# *Towards TMD splitting functions and the corresponding system of evolution equations*

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*Based on papers:*

*M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174*

*M. Hentschinski, A. Kusina, K.K; Phys. Rev. D 94, 114013 (2016)*

*O. Gituliar, M. Hentschinski, K.K; JHEP 1601 (2016) 181*

# *(Some of) limitations of high-energy factorization framework:*

*Splitting kernel of basic evolution equations limited to small “z”*

*Gluon density does not get contribution from quark channels i.e. there is no system of equations for quarks and gluons*

*For hard processes like: $qq \rightarrow Z$ initiated by quarks the appropriate unintegrated quark densities are needed*

# So far we can

*Use CCFM includes “1/z” and “1/(1-z)” terms of splitting function, depends on hard scale but:*
*does not allow to account for finite terms like “z(1-z)”. Jumps from low z to large z. Framework limited only to gluons. Limited description of data.*

*Framework by Balitsky and Tarasov: large “z”, small “z”, moderate “z”, Sudakov, nonlinearity but:*
*limited so far to gluons only. Not obvious (at present) how to deal with it numerically.*

*Kimber,Martin, Ryskin, Watt or “parton branching” Jung at. al 1804.11152 provides full set of TMD pdfs.*
*but:*
*DGLAP based. It should be at least refitted.*

*Ciafaloni,Colferai,Staśto,Salam JHEP 0708:046,2007 provides system of equations unifying DGLAP and BFKL.*
*but*
*quark splitting functions are $k_t$ independent. This is the framework to which we should at some point compare our results.*

# *The goal*

- *go beyond DGLAP and BFKL by generalized splitting kernel*
- *coverage of all "z" regions*
- *extend evolution towards large x*
- *reproduce collinear limit (DGLAP)*
- *reproduce BFKL in low z limit*
- *$k_T$-dependent splitting functions*
- *in longer term goal: to describe large class of exclusive processes*

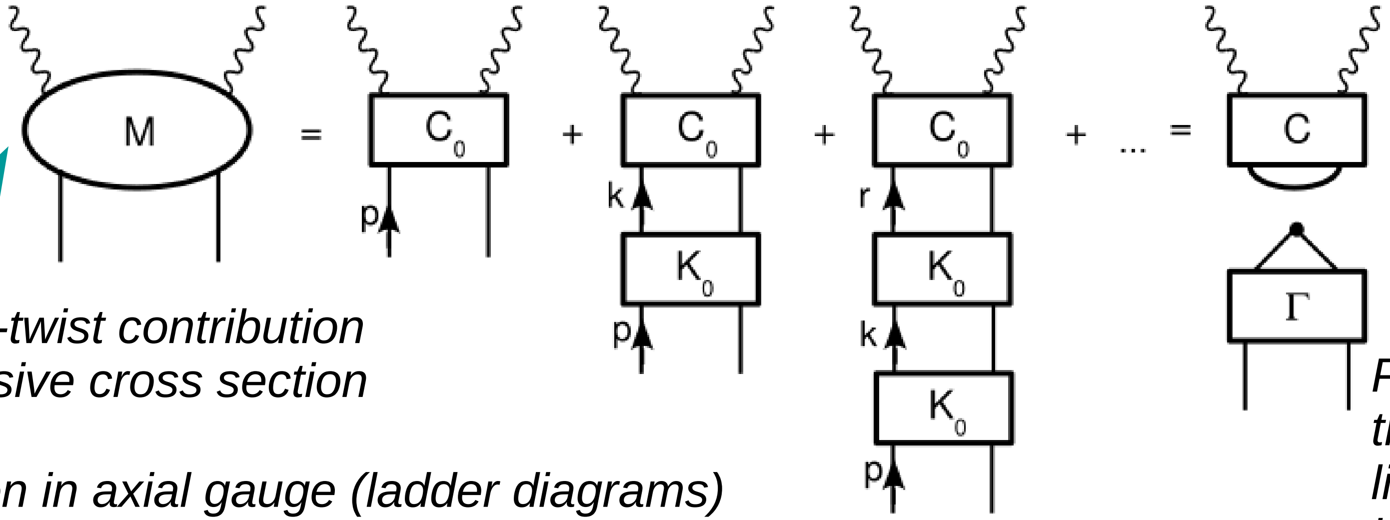
*We aim at will achieving this goals by using Curci-Furmanski-Petronzio (CFP) and Catani-Hautmann (CH) formalisms.*

