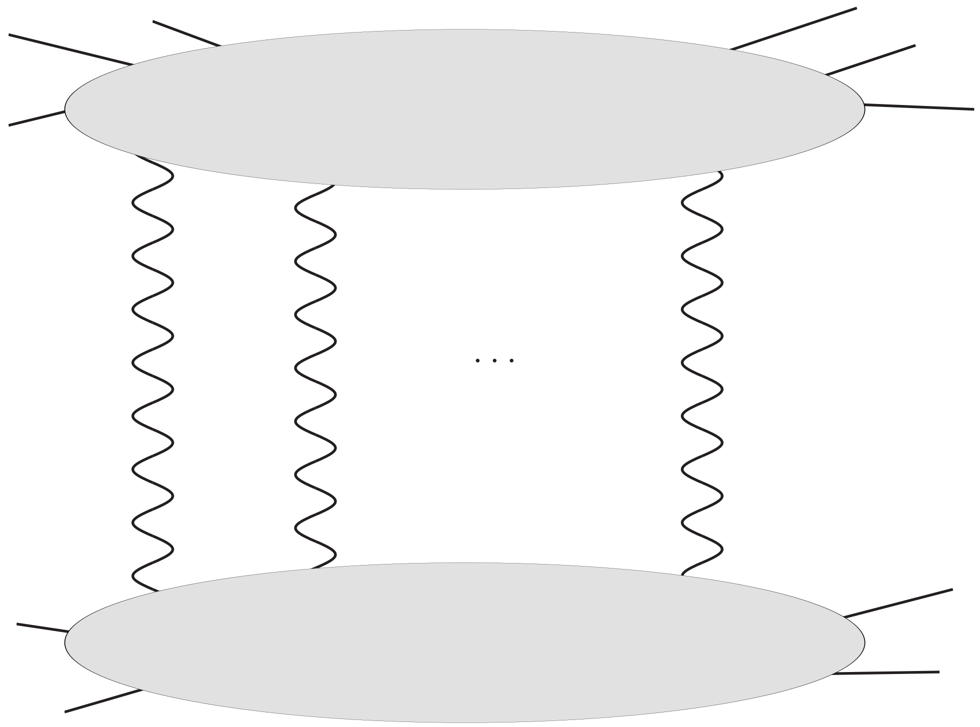


- still missing: summation into Wilson line requires shock wave form→ is this case?

from the reggeized gluon to the shock wave …



reggeized gluon

$\updownarrow$

interaction between local clusters, significantly separated in rapidity $y=\ln(k^+/k^-)$

- reggeized gluon field obeys kinematic constraints

$$\partial_- A_+(x) = 0,$$
$$\partial_+ A_-(x) = 0,$$

- implies parametrization, $x_0^\pm$ arbitrary constants

$$A_+(x) = A_+(x_0^-, \boldsymbol{x}, x^+)$$
$$A_-(x) = A_-(x_0^+, \boldsymbol{x}, x^+)\,,$$

$$\langle A_+(x)A_-(y)\rangle = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{2i}{\boldsymbol{q}^2}$$

the 4dim propagator

$$= \frac{1}{2}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \int \frac{dq^+}{2\pi} e^{-iq^+(x_0^- - y^-)/2} \int \frac{dq^-}{2\pi} e^{-iq^-(x^+ - x_0^+)/2} e^{i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} \frac{2i}{\boldsymbol{q}^2}$$

$$= 4\delta(y^- - x_0^-)\delta(x^+ - x_0^+) \cdot \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} \frac{i}{\boldsymbol{q}^2}.$$

…. relates to 2dim propagator

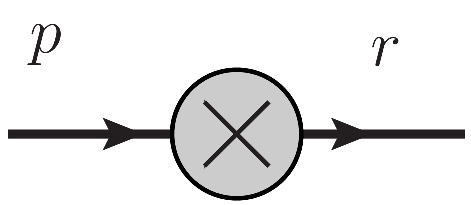
$$\langle A_+(x)A_-(y)\rangle = 4\delta(x^+ - x_0^+)\delta(y^- - x_0^-) \cdot \langle \alpha(\boldsymbol{x})\alpha(\boldsymbol{y})\rangle,$$

with $\langle \alpha(\boldsymbol{x})\alpha(\boldsymbol{0})\rangle = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \frac{ie^{i\boldsymbol{q}\cdot(\boldsymbol{x})}}{\boldsymbol{q}^2}.$

