Double and triple inclusive gluon production: Quantum interference in pA collisions

Tolga Altinoluk

National Centre for Nuclear Research, Warsaw

Probing QCD at the high energy frontier
ECT*, Trento, Italy

May 23, 2018

T. A. , N. Armesto, A. Kovner, M. Lublinsky - arXiv:1805.07739 [hep-ph].

Narodowe Centrum Badań Jądrowych
National Centre for Nuclear Research
ŚWIERK
JRC collaboration partner

Outline

- short introduction
- double inclusive gluon production - Quantum interference effects
- triple inclusive gluon production - Quantum interference effects
- conclusions

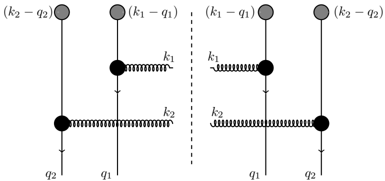
Correlations within the CGC - I

Final state correlations carry the imprint of the partonic correlations that exist in the initial state:

"Glasma Graph" contributions to particle production:

- A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A 810 (2008) 91
- A. Dumitru, F. Gelis, J. Jalilian-Marian, T. Lappi, Phys.Lett. B697 (2011) 21
- K. Dusling, R. Venugopalan, Phys.Rev.Lett. 108 (2012) 262001
- K. Dusling, R. Venugopalan, Phys.Rev. D87 (2013) 051502
- K. Dusling, R. Venugopalan, Phys.Rev. D87 (2013) 054014
- K. Dusling, R. Venugopalan, Phys. Rev. D 87 (2013) no.9, 094034
- K. Dusling, M. Mace, R. Venugopalan, Phys. Rev. Lett. 120 (2018) no.4, 042002
- K. Dusling, M. Mace, R. Venugopalan, Phys. Rev. D 97 (2018) no.1, 016014

Correlations within the CGC - II

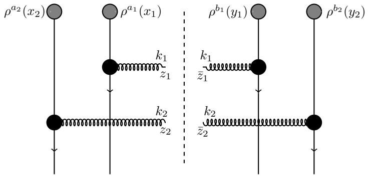


Glasma graph calculation contains two physical effects:

- Bose enhancement of the gluons in the projectile wave function.
  T.A., N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, Phys.Lett. B751 (2015) 448-452
  $\sigma \propto \left[\delta^{(2)}(k_1 - q_1 - k_2 + q_2) + \delta^{(2)}(k_1 - q_1 + k_2 - q_2)\right]$
- Hanbury-Brown-Twiss (HBT) correlations between gluons far separated in rapidity.
  $\sigma \propto \left[\delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2)\right]$
  - k_T-factorized approach
    Y. V. Kovchegov, D. E. Wertepny, Nucl. Phys. A 906 (2013) 50
    Y. V. Kovchegov, D. E. Wertepny, Nucl. Phys. A 925 (2014) 254
  - Glasma graph approach:
    T.A., N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, Phys.Lett. B752 (2016) 113-121

Glasma graph approach dilute-dense collisions: k_T-factorized approach
T.A., N. Armesto, D. E. Wertepny, arXiv:1804.02910 [hep-ph]

Double inclusive gluon production



$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)}$$

$$\times \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)\Big\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\Big\rangle_P$$

$$\times\Big\langle \big[U(z_1)-U(x_1)\big]^{a_1c}\big[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\big]^{cb_1}\big[U(z_2)-U(x_2)\big]^{a_2d}\big[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\big]^{db_2}\Big\rangle_T$$

Weiszacker-Williams field A^i is given by

$$A^i(x-y) = -\frac{1}{2\pi}\frac{(x-y)_i}{(x-y)^2} = \int \frac{d^2k}{(2\pi)^2} e^{-ik\cdot(x-y)}\frac{k^i}{k^2}$$

Projectile averaging in double inclusive gluon production

The color charge densities factorizes into a product of all possible Wick contractions:

$$\left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P = \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\right\rangle_P \left\langle \rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$
$$+ \left\langle \rho^{a_1}(x_1)\rho^{b_1}(y_1)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_2}(y_2)\right\rangle_P + \left\langle \rho^{a_1}(x_1)\rho^{b_2}(y_2)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_1}(y_1)\right\rangle_P$$

Average of two projectile color charges we take a general form:

$$\left\langle \rho^{a}(x)\rho^{b}(y)\right\rangle_P = \delta^{ab}\mu^2(x,y)$$

The double inclusive production cross section:

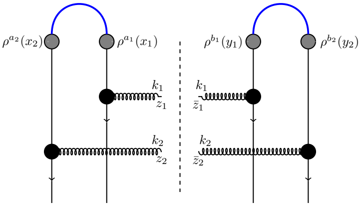
$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$
$$\times \Big\{ \mu^2(x_1,x_2)\,\mu^2(y_1,y_2) \left\langle \mathrm{tr}\left\{ \left[U(z_1)-U(x_1)\right]\left[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\right]\left[U(\bar{z}_2)-U(y_2)\right]\left[U^\dagger(z_2)-U^\dagger(x_2)\right]\right\}\right\rangle_T$$
$$+\mu^2(x_1,y_1)\,\mu^2(x_2,y_2) \left\langle \mathrm{tr}\left\{ \left[U(z_1)-U(x_1)\right]\left[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\right]\right\} \mathrm{tr}\left\{\left[U(z_2)-U(x_2)\right]\left[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\right]\right\}\right\rangle_T$$
$$+\mu^2(x_1,y_2)\,\mu^2(x_2,y_1) \left\langle \mathrm{tr}\left\{ \left[U(z_1)-U(x_1)\right]\left[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\right]\left[U(z_2)-U(x_2)\right]\left[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\right]\right\}\right\rangle_T \Big\}$$

Double inclusive gluon production

For a translationally invariant target:

$$\langle D(x_1,x_2)\rangle_T = \int \frac{d^2q_1}{(2\pi)^2} e^{-iq_1\cdot(x_1-x_2)}\langle D(q_1)\rangle_T$$

$$\langle Q(x_1,x_2,x_3,x_4)\rangle_T = \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2\Delta}{(2\pi)^2} e^{-iq_1\cdot(x_1-x_2)} e^{-iq_2\cdot(x_3-x_4)} e^{-i\frac{\Delta}{2}\cdot(x_1+x_2-x_3-x_4)}\langle Q(q_1,q_2,\Delta)\rangle_T$$



