Gluon TMDs in forward dijet+photon production in the CGC

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3DSPIN MAPPING THE PROTON IN 3D

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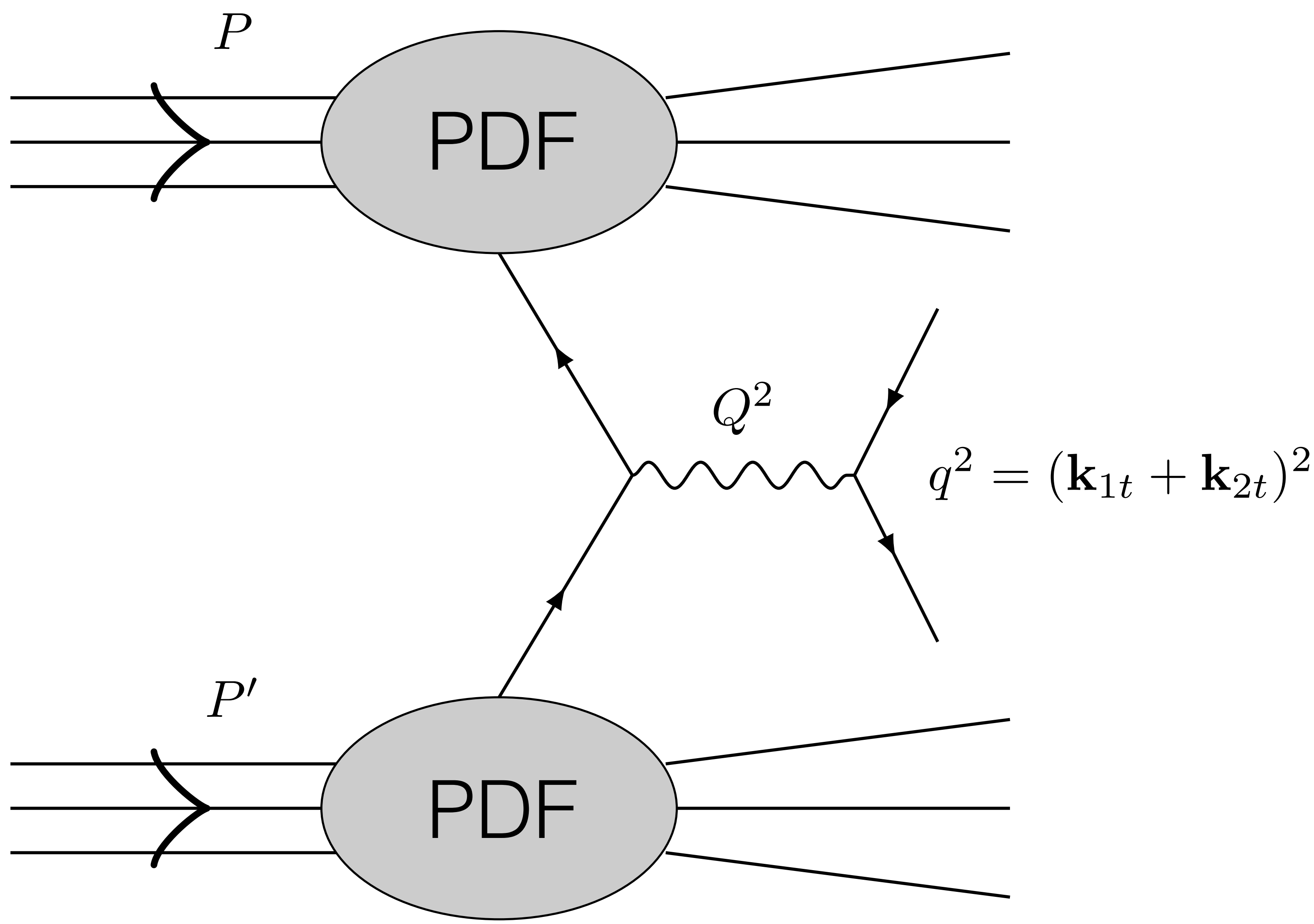
Transverse momentum dependent parton distribution functions (TMDs)

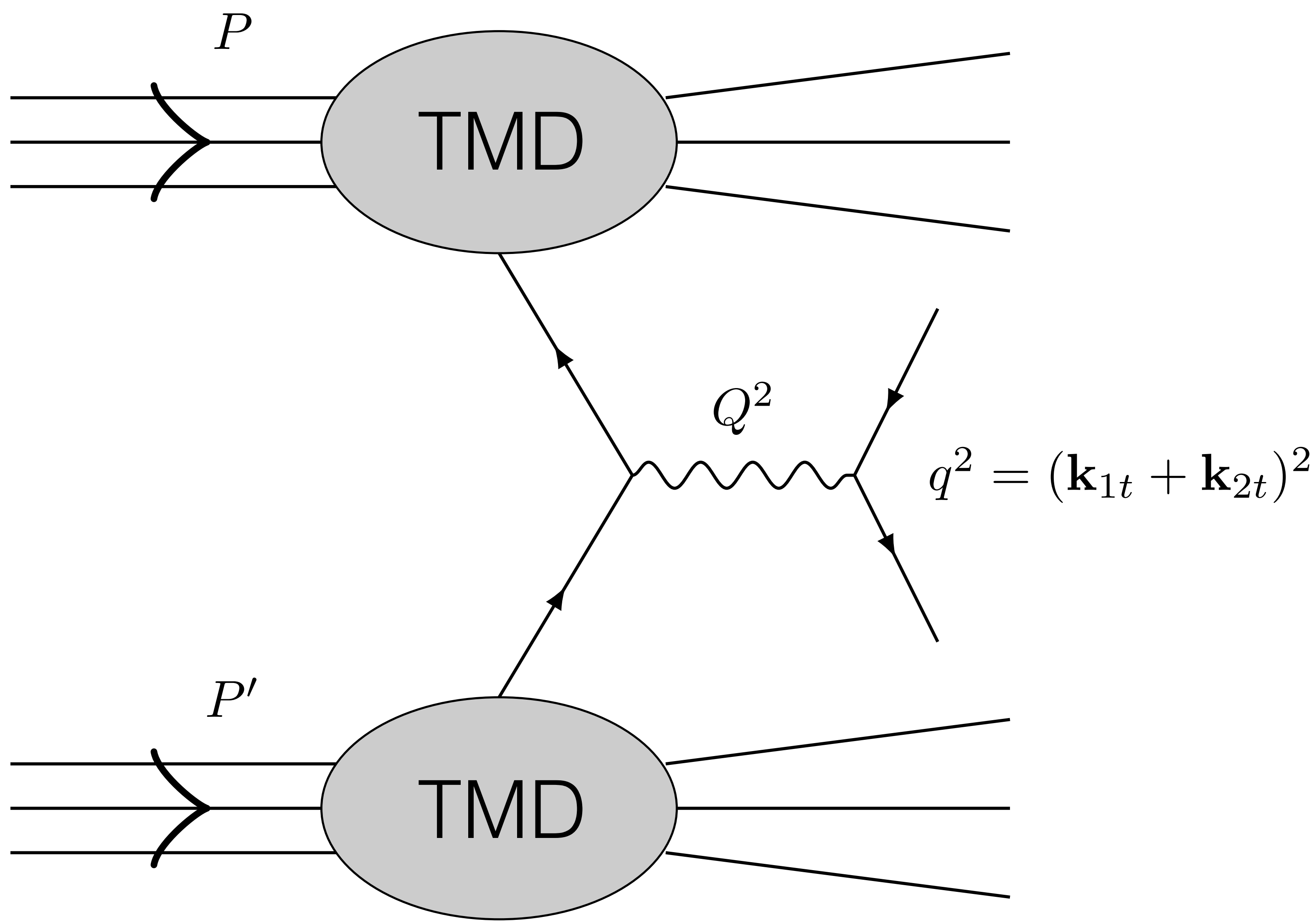
Drell-Yan (collinear)



$Q^2 \simeq q^2$ Transverse momentum comes from hard emissions: **collinear factorization** (NLO)

$$\sigma_{DY,coll} = \hat{\sigma} \otimes f(x_a, Q^2) \otimes f(x_b, Q^2)$$

Drell-Yan (TMD)



$Q^2 \gg q^2$ Transverse momentum comes from *intrinsic* momentum of the quarks: **TMD factorization** (LO)

$$\sigma_{DY,TMD} = \hat{\sigma} \otimes f(x_a, k_\perp, Q^2) \otimes f(x_b, q_\perp - k_\perp, Q^2)$$

Operator definitions of (TMD) PDFs

Collins (2011)

Collinear gluon PDF:

$$xg\left(x,Q^2\right) \equiv \int \frac{\mathrm{d}\xi^-}{\pi p^+} e^{ixp^+\xi^-} \mathrm{Tr}\left\langle P \middle| F^{i+}\left(\xi^-\right) U^\dagger_{[\xi^-,0^-]}(\mathbf{0})\, F^{i+}\left(0^-\right) U_{[\xi^-,0^-]}(\mathbf{0}) \middle| P \right\rangle$$

gauge links/Wilson lines to preserve gauge invariance

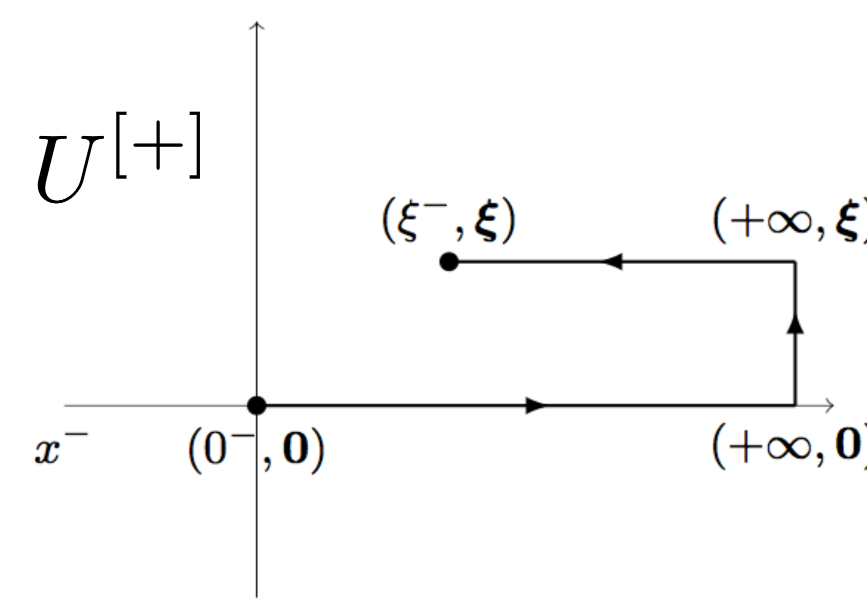
$$U^\dagger_{[\xi^-,0^-]}(\mathbf{0}) = \mathcal{P} e^{-ig_s \int \mathrm{d}z^- A^+(z^-,\mathbf{0})}$$

Gluon TMD:

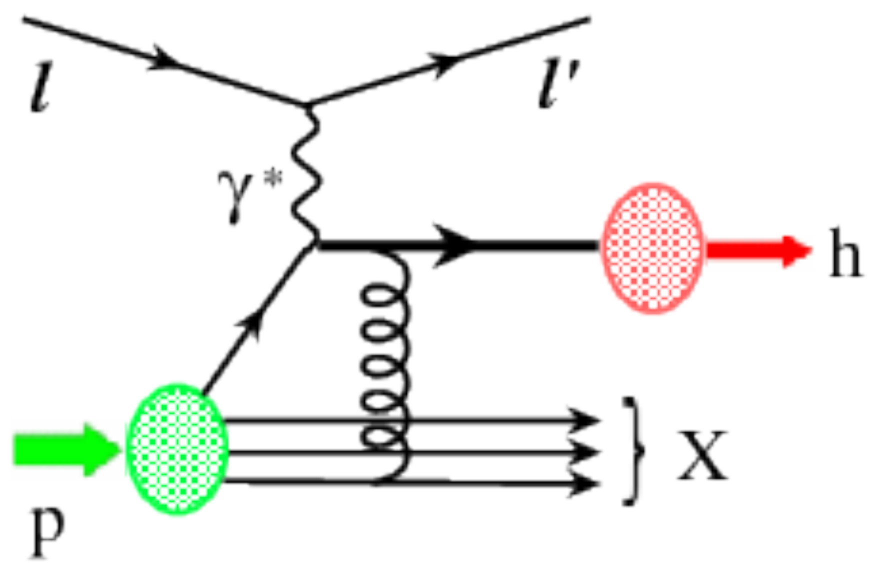
$$\mathcal{F}_{\mathrm{WW}}\left(x,k_\perp\right) \equiv 2 \int \frac{\mathrm{d}^3\xi}{(2\pi)^3 p^+} e^{ixp^+\xi^-} e^{-i\mathbf{k}_\perp \boldsymbol{\xi}} \mathrm{Tr}\langle P | F^{i+}(\vec{\xi}) U^{[+]\dagger} F^{i+}(\vec{0}) U^{[+]} | P \rangle$$

$$U^\dagger_\Gamma = \mathcal{P} \exp\left(-ig_s \int_\Gamma \mathrm{d}x^\mu A_\mu(x) \right)$$

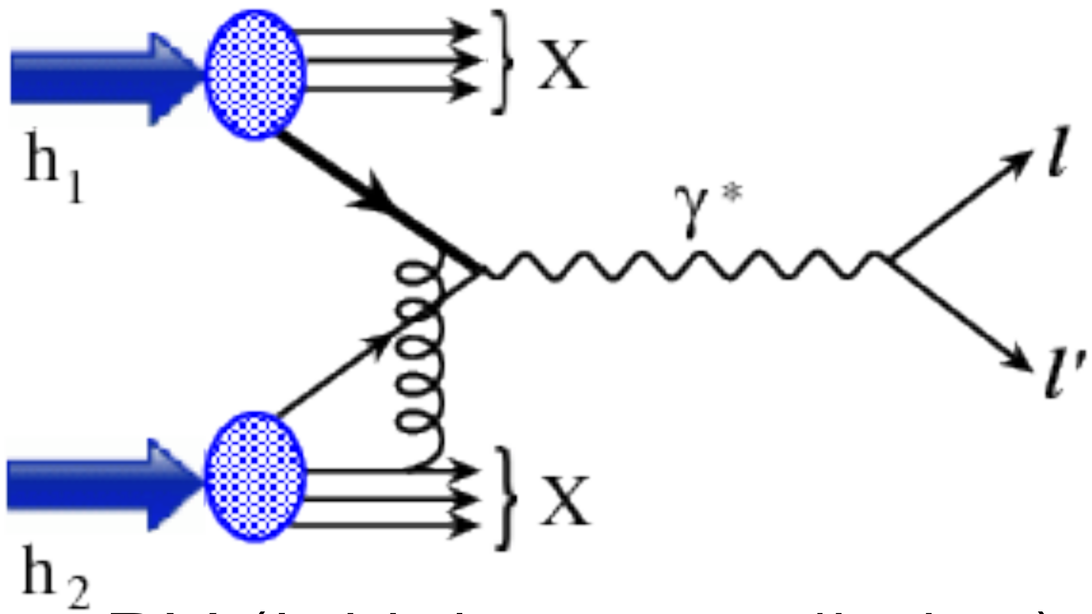
different possible paths in $(x^-, x_\perp)$-plane to connect field strengths



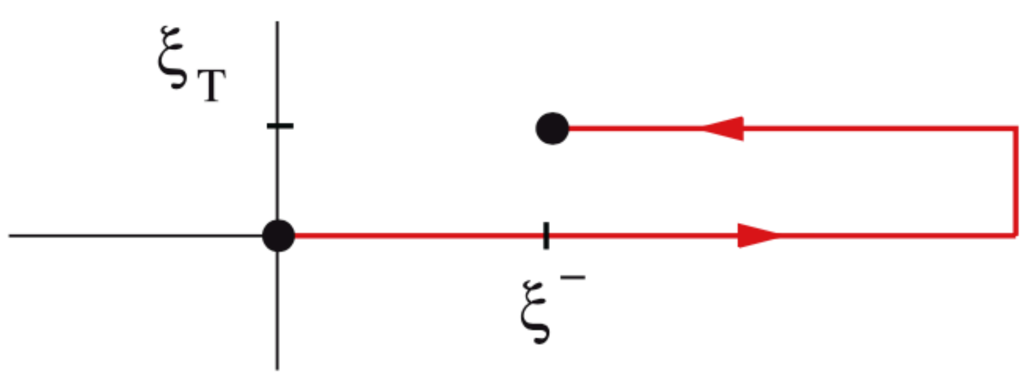
Path dependence of the TMDs

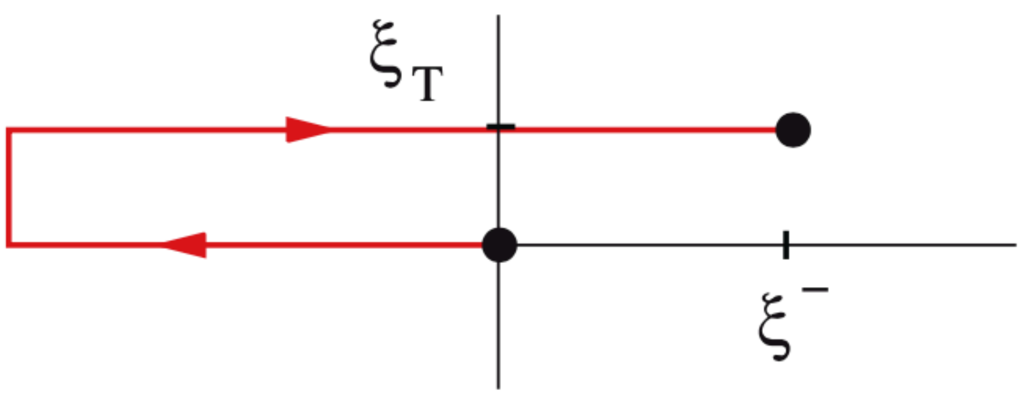


SIDIS (Final state radiation)



DY (Initial state radiation)





Bomhof, Mulders, Pijlman (2006)

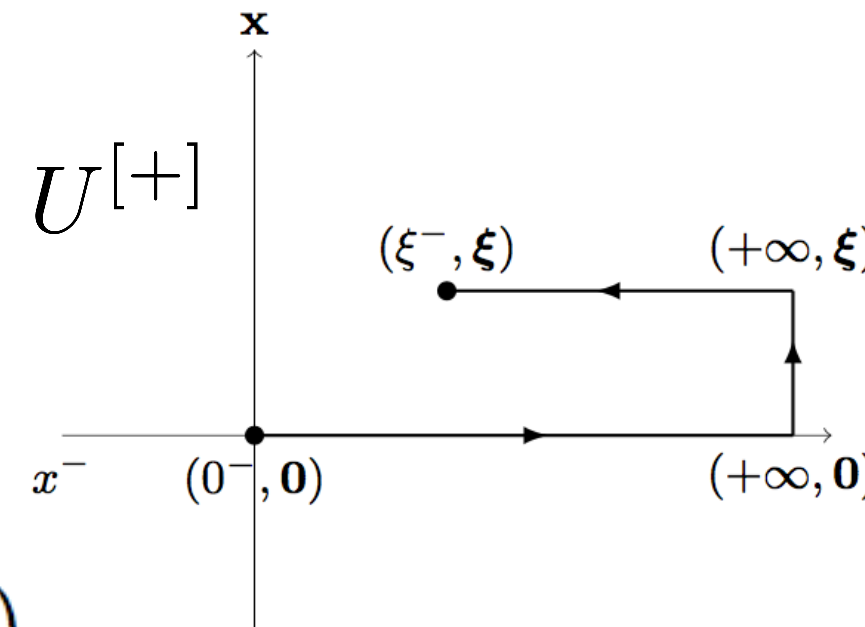
Gluon TMDs at small-x

Weizsäcker-Williams distribution

$$\mathcal{F}_{\mathrm{WW}}(x,k_\perp) \equiv 2\int \frac{\mathrm{d}^3\xi}{(2\pi)^3 p^+} e^{ixp^+\xi^-} e^{-i\mathbf{k}_\perp \boldsymbol{\xi}} \mathrm{Tr}\langle P|F^{i+}(\vec{\xi})U^{[+]\dagger}F^{i+}(\vec{0})U^{[+]}|P\rangle$$

with gauge choice:

$$A^+ = 0 \qquad A_\perp(x^- = +\infty) = 0$$



in light-cone Fock state

$$\langle \mathcal{O}\rangle_x = \frac{\langle A|\mathcal{O}|A\rangle}{\langle A|A\rangle} \qquad F_a^{i+}(\vec{k}) = ik^+ A_a^i(\vec{k})$$

$$\mathcal{F}_{\mathrm{WW}}(x,k_\perp) = 4\int \frac{\mathrm{d}^3v\,\mathrm{d}^3w}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{v}-\vec{w})} \mathrm{Tr}\langle F^{i+}(\vec{v})F^{i+}(\vec{w})\rangle_x$$

$$= \frac{1}{4\pi^3}\langle F_a^{i+}(\vec{k})F_a^{i+}(-\vec{k})\rangle_x$$

$$= k^+ \sum_\lambda \langle a_a^{\lambda\dagger}(\vec{k}) a_a^\lambda(\vec{k})\rangle_x = \frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2k_\perp}$$

WW distribution
~ number operator
in Fock space

Weizsäcker-Williams distribution

in small x limit: $e^{ixP^+\xi^-} \simeq 1$ and gauge choice: $F_a^{i+} = \partial^i A_a^+$

rewrite only in terms of Wilson lines on the LC!

$$\partial^i U_x = i g_s \int \mathrm{d}x^- U_{[-\infty,x^-]}(x) F^{i+}(\vec{x}) U_{[x^-,+\infty]}(x)$$

$$U_x \equiv U_{[-\infty,+\infty]}(x) = \mathcal{P}\exp\left(i g_s \int \mathrm{d}x^- A_a^+(x^-,x) t^a\right)$$

$$\mathcal{F}_{\mathrm{WW}}(x,k_\perp) = 4\int \frac{\mathrm{d}^3v\mathrm{d}^3w}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{v}-\vec{w})} \mathrm{Tr}\left\langle F^{i+}(\vec{v}) U_{\vec{v}}^{[+]\dagger} F^{i+}(\vec{w}) U_{\vec{w}}^{[+]}\right\rangle_x$$