*Curci, Furmanski, Petronzio Nucl. Phys. B175 (1980) 27*

*Catani, Hautmann NPB427 (1994) 475524*

# Curci-Furmanski-Petronzio method

*Factorization based on generalized ladder expansion (in terms of Two Particle Irreducible (2PI) kernels)*




*Renormalized and finite hard matrix element*

*the leading-twist contribution to the inclusive cross section*

*Process independent transition function linked to splitting kernel*

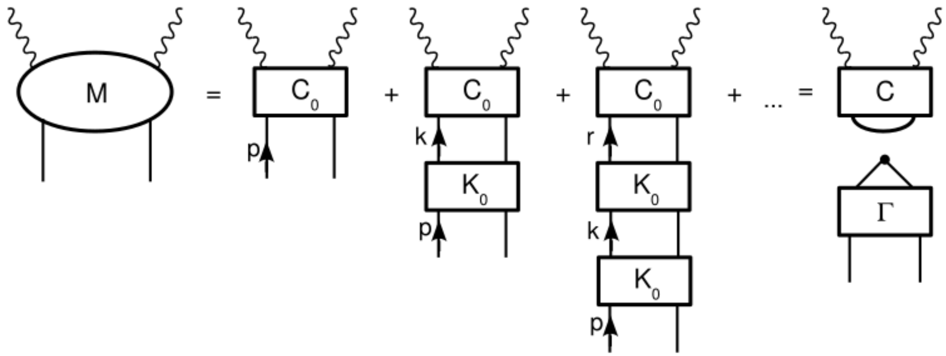
*formulation in axial gauge (ladder diagrams)*

*$C_0$ hard scattering coefficient function*

*$K_0$ 2PI kernels connected only by convolution in x this is achieved by introducing appropriate projector operators*

*How does it work….*

# Curci-Furmanski-Petronzio - factorization




notation from CFP paper
*they studied Pqq*

$$k_\mu = x_\mu + \alpha n_\mu + k_{\perp\,\mu}$$

*finite*

$$M = \frac{1}{2} C_0 K_0 \not{p} = \frac{1}{2} C_0 \mathbb{P} K_0 \not{p} + \frac{1}{2} C_0 (1-\mathbb{P}) K_0 \not{p}$$

$$\frac{1}{2} C_0 \mathbb{P} K_0 \not{p} = \frac{1}{2} C_0 \mathbb{P}_\epsilon \mathbb{P}_s K_0 \not{p}$$

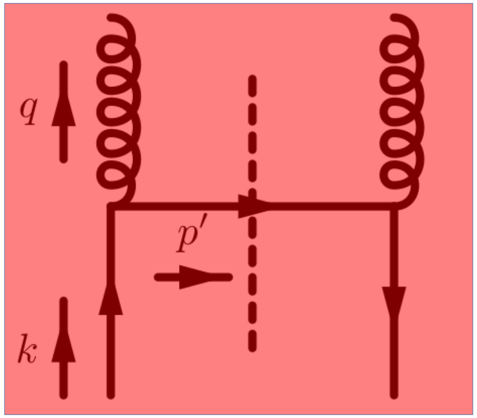
$$= \frac{1}{2} C_0 \mathbb{P}_\epsilon \mathbb{P}_{out} K_0 \mathbb{P}_{in}$$

*the projector performs integral over phase space of "k" and extracts poles*

*factorization convolution only in "x"*

$$= \int \frac{dx}{x} \frac{1}{2} C_0 \mathbb{P}_{out}|_{k^2=0} \; \Gamma\left(\frac{Q^2}{\mu_F^2}, x, \frac{1}{\epsilon}\right)$$

# Curci-Furmanski-Petronzio - splitting function




- *incoming propagators amputated*
- *contains propagator of outgoing parton + incoming on-shell*