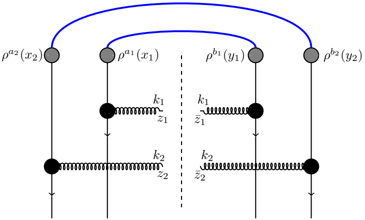
TYPE A

$$\propto \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\right\rangle_P \left\langle \rho^{b_1}(y_1)\rho^{b_2}(y_2)\right\rangle_P$$

with $L^i(k,q) \equiv \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}$

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2}\right|_{\text{type A}} = \alpha_s^2(4\pi)^2(N_c^2-1)\int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2\Delta}{(2\pi)^2}\,\langle Q(q_1,q_2,\Delta)\rangle_T$$
$$\times\, \mu^2\Big[(k_1-q_1-\Delta/2),(k_2+q_2+\Delta/2)\Big]\mu^2\Big[-(k_1-q_1+\Delta/2),-(k_2+q_2-\Delta/2)\Big]$$
$$\times\, L^i(k_1,q_1+\Delta/2)L^i(k_1,q_1-\Delta/2)\,L^j(k_2,-q_2+\Delta/2)L^j(k_2,-q_2-\Delta_2)$$

Double inclusive gluon production

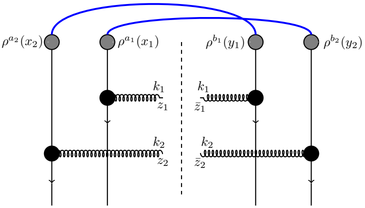


TYPE B

$$\propto \left\langle \rho^{a_1}(x_1)\rho^{b_1}(y_1)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_2}(y_2)\right\rangle_P$$

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2}\bigg|_{\text{type B}} = \alpha_s^2(4\pi)^2(N_c^2-1)^2 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\langle D(q_1)D(q_2)\rangle_T$$
$$\times\, \mu^2\Big[(k_1-q_1),-(k_1-q_2)\Big]\mu^2\Big[(k_2-q_2),-(k_2-q_2)\Big] L^i(k_1,q_1)L^i(k_1,q_1)\; L^j(k_2,q_2)L^j(k_2,q_2)$$

Projectile averaging in double inclusive gluon production



TYPE C

$$\propto \left\langle \rho^{a_1}(x_1)\rho^{b_2}(y_2)\right\rangle_P \left\langle \rho^{a_2}(x_2)\rho^{b_1}(y_1)\right\rangle_P$$

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2}\bigg|_{\text{type C}} = \alpha_s^2(4\pi)^2(N_c^2-1)\int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2\Delta}{(2\pi)^2}\,\langle Q(q_1,q_2,\Delta)\rangle_T$$
$$\times\, \mu^2\Big[-(k_1-q_1-\Delta/2),(k_2-q_2-\Delta/2)\Big]\mu^2\Big[-(k_2-q_2+\Delta/2),(k_1-q_1+\Delta/2)\Big]$$
$$\times\, L^i(k_1,q_1+\Delta/2)L^i(k_1,q_1-\Delta/2)\,L^j(k_2,q_2-\Delta/2)L^j(k_2,q_2+\Delta/2)$$

Target Averaging in double inclusive production

- The cross section involves integration over the four coordinates $\Rightarrow$ the main contribution will come from the region where as many points are far away from each other as possible .
- However, all four points can not be far away from each other since the target field ensemble has to be color invariant.
- color neutralization in the target ensemble is achieved on transverse distance scales of order $1/Q_s$.
- in order to have a non vanishing S-matrix, the objects that scatter must be color singlets of size of $O(1/Q_s)$.

$\Rightarrow$**the maximal contribution must come from the configurations where the four points are combined into pairs, such that each pair is a singlet and the distance between the pairs is large.**

Taking into account only such configurations is equivalent to the calculation of target averages in which one factorizes the average of a product of any number of U matrices into averages of pairs with the basic "Wick contraction" given by

$$\left\langle U^{ab}(x)U^{cd}(y)\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{(N_c^2-1)^2}\left\langle \mathrm{tr}[U(x)U^\dagger(y)]\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{N_c^2-1}d(x,y)$$

where

$$d(x,y) \equiv \langle D(x,y)\rangle_T$$

Target Averaging in double inclusive production

Using these physical assumptions

$$\langle Q(x,y,z,v)\rangle_T = d(x,y)d(z,v) + d(x,v)d(z,y) + \frac{1}{N_c^2-1}d(x,z)d(y,v)$$

$$\langle D(x,y)D(z,v)\rangle_T = d(x,y)d(z,v) + \frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right]$$

should be plugged in the double inclusive gluon production cross section

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)} \int_{x_1x_2y_1y_2} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$

$$\times\Big\{\mu^2(x_1,x_2)\,\mu^2(y_1,y_2)\Big\langle \mathrm{tr}\Big\{\big[U(z_1)-U(x_1)\big]\big[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\big]\big[U(\bar{z}_2)-U(y_2)\big]\big[U^\dagger(z_2)-U^\dagger(x_2)\big]\Big\}\Big\rangle_T$$

$$+\mu^2(x_1,y_1)\,\mu^2(x_2,y_2)\Big\langle \mathrm{tr}\Big\{\big[U(z_1)-U(x_1)\big]\big[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\big]\Big\}\mathrm{tr}\Big\{\big[U(z_2)-U(x_2)\big]\big[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\big]\Big\}\Big\rangle_T$$

$$+\mu^2(x_1,y_2)\,\mu^2(x_2,y_1)\Big\langle \mathrm{tr}\Big\{\big[U(z_1)-U(x_1)\big]\big[U^\dagger(\bar{z}_1)-U^\dagger(y_1)\big]\big[U(z_2)-U(x_2)\big]\big[U^\dagger(\bar{z}_2)-U^\dagger(y_2)\big]\Big\}\Big\rangle_T\Big\}$$

Factorized cross section

Plug in the factorized double-dipole and quadrupole amplitudes in the double inclusive cross section:

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2(N_c^2-1)^2 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2} d(q_1)d(q_2)\left\{ I_0 + \frac{1}{N_c^2-1}I_1 + \frac{1}{(N_c^2-1)^2}I_2 \right\}$$

with

$$I_0 = \mu^2\Big[(k_1-q_1),(q_1-k_1)\Big]\mu^2\Big[(k_2-q_2),(q_2-k_2)\Big]L^i(k_1,q_1)L^i(k_1,q_1)L^j(k_2,q_2)L^j(k_2,q_2)$$

$$\begin{aligned} I_1 =\; & \mu^2\Big[(k_1-q_1),(q_2-k_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_1)\Big]L^i(k_1,q_1)L^i(k_1,q_1)L^j(k_2,q_2)L^j(k_2,q_2) \\ & + \mu^2\Big[(k_1-q_1),(q_1-k_2)\Big]\mu^2\Big[(k_2-q_2),-(k_1-q_2)\Big]L^i(k_1,q_1)L^i(k_1,q_2)L^j(k_2,q_1)L^j(k_2,q_2) \\ & + (k_2\to -k_2) \end{aligned}$$

$$\begin{aligned} I_2 =\; & \mu^2\Big[(k_1-q_1),-(k_1-q_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_2)\Big]L^i(k_1,q_1)L^i(k_1,q_2)L^j(k_2,-q_1)L^j(k_2,-q_2) \\ & + (k_2\to -k_2) \\ & + \mu^2\Big[(k_1-q_1),-(k_2-q_2)\Big]\mu^2\Big[-(k_1+q_2),(q_1+k_2)\Big]L^i(k_1,q_1)L^i(k_1,-q_2)L^j(k_2,-q_1)L^j(k_2,q_2) \\ & + (k_2\to -k_2) \end{aligned}$$

