BOSE ENHANCEMENT IN THE DILUTE-DENSE LIMIT

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Probing QCD at the high energy frontier

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Universitdade de Santiago de Compostela

Collaboration with T. Altinoluk and N. Armesto

Based off: arXiv: 1801.08986

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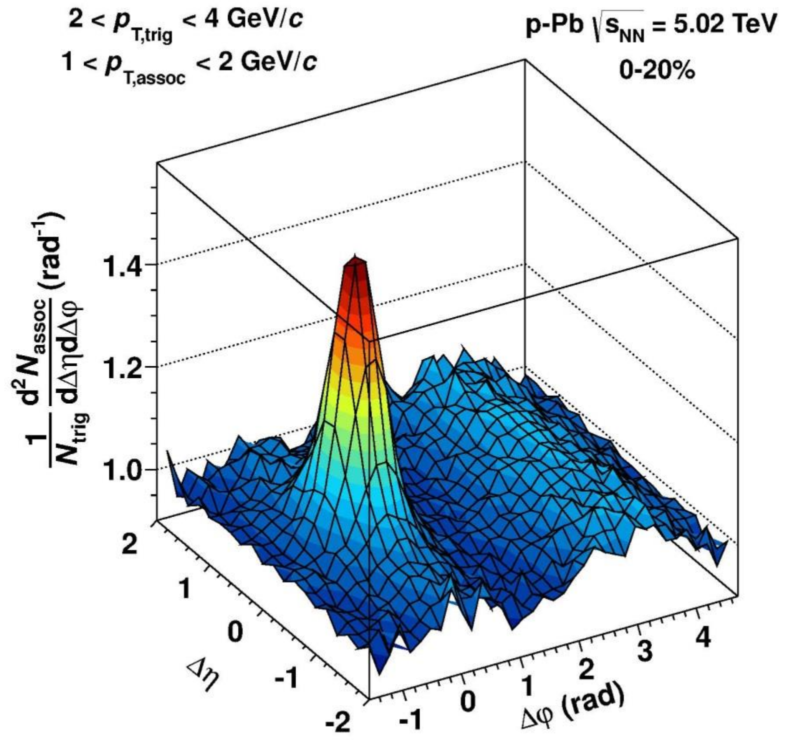
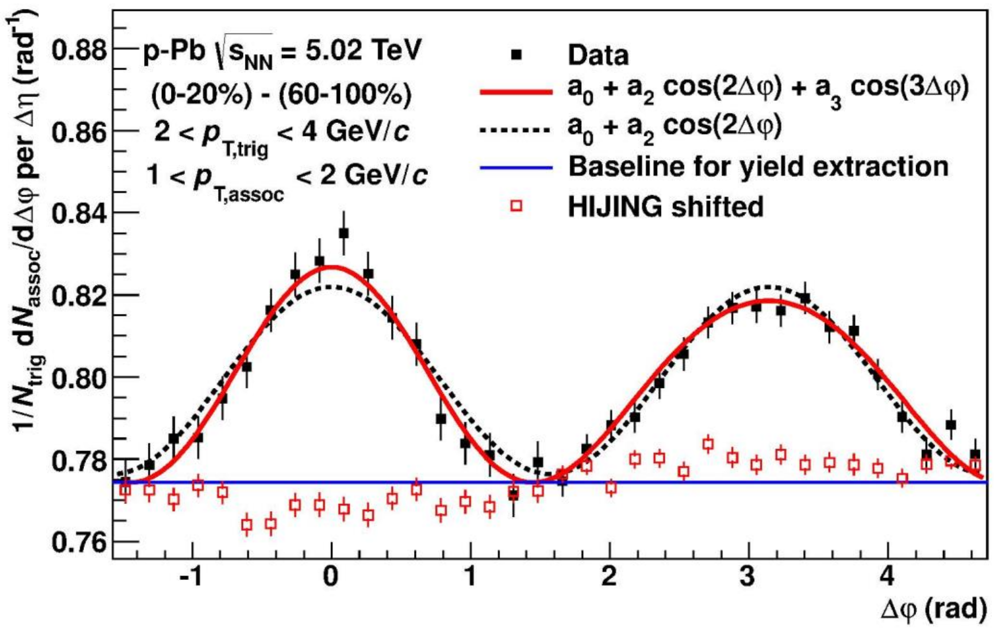
Outline

- We will examine bose enhancement in two-gluon correlations and its relation to the “ridge”.

- Review of the ridge.
- Review two-gluon production in the dilute-dense limit.
- Various important contributions at the dilute-dilute limit.
- Examining these terms in the dilute-dense limit.
- Bose enhancement and its effect on the ridge.

The “ridge” – ALICE data, p+Pb

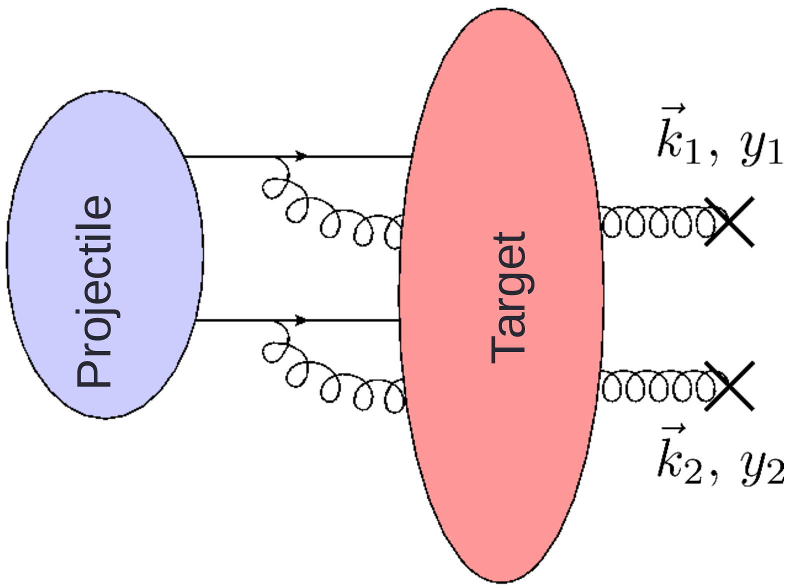




- Observed in A-A, p-A, p-p collisions
- Correlation between two particles
- Long-range in rapidity, near- and away-side in azimuthal angle
- ALICE collaboration (2012) data for p+Pb collisions.

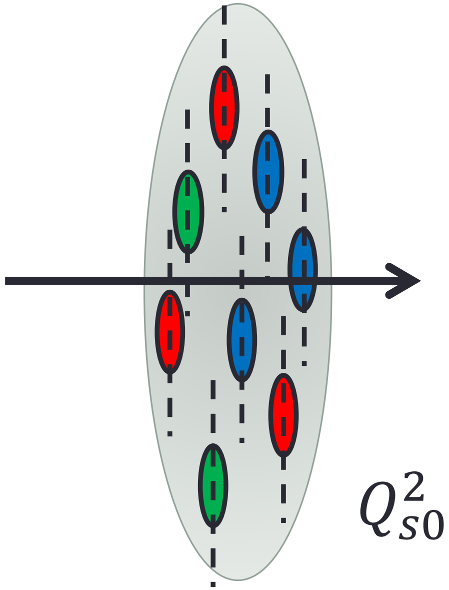
Separating the gluon emission from the interaction

- In the dilute-dense limit two different sources from the projectile emit a gluon. This is more likely than one quark emitting two gluons.
- The gluons and sources proceed to interact with all of the gluon fields in the target.
- View the emission of the gluon and the interaction in the nucleus as two separate events since emission is on a much larger time scale than the interaction.



Modeling the interaction through Wilson lines

- A quark or gluon propagating through a nucleus at high energy can be thought of as a Wilson line. The following is for a gluon. The gluon is high energy and recoilless in transverse spatial coordinate. Interacts with many different color patches whose weight is described by the saturation scale. This can be thought of as a rotation in color space giving a net phase.



$Q_{s0}^2 \sim A^{1/3}$

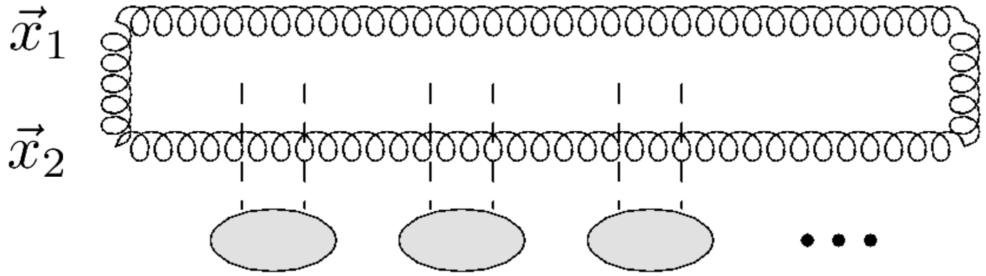
$$U_{\vec{x}} = \mathrm{P}\exp\left\{ i\,g \int_{-\infty}^{\infty} dx^+ \mathcal{A}^-(x^+, x^- = 0, \vec{x}) \right\}$$

$$\mathcal{A}^- = \sum_i T^a A_i^{a-}$$

Analysis of the gluon dipole – Saturation effects

- Modeling the gluon dipole as a series of tree level scatterings off many nucleons. The dotted lines represent gluons.

$$S_G(\vec{x}_1, \vec{x}_2; y) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\vec{x}_1} U^{\dagger}_{\vec{x}_2}] \right\rangle (y)$$



$$S_G(\vec{x}_1, \vec{x}_2; y = 0) = \exp\left[-\frac{1}{4} \left|\vec{x}_1 - \vec{x}_2\right|^2 Q_{s0}^2 \left(\frac{\vec{x}_1 + \vec{x}_2}{2} \right) \ln\left(\frac{1}{\left|\vec{x}_1 - \vec{x}_2\right| \Lambda} \right) \right]$$

- Here we used the McLerran Venugopalan (MV) model.
- The forward scattering amplitude is given by.

$$N_G(\vec{x}_1, \vec{x}_2; y = 0) = 1 - S_G(\vec{x}_1, \vec{x}_2; y = 0)$$

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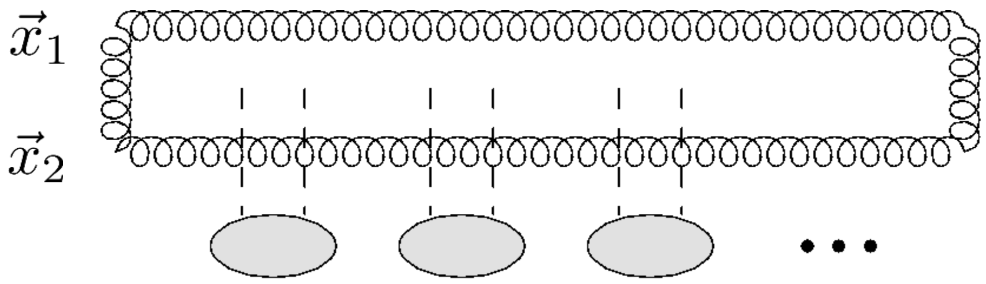


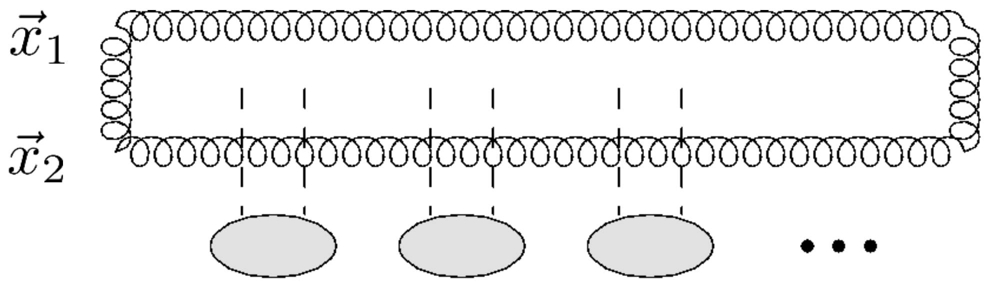
- The forward scattering amplitude is used to define the unintegrated gluon distribution function.

$$\phi_A(\vec{q};y) = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2b\, d^2r\; e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, N_G(\vec{b}+\vec{r},\vec{b};y)$$

$$\left\langle \frac{d\phi_A(\vec{q};y)}{d^2b} \right\rangle_A = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2r\; e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, N_G(\vec{b}+\vec{r},\vec{b};y)$$

Analysis of the gluon dipole – Saturation effects

- Modeling the gluon dipole as a series of tree level scatterings off many nucleons. The dotted lines represent gluons.