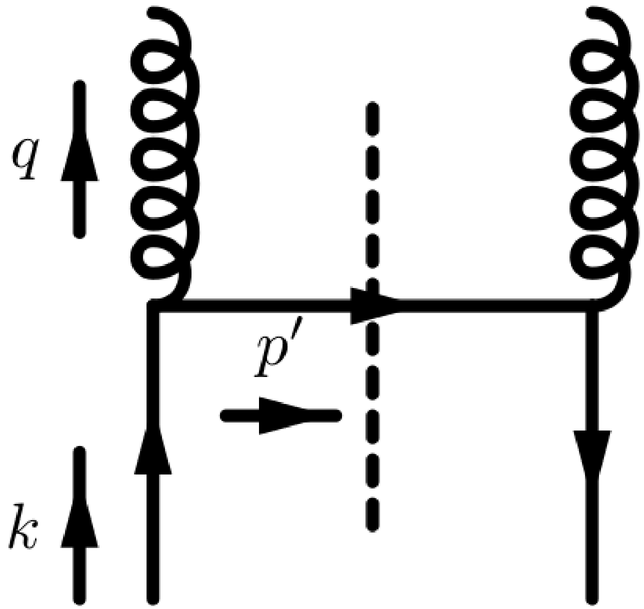
*The CFP method applied to construct splitting functions*

$$\Gamma \sim \hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = z \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 + q^2)\mathbb{P}_{j,\,\mathrm{in}} \otimes \hat{K}_{ij}^{(0)}(q,k) \otimes \mathbb{P}_{i,\,\mathrm{out}}$$

$$= \frac{\alpha_S}{2\pi\Gamma(1+\epsilon)} z \int_0^{\mu_F^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \left(\frac{e^{-\gamma_E}\boldsymbol{q}^2}{\mu^2}\right)^{\epsilon} P_{ij}^{(0)}(z;\epsilon)$$

*The method can be used to prove factorization and to derive evolution equations*

# Generalization to HEF kinematics




$$q^\mu = x p^\mu + q_\perp^\mu + \frac{q^2 + \boldsymbol{q}^2}{2x\, p\cdot n} n^\mu$$

$$k^\mu = y p^\mu + k_\perp^\mu$$

*ordering in "-" components*

*Catani, Hautmann NPB427 (1994) 475524*

*F. Hautmann, M. Hentschinski, H. Jung*
*Nucl.Phys. B865 (2012) 54-66*

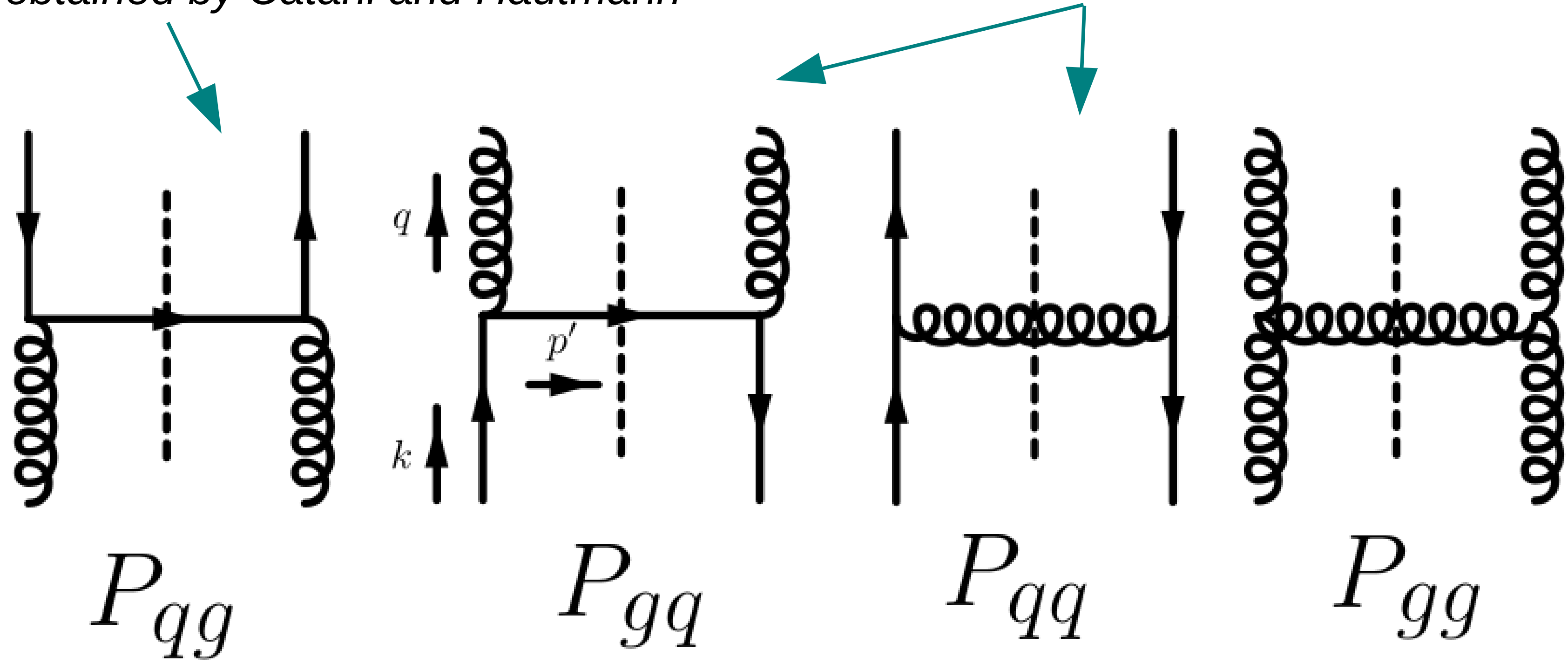
*We will define and constrain splitting functions by requiring:*