Type A - quadrupole
Type B - double dipole
Type C - quadrupole

Identifying terms in double inclusive production - I

$$\mu^2(k,p) = T\left(\frac{k-p}{2}\right) F[(k+p)R]$$

$R \equiv$ radius of the projectile.
$T \approx$ transverse dependent distribution of the valence charges.
$F(x) \equiv$ soft form factor which is maximal at $x = 0$.

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2(N_c^2-1)^2 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2} d(q_1)d(q_2)\left\{I_0 + \frac{1}{N_c^2-1}I_1 + \frac{1}{(N_c^2-1)^2}I_2\right\}$$

$I_0 \propto \mu^2\Big[(k_1-q_1),(q_1-k_1)\Big]\mu^2\Big[(k_2-q_2),(q_2-k_2)\Big]$

- *classical contribution: emission of two independent gluons*
- originates from Type B - double dipole contribution

I_1 has two contributions:

(i) $\mu^2\Big[(k_1-q_1),(q_2-k_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_1)\Big] \propto F^2\{[(k_1-q_1)-(k_2-q_2)]R\}$

- *forward Bose enhancement of gluons $k_1 - q_1$ and $k_2 - q_2$ in the projectile wave function.*
- originates from Type C - quadrupole contribution

(ii) $\mu^2\Big[(k_1-q_1),(q_1-k_2)\Big]\mu^2\Big[(k_2-q_2),-(k_1-q_2)\Big] \propto F^2[(k_1-k_2)R]$

- *contribution to forward HBT correlations of gluons k_1 and k_2.*
- originates from Type C - quadrupole contribution

• The mirror image $k_2 \to -k_2$ originates from Type A- quadrupole term: Gives contribution to backward HBT/Bose enhancement

Identifying terms in double inclusive production - II

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2(4\pi)^2(N_c^2-1)^2 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2} d(q_1)d(q_2)\left\{ I_0 + \frac{1}{N_c^2-1} I_1 + \frac{1}{(N_c^2-1)^2} I_2 \right\}$$

I_2 has two contributions:

(i) $\mu^2\Big[(k_1-q_1), -(k_1-q_2)\Big]\mu^2\Big[(k_2-q_2),(q_1-k_2)\Big] \propto F^2[(q_1-q_2)R]$

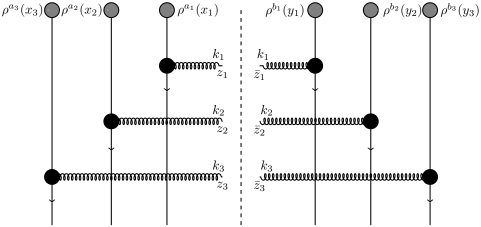
- *forward Bose enhancement of gluons q_1 and q_2 in the target wave function.*
- originates from Type B - double-dipole contribution

(ii) $\mu^2\Big[(k_1-q_1), -(k_2-q_2)\Big]\mu^2\Big[-(k_1+q_2),(q_1+k_2)\Big] \propto F^2[(k_1-q_1)-(k_2-q_2)R]$

- *$O(1/N_c^2)$ correction to the forward Bose enhancement of gluons k_1-q_1 and k_2-q_2.*
- originates from Type C - quadrupole contribution
- its mirror image $k_2 \to -k_2$ originates from Type A - quadrupole term gives $O(1/N_c^2)$ correction to the backward Bose enhancement.

1. *We have identified all the terms in the double inclusive production.*
2. *Leading N_c term gives uncorrelated/classical production.*
3. *The quantum interference effects that we are looking for appears at $O(1/N_c^2)$.*
4. *$O(1/N_c^4)$ terms give target Bose enhancement contribution and N_c-surpassed correction to projectile Bose enhancement contribution.*

Triple inclusive gluon production



$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3 \int_{z_1\bar{\mathbf{z}}_1 z_2\bar{\mathbf{z}}_2 z_3\bar{\mathbf{z}}_3} e^{ik_1\cdot(z_1-\bar{\mathbf{z}}_1)+ik_2\cdot(z_2-\bar{\mathbf{z}}_2)+ik_3\cdot(z_3-\bar{\mathbf{z}}_3)}$$

$$\times \int_{x_1y_1x_2y_2x_3y_3} A^i(x_1-z_1)A^i(\bar{\mathbf{z}}_1-y_1)A^j(x_2-z_2)A^j(\bar{\mathbf{z}}_2-y_2)\,A^k(x_3-z_3)A^k(\bar{\mathbf{z}}_3-y_3)$$

$$\times \left\langle \rho^{a_1}(x_1)\rho^{a_2}(x_2)\rho^{a_3}(x_3)\rho^{b_1}(y_1)\rho^{b_2}(y_2)\rho^{b_3}(y_3)\right\rangle_P$$

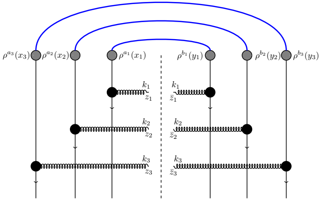
$$\times\left\langle \left(U_{z_1}-U_{x_1}\right)^{a_1c_1}\left(U^\dagger_{\bar{\mathbf{z}}_1}-U^\dagger_{y_1}\right)^{c_1b_1}\left(U_{z_2}-U_{x_2}\right)^{a_2c_2}\left(U^\dagger_{\bar{\mathbf{z}}_2}-U^\dagger_{y_2}\right)^{c_2b_2}\left(U_{z_3}-U_{x_3}\right)^{a_3c_3}\left(U^\dagger_{\bar{\mathbf{z}}_3}-U^\dagger_{y_3}\right)^{c_3b_3}\right\rangle_T$$