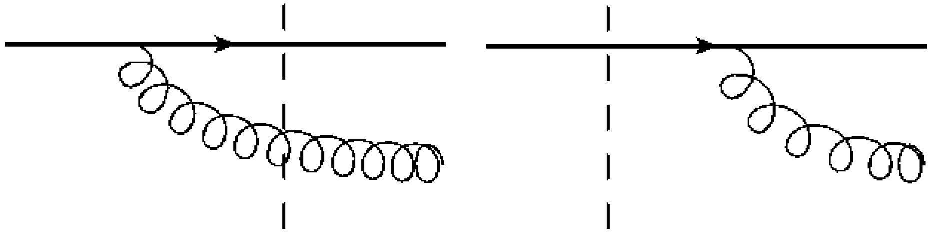


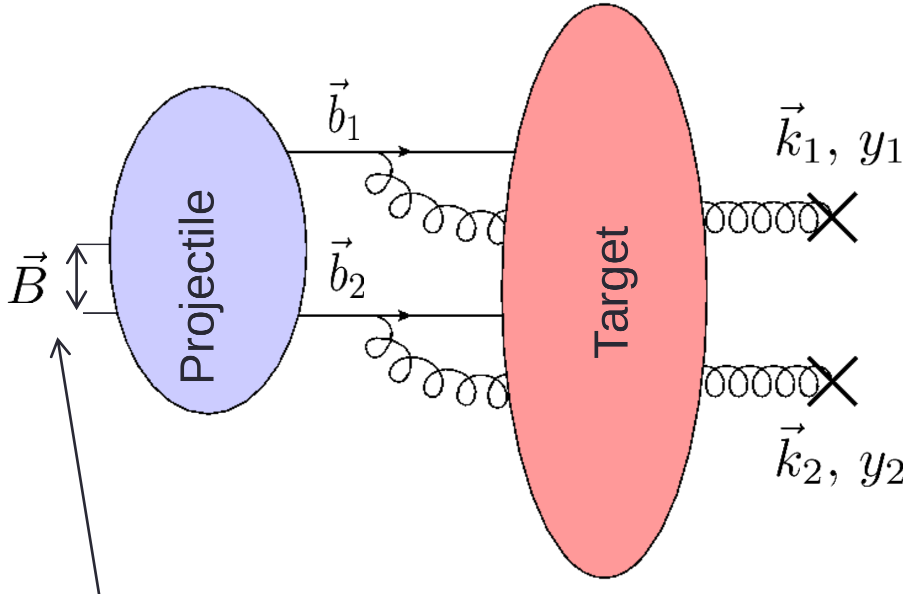
- If the saturation scale does not depend on the position of the dipole (assuming translational invariance).

$$\left\langle \frac{d\phi_A(\vec{q};y)}{d^2b} \right\rangle_A = \frac{1}{S_\perp}\phi_A(\vec{q};y)$$

Calculating the cross-section

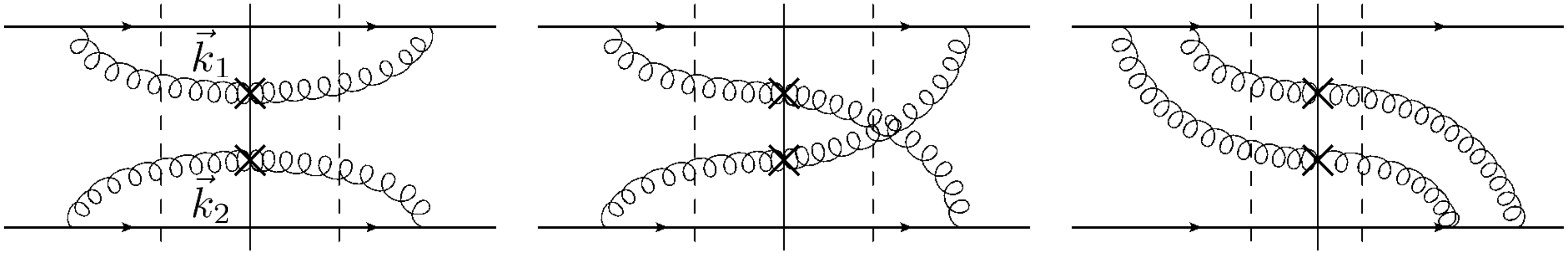
- Each gluon can be emitted from each source either before or after the interaction, which is represented as a dashed line and modeled as a Wilson line.





Impact parameter between the target and the projectile

- Calculate the amplitude squared, which involves 3 classes of diagrams



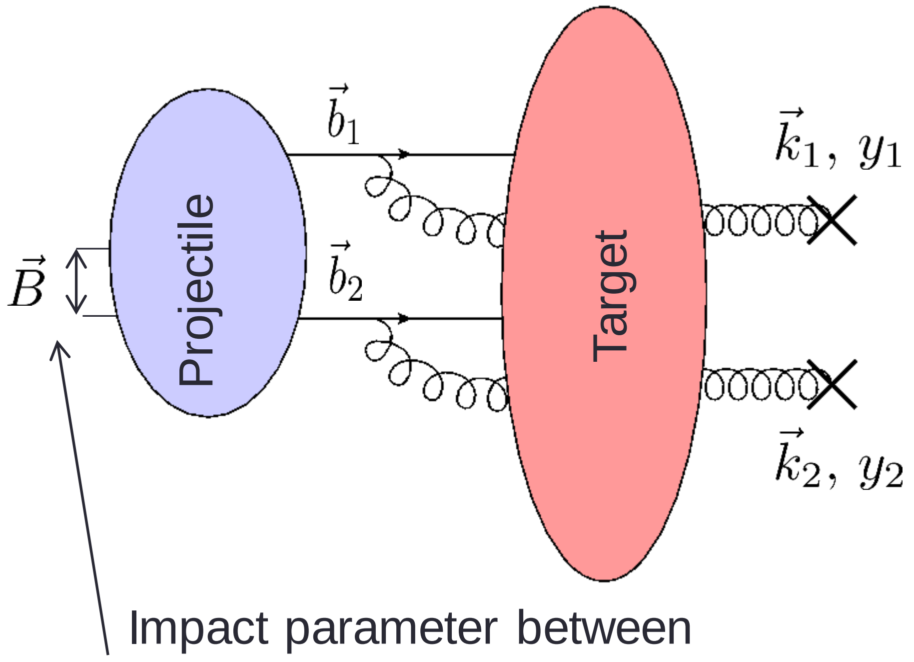
Full dilute-dense expression

- The two-gluon production cross section can be written as

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right.$$
$$\left. + e^{-i(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1,\vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2(\vec{k}_2-\vec{q}_2)^2} \Big\{ k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2$$
$$- k_1^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_2\cdot\vec{q}_1)\,q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\,q_1^2 - q_1^2\,q_2^2\right]$$
$$- k_2^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_1\cdot\vec{q}_1)\,q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\,q_1^2 - q_1^2\,q_2^2\right]$$
$$+ q_1^2\,q_2^2\left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1)\right]\Big\}$$

- Complicated expression with important features



Impact parameter between the target and the projectile

Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

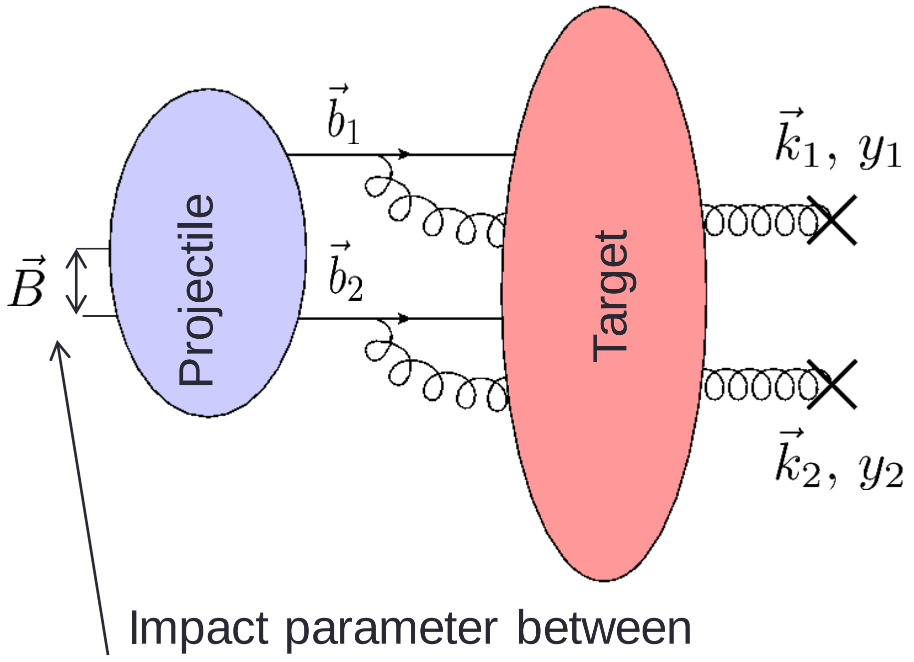
Full dilute-dense expression

- The two-gluon production cross section can be written as

$$
\begin{aligned}
\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} &= \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\, k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2 \\
&\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right. \\
&\left. + e^{-i\,(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)}\, \frac{\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} \Big\{ & k_1^2\, k_2^2 (\vec{q}_1 \cdot \vec{q}_2)^2 \\
& - k_1^2\, (\vec{q}_1 \cdot \vec{q}_2) \left[(\vec{k}_2 \cdot \vec{q}_1)\, q_2^2 + (\vec{k}_2 \cdot \vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& - k_2^2\, (\vec{q}_1 \cdot \vec{q}_2) \left[(\vec{k}_1 \cdot \vec{q}_1)\, q_2^2 + (\vec{k}_1 \cdot \vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& + q_1^2\, q_2^2 \left[(\vec{k}_1 \cdot \vec{q}_1)(\vec{k}_2 \cdot \vec{q}_2) + (\vec{k}_1 \cdot \vec{q}_2)(\vec{k}_2 \cdot \vec{q}_1) \right] \Big\}
\end{aligned}
$$

- Different gluon distribution functions



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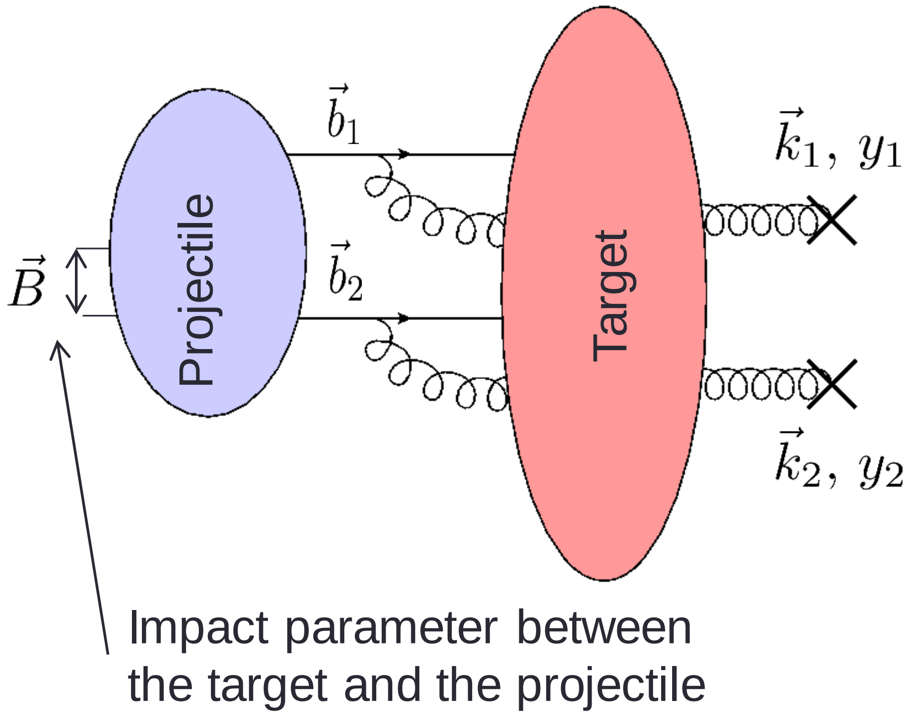
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right.$$
$$\left. + e^{-i(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} \left\{ k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2 \right.$$
$$- k_1^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_2\cdot\vec{q}_1)\,q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\,q_1^2 - q_1^2\,q_2^2\right]$$
$$- k_2^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_1\cdot\vec{q}_1)\,q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\,q_1^2 - q_1^2\,q_2^2\right]$$
$$\left. + q_1^2\,q_2^2\left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1)\right]\right\}$$