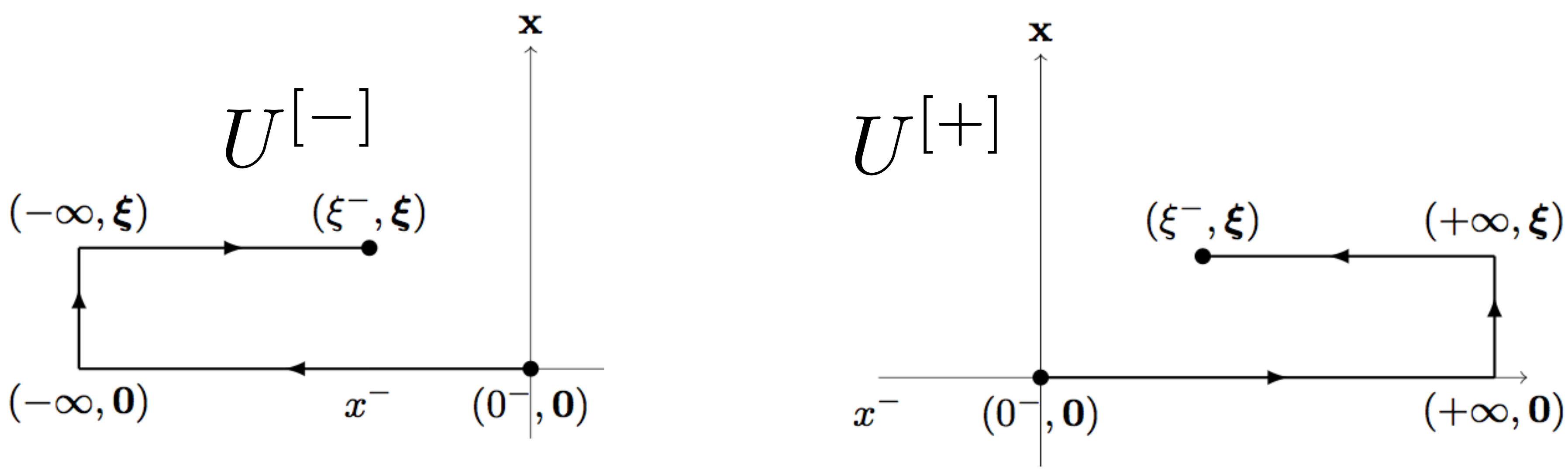
$$= -\frac{4}{g_s^2}\int \frac{\mathrm{d}^2v\mathrm{d}^2w}{(2\pi)^3} e^{ik_\perp(v-w)} \mathrm{Tr}\left\langle (\partial^i U_v) U_w^\dagger (\partial^i U_w) U_v^\dagger\right\rangle_x$$

can be calculated in e.g. the MV model:

$$\mathcal{F}_{\mathrm{WW}}(x,k_\perp) = \frac{2C_F S_\perp}{\alpha_s \pi^2}\int \frac{\mathrm{d}^2r}{(2\pi)^2} e^{-ik_\perp r} \frac{1}{r^2}\left(1 - e^{-\frac{r^2}{4}Q_{sg}^2(r)}\right)$$

Dipole distribution

$$\mathcal{F}_{\rm dip}(x,k_\perp) \equiv 2\int \frac{{\rm d}^3\xi}{(2\pi)^3 p^+} e^{ixp^+\xi^-} e^{-i\mathbf{k}_\perp \boldsymbol{\xi}} {\rm Tr}\langle P|F^{i+}(\vec{\xi})U^{[-]\dagger}F^{i+}(\vec{0})U^{[+]}|P\rangle$$



$$\mathcal{F}_{\rm dip}(x,k_\perp) = \frac{4}{g_s^2}\int \frac{{\rm d}^2v{\rm d}^2w}{(2\pi)^3} e^{ik_\perp(v-w)} {\rm Tr}\langle(\partial^i U_v^\dagger)(\partial^i U_w)\rangle_x$$

$$= \frac{N_c k_t^2}{2\pi^2\alpha_s}\int \frac{{\rm d}^2v{\rm d}^2w}{(2\pi)^2} e^{ik_\perp(v-w)} \frac{1}{N_c}{\rm Tr}\langle U_v^\dagger U_w\rangle_x$$

in the MV model:

$$\mathcal{F}_{\rm dip}(x,k_\perp) = \frac{N_c S_\perp k_t^2}{2\alpha_s\pi^2}\int \frac{{\rm d}^2r}{(2\pi)^2} e^{-ik_t r} e^{-\frac{r^2}{4}Q_s^2(r)}$$

Multitude of gluon TMDs

in the large N_c limit:

	DIS and DY	SIDIS	hadron in pA	photon-jet in pA	Dijet in DIS	Dijet in pA
$xG^{(1)}$ (WW)	x	x	x	x	√	√
$xG^{(2)}$ (dipole)	√	√	√	√	x	√

Dominguez, Marquet, Xiao, Yuan (2011)

at finite N_c, many more….

for example:

$$\frac{d\sigma(pA\to q\bar{q}X)}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{2C_F}\frac{z(1-z)}{P_t{}^4}x_1 g(x_1,\mu^2)P_{qg}(z)\Big\{[(1-z)^2+z^2]\mathcal{F}_{gg}^{(1)}(x_2,k_t) + 2z(1-z)\mathrm{Re}\,\mathcal{F}_{gg}^{(2)}(x_2,k_t) - \frac{1}{N_c^2}\mathcal{F}_{gg}^{(3)}(x_2,k_t)\Big\},$$

Kotko, Kutak, Marquet, Petreska, Sapeta & van Hameren (2015)

Marquet, Petreska, Roiesnel (2016)

From CGC to TMD factorization

example: dijet photoproduction in the CGC:

$$\frac{d\sigma^{\gamma A\to q\bar{q}X}}{d^3k_1 d^3k_2} = N_c \alpha_e e_q^2 \delta\left(k_1^+ + k_2^+ - p^+\right) \int \frac{d^2x d^2y d^2x' d^2y'}{(2\pi)^8} e^{ik_{1\perp}(y-y')} e^{ik_{2\perp}(x-x')}$$
$$\times \sum_{\lambda s s'} \phi^{\lambda *}_{s's}(p^+, k_1^+, x-y) \phi^{\lambda}_{s's}\left(p^+, k_1^+, x'-y'\right)$$
$$\times \left(\frac{1}{N_c} \mathrm{Tr}\langle U_x U^\dagger_{x'} U_{y'} U^\dagger_y \rangle_x - \frac{1}{N_c} \mathrm{Tr}\langle U_y U^\dagger_x \rangle_x - \frac{1}{N_c} \mathrm{Tr}\langle U_{x'} U^\dagger_{y'} \rangle_x + 1 \right)$$

three scales: $Q_s \lesssim q_\perp \lesssim P_\perp$ $\qquad q_\perp = |\mathbf{k}_{1t} + \mathbf{k}_{2t}|$

$P_\perp \sim k_{1t} \sim k_{2t}$

CGC result in terms of Wilson line averages,
contains both the dilute regime $q_\perp \sim P_\perp$
and the saturation regime $q_\perp \ll P_\perp$ → **validity domain of TMD factorization**

Dominguez, Xiao, Yuan (2011)
Dominguez, Marquet, Xiao, Yuan (2011)

From CGC to TMD factorization

correlation limit:

$$e^{ik_{1\perp}(y-y')}e^{ik_{2\perp}(x-x')} = e^{iq_\perp(v-v')}e^{iP_\perp(u-u')}$$

Taylor expand in $u, u' \ll v, v'$

is tantamount to a twist expansion $\dfrac{q_\perp^2}{P_\perp^2}$

and the cross section can be written in an explicitly TMD factorized form:

$$\frac{\mathrm{d}\sigma^{\gamma A\to q\bar{q}X}}{\mathrm{d}\mathcal{P}.\mathcal{S}.} = \alpha_s\alpha_e e_q^2\delta\left(x_{\gamma^*}-1\right)\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\mathcal{F}_{gg}^{(3)}\left(x,q_\perp\right)$$

Does such a procedure work in 2→3 processes, in general kinematics?

The process $pA \to \mathrm{jet} + \mathrm{jet} + \gamma$ in the correlation limit

Light cone perturbation theory

similar to 'old fashioned perturbation theory' in interaction picture

$$|\psi(0)\rangle_I = T\exp\Big(-i\int_{-\infty}^{0} \mathrm{d}t\mathcal{H}_I(t)\Big)|\omega_0\rangle$$
$$= |\omega_0\rangle + \sum_{\omega}|\omega\rangle\frac{\langle\omega|H_{\text{int}}|\omega_0\rangle}{\omega_0-\omega} + \sum_{\omega,\omega'}|\omega\rangle\frac{\langle\omega|H_{\text{int}}|\omega'\rangle\langle\omega'|H_{\text{int}}|\omega_0\rangle}{(\omega_0-\omega')(\omega_0-\omega)} + \ldots$$

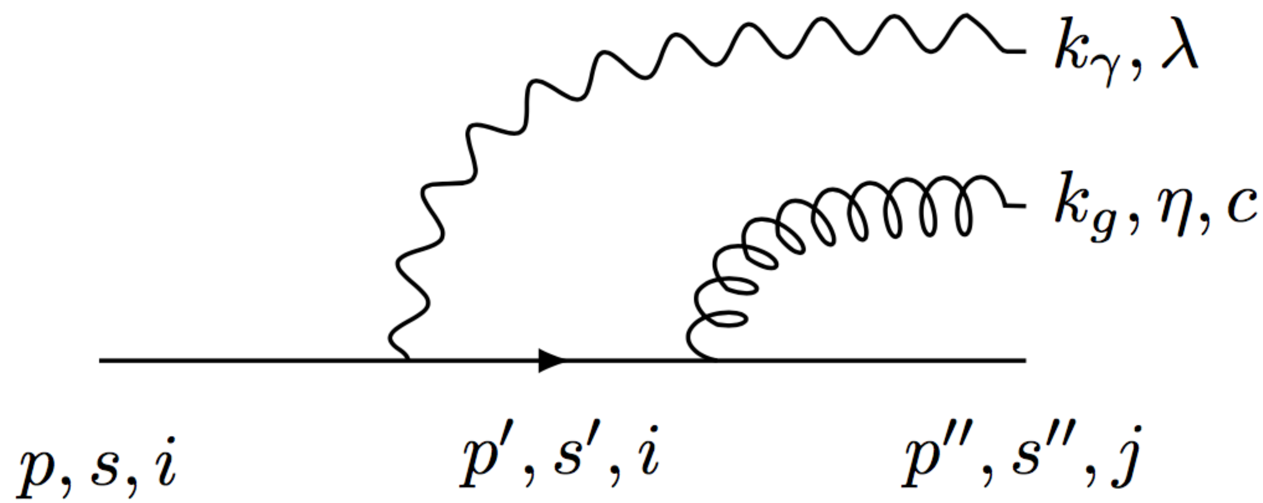
leads to picture of a 'dressed' state which interacts at time $t = 0$

$$|q\rangle_{\text{dressed}} = |q\rangle + g_e F_{q\gamma}|q\gamma\rangle + g_s F_{qg}|qg\rangle + g_e g_s F_{q\gamma g}|q\gamma g\rangle + \mathcal{O}(g_{e,s}^2)\ldots$$

on the light cone $t = x^+$ and $\omega = p^-$

Kogut, Soper (1970), Bjorken, Kogut, Soper (1971),
Brodsky, Pauli, Pinsky (1996), Zhang (1994), Heinzl (2000)