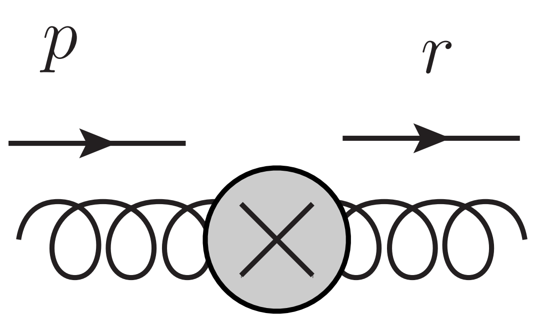
suggests shock-wave form of reggeized gluon field $A_+(x) = 2\cdot\alpha(\boldsymbol{x})\delta(x^+ - x_0^+)$

sum ∞ # of reggeized gluons into Wilson line ✓

result: vertices which resum interaction with an arbitrary # of reggeized gluon fields



$$= \tau_F(q,-r) = 2\pi\delta(p^+ - r^+)\not{n}^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})} \cdot \Big[\theta(p^+)\left[W(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[[W(\boldsymbol{z})]^\dagger - 1\right]\Big].$$



$$= \tau^{ab}_{G,\nu\mu}(p,-r) = -4\pi\delta(p^+ - r^+)\Gamma_{\nu\mu}(r,p) \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{r})} \cdot \Big[\theta(p^+)\left[U^{ba}(\boldsymbol{z}) - \delta^{ab}\right] - \theta(-p^+)\left[[U^{ba}(\boldsymbol{z})]^\dagger - \delta^{ab}\right]\Big].$$

+ interaction resumed into Wilson lines

$$U^{ab}(\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dz^+ \tilde{A}_+\right) \qquad W(\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dz^+ A_+\right)$$

Comparison to CGC expressions

- usually given for summed up propagators

$$S_F(p,q) = S_F^{(0)}(p)(2\pi)^4\delta^{(4)}(p-q) \,+\, S_F^{(0)}(p) \,\cdot\, \tau_F(p,q) \,\cdot\, S_F^{(0)}(q)\,,$$
$$G_{\mu\nu}^{ad}(p,q) = G_{\mu\nu}^{(0),ab}(p)(2\pi)^4\delta^{(4)}(p-q) \,+\, G_{\mu\alpha}^{(0),ab}(p) \,\cdot\, \tau_G^{\alpha\beta,bc}(p,q) \,\cdot\, G_{\beta\nu}^{(0),cd}(q)$$

- quark case: immediate agreement, but with Wilson line in the background field (here: light-cone gauge)

$$W[b](\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dx^+ b^{-,c}(x^+,\boldsymbol{z})t^c\right), \qquad b^-(x^+,\boldsymbol{z}) = -ib^{-,c}(x^+,\boldsymbol{z})t^c$$
$$U[b](\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dx^+ b^{-,c}(x^+,\boldsymbol{z})T^c\right), \qquad \tilde{b}^-(x^+,\boldsymbol{z}) = -ib^{-,c}(x^+,\boldsymbol{z})T^c\,.$$

→ difference matters for average over target configuration, does not matter for calculation of (projectile) correlators

gluon case

difference in the Lorentz-structure

$$p^+ g_{\nu\mu} \quad \Longleftrightarrow \quad \Gamma_{\nu\mu}(r,p)$$

background field in light-cone gauge

reggeized gluon/ Lipatov action → any gauge

good news: if light-cone gauge $v \cdot n^+ = 0$ is used ⇒ all terms, but the first vanish for propagator

$$\Gamma_+^{\nu\mu}(r,p) = p^+ g^{\mu\nu} - (n^+)^\mu p^\nu - (n^+)^\nu r^\mu + \frac{r \cdot p}{p^+}(n^+)^\mu (n^+)^\nu$$

⇒ both expressions agree for $v \cdot n^+ = 0$ light-cone gauge ✓

What about the Wilson line ?

