Correlations anf fluctuations in gluon production

Alex Kovner

University of Connecticut

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A.K. and Amir Rezaeian Phys.Rev. D96 (2017) no.7, 074018
A.K. and Vladi Skokov - paper I and paper II

Correlated gluon production

Observation of Ridge → study of correlated particle production.

Here: no final state interactions are considered - neither classical fields nor hydro like.

Work entirely (and consistently) within the dilute-dense CGC approach: "p-A at mid rapidity".

Concentrate on Quantum interference (quantum statistics) effects on parton level.

Two questions:

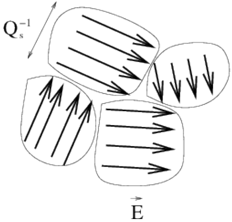
1. How does spatial anisotropy affect v_2 in gluon production?

2. How does quantum statistics affect multiplicity fluctuations?

General mechanisms I

Two distinct types of mechanisms for generating correlations within CGC: "classical" and "quantum".

Classical 1: **Local anisotropy** - M. Lublinsky and A.K.
Q_s - color correlation length in the target: "color field domains".



Purely classical correlation: gluons that hit the same domain scatter in the same direction (if they have the same charge.)

General mechanisms II

Classical II: Density variation - E. Levin, A. Rezaeian
If the density profile is not constant, a dipole scatters differently depending on its orientation. Scattering is more efficient for dipole oriented along the density gradient rather than perpendicular to it.

More particles are produced with momentum parallel to density gradient.

Both mechanisms are purely classical: in order to produce correlated particles, the incoming gluons have to sit close to each other in the transverse plain so that they feel the same local structure of the target.

General mechanisms III

Quantum: "Glasma graphs"- - Dumitru, Gelis, Jalilian-Marian, Lappi, McLerran, Venugopalan;
Numerics: K. Dusling and R. Venugopalan, Phys.Rev. D87 (2013) no.9, 094034 and more

Quantum statistics (quantum interference) is the physics origin of "Glasma graph" correlated production.

Two main routes to interfere: HBT of emitted gluons ("final sate" interference) and Bose enhancement (BE) of gluons in the initial wave function ("initial state" interference).

T. Altinoluk, N. Armesto, G. Beuf, A.K., M. Lublinsky, Phys.Lett. B752 (2016) 113-121 and more

Also on quantum interference: L. McLerran and V. Skokov, Nucl.Phys. A947 (2016) 142-154, B. Blok, C. Jakel, M. Strikman, U. Widemann, JHEP 1712 (2017) 074

How to separate classical from quantum?

Double inclusive gluon production

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2 (4\pi)^2 \int_{z_1\bar{z}_1 z_2\bar{z}_2; x_1 x_2 y_1 y_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)}$$
$$\times A^i(x_1 - z_1)A^i(\bar{z}_1 - y_1)A^j(x_2 - z_2)A^j(\bar{z}_2 - y_2)$$
$$\times \Big\langle \rho^{a_1}(x_1)\left[U(z_1) - U(x_1)\right]\left[U^\dagger(\bar{z}_1) - U^\dagger(y_1)\right]^{a_1 b_1} \rho^{b_1}(y_1)$$
$$\times \rho^{a_2}(x_2)\left[U(z_2) - U(x_2)\right]\left[U^\dagger(\bar{z}_2) - U^\dagger(y_2)\right]^{a_2 b_2} \rho^{b_2}(y_2)\Big\rangle_{P,T}.$$

with the Weizsacker-Williams field

$$A^i(x-y) = -\frac{1}{2\pi}\frac{(x-y)_i}{(x-y)^2}$$

Points close together - classical; points far away - quantum.

Separating quantum I

A.K. and Amir Rezaeian, Phys.Rev. D96 (2017) no.7, 074018

Concentrate on target averages.

E.g.:

$$\int_{z,\bar{z}} F(z,\bar{z})\langle [U(z_1)U^\dagger(\bar{z}_1)]^{ab}[U(z_2)U^\dagger(\bar{z}_2)]^{cd}\rangle_T$$

Roughly: The further apart the points are, the larger the contribution due to integration (no area suppression.)