- *gauge invariance/current conservation of vertices*
- *collinear limit (LO)*
- *HEF limit (LO)*

# Generalization to HEF kinematics

*O. Gituliar, M. Hentschinski, K.K; JHEP 1601 (2016) 181*

*Kernel obtained by Catani and Hautmann*




*One needs:*

- *appropriate projector operators*
- *generalize QCD vertices (can be obtained form Lipatov effective action or equivalently by spin helicity method)*

# CH kernel

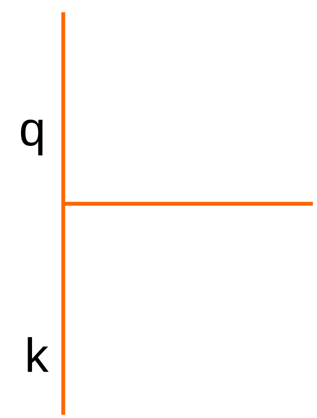
*Application of method proposed by Catani-Hautmann for $P_{qg}$*

*Usage of axial gauge. The outgoing projector is the same for quark  as in the original CFP*

*The projector for incoming gluons obtained from*

$$\mathcal{M}^{g^* g^* \to q\bar{q}}(k_1, k_2; p_3, p_4) = \frac{2\, y_1\, y_2\, p_1^{\mu_1}\, p_2^{\mu_2}}{\sqrt{k_{1\perp}^2\, k_{2\perp}^2}}\, d_{\mu_1 \nu_1}(k_1)\, d_{\mu_2 \nu_2}(k_2)\, \hat{\mathcal{M}}^{g^* g^* \to q\bar{q}}_{\mu_1, \mu_2}(k_1, k_2; p_3, p_4)$$

$$y_1 p_1^{\mu_1} d_{\mu_1 \nu_1}(k_1) = k_{1\perp\, \nu_1} \qquad y_2 p_2^{\mu_2} d_{\mu_2 \nu_2}(k_2) = k_{2\perp\, \nu_2}$$




$$\mathbb{P}^{s\,\mu\nu}_{g,in} = -\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \qquad \mathbb{P}^{s}_{q,out} = \frac{\not{n}}{2q \cdot n}$$

$$\tilde{\boldsymbol{q}} = \boldsymbol{q} - z\boldsymbol{k}$$

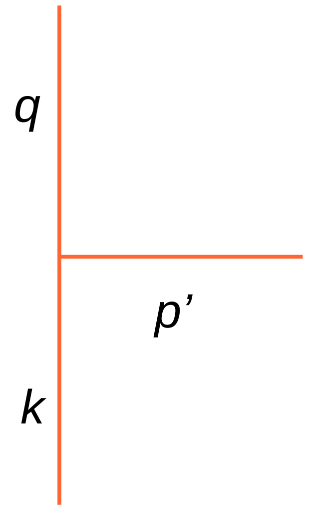
$$\bar{P}^{(0)}_{qg} \;=\; T_R \left( \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[ z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2} \right]$$