one in gluon background field,
one in reggeized gluon field

- in both cases, we have a remaining path integral over the regarding field → "weight-functional" over target configuration
- Lipatov's effective action: there's an explicit action to calculate this average
- Note: also the Wilson line itself differs

$$W[V](x) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dx^+ V_+(x)\right)$$

generic path-ordered exponential

$$= \sum_{n=0}^{\infty}\left(\frac{-g}{2}\right)^n \prod_{i=1}^{n}\int dx_i^+ V_+(x_1)\ldots V_+(x_n)\theta(x^+ - x_1^+)\ldots\theta(x_{n-1}^+ - x_n^+).$$

$$= \sum_{n=0}^{\infty}\frac{(-g)^n}{2^n n!}\int\prod_{i=1}^{n} dx_i^+$$

rewrite as sum over all permutations

$$\left[V_+(x_1)\ldots V_+(x_n)\theta(x_1^+ - x_2^+)\ldots\theta(x_{n-1}^+ - x_n^+) + \text{permutations}\right]$$

now $V_+(x) = A_+(x) = -2i\delta(x^+ - x_0^+)\alpha^a(\boldsymbol{x})t^a$

x^+-dependence of all fields identical

fields independent of ordering

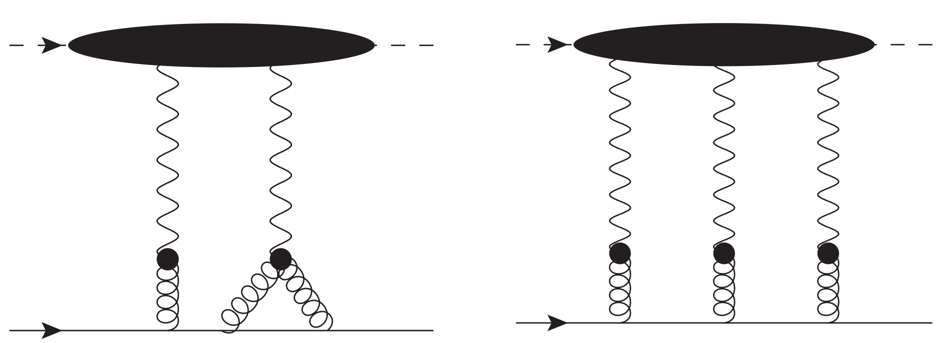
Wilson line of a reggeized gluon = a simple matrix exponential

$$W[A](x) = e^{ig\alpha^a(\boldsymbol{x})t^a}$$

objection: this would then also apply to a gluon background field in shock-wave form

in reality: such an interpretation is usually avoided → would miss certain configurations

Lipatov's effective action:



- reggeized gluon coupling to Wilson line symmetric color
- anti-symmetric configuration through induced vertices on target side

Wilson lines and reggeized gluons

proposal in **[Caron-Huot, 1309.6521]** :
reggeized gluon as the log of an adjoint Wilson line, subject to Balitsky-JIMWLK evolution

$$R^a(\boldsymbol{z}) \equiv \frac{1}{gN_c} f^{abc} \log U^{bc}(\boldsymbol{z})$$

applying this idea to Lipatov's action:

$$R^a(\boldsymbol{z}) = \frac{1}{gN_c} f^{abc} \left[ig\alpha^d(\boldsymbol{z}) T^d_{bc}\right] = \alpha^a(\boldsymbol{z}) = \frac{1}{2}\int dx^+ A^a_+(x^+, \boldsymbol{z}),$$

reggeized gluon of Caron-Hout ≡ integrated reggeized gluon of the high energy effective action

within the effective action this works also with a fundamental Wilson line!

Can we re-obtain Balitsky-JIMWLK evolution form Lipatov's action? → Yes

- quantum fluctuations of Wilson lines within Lipatov's action →Balitsky-JIMWLK evolution (so far LL)
- effective action for central production processes → color decomposition imposed of effective action gives complication
- similar problems in deriving the Triple Pomeron vertex [MH, 0908.2576])
- here: investigate quantum corrections to quasi-elastic regime;
  high energy divergence = JIMWLK-Kernel

Baltisky-JIMWLK evolution

- consider quantum fluctuations of an ensemble of Wilson lines

$$W[A_+](\boldsymbol{z}_1) \otimes \ldots \otimes W[A_+](\boldsymbol{z}_n).$$

- 1 Wilson line from action for 1-dim complex (auxiliary) scalar field $\boldsymbol{\varphi}$, charged in fundamental rep. of SU(N_C)

$$S[\varphi, V] = \int dx^+ \varphi^\dagger \left[i\partial_+ + igv_+ \right] \varphi$$