- Different gluon distribution functions
  - Single-gluon distribution function

Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

Single-gluon distribution functions

- For the projectile nucleus

$$\left\langle \frac{d\phi_{A_1}(\vec{q};y)}{d^2b} \right\rangle_{A_1} = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2r\, e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, n_G(\vec{b}+\vec{r},\vec{b};y) \qquad \phi_{A_1}(\vec{q};y) = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2b\, d^2r\, e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, n_G(\vec{b}+\vec{r},\vec{b};y)$$

$$n_G(\vec{b}+\vec{r},\vec{b};y=0) = \frac{1}{4} Q^2_{s,1}(\vec{b})\, r^2 \ln\left(\frac{1}{r\,\Lambda}\right)$$

- For the target nucleus

$$\left\langle \frac{d\phi_{A_2}(\vec{q};y)}{d^2b} \right\rangle_{A_2} = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2r\, e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, N(\vec{b}+\vec{r},\vec{b};y) \qquad \phi_{A_2}(\vec{q};y) = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2b\, d^2r\, e^{-i\vec{q}\cdot\vec{r}}\, \nabla^2_{\vec{r}}\, N(\vec{b}+\vec{r},\vec{b};y)$$

$$N_G(\vec{x}_1,\vec{x}_2;y) = 1 - S_G(\vec{x}_1,\vec{x}_2;y)$$

- Translational invariance

$$\left\langle \frac{d\phi_{A_i}(\vec{q};y)}{d^2b} \right\rangle_{A_i} = \frac{1}{S_{\perp,i}} \phi_{A_i}(\vec{q};y)$$

Full dilute-dense expression

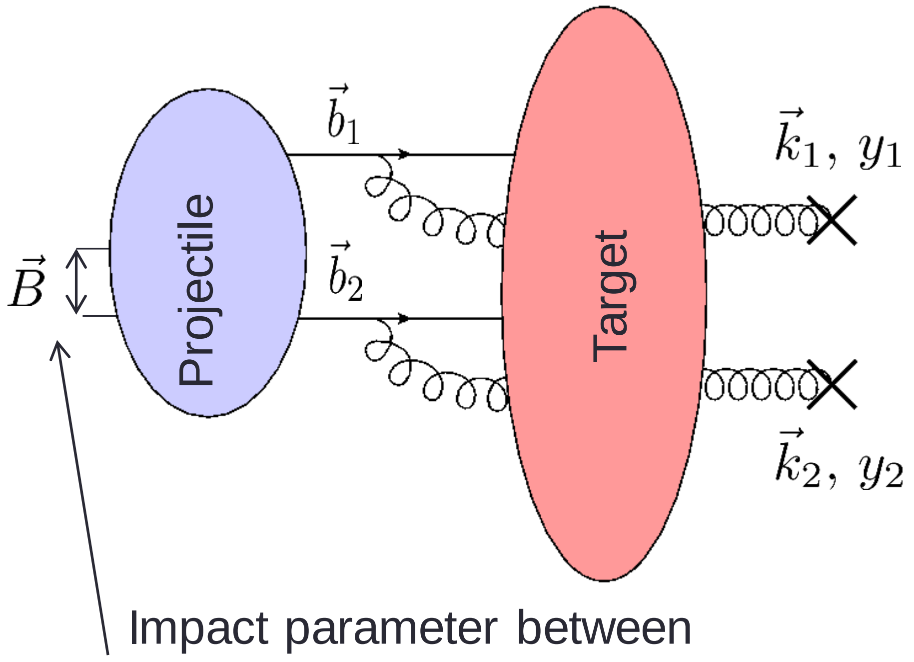
- The two-gluon production cross section can be written as

$$\frac{d\sigma}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\, k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2$$

$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right.$$

$$\left. + e^{-i\,(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)}\, \frac{\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\begin{aligned}
\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} \Big\{ & k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2 \\
& - k_1^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_2\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& - k_2^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_1\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& + q_1^2\, q_2^2 \left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1) \right] \Big\}
\end{aligned}$$



Impact parameter between the target and the projectile

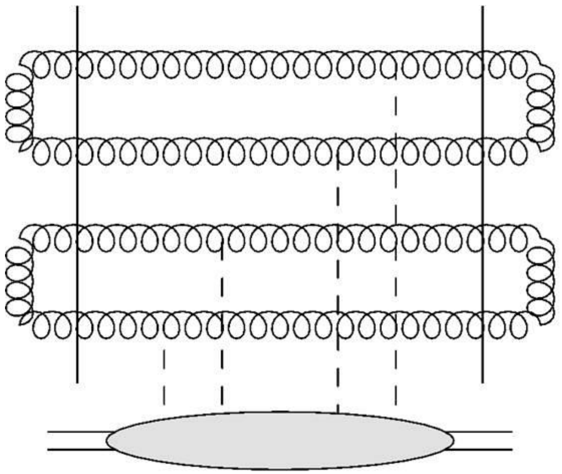
- Different gluon distribution functions
  - Two different two-gluon distribution functions

Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

Two-gluon distribution functions

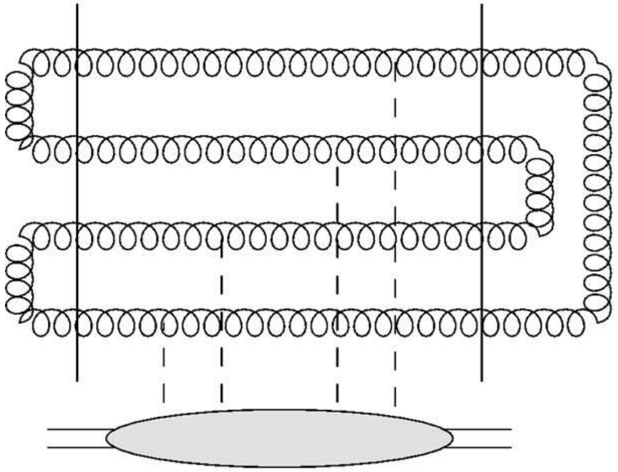
- Double-dipole

$$\left\langle \frac{d\phi^D_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\ d^2b_2}\right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s(2\pi)^3}\right)^2 \int d^2r_1\ d^2r_2\ e^{-i\vec{q}_1\cdot\vec{r}_1-i\vec{q}_2\cdot\vec{r}_2}$$
$$\times\ \nabla^2_{\vec{r}_1}\ \nabla^2_{\vec{r}_2}\ \frac{1}{(N_c^2-1)^2}\ \left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_1}U^\dagger_{\vec{b}_1}\right]\mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_2}U^\dagger_{\vec{b}_2}\right]\right\rangle_{A_2}(y)$$



- Quadrupole

$$\left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\ d^2b_2}\right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s(2\pi)^3}\right)^2 \int d^2r_1\ d^2r_2\ e^{-i\vec{q}_1\cdot\vec{r}_1-i\vec{q}_2\cdot\vec{r}_2}$$
$$\times\ \nabla^2_{\vec{r}_1}\ \nabla^2_{\vec{r}_2}\ \frac{1}{(N_c^2-1)^2}\ \left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_1}U^\dagger_{\vec{b}_1}U_{\vec{r}_1+\vec{b}_2}U^\dagger_{\vec{b}_2}\right]\right\rangle_{A_2}(y)$$



Full dilute-dense expression

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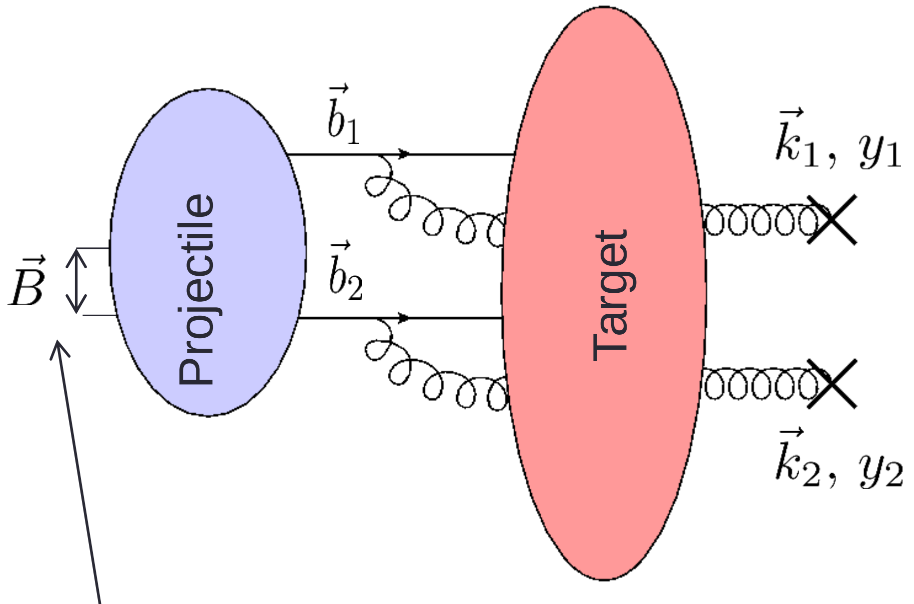
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right.$$
$$\left. + e^{-i(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \boxed{\frac{\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)}{N_c^2-1}} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\boxed{\begin{aligned} \mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2(\vec{k}_2-\vec{q}_2)^2} & \Big\{ k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2 \\ & - k_1^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_2\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\ & - k_2^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_1\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\ & + q_1^2\, q_2^2 \left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1) \right] \Big\} \end{aligned}}$$

- Kernel associated with the crossed diagrams



Impact parameter between the target and the projectile

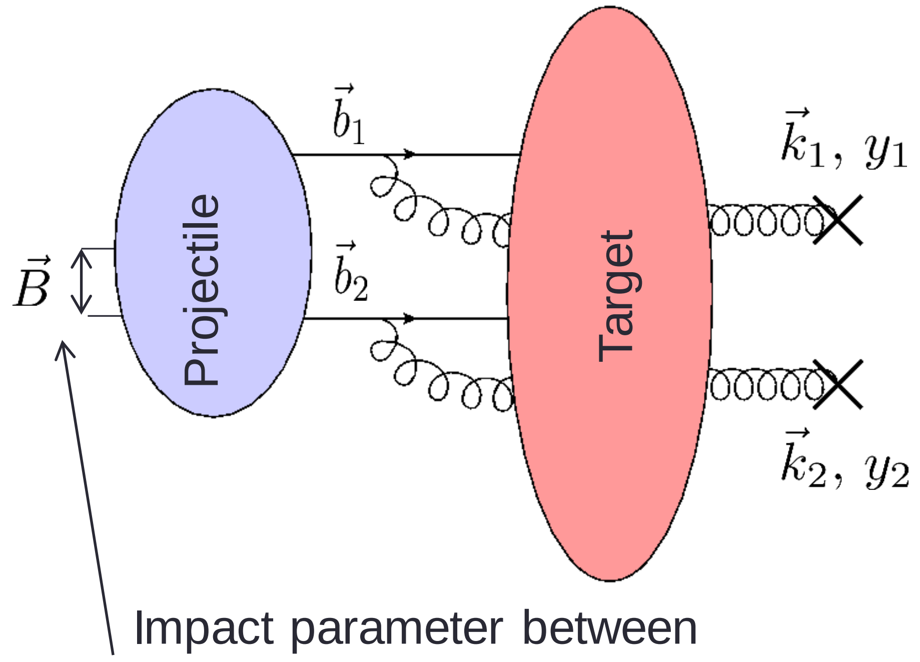
Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

Full dilute-dense expression

- The two-gluon production cross section can be written as

$$
\begin{aligned}
\frac{d\sigma}{d^2k_1\, dy_1\, d^2k_2\, dy_2} &= \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\, k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2 \\
&\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right. \\
&\left. + e^{-i(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} & \Big\{ k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2 \\
& - k_1^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_2\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& - k_2^2\, (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_1\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2 \right] \\
& + q_1^2\, q_2^2 \left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1) \right] \Big\}
\end{aligned}
$$



Impact parameter between the target and the projectile

- Integration over the impact parameter between the target and the projectile.
- Integration over the positions of the source quarks in the projectile nucleus.

Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

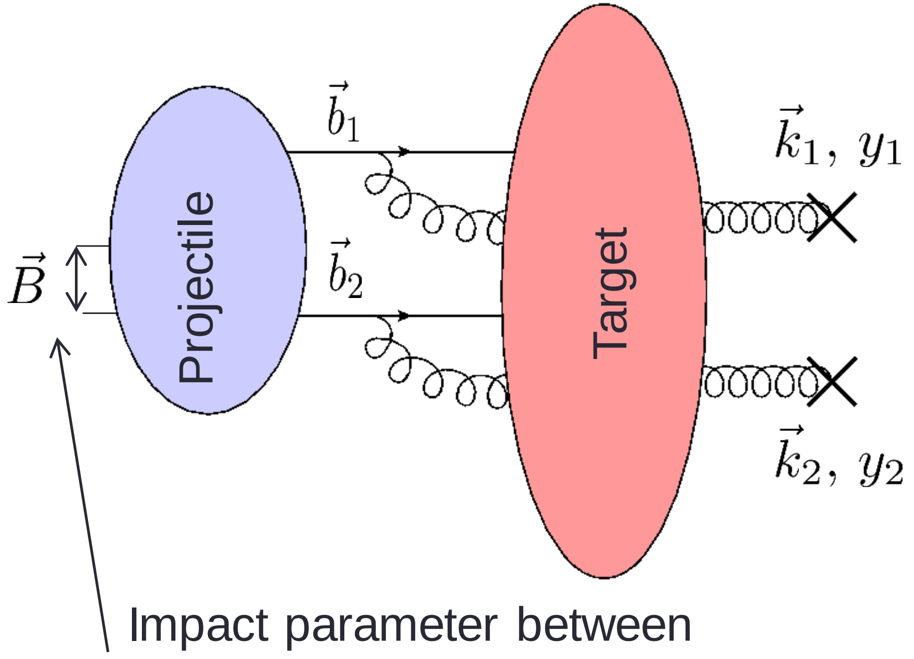
Full dilute-dense expression

- The two-gluon production cross section can be written as

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right.$$
$$\left. + e^{-i(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\, d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2) = \frac{1}{q_1^2\, q_2^2\, (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} \left\{ k_1^2\, k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2 \right.$$
$$- k_1^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_2\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_2\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2\right]$$
$$- k_2^2\,(\vec{q}_1\cdot\vec{q}_2)\left[(\vec{k}_1\cdot\vec{q}_1)\, q_2^2 + (\vec{k}_1\cdot\vec{q}_2)\, q_1^2 - q_1^2\, q_2^2\right]$$
$$\left. + q_1^2\, q_2^2 \left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) + (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1)\right] \right\}$$

- Integration over momenta of the gluons emitted from the projectile.



Impact parameter between the target and the projectile

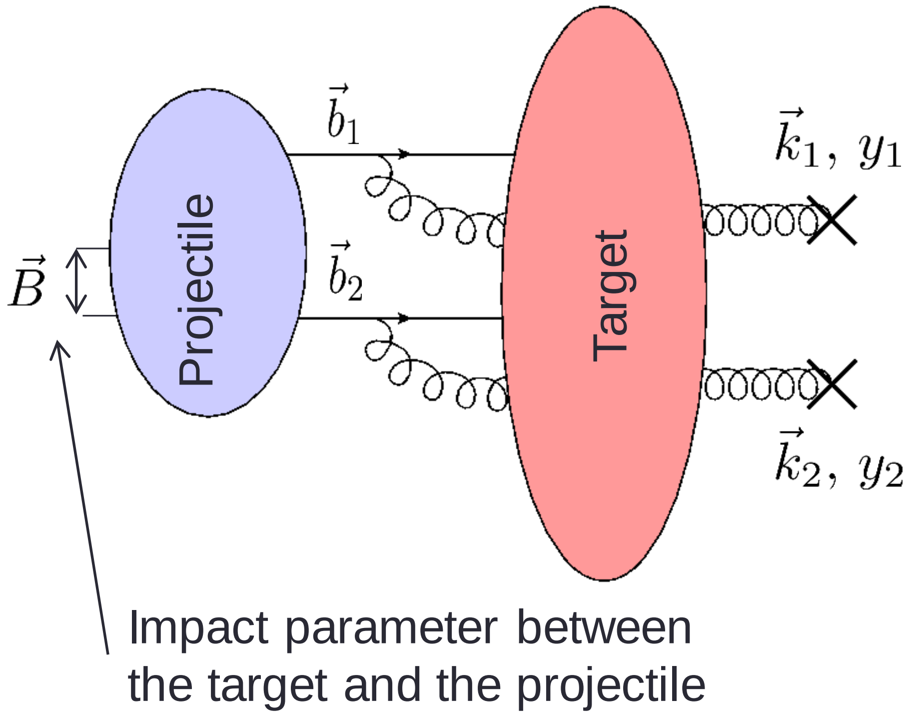
Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

Full dilute-dense expression

- The two-gluon production cross section can be written as

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\; d^2b_1\; d^2b_2 \int d^2q_1\; d^2q_2$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\{ \left\langle \frac{d\phi^D_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2, ; y)}{d^2b_1\; d^2b_2} \right\rangle_{A_2} \right.$$
$$\left. + e^{-i\,(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2)}{N_c^2-1} \left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1-\vec{k}_1, \vec{q}_2-\vec{k}_2; y)}{d^2b_1\; d^2b_2} \right\rangle_{A_2} \right\} + (\vec{k}_2 \to -\vec{k}_2)$$

$$\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q}_1, \vec{q}_2) = \frac{1}{q_1^2\; q_2^2\; (\vec{k}_1-\vec{q}_1)^2 (\vec{k}_2-\vec{q}_2)^2} \Big\{ k_1^2\; k_2^2 (\vec{q}_1\cdot\vec{q}_2)^2$$
$$- \; k_1^2\; (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_2\cdot\vec{q}_1)\; q_2^2 \; + \; (\vec{k}_2\cdot\vec{q}_2)\; q_1^2 \; - \; q_1^2\; q_2^2 \right]$$
$$- \; k_2^2\; (\vec{q}_1\cdot\vec{q}_2) \left[(\vec{k}_1\cdot\vec{q}_1)\; q_2^2 \; + \; (\vec{k}_1\cdot\vec{q}_2)\; q_1^2 \; - \; q_1^2\; q_2^2 \right]$$
$$+ \; q_1^2\; q_2^2 \left[(\vec{k}_1\cdot\vec{q}_1)(\vec{k}_2\cdot\vec{q}_2) \; + \; (\vec{k}_1\cdot\vec{q}_2)(\vec{k}_2\cdot\vec{q}_1) \right] \Big\}$$



- Unwieldy to deal with
- It is helpful to isolate important contributions

Y. Kovchegov and D. Wertepny, (2013) arXiv:1310.6701

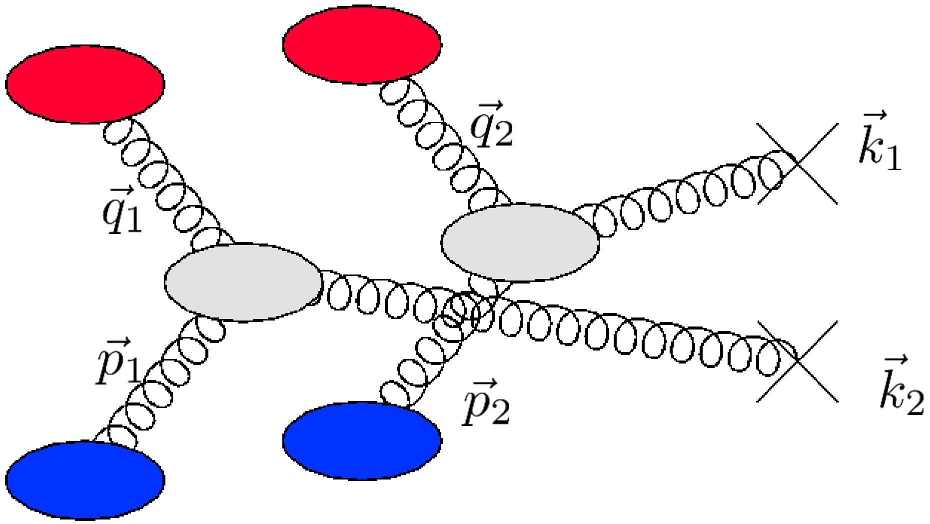
VARIOUS CONTRIBUTIONS IN THE DILUTE-DILUTE LIMIT

Dilute dilute limit, various terms

- Considered two sources in the projectile and the target. Known as "Glasma" graphs: Dumitru, Gelis, McLerran, Venugopalan '08.
- Found terms where both of the final state gluons are emitted independently. Known as classical (uncorrelated) terms (classical, since the gluons behave as if they are distinguishable).

Projectile Distributions

Target Distributions



$\propto 1$

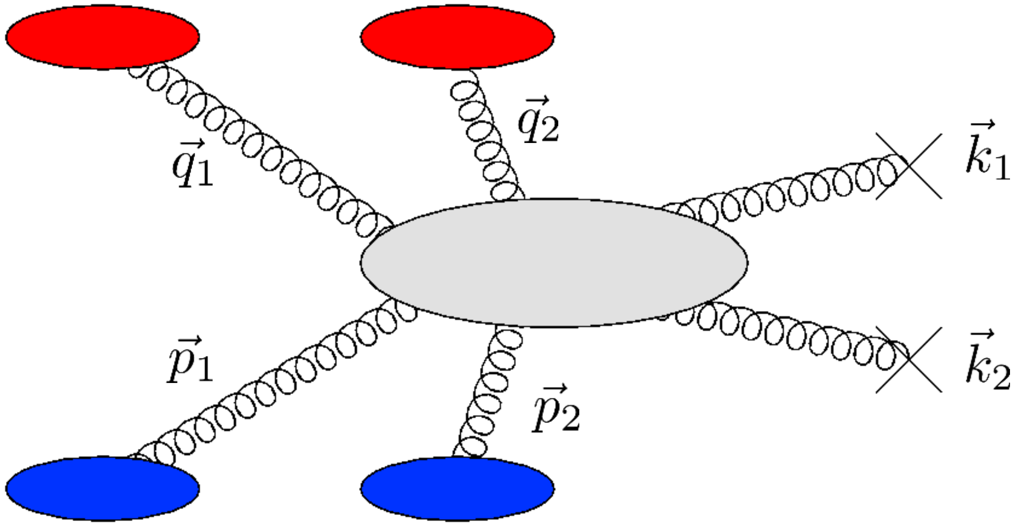
Classical

- Contains no non-trivial correlations.
- Survives the multiple rescatterings in the target.

Dilute dilute limit, various terms

- Considered two sources in the projectile and the target. Known as “Glasma” graphs: Dumitru, Gelis, McLerran, Venugopalan ‘08.
- Found terms where both of the final state gluons have the same momentum. Known as Hanbury, Brown and Twiss (HBT) correlations. Y. Kovechgov and D. Wertepny, (2012) arXiv:1212.1195

Projectile Distributions



Target Distributions

$$\propto \frac{1}{N_C^2 - 1}\frac{1}{S_\perp}\delta(\vec{k}_1 - \vec{k}_2)$$

HBT

- Leads to a ridge structure.
- Survives the multiple rescatterings in the target.

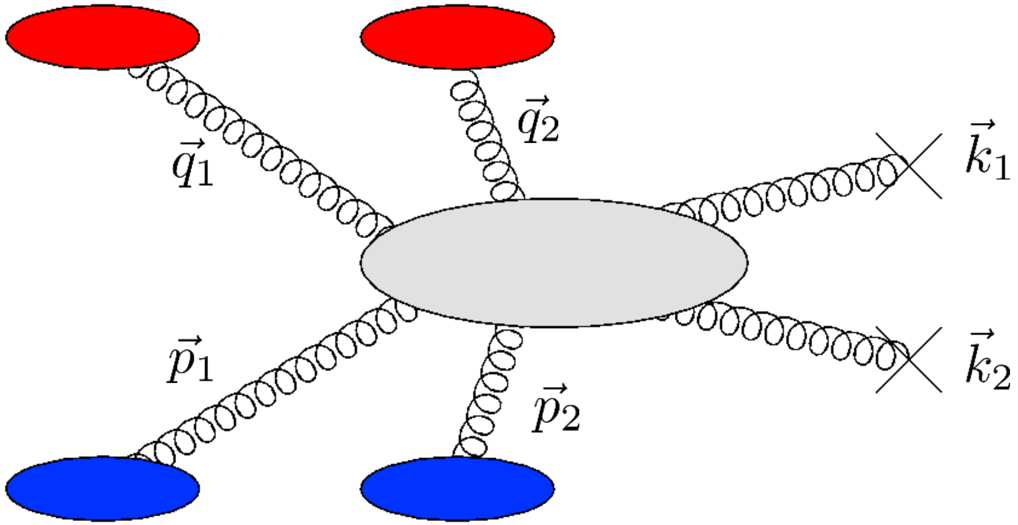
Dilute dilute limit, various terms

- Considered two sources in the projectile and the target. Known as “Glasma” graphs: Dumitru, Gelis, McLerran, Venugopalan ‘08.