Projectile averaging in triple inclusive gluon production

$$\left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle_P = \langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\rangle\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\rangle\langle \rho^{a_3}_{x_3}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\rangle\Big[\langle \rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\rangle\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_2}_{x_2}\rho^{b_3}_{y_3}\rangle\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\rangle\Big]$$
$$+ \langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\rangle\Big[\langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\rangle\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\rangle\Big] + \langle \rho^{a_3}_{x_3}\rho^{b_3}_{y_3}\rangle\Big[\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\rangle + \langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\rangle\langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rangle\Big]$$
$$+ \langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rangle\Big[\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\rangle\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\rangle\Big] + \langle \rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\rangle\Big[\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\rangle\Big]$$
$$+ \langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rangle\Big[\langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\rangle\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\rangle + \langle \rho^{a_1}_{x_1}\rho^{b_3}_{y_3}\rangle\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\rangle\Big] + \langle \rho^{a_2}_{x_2}\rho^{b_3}_{y_3}\rangle\Big[\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\rangle\langle \rho^{a_3}_{x_3}\rho^{b_1}_{y_1}\rangle + \langle \rho^{a_1}_{x_1}\rho^{a_3}_{x_3}\rangle\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\rangle\Big]$$

- three-dipole contribution to the three-gluon production
- dipole-quadrupole contribution to the three-gluon production
- sextupole contribution to the three-gluon production

ddd-contribution to three-gluon production



$\propto \left\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\right\rangle\left\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_3}_{y_3}\right\rangle$

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\bigg|_{\rm ddd} = \alpha_s^3\,(4\pi)^3 \int_{z_1\bar{\mathbf{z}}_1 z_2\bar{\mathbf{z}}_2 z_3\bar{\mathbf{z}}_3} e^{ik_1\cdot(z_1-\bar{\mathbf{z}}_1)+ik_2\cdot(z_2-\bar{\mathbf{z}}_2)+ik_3\cdot(z_3-\bar{\mathbf{z}}_3)} \int_{x_1y_1x_2y_2x_3y_3}$$
$$\times A^i(x_1-z_1)A^i(\bar{\mathbf{z}}_1-y_1)A^j(x_2-z_2)A^j(\bar{\mathbf{z}}_2-y_2)\,A^k(x_3-z_3)A^k(\bar{\mathbf{z}}_3-y_3)\;\mu^2(x_1,y_1)\mu^2(x_2,y_2)\mu^2(x_3,y_3)$$
$$\times\Big\langle \mathrm{tr}\left\{[U_{z_1}-U_{x_1}][U^\dagger_{\bar{\mathbf{z}}_1}-U^\dagger_{y_1}]\right\}\;\mathrm{tr}\left\{[U_{z_2}-U_{x_2}][U^\dagger_{\bar{\mathbf{z}}_2}-U^\dagger_{y_2}]\right\}\;\mathrm{tr}\left\{[U_{z_3}-U_{x_3}][U^\dagger_{\bar{\mathbf{z}}_3}-U^\dagger_{y_3}]\right\}\Big\rangle_T$$

Pairwise factorization of a generic three-dipole amplitude:

$$\Big\langle D(x_1,x_1')D(x_2,x_2')D(x_3,x_3')\Big\rangle_T = d(x_1,x_1')d(x_2,x_2')d(x_3,x_3')$$
$$+\frac{1}{(N_c^2-1)^2}\Big\{d(x_1,x_1')\Big[d(x_2,x_3)d(x_2',x_3')+d(x_2,x_3')d(x_2',x_3)\Big]+d(x_2,x_2')\Big[d(x_1,x_3)d(x_1',x_3')+d(x_1,x_3')d(x_1',x_3)\Big]$$
$$+d(x_3,x_3')\Big[d(x_1,x_2)d(x_1',x_2')+d(x_1,x_2')d(x_1',x_2)\Big]\Big\}+O\left(\frac{1}{(N_c^2-1)^4}\right)$$

ddd-contribution to three-gluon production

the three-dipole contribution to the triple inclusive gluon production cross section as

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\bigg|_{\rm ddd} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\, d(q_1)d(q_2)d(q_3)$$
$$\times\left\{ I_{\rm ddd,0} + \frac{1}{(N_c^2-1)^2}\Big[I_{\rm ddd,1}+I_{\rm ddd,2}+I_{\rm ddd,3}\Big] + \mathcal{O}\left(\frac{1}{(N_c^2-1)^4}\right)\right\}$$

where we have defined $I_{\rm ddd,0}$ as

$$I_{\rm ddd,0} = \mu^2(k_1-q_1, q_1-k_1)\,\mu^2(k_2-q_2, q_2-k_2)\,\mu^2(k_3-q_3, q_3-k_3)$$
$$\times\, L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)$$

$\mathcal{O}\left(1/N_c^4\right)$ terms:

$$I_{\rm ddd,1} = \tilde{I}_{\rm ddd,1} + (k_3 \to -k_3)$$

with

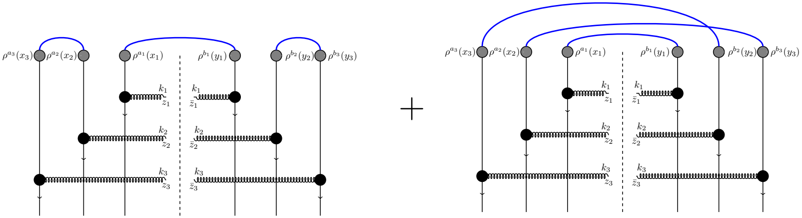
$$\tilde{I}_{\rm ddd,1} = \mu^2(k_1-q_1, q_1-k_1)\,\mu^2(k_2-q_2, q_3-k_2)\,\mu^2(k_3-q_3, q_2-k_3)$$
$$\times\, L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_3)L^k(k_3,q_3)L^k(k_3,q_2)$$