Dressed quark state

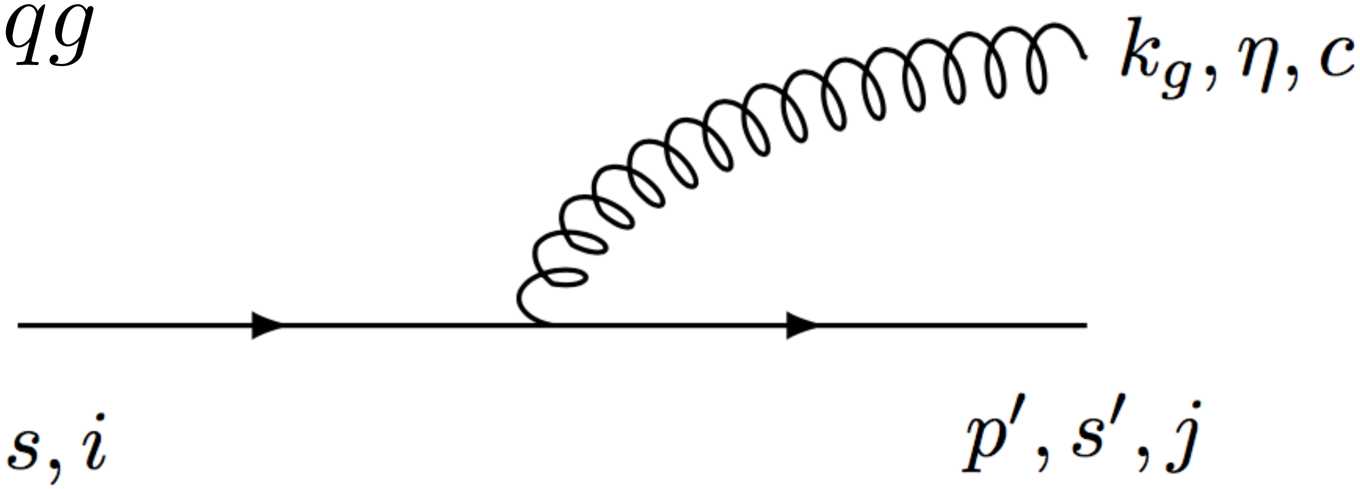


$$
\begin{aligned}
|(\boldsymbol{q})[\vec{p}]^i_s\rangle_D = {} & |(\boldsymbol{q})[\vec{p}]^i_s\rangle + g_s t^c_{ij} \int_{\vec{k}_g} |(\boldsymbol{q})[\vec{p} - \vec{k}_g]^j_{s'}; (\boldsymbol{g})[\vec{k}_g]^c_\eta\rangle \times F_{qg}\Big[(q)[\vec{p} - \vec{k}_g]^j_{s's}; (g)[\vec{k}_g]^c_\eta\Big] \\
& + g_e \int_{\vec{k}_\gamma} |(\boldsymbol{q})[\vec{p} - \vec{k}_\gamma]^i_{s'}; (\boldsymbol{\gamma})[\vec{k}_\gamma]_\lambda\rangle \times F_{q\gamma}\Big[(q)[\vec{p} - \vec{k}_\gamma]^i_{s's}; (\gamma)[\vec{k}_\gamma]_\lambda\Big] \\
& + g_e g_s t^c_{ij} \int_{\vec{k}_\gamma, \vec{k}_g} |(\boldsymbol{q})[\vec{p} - \vec{k}_\gamma - \vec{k}_g]^j_{s'}; (\boldsymbol{\gamma})[\vec{k}_\gamma]_\lambda; (\boldsymbol{g})[\vec{k}]^c_\lambda\rangle \\
& \times \big(F_{q\gamma g} + F_{qg\gamma}\big)\Big[(q)[\vec{p} - \vec{k}_\gamma - \vec{k}_g]^j_{s'}; (\gamma)[\vec{k}_\gamma]_\lambda; (g)[\vec{k}]^c_\lambda\Big]
\end{aligned}
$$

Altinoluk, Armesto, Kovner, Lublinsky & Petreska (2018)

Light cone wave functions

example: $q \to qg$



$$F_{qg}\left[(q)[\vec{p}-\vec{k}_g]^{j}_{s's};(g)[\vec{k}_g]^{c}_{\eta}\right] = \frac{1}{2k_g^+ 2\left(p^+ - k_g^+\right)} \frac{\bar{u}^{s'}(\vec{p}-\vec{k}_g)\epsilon^*_\lambda(\vec{k}_g)u^s(\vec{p})}{p^- - k_g^- - p'^-}$$

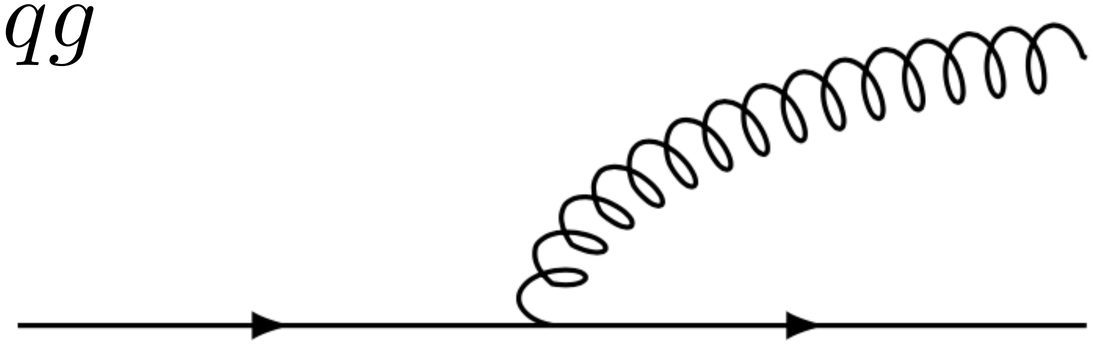
$$\mathcal{P}_B = \frac{\gamma^0\gamma^-}{\sqrt{2}},\ \mathcal{P}_G = \frac{\gamma^0\gamma^+}{\sqrt{2}}$$

LC quantization implies that only ‘good’ parts of the spinors are independent:

$$u^s_B(\vec{k}) = \frac{\gamma^+}{2k^+}\mathbf{k}\cdot\boldsymbol{\gamma}\, u^s_G(\vec{k}), \quad v^s_B(\vec{k}) = \frac{\gamma^+}{2k^+}\mathbf{k}\cdot\boldsymbol{\gamma}\, v^s_G(\vec{k})$$

Light cone wave functions

example: $q \to qg$



$$F_{qg}\left[(q)[\vec{p}-\vec{k}_g]^j_{s's};(g)[\vec{k}_g]^c_\eta\right] = \frac{1}{2k_g^+ 2\left(p^+ - k_g^+\right)} \frac{\bar{u}^{s'}(\vec{p}-\vec{k}_g)\epsilon^*_\lambda(\vec{k}_g)u^s(\vec{p})}{p^- - k_g^- - p'^-}$$

$$= -\phi^{\bar{\eta}\eta}_{s's}(\xi_g)\frac{\left(\xi_g p - k_g\right)^{\bar{\eta}}}{\left(\xi_g p - k_g\right)^2}$$

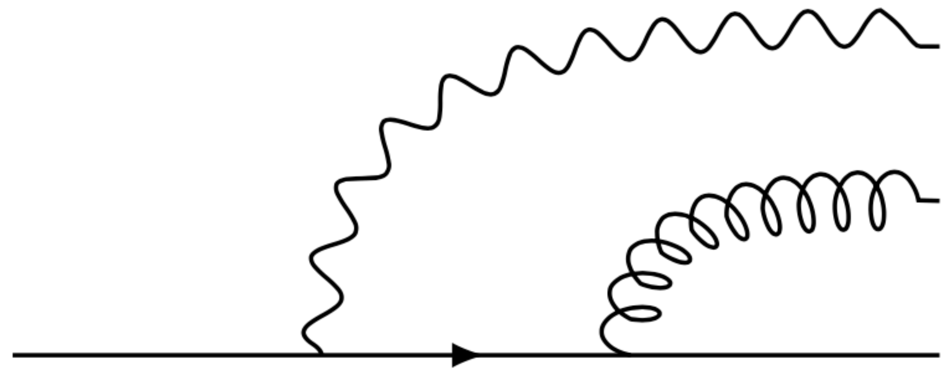
where:

$$\phi^{\bar{\lambda}\lambda}_{s's}(\xi_g) = \frac{1}{2k_g^+ 2p'^+}\epsilon^j_\lambda \bar{u}^{s'}_G(\vec{p'})\gamma^+\left[(2-\xi_g)\,\delta^{\bar{\lambda}j} + i\xi_g\sigma^{\bar{\lambda}j}\right]u^s_G(\vec{p})$$

$$\xi_g = \frac{k_g^+}{p^+} \qquad \sigma^{ij} = \frac{i}{2}[\gamma^i,\gamma^j]$$

Beuf (2016)

Light cone wave functions



$$F_{q\gamma g}\left[(q)[\vec{p}-\vec{k}_\gamma-\vec{k}_g]^j_{s'};(\gamma)[\vec{k}_\gamma]_\lambda;(g)[\vec{k}]^c_\lambda\right]$$

$$=\sum_{s'}\phi^{\lambda\bar{\lambda}}_{s's}(\xi_\gamma)\tilde{\phi}^{\eta\bar{\eta}}_{s''s'}(\xi_\gamma,\xi_g)\frac{(\xi_\gamma p-k_\gamma)^{\bar{\lambda}}}{(\xi_\gamma p-k_\gamma)^2}\frac{(\xi_g(p-k_\gamma)-\bar{\xi}_g k_g)^{\bar{\eta}}}{\xi_g(\xi_\gamma p-k_\gamma)^2+\xi_\gamma(\xi_g p-k_g)^2-(\xi_g k_\gamma-\xi_\gamma k_g)^2}$$

$$F_{qg\gamma}=F_{q\gamma g}(\xi_g\leftrightarrow\xi_\gamma,\, k_g\leftrightarrow k_\gamma)$$

Dressed quark state in mixed Fourier space

$$F_{qg}\left[(q)[p^+ - k_g^+, v]^j_{s's}; (g)[k_g^+, x_g]^c_\eta\right] = -i\phi^{\bar{\eta}\eta}_{s's}(\xi_g)\, A^{\bar{\eta}}(v - x_g)\, \delta^{(2)}(\omega - \bar{\xi}_g v - \xi_g x_g)$$

$$F_{q\gamma g}\left[(q)[p^+ - k_\gamma^+ - k_g^+, x_q]^j_{s'}; (\gamma)[k_\gamma^+, x_\gamma]_\lambda; (g)[k_g, x_g]^c_\lambda\right] = \left[-i\phi^{\lambda\bar{\lambda}}_{s's}(\xi_\gamma)\right]\left[\frac{-i\tilde{\phi}^{\eta\bar{\eta}}_{s''s'}(\xi_\gamma, \xi_g)}{\xi_\gamma}\right]$$
$$\times \int_v \delta^{(2)}(\omega - \xi_\gamma x_\gamma - \bar{\xi}_\gamma v)\, \delta^{(2)}(v - \bar{\tilde{\xi}}_g x_q - \tilde{\xi}_g x_g)\, A^{\bar{\eta}}(x_q - x_g)\, \mathcal{A}^{\bar{\lambda}}(v - x_\gamma)$$

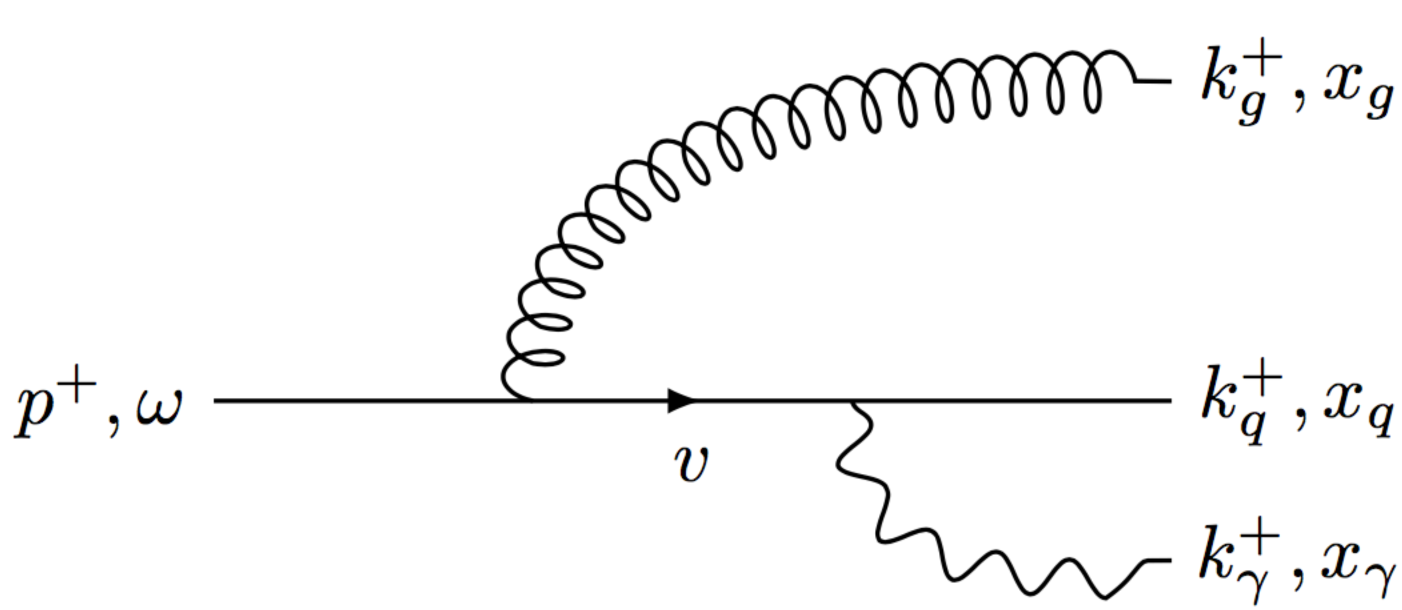
$$A^{\bar{\eta}}(v - x_g) = -\frac{1}{2\pi}\frac{(v - x_g)^{\bar{\eta}}}{(v - x_g)^2}$$ Weizsäcker-Williams field

$$\mathcal{A}^{\bar{\lambda}}(v - x_\gamma) = -\frac{1}{2\pi}\frac{\xi_\gamma (v - x_\gamma)^{\bar{\lambda}}}{\xi_\gamma (v - x_\gamma)^2 + \tilde{\xi}_g \bar{\tilde{\xi}}_g (x_q - x_g)^2}$$ modified Weizsäcker-Williams field