- Found terms where both of the gluons emitted from the projectiles have the same momentum. Known as bose enhanced terms. T. Altinoluk et al., (2015) arXiv:1503.07126

Projectile Distributions

Target Distributions



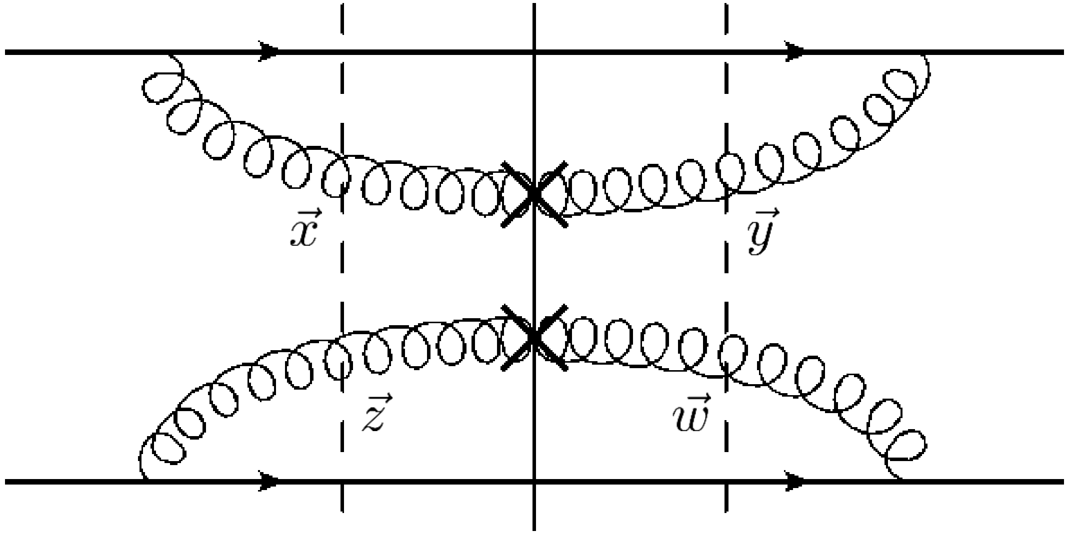
$$\propto \frac{1}{N_C^2 - 1} \frac{1}{S_\perp} \delta(\vec{q}_1 - \vec{q}_2)$$

Bose

- Leads to a ridge structure.
- Does this survive the multiple rescatterings in the target?

VARIOUS CONTRIBUTIONS IN THE DILUTE-DENSE LIMIT

Classical contribution

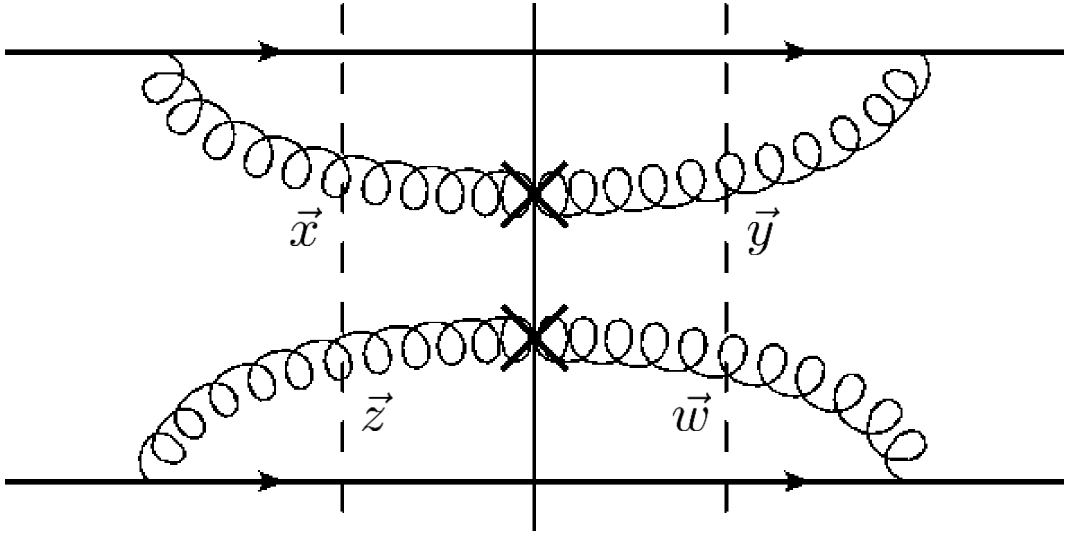


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$=\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- The double-dipole has a component that factorizes into two independent gluon dipoles.

$$\left\langle \frac{d\phi^D_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\, d^2b_2}\right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s(2\pi)^3}\right)^2 \int d^2r_1\, d^2r_2\, e^{-i\vec{q}_1\cdot\vec{r}_1 - i\vec{q}_2\cdot\vec{r}_2}$$
$$\times \nabla^2_{\vec{r}_1}\,\nabla^2_{\vec{r}_2}\,\frac{1}{(N_c^2-1)^2}\left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_1}U^\dagger_{\vec{b}_1}\right]\mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_2}U^\dagger_{\vec{b}_2}\right]\right\rangle_{A_2}(y)$$

Classical contribution

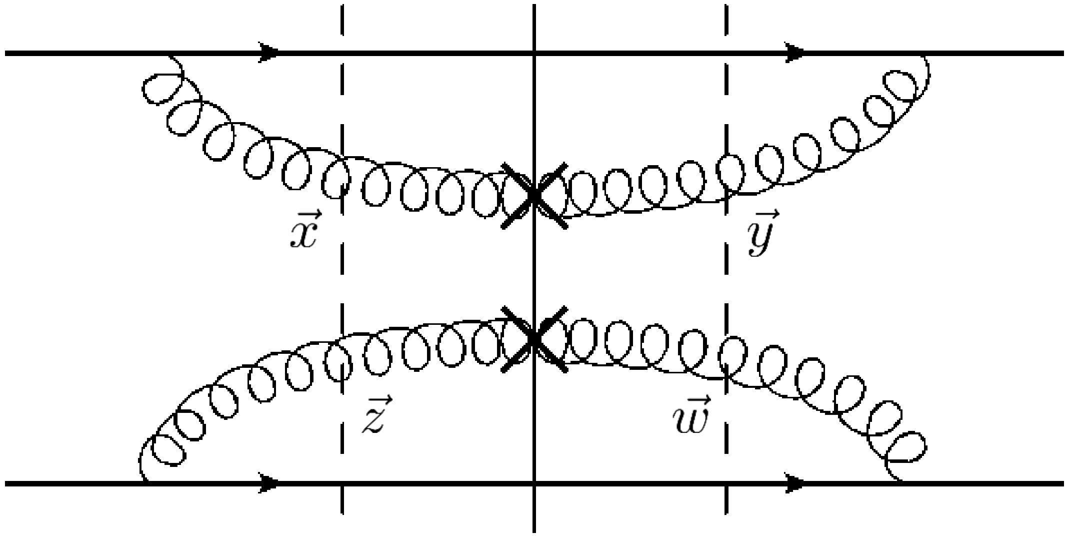


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}} U^\dagger_{\vec{y}}\right] \mathrm{Tr}\left[U_{\vec{z}} U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$= \left\langle \mathrm{Tr}\left[U_{\vec{x}} U^\dagger_{\vec{y}}\right]\right\rangle_{A_2}(y) \left\langle \mathrm{Tr}\left[U_{\vec{z}} U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- We can insert this component into the double-dipole distribution function.

$$\left\langle \frac{d\phi^D_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\, d^2b_2}\right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s(2\pi)^3}\right)^2 \int d^2r_1\, d^2r_2\, e^{-i\vec{q}_1\cdot\vec{r}_1 - i\vec{q}_2\cdot\vec{r}_2}$$
$$\times \nabla^2_{\vec{r}_1}\,\nabla^2_{\vec{r}_2}\,\frac{1}{(N_c^2-1)^2}\left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_1} U^\dagger_{\vec{b}_1}\right]\right\rangle_{A_2}\left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_2} U^\dagger_{\vec{b}_2}\right]\right\rangle_{A_2}(y)$$

Classical contribution

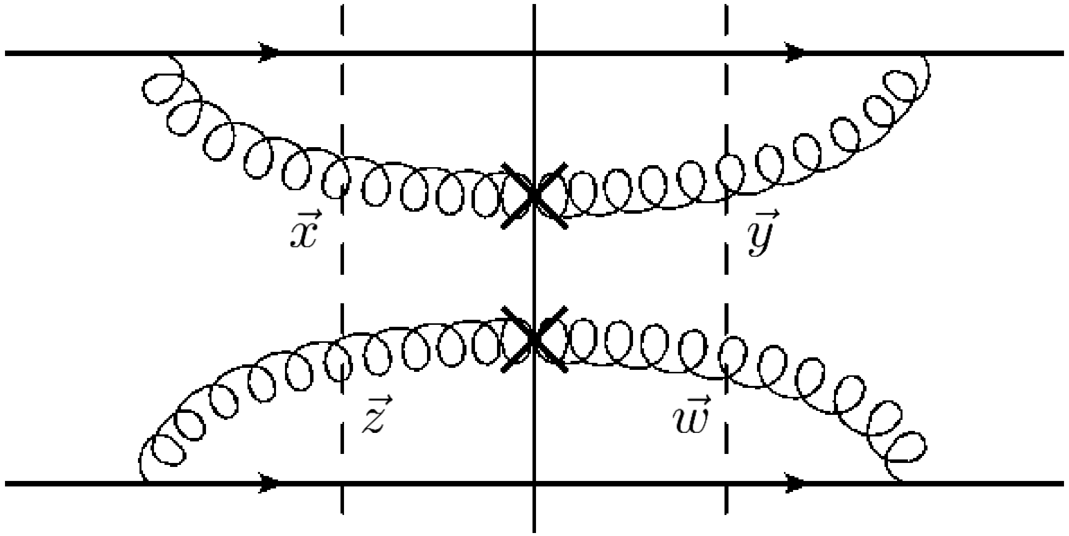


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^{\dagger}_{\vec{y}}\right]\mathrm{Tr}\left[U_{\vec{z}}U^{\dagger}_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$=\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^{\dagger}_{\vec{y}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^{\dagger}_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- This then factorizes into two single-gluon distribution functions.

$$\left\langle \frac{d\phi^{D}_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\; d^2b_2}\right\rangle_{A_2}=\left\langle \frac{d\phi_{A_2}(\vec{q}_1-\vec{k}_1;y)}{d^2b_1}\right\rangle_{A_2}\left\langle \frac{d\phi_{A_2}(\vec{q}_2-\vec{k}_2;y)}{d^2b_2}\right\rangle_{A_2}$$

Classical contribution



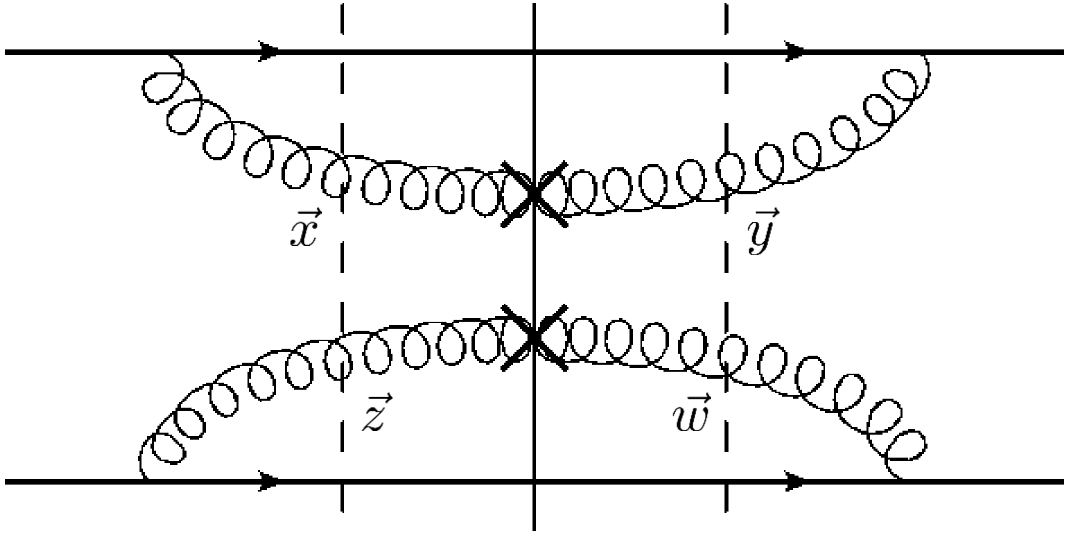
$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$= \left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- Assuming the saturation scale is independent of the transverse position we have

$$\left\langle \frac{d\phi^D_{Classical}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\ d^2b_2}\right\rangle_{A_2} = \frac{1}{S^2_{\perp,2}}\phi_{A_2}(\vec{q}_1-\vec{k}_1;y)\phi_{A_2}(\vec{q}_2-\vec{k}_2;y)$$

- The final result corresponds to the classical term.