# *Full set of projectors*

*Constrained by Ward identities and appropriate limits  the splitting functions should have correct DGLAP and BFKL limits we have the following projectors*

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{in}} = -y^2\,\frac{p^\mu p^\nu}{k_\perp^2}$$

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} - k^2\,\frac{n_\mu n_\nu}{(k\cdot n)^2}$$




$$\mathbb{P}^{s}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}$$

$$\mathbb{P}^{s}_{q,\,\mathrm{out}} = \frac{\not{n}}{2\,n\cdot l}$$

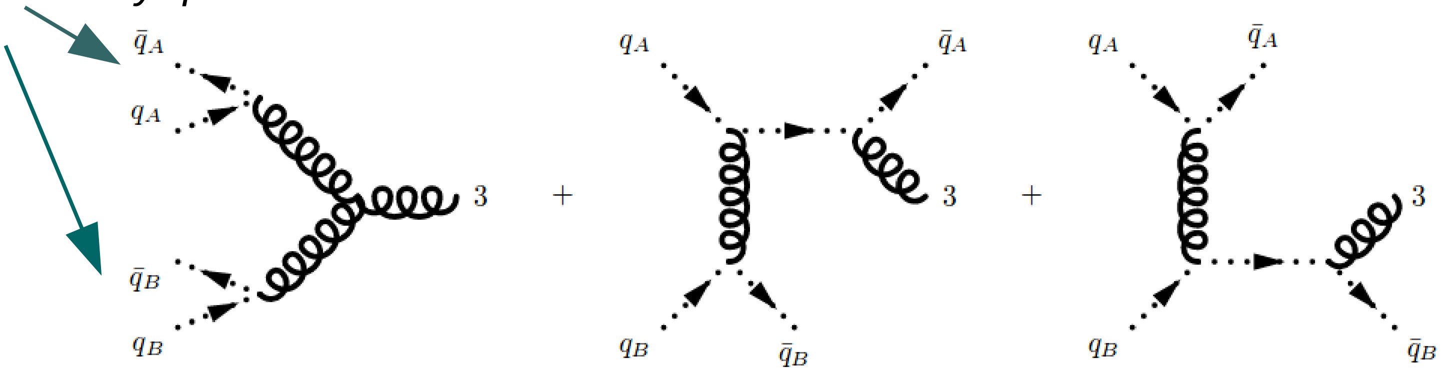
$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \boldsymbol{q}^2}{2x\,p\cdot n}n^\mu$$

$$k^\mu = yp^\mu + k_\perp^\mu$$

# Vertices – example derivation

*M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174*

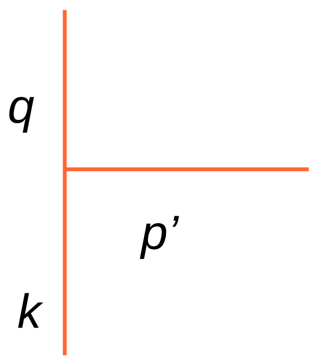
*auxiliary quarks*




$$\mathcal{A}(q,k,p') = (\sqrt{2})\,\frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2} \Big\{ \mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k)$$

$$+\, d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'} \Big\}$$

$$\equiv (\sqrt{2})\,\frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2}\, \Gamma^{\mu_1\mu_2\mu_3}(q,k,p')$$




$d_{\mu\nu}$ *can not be factorized, so it is a part of the vertex*

# Full set of vertices

*M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174*

$$\Gamma^{\mu}_{q^* g^* q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(k)\,\left(\gamma^{\nu} - \frac{n^{\nu}}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^* q^* q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(q)\,\left(\gamma^{\nu} - \frac{p^{\nu}}{p\cdot q}\not{k}\right)$$

$$\Gamma^{\mu}_{q^* q^* g}(q,k,p') = igt^a\left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot p'}\not{k} + \frac{n^{\mu}}{n\cdot p'}\not{q}\right)$$