The remaining terms can be defined by using the explicit expression of $I_{\rm ddd,1}$ and the symmetry properties:

$$I_{\rm ddd,2} \equiv \tilde{I}_{\rm ddd,1}(1\leftrightarrow 2) + (k_3\to -k_3)$$
$$I_{\rm ddd,3} \equiv \tilde{I}_{\rm ddd,1}(1\leftrightarrow 3) + (k_2\to -k_2)$$

dQ contribution to three-gluon production

$$\propto \left\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\right\rangle \left[\left\langle \rho^{a_2}_{x_2}\rho^{a_3}_{x_3}\right\rangle\left\langle \rho^{b_2}_{y_2}\rho^{b_3}_{y_3}\right\rangle + \left\langle \rho^{a_2}_{x_2}\rho^{b_3}_{y_3}\right\rangle\left\langle \rho^{a_3}_{x_3}\rho^{b_2}_{y_2}\right\rangle\right] + (1 \leftrightarrow 2) + (1 \leftrightarrow 3)$$



$$\left.\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\right|_{\rm dQ} = \alpha_s^3\,(4\pi)^3 \int_{z_1\bar{\mathbf{z}}_1 z_2\bar{\mathbf{z}}_2 z_3\bar{\mathbf{z}}_3} e^{ik_1\cdot(z_1-\bar{\mathbf{z}}_1)+ik_2\cdot(z_2-\bar{\mathbf{z}}_2)+ik_3\cdot(z_3-\bar{\mathbf{z}}_3)} \int_{x_1y_1x_2y_2x_3y_3}$$

$$\times A^i(x_1-z_1)A^i(\bar{\mathbf{z}}_1-y_1)A^j(x_2-z_2)A^j(\bar{\mathbf{z}}_2-y_2)\,A^k(x_3-z_3)A^k(\bar{\mathbf{z}}_3-y_3)$$

$$\times\Big\langle \mu^2(x_1,y_1)\,\mathrm{tr}\left\{[U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]\right\}$$

$$\times\ \Big(\mu^2(x_2,x_3)\mu^2(y_2,y_3)\ \mathrm{tr}\left\{[U_{\bar{z}_2}-U_{y_2}][U^\dagger_{z_2}-U^\dagger_{x_2}][U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}]\right\}$$

$$+\ \mu^2(x_2,y_3)\mu^2(x_3,y_2)\ \mathrm{tr}\left\{[U_{z_2}-U_{x_2}][U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}][U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}]\right\}\Big)\Big\rangle_T$$

$$+(1\leftrightarrow 2)+(1\leftrightarrow 3)$$

dQ contribution to three-gluon production

Pairwise contraction for a generic dipole-quadrupole term reads

$$\Big\langle D(x_1,x_1')Q(x_2,x_2',x_3,x_3')\Big\rangle_T \approx d(x_1,x_1')\Big[d(x_2,x_2')d(x_3,x_3')+d(x_2,x_3')d(x_3,x_2')\Big]+\frac{1}{N_c^2-1}d(x_1,x_1')d(x_2,x_3)d(x_2',x_3')$$

dQ contribution to the three gluon production:

$$\frac{d\sigma}{d^2k_1d\eta_1\,d^2k_2d\eta_2\,d^2k_3d\eta_3}\bigg|_{\rm dQ} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^2\int\frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\;d(q_1)d(q_2)d(q_3)$$
$$\times\bigg\{\Big[I_{\rm dQ,1}+I_{\rm dQ,2}+I_{\rm dQ,3}\Big]+\frac{1}{N_c^2-1}\Big[I'_{\rm dQ,1}+I'_{\rm dQ,2}+I'_{\rm dQ,3}\Big]+\mathcal{O}\left(\frac{1}{(N_c^2-1)^2}\right)\bigg\}$$

where we have introduced the same notation used in the three-dipole contribution and we define

$$I_{\rm dQ,1}=\tilde I_{\rm dQ,1}+(k_2\to -k_2)$$
$$I'_{\rm dQ,1}=\tilde I'_{\rm dQ,1}+(k_2\to -k_2)$$

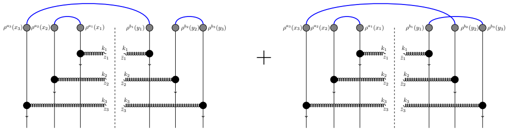
with

$$\begin{aligned}\tilde I_{\rm dQ,1} =\; &\mu^2(k_1-q_1,q_1-k_1)\,\mu^2(k_2-q_2,q_3-k_3)\,\mu^2(k_3-q_3,q_2-k_2)\\ &\times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)\\ &+\mu^2(k_1-q_1,q_1-k_1)\,\mu^2(k_2-q_2,q_2-k_3)\,\mu^2(k_3-q_3,q_3-k_2)\\ &\times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_3)L^k(k_3,q_3)L^k(k_3,q_2)\end{aligned}$$

$$\begin{aligned}\tilde I'_{\rm dQ,1} =\; &\mu^2(k_1-q_1,q_1-k_1)\,\mu^2(-k_2-q_3,q_2+k_3)\,\mu^2(k_2-q_2,q_3-k_3)\\ &\times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,-q_3)L^k(k_3,q_3)L^k(k_3,-q_2)\end{aligned}$$

X contribution to three-gluon production

Sextupole contribution:



$$
\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\Bigg|_{\mathrm{X}} = \alpha_s^3\,(4\pi)^3 \int_{z_1\bar{\mathbf{z}}_1 z_2\bar{\mathbf{z}}_2 z_3\bar{\mathbf{z}}_3} e^{ik_1\cdot(z_1-\bar{\mathbf{z}}_1)+ik_2\cdot(z_2-\bar{\mathbf{z}}_2)+ik_3\cdot(z_3-\bar{\mathbf{z}}_3)} \int_{x_1 y_1 x_2 y_2 x_3 y_3}
$$

$$
\times A^i(x_1-z_1)A^i(\bar{\mathbf{z}}_1-y_1)A^j(x_2-z_2)A^j(\bar{\mathbf{z}}_2-y_2)\,A^k(x_3-z_3)A^k(\bar{\mathbf{z}}_3-y_3)
$$

$$
\times\Big\langle \mu^2(x_2,x_3)\Big(\mu^2(x_1,y_2)\,\mu^2(y_1,y_3)\;\mathrm{tr}\big\{[U_{\bar{z}_1}-U_{y_1}][U^\dagger_{z_1}-U^\dagger_{x_1}][U_{\bar{z}_2}-U_{y_2}][U^\dagger_{z_2}-U^\dagger_{x_2}][U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}]\big\}
$$

$$
+\mu^2(x_1,y_3)\mu^2(y_1,y_2)\;\mathrm{tr}\big\{[U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}][U_{\bar{z}_2}-U_{y_2}][U^\dagger_{z_2}-U_{x_2}][U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}]\big\}\Big)
$$

$$
+\mu^2(x_2,x_1)\Big(\mu^2(x_3,y_2)\mu^2(y_3,y_1)\;\mathrm{tr}\big\{[U_{\bar{z}_3}-U_{y_3}][U^\dagger_{z_3}-U^\dagger_{x_3}][U_{\bar{z}_2}-U_{y_2}][U^\dagger_{z_2}-U^\dagger_{x_2}][U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]\big\}
$$

$$
+\mu^2(x_3,y_1)\mu^2(y_3,y_2)\;\mathrm{tr}\big\{[U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}][U_{\bar{z}_2}-U_{y_2}][U^\dagger_{z_2}-U^\dagger_{x_2}][U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]\big\}\Big)
$$