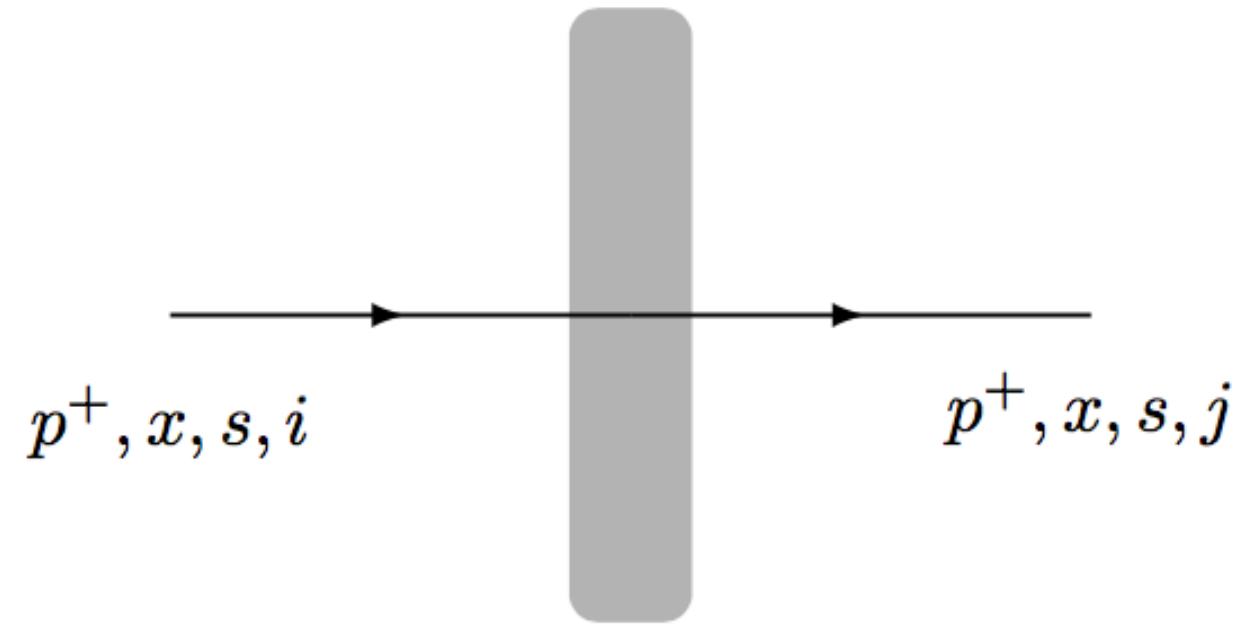
$$\tilde{\xi}_\gamma = \frac{\xi_\gamma}{\bar{\xi}_g}, \quad \tilde{\xi}_g = \frac{\xi_g}{\bar{\xi}_\gamma}$$

Dressed quark state in mixed Fourier space

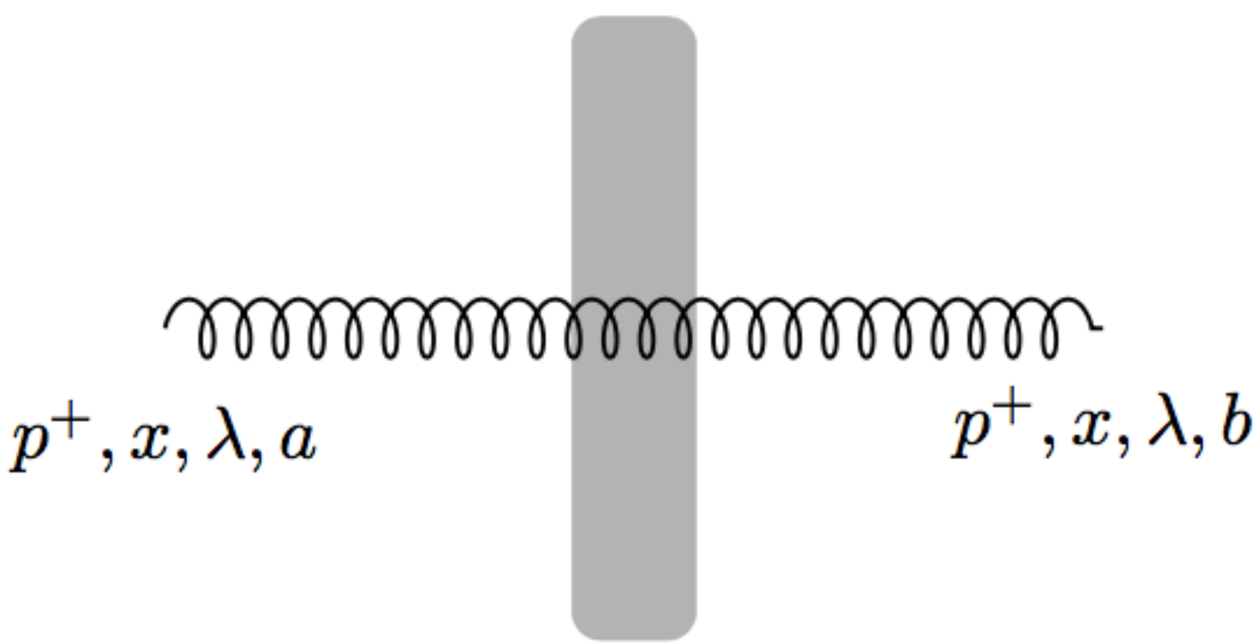
$$\left|(\boldsymbol{q})[p^+,0]^i_s\right\rangle_D = \int_\omega \left|(\boldsymbol{q})[p^+,\omega]^i_s\right\rangle$$
$$+ g_s t^c_{ij} \int \frac{\mathrm{d}k^+_g}{2\pi} \int_{\omega,v,x_g} \left[-i\phi^{\bar\eta\eta}_{s's}(\xi_g)\right] A^{\bar\eta}(v-x_g)\delta^{(2)}(\omega-\xi_g x_g-\bar\xi_g v)\left|(\boldsymbol{q})[p^+-k^+_g,v]^j_{s'};(\boldsymbol{g})[k^+_g,x_g]^c_\eta\right\rangle$$
$$+ g_e \int \frac{\mathrm{d}k^+_\gamma}{2\pi} \int_{\omega,v,x_\gamma} \left[-i\phi^{\bar\lambda\lambda}_{s's}(\xi_\gamma)\right] A^{\bar\lambda}(v-x_\gamma)\delta^{(2)}(\omega-\xi_\gamma x_\gamma-\bar\xi_\gamma v)\left|(\boldsymbol{q})[p^+-k^+_\gamma,v]^i_{s'};(\boldsymbol{\gamma})[k^+_\gamma,x_\gamma]_\lambda\right\rangle$$
$$+ g_e g_s t^c_{ij} \int \frac{\mathrm{d}k^+_g}{2\pi} \int \frac{\mathrm{d}k^+_\gamma}{2\pi} \int_{\omega,v,x_\gamma,x_g,x_q} \left[-i\phi^{\bar\lambda\lambda}_{s's}(\xi_\gamma)\right]\left[-i\frac{\phi^{\bar\eta\eta}_{s''s'}(\xi_\gamma,\xi_g)}{\xi_\gamma}\right] A^{\bar\eta}(x_q-x_g)\mathcal{A}^{\bar\lambda}(v-x_\gamma)$$
$$\times\,\delta^{(2)}(\omega-\xi_\gamma x_\gamma-\bar\xi_\gamma v)\delta^{(2)}(v-\bar{\tilde\xi}_g x_q-\tilde\xi_g x_g)\left|(\boldsymbol{q})[p^+-k^+_\gamma-k^+_g,x_q]^j_{s'};(\boldsymbol{\gamma})[k^+_\gamma,x_\gamma]_\lambda;(\boldsymbol{g})[k_g,x_g]^c_\lambda\right\rangle$$
$$+ \left(\xi_g\leftrightarrow\xi_\gamma,\ x_\gamma\leftrightarrow x_g,\ \eta\leftrightarrow\lambda\right)\left|(\boldsymbol{q})[p^+-k^+_\gamma-k^+_g,x_q]^j_{s'};(\boldsymbol{\gamma})[k^+_\gamma,x_\gamma]_\lambda;(\boldsymbol{g})[k_g,x_g]^c_\lambda\right\rangle$$



Scattering off the CGC



$$\mathcal{U}(x) = \mathcal{P}\exp\Big(ig_s \int \mathrm{d}x^+ A_c^-(x^+, x) t^c\Big)$$



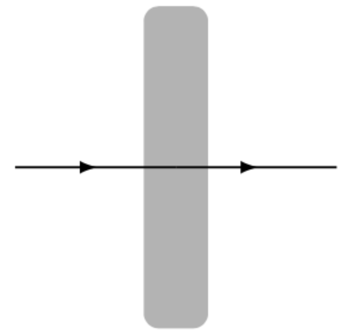
$$\mathcal{W}(x) = \mathcal{P}\exp\Big(ig_s \int \mathrm{d}x^+ A_c^-(x^+, x) T^c\Big)$$