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^* g^* g}(q,k,p') = i\,g\,f^{abc}\left\{\mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\,d^{\mu_1}{}_{\lambda}(q)\,d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}\right\}$$

*Obtained using spinor helicity methods*

*Van Hameren, Kotko, Kutak, JHEP 1301 (2013) 078*

*remark: can be obtained*
*from Lipatov effective action*

# Example calculation of splitting function: $P_{gg}$ case

*M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174*

$$\mathbb{P}_{g,\,\mathrm{in}} \otimes \hat{K}^{(0)}_{gg}(q,k) \otimes \mathbb{P}_{g,\,\mathrm{out}} =$$

$$\mathbb{P}^{\beta\beta'}_{g,\,\mathrm{in}}(k)\,\mathbb{P}^{\mu'\nu'}_{g,\,\mathrm{out}}(q)(\Gamma^{\beta\mu\alpha}_{g^*g^*g})^{\dagger}\Gamma^{\nu\beta'\alpha'}_{g^*g^*g}\,\frac{-id^{\mu\mu'}(q)}{q^2-i\epsilon}\,\frac{id^{\nu\nu'}(q)}{q^2+i\epsilon}\,d^{\alpha\alpha'}(k-q)$$

$$\tilde{P}^{(0)}_{gg}(z,\tilde{\boldsymbol{q}},\boldsymbol{k}) = 2C_A\left\{\frac{\tilde{\boldsymbol{q}}^4}{(\tilde{\boldsymbol{q}}-(1-z)\boldsymbol{k})^2\,[\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2]}\left[\frac{z}{1-z}+\frac{1-z}{z}+\right.\right.$$
$$\left.\left.+(3-4z)\frac{\tilde{\boldsymbol{q}}\cdot\boldsymbol{k}}{\tilde{\boldsymbol{q}}^2}+z(3-2z)\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right]+\frac{(1+\epsilon)\tilde{\boldsymbol{q}}^2 z(1-z)[2\tilde{\boldsymbol{q}}\cdot\boldsymbol{k}+(2z-1)\boldsymbol{k}^2]^2}{2\boldsymbol{k}^2[\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2]^2}\right\}$$

*The difference to BFKL comes from exact kinematics*

# Results

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

*For sake of presentation only angular averaged kernels*

$$\bar{P}_{qg}^{(0)} = T_R \left( \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[ z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2} \right]$$

$$\bar{P}_{gq}^{(0)} = C_F \left[ \frac{2\tilde{\boldsymbol{q}}^2}{z|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{(2-z)\tilde{\boldsymbol{q}}^4 + z(1-z^2)\boldsymbol{k}^2\tilde{\boldsymbol{q}}^2}{\left(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2\right)^2} \right]$$

$$\bar{P}_{qq}^{(0)} = C_F \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[ \frac{\tilde{\boldsymbol{q}}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\tilde{\boldsymbol{q}}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]$$

$$\bar{P}_{gg}^{(0)} = C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[ \frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\,|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]$$

# Kinematic limits $P_{gg}$

$$\tilde{\boldsymbol{p}} = \frac{\boldsymbol{k}-\boldsymbol{q}}{1-z}$$

*with this variable one can disentangle singularities*

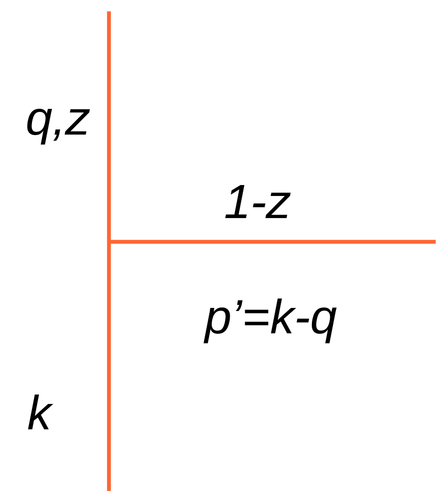
$$z \to 1 \qquad \tilde{\boldsymbol{q}} \to (1-z)\boldsymbol{k}$$