$$
+\mu^2(x_2,y_1)\Big(\mu^2(x_1,x_3)\mu^2(y_2,y_3)\;\mathrm{tr}\big\{[U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}][U_{z_2}-U_{x_2}][U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}][U_{\bar{z}_3}-U_{y_3}][U^\dagger_{z_3}-U^\dagger_{x_3}]\big\}
$$

$$
+\mu^2(x_1,y_3)\mu^2(x_3,y_2)\;\mathrm{tr}\big\{[U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}][U_{z_2}-U_{x_2}][U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}][U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}]\big\}\Big)
$$

$$
+\mu^2(x_2,y_3)\Big(\mu^2(x_3,x_1)\mu^2(y_2,y_1)\;\mathrm{tr}\big\{[U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}][U_{z_2}-U_{x_2}][U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}][U_{\bar{z}_1}-U_{y_1}][U^\dagger_{z_1}-U^\dagger_{x_1}]\big\}
$$

$$
+\mu^2(x_3,y_1)\mu^2(x_1,y_2)\;\mathrm{tr}\big\{[U_{z_3}-U_{x_3}][U^\dagger_{\bar{z}_3}-U^\dagger_{y_3}][U_{z_2}-U_{x_2}][U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}][U_{z_1}-U_{x_1}][U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]\big\}\Big)\Big\rangle_T .
$$

X contribution to three-gluon production

Pairwise contraction of a generic sextupole term:

$$\left\langle X(x_1,x_1',x_2,x_2',x_3,x_3')\right\rangle_T = d(x_1,x_1')d(x_2,x_2')d(x_3,x_3') + d(x_1,x_3')d(x_2,x_1')d(x_3,x_2')$$
$$+ d(x_1,x_1')d(x_2,x_3')d(x_3,x_2') + d(x_2,x_2')d(x_3,x_1')d(x_1,x_3') + d(x_3,x_3')d(x_1,x_2')d(x_2,x_1') + O\left(\frac{1}{N_c^2-1}\right)$$

X-contribution to the three gluon production cross section

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3}\right|_{\mathrm{X}} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)\int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$
$$\times\left\{\left[I_{\mathrm{X},1}+I_{\mathrm{X},2}+I_{\mathrm{X},3}+I_{\mathrm{X},4}+I_{\mathrm{X},5}\right] + \mathcal{O}\left(\frac{1}{(N_c^2-1)}\right) + \mathcal{O}\left(\frac{1}{(N_c^2-1)^2}\right)\right\}$$

where

$$I_{\mathrm{X},1} = \left[\tilde{I}_{\mathrm{X},1} + (k_3\to -k_3)\right] + \left[\tilde{I}'_{\mathrm{X},1} + (k_1\to -k_1)\right]$$

The terms $I_{\mathrm{X},2}$ and $I_{\mathrm{X},3}$ can again be defined by using the symmetry properties as

$$I_{\mathrm{X},2} = \left[\tilde{I}_{\mathrm{X},1}(1\to 2, 2\to 3, 3\to 1) + (k_3\to -k_3)\right] + \left[\tilde{I}'_{\mathrm{X},1}(1\to 2, 2\to 3, 3\to 1) + (k_1\to -k_1)\right]$$
$$I_{\mathrm{X},3} = \left[\tilde{I}_{\mathrm{X},1}(1\to 3, 3\to 2, 2\to 1) + (k_3\to -k_3)\right] + \left[\tilde{I}'_{\mathrm{X},1}(1\to 3, 3\to 2, 2\to 1) + (k_1\to -k_1)\right]$$

Identifying terms in triple inclusive gluon production

Three-gluon cross section can be organized as

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$
$$\times\Big\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\Big[I_{\mathrm{dQ},1}+I_{\mathrm{dQ},2}+I_{\mathrm{dQ},3}\Big]$$
$$+\frac{1}{(N_c^2-1)^2}\Big(\Big[I_{\mathrm{ddd},1}+I_{\mathrm{ddd},2}+I_{\mathrm{ddd},3}\Big]+\Big[I'_{\mathrm{dQ},1}+I'_{\mathrm{dQ},2}+I'_{\mathrm{dQ},3}\Big]+\Big[I_{\mathrm{X},1}+I_{\mathrm{X},2}+I_{\mathrm{X},3}+I_{\mathrm{X},4}+I_{\mathrm{X},5}\Big]\Big)\Big\}$$

$I_{ddd,0} \propto \mu^2(k_1-q_1,q_1-k_1)\mu^2(k_2-q_2,q_2-k_2)\mu^2(k_3-q_3,q_3-k_3)$

- *classical contribution*: emission of three independent gluons.

$I_{dQ,1}$ has two contributions:

(i) $\mu^2(k_1-q_1,q_1-k_1)\,\mu^2(k_2-q_2,q_3-k_3)\,\mu^2(k_3-q_3,q_2-k_2) \propto F^2\,\{[(k_2-q_2)-(k_3-q_3)]\,R\}$

- *Independent emission of k_1 and Bose enhancement of k_2-q_2 and k_3-q_3.*

(ii) $\mu^2(k_1-q_1,q_1-k_1)\,\mu^2(k_2-q_2,q_2-k_3)\,\mu^2(k_3-q_3,q_3-k_2) \propto F^2\,[(k_2-k_3)R]$

- *Independent emission of k_1 and HBT of k_2 and k_3.*

- $I_{\mathrm{dQ},2}$ and $I_{\mathrm{dQ},3}$ exhibit the same behavior as $I_{\mathrm{dQ},1}$ but with gluons interchanged.

Identifying terms in triple inclusive gluon production - II

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$
$$\times\Big\{ I_{\rm ddd,0} + \frac{1}{N_c^2-1}\Big[I_{\rm dQ,1}+I_{\rm dQ,2}+I_{\rm dQ,3}\Big]$$
$$+\frac{1}{(N_c^2-1)^2}\bigg(\Big[I_{\rm ddd,1}+I_{\rm ddd,2}+I_{\rm ddd,3}\Big]+\Big[I'_{\rm dQ,1}+I'_{\rm dQ,2}+I'_{\rm dQ,3}\Big]+\Big[I_{\rm X,1}+I_{\rm X,2}+I_{\rm X,3}+I_{\rm X,4}+I_{\rm X,5}\Big]\bigg)\Big\}$$

$$I_{\rm ddd,1} \Rightarrow \mu^2(k_1-q_1, q_1-k_1)\,\mu^2(k_2-q_2, q_3-k_2)\,\mu^2(k_3-q_3, q_2-k_3) \propto F^2\,[(q_3-q_2)R]$$

- *Independent emission of k_1 and Bose enhancement of q_3 and q_2 in the target.*

- $I_{\rm ddd,2}$ and $I_{\rm ddd,3}$ exhibit the same behavior as $I_{\rm ddd,1}$ but with gluons interchanged.

$$I'_{\rm dQ,1} \Rightarrow \mu^2(k_1-q_1, q_1-k_1)\,\mu^2(-k_2-q_3, q_2+k_3)\,\mu^2(k_2-q_2, q_3-k_3)$$
$$\propto F^2\,\{[(k_2-q_2)-(k_3-q_3)]\,R\}$$

- *Independent emission of k_1 and Bose enhancement of k_2-q_2 and k_3-q_3.*

- $I'_{\rm dQ,2}$ and $I'_{\rm dQ,3}$ exhibit the same behavior as $I'_{\rm dQ,1}$ but with gluons interchanged.
- N_c-supressed correction to first term of $I_{\rm dQ,1}$.