Classical contribution



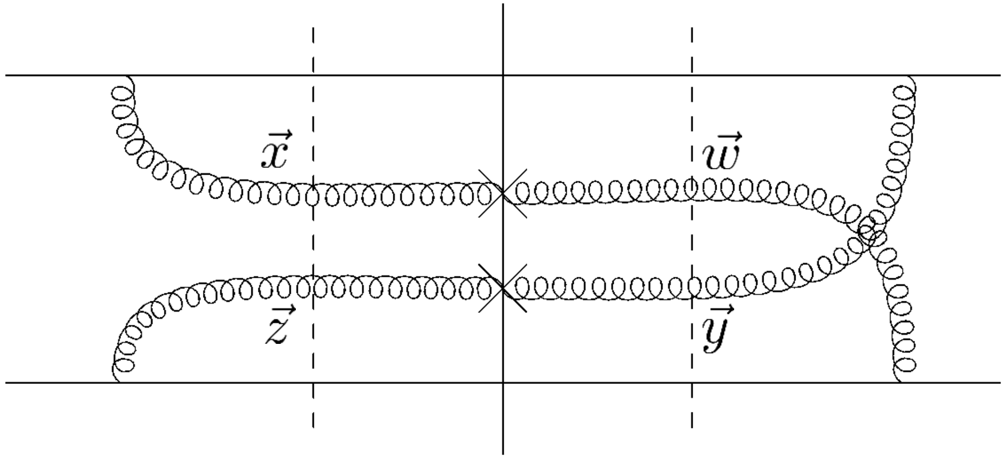
$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$= \left\langle \mathrm{Tr}\left[U_{\vec{x}}U^\dagger_{\vec{y}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^\dagger_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- The total cross section is just two single-gluon production cross sections divided by the transverse area of the target.

$$\frac{d\sigma_{classical}}{d^2k_1dy_1d^2k_2dy_2} = \frac{1}{S_{\perp,2}}\frac{d\sigma_g}{d^2k_1dy_1}\frac{d\sigma_g}{d^2k_2dy_2}$$

- This is called classical because this is equivalent to the two produced gluons being distinguishable, no interference.

Quadrupole

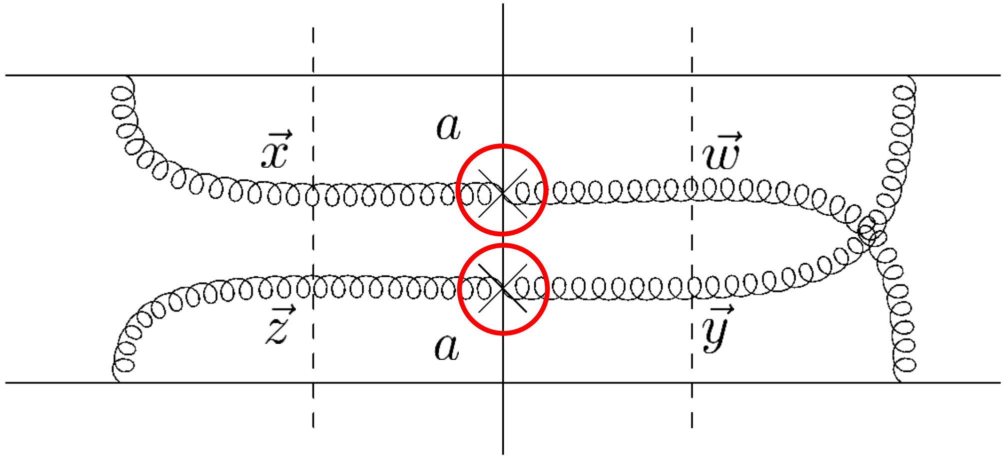


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{y}}^\dagger U_{\vec{z}} U_{\vec{w}}^\dagger\right]\right\rangle_{A_2}(y)$$
$$= \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{y}}^\dagger\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}} U_{\vec{w}}^\dagger\right]\right\rangle_{A_2}(y)$$
$$+ \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{w}}^\dagger\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}} U_{\vec{y}}^\dagger\right]\right\rangle_{A_2}(y)$$
$$+ \cdots$$

- Split it up into different possible two Wilson lines pairs.
- Total quadrupole can be written as this plus other terms.
- Each of these factorizations correspond to either HBT or Bose enhancement.

$$\left\langle \frac{d\phi^Q_{A_2}(\vec{q}_1,\vec{q}_2;y)}{d^2b_1\, d^2b_2}\right\rangle_{A_2} = \left(\frac{C_F}{\alpha_s(2\pi)^3}\right)^2 \int d^2r_1\, d^2r_2\, e^{-i\vec{q}_1\cdot\vec{r}_1 - i\vec{q}_2\cdot\vec{r}_2}$$
$$\times \nabla^2_{\vec{r}_1}\, \nabla^2_{\vec{r}_2}\, \frac{1}{(N_c^2-1)^2}\left\langle \mathrm{Tr}\left[U_{\vec{r}_1+\vec{b}_1} U^\dagger_{\vec{b}_1} U_{\vec{r}_1+\vec{b}_2} U^\dagger_{\vec{b}_2}\right]\right\rangle_{A_2}(y)$$

HBT contribution

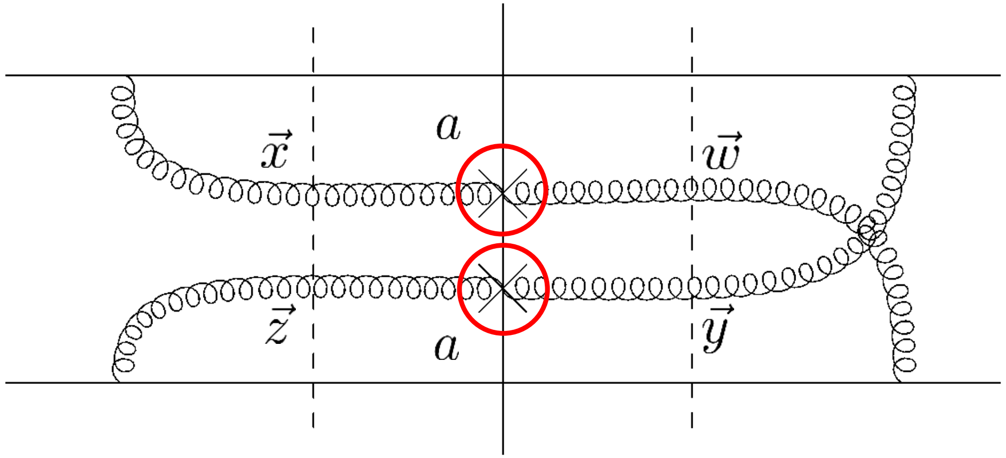


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U_{\vec{y}}^{\dagger}U_{\vec{z}}U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$= \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}}U_{\vec{y}}^{\dagger}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$+ \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}}U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U_{\vec{y}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- Final state gluons have the same color
- HBT contribution to the quadrupole distribution function

$$\left\langle \frac{\phi^{Q}_{HBT}(\vec{q}_1-\vec{k}_1,\vec{q}_2-\vec{k}_2;y)}{d^2b_1 d^2b_2}\right\rangle_{A_2} = \frac{1}{S_{\perp,2}^2}\phi_{A_2}(\vec{q}_1-\vec{k}_1;y)\phi_{A_2}(\vec{q}_2-\vec{k}_2;y)$$

HBT contribution

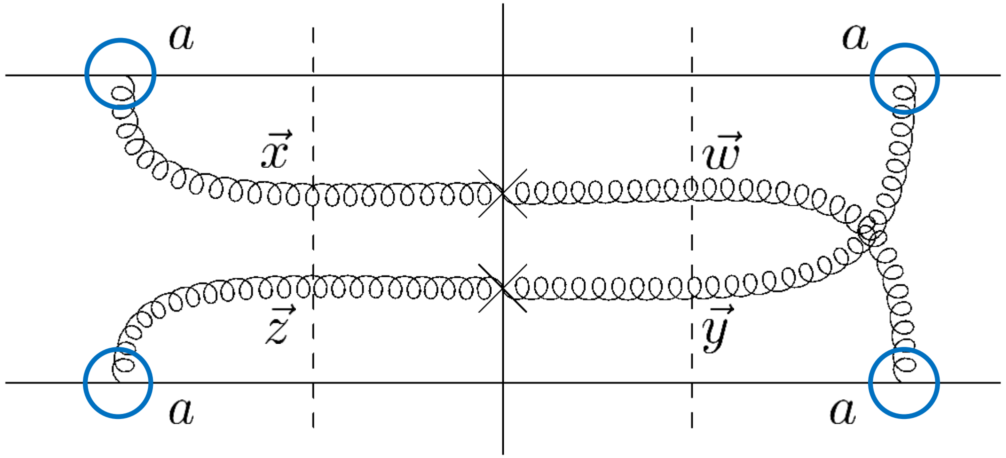


$$\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{y}}^{\dagger} U_{\vec{z}} U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$= \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{y}}^{\dagger}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}} U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$+ \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}} U_{\vec{w}}^{\dagger}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}} U_{\vec{y}}^{\dagger}\right]\right\rangle_{A_2}(y)$$
$$+\cdots$$

- Final state gluons have the same color
- We can see the HBT nature of this term in the delta functions.

$$\frac{d\sigma_{HBT}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{S_{\perp,1}S_{\perp,2}}\left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2}\int d^2q_1\,d^2q_2$$
$$\times\,\phi_{A_1}(\vec{q}_1; y=0)\,\phi_{A_1}(\vec{q}_2; y=0)\,\phi_{A_2}(\vec{q}_1-\vec{k}_1; y)\,\phi_{A_2}(\vec{q}_2-\vec{k}_2; y)$$
$$\times\,\frac{2\pi}{N_c^2-1}\left(\delta(\vec{k}_1-\vec{k}_2)+\delta(\vec{k}_1+\vec{k}_2)\right)\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)$$

Bose enhancement contribution



$$\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^{\dagger}_{\vec{y}}U_{\vec{z}}U^{\dagger}_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$= \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^{\dagger}_{\vec{y}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^{\dagger}_{\vec{w}}\right]\right\rangle_{A_2}(y)$$
$$+ \boxed{\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[U_{\vec{x}}U^{\dagger}_{\vec{w}}\right]\right\rangle_{A_2}(y)\left\langle \mathrm{Tr}\left[U_{\vec{z}}U^{\dagger}_{\vec{y}}\right]\right\rangle_{A_2}(y)}$$
$$+ \cdots$$

- Gluons emitted from the projectile sources have the same color
- The Bose enhancement part of the quadrupole distribution function