*DGLAP limit:*

$$\lim_{\boldsymbol{k}^2\to 0} \int_0^{2\pi} d\phi P(z, \boldsymbol{k}^2, \tilde{\boldsymbol{p}}^2) = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z)\right]$$

*BFKL limit:*

$$\lim_{z\to 0} \hat{K}_{gg}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s C_A}{\pi(e^{\gamma_E}\mu^2)^\epsilon} \int \frac{d^{2+2\epsilon}\tilde{\boldsymbol{p}}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - (\boldsymbol{k}-\tilde{\boldsymbol{p}})^2\right) \frac{1}{\tilde{\boldsymbol{p}}^2}$$
$$= \int \frac{d^{2+2\epsilon}\boldsymbol{q}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - \boldsymbol{q}^2\right) \frac{\alpha_s C_A}{\pi(e^{\gamma_E}\mu^2)^\epsilon} \frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2},$$




*CCFM limit:*

$$\lim_{\tilde{\boldsymbol{p}}\to 0} \hat{K}_{gg}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, 0, \alpha_s\right) = z \int_{\tilde{\boldsymbol{p}}^2_{min}}^{\tilde{\boldsymbol{p}}^2_{max}} \frac{d\tilde{\boldsymbol{p}}^2}{\tilde{\boldsymbol{p}}^2} \frac{\alpha_s C_a}{\pi} \left[\frac{1}{z} + \frac{1}{1-z} + \mathcal{O}\left(\frac{\tilde{\boldsymbol{p}}^2}{\boldsymbol{k}^2}\right)\right]$$

# *Towards evolution equation*

*For now we have real part emissions of the splitting functions.*

- *The non diagonal splitting functions do not have virtual contribution at the LO.*
- *They are divergent when p' → 0. The diagonal once have virtual contributions.*
- *However, the distribution of gluons gets contribution from quarks….*

# Towards evolution equation

*Real part of Pqq to be complemented by virtual corrections → can expect cancellations of singularities but Pgq is divergent*

*For gluonic part we use low z limit part of Pgg i.e. LO BFKL equation*

$$\mathcal{F}(x,\boldsymbol{q}^2) = \mathcal{F}^0(x,\boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \left[ \mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) - \theta(\boldsymbol{q}^2 - \boldsymbol{p}^2)\mathcal{F}\left(\frac{x}{z}, \boldsymbol{q}^2\right)\right]$$

*add quark induced contribution*

$$+\frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z,\boldsymbol{p},\boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

# Towards evolution equation- BFKL with Regge form factor

*Use simplified Pgg kernel i.e. BFKL limit. Introduce phase space slicing parameter to separate resolved and unresolved emissions*

$$\mathcal{F}(x,\boldsymbol{q}^2) = \mathcal{F}^0(x,\boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z}\left[\int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right) - \ln\frac{\boldsymbol{q}^2}{\mu^2}\mathcal{F}\left(\frac{x}{z},\boldsymbol{q}^2\right)\right]$$

*Using Mellin transforms and some algebra we get*

$$\mathcal{F}(x,\boldsymbol{q}^2) = \underbrace{\tilde{\mathcal{F}}^0(x,\boldsymbol{q}^2)}_{\text{modified}} + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \underbrace{\Delta_R(z,\boldsymbol{q}^2,\mu^2)}_{\exp(-\overline{\alpha}_s \ln 1/z \ln \boldsymbol{q}^2/\mu^2)} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)$$

*Stable in μ → 0*

# Towards evolution equation

*M. Hentschinski, A. Kusina, K.K; Phys. Rev. D 94, 114013 (2016)*

*For quark part the crucial difference: no virtual corrections*

$$\int \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2} \to \int_{\mu^2} \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2}$$

$$\text{‘BFKL’} + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z, \boldsymbol{p}, \boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