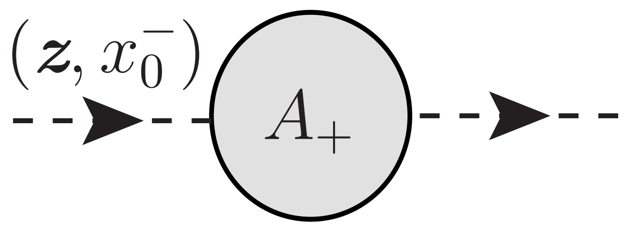
$$\delta_L \varphi = -\chi_L \varphi$$

- propagator

$$\left\langle x^- \left| \frac{1}{1 + \frac{g}{\partial_+ + \epsilon} V_+} \frac{1}{\partial_+ + \epsilon} \right| y^- \right\rangle = \mathrm{P}\exp\left(\frac{-g}{2} \int_{y^+}^{x^+} dz^+ v_+ \right).$$

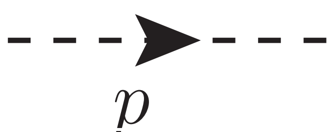
$$v^\mu \to V^\mu = v^\mu + \frac{1}{2}(n_-)^\mu \left(A_+ + [A_+, \frac{g}{\partial_-} v_-] \right) + \mathcal{O}(v_-^2).$$