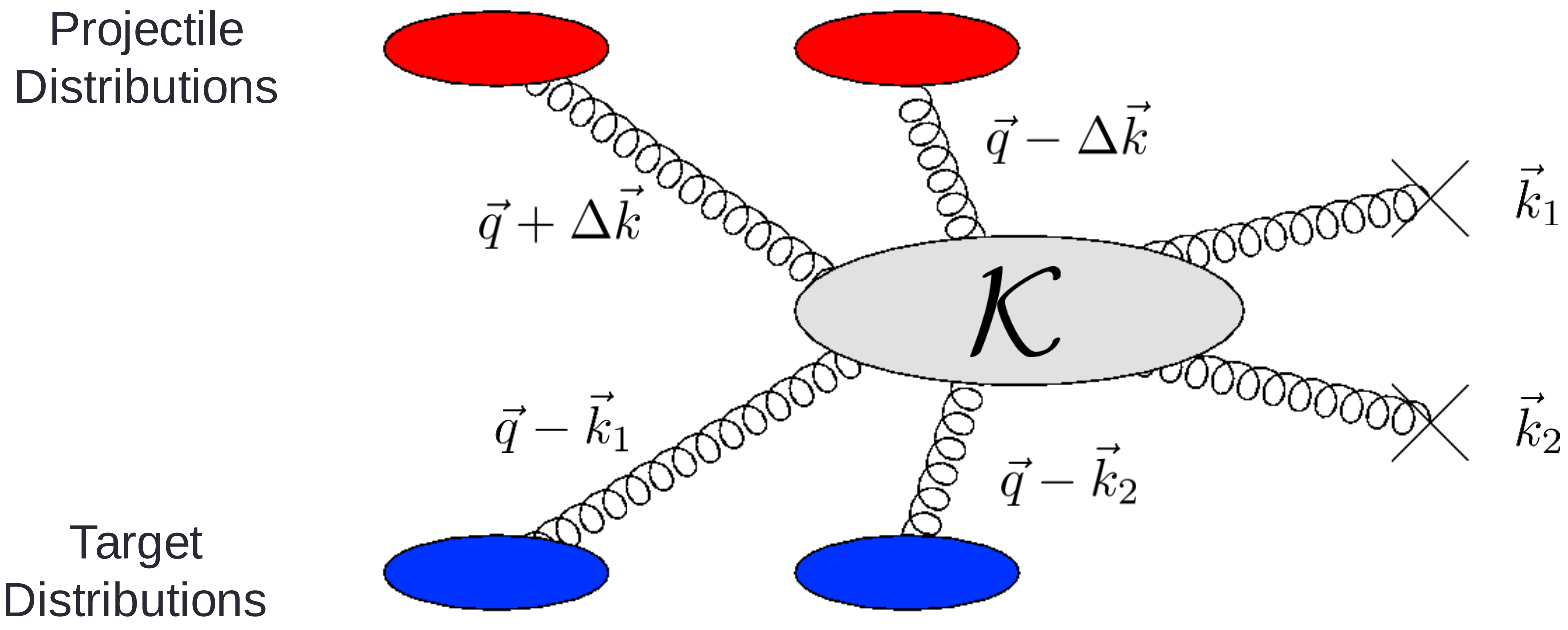
$$\left\langle \frac{\phi^{Q}_{Bose}(\vec{q}_1-\vec{k}_1,\vec{q}_2-\vec{k}_2;y)}{d^2b_1d^2b_2}\right\rangle_{A_2} = \frac{1}{S^2_{\perp,2}}e^{-i(\Delta\vec{b})\cdot(\vec{q}_2-\vec{k}_2-\vec{q}_1+\vec{k}_1)}\phi_{A_2}(\vec{q}_1-\vec{k}_1;y)\phi_{A_2}(\vec{q}_2-\vec{k}_2;y)$$

Bose enhancement cross section

- Bose enhancement when the momenta of the gluons are equal or opposite.

$$\frac{d\sigma_{Bose}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2}\,\frac{1}{S_{\perp,1}S_{\perp,2}}\,\frac{1}{N_c^2-1}\int d^2q\,\mathcal{K}\left(\vec{k}_1,\vec{k}_2,\vec{q}+\Delta\vec{k},\vec{q}-\Delta\vec{k}\right)$$

$$\times\,\phi_{A_1}(\vec{q}+\Delta\vec{k};y=0)\,\phi_{A_1}(\vec{q}-\Delta\vec{k};y=0)\,\phi_{A_2}(\vec{q}-\vec{k}_1;y)\,\phi_{A_2}(\vec{q}-\vec{k}_2;y) + (\vec{k}_2\to-\vec{k}_2)$$



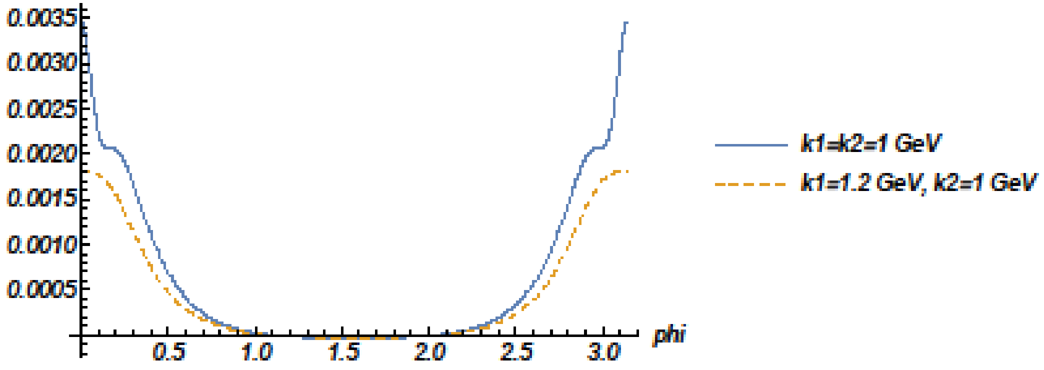
Toy model

- We use the single gluon emission result for the projectile distribution and the Golec-Biernat-Wüsthoff (GWB) model for the target distributions.
- Results in the analytic formula with gaussian functions.

$$\frac{d\sigma_{Bose}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{C_F}{\alpha_s}\right)^2 16\, \frac{1}{(2\pi)^8} S_{\perp,1} S_{\perp,2} \left(\frac{Q_1^2}{Q_2^2}\right) \frac{1}{k_1^2 k_2^2} \int d^2q \frac{\left(\vec{q} - \bar{k} - \frac{\Delta\vec{k}}{2}\right)^2 \left(\vec{q} - \bar{k} + \frac{\Delta\vec{k}}{2}\right)^2}{(\vec{q} + \Delta\vec{k})^2 (\vec{q} - \Delta\vec{k})^2}$$

$$\times \frac{2\pi}{N_c^2 - 1} e^{\left\{-\frac{2}{Q_2^2}(\vec{q}-\bar{k})^2 - \frac{1}{Q_2^2}(\Delta\vec{k})^2\right\}} \mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q} + \Delta\vec{k}, \vec{q} - \Delta\vec{k}) + (\vec{k}_2 \to -\vec{k}_2)$$

- Plotting this function we can see a near and away-side ridge structure.



$$Q_1 = 0.2\, GeV, \quad Q_2 = 1\, GeV$$

Normalized by

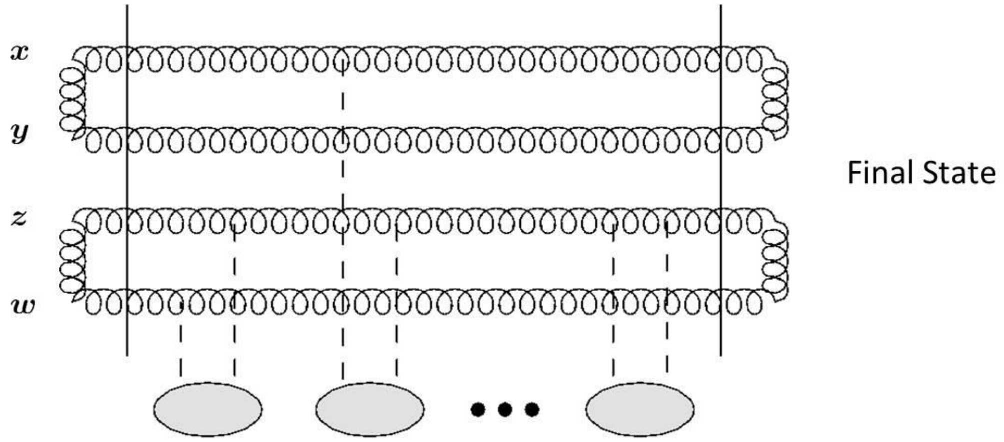
$$\left(\frac{C_F}{\alpha_s}\right)^2 16\, \frac{1}{(2\pi)^8} S_{\perp,1} S_{\perp,2} \frac{1}{k_1^2 k_2^2}$$

Conclusions

- Explored the physical origin of the ridge.
- Many important contributions that existed in the dilute-dilute limit also exist in the dilute-dense limit.
  - Classical (uncorrelated)
  - HBT
  - Bose enhanced
- Isolated these various contributions in the dilute-dense limit.
- Showed that Bose enhanced contributions gives rise to a ridge structure.

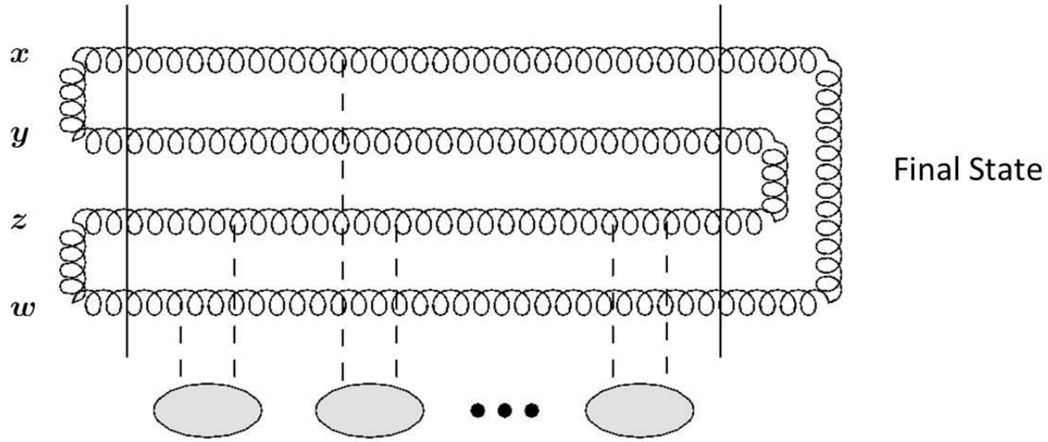
BACKUP SLIDES

More Complicated Wilson Line Operators



Two-Dipoles

$$\left\langle Tr[U_{\vec{x}}U^{\dagger}_{\vec{y}}]Tr[U_{\vec{z}}U^{\dagger}_{\vec{w}}]\right\rangle$$



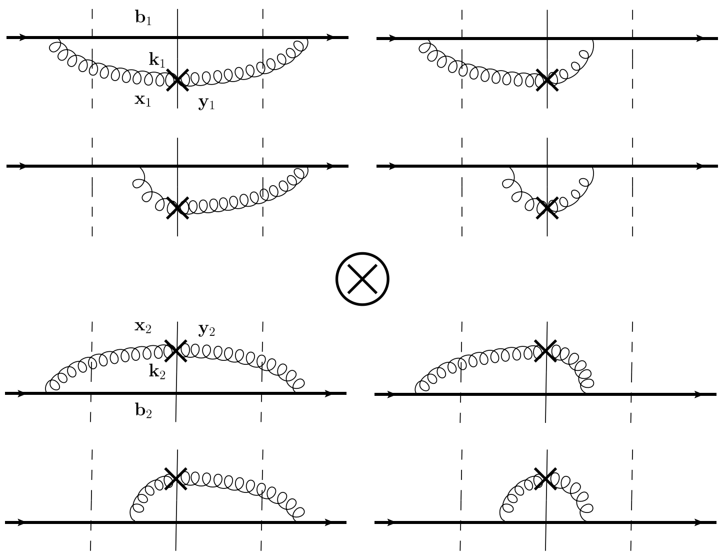
Quadrupole

$$\left\langle Tr[U_{\vec{x}}U^{\dagger}_{\vec{y}}U_{\vec{z}}U^{\dagger}_{\vec{w}}]\right\rangle$$

Two-gluon production cross section

- The "separated" diagrams give

$$\frac{d\sigma}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{\alpha_s^2 C_F^2}{16\,\pi^8} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2\, e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_1)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_2)}$$
$$\times \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}\, \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}$$
$$\times \left\langle \left(\frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_1} U_{\mathbf{y}_1}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_1} U_{\mathbf{b}_1}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{b}_1} U_{\mathbf{y}_1}^\dagger] + 1 \right)\right.$$
$$\times \left.\left(\frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_2} U_{\mathbf{y}_2}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{x}_2} U_{\mathbf{b}_2}^\dagger] - \frac{1}{N_c^2-1}\, Tr[U_{\mathbf{b}_2} U_{\mathbf{y}_2}^\dagger] + 1 \right)\right\rangle$$



(cf. Kovner & Lublinsky, '12)