Identifying terms in triple inclusive gluon production - III

$$\frac{d\sigma}{d^2k_1 d\eta_1\, d^2k_2 d\eta_2\, d^2k_3 d\eta_3} = \alpha_s^3\,(4\pi)^3\,(N_c^2-1)^3 \int \frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\; d(q_1)d(q_2)d(q_3)$$
$$\times\Bigg\{ I_{\mathrm{ddd},0} + \frac{1}{N_c^2-1}\Big[I_{\mathrm{dQ},1}+I_{\mathrm{dQ},2}+I_{\mathrm{dQ},3}\Big]$$
$$+\frac{1}{(N_c^2-1)^2}\Bigg(\Big[I_{\mathrm{ddd},1}+I_{\mathrm{ddd},2}+I_{\mathrm{ddd},3}\Big]+\Big[I'_{\mathrm{dQ},1}+I'_{\mathrm{dQ},2}+I'_{\mathrm{dQ},3}\Big]+\Big[I_{\mathrm{X},1}+I_{\mathrm{X},2}+I_{\mathrm{X},3}+I_{\mathrm{X},4}+I_{\mathrm{X},5}\Big]\Bigg)\Bigg\}$$

All gluons are correlated, no independent production!

$I_{\mathrm{X},1}$ has four contributions: $\propto F\,[(k_2 \pm k_1)R]\,F^2\,\{[(k_1-q_1)\pm(k_3-q_3)]\,R\}$

- *(forward-backward) HBT of k_1 and k_2 with (forward-backward) Bose enhancement of $k_1 - q_1$ and $k_3 - q_3$.*

- $I_{\mathrm{X},2}$ and $I_{\mathrm{X},3}$ exhibit the same behavior as $I_{\mathrm{X},1}$ with the gluons intercahnged.

$I_{\mathrm{X},4}$ has four contributions:
$\propto F\{[(k_2-q_2)-(k_3-q_3)]R\}F\{[(k_1-q_1)-(k_3-q_3)]R\}\,F\{[(k_1-q_1)-(k_2-q_2)]R\}$

- *Forward Bose enhancement of k_1-q_1, k_2-q_2 and k_3-q_3.*

$I_{\mathrm{X},5}$ has four contributions: $\propto F[(k_2-k_1)R]\,F[(k_1-k_3)R]\,F[(k_2-k_3)R]$

- *Forward HBT of k_1, k_2 and k_3.*

Conclusions and Outlook

- We have calculated the double and triple inclusive gluon production in glasma graph approximation for dilute-dense collisions.
- We have identified all the quantum interference terms both in double and triple inclusive gluon production. We have shown that in the double inclusive case the Bose enhancement and HBT correlations stems from the quadrupole term where as in the triple inclusive case these these effects stem from the sextupole term.
- In the glasma graph calculation, which is based on the dilute-dilute limit, and is therefore symmetric between the projectile and the target, the Bose enhancement in the target wave function is of the leading order in $1/N_c$, just like the Bose enhancement in the projectile. In the complete dilute-dense framework, although present, is suppressed as $1/N_c^2$ relative to the projectile Bose enhancement effect.

BACKUP SLIDES

$$
\begin{aligned}
\Big\langle D(x_1,x_1')D(x_2,x_2')D(x_3,x_3')\Big\rangle_T &= d(x_1,x_1')d(x_2,x_2')d(x_3,x_3') \\
+\frac{1}{(N_c^2-1)^2}\Big\{ & d(x_1,x_1')\Big[d(x_2,x_3)d(x_2',x_3')+d(x_2,x_3')d(x_2',x_3)\Big] \\
& + d(x_2,x_2')\Big[d(x_1,x_3)d(x_1',x_3')+d(x_1,x_3')d(x_1',x_3)\Big] \\
& + d(x_3,x_3')\Big[d(x_1,x_2)d(x_1',x_2')+d(x_1,x_2')d(x_1',x_2)\Big]\Big\} \\
+\frac{1}{(N_c^2-1)^4}\Big\{ & d(x_1,x_2)\Big[d(x_3,x_1')d(x_2',x_3')+d(x_1',x_3')d(x_3,x_2')\Big] \\
& + d(x_1,x_2')\Big[d(x_3,x_1')d(x_2,x_3')+d(x_1',x_3')d(x_3,x_2)\Big] \\
& + d(x_1,x_3)\Big[d(x_2,x_1')d(x_3',x_2')+d(x_1',x_2')d(x_2,x_3')\Big] \\
& + d(x_1,x_3')\Big[d(x_2,x_1')d(x_3,x_2')+d(x_1',x_2')d(x_2,x_3)\Big]\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
\Big\langle D(x_1,x_1')Q(x_2,x_2',x_3,x_3')\Big\rangle_T &= d(x_1,x_1')\Big[d(x_2,x_2')d(x_3,x_3') + d(x_2,x_3')d(x_3,x_2')\Big] \\
&+ \frac{1}{N_c^2-1} d(x_1,x_1')d(x_2,x_3)d(x_2',x_3') \\
+\frac{1}{(N_c^2-1)^2}\Big\{ & d(x_1,x_2)\Big[d(x_1',x_2')d(x_3,x_3') + d(x_1',x_3')d(x_2',x_3)\Big] \\
&+ d(x_1,x_2')\Big[d(x_1',x_2)d(x_3,x_3') + d(x_1',x_3)d(x_2,x_3')\Big] \\
&+ d(x_1,x_3)\Big[d(x_1',x_3')d(x_2,x_2') + d(x_1',x_2')d(x_3',x_2)\Big] \\
&+ d(x_1,x_3')\Big[d(x_1',x_3)d(x_2,x_2') + d(x_1',x_2)d(x_3,x_2')\Big]\Big\} \\
+\frac{1}{(N_c^2-1)^3}\Big\{ & d(x_2',x_3')\Big[d(x_1,x_2)d(x_1',x_3) + d(x_1,x_3)d(x_1',x_2)\Big] \\
&+ d(x_2,x_3)\Big[d(x_1,x_2')d(x_1',x_3') + d(x_1',x_2')d(x_1,x_3')\Big]\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
\Big\langle X(x_1,x_1',x_2,x_2',x_3,x_3')\Big\rangle_T &= d(x_1,x_1')d(x_2,x_2')d(x_3,x_3') + d(x_1,x_3')d(x_2,x_1')d(x_3,x_2') \\
&\quad + d(x_1,x_1')d(x_2,x_3')d(x_3,x_2') + d(x_2,x_2')d(x_3,x_1')d(x_1,x_3') + d(x_3,x_3')d(x_1,x_2')d(x_2,x_1') \\
&+\frac{1}{N_c^2-1}\Big\{ d(x_1,x_1')d(x_2,x_3)d(x_2',x_3') + d(x_2,x_2')d(x_3,x_1)d(x_3',x_1') + d(x_3,x_3')d(x_1,x_2)d(x_1',x_2') \\
&\quad + d(x_2,x_3)d(x_1,x_3')d(x_1',x_2') + d(x_3,x_1)d(x_2,x_1')d(x_2',x_3') + d(x_1,x_2)d(x_3,x_2')d(x_3',x_1')\Big\} \\
&+\frac{1}{(N_c^2-1)^2}\Big\{ d(x_1,x_2)d(x_3,x_1')d(x_2',x_3') + d(x_2,x_3)d(x_1,x_2')d(x_3',x_1') + d(x_3,x_1)d(x_2,x_3')d(x_1',x_2') \\
&\quad + d(x_1,x_2')d(x_3,x_1')d(x_2,x_3')\Big\}
\end{aligned}
$$