shift + quantum fluctuations around reggeized gluon field → Feynman rules



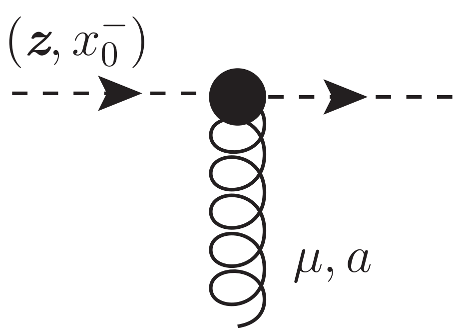
$= W[A_+](\boldsymbol{z}, x_0^-)$

the Wilson line



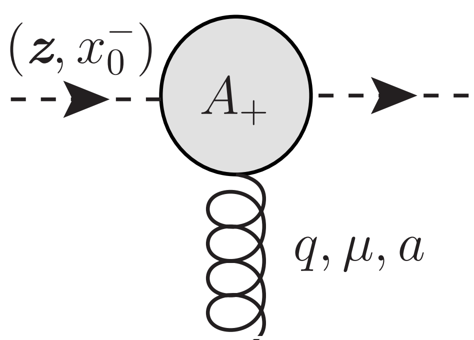
$= \dfrac{i}{p^- + i\epsilon}$

free propagator



$= igt^a(n^-)^\mu e^{-iq^+ x_0^-/2 + i\boldsymbol{q}\cdot\boldsymbol{z}}.$

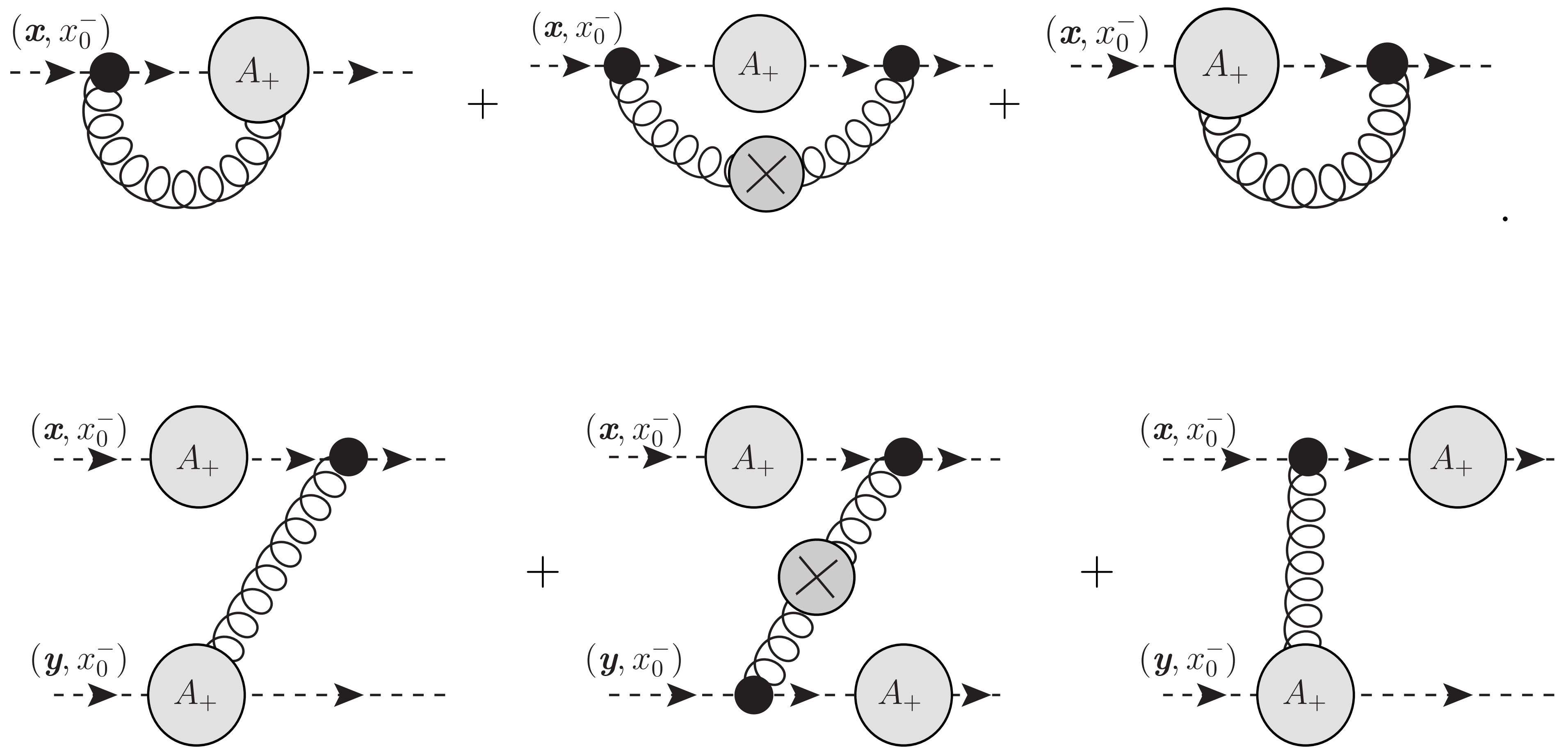
gluon - Wilson line vertex



$= \dfrac{g}{q^+}(n^+)^\mu e^{-iq^+ x_0^-/2 + i\boldsymbol{q}\cdot\boldsymbol{z}} \cdot \left[W[A](\boldsymbol{z}, x_0^-), t^a \right]$

2nd gluon - Wilson line vertex

B-JIMWLK in covariant gauge



$$\int_{s_0/\Lambda^b}^{\Lambda^a} \frac{dp^+}{p^+} = \ln\left(\frac{\Lambda_a\Lambda_b}{s_0}\right)$$

regularize high energy divergency with $\Lambda_{a,b} \to \infty$

re-obtain B-JIMWLK evolution

$$-\Lambda_a \frac{d}{d\Lambda_a}\left[W(\boldsymbol{x}_1)\otimes\ldots\otimes W(\boldsymbol{x}_n)\right] = \sum_{i,j=1} H_{ij}\left[W(\boldsymbol{x}_1)\otimes\ldots\otimes W(\boldsymbol{x}_n)\right]$$

$$H_{ij} = \frac{\alpha_s\Gamma^2(1+\epsilon)}{2\pi^2\Gamma(1-\epsilon)}\left(\frac{4}{\pi\mu^2}\right)^{\epsilon}\int d^{2+2\epsilon}\boldsymbol{z}\,\frac{(\boldsymbol{x}_i-\boldsymbol{z})\cdot(\boldsymbol{x}_j-\boldsymbol{z})}{[(\boldsymbol{x}_i-\boldsymbol{z})^2]^{1+\epsilon}[(\boldsymbol{x}_j-\boldsymbol{z})^2]^{1+\epsilon}}$$
$$\left[T^a_{i,L}T^a_{j,L}+T^a_{i,R}T^a_{j,R}-U^{ab}(\boldsymbol{z})\left(T^a_{i,L}T^b_{j,R}+T^a_{j,L}T^b_{i,R}\right)\right]$$

group generators acting to the left (L) or to the right (R)