But has to be color invariant!

Color neutralization in the target on distance scales $r \sim 1/Q_s$.

So points should be pairwise close to each other - otherwise the average vanishes.

Any product of U's factorizes *a la* Wick with basic "contraction"

$$\left\langle U^{ab}(x)U^{cd}(y)\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{N_c^2-1}d(x,y),$$

Separating quantum - II

So, for example

$$
\begin{aligned}
\langle Q(x,y,z,v)\rangle_T &= d(x,y)d(z,v) + d(x,v)d(z,y) \\
&+ \frac{1}{N_c^2-1} d(x,z)d(y,v), \\
\langle D(x,y)D(z,v)\rangle_T &= d(x,y)d(z,v) \\
&+ \frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right].
\end{aligned}
$$

Not large N_c limit - rather dense target limit!

All points close together - factorization does not hold. This is where "classical" contributions come from. We do not consider those.

Same factorization used in Y. Kovchegov, D. Wertepny Nucl. Phys. A906, 50 (2013)

BE vs HBT

$$\langle Q(z_1,\bar z_1,z_2,\bar z_2)\rangle_T = d(z_1,\bar z_1)d(z_2.\bar z_2)+d(z_1,\bar z_2)d(z_2,\bar z_1) + \frac{1}{N_c^2-1}d(z_1,z_2)d(\bar z_1,\bar z_2),$$

First term: two gluons scatter independently. But arise with larger probability from the wave function. Bose enhancement.

Second term: correlates directly momenta of produced gluons. Hanbury Brown - Twiss effect.

More details and discussion in Wertepny's and Altinoluk's talks.

How about geometry dependence?

Both effects produce angular correlations. What can we tell about effect of spatial eccentricity on the magnitude of angular correlations?

A.K. and Vladi Skokov, paper I

In p-A (dense nucleus) geometry means geometry of the proton wave function, since fluctuations in the nucleus are suppressed.

Proton geometry

Proton geometry enters via MV model

$$\mu^2(\underline{p},\underline{k}) = \mu^2(\underline{p}+\underline{k})F\left(\frac{(\underline{p}-\underline{k})^2}{\Lambda^2}\right).$$

$\mu^2(\underline{p})$ - charge density in the transverse plain.

$$\mu^2(\underline{p}) = \int d^2b e^{i\underline{p}\underline{b}}\mu^2(\underline{b}).$$

In the original MV model $\mu^2(\underline{b}) = const$.
We take:

$$\mu^2(\underline{b}) = C(a)e^{-\frac{b_1^2}{a^2R^2}}e^{-\frac{a^2b_2^2}{R^2}}$$

We keep the total number of sources independent of a - shape fluctuations of "incompressible" proton

$$\int d^2b\mu^2(\underline{b}) = S_\perp\mu_0^2.$$

TMD

$F(p)$ - roughly is the TMD of valence particles.

Original MV model: $F(p) = const$

We use this for numerical calculations.

For analytic estimates we use "parton model" type TMD:

$$F(p) \rightarrow_{p^2 > \Lambda^2} 0$$

HBT $v_2\{2\}$ I

The HBT contribution to double inclusive

$$\left[\frac{d^2N}{d^2k_1dy_1d^2k_2dy_2}\right]_{\rm HBT} = (N_c^2-1)|\mu^2(\underline{k}_1-\underline{k}_2)|^2\left(\frac{2g^2}{(2\pi)^3}\right)^2$$
$$\times\int\frac{d^2q}{(2\pi)^2}\frac{d^2p}{(2\pi)^2}\Gamma(\underline{k}_1,\underline{q},\underline{k}_1-\underline{k}_2+\underline{p})\Gamma(\underline{k}_2,\underline{p},\underline{k}_2-\underline{k}_1+\underline{q})$$
$$\times D(\underline{k}_1-\underline{q})D(\underline{p}-\underline{k}_2)+[\underline{k}_2\to-\underline{k}_2]\ .$$

where

$$\Gamma(\underline{k},\underline{q},\underline{q}')=\left(\frac{\underline{q}}{\underline{q}^2}-\frac{\underline{k}}{\underline{k}^2}\right)\cdot\left(\frac{\underline{q}'}{\underline{q}'^2}-\frac{\underline{k}}{\underline{k}^2}\right)\ .$$