Outgoing quark state

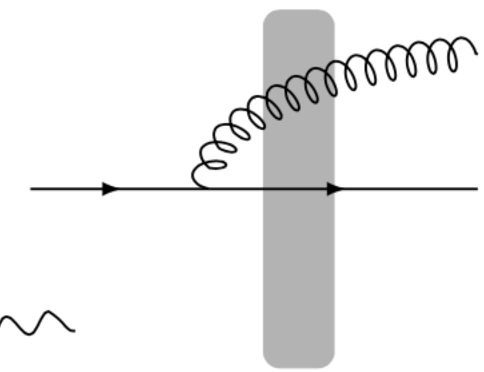
$$\big|(\boldsymbol{q})[p^+,0]^i_s\big\rangle_{\rm out} = \int_\omega \mathcal{U}_{ij}(\omega)\big|(\boldsymbol{q})[p^+,\omega]^j_s\big\rangle$$
$$+ g_s \int \frac{\mathrm{d}k_g^+}{2\pi} \int_{\omega,v,x_g} \Big[-i\phi^{\bar\eta\eta}_{s's}(\xi_g)\Big] A^{\bar\eta}(v-x_g)\delta^{(2)}(\omega-\xi_g x_g-\bar\xi_g v)\, t^d_{il}\mathcal{U}_{lj}(v)\mathcal{W}_{dc}(x_g)\big|(\boldsymbol{q})[p^+-k_g^+,v]^i_{s'};(\boldsymbol{g})[k_g^+,x_g]^c_\eta\big\rangle$$
$$+ g_e \int \frac{\mathrm{d}k_\gamma^+}{2\pi} \int_{\omega,v,x_\gamma} \Big[-i\phi^{\bar\lambda\lambda}_{s's}(\xi_\gamma)\Big] A^{\bar\lambda}(v-x_\gamma)\delta^{(2)}(\omega-\xi_\gamma x_\gamma-\bar\xi_\gamma v)\,\mathcal{U}_{ij}(v)\big|(\boldsymbol{q})[p^+-k_\gamma^+,v]^i_{s'};(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma]_\lambda\big\rangle$$
$$+ g_e g_s \int \frac{\mathrm{d}k_g^+}{2\pi} \int \frac{\mathrm{d}k_\gamma^+}{2\pi} \int_{\omega,v,x_\gamma,x_g,x_q} \Big[-i\phi^{\bar\lambda\lambda}_{s's}(\xi_\gamma)\Big]\Big[\frac{-i\phi^{\bar\eta\eta}_{s''s'}(\xi_\gamma,\xi_g)}{\xi_\gamma}\Big] A^{\bar\eta}(x_q-x_g)\mathcal{A}^{\bar\lambda}(v-x_\gamma)$$
$$\times\,\delta^{(2)}(\omega-\xi_\gamma x_\gamma-\bar\xi_\gamma v)\delta^{(2)}(v-\bar{\tilde\xi}_g x_q-\tilde\xi_g x_g)t^d_{il}\mathcal{U}_{lj}(x_q)\mathcal{W}_{dc}(x_g)\big|(\boldsymbol{q})[p^+-k_\gamma^+-k_g^+,x_q]^j_{s'};(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma]_\lambda;(\boldsymbol{g})[k_g,x_g]^c_\lambda\big\rangle$$
$$+ \big(\xi_g\leftrightarrow\xi_\gamma,\; x_\gamma\leftrightarrow x_g,\; \eta\leftrightarrow\lambda\big)\big|(\boldsymbol{q})[p^+-k_\gamma^+-k_g^+,x_q]^j_{s'};(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma]_\lambda;(\boldsymbol{g})[k_g,x_g]^c_\lambda\big\rangle$$

Outgoing quark state

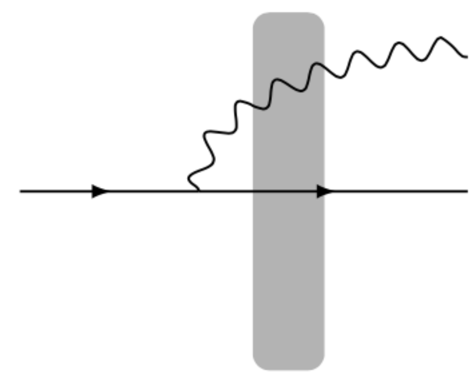
$$\big|(\boldsymbol{q})[p^+,0]\big\rangle_{\mathrm{out}} = \int_\omega \mathcal{U}(\omega)\big|(\boldsymbol{q})[p^+,\omega]\big\rangle$$



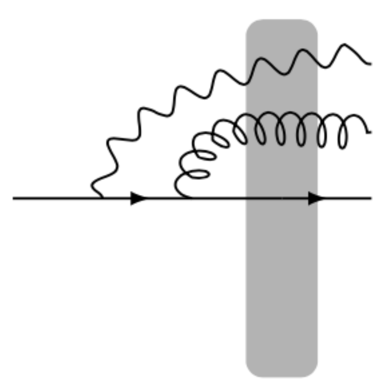
$$+\, g_s \int_{k_g^+}\int_{\omega,v,x_g} F_{qg}\, t^c\,\mathcal{U}(v)\mathcal{W}(x_g)\big|(\boldsymbol{q})[p^+-k_g^+,v];(\boldsymbol{g})[k_g^+,x_g]\big\rangle$$

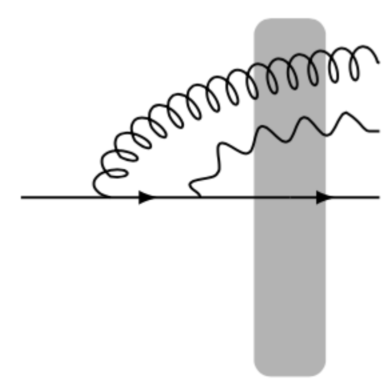


$$+\, g_e \int_{k_\gamma^+}\int_{\omega,v,x_\gamma} F_{q\gamma}\,\mathcal{U}(v)\big|(\boldsymbol{q})[p^+-k_\gamma^+,v];(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma]\big\rangle$$

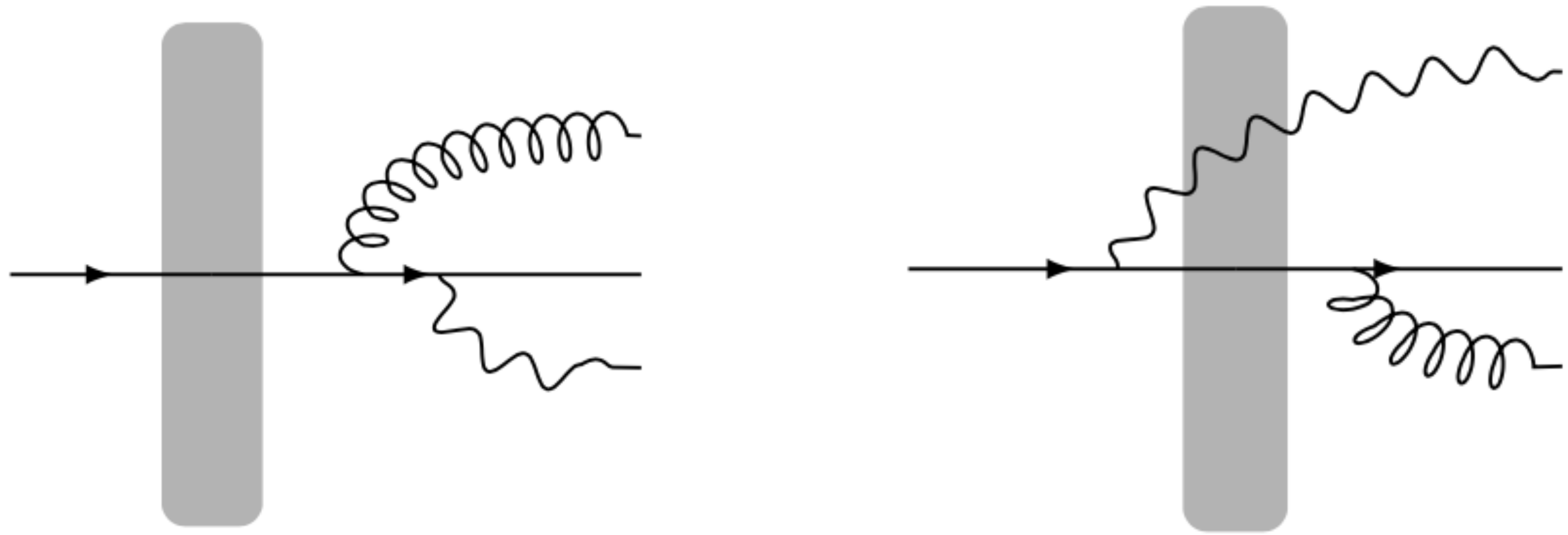


$$+\, g_e g_s \int_{k_g^+,k_\gamma^+}\int_{\omega,v,x_\gamma,x_g,x_q} \big(F_{q\gamma g}+F_{qg\gamma}\big)\, t^c\,\mathcal{U}(x_q)\mathcal{W}(x_g)\big|(\boldsymbol{q})[p^+-k_\gamma^+-k_g^+,x_q];(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma];(\boldsymbol{g})[k_g,x_g]\big\rangle$$





What about scattering of intermediate states?



‘hidden’ in the bare states…

$$|q\rangle = |q\rangle_{\text{dressed}} - g_e F_{q\gamma}|q\gamma\rangle - g_s F_{qg}|qg\rangle - g_e g_s F_{q\gamma g}|q\gamma g\rangle + \mathcal{O}(g_{e,s}^2)...$$

$$|qg\rangle = |qg\rangle_{\text{dressed}} - g_e F_{q\gamma}|q\gamma g\rangle + \mathcal{O}(g_{e,s}^2)...$$

$$|q\gamma\rangle = |q\gamma\rangle_{\text{dressed}} - g_s F_{qg}|q\gamma g\rangle + \mathcal{O}(g_{e,s}^2)...$$

$$|q\gamma g\rangle = |q\gamma g\rangle_{\text{dressed}} + \mathcal{O}(g_{e,s}^2)...$$

Complete outgoing quark state

$$\big|(\boldsymbol{q})[p^+,0]^i_s\big\rangle_{\text{out}} = g_e g_s \int_{k_g^+,k_\gamma^+}\int_{\omega,v,x_\gamma,x_g,x_q}\big|(\boldsymbol{q})[p^+-k_\gamma^+-k_g^+,x_q]^j_{s'};(\boldsymbol{\gamma})[k_\gamma^+,x_\gamma]_\lambda;(\boldsymbol{g})[k_g,x_g]^c_\lambda\big\rangle$$
$$\times\Big\{\delta^{(2)}\big(\omega-\xi_\gamma x_\gamma-\bar{\xi}_\gamma v\big)\delta^{(2)}\big(v-\bar{\tilde{\xi}}_g x_q-\tilde{\xi}_g x_g\big)\Big[-i\phi^{\bar{\lambda}\lambda}_{s's}(\xi_\gamma)\Big]\Big[-i\phi^{\bar{\eta}\eta}_{s''s'}(\xi_g)\Big]A^{\bar{\eta}}(x_q-x_g)$$
$$\times\Big[\mathcal{A}^{\bar{\lambda}}(v-x_\gamma)\Big(t^d_{il}\mathcal{U}_{lj}(x_q)\mathcal{W}_{dc}(x_g)-\mathcal{U}_{il}(\omega)t^c_{lj}\Big)-A^{\bar{\lambda}}(v-x_\gamma)\Big(\mathcal{U}_{il}(v)t^c_{lj}-\mathcal{U}_{il}(\omega)t^c_{lj}\Big)\Big]$$
$$+\delta^{(2)}\big(\omega-\xi_g x_g-\bar{\xi}_g v\big)\delta^{(2)}\big(v-\bar{\tilde{\xi}}_\gamma x_\gamma-\tilde{\xi}_\gamma x_\gamma\big)\Big[-i\phi^{\bar{\lambda}\lambda}_{s's}(\xi_g)\Big]\Big[-i\phi^{\bar{\eta}\eta}_{s''s'}(\xi_\gamma)\Big]A^{\bar{\eta}}(x_q-x_\gamma)$$
$$\times\Big[\mathcal{A}^{\bar{\lambda}}(v-x_g)\Big(t^d_{il}\mathcal{U}_{lj}(x_q)\mathcal{W}_{dc}(x_g)-\mathcal{U}_{il}(\omega)t^c_{lj}\Big)-A^{\bar{\lambda}}(v-x_g)\Big(t^d_{il}\mathcal{U}_{lj}(x_q)\mathcal{W}_{dc}(x_g)-\mathcal{U}_{il}(\omega)t^c_{lj}\Big)\Big]\Big\}$$