X contribution to three-gluon production

$$I_{X,1} = \left[\tilde{I}_{X,1} + (k_3 \to -k_3)\right] + \left[\tilde{I}'_{X,1} + (k_1 \to -k_1)\right]$$

with $\tilde{I}_{X,1}$ and $\tilde{I}'_{X,1}$ defined as

$$\begin{aligned}
\tilde{I}_{X,1} = &\, \mu^2(k_2 - q_2, q_2 - k_1)\, \mu^2(k_1 - q_1, q_3 - k_3)\, \mu^2(k_3 - q_3, q_1 - k_2) \\
& \times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_2) L^j(k_2, q_1) L^k(k_3, q_3) L^k(k_3, q_3) \\
& + \mu^2(k_2 + q_2, k_1 - q_2)\, \mu^2(k_3 - q_3, q_1 - k_1)\, \mu^2(q_3 - k_3, -q_1 - k_2) \\
& \times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, -q_1) L^i(k_2, -q_2) L^k(k_3, q_3) L^k(k_3, q_3)
\end{aligned}$$

$$\begin{aligned}
\tilde{I}'_{X,1} = &\, \mu^2(k_1 - q_2, q_2 - k_2)\, \mu^2(k_2 - q_1, k_3 - q_3)\, \mu^2(q_1 - k_1, q_3 - k_3) \\
& \times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_1) L^j(k_2, q_2) L^k(k_3, q_3) L^k(k_3, q_3) \\
& + \mu^2(-k_1 - q_2, q_2 - k_2)\, \mu^2(k_2 + q_1, q_3 - k_3)\, \mu^2(k_3 - q_3, k_1 - q_1) \\
& \times L^i(k_1, -q_2) L^i(k_1, q_1)\, L^j(k_2, -q_1) L^j(k_2, q_2) L^k(k_3, q_3) L^k(k_3, q_3)
\end{aligned}$$

X contribution to three-gluon production

$$
\begin{aligned}
I_{[X,4]} = {} & \mu^2(k_2-q_2,q_1-k_1)\,\mu^2(k_1-q_1,q_3-k_3)\,\mu^2(k_3-q_3,q_2-k_2)\\
& \times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)+(k_3\to -k_3)\\
& +\mu^2(k_2-q_2,q_3-k_3)\,\mu^2(k_3-q_3,k_1-q_1)\,\mu^2(q_1-k_1,q_2-k_2)\\
& \times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)+(k_1\to -k_1)\\
& +\mu^2(k_2-q_2,k_1-q_1)\,\mu^2(k_3-q_3,q_1-k_1)\,\mu^2(q_3-k_3,q_2-k_2)\\
& \times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)+(k_3\to -k_3)\\
& +\mu^2(q_1-k_1,q_3-k_3)\,\mu^2(k_1-q_1,q_2-k_2)\,\mu^2(k_2-q_2,k_3-q_3)\\
& \times L^i(k_1,q_1)L^i(k_1,q_1)\,L^j(k_2,q_2)L^j(k_2,q_2)L^k(k_3,q_3)L^k(k_3,q_3)+(k_1\to -k_1)
\end{aligned}
$$

$$
\begin{aligned}
I_{[X,5]} = {} & \mu^2(k_2-q_2,q_2-k_1)\,\mu^2(k_1-q_1,q_1-k_3)\,\mu^2(k_3-q_3,q_3-k_2)\\
& \times L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,q_3)L^k(k_3,q_3)L^k(k_3,q_1)+(k_3\to -k_3)\\
& +\mu^2(k_2-q_2,q_2-k_3)\,\mu^2(k_3-q_3,q_3+k_1)\,\mu^2(q_1-k_2,-k_1-q_1)\\
& \times L^i(k_1,-q_1)L^i(k_1,-q_3)\,L^j(k_2,q_2)L^j(k_2,q_1)L^k(k_3,q_3)L^k(k_3,q_2)+(k_1\to -k_1)\\
& +\mu^2(k_2+q_1,k_1-q_1)\,\mu^2(k_3-q_3,q_3-k_1)\,\mu^2(q_2-k_3,-k_2-q_2)\\
& \times L^i(k_1,q_1)L^i(k_1,q_3)\,L^j(k_2,-q_2)L^j(k_2,-q_1)L^k(k_3,q_3)L^k(k_3,q_2)+(k_3\to -k_3)\\
& +\mu^2(k_2+q_3,k_3-q_3)\,\mu^2(-k_1-q_1,q_1-k_3)\,\mu^2(k_1-q_2,q_2-k_2)\\
& \times L^i(k_1,-q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,-q_3)L^k(k_3,q_3)L^k(k_3,q_1)+(k_1\to -k_1)
\end{aligned}
$$