For $v_2\{2\}$

$$\int\frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi}e^{2i(\phi_1-\phi_2)}\left|\int d^2b\, e^{i\underline{b}\cdot(\underline{k}_1-\underline{k}_2)}\mu^2(\underline{b})\right|^2\ .$$

HBT $v_2\{2\}$ II

With the Gaussian density profile

$$|\mu^2(\underline{k}_1 - \underline{k}_2)|^2 = (S_\perp \mu_0^2)^2 e^{-(Rk)^2\left(a^2[\cos\phi_1 - \cos\phi_2]^2 + a^{-2}[\sin\phi_1 - \sin\phi_2]^2\right)} .$$

For $Rk \gg 1$ we expand in $\Delta\phi$, $\phi_1 = \Delta\phi + \phi_2$

$$\int \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi} e^{2i(\phi_1-\phi_2)} \left| \int d^2 b e^{i\underline{b}\cdot(\underline{k}_1-\underline{k}_2)} \mu^2(\underline{b}) \right|^2$$

$$= \frac{(S_\perp \mu_0^2)^2}{2\pi} \int \frac{d\phi_2}{2\pi} \frac{\sqrt{2\pi}a}{kR\sqrt{a^4 \sin^2(\phi_2) + \cos^2(\phi_2)}} + \mathcal{O}\left((kR)^0\right) .$$

For small a

$$\lim_{a\to 0} \int \frac{d\phi_1}{2\pi}\frac{d\phi_2}{2\pi} e^{2i(\phi_1-\phi_2)} \left| \int d^2 b e^{i\underline{b}\cdot(\underline{k}_1-\underline{k}_2)} \mu^2(\underline{b}) \right|^2$$

$$= (S_\perp \mu_0^2)^2 \frac{8\sqrt{2\pi}a \ln\left(\frac{2}{a}\right)}{(2\pi)^2 kR} + \mathcal{O}\left((kR)^0\right) .$$

Small a - small $v_2\{2\}$!

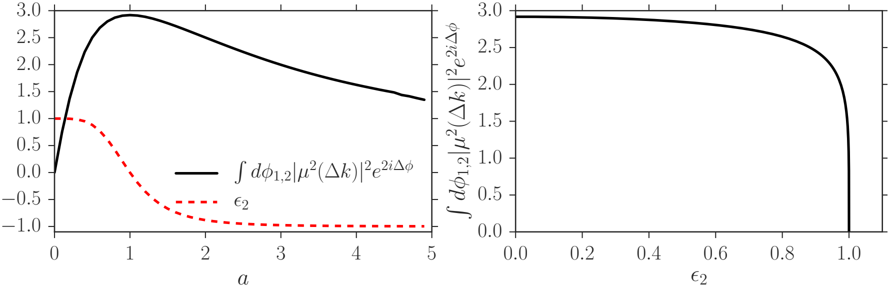


Figure: $v_2^2\{2\}$ versus spatial eccentricity for the HBT contribution. Left panel: eccentricity and $v_2^2\{2\}$ plotted versus parameter *a*. Right panel $v_2^2\{2\}$ plotted versus the spatial eccentricity.

BE behaves in the same way!