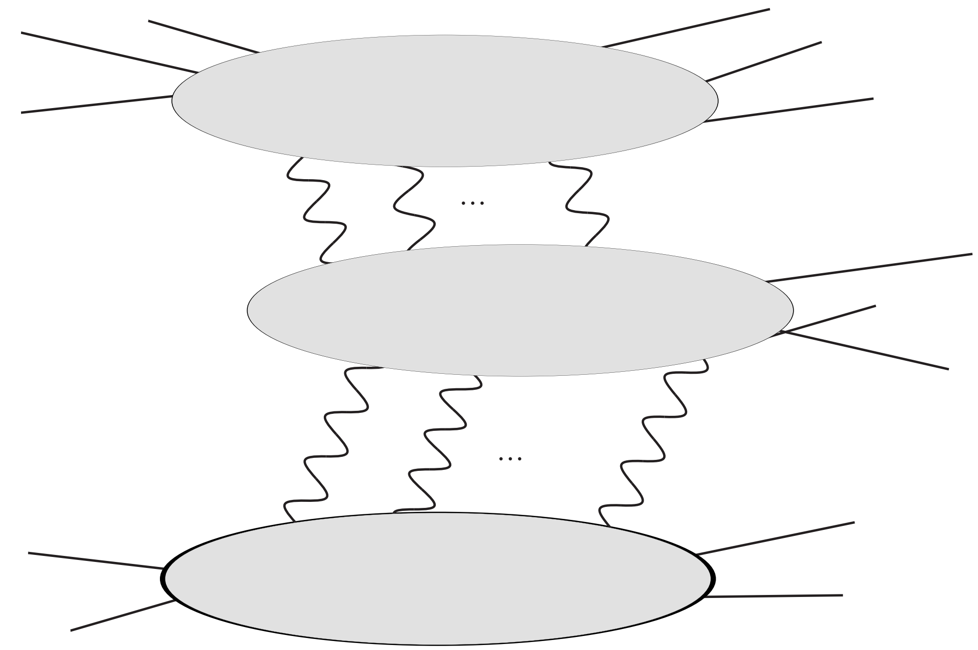
$$T^a_{L,i}[W(\boldsymbol{z}_i)] \equiv t^a W(\boldsymbol{z}_i)$$

$$T^a_{R,i}[W(\boldsymbol{z}_i)] \equiv W(\boldsymbol{z}_i)t^a$$

Conclusion

- up to quadratic order in quantum fluctuations, Lipatov's high energy effective action and standard expressions for propagators in presence of a strong background field agree
- important difference: gluon background field vs. reggeized gluon → different gauge transformation + symmetry properties
- leading order Baltisky-JIMWLK evolution has been re-obtained
- future: higher flexibility concerning choosing gauges

Conclusion



- so far paradigm: get high energy divergence from effective action for *central* production → we didn't achieve this so far

- confirmed so far the leading order, NLO remains to be verified

what about 2 strong reggeized gluon fields?

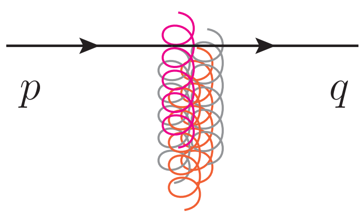
… work in progress …

Appendix

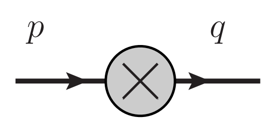
Theory: Propagators in background field

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

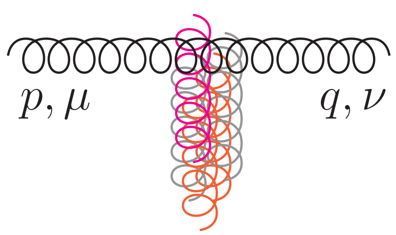


$$= (2\pi)^d \delta^{(d)}(p-q) \tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

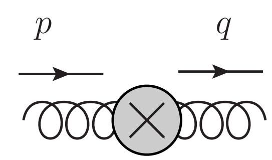


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q) \tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



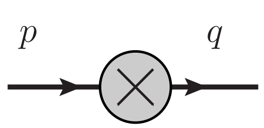
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

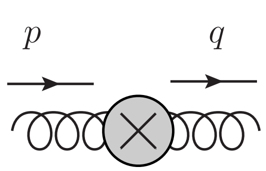


$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\, \not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right] \right\}$$

strong background field resummed into path ordered exponentials (Wilson lines)

$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$



$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right] \right\}$$