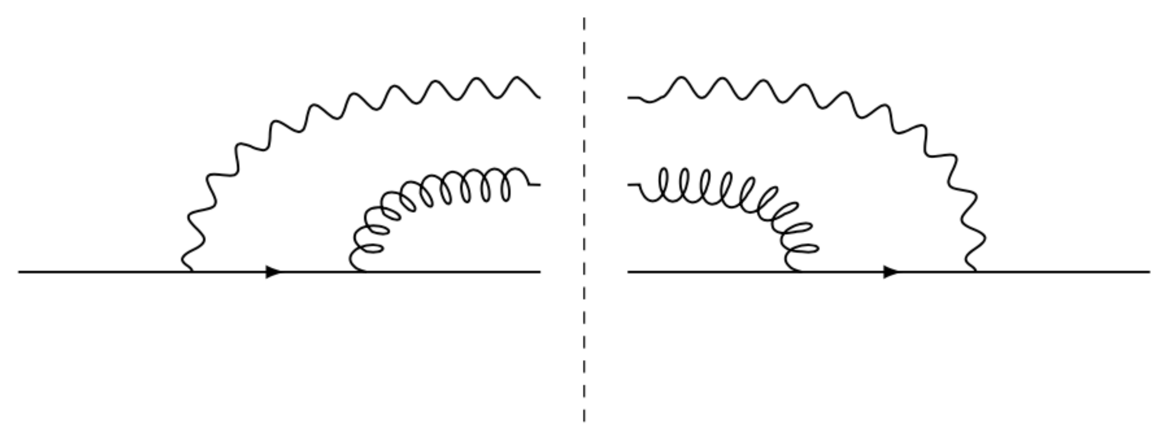
obtain cross section from acting with counting operators $N_q,\, N_g,\, N_\gamma$ on this outgoing state:

$$2k_q^+2k_\gamma^+2k_g^+\,(2\pi)^9\,2p^+2\pi\delta\big(p^+-k_q^+-k_g^+-k_\gamma^+\big)\frac{\mathrm{d}\sigma^{qA\to qg\gamma}}{\mathrm{d}^3\vec{k}_q\mathrm{d}^3\vec{k}_g\mathrm{d}^3\vec{k}_\gamma}=\frac{1}{2N_c}\langle\text{out}|N_qN_gN_\gamma|\text{out}\rangle$$

One contribution to the cross section

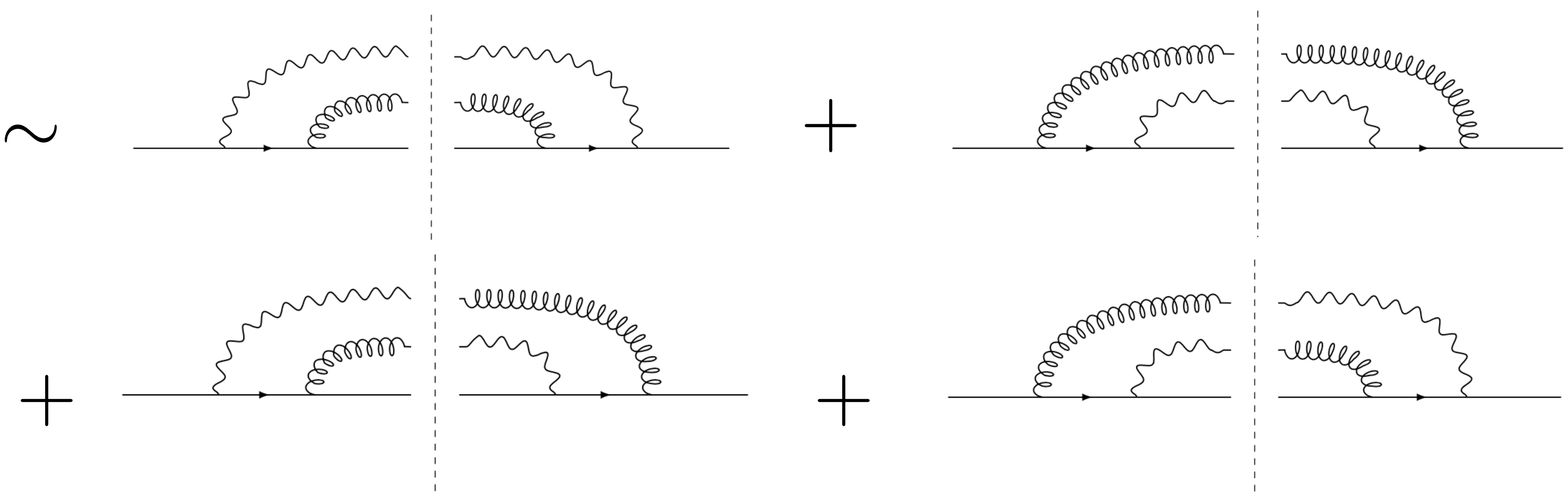
$$
\begin{aligned}
&(2\pi)^9 \frac{\mathrm{d}\sigma^{qA\to q\gamma g+X}}{\mathrm{d}^3\vec{q}_1\mathrm{d}^3\vec{q}_2\mathrm{d}^3\vec{q}_3} = \frac{g_s^2 g_e^2}{N_c} 2\pi\delta\left(p^+ - q_1^+ - q_2^+ - q_3^+\right) \sum_{i=1,2,3} \int_{z_i,y_i} e^{iq_i(y_i-z_i)} \frac{1}{(p^+)^2} \frac{1}{\xi_1\xi_2} \left[\Psi^{\mathrm{bb}}_{\mathrm{even}}\delta^{kk'}\delta^{ll'} + \Psi^{\mathrm{bb}}_{\mathrm{odd}}\epsilon^{kk'}\epsilon^{ll'}\right] \\
&\times \int_{\omega,v,\omega',v',z_i,y_i} \delta^{(2)}\left[\omega - \xi_1 z_1 - \bar{\xi}_1 v\right] \delta^{(2)}\left[\omega' - \xi_1 y_1 - \bar{\xi}_1 v'\right] \delta^{(2)}\left[v - \tilde{\xi}_2 z_2 - \bar{\tilde{\xi}}_2 z_3\right] \delta^{(2)}\left[v' - \tilde{\xi}_2 y_2 - \bar{\tilde{\xi}}_2 y_3\right] \\
&\times A^{k'}\left(z_3 - z_2\right) A^{k}\left(y_3 - y_2\right) \Bigg\{ \\
&\mathcal{A}^{l'}\left(v - z_1\right) \mathcal{A}^{l}\left(v' - y_1\right) \left[\frac{N_c^2}{2}\Big(Q\left(y_3,y_2,z_2,z_3\right) s\left(y_2,z_2\right) - s\left(y_3,y_2\right) s\left(y_2,\omega\right) - s\left(z_2,z_3\right) s\left(\omega',z_2\right) + s\left(\omega',\omega\right)\Big)\right. \\
&\left. + \frac{1}{2}\Big(s\left(\omega',z_3\right) + s\left(y_3,\omega\right) - s\left(y_3,z_3\right) - s\left(\omega',\omega\right)\Big)\right] \\
&+ A^{l'}\left(v - z_1\right) A^{l}\left(v' - y_1\right) \frac{N_c^2 - 1}{2}\Big(s\left(v',v\right) - s\left(v',\omega\right) - s\left(\omega',v\right) s + s\left(\omega',\omega\right)\Big) \\
&- A^{l'}\left(v - z_1\right) \mathcal{A}^{l'}\left(v' - y_1\right) \left[\frac{N_c^2}{2}\Big(s\left(y_3,y_2\right) s\left(y_2,v\right) - s\left(y_3,y_2\right) s\left(y_2,\omega\right) - s\left(\omega',v\right) + s\left(\omega',\omega\right)\Big)\right. \\
&\left. + \frac{1}{2}\Big(s\left(y_3,\omega\right) - s\left(y_3,v\right) + s\left(\omega',v\right) - s\left(\omega',\omega\right)\Big)\right] \\
&- \mathcal{A}^{l'}\left(v - z_1\right) A^{l'}\left(v' - y_1\right) \left[\frac{N_c^2}{2}\Big(s\left(z_2,z_3\right) s\left(v',z_2\right) - s\left(z_2,z_3\right) s\left(\omega',z_2\right) - s\left(v',\omega\right) + s\left(\omega',\omega\right)\Big)\right. \\
&\left. + \frac{1}{2}\Big(s\left(\omega',z_3\right) - s\left(v',z_3\right) + s\left(v',\omega\right) - s\left(\omega',\omega\right)\Big)\right]\Bigg\} .
\end{aligned}
$$

photon emitted before gluon

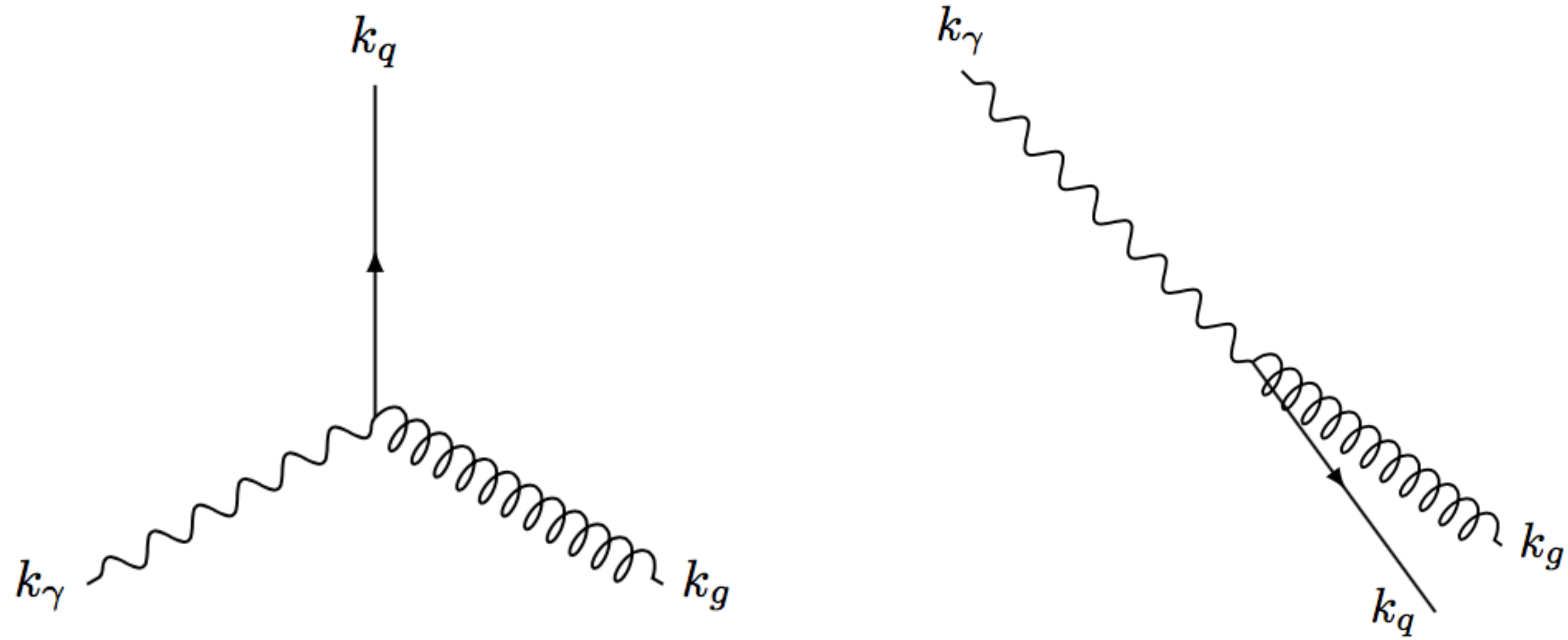


Cross section (schematic)