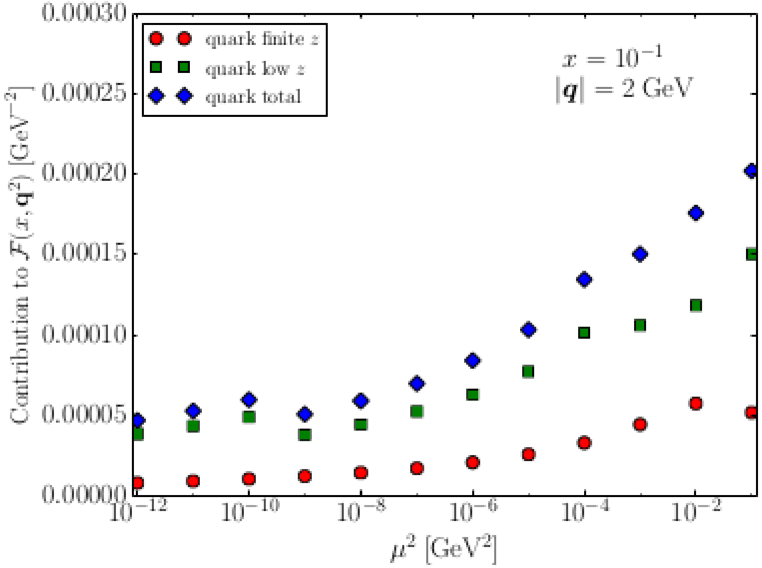
*The equation for gluon reads:*

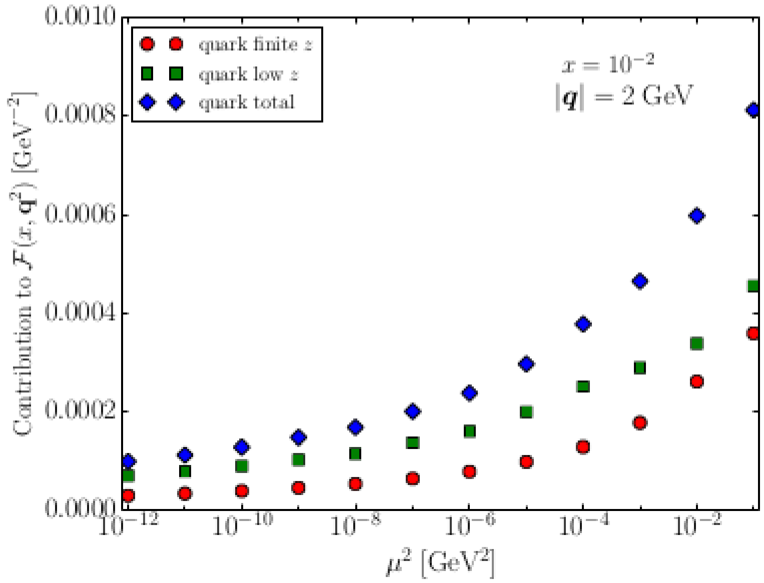
$$\mathcal{F}(x, \boldsymbol{q}^2) = \tilde{\mathcal{F}}^0(x, \boldsymbol{q}^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \theta(\boldsymbol{p}^2 - \mu^2) \Bigg[ \Delta_R(z, \boldsymbol{q}^2, \mu^2) \left( 2C_A \mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) + C_F \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) \right) - \int_z^1 \frac{dz_1}{z_1} \Delta_R(z_1, \boldsymbol{q}^2, \mu^2) \left[ \tilde{P}'_{gq}\left(\frac{z}{z_1}, \boldsymbol{p}, \boldsymbol{q}\right) \frac{z}{z_1} \right] \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) \Bigg]$$

*where*

$$P_{gq} = \tilde{P}_{gq}/z$$

# Towards evolution equation - stability




*Resummation of* $\ln \boldsymbol{q}^2/\mu^2$ in $\Delta_R = \left(\frac{\mu^2}{\boldsymbol{q}^2}\right)^{\overline{\alpha}_s \ln 1/z}$ *cuts of μ → region*

# Conclusions and outlook

- *We have applied CFP and CH technics to calculate real emissions splitting functions*
- *We used the splitting functions to construct equation for gluon density receiving contributions from quarks*
- *We found that found that resummation of virtual contributions to $P_{gg}$ at low $x$ helps with treatment of singularity of $P_{gq}$ splitting function*
- *Obtain virtual contributions to $P_{gg}$ and $P_{qq}$ should be computed using the same formalism*
- *Evolution variable: seems that angle would be optimal*
- *The full set of evolution equations*
- *Relation to operator definition of TMD*
- *Solution*
- *Monte Carlo implementation*