Numerical results

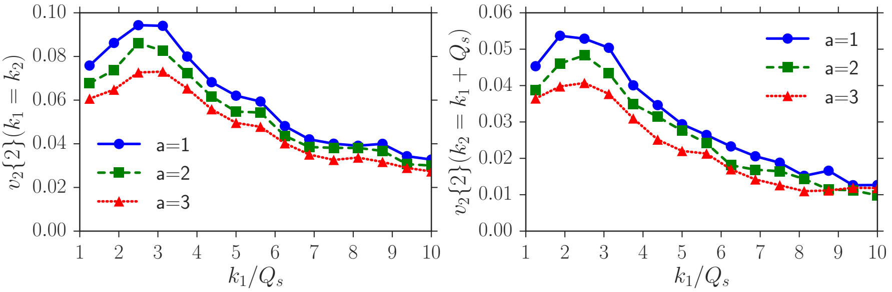


Figure: $v_2\{2\}$ as a function of momentum and the anisotropy of the projectile a. The total area of the projectile is kept independent of a. The left panel: both gluons are at the same absolute value of momenta $k_1 = k_2$. The right panel: the difference between the momenta of gluons is given by the saturation momentum of the target; this excludes HBT contribution. $\mu = \mu_t/4$.

Conclusions 1

Spatial eccentricity decreases $v_2\{2\}$ in the CGC approach. The effect is mild, but unmistakeable.

Mulitplicity fluctuations.

Precursor: F. Gelis, T. Lappi and L. McLerran, Nucl. Phys. A828, 2009, 149 - Glittering Glasma

Dense-Dilute single inclusive (single event) :

$$\left.\frac{dN}{d^2k dy}\right|_{\rho_{\rm p},\rho_{\rm t}} = \frac{2g^2}{(2\pi)^3}\int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\Gamma(\underline{k},\underline{q},\underline{q}') \times \rho^a_{\rm p}(-\underline{q}')\left[U^\dagger(\underline{k}-\underline{q}')U(\underline{k}-\underline{q})\right]_{ab}\rho^b_{\rm p}(\underline{q}),$$

The square of Lipatov vertex:

$$\Gamma(\underline{k},\underline{q},\underline{q}') = \left(\frac{\underline{q}}{q^2}-\frac{\underline{k}}{k^2}\right)\cdot\left(\frac{\underline{q}'}{q'^2}-\frac{\underline{k}}{k^2}\right).$$

Calculate directly multiplicity generating function

$$G(t) = \left\langle\left\langle \exp\left[t\int_{k_{\rm min}} d^2k \left.\frac{dN}{d^2k dy}\right|_{\rho_{\rm p},\rho_{\rm t}}\right]\right\rangle_{\rm p}\right\rangle_{\rm t},$$

What is important?

Hark back to double inclusive.

$$\left[\frac{d^2N}{dy_1 dy_2}\right]_{\mathrm{HBT}} = 2(N_c^2-1)S_\perp \left(\frac{2g^2}{(2\pi)^3}\right)^2$$
$$\times \int_{k_{\min}} d^2k \int d^2q d^2p \; \Gamma(\underline{k},\underline{q},\underline{p})\Gamma(\underline{k},\underline{p},\underline{q})\mu_{\mathrm{p}}^2(\underline{q})\mu_{\mathrm{p}}^2(\underline{p})D(\underline{k}-\underline{q})D(\underline{k}-\underline{p})\,.$$

$$\left[\frac{d^2N}{dy_1 dy_2}\right]_{\mathrm{BE}} = 2(N_c^2-1)S_\perp$$
$$\times \int d^2q \; |\mu_{\mathrm{p}}^2(\underline{q})|^2 \left|\frac{2g^2}{(2\pi)^3}\int_{k_{\min}} d^2k\Gamma(\underline{k},\underline{q},\underline{q})D(\underline{q}-\underline{k})\right|^2\,.$$

Largest contribution comes from BE at small q:

$$\Gamma(\underline{k},\underline{q},\underline{q}) = \frac{(\underline{k}-\underline{q})^2}{k^2q^2}\,.$$

BE contribution

HBT dominates correlations, but BE dominates fluctuations!

$$\left[\frac{d^2N}{dy_1 dy_2}\right]_{\rm BE} \approx 2(N_c^2-1)S_\perp \int d^2q \; \frac{|\mu_{\rm p}^2(\underline{q})|^2}{q^4} \left|\frac{2g^2}{(2\pi)^3}\int_{k_{\rm min}} d^2kD(\underline{k})\right|^2 .$$

In MV model, $\mu_{\rm p}^2 = {\rm const}$ strong IR divergence

$$\int d^2q \; \frac{\mu_{\rm p}^4}{q^4}$$

Regulated by finite projectile area $\Lambda = \frac{1}{R_{\rm p}}$, so

$$\int d^2q \; \frac{\mu_{\rm p}^4}{q^4} \propto \mu_{\rm p}^4 S_\perp \, ,$$

Same happens for higher moments: BE contribution to $\frac{d^mN}{dk^m}$ scales with area as $\left(\frac{dN}{dk}\right)^m$.