Symmetric under: $\boldsymbol{k}_2 \to -\boldsymbol{k}_2$

Two-gluon production cross section

- The "crossed" diagrams give

$$\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2$$

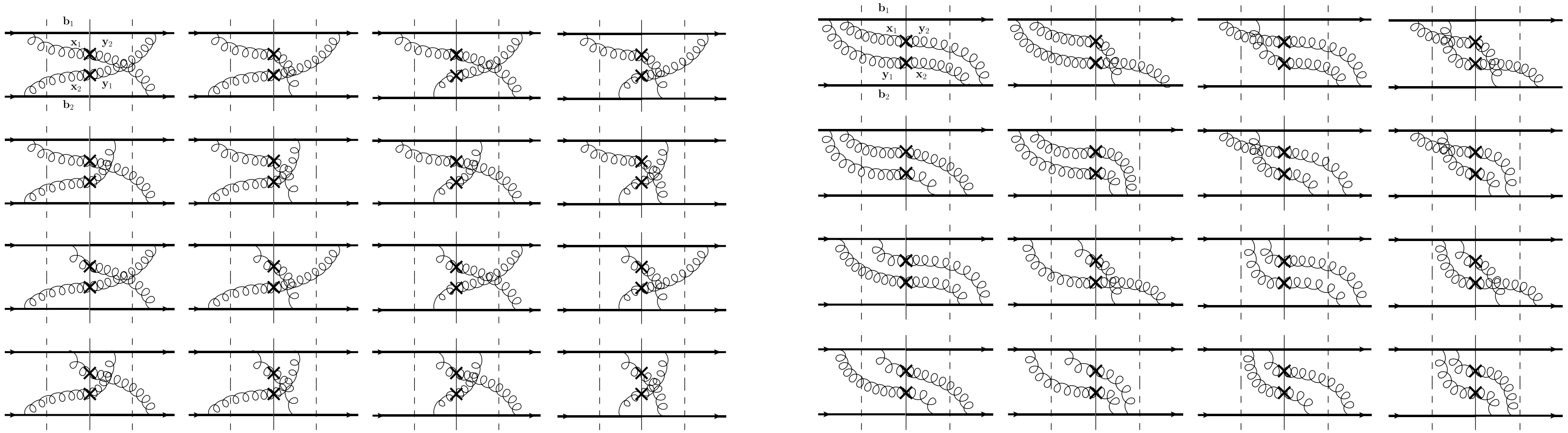
$$\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]$$

$$\times \frac{16\,\alpha_s^2}{\pi^2}\,\frac{C_F}{2N_c}\,\frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2}\cdot\frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2}\,\frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2}\cdot\frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}$$

$$\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)$$

$$+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)$$

$$+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]$$



Two-gluon production cross section

- The "crossed" diagrams give

$$\frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b_1\, d^2b_2\, T_1(\mathbf{B}-\mathbf{b}_1)\, T_1(\mathbf{B}-\mathbf{b}_2)\, d^2x_1\, d^2y_1\, d^2x_2\, d^2y_2$$
$$\times \left[e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)-i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} + e^{-i\,\mathbf{k}_1\cdot(\mathbf{x}_1-\mathbf{y}_2)+i\,\mathbf{k}_2\cdot(\mathbf{x}_2-\mathbf{y}_1)} \right]$$
$$\times \frac{16\,\alpha_s^2}{\pi^2} \frac{C_F}{2N_c} \frac{\mathbf{x}_1-\mathbf{b}_1}{|\mathbf{x}_1-\mathbf{b}_1|^2} \cdot \frac{\mathbf{y}_2-\mathbf{b}_2}{|\mathbf{y}_2-\mathbf{b}_2|^2} \frac{\mathbf{x}_2-\mathbf{b}_2}{|\mathbf{x}_2-\mathbf{b}_2|^2} \cdot \frac{\mathbf{y}_1-\mathbf{b}_1}{|\mathbf{y}_1-\mathbf{b}_1|^2}$$
$$\times \Big[Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2) - Q(\mathbf{x}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) + S_G(\mathbf{x}_1,\mathbf{y}_1) - Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{y}_2)$$
$$+ Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{x}_2,\mathbf{b}_2) + Q(\mathbf{x}_1,\mathbf{b}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{x}_1,\mathbf{b}_1) - Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2) + Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{b}_2)$$
$$+ Q(\mathbf{b}_1,\mathbf{y}_1,\mathbf{b}_2,\mathbf{y}_2) - S_G(\mathbf{b}_1,\mathbf{y}_1) + S_G(\mathbf{x}_2,\mathbf{y}_2) - S_G(\mathbf{x}_2,\mathbf{b}_2) - S_G(\mathbf{b}_2,\mathbf{y}_2) + 1 \Big]$$

- where

$$S_G(\mathbf{x}_1,\mathbf{x}_2,y) \equiv \frac{1}{N_c^2-1} \left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2}] \right\rangle$$
$$Q(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) \equiv \frac{1}{N_c^2-1} \left\langle Tr[U_{\mathbf{x}_1} U^\dagger_{\mathbf{x}_2} U_{\mathbf{x}_3} U^\dagger_{\mathbf{x}_4}] \right\rangle$$

Symmetric under: $\boldsymbol{k}_2 \to -\boldsymbol{k}_2$

Classical Result

- The classical term contains only geometric contributions.

$$\frac{d\sigma_{classical}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_2}(\vec{q}_1-\vec{k}_1; y)}{d^2b_1} \right\rangle_{A_2} \left\langle \frac{d\phi_{A_2}(\vec{q}_2-\vec{k}_2; y)}{d^2b_2} \right\rangle_{A_2}$$

- Assuming translational invariance makes the classical nature of the result clearer.

$$\frac{d\sigma_{classical}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{S_{\perp,2}} \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2q_1\, d^2q_2$$
$$\times\, \phi_{A_1}(\vec{q}_1; y=0)\, \phi_{A_1}(\vec{q}_2; y=0)\, \phi_{A_2}(\vec{q}_1-\vec{k}_1; y)\, \phi_{A_2}(\vec{q}_2-\vec{k}_2; y)$$

$$\frac{d\sigma_{classical}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{S_{\perp,2}} \frac{d\sigma_g}{d^2k_1 dy_1} \frac{d\sigma_g}{d^2k_2 dy_2}$$

HBT Result

- The HBT contribution to the cross section.

$$\frac{d\sigma_{HBT}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q_1\, d^2q_2$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}_1; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}_2; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_2}(\vec{q}_1-\vec{k}_1; y)}{d^2b_1} \right\rangle_{A_2} \left\langle \frac{d\phi_{A_2}(\vec{q}_2-\vec{k}_2; y)}{d^2b_2} \right\rangle_{A_2}$$
$$\times \left[e^{-i\,(\vec{k}_1-\vec{k}_2)\cdot(\vec{b}_1-\vec{b}_2)} \frac{\mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)}{N_c^2-1} \right] + (\vec{k}_2 \to -\vec{k}_2)$$

- Assuming translational invariance the HBT nature becomes obvious.

$$\frac{d\sigma_{HBT}}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{1}{S_{\perp,1} S_{\perp,2}} \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2q_1\, d^2q_2$$
$$\times\, \phi_{A_1}(\vec{q}_1; y=0)\, \phi_{A_1}(\vec{q}_2; y=0)\, \phi_{A_2}(\vec{q}_1-\vec{k}_1; y)\, \phi_{A_2}(\vec{q}_2-\vec{k}_2; y)$$
$$\times \frac{2\pi}{N_c^2-1} \Big(\delta(\vec{k}_1-\vec{k}_2) + \delta(\vec{k}_1+\vec{k}_2)\Big)\, \mathcal{K}(\vec{k}_1,\vec{k}_2,\vec{q}_1,\vec{q}_2)$$

Bose Enhancement Result

- The bose enhancement contribution to the cross section.

$$\frac{d\sigma_{Bose}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2} \int d^2B\, d^2b_1\, d^2b_2 \int d^2q\, d^2\Delta q\, \frac{1}{N_c^2-1}\, e^{i\Delta\vec{b}\cdot\Delta\vec{q}}$$
$$\times \left\langle \frac{d\phi_{A_1}(\vec{q}+\Delta\vec{q}/2-\Delta\vec{k}; y=0)}{d^2(\vec{B}-\vec{b}_1)} \right\rangle_{A_1} \left\langle \frac{d\phi_{A_1}(\vec{q}-\Delta\vec{q}/2-\Delta\vec{k}; y=0)}{d^2(\vec{B}-\vec{b}_2)} \right\rangle_{A_1}$$
$$\times \left\langle \frac{d\phi_{A_2}(\vec{q}-\Delta\vec{q}/2-\vec{k}_1; y)}{d^2b_1} \right\rangle_{A_2} \left\langle \frac{d\phi_{A_2}(\vec{q}+\Delta\vec{q}/2-\vec{k}_2; y)}{d^2b_2} \right\rangle_{A_2}$$
$$\times\ \mathcal{K}\left(\vec{k}_1, \vec{k}_2, \vec{q}+\frac{\Delta\vec{q}}{2}+\Delta\vec{k}, \vec{q}-\frac{\Delta\vec{q}}{2}-\Delta\vec{k}\right)\ +\ (\vec{k}_2 \to -\vec{k}_2)$$

- Becomes clear after assuming translational invariance.

$$\frac{d\sigma_{Bose}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{2\,\alpha_s}{C_F}\right)^2 \frac{1}{k_1^2\,k_2^2}\,\frac{1}{S_{\perp,1}S_{\perp,2}}\,\frac{1}{N_c^2-1} \int d^2q\, \mathcal{K}\left(\vec{k}_1, \vec{k}_2, \vec{q}+\Delta\vec{k}, \vec{q}-\Delta\vec{k}\right)$$
$$\times\, \phi_{A_1}(\vec{q}+\Delta\vec{k}; y=0)\, \phi_{A_1}(\vec{q}-\Delta\vec{k}; y=0)\, \phi_{A_2}(\vec{q}-\vec{k}_1; y)\, \phi_{A_2}(\vec{q}-\vec{k}_2; y)\ +\ (\vec{k}_2 \to -\vec{k}_2)\,.$$

- The gluons originating from the source are Bose Enhanced when the emitted gluons have equal or opposite transverse momentum.

Toy Model – Analytic Results

- Models for the distribution functions
  - Single gluon emission for the projectile distributions

$$\phi_{A_1}(\vec{q}) \approx \frac{C_F\, S_{\perp,1}\, Q_1^2}{\alpha_s\,(2\pi)^3}\,\frac{1}{4}\int d^2r\, e^{-i\vec{q}\cdot\vec{r}}\,\nabla_{\vec{r}}^2\left[r^2 \ln\left(\frac{1}{r\,\Lambda}\right)\right] = \frac{C_F\, S_{\perp,1}\, Q_1^2}{\alpha_s\,(2\pi)^3}\,\frac{2\pi}{q^2}$$

  - Golec-Biernat-Wüsthoff (GWB) model for the target distributions

$$\phi_{A_2}(\vec{q}) \approx \frac{C_F}{\alpha_s(2\pi)^3}\int d^2r d^2b\, e^{-i\vec{q}\cdot\vec{r}}\,\nabla_{\vec{r}}^2\left(1 - e^{-\frac{Q_2^2}{4}r^2}\right) = \frac{C_F}{\alpha_s(2\pi)^3}\, S_{\perp,2}\,\frac{q^2}{Q_2^2}\,4\pi\, e^{-\frac{q^2}{Q_2^2}}$$

- Final Result

$$\frac{d\sigma_{Bose}}{d^2k_1 dy_1 d^2k_2 dy_2} = \left(\frac{C_F}{\alpha_s}\right)^2 16\,\frac{1}{(2\pi)^8}\, S_{\perp,1} S_{\perp,2}\left(\frac{Q_1^2}{Q_2^2}\right)\frac{1}{k_1^2 k_2^2}\int d^2q\,\frac{\left(\vec{q} - \bar{k} - \frac{\Delta\vec{k}}{2}\right)^2\left(\vec{q} - \bar{k} + \frac{\Delta\vec{k}}{2}\right)^2}{(\vec{q} + \Delta\vec{k})^2(\vec{q} - \Delta\vec{k})^2}$$

$$\times\,\frac{2\pi}{N_c^2 - 1}\, e^{\left\{-\frac{2}{Q_2^2}(\vec{q}-\bar{k})^2 - \frac{1}{Q_2^2}(\Delta\vec{k})^2\right\}}\,\mathcal{K}(\vec{k}_1, \vec{k}_2, \vec{q} + \Delta\vec{k}, \vec{q} - \Delta\vec{k}) + (\vec{k}_2 \to -\vec{k}_2)$$