$$\mathrm{d}\sigma \sim$$



Correlation limit in momentum space



$$q_T = k_q + k_g + k_\gamma$$

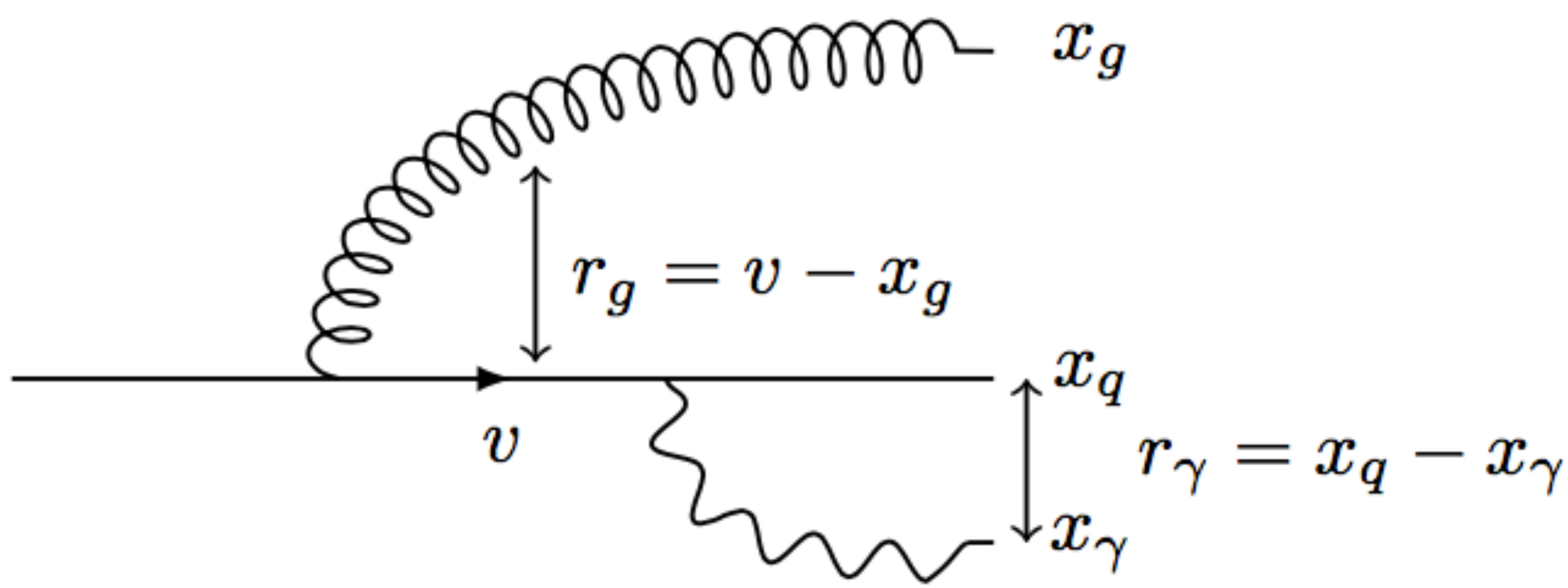
$$q_T \sim Q_s \ll P_T \sim k_q \sim k_g \sim k_\gamma$$

rewrite phases in cross section as, e.g.

$$\int_{x_\gamma, x_g, x_q, x'_\gamma, x'_g, x'_q} e^{ik_q(x_q - x'_q)} e^{ik_g(x_g - x'_g)} e^{ik_\gamma(x_\gamma - x'_\gamma)} = \int_{r_g, r_\gamma, \bar{r}_g, \bar{r}_\gamma, v, v'} e^{iq_T\left(v' - v\right)} e^{iK\left(\bar{r}_\gamma - r_\gamma\right)} e^{iQ\left(\bar{r}_\gamma - r_\gamma\right)}$$

$$K = k_\gamma \qquad Q = \bar{\tilde{\xi}}_g k_g - \tilde{\xi}_g k_q$$

Correlation limit in coordinate space



Expansion of the cross section in the small-dipole limit:

$$r_g, r_\gamma \ll 1$$

is the leading contribution to the cross section since $A^i(r) \propto \frac{r^i}{r^2}$

Cross section in the correlation limit

$$\frac{\mathrm{d}\sigma^{pA\to q\gamma g+X}}{\mathcal{P.S.}} = \frac{\alpha_s^2\alpha_e}{(2\pi)^2} x_p g(x_p, Q^2)(1-\xi_\gamma-\xi_g)$$

gluon channel

$$\times \sum_{l=1}^{3} H_l^{ij}(k_q, k_g, k_\gamma, \xi_g, \xi_\gamma)\left[\delta^{ij}\mathcal{F}_{gg}^{(l)}(x, q_T) + \left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}_{gg}^{(l)}(x, q_T)\right]$$

$$+ \frac{\alpha_s^2\alpha_e}{(2\pi)^2} x_p f_q(x_p, Q^2)(1-\xi_\gamma-\xi_g)$$

quark channel

$$\times \sum_{l=1}^{2} H_l^{ij}(k_q, k_g, k_\gamma, \xi_g, \xi_\gamma)\left[\delta^{ij}\mathcal{F}_{qg}^{(l)}(x, q_T) + \left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}_{qg}^{(l)}(x, q_T)\right]$$

sensitive to linearly polarized gluons!

$$\mathcal{H}^{(i)}(x, q_T)$$

Akcakaya, Schaefer, Zhou (2013)
Dumitru, Lappi, Skokov (2015)
Marquet, Petreska, Roiesnel (2016)
Marquet, Roiesnel, PT (2017)

Cross section in the correlation limit

$$\int_{v,v'} e^{iq_T(v'-v)}\mathrm{Tr}\left[(\partial^i U^\dagger_{v'})(\partial^j U_v)\right] = \alpha_s 8\pi^4 N_c\left[\frac{1}{2}\delta^{ij}\mathcal{F}^{(1)}_{qg}(x,q_T) + \frac{1}{2}\left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}^{(1)}_{qg}(x,q_T)\right]$$

$$\int_{v,v'} e^{iq_T(v'-v)}\mathrm{Tr}\left[(\partial^i U^\dagger_{v'})U_{v'}(\partial^j U^\dagger_v)U_v\right]\mathrm{Tr}\left[U^\dagger_{v'}U_v\right] = \alpha_s 8\pi^4 N_c\left[\frac{1}{2}\delta^{ij}\mathcal{F}^{(2)}_{qg}(x,q_T) + \frac{1}{2}\left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}^{(2)}_{qg}(x,q_T)\right]$$

$$\int_{v,v'} e^{iq_T(v'-v)}\left(\mathrm{Tr}\left[U^\dagger_v U_{v'}\right]\mathrm{Tr}\left[(\partial^i U^\dagger_{v'})(\partial^j U_v)\right] + h.c.\right) = \alpha_s 8\pi^4 N_c\left[\frac{1}{2}\delta^{ij}\mathcal{F}^{(1)}_{gg}(x,q_T) + \frac{1}{2}\left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}^{(1)}_{gg}(x,q_T)\right]$$

$$\int_{v,v'} e^{iq_T(v'-v)}\left(\mathrm{Tr}\left[U_v(\partial^i U^\dagger_{v'})\right]\left[(U^\dagger_v(\partial^j U_{v'})\right] + h.c.\right) = \alpha_s 8\pi^4 N_c\left[\frac{1}{2}\delta^{ij}\mathcal{F}^{(2)}_{gg}(x,q_T) + \frac{1}{2}\left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}^{(2)}_{gg}(x,q_T)\right]$$

$$\int_{v,v'} e^{iq_T(v'-v)}\mathrm{Tr}\left[(\partial^i U^\dagger_{v'})U_{v'}(\partial^j U^\dagger_v)U_v\right] = \alpha_s 8\pi^4 N_c\left[\frac{1}{2}\delta^{ij}\mathcal{F}^{(3)}_{gg}(x,q_T) + \frac{1}{2}\left(\frac{2q_T^i q_T^j}{q_T^2} - \delta^{ij}\right)\mathcal{H}^{(3)}_{gg}(x,q_T)\right]$$

Cross section in the correlation limit

$$H_l^{ij} = \mathcal{I}^{(l)}_{KK} K^i K^j + \mathcal{I}^{(l)}_{QQ} Q^i Q^j + \mathcal{I}^{(l)}_{\tilde{K}\tilde{K}} \tilde{K}^i \tilde{K}^j + \mathcal{I}^{(l)}_{\tilde{Q}\tilde{Q}} \tilde{Q}^i \tilde{Q}^j + \mathcal{I}^{(l)}_{K\tilde{K}} K^{\{i} \tilde{K}^{j\}} + \mathcal{I}^{(l)}_{Q\tilde{Q}} Q^{\{i} \tilde{Q}^{j\}}$$
$$+ \mathcal{I}^{(l)}_{KQ} K^{\{i} Q^{j\}} + \mathcal{I}^{(l)}_{\tilde{K}\tilde{Q}} \tilde{K}^{\{i} \tilde{Q}^{j\}} + \mathcal{I}^{(l)}_{K\tilde{Q}} K^{\{i} \tilde{Q}^{j\}} + \mathcal{I}^{(l)}_{\tilde{K}Q} \tilde{K}^{\{i} Q^{j\}} + \mathcal{I}^{(l)}_{\delta} \delta^{ij} \quad j = 1, 2\,,$$

$$\mathcal{I}^{(1)}_{KQ} = \frac{1}{N_c}\Bigg[\frac{C_\gamma \xi_\gamma \left(N_c^2 \bar{\tilde{\xi}}_g + \tilde{\xi}_g\right)\left(\Psi^{\rm bb}_{\rm odd}\left(Q^2 + C_\gamma K^2\right)^2 - \Psi^{\rm bb}_{\rm even}\left(Q^2 - C_\gamma K^2\right)^2\right)}{2K^2 Q^2 \left(Q^2 + C_\gamma K^2\right)^4}$$
$$+ \frac{C_\gamma^2 K \xi_\gamma \tilde{\xi}_\gamma \Psi^{\rm cross}_{\rm odd} \cos\left(\widehat{K\tilde{Q}}\right)}{\tilde{Q} Q^2 \left(Q^2 + C_\gamma K^2\right)^2 \left(\tilde{K}^2 + C_g \tilde{Q}^2\right)} + \frac{C_\gamma^2 C_g K \xi_\gamma \left(\bar{\xi}_g N_c^2 + \xi_g\right) \Psi^{\rm cross}_{\rm even} \cos\left(\widehat{K\tilde{K}}\right)}{\tilde{K} Q^2 \left(Q^2 + C_\gamma K^2\right)^2 \left(\tilde{K}^2 + C_g \tilde{Q}^2\right)}$$
$$- \frac{C_g Q \left(\bar{\xi}_g \bar{\tilde{\xi}}_g N_c^2 - \xi_g \tilde{\xi}_g\right) \Psi^{\rm cross}_{\rm odd} \cos\left(\widehat{\tilde{K}Q}\right)}{\tilde{K} K^2 \left(Q^2 + C_\gamma K^2\right)^2 \left(\tilde{K}^2 + C_g \tilde{Q}^2\right)} + \frac{\tilde{\xi}_\gamma \tilde{\xi}_g Q \Psi^{\rm cross}_{\rm even} \cos\left(\widehat{Q\tilde{Q}}\right)}{\tilde{Q} K^2 \left(Q^2 + C_\gamma K^2\right)^2 \left(\tilde{K}^2 + C_g \tilde{Q}^2\right)}\Bigg]$$

$$C_\gamma = \frac{\tilde{\xi}_g \bar{\tilde{\xi}}_g}{\xi_\gamma},$$

$$C_g = \frac{\tilde{\xi}_\gamma \bar{\tilde{\xi}}_\gamma}{\xi_g}$$

Conclusions

- CGC calculation can be mapped onto a TMD factorized expression
- ‘sensitive’ to linearly polarized gluons
- first step towards more complicated calculations such as three-jet production and NLO photon-jet pair production

Thanks to the organizers for the nice conference,
thanks to you for your attention!