Projectile averaging

With the factorized target averaging:

$$G(t) = \int D\rho_{\rm p} DU e^{-\int \frac{d^2q}{(2\pi)^2}\left(\rho^a_{\rm p}(-\underline{q})\frac{1}{2\mu^2_{\rm p}(q)}\rho^a_{\rm p}(\underline{q})+\frac{1}{2D(q)}{\rm tr}[U^\dagger(q)U(-q)]\right)} \times e^{t\int_{k_{\rm min}} d^2k \left.\frac{dN}{d^2k dy}\right|_{\rho_{\rm p},\rho_{\rm t}}}.$$

Averaging over the projectile

$$G(t) = \int DU \exp\left[-\int \frac{d^2q}{(2\pi)^2}\frac{1}{2D(q)}{\rm tr}[U^\dagger(q)U(-q)] - \frac{1}{2}{\rm tr}\ln\left[1 - tM\right]\right]$$

where

$$M_{ab}(q', q) = \frac{4g^2}{(2\pi)^3}\mu^2(q)\int_{k_{\rm min}} \frac{d^2k}{(2\pi)^2}\Gamma(k, q, q')\left[U^\dagger(\underline{k}-\underline{q}')U(\underline{k}-\underline{q})\right]_{ab}.$$

BE dominance

Keeping only BE contributions is equivalent to contracting the U's within the same vertex". This is like mean field!

$$G_{\rm LO}(t) = \exp\left[-\frac{1}{2}{\rm tr}\ln\left[1 - t\langle M\rangle_t\right]\right]$$
$$= \exp\left[-\frac{1}{2}(N_c^2-1)S_\perp\int_\Lambda^{k_{\rm min}}\frac{d^2q}{(2\pi)^2}\ln\left(1 - t\,\frac{\mu_{\rm p}^2(q)\mathfrak{D}}{q^2}\right)\right]$$

with

$$\mathfrak{D} = \frac{4g^2}{(2\pi)^3}\int_{k_{\rm min}} d^2k D(\underline{k})\,.$$

Keeping only leading terms at $k_{min} \gg \Lambda$

$$\ln G_{\rm LO}(t) \approx \frac{1}{8\pi}(N_c^2-1)S_\perp\mu_{\rm p}^2\mathfrak{D}\left[\ln\frac{k_{\rm min}^2}{\Lambda^2}\;t + \sum_{n=2}^{\infty}\frac{1}{n(n-1)}\left(\frac{\mu_{\rm p}^2\mathfrak{D}}{\Lambda^2}\right)^{n-1}t^n\right]$$

Some properties

Cumulants

$$\kappa_1 = \frac{1}{8\pi}(N_c^2 - 1)S_\perp \mu_{\rm p}^2 \mathfrak{D} \ln \frac{k_{\rm min}^2}{\Lambda^2} \,.$$

higher order cumulants

$$\kappa_{n\geq 2} = \frac{\partial}{\partial t^n} \ln G_{\rm LO}(t)\Big|_{t=0} = (n-2)! \frac{(N_c^2 - 1)S_\perp \Lambda^2}{8\pi} \left(\frac{\mu_{\rm p}^2 \mathfrak{D}}{\Lambda^2}\right)^n \,.$$

Due to increasing powers of $1/\Lambda^2$, $\kappa_{n+1} \gg \kappa_n$, and the factorial cumulants are equal to cumulants

$$c_{n>2} = \frac{\partial}{\partial z^n} \ln G_{\rm LO}(t = \ln z)\Big|_{z=1}$$

$$c_n \approx \kappa_n$$

Very close to those of the γ-distribution

$$\bar{x} P(z = x/\bar{x}) = \frac{\alpha}{\Gamma(\alpha)} e^{-z\alpha} (\alpha z)^{\alpha - 1}; \qquad \kappa_n = (n-1)!\, \alpha \left(\frac{\bar{x}}{\alpha}\right)^n$$

KNO scaling and Λ/k_{min}

γ-distribution is known to exhibit KNO scaling.

No approximations: exact numerics with MV for both objects.

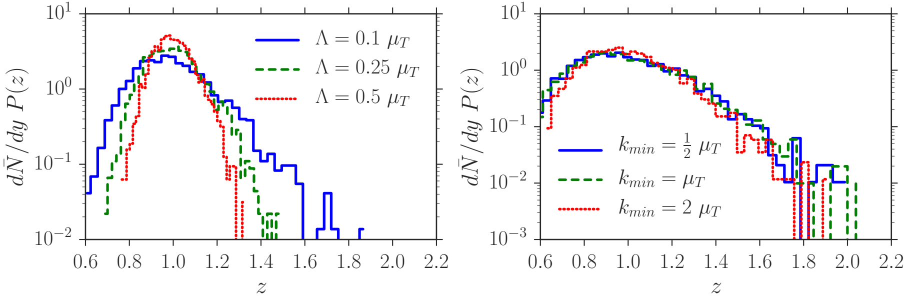


Figure: $z = dN/dy/(d\bar{N}/dy)$ KNO scaling, more statistics; tails sensitivity to Λ (for large Λ BE IR is not important) ; insensitivity to $k_{\rm min}$.

Dense-dilute vs Dilute Glitter

- Half of the contributions: no Bose enhancement in the target: suppressed by $1/N_c^2$.
- Gamma distribution rather than Negative Binomial Distribution - factor $n-1$ in $n-th$ cumulant. This difference in treating "IR divergent" integrals. This work: finite projectile area regulates IR. Glitter: $\mu_{\rm p}^2(p) \sim p^2$ for $p^2 < Q_{Ps}^2$ - color charge correlations nonlocal in coordinate space.

$$\int dq^2 \frac{\mu_{\rm p}^{2n}}{q^{2n}} = \frac{1}{n-1} \frac{\mu_{\rm p}^{2n}}{\Lambda^{2(n-1)}} \quad \text{vs} \quad \frac{\mu_p^{2n}}{Q_s^{2(n-1)}}$$

- The parameter of the distribution is $\alpha = \frac{N_c^2-1}{8\pi} S_\perp \Lambda^2$ where $S_\perp \Lambda^2 \approx 1$. Does not depend on energy for the dilute projectile! α is approximately energy independent.
  Glitter: $\Lambda^2 \to Q_s^2$. This has strong dependence on energy.

More fun

- Log in the generating function - the same as Log in Liouville effective action for composite operator/saturation momentum fluctuations!
  Introduced "on a hunch": E. Iancu and L. McLerran, Nucl. Phys. A793, 2007, 96
  Derived for the projectile wave function: A. Dumitru and V. Skokov, Phys. Rev. D96, 2017, 5, 056029
- Calculated corrections due to the leading HBT contribution. Area suppressed when $k_{min} \gg \Lambda$, but not otherwise.

$$\mathfrak{D} \to (\mathfrak{D} + tZ)$$

Recall

$$\mathfrak{D} = \frac{4g^2}{(2\pi)^3} \int_{k_{\rm min}} d^2k D(k)\,.$$

while

$$Z = \left(\frac{g^2}{\pi^3}\right)^2 \int_{\Lambda}^{k_{\rm min}} \frac{d^2q'}{(2\pi)^2} \int d^2k D(k) D(k-q') \frac{\hat{\mu}_{\rm p}^2(\underline{q}')}{q'^2}$$

Conclusions II

Bose enhancement - the main source of multiplicity fluctuations.

Details of distribution (γ or NBD or...) depend on IR (1/R or Q_s or ...). But either way the distribution has long tails for large multiplicities just like the Bose-Einstein distribution